ANOMALY MEDIATED SUSY BREAKING IN SU(N) & Sp(N) GAUGE THEORIES

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Giudice, Luty, Murayama, and Rattazzi JHEP 12 (1998) 027
Murayama, Phys.Rev.Lett. 126 (2021) 25, 251601
Csáki, Gomes, Murayama, Noether, **DRV**, Telem, Phys.Rev.D 107 (2023) 5, 054015
Kondo, Murayama, Noether, **DRV**, JHEP 04 (2025) 152
Gu, Murayama, Noether, **DRV** (coming soon)

Overview

- Strongly coupled theories and QCD
  - Confinement and Chiral Symmetry Breaking
- SUSY: Formalism & Advantages
- AMSB: Mechanism & Effects
- QCD-like gauge theories with fundamentals
  - Compare IR Phases in SU(N) vs Sp(N)
  - Seiberg Duality
  - Deriving χSB vacua in softly-broken SUSY limit
    - ADS
    - QM
    - S-confining
    - Free Magnetic
    - Conformal Window
- Conclusion

Problem of Predicting Phases

SU(N), Sp(N) or SO(N) with fermions

↓ *IR strong dynamics (non-perturbative)*

SUSY: Confinement? Coulomb Phase? IR Fixed Point? ☺

Non-SUSY: Chiral Sym. Breaking? Massless fermions? HARD!

Objective: *Analytically "prove" quark confinement into hadrons, χSB and light NGBs in QCD-like theories*

Method: *Work with controlled deformations of SUSY theories to enhance calculability and* ***obtain EXACT results*** *in the non-perturbative regime (where it's generically almost impossible to get a handle on dynamics) when the amount of SUSY breaking is small.*
SUSY is treated purely as a theoretical tool, not necessarily required to be a part of reality.

Chiral Symmetry in massless QCD

- Keeping only the three light quark flavors *u*, *d* and *s*, QCD is described by

$$\mathcal{L}_{\rm QCD} = -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} + \sum_{f=1}^{3} \left[\bar{q}^f_L \, i\gamma^\mu D_\mu \, q^f_L + \bar{q}^f_R \, i\gamma^\mu D_\mu \, q^f_R - \left(\bar{q}^f_L m_f q^f_R + \bar{q}^f_R m_f q^f_L \right) \right]$$

- In the limit of vanishing quark masses, this is invariant under independent SU(3) rotations $\psi \to \psi e^{i\alpha^a T^a}$ on the left and right chirality quark fields
  - This $SU(3)_L \times SU(3)_R$ global symmetry is known as the chiral symmetry
- *u, d, s* masses < Λ_{QCD} → $SU(3)_L \times SU(3)_R$ symmetry of the massless limit is a good approximation. But since such a symmetry is not observed in the spectrum, it must be spontaneously broken

Chiral Symmetry Breaking in QCD

- Broken into a diagonal $SU(3)_V$ (identical SU(3) rotations for both quark chiralities) by nonperturbative QCD dynamics that yield nonvanishing chiral condensates (quark bilinears) $U_{fg} = \langle \bar{q}_{f_L} q_{g_R} \rangle$
  - Quarks acquire dynamical effective masses
  - Chiral PT → quark mass ratios and meson masses

$$B = \frac{U}{f_\pi^2}$$

$$\begin{aligned} m^2_{\pi^0} &= B\,(m_u + m_d)\,, \\ m^2_{\pi^\pm} &= B\,(m_u + m_d) + \Delta_{\rm em}, \\ m^2_{K^0} &= m^2_{\overline{K}^0} = B\,(m_d + m_s)\,, \\ m^2_{K^\pm} &= B\,(m_u + m_s) + \Delta_{\rm em}, \\ m^2_\eta &= \frac{1}{3} B\,(m_u + m_d + 4m_s) \end{aligned}$$

- The 8 pseudo-NGBs are the 8 light pseudoscalar mesons
  - Not exactly massless due to the small u, d, s masses
  - *Light* pions crucial for mediating strong force far enough to bind nuclei together
- A special kind of SUSY breaking called AMSB allows to make progress towards an analytical derivation of the spontaneous chiral symmetry breaking

Supersymmetric Lagrangian

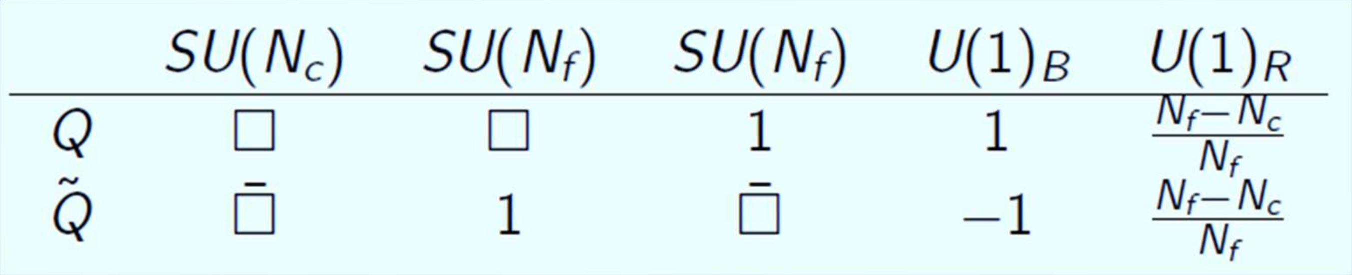
- General renormalizable $\mathcal{L}$ for a non-abelian SUSY gauge theory:

$$\mathcal{L} = \left[\Phi^{*i}\left(e^{2g_a T^a V^a}\right)_i^{\ j}\Phi_j\right]_D + \left(\left(\frac{1}{4} - i\frac{g_a^2\Theta_a}{32\pi^2}\right)[\mathcal{W}^{a\alpha}\mathcal{W}^a_\alpha]_F + [W(\Phi_i)]_F + c.c.\right)$$

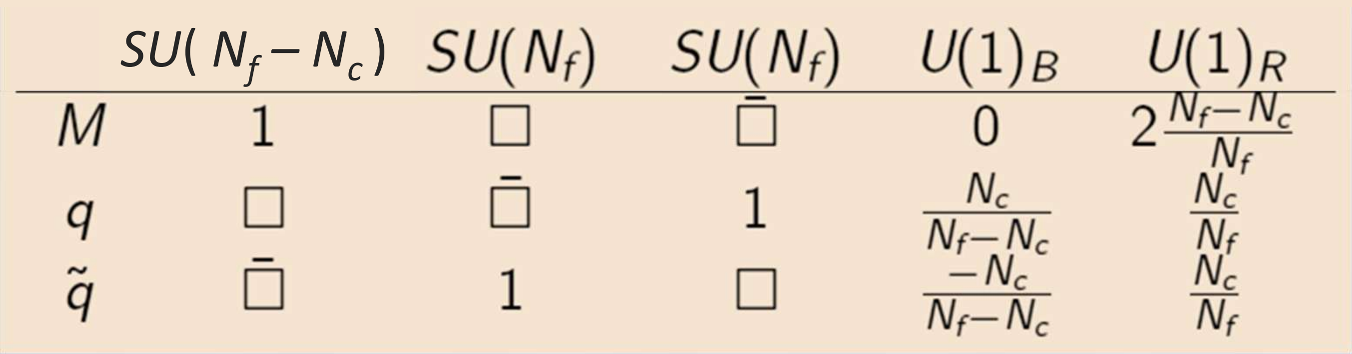
- Often have manifolds (*moduli spaces*) of inequivalent vacua parameterized by VEVs
- Holomorphic coupling $\tau \equiv \left(\frac{1}{g_a^2} - i\frac{\Theta_a}{8\pi^2}\right)$ can be treated as a chiral background superfield
  - receives only 1-loop and non-perturbative corrections
- Non-renormalization Theorem
  - The superpotential is not renormalized due to holomorphy
  - Couplings run due to wavefunction renormalization

Seiberg Duality for SU(N)

- Dual gauge theories that describe the same IR physics
- The *electric theory* is an SU(N_c) $\mathcal{N}$= 1 SUSY theory with N_f flavors of quarks
- The *magnetic theory* is an SU($N_f - N_c$) $\mathcal{N}$= 1 SUSY theory with N_f flavors of dual quarks and a scalar dual meson field.



	$SU(N_c)$	$SU(N_f)$	$SU(N_f)$	$U(1)_B$	$U(1)_R$
Q	$\square$	$\square$	1	1	$\frac{N_f-N_c}{N_f}$
$\tilde{Q}$	$\bar{\square}$	1	$\bar{\square}$	-1	$\frac{N_f-N_c}{N_f}$



	$SU(N_f-N_c)$	$SU(N_f)$	$SU(N_f)$	$U(1)_B$	$U(1)_R$
M	1	$\square$	$\bar{\square}$	0	$2\frac{N_f-N_c}{N_f}$
q	$\square$	$\bar{\square}$	1	$\frac{N_c}{N_f-N_c}$	$\frac{N_c}{N_f}$
$\tilde{q}$	$\bar{\square}$	1	$\square$	$\frac{-N_c}{N_f-N_c}$	$\frac{N_c}{N_f}$

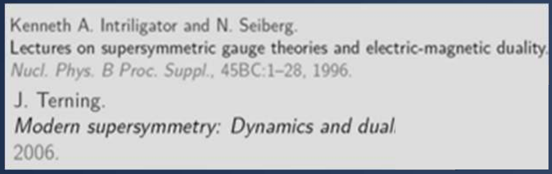
For rest of the talk, $N_c \equiv N$, $N_f \equiv F$

Theoretical control in $\mathcal{N}$= 1 SUSY Gauge Theories

- Non-renormalization Theorem
- Extra U(1)$_R$ symmetry i.e. more constraints
- IR DOF (parametrize the moduli space)
  - Gauge invariant holomorpic functions of UV fields
  - Fixed by global symmetries and anomaly matching
- Holomorphy, symmetries and weak coupling limit fix W$_{IR}$
- Seiberg Dualities

$$M_i^j = \bar{\Phi}^{jn}\Phi_{ni}$$

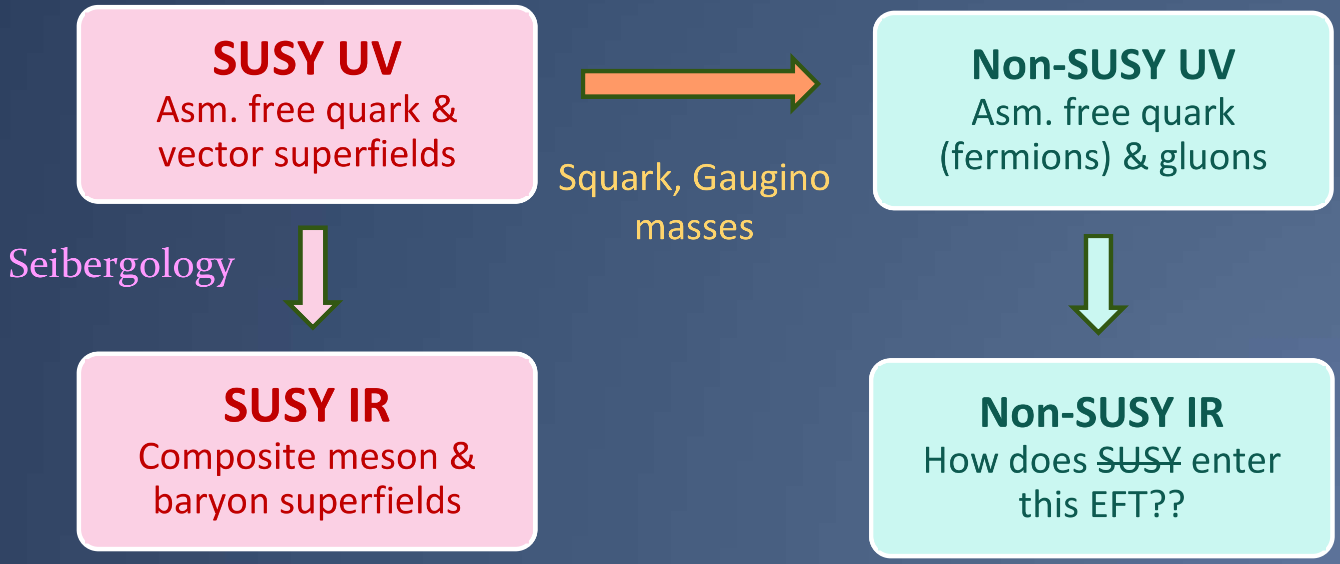
$$B_{i_1,\cdots,i_N} = \Phi_{n_1 i_1}\cdots\Phi_{n_N i_N}\epsilon^{n_1,\cdots,n_N}$$



Kenneth A. Intriligator and N. Seiberg.
Lectures on supersymmetric gauge theories and electric-magnetic duality.
Nucl. Phys. B Proc. Suppl., 45BC:1–28, 1996.

J. Terning.
Modern supersymmetry: Dynamics and dual
2006.

Exactly analyzed by Seiberg, Intriligator, Pouliot, et. al. in 90s



SUSY UV
Asm. free quark & vector superfields

Squark, Gaugino masses

Non-SUSY UV
Asm. free quark (fermions) & gluons

Seibergology

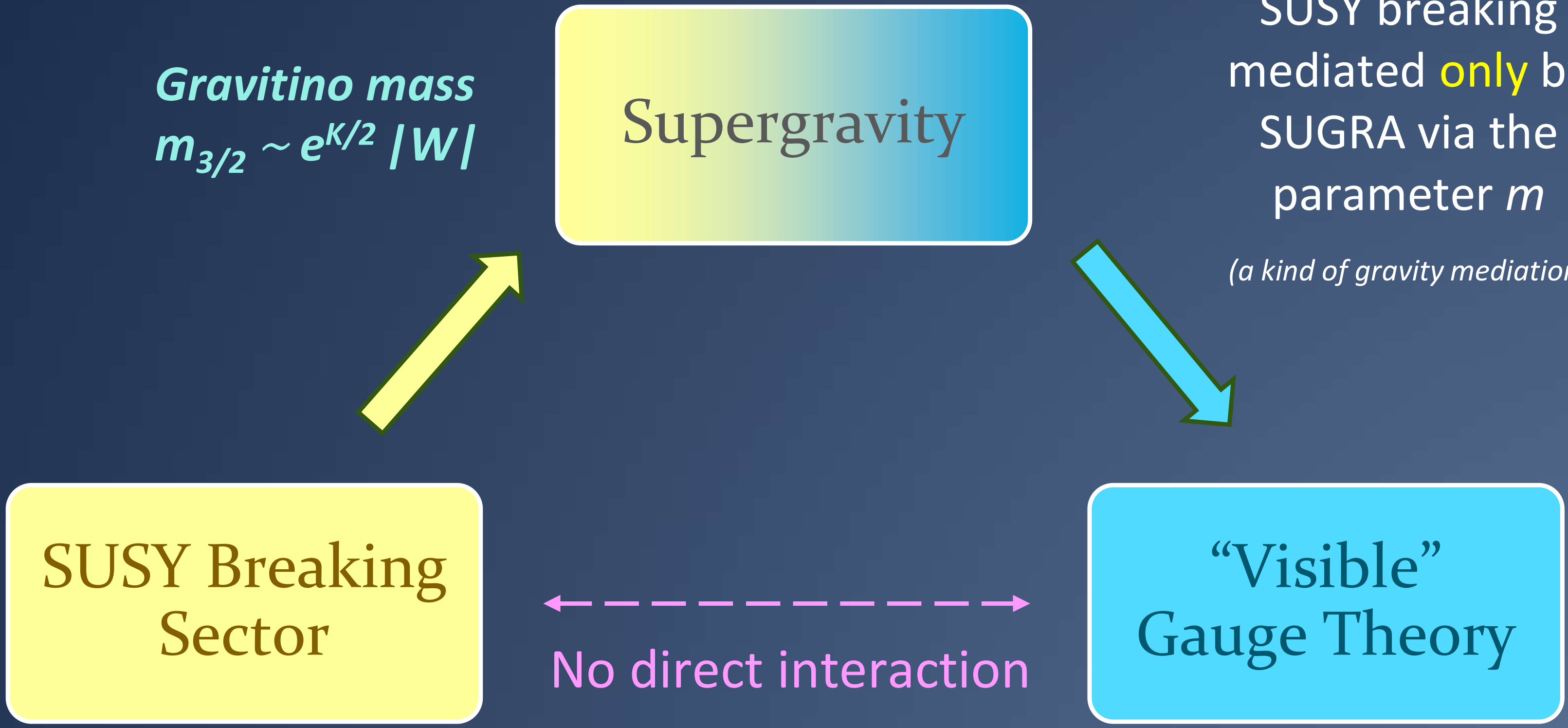
SUSY IR
Composite meson & baryon superfields

Non-SUSY IR
How does ~~SUSY~~ enter this EFT??

AMSB: Sequestering

Randall, Sundrum '99;
Giudice, Luty, Murayama, Rattazzi '98;
Arkani-Hamed and R. Rattazzi '99

*Gauged U(1)R



Supergravity

SUSY Breaking Sector

"Visible" Gauge Theory

No direct interaction

Gravitino mass
$m_{3/2} \sim e^{K/2} |W|$

SUSY breaking mediated only by SUGRA via the parameter *m*

(a kind of gravity mediation)

$V = |F|^2 - 3e^K |W|^2 + D^2/2$

F = auxiliary of matter chiral superfields, $M_{pl} \rightarrow 1$

$m \sim W/(M_{Pl})^2$

e.g. MSSM is confined to this brane

Weyl Compensator

Gates, Siegel '79;
Pamarol, Ratazzi '99

- Non-dynamical chiral sf spurion ξ , coupling constrained by super-Weyl inv.
  - *Compensates* for the theory not being superconformal invariant
  - Can be removed by rescaling fields if the SUSY theory is conformal. Then ξ effectively couples only to non-marginal terms
- Attaches to fundamental scales e.g. ξ Λ and in

$$\mathcal{L} = \int d^4\theta \mathcal{E}^* \mathcal{E} K + \int d^2\theta \mathcal{E}^3 W + c.c. \,|\, \Phi \to \Phi / \xi$$

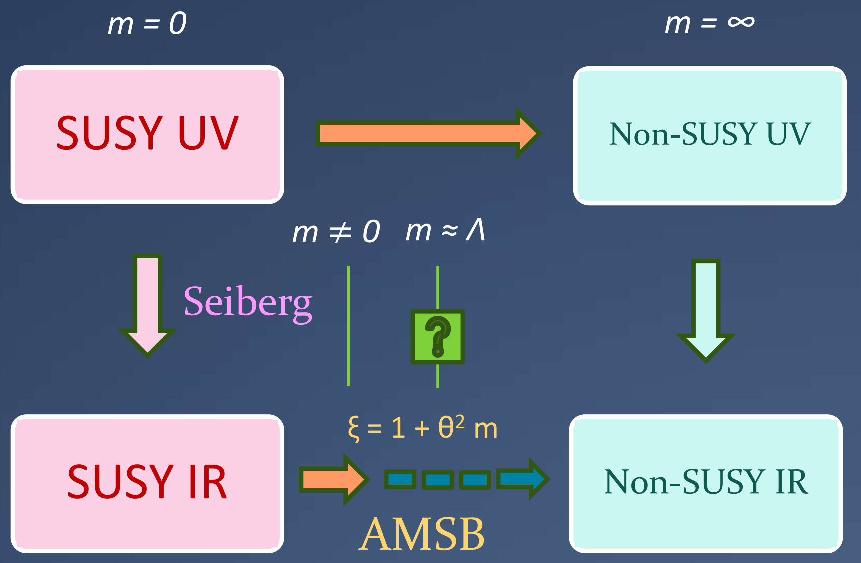
(just replace K_{IR} & W_{IR})

- Universal coupling at all energy scales → UV insensitivity → IR predictability
- Now the theory can couple to conformal version of SUGRA
  - Give ξ a VEV (scalar → M_{pl}) to regain Poincaré SUGRA
  - SUSY breaking sector triggers an F-term for ξ via SUGRA: $\xi = 1 + \theta^2\, m$

All ~~SUSY~~ effects encapsulated here

- Violation of conformal invariance in SUSY theory → ξ → m effects
  - Hence (superconformal) "anomaly" mediated SUSY breaking

AMSB: Effects



m = 0

m = ∞

SUSY UV

Non-SUSY UV

m ≠ 0   *m ≈ Λ*

Seiberg

SUSY IR

ξ = 1 + θ² m

AMSB

Non-SUSY IR

$$\mathcal{L}_{IR} = \int d^4\theta \mathcal{E}^* \mathcal{E} K_{IR} + \int d^2\theta \mathcal{E}^3 W_{IR} + c.c.$$

TREE LEVEL

(due to tree level dimensionful parameters, like mass)

$$\mathcal{L}_{\text{tree}} = m\left(\phi_i \frac{\partial W}{\partial \phi_i} - 3W\right) + c.c.$$

LOOP LEVEL

(due to dimensionful RGE cutoff Λ)

Determined only by physics at the energy scale of interest

ln(μ) -derivative of anomalous dimension

Scalar Mass, e.g. for squarks : $m_i^2(\mu) = -\frac{1}{4}\dot{\gamma}_i(\mu)m^2$

Gaugino Mass : $m_\lambda(\mu) = -\frac{\beta_g}{2g^2}(\mu)m$

NSVZ beta function β(g²)

A-terms: $A_{ijk}(\mu) = -\frac{1}{2}(\gamma_i + \gamma_j + \gamma_k)(\mu)m$

Pomarol, Rattazzi '99

An example: Generalized QCD in the UV

- SU(N) gauge theory with F quarks in fundamental and F antiquarks in antifundamental
- $F < 3N$
  - No tree level AMSB effects as $W = 0$ (asymptotic freedom)
  - Loop effects:

$$m_Q^2 = m_{\widetilde{Q}}^2 = \frac{g^4}{(8\pi^2)^2} 2C_F(3N-F)m^2$$

$$m_\lambda = \frac{g^2}{16\pi^2}(3N-F)m$$

$C_F = (N^2 - 1)/2N$

- Positive masses for squarks and gauginos (true in UV for any theory with $\beta_g < 0$)
  - Superpartners integrated out as m → ∞
  - Asymptotically free QCD-like non-SUSY theory

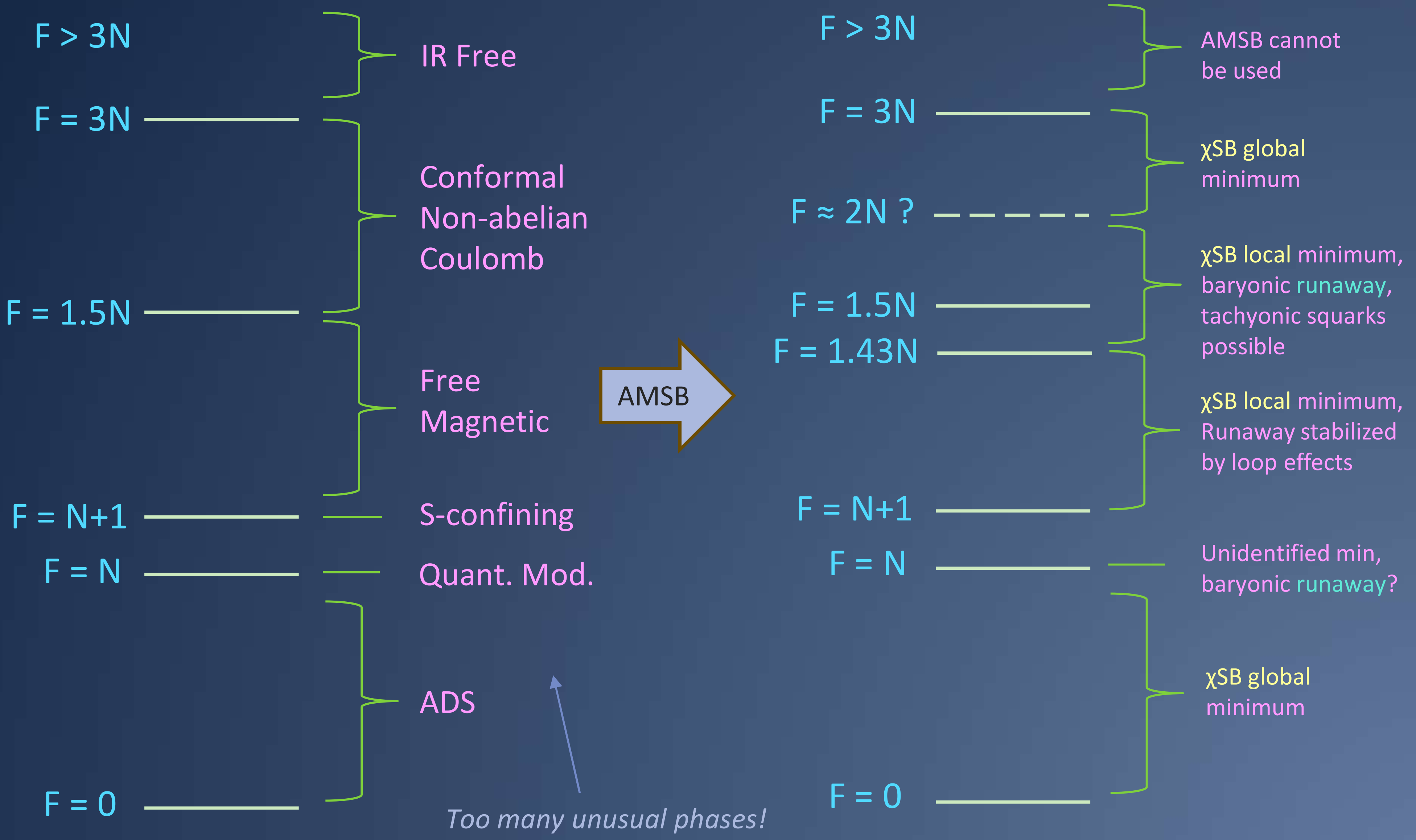
Csaki, Gomes, Murayama,
*Noether, **DRV**, Telem ‘23*

IR Phases of SU(N) QCD

SUSY χ : SU(F)$_L$ x SU(F)$_R$

Non-SUSY

*Runaways are to O(Λ) field values, not -∞



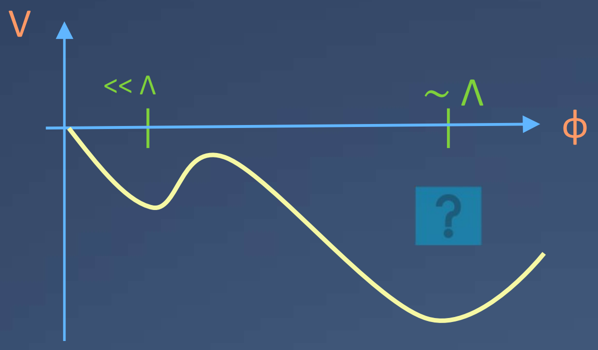
F > 3N
IR Free
F = 3N
Conformal Non-abelian Coulomb
F = 1.5N
Free Magnetic
F = N+1 — S-confining
F = N — Quant. Mod.
ADS
F = 0

AMSB

Too many unusual phases!

F > 3N
AMSB cannot be used
F = 3N
χSB global minimum
F ≈ 2N ?
χSB local minimum, baryonic runaway, tachyonic squarks possible
F = 1.5N
F = 1.43N
χSB local minimum, Runaway stabilized by loop effects
F = N+1
F = N — Unidentified min, baryonic runaway?
χSB global minimum
F = 0

Note on baryonic "runaways"



- A scenario in the magnetic description of IR when
  - Dual squarks/baryons get VEVs, and
  - Far from the origin of moduli space, $V_{SUSY} + V_{AMSB}$ becomes increasing negative as the VEVs are made larger (local minima may exist near origin)
- Does not imply a *true* runaway to infinite field values
  - V is calculated (and trusted) only in the weakly coupled regime where the mesonic/baryonic VEVs are small compared to the scale Λ
  - When VEVs increase to O(Λ), electric & magnetic descriptions are strongly coupled
    - We lose control on the Kähler potential (unknown higher order terms)
    - For asymptotically free theories, it is known that V has a +ve slope in the UV electric description (VEV $\gg$ Λ). Then the *global* min. resides in O(Λ) regime but is incalculable.

*Unitary symplectic Sp(N) ⊆ SU(2N)

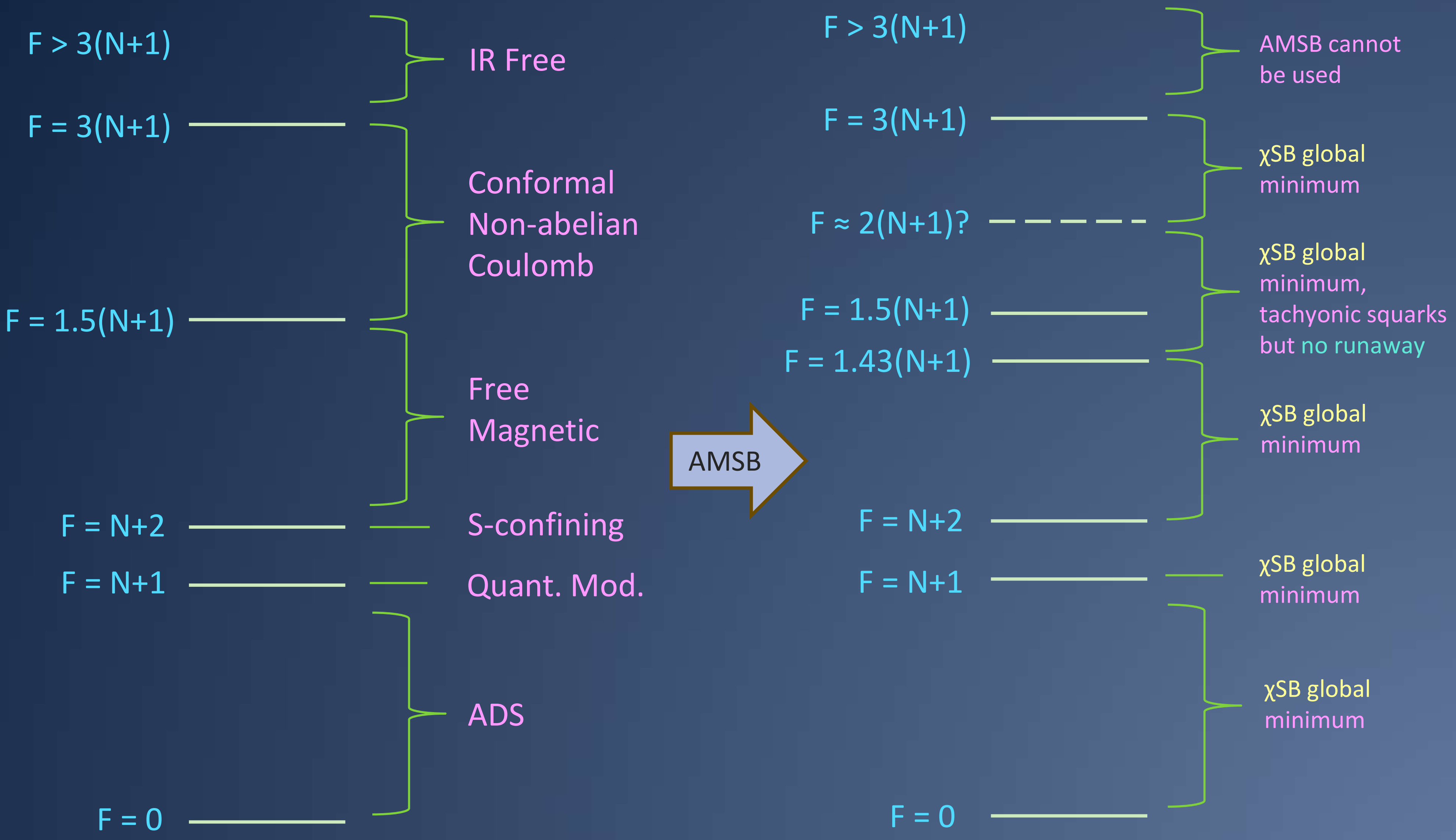
Why Sp(N)*?

- SU(N) >> includes baryonic runaways to regime of O(Λ) field VEVs
  - Unknown coeff of higher order flavor invariants in ***K*** → we lose control
  - Often the χSB minimum was only local (hope to be global when m → Λ & continuously connect to QCD)
- Sp(N) >> free of baryonic runaways
  - No Baryons – no totally antisymmetric invariant tensor over more than 2 indices
  - Tree level SUSY potential prevents runaway by tachyonic 2-loop masses
  - QCD-like: Same SUSY phases as SU(N)
    - Asymptotically free for F < 3(N+1)
    - 2F quark superfields in the 2N dim. fund. rep.
    - Global χSB ~~SUSY~~ minima for all F

I will focus on the neater Sp(N) results, and draw brief comparisons to SU(N)

***DRV**, Gu, Noether, Murayama '25(?)*

IR Phases of Sp(N) QCD



SUSY χ : SU(2F)

F > 3(N+1)	IR Free
F = 3(N+1)	
	Conformal Non-abelian Coulomb
F = 1.5(N+1)	
	Free Magnetic
F = N+2	S-confining
F = N+1	Quant. Mod.
	ADS
F = 0	

AMSB

Non-SUSY

F > 3(N+1)	AMSB cannot be used
F = 3(N+1)	
	χSB global minimum
F ≈ 2(N+1)?	
	χSB global minimum, tachyonic squarks but no runaway
F = 1.5(N+1)	
F = 1.43(N+1)	
	χSB global minimum
F = N+2	
F = N+1	χSB global minimum
	χSB global minimum
F = 0	

$\mathcal{N}$ = 1 SUSY Sp(N)

There is also $U(1)_B$ in SU(N)

- Global symmetry: SU(2F) x $U(1)_R$ (anomaly-free)
- F ‘flavors’ : superfields $\mathbb{Q}_i$, i = 1 … 2F , in $\{\mathbf{2N}, \mathbf{2F}, (F\text{-}N\text{-}1)/F\}$ rep.
- Invariant antisymm tensor $J = 1_N \otimes i\sigma_2$
- Gauge invariants $M_{ij} = \mathbb{Q}_{ic}\, \mathbb{Q}_{jd}\, J^{cd}$ in $\{\mathbf{1}, \mathbf{F(2F\text{-}1)}, 2(F\text{-}N\text{-}1)/F\}$ rep.
- D-flat constraints: Classical moduli space of degenerate SUSY vacua for
  - F ≥ N+1: labelled by antisymm $\langle M_{ij} \rangle$ with rank(M_{ij}) ≤ 2N
  - F < N+1: parametrized by $\langle \mathbb{Q} \rangle$ …. rank(M_{ij}) is arbitrary

Superfield contains boson and fermion!

$\mathbb{Q}_i$ fit into a 2Nx2F matrix

$$Q = \begin{pmatrix} a_1 & & & \\ & a_2 & & \\ & & \ddots & \\ & & & a_{N_f} \end{pmatrix} \otimes \mathbf{1}_2 \qquad M = \begin{pmatrix} 0 & a_1^2 & & & \\ -a_1^2 & 0 & & & \\ & & \ddots & & \\ & & & 0 & a_{N_f}^2 \\ & & & -a_{N_f}^2 & 0 \end{pmatrix}$$

*Upto gauge & flavor rotations

F < N+1 : ADS Phase

Same as in the SU(N) counterpart

All superpotenials: Intriligator, Pouliot '95

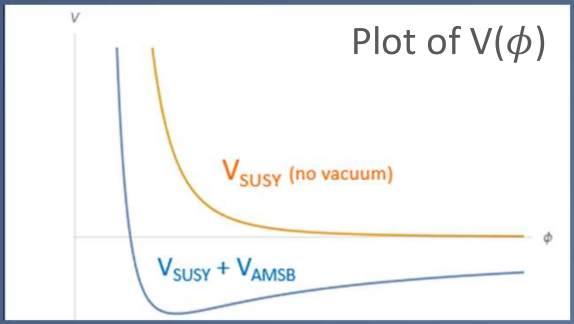
- Affleck-Dine-Seiberg superpotential (set root of unity = 1)

$$W = (N+1-F)\left(\frac{2^{N-1}\Lambda_{N,F}^{3(N+1)-F}}{Pf(M)}\right)^{1/(N+1-F)}$$

Flavor bifundamental

$$M = \begin{pmatrix} 0 & \phi^2 & & & \\ -\phi^2 & 0 & & & \\ & & \ddots & & \\ & & & 0 & \phi^2 \\ & & & -\phi^2 & 0 \end{pmatrix}$$

- Pushes $\langle M \rangle >> \Lambda^2$ into the weakly coupled regime
- Kähler potential is canonical in quark superfields
- $V = V_{SUSY} + V_{AMSB} = |\partial W/\partial \varphi_i|^2 - m(\varphi_i\, \partial W/\partial \varphi_i - 3W) - \text{c.c.}$
- Minimize along D-flat direction, $M = \varphi\, 1_N \otimes i\sigma_2$
  - $\varphi_{min} \propto \Lambda^2(\Lambda/m)^{1-F/(N+1)}$ --- for small m, $\varphi >> \Lambda^2$
- SU(2F) flavor symm dynamically broken to Sp(F)



Derive

F = N+1: Quantum Modified Constraint

Set Λ = 1

- W = 0 with Quantum constraint $Pf(M) = 2^{N-1}\Lambda_{N,N+1}^{2(N+1)}$
- $\langle M \rangle$ = O(Λ): Unsuppressed higher order K terms → V_{AMSB} can't be trusted
- Examine stability by fluctuating the a_i at Meson point: $M = 2^{(N-1)/(N+1)}\, 1_{N+1} \otimes i\sigma_2$
  - $M = 2^{(N-1)/(N+1)}\, e^{\Pi} \otimes i\sigma_2$ Π = diagonal, traceless: $\det(e^{\Pi}) = 1$
  - Similar quadratic form in Π for all flavor inv. in K :

$$TrM^\dagger M, (TrM^\dagger M)^2, TrM^\dagger M M^\dagger M \propto \; Tr\Pi^\dagger\Pi + \frac{1}{2}Tr\Pi^2 + \frac{1}{2}Tr\Pi^{\dagger 2}$$

- Physical kinetic term for Π → higher K term signs such that c > 0

$$V_{\text{tree}} = \partial_i W g^{ij^*} \partial_j^* W^* + m^* m \left(\partial_i K g^{ij^*} \partial_j^* K - K\right)$$

$$K \sim c(\Pi^\dagger\Pi + \frac{\eta}{2}(\Pi^2 + \Pi^{\dagger 2}))$$

$$+m\left(\partial_i W g^{ij^*} \partial_j^* K - 3W\right) + c.c.$$

Positive mass
No runaway!

Massless χSB NGBs
when η = 1

$$V_{\text{AMSB}} = c(\eta^2 + \eta)m^2(Re(\Pi))^2 + c(\eta^2 - \eta)m^2(Im(\Pi))^2$$

  - MP is a stable χSB min in the physically allowed moduli space

N = 1

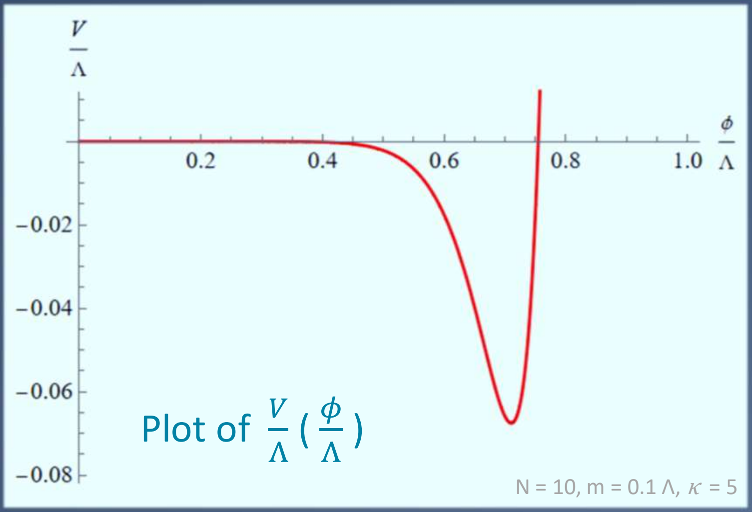
F = N+2: S-Confinement (N>1)

Λ = 1, κ = O(1) coupling to make K canonical

- $W = -\kappa\,\mathrm{Pf}(M)$: For M << Λ , at leading order take K canonical in M
- Unlike SU(N): No Yukawa term in W ↔ no $V_{2\text{-loop}}$ from AMSB
- SUSY vacuum: Confinement without χSB
- With AMSB, look in direction $M = diag\{v_1, \ldots, v_F\} \otimes i\sigma_2$
  - AMGM & real m: $M = v\, 1_{N+2} \otimes i\sigma_2$, $v \in \mathbb{R}$:

$$V_{min} = -\mathcal{O}(m^{(2N+2)/N})$$

$$\phi_{min} = \left(\frac{2m(N_c-1)}{\kappa(N_c+1)}\right)^{\frac{1}{N_c}} < \ 1(\equiv \Lambda)$$



Plot of $\frac{V}{\Lambda}(\frac{\phi}{\Lambda})$

N = 10, m = 0.1 Λ, κ = 5

- Higher K terms contribute at $\mathcal{O}(m^{(2N+4)/N})$ → Neglected for small *m*
- Global χSB minimum!

F = N+1 with AMSB in SU(N)

x, v << Λ = 1

$$W = \alpha \bar{B} M B - \beta det M$$

$$B = \bar{B} = \begin{pmatrix} b \\ 0 \\ \cdot \\ \cdot \\ \cdot \end{pmatrix} \qquad M = \begin{pmatrix} x & & \\ & v & \\ & & \ddots \end{pmatrix}$$

- Baryon no. breaking b ≠ 0:

Luzio, Xu '22

$$V_{tree} = -\frac{(N-2)^2\beta}{2\alpha} m^2 v^N \qquad V_{loop} = +\mathcal{O}(m^2 v^2)$$

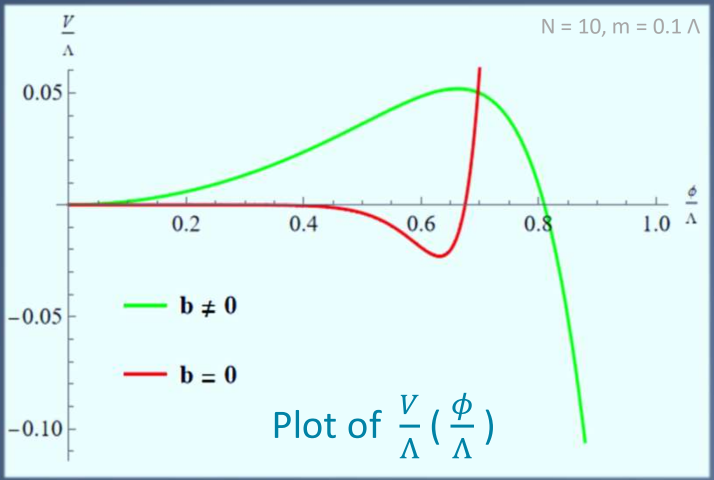
  - AMSB loop effects beat tree level runaway → stabilized near origin

- Baryon no. conserving b = 0:

$$V_{tree}, V_{loop} = -\mathcal{O}(m^{2N/(N-1)})$$

  - No tree level runaway
  - Loop effects same order in m, but loop suppressed

- Local χSB minimum from b = 0 branch



Plot of $\frac{V}{\Lambda}(\frac{\phi}{\Lambda})$

Magnetic Dual for Sp(N) (Intriligator & Pouliot)

There is also $U(1)_B$ in SU(N)

- Dual "Magnetic" description of IR physics for $F > N+1$
- $Sp(\tilde{N} = F - N - 2)$ gauge, same global symmetry: $SU(2F) \times U(1)_R$
- F 'flavors' : dual quarks q_i $i = 1 \ldots 2F$, in $\{2\tilde{N}, \overline{\mathbf{2F}}, (N+1)/F\}$ rep.
- Antisymm gauge singlets M_{ij} in $\{1, \mathbf{F(2F-1)}, 2(F-N-1)/F\}$

Transforms like electric M

- $W = \frac{\lambda}{2} M_{ij}\, q_{ic}\, q_{jd}\, J^{cd}$ canonical K for M and q
  - Marginal W → No tree level AMSB effects
  - $\lambda M_{ij}\, q_{ic}\, \tilde{q}_{jc}$ is the SU(N) analogue

UV ↔ IR

Higgs ↔ Confinement

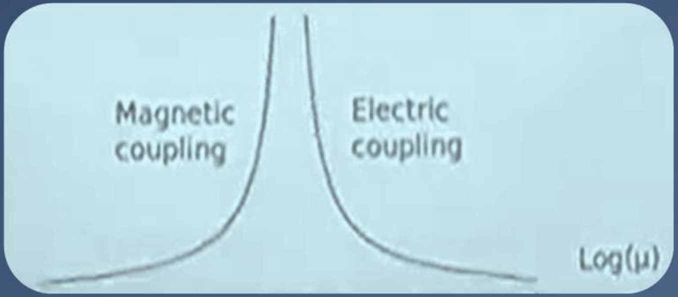
- As electric theory gets stronger, magnetic theory gets weaker

AMSB
Overview
NSVZ

N+3 ≤ F ≤ 1.5(N+1): Free Magnetic Phase **(Non-Abelian)**

UV: Electric
IR: Magnetic

- $\tilde{b} = 3\tilde{N} - F < 0$ → IR-free magnetic theory
  - Very strong coupling in electric IR → breaks down
  - Trivial fixed point : g, λ → 0 in the deep IR
  - Coupled β's allow us to write λ(g) at 1-loop near origin: $\frac{\beta_g}{g^2} = \frac{\beta_\lambda}{\lambda^2}$



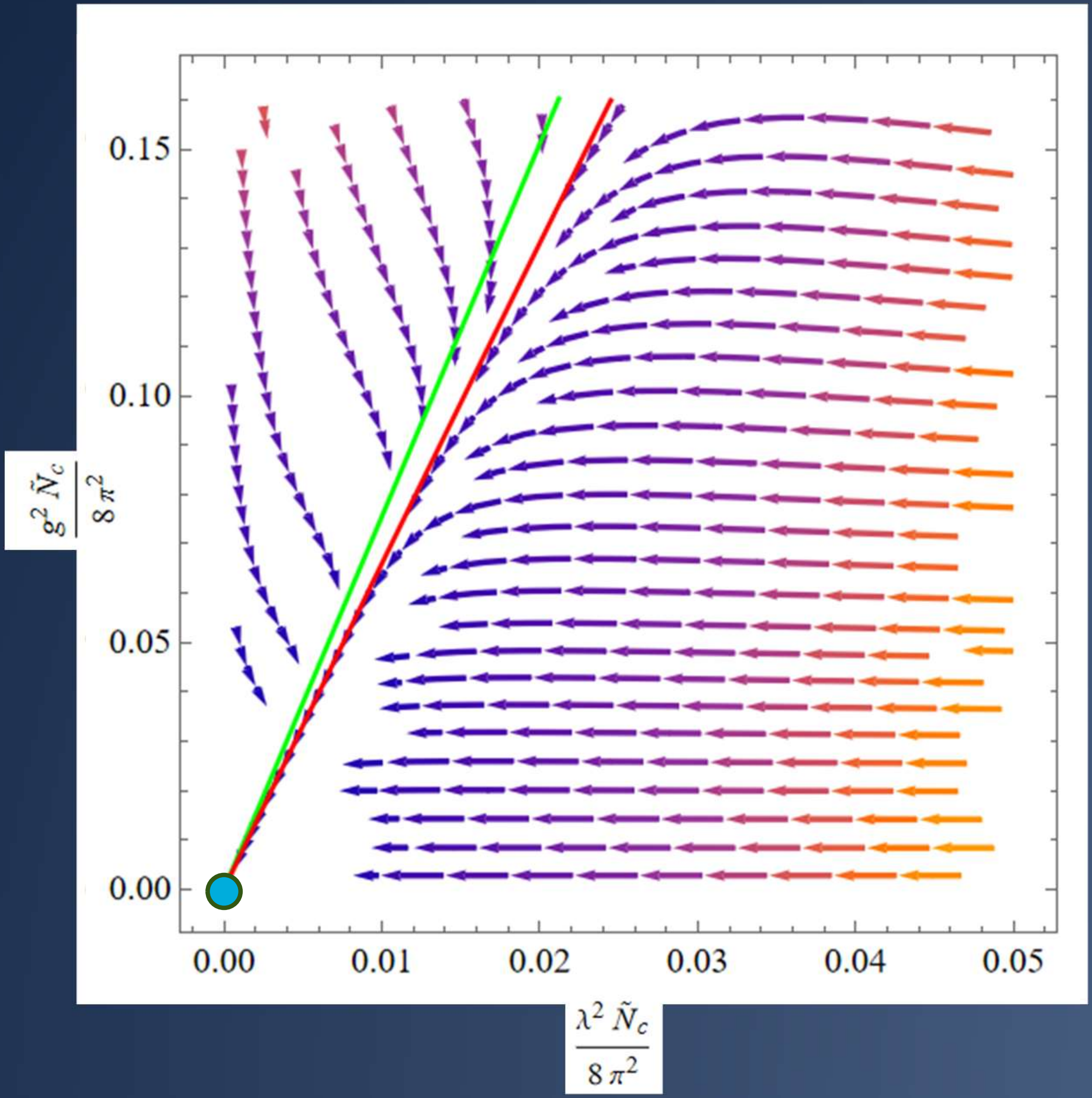
- RGE -

$$\gamma_M = -\frac{2\widetilde{N}\lambda^2}{8\pi^2} \qquad \gamma_q = \frac{1}{8\pi^2}(2C_F g^2 - (2F-1)\lambda^2) \qquad \beta_g = -\frac{g^4}{8\pi^2}\frac{3(\widetilde{N}+1) - F - F\gamma_q}{1-(\widetilde{N}+1)g^2/(8\pi^2)}$$

$$\beta_\lambda = -(\gamma_M + 2\gamma_q)\lambda^2$$

- AMSB-induced scalar mass-squared for M and q are positive for F ≤ 1.43(N+1), but negative for q when 1.43(N+1) ≤ F ≤ 1.5(N+1) → *Tachyonic runaway?*

RG Flow in Free Magnetic Phase



When $F \approx 1.43(N+1)$, expect

- $m_q^2 \approx 0$ as it switches from being positive to negative
- $m_M^2 > 0$ as it is positive for entire free magnetic range

Alongside is a plot for $F = 330$, $N = 230$, i.e. $F = 1.429(N+1)$

- Couplings clearly flow to the fixed point (0, 0) along a straight-line trajectory
- The red line is a plot of $m_q^2 = 0$ and indeed the trajectory converges onto it near (0, 0)
- Region below the green line satisfies $m_M^2 > 0$; the trajectory completely lies in this region

1.43(N+1) ≤ F ≤ 1.5(N+1): Mixed q & M Branch

- Direction with D-flat dual squark VEVs trigger condensation of mesons as well

$$q = \phi_q \mathbf{1} \otimes \mathbf{1}_2$$
$$M = \phi_M \mathbf{1} \otimes i\sigma_2$$

Runaways in ϕ_q direction for SU(N) as no stabilizing +ve quartic there

$$V = \frac{(\lambda \tilde{N}_c \phi_q^2)^2}{N_f} + \frac{(2\lambda \tilde{N}_c \phi_M \phi_q)^2}{2\tilde{N}_c} + m\left(\gamma_M + 2\gamma_q\right)\left(\lambda \tilde{N}_c \phi_M \phi_q^2\right) - 2\tilde{N}_c m^2 \frac{\dot{\gamma}_q}{4}\phi_q^2 - N_f m^2 \frac{\dot{\gamma}_M}{4}\phi_M^2$$

  - Not minimized at $\phi_M = 0$; can't integrate out M → Analytical calc hard
  - Numerically explore for various F, N and m choices

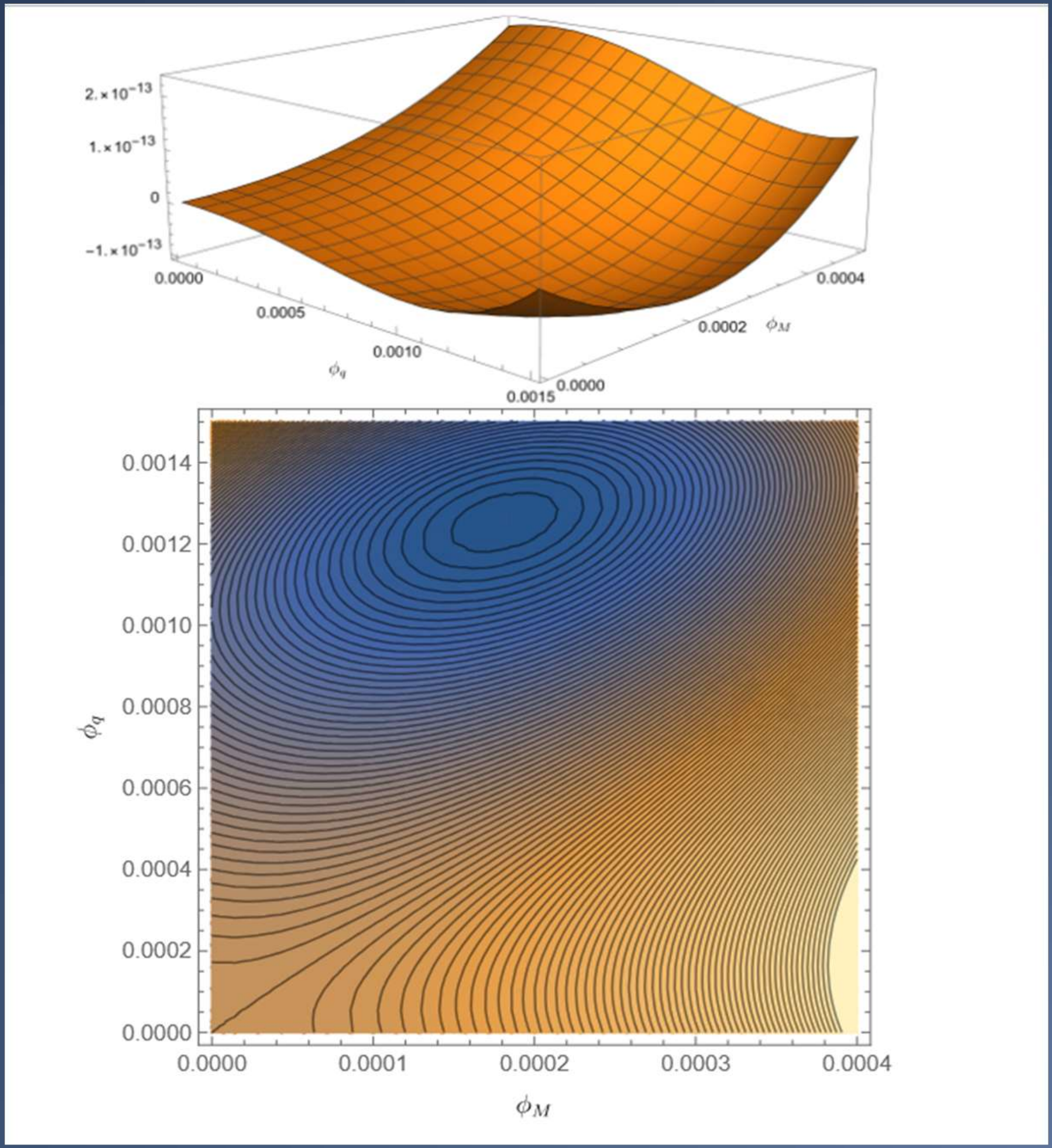
$$\frac{g^2}{8\pi^2} \sim \frac{1}{(3\tilde{N}_c + 3 - N_f)\ln\mu} \qquad m_q^2 = \frac{(-\tilde{b})g^4}{(16\pi^2)^2}\frac{2N_f^2 - (6\tilde{N}_c + 7)N_f - 2\tilde{N}_c^2 + 2\tilde{N}_c + 3}{2(\tilde{N}_c + 2N_f - 1)}m^2$$

$$\frac{\lambda^2}{4\pi^2} \sim \frac{N_f - \tilde{N}_c - 2}{(3\tilde{N}_c + 3 - N_f)(\tilde{N}_c + 2N_f - 1)\ln\mu} \qquad m_M^2 = \frac{(-\tilde{b})2\tilde{N}_c g^2\lambda^2}{(16\pi^2)^2}m^2$$

- Always single min on ϕ_M - ϕ_q contour plots for finite non-zero ϕ_M & ϕ_q

- No runway despite tachyonic dual squarks for F $\gtrsim$ 1.43(N+1)
    - SU(2F) broken to Sp(Ñ) x Sp(2F - 2Ñ) at this local minimum
- Global min will come from a pure mesonic branch on accounting for non-perturbative dynamics
    - Integrate out q, derive the ϕ_M vacuum of low energy pure SYM-like theory, plug it back into mixed branch V, numerically get an eff pot. for ϕ_q that turns out to have min at $\phi_q = 0$. So, it is consistent to integrate out q.

N = 14, F = 22 = 1.467(N+1), m = 0.1 Λ



Mixed Mesonic Branch

N+3 ≤ F ≤ 1.5(N+1): Mesonic Branch

- Full rank meson VEV M = $diag\{v_1, \dots, v_F\} \otimes i\sigma_2$ & integrate out all q_i
  - Low energy EFT (pure SYM-like)

$$W = (\widetilde{N}+1)2 \cdot 2^{\frac{-2}{\widetilde{N}+1}} \Lambda_L^3$$

New confinement scale

$$\Lambda_L^{3(\widetilde{N}+1)} = 2^F \widetilde{\Lambda}^{3(\widetilde{N}+1)-F} Pf(M)$$

- With tree level AMSB:
  - AMGM: Homogeneous meson VEVs $V_{min} = -\mathcal{O}\left(m^{2\frac{F-(\widetilde{N}+1)}{F-2(\widetilde{N}+1)}}\right)$
- F ≤ 1.43(N+1) : Conclusively global χSB min from mesonic branch
- 1.43(N+1) ≤ F ≤ 1.5(N+1) : Numerically deeper than the mixed branch min (checked for various N), thus conjectured global min here too

SU(2F) completely broken to Sp(F)

*I will discuss CW for SU(N) based on Kondo, Murayama, Noether, **DRV** '25*

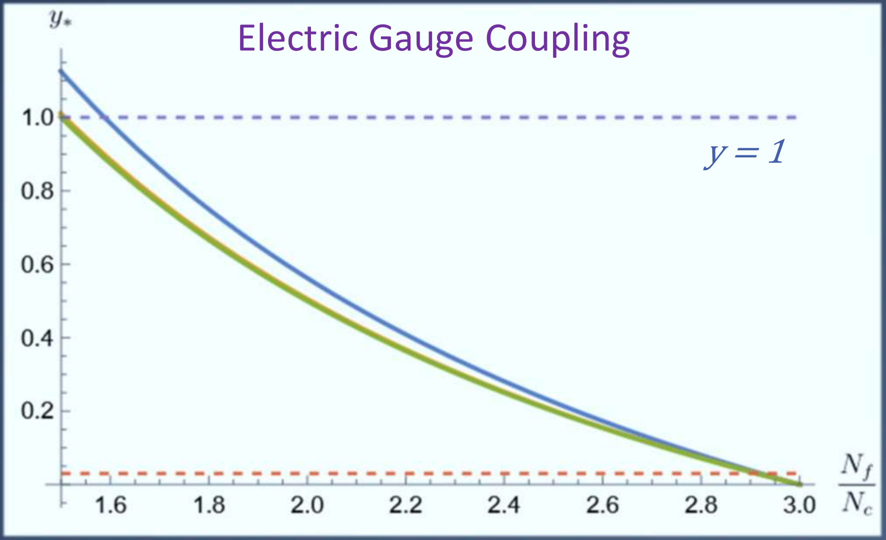
1.5 N < F < 3 N : Conformal Window of SU(N)

- Non-trivial IR fixed point (β_g , $\beta_\lambda = 0$ when $g, \lambda \neq 0$)
- Electric and magnetic descriptions have identical fixed pt.
- $D = \frac{3}{2} R$ for chiral primary operators

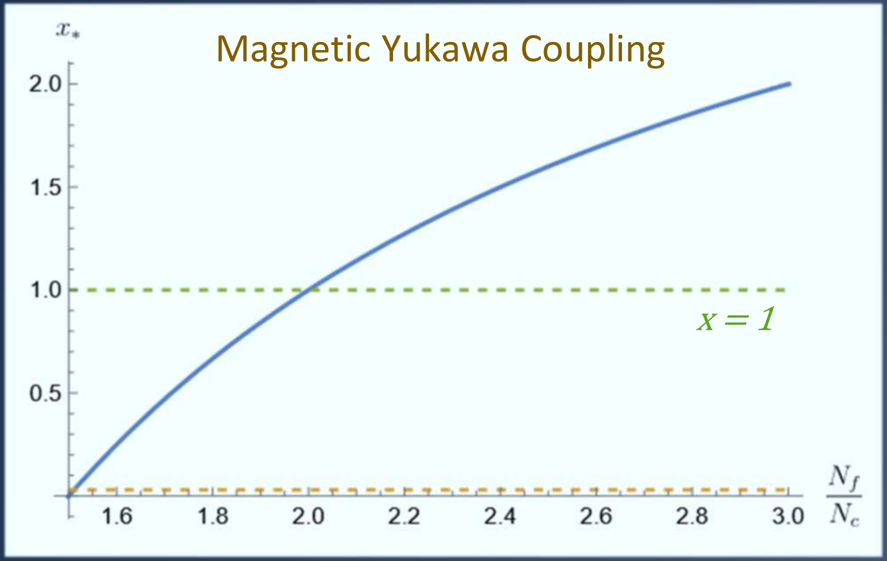
  $R(\mathbb{Q}) = (F - N)/F$, $R(q) = (F - \tilde{N})/F$
  $R(M) = 2\,\tilde{N}/F$
- If AMSB masses scale sufficiently slowly as μ → 0, AMSB effects are relevant and change the IR dynamics by deflecting RGE flow to χSB
- Goal: Study μ-dependence of masses & look for χSB minima
  - Perturbative analysis needs small electric (magnetic) coupling(s)
  - Possible only at upper (lower) edge of CW → BZ fixed pts.

    Banks, Zaks '82
  - Conjecture about intermediate strongly coupled region

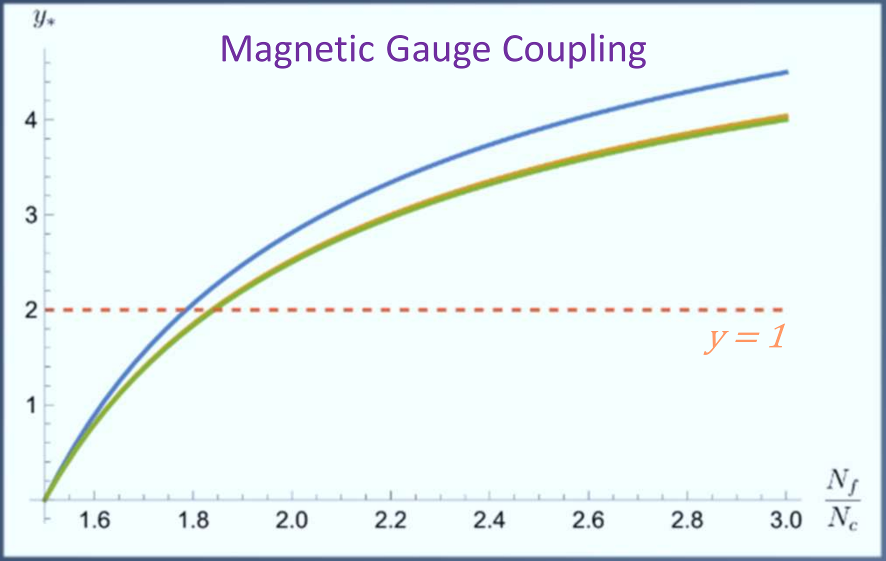
t’Hooft couplings at the fixed point in the electric (magnetic) theories are small ($\ll 1$) only at the upper (lower) edges of the conformal window, as seen alongside (below) for various N



Electric Gauge Coupling



Magnetic Yukawa Coupling



Magnetic Gauge Coupling

Lower Conformal Window of SU(N) F = 3Ñ/(1+ϵ) $\approx \frac{3}{2}$N

- Ñ, F $\gg$ 1, $\epsilon \ll$ 1, $x \equiv \frac{\tilde{N}}{8\pi^2}\lambda^2, y \equiv \frac{\tilde{N}}{8\pi^2}g^2$

$$\delta x = x - 2\epsilon$$

$$\delta y = y - 7\epsilon$$

$$\beta(y) = -3y^2(\epsilon - y + 3x) \qquad \beta(x) = x(-2y + 7x)$$

- Coupled β's → approach fixed pt. (2ϵ, 7ϵ)
  along eigenvec trajectory with lesser eigenval

→ $$\delta x \approx \frac{2}{7}\left(1 + \frac{3\epsilon}{2}\right)\delta y$$

→ $$\delta x, \delta y \sim \mu^{21\epsilon^2}$$

Slower than μ^2
if 7$\epsilon \ll$ 1* !

Negative $$m_M^2 = \frac{3}{2}\epsilon^2 \delta y m^2$$

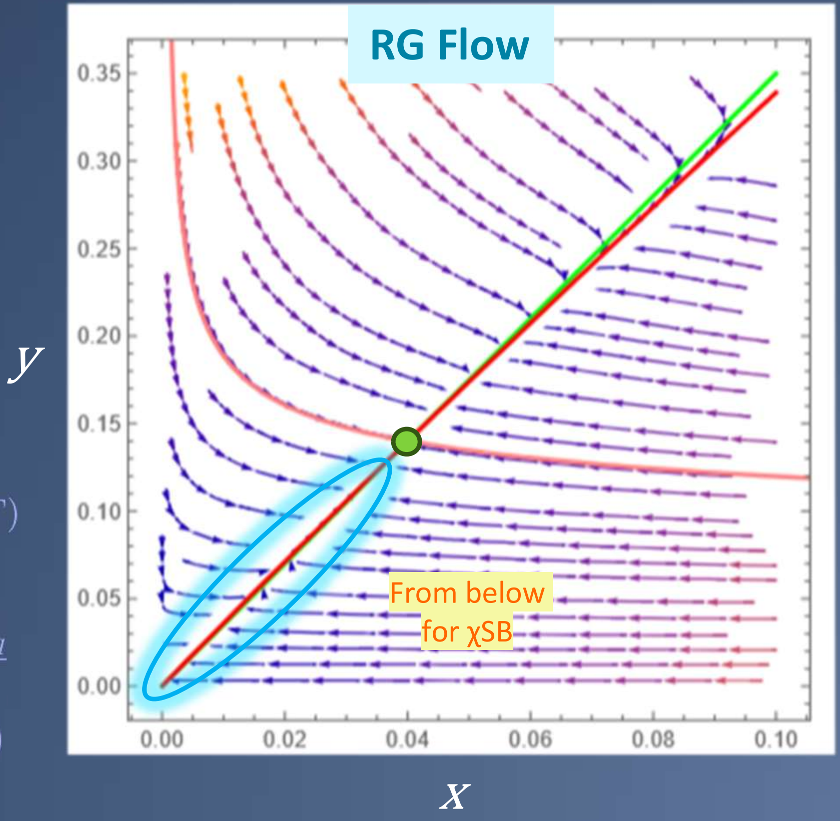
Positive $$m_q^2 = -\frac{3}{4}\epsilon^2 \delta y m^2$$

$$\gamma_q = \frac{1}{8\pi^2}(2C_F g^2 - \lambda^2 F)$$

$$\gamma_M = -\frac{1}{8\pi^2}\tilde{N}\lambda^2$$

$$\beta_g = -g^4 \frac{3\tilde{N} - F - F\gamma_q}{8\pi^2 - \tilde{N}g^2}$$

$$\beta_\lambda = -\lambda^2(2\gamma_q + \gamma_M)$$



*or F < 1.77N

χSB minimum from Mesonic branch (lower edge)

- Similar to free Mag: $\langle M \rangle$ → integrate out dual squarks & EFT is

$$W_{\rm eff} \quad = \tilde{N}\left(\tilde{\Lambda}^{3\tilde{N}-F}\,\det(\lambda M)\right)^{\frac{1}{\tilde{N}}}$$

(exact, non-perturbatively generated)

- $Z_M(\mu) = c_m\left(\frac{\mu}{\Lambda_m}\right)^{\gamma_M}$ is not logarithmic; appears in *K* and hence affects *V*

$$V = -\frac{F}{4}c_m Z_m \dot{\gamma}_M m^2\phi^2 + m\left(1-\frac{\gamma_M}{2}\right)\left(\phi\frac{\partial W}{\partial\phi}+\phi^*\frac{\partial W^*}{\partial\phi^*}\right) - 3mW - 3mW^* + \frac{1}{F c_m Z_m}\left|\frac{\partial W}{\partial\phi}\right|^2$$

- Deviations from the CFP parametrized by $\gamma_M = -2\frac{3\tilde{N}-F}{F} + c_\gamma\left(\frac{\mu}{\Lambda_m}\right)^{\alpha}$
- Taking $M_{ij} = \phi\delta_{ij}$ (near lower edge of CW, $\alpha \sim 21\epsilon^2$):

$$\gamma_M \quad = -\tfrac{1}{8\pi^2}\tilde{N}\lambda^2$$

$$\frac{\phi_{min}}{\Lambda_m} \propto \left(\frac{m}{\Lambda_m}\right)^{\frac{2}{2-\alpha}}$$

→ χSB minimum with no baryonic instability as $m_q^2 > 0$

Upper Conformal Window

F = 3N / (1+ϵ)

$$C_F = (N^2 - 1)/2N$$

$$\gamma_Q = \frac{1}{8\pi^2}(2g^2 C_F)$$

$$\beta(g) = -g^4 \frac{3N - F - F\gamma_Q}{8\pi^2 - Ng^2}$$

- N, F ≫ 1, $\epsilon \ll 1$

$$\beta(y) = -3y^2(\epsilon - y)$$

- Fixed point when $y = \epsilon$
- Electric theory weakly coupled in UV → $\epsilon > y$ on RG flow
- Near fixed pt. $y(t) - \epsilon = (y(0) - \epsilon)\mu^{3\epsilon^2}$

$\epsilon^2 < \frac{3}{2} \leftrightarrow F > 1.65N$

$$m_Q^2 = \frac{3}{4} y^2 (\epsilon - y) m^2 \qquad m_\lambda = \frac{3}{2}(\epsilon - y) m \longleftarrow$$

- Positive
- Slower than μ^2 if $\epsilon \ll 1$

- ➢ $W_{elec} = 0$ → Min of squark branch at origin
- ➢ When mass > μ, $\mathbb{Q}$ & λ integrated out → SC phase destroyed → non-SUSY QCD (strongly coupled)
  - Low energy description: Twice-dual theory

AMSB

χSB minimum from Twice-Dual (upper edge)

- $W = N^{ij}(Q_i\tilde{Q}_j - M_{ij})$ where *N* is a Lagrange multiplier
  - SUSY EOM set *M* = *Q*$\tilde{Q}$ and *N* = 0 reproducing original electric theory. But *N* ≠ 0 with AMSB.
- RGE analysis is identical to the Magnetic one that was done near the lower edge of CW, with Yukawa coupling *Y* for *NQ*$\tilde{Q}$ replacing *λ*
- With initial conditions of RG flow consistent with electric result, $m_Q^2 > 0$ and $m_N^2 < 0$. After decoupling squarks, the EFT is : $W = N\Lambda^3\left(\frac{Y\phi_N}{\Lambda}\right)^{\frac{F}{N}} - FY\Lambda\phi_N\phi_M$
- Keeping track of Z's, *V* ends up symmetric in the VEVs, so $\phi_M = \phi_N \equiv \phi$
- For F = 3N/(1+ϵ), we find the stable χSB minimum at $\frac{\phi_{min}}{\Lambda} \propto \left(\frac{m}{\Lambda}\right)^{\frac{1}{\epsilon}}$

F ≥ 3(N+1): Free Electric Phase

- 2-loop squark & gaugino masses from AMSB are negative

$$m_Q^2 = \frac{g^4}{2(8\pi^2)^2}(N+1)[3(N+1)-F]m^2$$

$$m_\lambda = \frac{g^2}{16\pi^2}[3(N+1)-F]m^2$$

  - True runaway behavior

- AMSB cannot be used to understand the non-SUSY theory

Summary

- Stable χSB minima [SU(2F) → Sp(F)] are present for all $F < 3(N + 1)$ upon AMSB in the small SUSY-breaking limit (promising as controlled approximation to QCD)
- $F = N + 1$: strongly coupled nature of the quantum modified moduli space → non-linear analysis around meson point → stable χSB min. (no baryons)
- $F \gtrsim 1.43N$ theories protected from runaways to incalculable minima by tachyons (dual quark branch of free magnetic phase & lower CW), thanks to quartic in V_{SUSY} → *unlike SU(N)*
- Perturbative analysis performed for BZ fixed pts. At lower & upper edges of CW, with χSB min. seen using the magnetic and twice-dual descriptions
- OUTLOOK: Behavior in the (real-world) limit $m \gg \Lambda$? Phase transition? If ASQCD and QCD are in the same universality class, we have an analytic demonstration of χSB for QCD. The $m \sim \Lambda$ regime is an open question…

AMSB Conjecture: There exists some class of strongly coupled theories that are continuously connected to their AMSB counterparts, thus sharing the same vacuum universality class. When and how this conjecture fails is an active area of research.

Thank You