Thoroughly AMSB QCD

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In Preparation with:
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What is the IR behavior of gauge theories?

- In this talk: SU gauge theories with fundamentals
  - Asymptotically free quarks that confine
  - How to calculate IR quantities/behavior?
- IR phase? Chiral symmetry breaking (χSB), quark confinement?
  - This is our aim
  - Crucial if we want a model of QCD with quantitative predictability

Approaches

- Lattice, but would like an analytic derivation
- Symmetry, spurions (e.g. quark mass breaks χ symmetry)
- ‘t Hooft anomaly matching
- Non-SUSY results from deformation of SUSY theories!
  - Extra symmetry $U(1)_R$
  - Non-renormalization theorems
  - IR fields are gauge invariant, holomorphic functions of UV fields
  - Remarkable set of Seiberg dualities (dual descriptions of a gauge theory – see later)

The plan

- Deformation of SUSY gauge theories → Anomaly Mediated Supersymmetry Breaking (AMSB)
- An example: χSB for SU(N_c) with $N_F < N_c$
- First runaway and its stabilization ($N_F = N_c+1$)
- Finally address the range $N_F \leq 3N_c/2$

The method

- Prepare non-SUSY theory in UV
  - Give masses to gluinos and/or squarks, leaving the gluons and quarks
  - Ideally above confinement scale!
- Try to keep IR control
  - Runaways → SUSY potential flat directions often become tachyonic with SUSY breaking
- Novel use of AMSB to obtain χSB phase of QCD

Cheng, Shadmi 9801146
Luty Rattazzi 9908085
Abel, Buican, Komargodski 1105.2885
And others…

"Moduli space"

Murayama 2104.01179

None of these vacua known before last year!

AMSB the mechanism

Randall, Sundrum 9810155
Giudice, Luty, Murayama, Rattazzi 9810442
Arkani-Hamed, Rattazzi 9804068
Katz, Shadmi, Shirman 9906296

- Supergravity (SUGRA) with sequestering of visible and SUSY breaking sectors
  - Visible sector is where we will place our “QCD laboratory”
  - Only mediation via gravity
  - Universal coupling at all scales → UV insensitivity
  - Can be applied in the same way in UV and IR!
- All effects encapsulated in “Weyl compensator”

Why “anomaly”?
Breaking due to violation of scale invariance

Weyl compensator

Pomarol, Rattazzi 9903448

$$\Phi = 1 + \theta^2 m \qquad \mathcal{L} = \int d^4\theta \Phi^* \Phi K + \int d^2\theta \Phi^3 W + c.c.$$

- Φ a non-dynamical spurion of scale invariance
  - After canonical normalization, simply find it attached to fundamental scales as ΦΛ
- SUSY breaking m encoded in F-component
- Effects:

$$\mathcal{L}_{\text{tree}} = m\left(\phi_i \frac{\partial W}{\partial \phi_i} - 3W\right) + c.c.$$

Important! Couples to non-marginal terms

$$A_{ijk}(\mu) = -\frac{1}{2}(\gamma_i + \gamma_j + \gamma_k)(\mu) m,$$

$$m_i^2(\mu) = -\frac{1}{4}\dot{\gamma}_i(\mu) m^2,$$

Remember this one too!

$$m_\lambda(\mu) = -\frac{\beta(g^2)}{2g^2}(\mu) m.$$

Application to QCD

- Take SU(N_c) gauge theory with N_f fundamentals and anti-fundamentals
- W = 0 → only loop effects

$$m_Q^2 = m_{\tilde{Q}}^2 = \frac{g^4}{(8\pi^2)^2} 2C_i(3N_c - N_f)m^2,$$

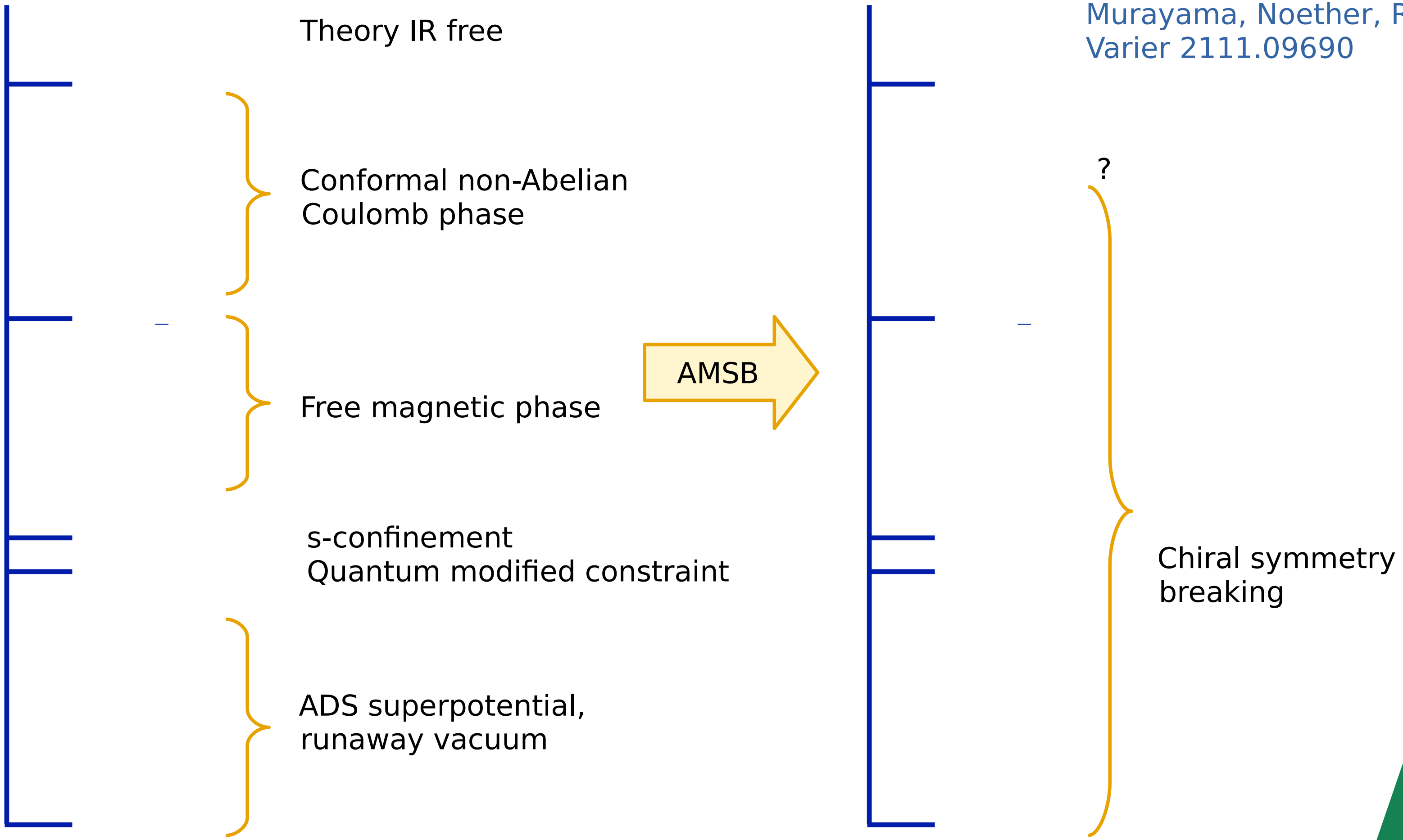
$$m_\lambda = \frac{g^2}{16\pi^2}(3N_c - N_f)m.$$

Pushes scalars to origin – incalculable in THIS description

- Exactly the theory we want (for asymptotically free theories where $3N_c > N_f$)!

Phases of SUSY QCD

Murayama 2104.01179
Murayama, Noether, Roy-Varier 2111.09690



We will talk about some of these

Example: $N_f < N_c$

Murayama 2104.01179

- Have Affleck-Dine-Seiberg (ADS) superpotential

$$W = (N_c - N_f)\left(\frac{\Lambda^{3N_c - N_f}}{\det M}\right)^{1/(N_c - N_f)} \qquad M_{ij} = Q_i^\alpha \tilde{Q}_{\alpha j}$$

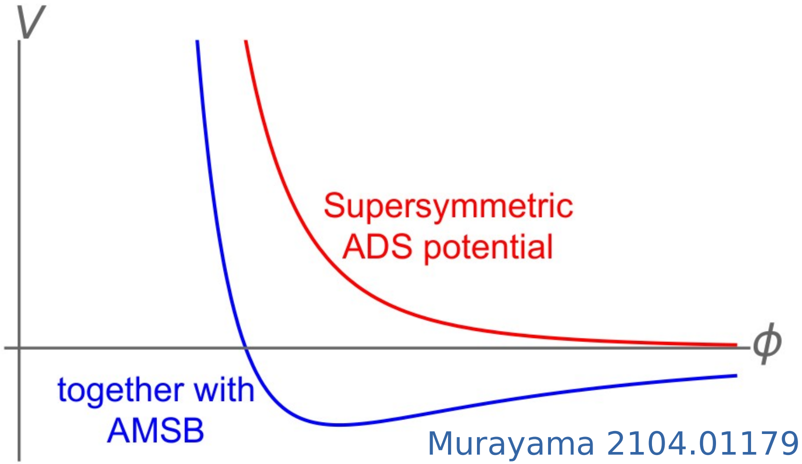
Superfield – contains boson and fermion!

- Non-perturbatively generated by instantons or gluino condensation
- Pushes $M \gg \Lambda$, Higgsed to $SU(N_c - N_f)$ before strong coupling, very different from non-SUSY QCD!

Example: $N_f < N_c$

$$Q = \tilde{Q} = \begin{pmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \\ \hline 0 & \cdots & 0 \\ \vdots & \vdots & \vdots \\ 0 & \cdots & 0 \end{pmatrix} \phi, \qquad M = \phi^2$$

$$V = \left| 2N_f \frac{1}{\phi} \left(\frac{\Lambda^{3N_c - N_f}}{\phi^{2N_f}} \right)^{1/(N_c - N_f)} \right|^2 - (3N_c - N_f) m \left(\frac{\Lambda^{3N_c - N_f}}{\phi^{2N_f}} \right)^{1/(N_c - N_f)} + c.c.$$



Murayama 2104.01179

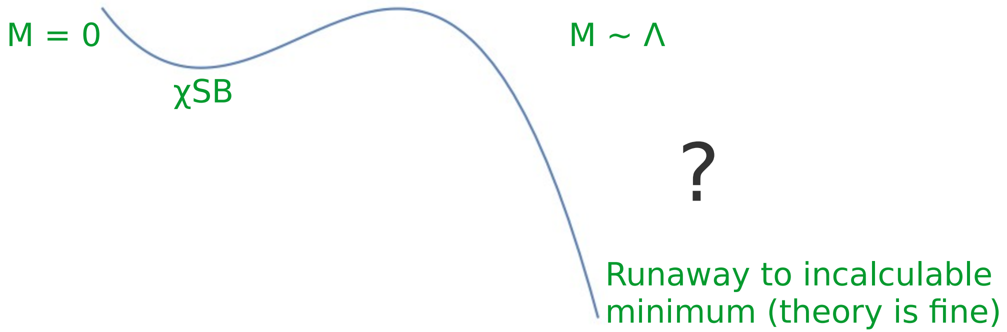
Example: $N_f < N_c$

$$M_{ij} = \Lambda^2 \left(\frac{4N_f(N_c + N_f)}{3N_c - N_f} \frac{\Lambda}{m} \right)^{(N_c - N_f)/N_c} \delta_{ij}$$

- M is a flavor bifundamental → demonstration of χSB!
- Get the massless adjoint pions expected from Goldstone's theorem
- No sign of phase transition as m → Λ, but ultimately incalculable in this limit (this will be typical)

What about those runaways?

- The case of $N_f < N_c$ was pretty tame (only mesons)
- More flavors → new species: baryons, dual quarks
- Can lead to potentials that runaway to Λ
  - If χSB minima are local, still useful since should become global when m → Λ
  - But would be great if they are global



$N_F = N_c+1$ ($N_c > 2$)

Why skip $N_F = N_c$?
Fields VEVs at Λ
→ strongly coupled

$$W = \alpha B M \bar{B} - \beta \det M \qquad B_i = \epsilon_{i j_1 \cdots j_N} Q^{j_1} \cdots Q^{j_N}$$

Λ = 1 throughout

- "s-confinement": meson flavor bifundamental, left and right baryon fundamentals
  - Unlike before, origin lies in moduli space (no χSB)
- Let's look at the potential along this direction

$$B = \begin{pmatrix} b \\ 0 \\ \vdots \end{pmatrix}, \bar{B} = \begin{pmatrix} \bar{b} \\ 0 \\ \vdots \end{pmatrix}, M = \begin{pmatrix} x & & \\ & v & \\ & & \ddots \end{pmatrix}$$

all real numbers, take baryons to be the same

$$V = 2\alpha^2 x^2 b^2 + (\alpha b^2 - \beta v^{N_c})^2 + N\beta^2 x^2 v^{2(N_c-1)} - 2(N_c-2)\beta m x v^{N_c}$$

Don't memorize me!

$N_F = N_c+1$, baryon number breaking

EOMs:

$$b^2 = \frac{\beta}{\alpha} v^{N_c} - 2x^2$$

$$x = \frac{(N_c-2)m}{2\alpha}$$

$$V|_{b,x} = -\frac{(N_c-2)^2\beta}{2\alpha} m^2 v^{N_c}$$

$\Lambda = 1 \rightarrow$ power suppressed

- Tree level runaway!

Luzio, Xu 2202.01239

- But we considered the AMSB loop effects

$$m_M^2 = \frac{(2N_c+3)\alpha(v)^4 m^2}{(16\pi^2)^2} \qquad m_b^2 = \frac{3\alpha(b)^4 m^2}{(16\pi^2)^2}$$

$$V_{2-\text{loop}} = \frac{m^2}{(16\pi^2)^2}\left[N_c(2N_c+3)\alpha(v)^4 v^2 + 6\alpha(b)^4 b^2\right]$$

- Loop suppression smaller than power suppression → no runaway!

$\alpha(v) \sim 1/\log(v)$

$N_F = N_c+1$, baryon number conserving

- Look at minima when b = 0

$$v = x = \left(\frac{(N_c-2)m}{N_c\beta}\right)^{1/(N_c-1)} \qquad V_{\min} = \mathcal{O}(m^{2N_c/(N_c-1)})$$

- The χSB minimum

$$V_{2-\text{loop}} = \frac{(N_c+1)(2N_c+3)\alpha(v)^4}{(16\pi^2)^2}m^2v^2$$

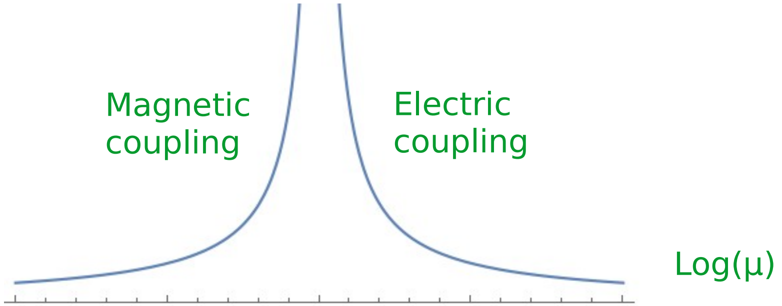
- Same order in m, but loop suppressed → perfect!

Testing our mettle: $N_c+1 < N_F \leq 3N_c/2$

$$W = \lambda \, \mathrm{Tr}\, q_i M_{ij} \bar{q}_j$$

In this context:
Electric = UV
Magnetic = IR

- "Free magnetic phase" of Seiberg duality
- q is a "dual quark" of the emergent gauge group $SU(\tilde{N}_c)$ ($\tilde{N}_c = N_F - N_c$)
- So many flavors versus colors → IR free $\widetilde{b} = 3\widetilde{N}_c - N_F$ Negative!
- Marginal superpotential → no tree level effects



Testing our mettle: $N_c+1 < N_F \leq 3N_c/2$

- So g, λ → 0 in the deep IR
- But beta functions are entwined!
- Look at the "IR attractor"

$$0 = \frac{d}{d\log\mu}\frac{g^2}{\lambda^2}$$

$$m_q^2 = \frac{(-\widetilde{b})g^4}{(16\pi^2)^2}\frac{N_F^2 - 3N_F\widetilde{N}_c - \widetilde{N}_c^2 + 1}{2N_F + \widetilde{N}_c}m^2$$

$$m_M^2 = \frac{(-\widetilde{b})\widetilde{N}_c\lambda^2 g^2}{(16\pi^2)^2}m^2$$

- Positive masses until $N_F \gtrsim 1.43\, N_c$

When the dual quark numerator flips sign!

$N_c+1 < N_F \leq 3N_c/2$, baryonic branch

- Concretely, consider direction where $\tilde{N}_c$ dual-quark VEVs are turned on "baryonic direction"

$$q = \widetilde{B}\begin{pmatrix} 1_{\widetilde{N}_c \times \widetilde{N}_c} \\ 0_{\widetilde{N}_c \times N_c} \end{pmatrix}$$

- Get runaway for $N_F \gtrsim 1.43\ N_c$
- But stable for $N_F \lesssim 1.43\ N_c$
  - Typically upon SUSY breaking, runaways are present throughout the free magnetic phase!

e.g. Abel, Buican, Komargodski 1105.2885

$N_c+1 < N_F \leq 3N_c/2$, mesonic branch

- What about χSB?
- Give M a full rank VEV → mass for dual quarks
- Gauge theory develops confinement scale dependent on scale of dual quark masses

$$\Lambda_L^{3\widetilde{N}_c} = \widetilde{\Lambda}^{3\widetilde{N}_c - N_F} \det M$$

Dual theory Landau pole

- Gluino condensate generates superpotential at Λ_L

$$W = \widetilde{N}_c \Lambda_L^3 = \widetilde{N}_c (\det M)^{1/\widetilde{N}_c}$$

- Tree level AMSB → stable χSB minimum as before

The story with SO

Csaki, AG, Murayama, Telem 2106.10288, 2107.02813

- First analytic demonstration of non-SUSY confinement and χSB
- SO has spinorial Wilson loop → order parameter of confinement
- Monopoles in Seiberg dual of $N_F = N_c - 2$ theory
- Shown to condense with AMSB → confinement via the dual Meissner effect
- Results extend to the range $N_F \leq 3(N_c - 2)/2$

Conclusions

- AMSB a powerful tool to learn about non-SUSY gauge theories:
- Find stable χSB minima for SU
- No runaways for $N_F \lesssim 1.43\, N_c$ ($N_F \neq N_c$)
  - Strong implication minima are global!
- Demonstrate χSB and confinement for SO
- Watch out for the paper!