Reviving chaotic inflation with fermion production: a SUGRA model

Lorenzo Sorbo

UMass Amherst

UC Davis, January 24, 2022

M Roberts, LS 2101.01796

PART I: MOTIVATION

Fact

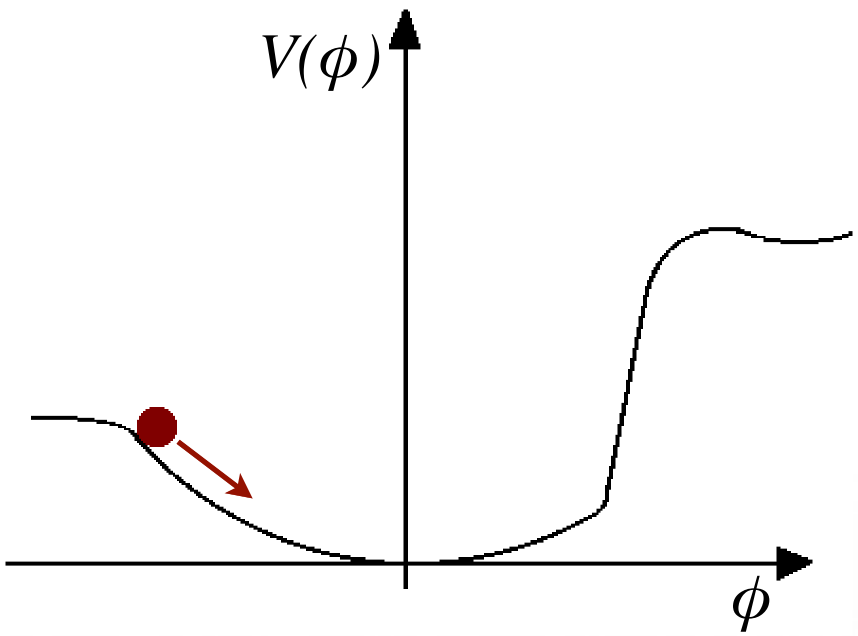
The structure we inhabit (Galaxies, clusters…) originates from the growth of a primordial spectrum of perturbations on the top of a homogeneous, isotropic distribution of matter

What are the observed properties of this primordial spectrum?

The primordial power spectrum

- Is quasi scale invariant, $n_s - 1 = -1/30 \pm 10\%$
- No observed running of spectral index
- Gaussian to 1 part in $\sim 10^4$
- Isocurvature modes are below $\sim 5\%$
- No observed tensor modes, $r < .04$

All properties (+ flatness of spatial slices of Universe)
that are in agreement with simple predictions of **inflation**



INFLATION

- ✓ very early Universe filled by scalar field ϕ, the ***inflaton***, with potential $V(\phi)>0$
- ✓ to give enough inflation, $V(\phi)$ must be *flat*

Inflation requires $|V'(\phi)| << V(\phi)/M_P$, $|V''(\phi)| << V(\phi)/M_P^{\,2}$

A simple (the simplest?) way of obtaining this:
monomial potential, with ϕ large enough

Famous example: quadratic potential *(chaotic inflation)*

Linde 1983

$$V(\phi)=m^2\,\phi^2\,/2$$

Amplitude of perturbations produced during inflation → $m \sim 10^{13}\ GeV$

A MODEL OF NATURAL QUADRATIC INFLATION...

Let me introduce you the 4-form…

Kaloper, LS 09
+Lawrence 11

(Higher rank relative of the electromagnetic field)

$$S_{4form} = -\frac{1}{48}\int F_{\mu\nu\varrho\lambda} F^{\mu\nu\varrho\lambda}\, d^4x$$

$$F_{\mu\nu\varrho\lambda} = \partial_{[\mu} A_{\nu\varrho\lambda]}$$

tensor structure in 4d $\Rightarrow F_{\mu\nu\varrho\lambda} = q(x^\alpha)\, \varepsilon_{\mu\nu\varrho\lambda}$

equations of motion $D^\mu F_{\mu\nu\varrho\lambda} = 0 \Rightarrow q(x^\alpha)$ = constant

(this is why particle physicists do not care about 4-forms:
trivial dynamics)

Sources for the 4-form: membranes

$$\mathcal{S}_{brane} \ni \frac{e}{6} \int d^3\xi \sqrt{\gamma} e^{abc} \partial_a x^\mu \partial_b x^\nu \partial_c x^\lambda A_{\mu\nu\lambda}$$

[$x^\alpha(\xi^a)$=membrane worldvolume]

e = charge per unit membrane surface

$q(x^\alpha)$ jumps by e across a membrane

$q(x^\alpha)$ is locally constant

and

quantized in units of e

Let us couple the 4-form to a pseudoscalar

$$\mathcal{S}_{bulk} = \int d^4x \sqrt{g}\Big(\frac{M_{Pl}^2}{2}R - \frac{1}{2}(\nabla\phi)^2 - \frac{1}{48}F_{\mu\nu\lambda\sigma}^2 + \frac{\mu\phi}{24}\frac{\epsilon^{\mu\nu\lambda\sigma}}{\sqrt{g}}F_{\mu\nu\lambda\sigma} + \ldots\Big)$$

Di Vecchia and Veneziano 1980
Quevedo and Trugenberger 1996
Dvali and Vilenkin 2001
Dvali 2005
Kaloper and LS 2008

Action invariant under shift symmetry:

under $\phi \to \phi + c$, $\mathcal{L} \to \mathcal{L} + c\,\mu\,\varepsilon^{\mu\nu\varrho\lambda} F_{\mu\nu\varrho\lambda}/24$

Let us couple the 4-form to a pseudoscalar

$$S_{bulk} = \int d^4x \sqrt{g}\Big(\frac{M_{Pl}^2}{2}R - \frac{1}{2}(\nabla\phi)^2 - \frac{1}{48}F_{\mu\nu\lambda\sigma}^2 + \frac{\mu\phi}{24}\frac{\epsilon^{\mu\nu\lambda\sigma}}{\sqrt{g}}F_{\mu\nu\lambda\sigma} + \ldots\Big)$$

Di Vecchia and Veneziano 1980
Quevedo and Trugenberger 1996
Dvali and Vilenkin 2001
Dvali 2005
Kaloper and LS 2008

Action invariant under shift symmetry:

under $\phi \to \phi + c, \mathcal{L} \to \mathcal{L} + c\,\mu\,\varepsilon^{\mu\nu\varrho\lambda} F_{\mu\nu\varrho\lambda}/24$

total derivative! $(F=dA)$

Equations of motion (away from branes)

Variation of the action

$$\nabla^{\mu} \left(F_{\mu\nu\varrho\lambda} - \mu\, \varepsilon_{\mu\nu\varrho\lambda}\, \phi\right) = 0$$

$$\nabla^2 \phi + \mu\, \varepsilon^{\mu\nu\varrho\lambda} F_{\mu\nu\varrho\lambda}/24 = 0$$

After simple manipulations

$$F_{\mu\nu\varrho\lambda} = \varepsilon_{\mu\nu\varrho\lambda} \left(q + \mu\, \phi\right)$$

$$\nabla^2 \phi - \mu^2 \left(\phi + q/\mu\right) = 0$$

q = integration constant

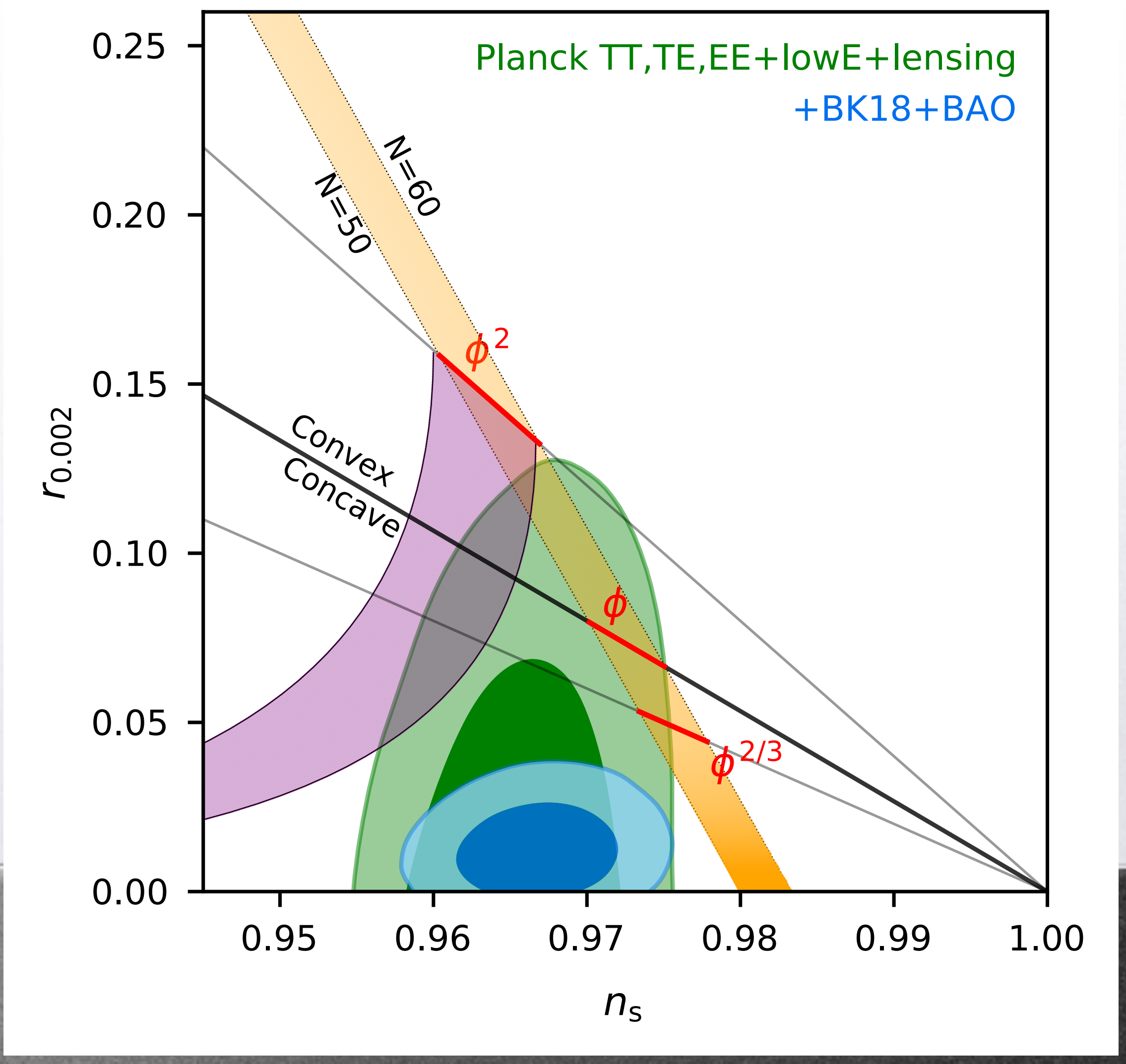
- $(\mu/24)\ \phi\ \varepsilon^{\mu\nu\varrho\lambda} F_{\mu\nu\varrho\lambda}$ is actually a mass term!
- *The theory is massive while retaining the shift symmetry!*
- No contributions $\propto\phi^4, \phi^6, \phi^8$... to potential.
- The symmetry is *broken spontaneously* when a solution is picked
- q changes by e across branes $\Rightarrow$ q is quantized

HOW ABOUT DATA?

Simple is beautiful…

…and quadratic inflation is in agreement with all the observed properties of the power spectrum (including the spectral index), **but is ruled out by non observation of tensors!**

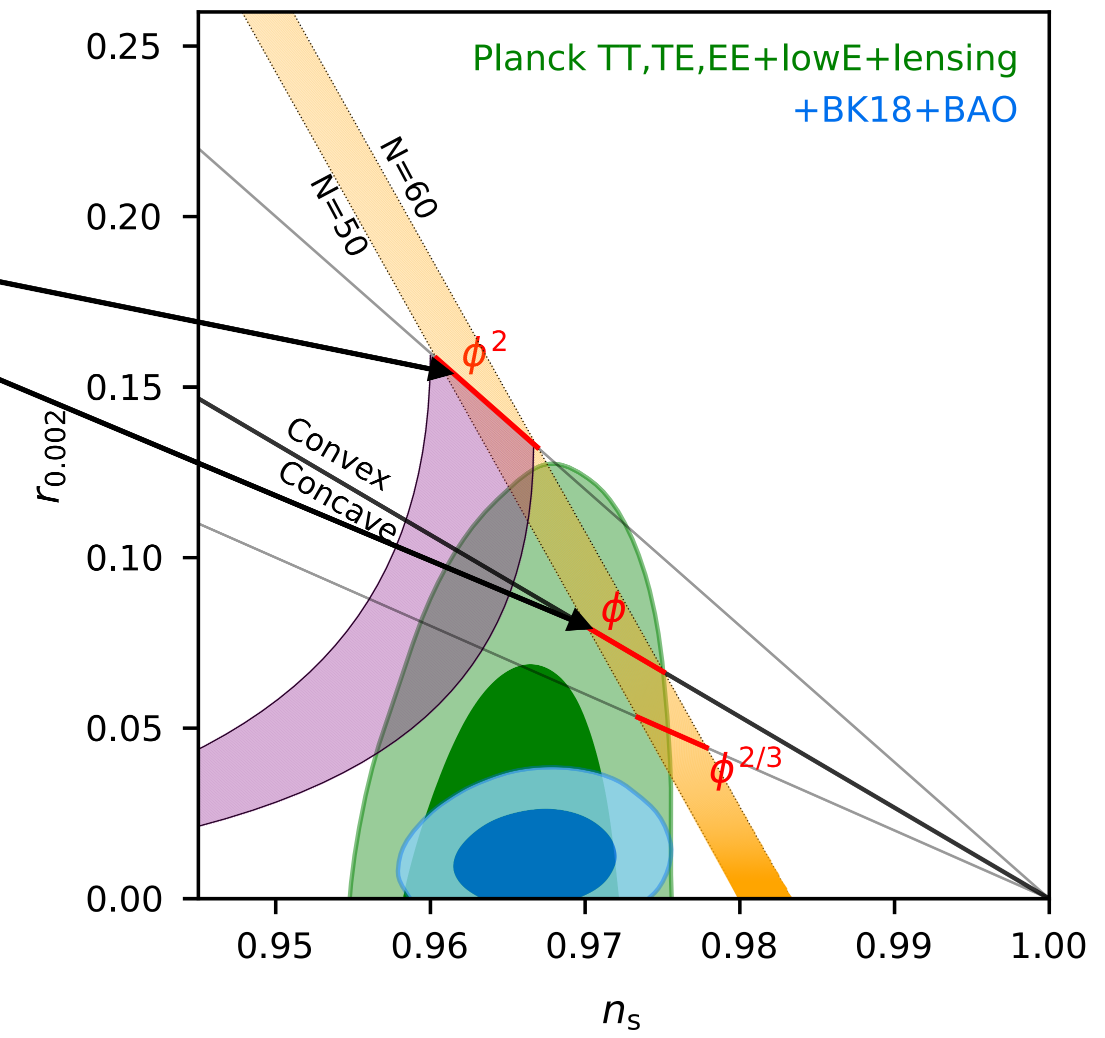
(figure from the BICEP-Keck 2021 paper)



Simple is beautiful…

…and quadratic inflation is in agreement with all the observed properties of the power spectrum (including the spectral index), **but is ruled out by non observation of tensors!**

These lines computed assuming metric perturbations generated by amplification of vacuum fluctuations



(figure from the BICEP-Keck 2021 paper)

Let us look more in detail into the models of quadratic inflation to see how robust this conclusion is...

Disclaimer:
I will make heavy use
of this one
theorist's prejudice

I- Supersymmetry

Even if we do not see SUSY at the *TeV* scale,
it might be there at the $\sim 10^{16}$ *GeV* inflationary scale…

A simple superpotential

$$W = \frac{\mu}{2}\Phi^2 \Longrightarrow V = \frac{\mu^2}{2}|\phi|^2$$

works great…
…but since the inflaton takes values $> M_P$,
must use full supergravity

II- Supergravity

Given superpotential $W(\Phi_i)$ and Kähler potential $K(\Phi_i, \Phi_i{}^*)$

$$V = e^{K/M_P^2}\left[\sum_i \left|\frac{\partial W}{\partial \Phi_i} + \frac{\partial K}{\partial \Phi_i}\frac{W}{M_P^2}\right|^2 - 3\frac{|W|^2}{M_P^2}\right]$$

makes V steep at large ϕ (“η problem”)

typically dominate at large ϕ

only term surviving in global SUSY

II- Supergravity

Given superpotential $W(\Phi_i)$ and Kähler potential $K(\Phi_i, \Phi_i{}^*)$

$$V = e^{K/M_P^2}\left[\sum_i \left|\frac{\partial W}{\partial \Phi_i} + \frac{\partial K}{\partial \Phi_i}\,\frac{W}{M_P^2}\right|^2 - 3\frac{|W|^2}{M_P^2}\right]$$

typically dominate at large ϕ

Problem solved in **stabilizer models**: $W=W(S,\Phi)=S\,f(\Phi)$

where the stabilizer $S=0$ during inflation, thanks to S-dependence of K

III- Shift symmetry

Given superpotential $W(\Phi_i)$ and Kähler potential $K(\Phi_i, \Phi_i^*)$

$$V = e^{K/M_P^2} \left[\sum_i \left| \frac{\partial W}{\partial \Phi_i} + \frac{\partial K}{\partial \Phi_i} \frac{W}{M_P^2} \right|^2 - 3 \frac{|W|^2}{M_P^2} \right]$$

makes V steep at large ϕ
("η problem")

Problem solved in **shift-symmetric models**: $K=K(\Phi+\Phi^*; S, S^*)$

If inflaton=$Im(\Phi)$, then Kähler does not contribute to V

SUGRA models of inflation

More complicated theory,
contains fermions and new interactions

INTERESTING PHENOMENOLOGY?

PART II: PHENOMENOLOGY OF FERMION PRODUCTION IN AXION INFLATION

Our system

Adshead, Pearce, Peloso, LS, Roberts 18

A rolling *pseudoscalar, shift symmetric inflaton* ϕ interacts with a fermion field Y of mass m_ψ via

$$\bar{Y}\left[i\,\gamma^\mu\,\partial_\mu - m_\psi a - \frac{1}{f}\gamma^\mu\,\gamma^5\,\partial_\mu\phi\right]Y$$

(f=constant with dimensions of a mass)

A useful field redefinition

$$Y = \mathrm{e}^{-i\gamma^5 \phi/f}\, \psi$$

allows to write the fermion Lagrangian as

$$\bar{\psi} \left\{ i\,\gamma^\mu\, \partial_\mu - m_\psi a \left[\cos\left(\frac{2\phi}{f}\right) - i\gamma^5 \sin\left(\frac{2\phi}{f}\right) \right] \right\} \psi$$

oscillating effective mass with amplitude m_ψ and frequency $2\dot{\phi}/f$

resonant production of fermions up to momenta $\sim\dot{\phi}/f$

Time-dependent ϕ → fermion generation

Assume $d\phi/dt$=constant,

$$\psi = \int \frac{\mathrm{d}^3 k}{(2\pi)^{\frac{3}{2}}} e^{i\mathbf{k}\cdot\mathbf{x}} \sum_{r=\pm} \left[U_r(\mathbf{k},\,\tau)\, a_r(\mathbf{k}) + V_r(-\mathbf{k},\,\tau)\, b_r^\dagger(-\mathbf{k}) \right]$$

$$U_r(\mathbf{k},\,\tau) = \frac{1}{\sqrt{2}} \begin{pmatrix} \chi_r(\mathbf{k})\, u_r(x) \\ r\chi_r(\mathbf{k})\, v_r(x) \end{pmatrix}, \quad V_r(\mathbf{k}) = C\, \bar{U}_r(\mathbf{k})^T, \qquad C = i\gamma^0\gamma^2 = \begin{pmatrix} 0 & i\sigma_2 \\ i\sigma_2 & 0 \end{pmatrix}$$

$$\chi_r(\mathbf{k}) \equiv \frac{(k + r\,\sigma\cdot\mathbf{k})}{\sqrt{2\,k\,(k+k_3)}}\,\bar{\chi}_r, \qquad \bar{\chi}_+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \bar{\chi}_- = \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$

$$u_r(x) = \frac{1}{\sqrt{2x}} \left[\mathrm{e}^{ir\hat{\phi}(x)}\, s_r(x) + \mathrm{e}^{-ir\hat{\phi}(x)}\, d_r(x) \right]$$

$$v_r(x) = \frac{1}{\sqrt{2x}} \left[\mathrm{e}^{ir\hat{\phi}(x)}\, s_r(x) - \mathrm{e}^{-ir\hat{\phi}(x)}\, d_r(x) \right]$$

$$s_r(x) = \mathrm{e}^{-\pi r\xi}\, W_{\frac{1}{2}+2ir\xi,\, i\sqrt{\tilde{\mu}^2+4\xi^2}}(-2ix), \qquad d_r(x) = -i\,\tilde{\mu}\,\mathrm{e}^{-\pi r\xi}\, W_{-\frac{1}{2}+2ir\xi,\, i\sqrt{\tilde{\mu}^2+4\xi^2}}(-2ix)$$

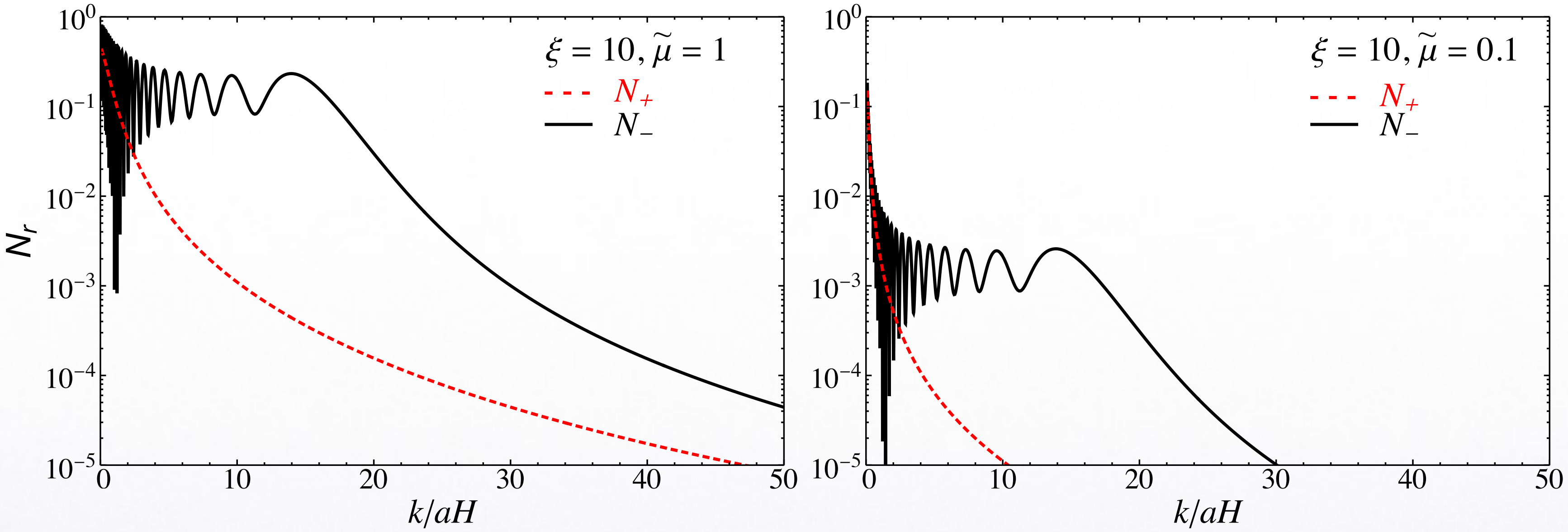
$$\hat{\phi}(x) \equiv \frac{\phi_0}{f} = -2\xi \log(x/x_{\mathrm{in}})$$

$$x \equiv -k\tau\,, \quad x_{\mathrm{in}} \equiv -k\tau_{\mathrm{in}}$$

$$\tilde{\mu} \equiv \frac{m_\psi}{H}, \qquad \xi \equiv \frac{\dot{\phi}_0}{2fH}$$

Time-dependent ϕ → fermion generation

Occupation numbers of fermions



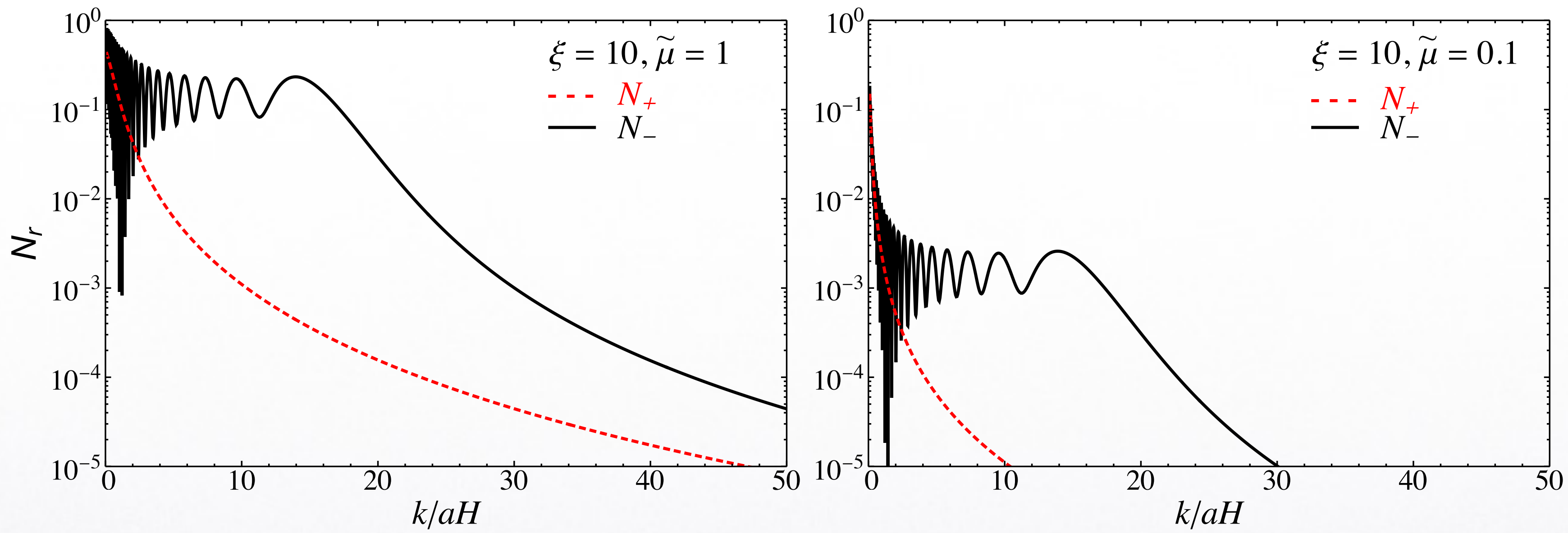
Different helicities ⇒ different occupation #s (parity violation)

(can be used for leptogenesis) Adshead and Sfakianakis 15

For $m_\psi \rightarrow 0$, neither helicity is produced

Time-dependent ϕ → fermion generation

Scalings, for $\xi \gg 1,\ \tilde{\mu} \lesssim 1$



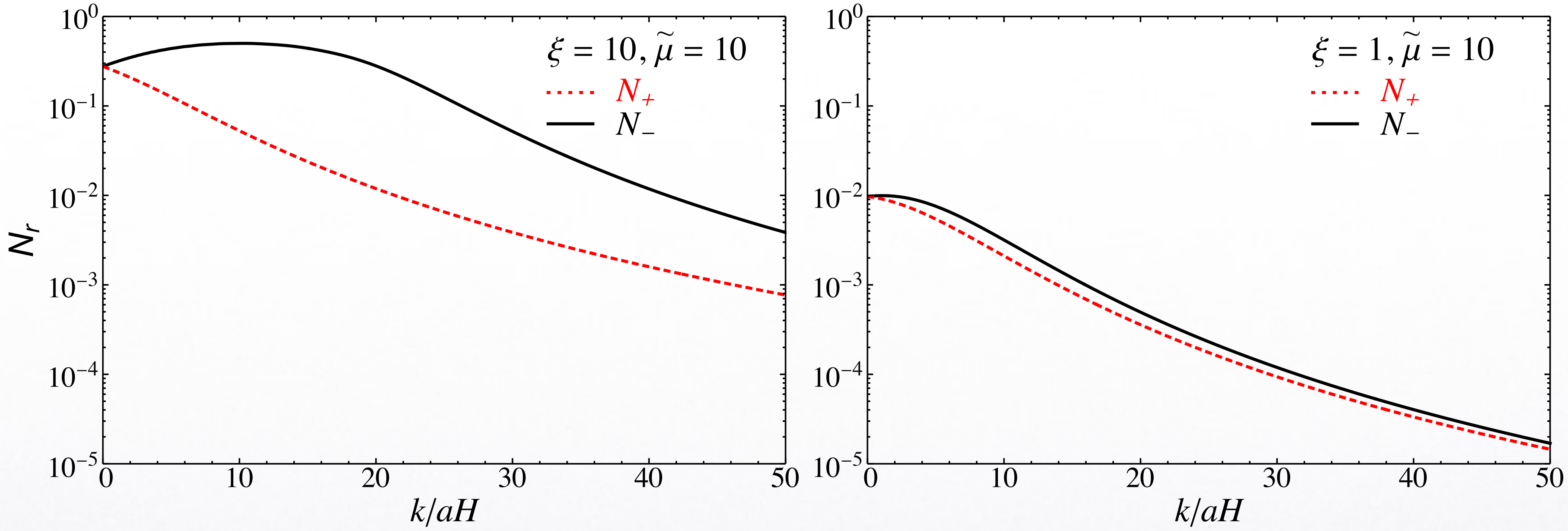
+ helicity: $N \sim 1$ for $k < am_\psi$, $N \sim 0$ for $k > am_\psi$

- helicity: $N \sim 1$ for $k < am_\psi$, $N \sim \mu^2/\xi$ for $am_\psi < k < 2aH\xi$, $N \sim 0$ for $k > 2aH\xi$

Total number density of -helicity $\sim \tilde{\mu}^2 \xi^2 H^3$, can be $>> H^3$!

Time-dependent ϕ → fermion generation

Occupation numbers of fermions



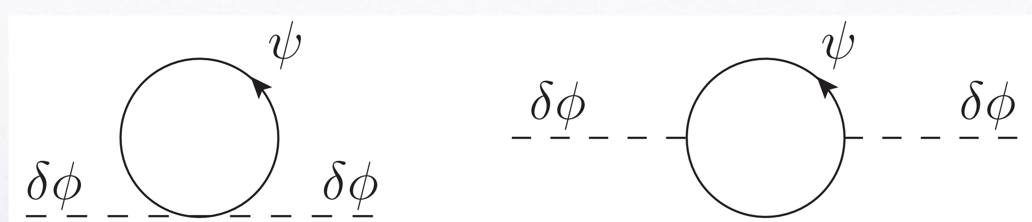
Even heavy $m_\psi >> H$ fermions copiously produced!

Effects of these fermions on CMB power spectrum

Using in-in formalism

$$\delta P_\zeta(\tau, k)\Big|_{-k\tau \ll 1} = \frac{k^3}{2\pi^2}\frac{H^2}{\dot{\phi}_0^2}\sum_{N=1}^{\infty}(-i)^N \int^{\tau} \mathrm{d}\tau_1 \ldots \int^{\tau_{N-1}} \mathrm{d}\tau_N \times \left\langle \left[\left[\cdots\left[\delta\phi^{(0)}(\tau, \mathbf{k})\,\delta\phi^{(0)}\left(\tau, \mathbf{k}'\right), H_{\rm int}(\tau_1)\right], \cdots\right], H_{\rm int}(\tau_N)\right]\right\rangle'$$

two leading order contributions



dominant,
and can be computed analytically!

Effects of these fermions on CMB power spectrum

The full result of the first diagram

$$\frac{\delta P_\zeta}{P^0} \approx \frac{4mH}{3\pi^2 f^2} \ln(x) \int \mathrm{d}y\, y \sum_r \mathrm{Re}[s_r(y) d_r^*(y)]$$

$$\begin{aligned}
\sum_r \int y \Re\left(s_r(y) d_r^*(y)\right)\, \mathrm{d}y = \tilde{\mu}\Bigg[&\frac{1}{2}\left(2\Lambda^2 + \frac{1}{4}\left(-8(\log(2\Lambda)+\gamma_E)\left(\tilde{\mu}^2 - 8\xi^2 + 1\right) + \tilde{\mu}^4 - 7\tilde{\mu}^2 + 12\right)\right) \\
&+\frac{1}{4}\left(\tilde{\mu}^2 - 8\xi - 6i\xi + 1\right)\Big[H_{-i\left(2\xi+\sqrt{\tilde{\mu}^2+4\xi^2}\right)}\left(\sinh(4\pi\xi)\operatorname{csch}\left(2\pi\sqrt{\tilde{\mu}^2+4\xi^2}\right)+1\right) \\
&\quad +H_{i\left(\sqrt{\tilde{\mu}^2+4\xi^2}-2\xi\right)}\left(1-\sinh(4\pi\xi)\operatorname{csch}\left(2\pi\sqrt{\tilde{\mu}^2+4\xi^2}\right)\right)\Big] \\
&+\frac{1}{4}\left(\tilde{\mu}^2 - 8\xi^2 + 6i\xi + 1\right)\Big[H_{i\left(2\xi+\sqrt{\tilde{\mu}^2+4\xi^2}\right)}\left(\sinh(4\pi\xi)\operatorname{csch}\left(2\pi\sqrt{\tilde{\mu}^2+4\xi^2}\right)+1\right) \\
&\quad +H_{-i\left(\sqrt{\tilde{\mu}^2+4\xi^2}-2\xi\right)}\left(1-\sinh(4\pi\xi)\operatorname{csch}\left(2\pi\sqrt{\tilde{\mu}^2+4\xi^2}\right)\right)\Big] \\
&+6\xi\sqrt{\tilde{\mu}^2+4\xi^2}\sinh(4\pi\xi)\operatorname{csch}\left(2\pi\sqrt{\tilde{\mu}^2+4\xi^2}\right) - \frac{\tilde{\mu}^4}{8} + \frac{11\tilde{\mu}^2}{8} - 12\xi^2\Bigg],
\end{aligned} \tag{D.7}$$

Effects of these fermions on CMB power spectrum

In the limit $\xi \gg 1$, $\tilde{\mu} \lesssim 1$

$$\delta\mathcal{P}_\zeta(k)\Big|_{\text{end of inflation}} \simeq \mathcal{P}_\zeta^{(0)}\,\frac{32\,m_\psi^2\,\xi^2\,\log\xi}{3\pi^2\,f^2}\,\log\left(H/k\right)$$

Effects of these fermions on CMB power spectrum

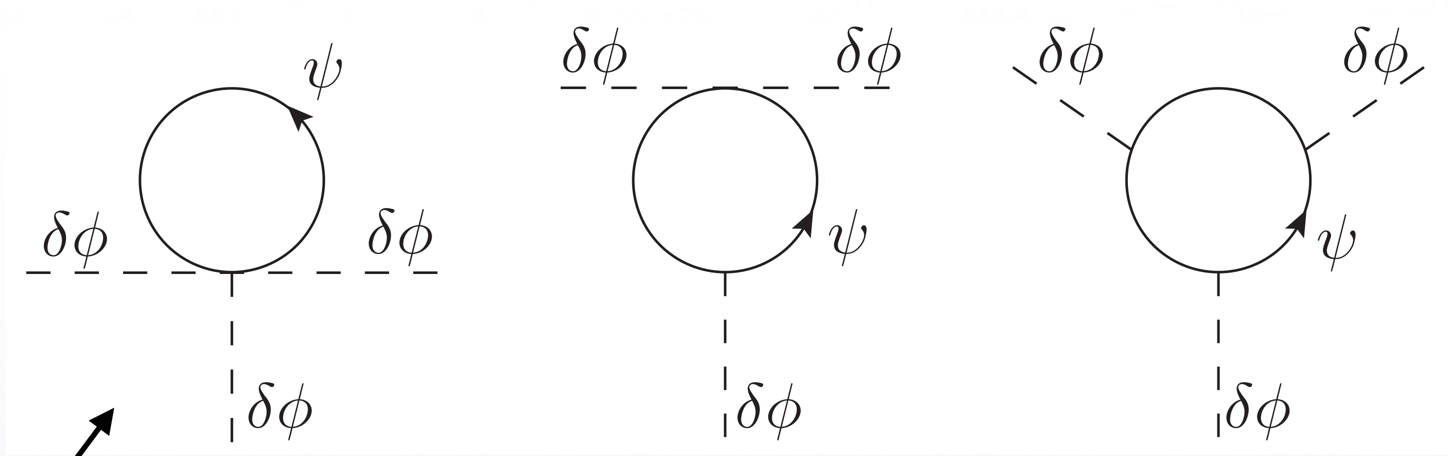
In the limit $\xi \gg 1$, $\tilde{\mu} \lesssim 1$

$$\delta\mathcal{P}_\zeta(k)\Big|_{\text{end of inflation}} \simeq \mathcal{P}_\zeta^{(0)} \, \frac{32\, m_\psi^2\, \xi^2 \, \log\xi}{3\pi^2 \, f^2} \log\left(H/k\right)$$

$$n_s - 1 = -3\epsilon - \frac{1}{N} + \frac{2\epsilon - \eta}{\log\xi}$$

Effects of these fermions on CMB bispectrum

Three leading order contributions

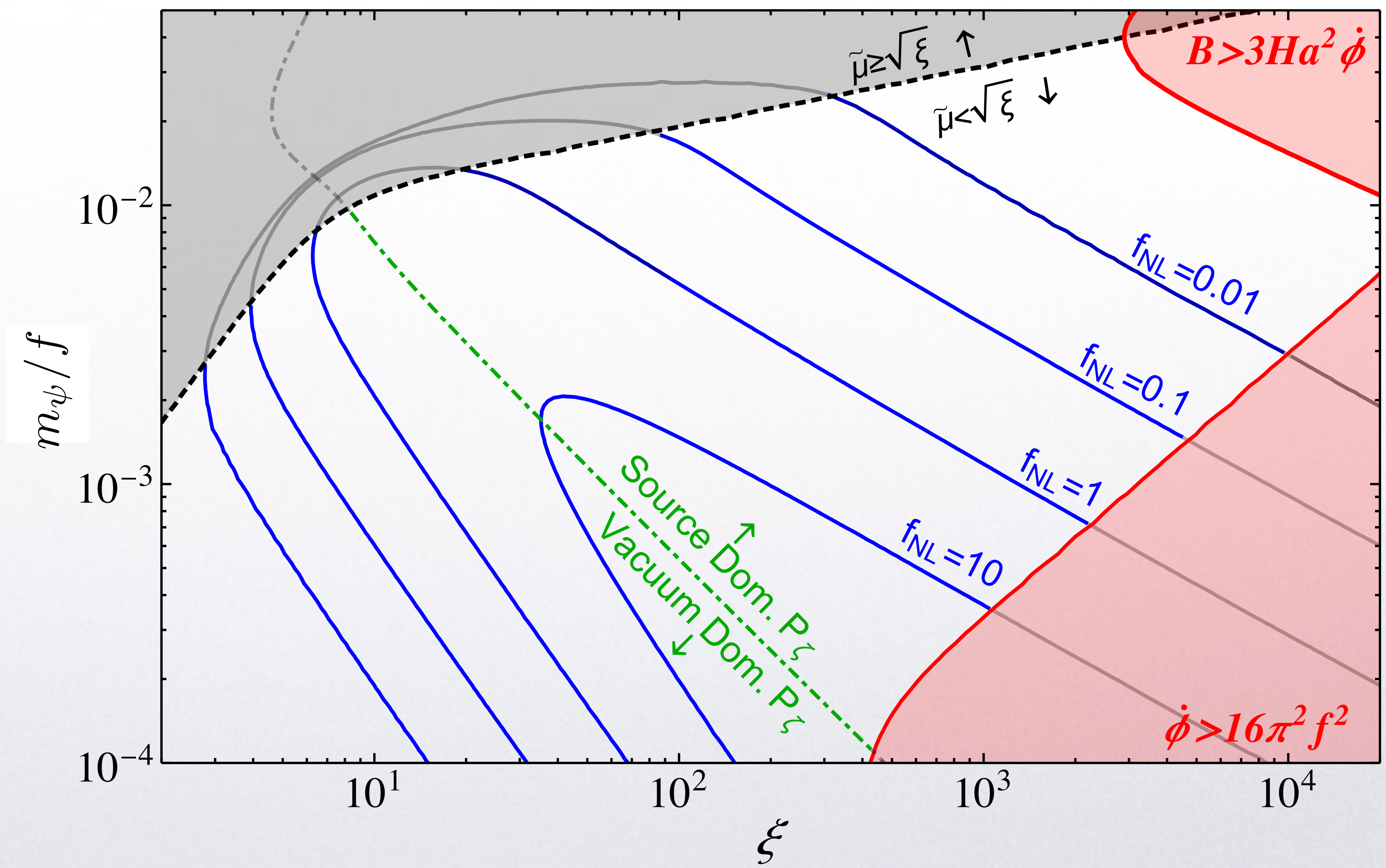


dominant,
and can be computed analytically!

Effects of these fermions on CMB bispectrum

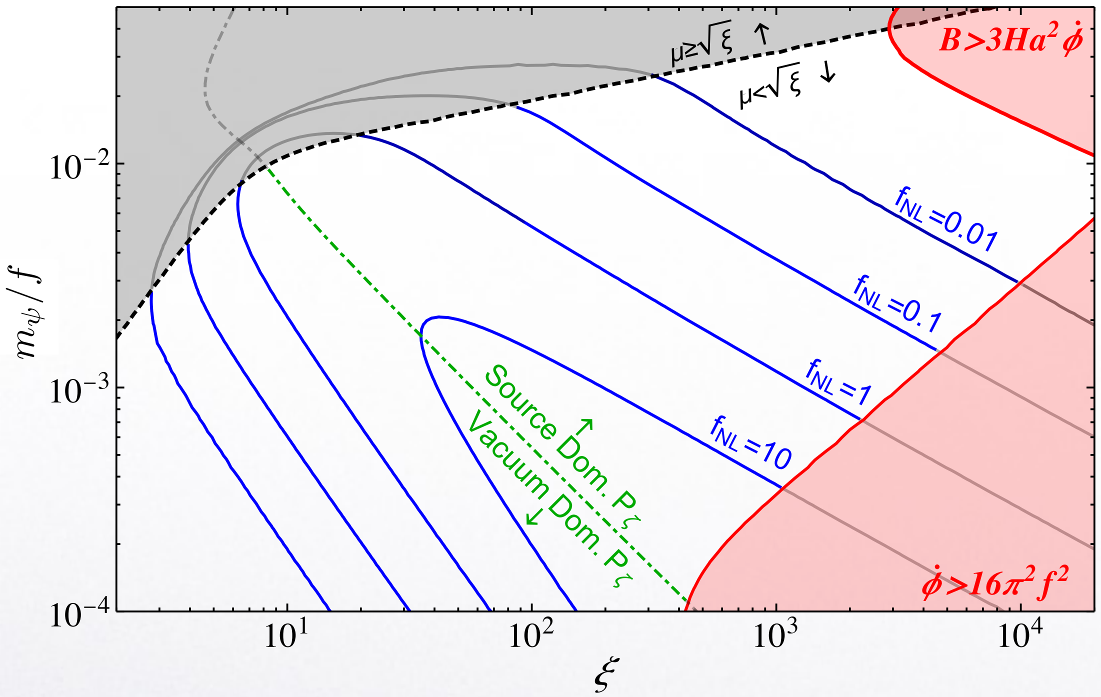
Since source of perturbations is sub horizon, expect equilateral bispectrum

$$f_{NL}^{eq} \simeq \frac{\frac{80\, H^2 \tilde{\mu}^2 \xi^3}{3\pi f^2} \log(H/k)}{\left(1 + \frac{32\, H^2\, \tilde{\mu}^2\, \xi^2\, \log \xi}{3\, \pi^2\, f^2} \log(H/k)\right)^2}$$



Effects of these fermions on CMB bispectrum

Since source of perturbations is sub horizon, expect equilateral bispectrum



Main message: can have spectrum dominated by sourced component and small f_{NL}

(Planck constrains $f_{NL}^{eq}\lesssim 40$)

Surprising: source quadratic in gaussian field ψ, so nongaussian

…but many many modes contribute→central limit→gaussian

So...

The system has a regime where Planck measures

$$\frac{H^2\, m_\psi^2}{f^4}$$

instead of the usual

$$\frac{H^4}{\dot{\phi}^2}$$

How about the tensors?

Computed them in Adshead, Pearce, Peloso, LS, Roberts 19:

The component sourced by the fermions
always subdominant with respect
to the standard one

so we keep the standard expression

$$\mathcal{P}_t = \frac{2}{\pi^2}\,\frac{H^2}{M_P^2}$$

PART III: REVIVING CHAOTIC INFLATION

M Roberts, LS 2101.01796

General study: equations of motion for fermions in models of inflation with stabilizer

General $N=1$, $d=4$ SUGRA with two superfields S, Φ with $W=S\,f(\Phi)$, $K=K(\Phi, \Phi^*)+g(SS^*)$

Stabilizer condition $S=0 \Longrightarrow W=0$

$$V = e^{K/M_P^2}\left|f(\phi)\right|^2$$

General study: equations of motion for fermions in models of inflation with stabilizer

Two matter fermions (one is goldstino, can be set to zero in unitary gauge)+helicity-*1/2* part of gravitino.
So two coupled fermions θ and Υ in the end

$$\mathcal{L}_f = -\frac{\alpha a^3}{4k^2}\bar{\theta}\Big[\Big(\gamma^0\hat{\partial}_0 + i\gamma^i k_i \hat{A} + \gamma^0\hat{B}\Big)\theta - \frac{4k^2}{a\alpha}\gamma^0\Upsilon\Big] +$$

$$-\frac{4a}{\alpha\Delta^2}\bar{\Upsilon}\Big[\Big(\gamma^0\hat{\partial}_0 - i\gamma^i k_i \hat{A} + \gamma^0\hat{B}^\dagger + a\gamma^0\hat{F} + 2\dot{a}\gamma^0 + \frac{a}{M_P^2}\gamma^0\mathbf{m}\gamma^0\Big)\Upsilon + \frac{1}{4}a\alpha\Delta^2\gamma^0\theta\Big],$$

(helicity-3/2 gravitino is decoupled and irrelevant here)

formulae from Kallosh, Kofman, Linde and Van Proeyen 00

Does not look simple…

$$m = \mathrm{e}^{\frac{K}{2M_P^2}} W\,,$$

$$\mathbf{m} = \Re\{m\} - i\,\Im\{m\}\,\gamma^5\,,$$

$$m^i = \left(\partial^i + \frac{1}{2M_P^2}\partial^i K\right) m\,,$$

$$m^{ij} = \left(\partial^i + \frac{1}{2M_P^2}\partial^i K\right) m^j - \Gamma^{ij}_k\, m^k\,,$$

$$\hat{\partial}_0 = \partial_0 - \frac{i}{2} A_0^B \gamma^5\,,$$

$$A_0^B = \frac{i}{2M_P^2}(\phi'^i \partial_i K - \phi'_i \partial^i K)\,,$$

$$H^2 = \frac{1}{3M_P^2}\left(|\dot{\phi}|^2 + V\right)\,,$$

$$V = m_i (g^{-1})^i_j m^j - 3\frac{|m|^2}{M_P^2}\,,$$

$$|\dot{\phi}|^2 \equiv g^i_j\,\dot{\phi}_i\,\dot{\phi}^j\,,$$

$$\alpha = 3M_P^2\left(H^2 + \frac{|m|^2}{M_P^4}\right),$$

$$\alpha_1 = -3M_P^2\left(H^2 + \frac{2}{3}\dot{H} + \frac{|m|^2}{M_P^4}\right),$$

$$\alpha_2 = 2\dot{\mathbf{m}}^\dagger\,,$$

$$\hat{A} = \frac{1}{\alpha}(\alpha_1 - \gamma^0 \alpha_2)\,,$$

$$\hat{B} = -\frac{3}{2}\dot{a}\hat{A} + \frac{a}{2M_P^2}\mathbf{m}\gamma^0(1 + 3\hat{A})\,,$$

$$\xi_i = m_i - \gamma^0 g^j_i \dot{\phi}_j\,,$$

$$\Delta = 2\frac{\sqrt{V}|\dot{\phi}|}{\alpha}\,,$$

But it is!

Fermions in models of inflation with stabilizer
Making a number of field redefinitions…

$$\mathcal{L} = (\bar{\chi}_1, \bar{\chi}_2) \left[-\gamma^0 \partial_0 + i\gamma \cdot k + a \begin{pmatrix} M_1 + iM_2\gamma^5 & 0 \\ 0 & -M_1 - iM_2\gamma^5 \end{pmatrix} \right] \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix}$$

…where M_1 and M_2 are reasonably complicated functions of the background fields

($M_2=0$ for $Im[\Phi]=0$)

Specializing to our system:
quadratic inflaton plus small instanton corrections

$$f(\Phi) = \mu\Phi + \hat{\Lambda}^2 \mathrm{e}^{-\frac{\sqrt{2}\Phi}{F}}\,, \qquad \mathcal{K}(\Phi,\,\bar{\Phi}) = \frac{1}{2}(\Phi + \bar{\Phi})^2$$

gives dominant quadratic potential

small "instanton" correction
(we'll require these to be negligible in V!)

no η problem for inflaton in imaginary part of Φ

To fix ideas…

$\mu = \mathcal{O}(10^{13})\,\mathrm{GeV}$ $\qquad \hat{\Lambda} = \mathcal{O}(10^{14})\,\mathrm{GeV}$ $\qquad F = \mathcal{O}(10^{15})\,\mathrm{GeV}$

Specializing to our system: quadratic inflaton plus small instanton corrections

Full scalar potential

$$V = \mathrm{e}^{\frac{\rho^2}{M_P^2}} \left[\frac{\mu^2}{2}(\rho^2 + \varphi^2) + \sqrt{2}\mu\,\hat{\Lambda}^2 \mathrm{e}^{-\frac{\rho}{F}} \left(\rho \cos\frac{\varphi}{F} - \varphi \sin\frac{\varphi}{F} \right) + \hat{\Lambda}^4 \mathrm{e}^{-\frac{2\rho}{F}} \right]$$

$$\phi = \frac{1}{\sqrt{2}}(\rho + i\varphi)$$

…but $\rho << F$, so we are left with

$$V \simeq \frac{\mu^2}{2}\varphi^2 - \sqrt{2}\mu\hat{\Lambda}^2 \varphi \sin\frac{\varphi}{F}$$

quadratic plus (small) wiggles

Specializing to our system: quadratic inflation plus small instanton corrections

And for the fermions, remind that we had

$$\mathcal{L} = (\bar{\chi}_1, \bar{\chi}_2) \left[-\gamma^0 \partial_0 + i\gamma \cdot k + a \begin{pmatrix} M_1 + iM_2\gamma^5 & 0 \\ 0 & -M_1 - iM_2\gamma^5 \end{pmatrix} \right] \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix}$$

in this regime

$$M_1 + iM_2\gamma^5 \simeq \mu\left(\frac{2M_P^2}{3\varphi^2} - \sqrt{2}\frac{\hat{\Lambda}^2}{\mu F}\left(\cos(\varphi/F) + i\sin(\varphi/F)\gamma^5 \right) \right)$$

negligible

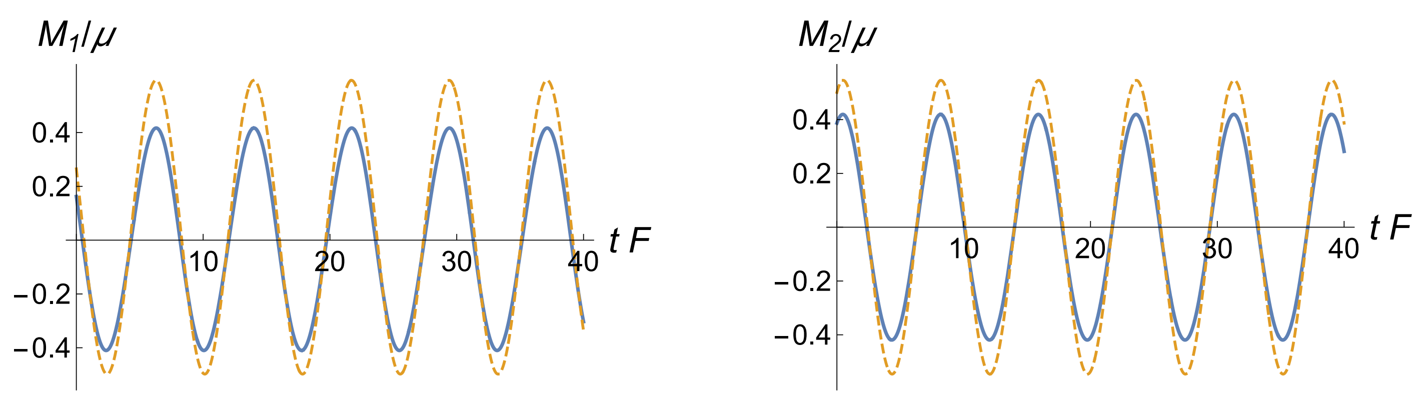


FIG. 2: Results of exact numerical integration (solid, blue) and analytical approximations, eq. (54), (dashed, orange) for the quantities $M_1(t)$ and $M_2(t)$. The parameters used for these plots are $\mu = 5 \times 10^{-6}\, M_P$, $F = 5 \times 10^{-4}\, M_P$. At these times $\varphi \simeq 13.9\, M_P$.

Specializing to our system: quadratic inflaton plus small instanton corrections

And for the fermions, remind that we had

$$\mathcal{L} = (\bar{\chi}_1, \bar{\chi}_2) \left[-\gamma^0 \partial_0 + i\gamma \cdot k + a \begin{pmatrix} M_1 + iM_2\gamma^5 & 0 \\ 0 & -M_1 - iM_2\gamma^5 \end{pmatrix} \right] \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix}$$

in this regime

$$M_1 + iM_2\gamma^5 \simeq \mu \Big(\frac{2M_P^2}{3\varphi^2} - \sqrt{2}\,\frac{\hat{\Lambda}^2}{\mu F} \big(\cos(\varphi/F) + i \sin(\varphi/F)\gamma^5 \big) \Big)$$

negligible

…equivalent to the system discussed in part II!

$$\bar{\psi} \left\{ i\,\gamma^\mu\,\partial_\mu - m_\psi a \left[\cos\left(\frac{2\phi}{f}\right) - i\gamma^5 \sin\left(\frac{2\phi}{f}\right) \right] \right\} \psi$$

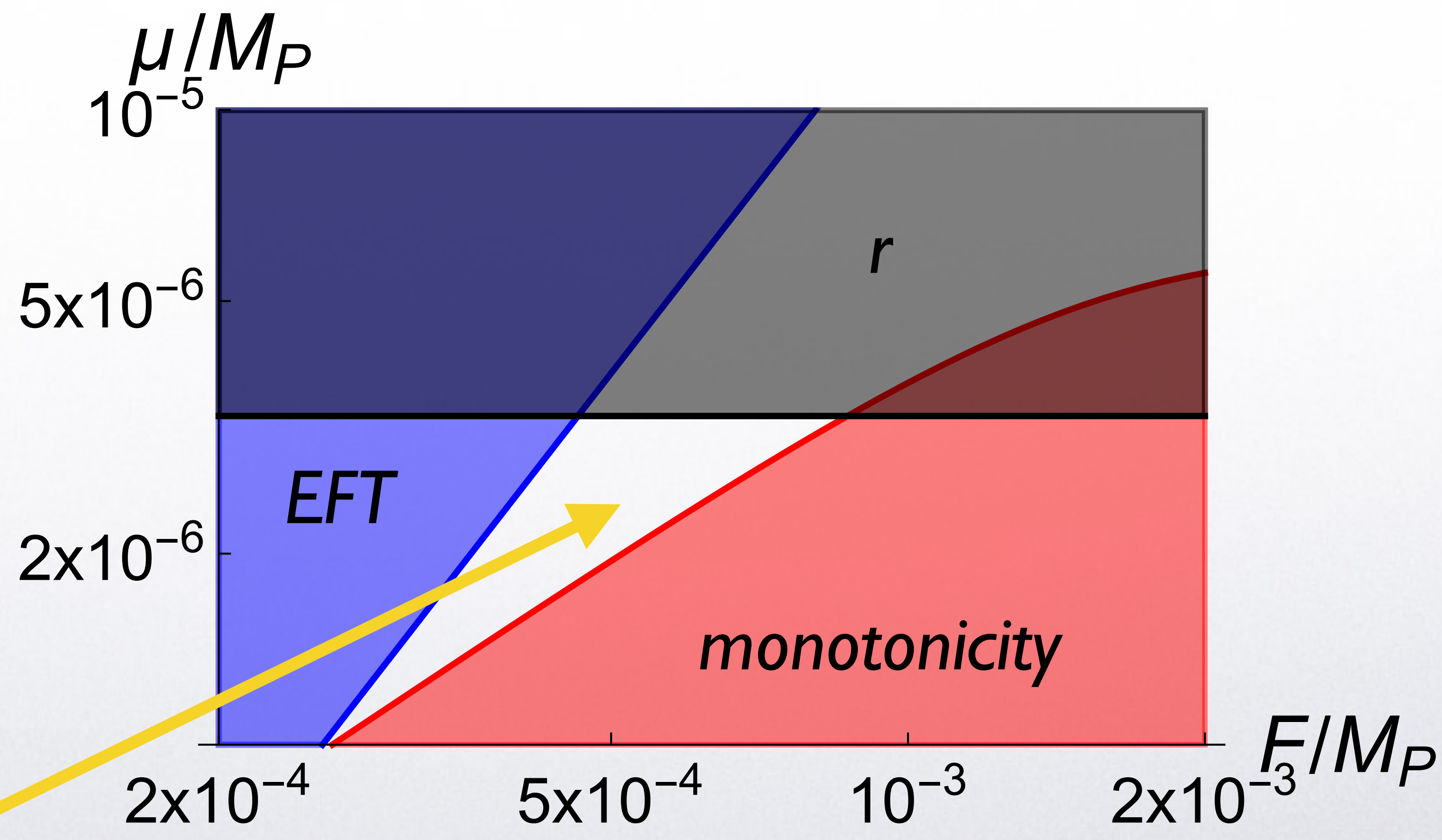
Importing the results from part II…

Imposing constraints:

- Monotonicity of potential
- Energies below $4\pi F$ cutoff
- $r<.04$
- Negligible backreaction of fermions on background
- No oscillations in scalar power spectrum
- No nongaussianities
- Scalar spectral index

Importing the results from part II…

Three parameters.
Eliminate $\hat{\Lambda}$ with normalization of scalar spectrum



ALLOWED!

Summing up

- Natural generalization of quadratic potential to sugra,
  with inclusion of instantons, in agreement with all existing data
- Lower bound on $r \gtrsim .004$, to be probed in next O(10) years
- Analysis easily generalizable to monomial potentials (monodromy)
- Oscillations in scalar power spectrum in monodromy models:
  do they survive in sugra models?

Conclusion

Monomial inflation is beautiful…
…but in its simplest form is ruled out by non observation of tensors

“Natural” embedding in supergravity can revive it…
…at least for a few years