Constraining BSM custodial violations beyond the oblique framework

UC Davis Joint Theory Seminar

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University of Oregon

arXiv: 2009.10725

with Graham D. Kribs, Adam Martin, and Tom Tong

$$\mathcal{L}_{\mathrm{UV}} = \mathcal{L}_{\mathrm{SM}} + \mathcal{L}_{\mathrm{BSM}}$$

custodial symmetric?

$$\alpha T \equiv \frac{\Pi_{WW}(0) - \Pi_{33}(0)}{m_W^2} \sim 0$$

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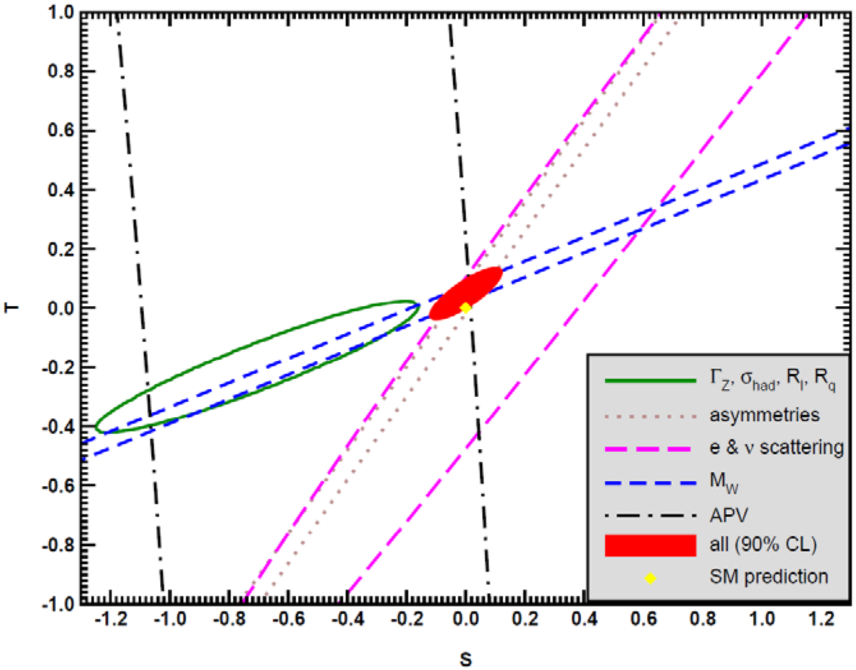

Particle Data Group Collaboration, P. Zyla et al., "Review of Particle Physics," *PETP* 2020 (2020) no. 8, 083C01.

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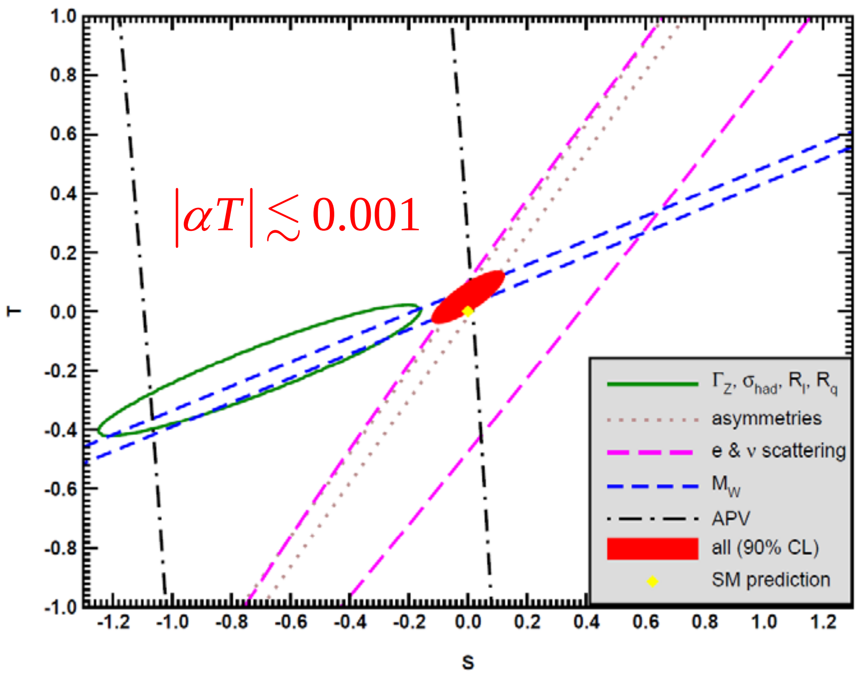

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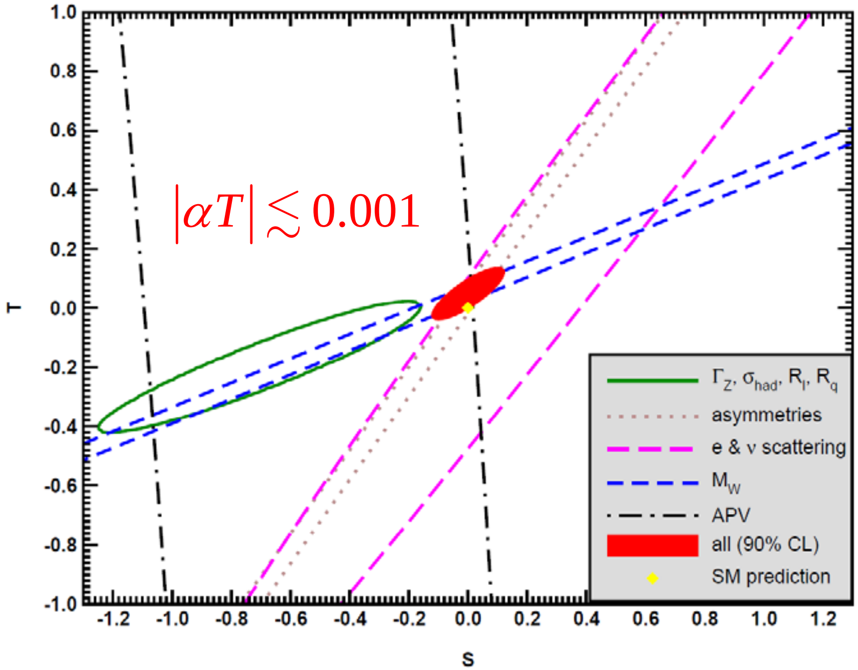

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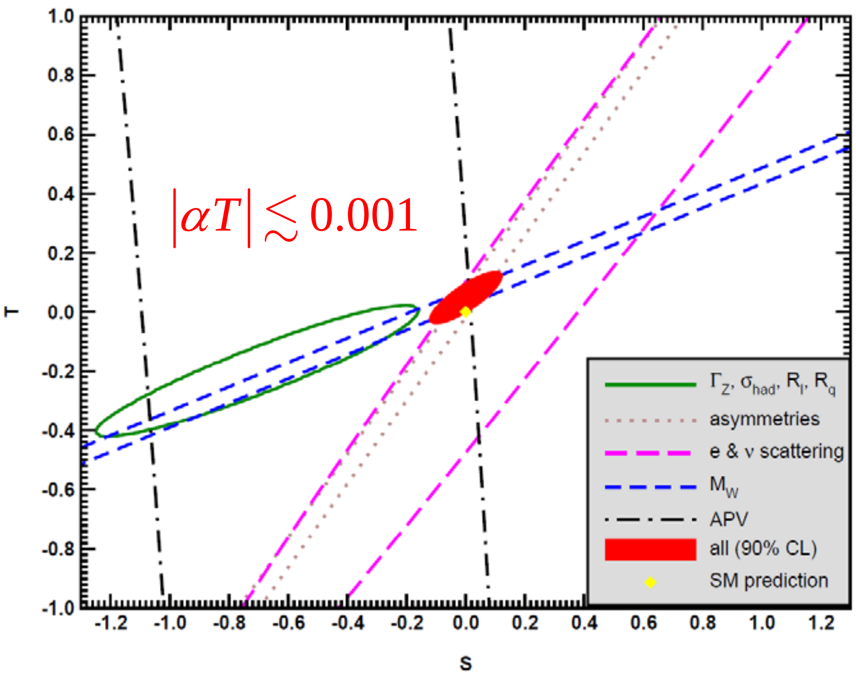

31 *10. Electroweak Model and Constraints on New Physics*

The dominant effect of many extensions of the SM can be described by the ρ_0 parameter,

$$\rho_0 \equiv \frac{M_W^2}{M_Z^2\, \hat{c}_Z^2\, \hat{\rho}}\,, \qquad (10.66)$$

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$$\rho_0 = 1.00038 \pm 0.00020\,, \qquad (10.67a)$$

$$\alpha_s(M_Z) = 0.1188 \ \pm 0.0017\,, \qquad (10.67b)$$

A heavy non-degenerate multiplet of fermions or scalars contributes positively to T as

$$\rho_0 - 1 = \frac{1}{1 - \hat{\alpha}(M_Z)T} - 1 \approx \hat{\alpha}(M_Z)T\,, \qquad (10.74)$$

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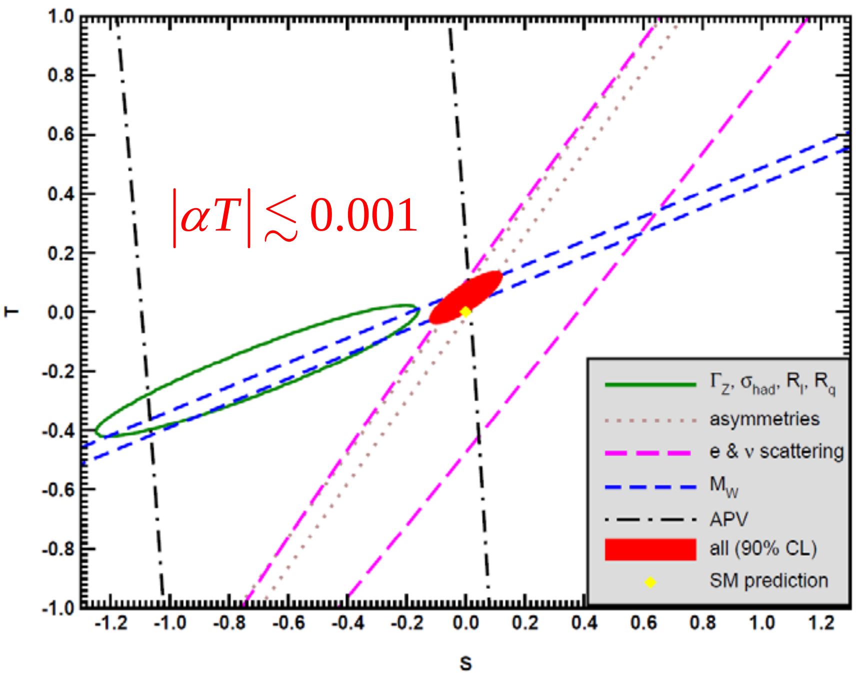

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$$\hat{\rho}_{\text{Veltman}} = 1 + \frac{\alpha}{c_{2\theta}}\left(-\frac{1}{2}S + c_\theta^2 T + \frac{c_{2\theta}}{4s_\theta^2}U\right)$$

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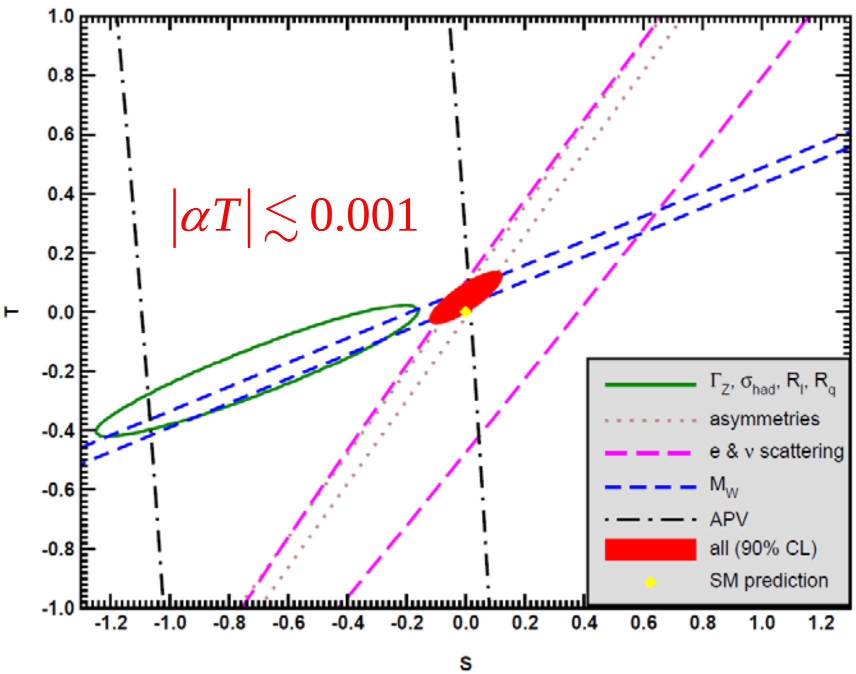

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$$\hat{\rho}_*(0) \sim \frac{\mathcal{M}_{\text{NC}}(0)}{\mathcal{M}_{\text{CC}}(0)}$$

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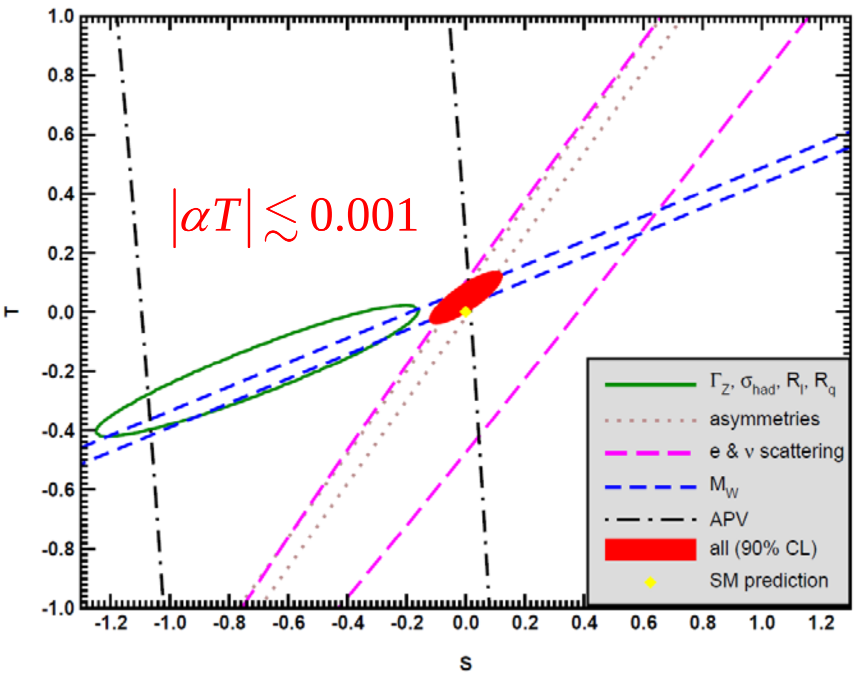

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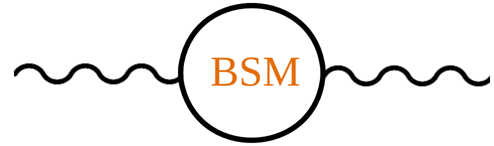
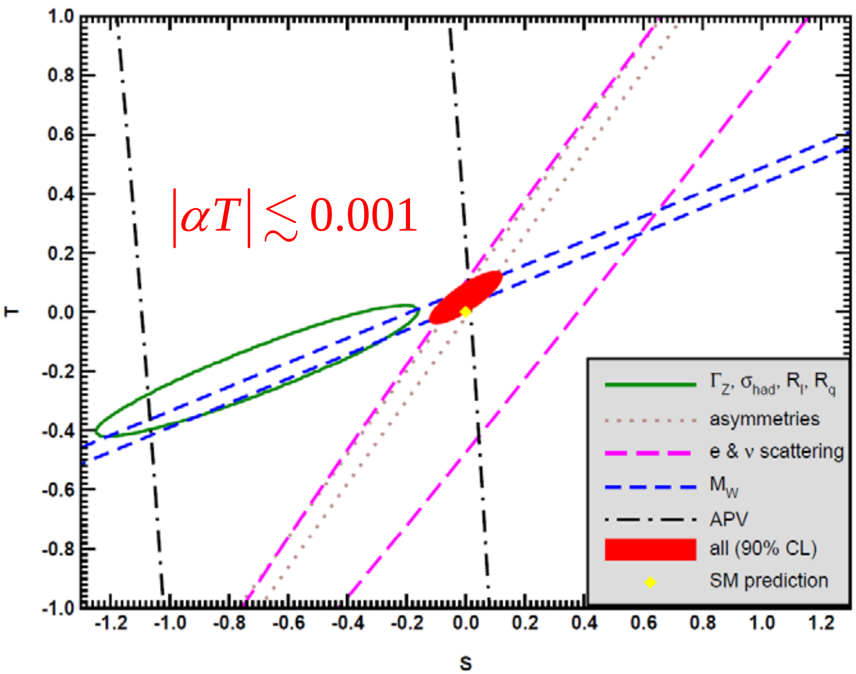


$$i\,\Pi_{XY}^{\mu\nu}\left(p^2\right) = i\left(\eta^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right)\Pi_{XY}\left(p^2\right)$$

Only applies to oblique BSM physics



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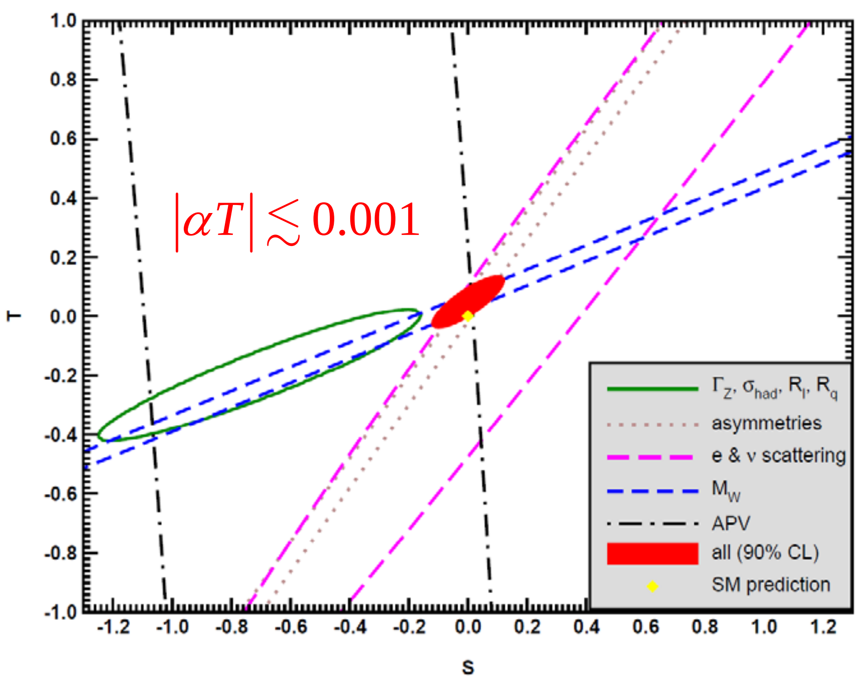

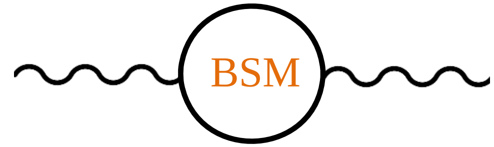

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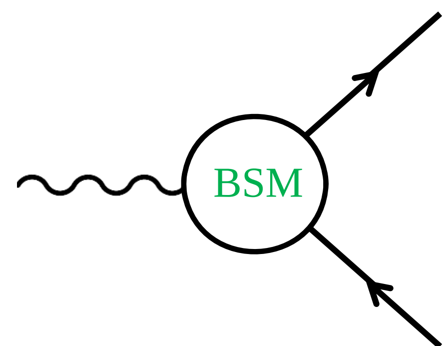
What about non-oblique BSM physics?



- T is no longer an observable
- T does not represent custodial violations

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A non-oblique example

$$\mathcal{L}_{\text{UV}} = \mathcal{L}_{\text{SM}} + \bar{N}\left(i\not{D} - M\right)N + \bar{E}\left(i\not{D} - M\right)E - \left(Y_N\,\bar{l}\,\tilde{H}\,N + Y_E\,\bar{l}\,H\,E + \text{h.c.}\right)$$

$$= \bar{l}\begin{pmatrix}\tilde{H} & H\end{pmatrix}\begin{pmatrix}Y_N & 0\\ 0 & Y_E\end{pmatrix}\begin{pmatrix}N\\ E\end{pmatrix}$$

If $|Y_N| = |Y_E|$, $U_R \in SU(2)_R$ is a symmetry

$$\Sigma \equiv \begin{pmatrix}\tilde{H} & H\end{pmatrix} \longrightarrow \Sigma U_R^\dagger \quad , \quad \begin{pmatrix}N\\ E\end{pmatrix} \longrightarrow U_R\begin{pmatrix}N\\ E\end{pmatrix}$$

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Custodial violating operator

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \frac{1}{4M^2}\left(\left|Y_N\right|^2 - \left|Y_E\right|^2\right)\left(H^\dagger i\overleftrightarrow{D}_\mu H\right)\left(\bar{l}\gamma^\mu l\right) + \cdots$$

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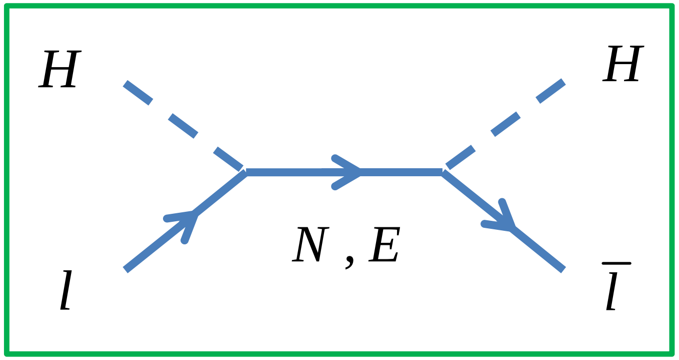

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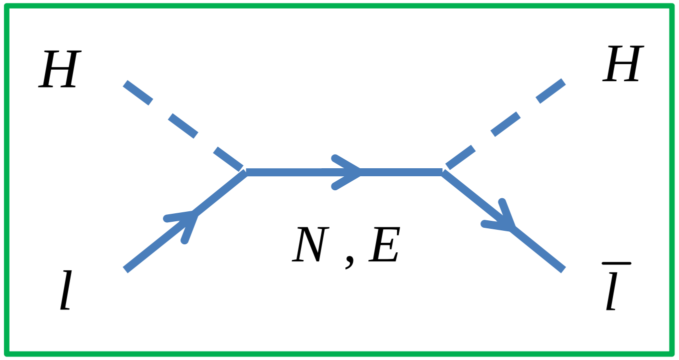
Need new indicators!



Outline

- Symmetries define the theory
- Review on custodial symmetry
- Custodial violations in oblique BSM physics
  --- electroweak precision parameters, *S*, *T*, *U*, *etc.*
- Custodial violations in non-oblique BSM physics
  --- *S*, *T*, *U*, *etc.* are no longer observables
  --- new framework: dim-6 SMEFT tree level; custodial basis
  --- SMEFT Reparameterization Invariance
  --- potential complication from EOM redundancies

Symmetries define the theory

$$\begin{cases} \text{field content } \{\phi_i\} \\ \text{symmetries} \end{cases}$$

$$\Downarrow$$

$$\mathcal{L}\left(\{\phi_i\}\right)$$

Symmetries define the theory

$$\begin{cases} \text{field content } \{\phi_i\} & \Rightarrow \quad \text{A real scalar } \phi \\ \text{symmetries} & \Rightarrow \quad Z_2\left(\phi \to -\phi\right) \end{cases}$$

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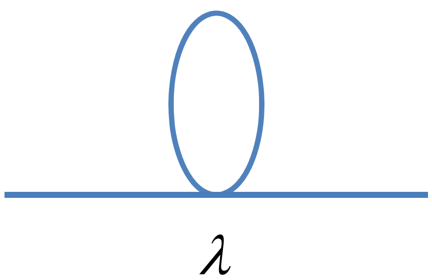


Standard Model

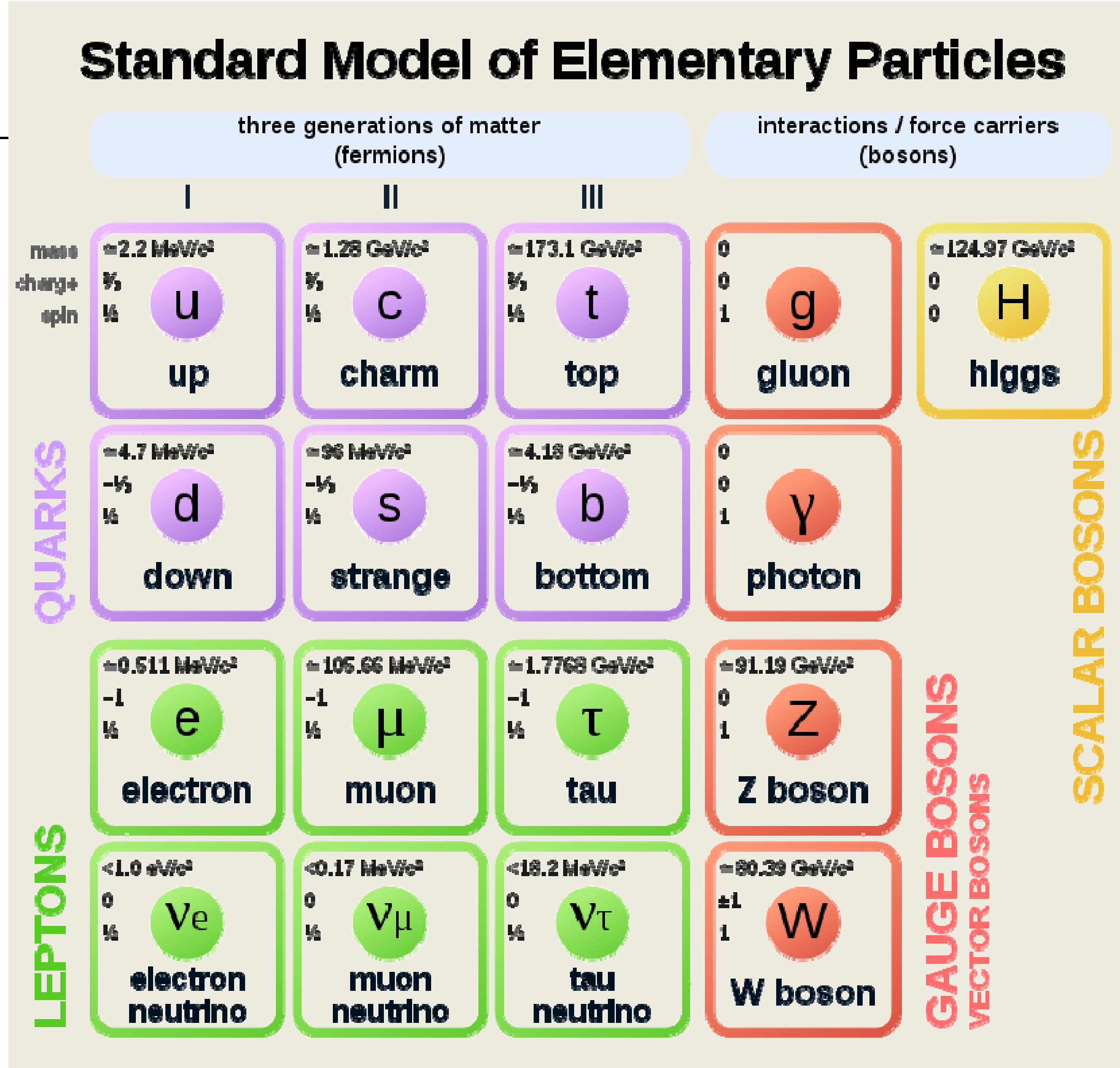
$$H$$

$$\psi \begin{cases} q \\ u \\ d \\ l \\ e \end{cases}$$

$$G^{A}_{\mu\nu}$$

$$W^{a}_{\mu\nu}$$

$$B_{\mu\nu}$$



Standard Model	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
H	1	2	$+\frac{1}{2}$
ψ q	3	2	$+\frac{1}{6}$
ψ u	3	1	$+\frac{2}{3}$
ψ d	3	1	$-\frac{1}{3}$
ψ l	1	2	$-\frac{1}{2}$
ψ e	1	1	-1
$G^A_{\mu\nu}$	8	1	0
$W^a_{\mu\nu}$	1	3	0
$B_{\mu\nu}$	1	1	0

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$B_{\mu\nu}$	1	1	0

$$\mathcal{L}_{\text{SM}} = |DH|^2 + \sum_{\psi=q,u,d,l,e} \bar{\psi} i \not{D} \psi - \frac{1}{4} G^A_{\mu\nu} G^{A\mu\nu} - \frac{1}{4} W^a_{\mu\nu} W^{a\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

$$- \lambda \left(|H|^2 - \frac{1}{2} v^2 \right)^2 - \left(\bar{q} Y_u \tilde{H} u + \bar{q} Y_d H d + \bar{l} Y_e H e + \text{h.c.} \right)$$

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Motivations beyond SM

- Neutrino mass
- Hierarchy problem
- Strong CP problem
- Dark matter
- Baryogenesis
- ……

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UV approach: $\mathcal{L}_{\text{UV}} = \mathcal{L}_{\text{SM}}\left(\phi_{\text{SM}}\right) + \mathcal{L}_{\text{BSM}}\left(\phi_{\text{SM}}, \Phi_{\text{BSM}}\right)$

new fields Φ_{BSM}?

new symmetries?

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UV approach: $\mathcal{L}_{\text{UV}} = \mathcal{L}_{\text{SM}}(\phi_{\text{SM}}) + \mathcal{L}_{\text{BSM}}(\phi_{\text{SM}}, \Phi_{\text{BSM}})$

$$\mathcal{L}_{\text{UV}} = \mathcal{L}_{\text{SM}} + \bar{N}(i\not{D} - M)N + \bar{E}(i\not{D} - M)E - \left(Y_N \bar{l}\tilde{H}N + Y_E \bar{l} H E + \text{h.c.}\right)$$

$$\Phi_{\text{BSM}} = \{N, E\}$$

Parameter	Custodial Symmetry
$\lvert Y_N \rvert = \lvert Y_E \rvert$	✓
$\lvert Y_N \rvert \neq \lvert Y_E \rvert$	×

UV approach: $\mathcal{L}_{\text{UV}} = \mathcal{L}_{\text{SM}}(\phi_{\text{SM}}) + \mathcal{L}_{\text{BSM}}(\phi_{\text{SM}}, \Phi_{\text{BSM}})$

new fields Φ_{BSM}?

new symmetries?

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EFT approach: $\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}}(\phi_{\text{SM}}) + \sum_i C_i Q_i(\phi_{\text{SM}})$

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EFT approach: $\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}}(\phi_{\text{SM}}) + \sum_i C_i Q_i(\phi_{\text{SM}})$

Symmetry restrictions?

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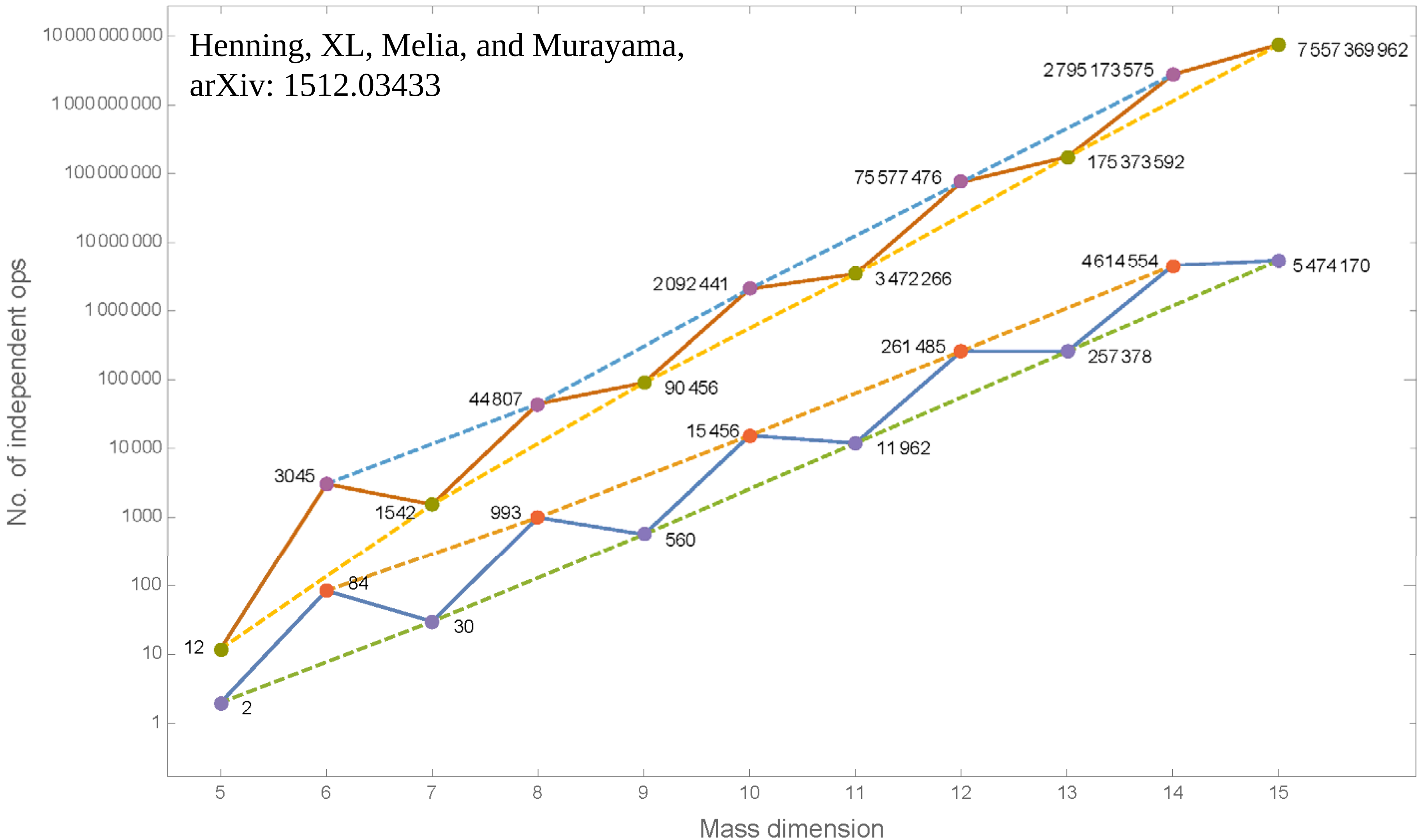
Symmetry restrictions?

dim-6 operators	Custodial Symmetry
$Q_{2W} \equiv -\frac{1}{2}\left(D^{\mu}W^{a}_{\mu\nu}\right)^2$	✓
$Q_{Hl}^{(3)} \equiv \left(H^{\dagger}\sigma^{a} i\overleftrightarrow{D}_{\mu} H\right)\left(\bar{l}\gamma^{\mu}\sigma^{a} l\right)$	✓
$Q_{Hl}^{(1)} \equiv \left(H^{\dagger} i\overleftrightarrow{D}_{\mu} H\right)\left(\bar{l}\gamma^{\mu} l\right)$	×

Number of SMEFT operators



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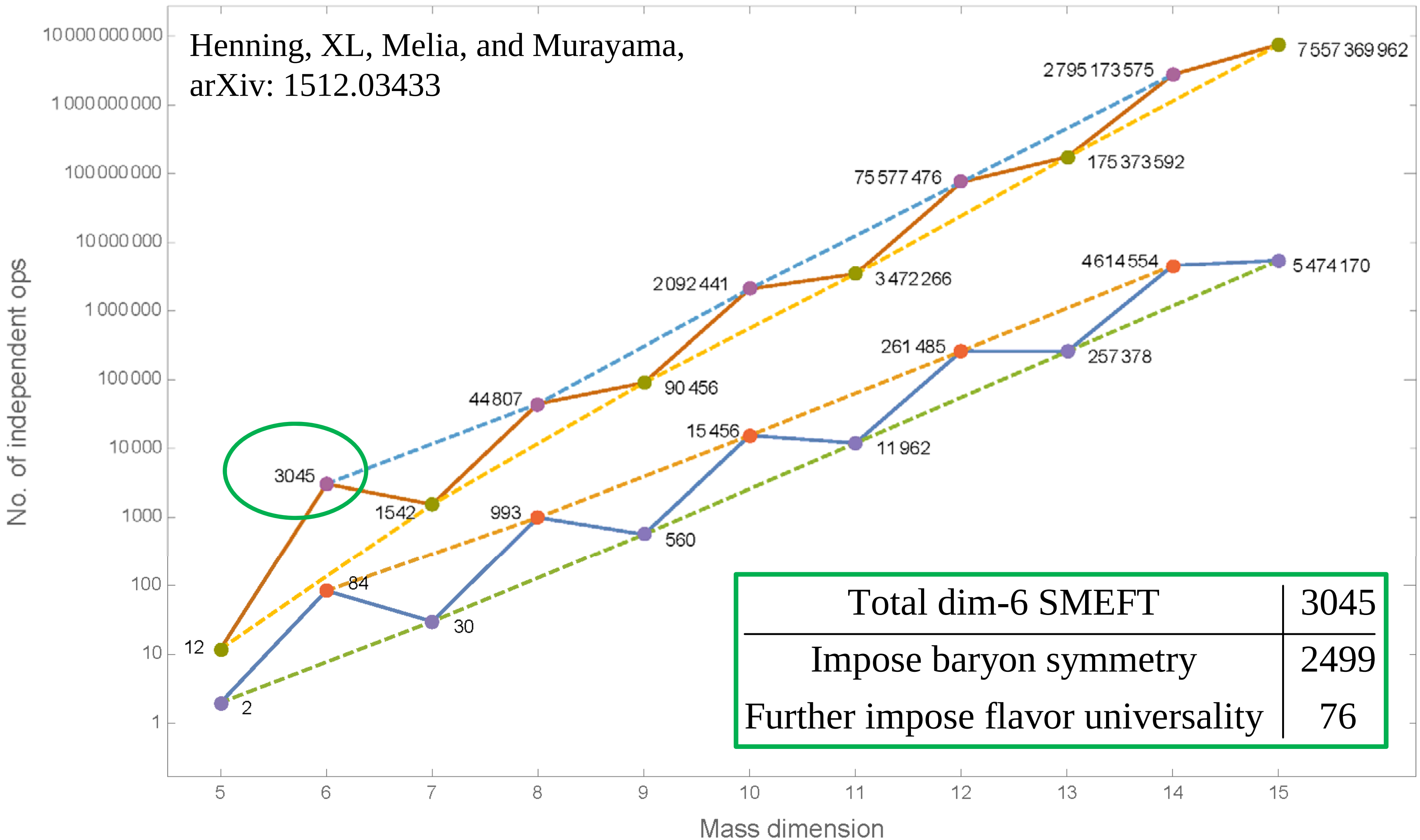

Total dim-6 SMEFT	3045
Impose baryon symmetry	2499
Further impose flavor universality	76

Custodial symmetry in SM

$$\mathcal{L}_{\text{SM}} = -\frac{1}{4}G^A_{\mu\nu}G^{A\mu\nu} - \frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + |DH|^2 - \lambda\left(|H|^2 - \frac{1}{2}v^2\right)^2$$

$$+ \sum_{\psi=q,u,d,l,e} \bar{\psi} i \not{D} \psi - \left(\bar{q} Y_u \tilde{H} u + \bar{q} Y_d H d + \bar{l} Y_e H e + \text{h.c.}\right)$$

$$\Sigma \equiv \begin{pmatrix} \tilde{H} & H \end{pmatrix} \qquad V(H) = \frac{\lambda}{4}\left[\text{tr}\left(\Sigma^\dagger \Sigma\right) - v^2\right]^2$$

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$$SU(2)_L \times SU(2)_{RH}$$ Custodial Symmetry

Two breaking sources in SM:

$$g_1 \neq 0 \quad \Rightarrow \quad \begin{cases} |DH|^2 = \frac{1}{2}\text{tr}\left[\left(D_\mu \Sigma\right)^\dagger \left(D^\mu \Sigma\right)\right] \not\rightarrow |DH|^2 \\ D_\mu \Sigma = \partial_\mu \Sigma - i g_2 W^a_\mu \frac{\sigma^a}{2}\Sigma + i g_1 B_\mu \Sigma \frac{\sigma^3}{2} \not\rightarrow U_L\left(D_\mu \Sigma\right) U_R^\dagger \end{cases}$$

Custodial symmetry in SM

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$$Y_u, Y_d, Y_e \neq 0 \quad \Rightarrow \quad \bar{q} Y_u \tilde{H} u + \bar{q} Y_d H d \not\to \bar{q} Y_u \tilde{H} u + \bar{q} Y_d H d$$

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{BSM}}$$

custodial symmetric?

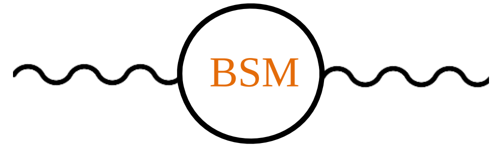
$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + \mathcal{L}_{\mathrm{BSM}}$ oblique

custodial symmetric?

$$i\,\Pi_{XY}^{\mu\nu}\left(p^2\right) = i\left(\eta^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right)\Pi_{XY}\left(p^2\right)$$

$$\Pi_{XY}\left(p^2\right) = \Pi_{XY}(0) + p^2\Pi'_{XY}(0) + \frac{1}{2}p^4\Pi''_{XY}(0) + \cdots$$

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + \mathcal{L}_{\mathrm{BSM}}$$ oblique



$$i\,\Pi_{XY}^{\mu\nu}\left(p^2\right) = i\left(\eta^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right)\Pi_{XY}\left(p^2\right)$$

Electroweak Precision Parameters

$$\alpha S = -4c_\theta s_\theta \Pi'_{3B}(0)$$

$$\alpha T = \frac{1}{m_W^2}\left[\Pi_{WW}(0) - \Pi_{33}(0)\right]$$

$$\alpha U = 4s_\theta^2\left[\Pi'_{WW}(0) - \Pi'_{33}(0)\right]$$

$$D_\mu = \partial_\mu - ig_2 W_\mu^a t^a - ig_1 B_\mu Y$$

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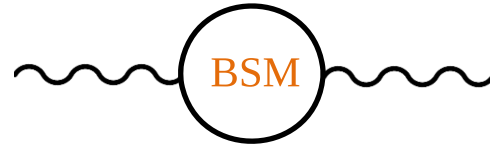
- M. E. Peskin and T. Takeuchi, “Estimation of oblique electroweak corrections,” Phys.Rev. D46 (1992) 381–409

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + \mathcal{L}_{\mathrm{BSM}}$$ oblique



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$$V = \frac{1}{2} m_W^2 \left[\Pi''_{WW}(0) - \Pi''_{33}(0)\right]$$

$$W = -\frac{1}{2} m_W^2 \Pi''_{33}(0)$$

$$Y = -\frac{1}{2} m_W^2 \Pi''_{BB}(0)$$

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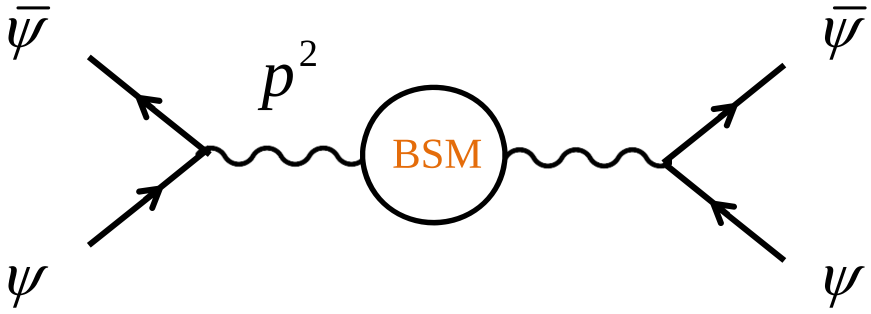
- M. E. Peskin and T. Takeuchi, "Estimation of oblique electroweak corrections," Phys.Rev. D46 (1992) 381–409
- I. Maksymyk, C. Burgess, and D. London, "Beyond S, T and U", arXiv: hep-ph/9306267
- C. Burgess, S. Godfrey, H. Konig, D. London, and I. Maksymyk, "A Global fit to extended oblique parameters", arXiv: hep-ph/9307337
- A. Kundu and P. Roy, "A General treatment of oblique parameters", arXiv: hep-ph/9603323
- R. Barbieri, A. Pomarol, R. Rattazzi, and A. Strumia, "Electroweak symmetry breaking after LEP-1 and LEP-2". arXiv: hep-ph/0405040

The oblique framework



$$\mathcal{M}_{\mathrm{NC}}\left(p^2\right) = e_*^2 Q \frac{1}{p^2} Q' + \frac{e_*^2}{c_*^2 s_*^2}\left(T_3 - s_*^2 Q\right)\frac{Z_{Z*}}{p^2 - m_{Z*}^2}\left(T_3' - s_*^2 Q'\right)$$

$$\mathcal{M}_{\mathrm{CC}}\left(p^2\right) = \frac{e_*^2}{2 s_*^2} T_+ \frac{Z_{W*}}{p^2 - m_{W*}^2} T_-$$

- D. C. Kennedy and B. W. Lynn, “Electroweak Radiative Corrections with an Effective Lagrangian: Four Fermion Processes”, Nucl. Phys. B322, I (1989)
- B. W. Lynn, M. E. Peskin, and R. G. Stuart, “Radiative Corrections in SU(2) x U(1): LEP / SLC”, Physics at LEP 1985, SLAC-PUB-3725

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$$\frac{m_W^2}{m_Z^2} - c_0^2 = \frac{\alpha c^2}{c^2 - s^2}\left[-\frac{1}{2}S + c^2 T + \frac{c^2 - s^2}{4s^2}U\right],$$

$$s_*^2(q^2) - s_0^2 = \frac{\alpha}{c^2 - s^2}\left(\tfrac{1}{4}S - s^2c^2T\right),$$

$$\boxed{\rho_*(0) - 1 = \alpha T\ ,} \tag{3.13}$$

$$Z_{Z*}(q^2) - 1 = \frac{\alpha}{4s^2c^2}S\ .$$

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M. E. Peskin and T. Takeuchi, “Estimation of oblique electroweak corrections,” Phys.Rev. D46 (1992) 381–409.

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$$\mathcal{M}_{\rm CC} = -\frac{1}{2}I_+\frac{1}{\left[\frac{v^2}{4}+\Pi_{11}(0)\right]}I_- \;. \tag{2.23}$$

These matrix elements should be compared with the standard form of the low-energy effective Lagrangian of weak interactions:

$$\mathcal{L}_{\rm eff} = -\frac{4G_F}{\sqrt{2}}\{J_+^\mu J_-^\mu + \rho_*(0)[J_3^\mu - s_*^2(0)J_Q^\mu]^2\} \;, \tag{2.24}$$

We denote the low-energy ratio of charged- to neutral-current amplitudes by $\rho_*(0)$, to avoid confusion with the Veltman definition (1.1). We can now make the

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M. E. Peskin and T. Takeuchi, “Estimation of oblique electroweak corrections,” Phys.Rev. D46 (1992) 381–409.

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$$\rho \equiv \frac{m_W^2}{m_Z^2 \cos^2\theta_W} = 1 \ . \tag{1.1}$$

Experimentally, this relation is satisfied to better than 1%, so that it is reasonable to assume that the correc-

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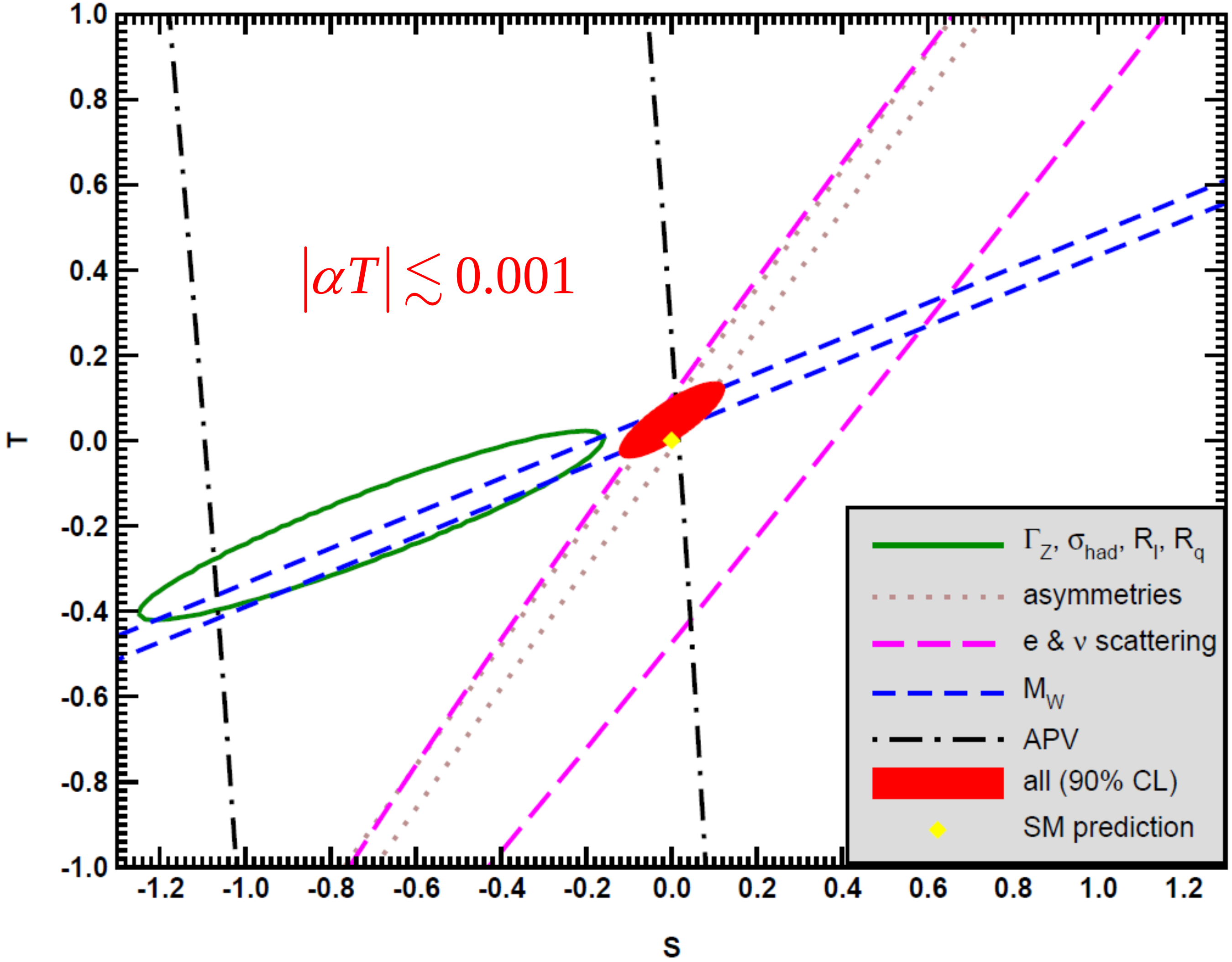
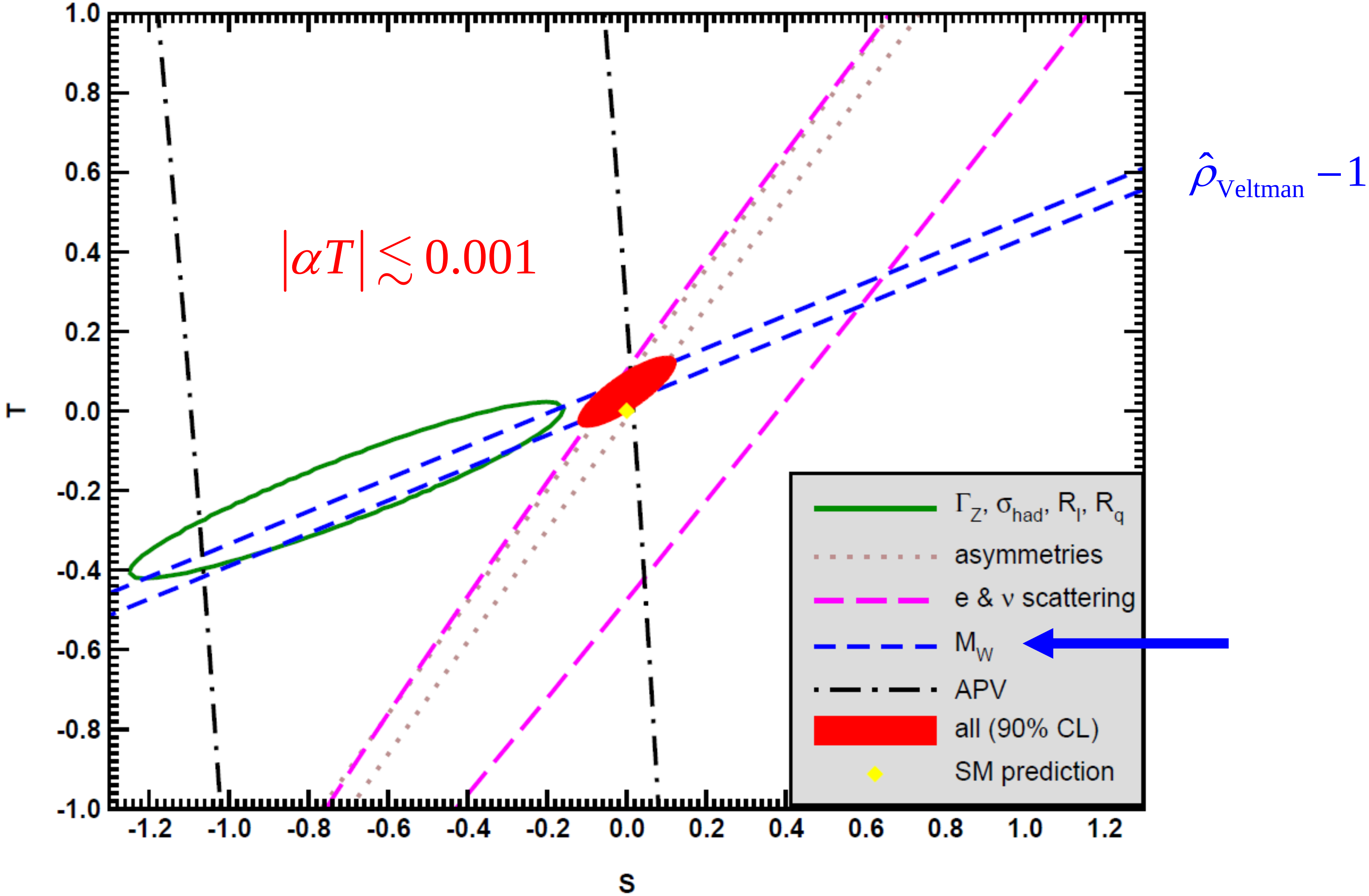
$$Z_{W*}(q^2) - 1 = \frac{\alpha}{4s^2}(S + U) \ .$$

M. E. Peskin and T. Takeuchi, “Estimation of oblique electroweak corrections,” Phys.Rev. D46 (1992) 381–409.



Particle Data Group Collaboration, P. Zyla et al., "Review of Particle Physics," *PETP* 2020 (2020) no. 8, 083C01.



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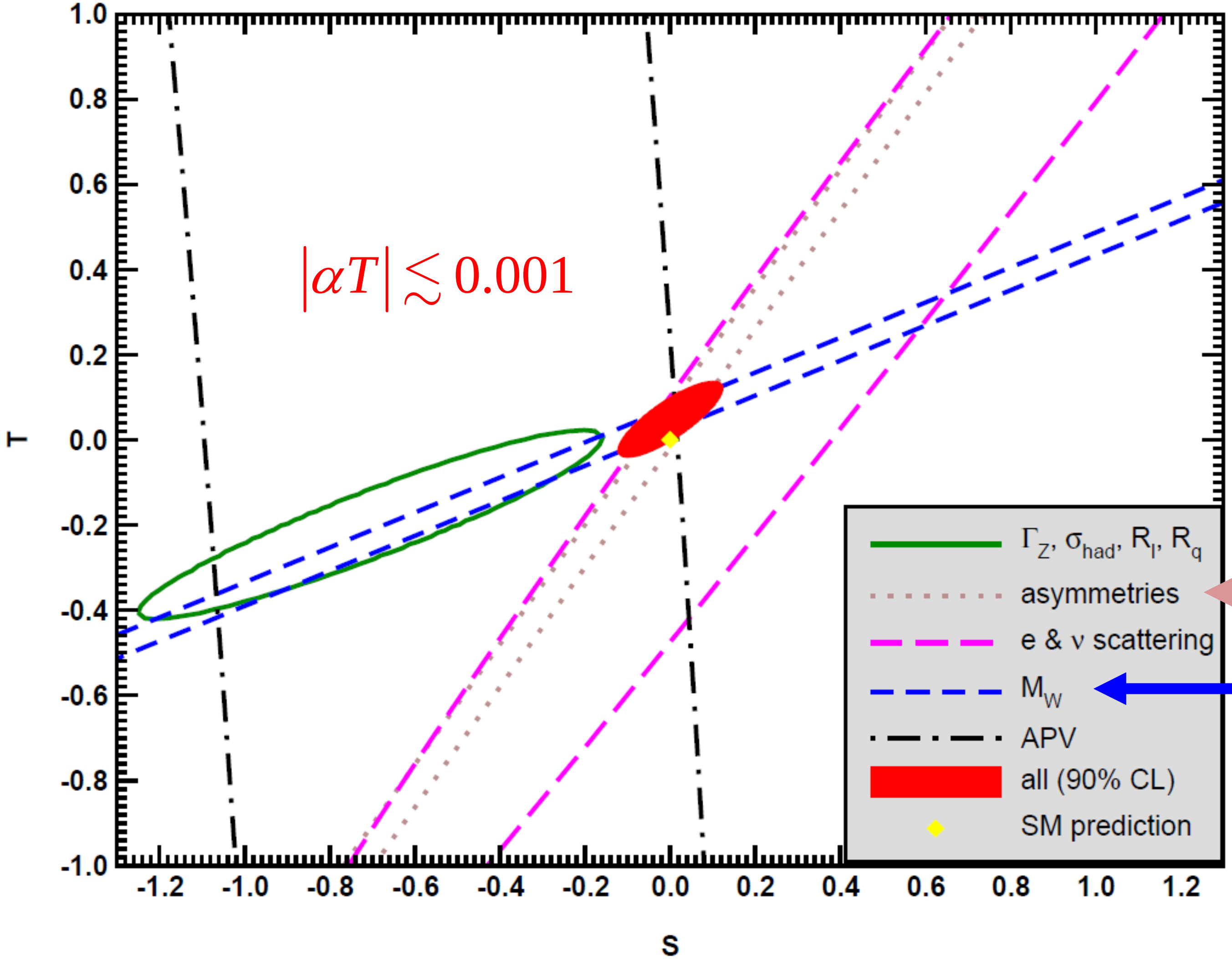

$s_*^2\left(m_Z^2\right) - s_0^2$

$\hat{\rho}_{\text{Veltman}} - 1$

$|\alpha T| \lesssim 0.001$

$$A_{LR}(q^2) = \frac{2[1-4s_*^2(q^2)]}{1+[1-4s_*^2(q^2)]^2}$$

Particle Data Group Collaboration, P. Zyla et al., "Review of Particle Physics," *PETP* 2020 (2020) no. 8, 083C01.

$$\hat{\rho}_{\text{Veltman}} \equiv \frac{\hat{m}_W^2}{\hat{m}_W^2 c_0^2} = 1 + \frac{\alpha}{c_{2\theta}}\left(-\frac{1}{2}S + c_\theta^2 T + \frac{c_{2\theta}}{4 s_\theta^2}U\right)$$

$$\hat{\rho}_*(0) \sim \frac{\mathcal{M}_{\text{NC}}(0)}{\mathcal{M}_{\text{CC}}(0)} = 1 + \alpha T$$

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$$\boxed{\frac{m_W^2}{m_Z^2} - c_0^2 = \frac{\alpha c^2}{c^2 - s^2}\left[-\frac{1}{2}S + c^2 T + \frac{c^2 - s^2}{4s^2}U\right],}$$

$$s_*^2(q^2) - s_0^2 = \frac{\alpha}{c^2 - s^2}\left(\tfrac{1}{4}S - s^2 c^2 T\right),$$

$$\boxed{\rho_*(0) - 1 = \alpha T\ ,} \qquad (3.13)$$

$$Z_{Z*}(q^2) - 1 = \frac{\alpha}{4 s^2 c^2}S\ .$$

$$Z_{W*}(q^2) - 1 = \frac{\alpha}{4 s^2}(S + U)\ .$$

M. E. Peskin and T. Takeuchi, “Estimation of oblique electroweak corrections,” Phys.Rev. D46 (1992) 381–409.

31 *10. Electroweak Model and Constraints on New Physics*

The dominant effect of many extensions of the SM can be described by the ρ_0 parameter,

$$\rho_0 \equiv \frac{M_W^2}{M_Z^2\, \hat{c}_Z^2\, \hat{\rho}}\,, \tag{10.66}$$

which describes new sources of SU(2) breaking that cannot be accounted for by the SM Higgs doublet or by m_t effects. $\hat{\rho}$ is calculated as in Eq. (10.18) assuming the validity of the SM. In the presence of $\rho_0 \neq 1$, Eq. (10.66) generalizes the second Eq. (10.18) while the first remains unchanged. Provided that the new physics which yields $\rho_0 \neq 1$ is a small perturbation which does not significantly affect other radiative corrections, ρ_0 can be regarded as a phenomenological parameter which multiplies G_F in Eqs. (10.21) and (10.41), as well as Γ_Z in Eq. (10.60c). There are enough data to determine ρ_0, M_H, m_t, and α_s, simultaneously. From the global fit,

$$\rho_0 = 1.00038 \pm 0.00020\,, \tag{10.67a}$$

$$\alpha_s(M_Z) = 0.1188 \quad \pm 0.0017\,, \tag{10.67b}$$

A heavy non-degenerate multiplet of fermions or scalars contributes positively to T as

$$\rho_0 - 1 = \frac{1}{1 - \hat{\alpha}(M_Z)T} - 1 \approx \hat{\alpha}(M_Z)T\,, \tag{10.74}$$

[15] Three additional parameters are needed if the new physics scale is comparable to M_Z [332]. Further generalizations, including effects relevant to LEP 2 and Drell-Yan production at the LHC, are described in Refs. [333] and [334], respectively.

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where $\rho_0 - 1$ is given in Eq. (10.69). The effects of non-standard Higgs representations cannot be separated from heavy non-degenerate multiplets unless the new physics has other consequences, such as vertex corrections. Most of the original papers defined T to include the effects of loops only. However, we will redefine T to include all new sources of SU(2) breaking, including non-standard Higgs, so that T and ρ_0 are equivalent by Eq. (10.74).

Particle Data Group Collaboration,
P. Zyla et al., "Review of Particle Physics,"
PETP 2020 (2020) no. 8, 083C01.

$$\hat{\rho}_{\text{Veltman}} \equiv \frac{\hat{m}_W^2}{\hat{m}_W^2 c_0^2} = 1 + \frac{\alpha}{c_{2\theta}}\left(-\frac{1}{2}S + c_\theta^2 T + \frac{c_{2\theta}}{4s_\theta^2}U\right)$$

$$\hat{\rho}_*(0) \sim \frac{\mathcal{M}_{\text{NC}}(0)}{\mathcal{M}_{\text{CC}}(0)} = 1 + \alpha T$$

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$$\begin{aligned}
&\boxed{\frac{m_W^2}{m_Z^2} - c_0^2 = \frac{\alpha c^2}{c^2 - s^2}\left[-\frac{1}{2}S + c^2 T + \frac{c^2 - s^2}{4s^2}U\right],}\\
&s_*^2(q^2) - s_0^2 = \frac{\alpha}{c^2 - s^2}\left(\tfrac{1}{4}S - s^2 c^2 T\right),\\
&\boxed{\rho_*(0) - 1 = \alpha T\,,}\\
&Z_{Z*}(q^2) - 1 = \frac{\alpha}{4s^2c^2}S\,.\\
&Z_{W*}(q^2) - 1 = \frac{\alpha}{4s^2}(S + U)\,.
\end{aligned} \tag{3.13}$$

M. E. Peskin and T. Takeuchi, "Estimation of oblique electroweak corrections," Phys.Rev. D46 (1992) 381–409.

An oblique Example : W'

$$\mathcal{L}_{\mathrm{UV}} \supset \sum_{\psi} \bar{\psi} i \not{D} \psi - \frac{1}{4} W_{A\mu\nu}^{a} W_{A}^{a\mu\nu} - \frac{1}{4} W_{B\mu\nu}^{a} W_{B}^{a\mu\nu} + \frac{1}{2} \mathrm{tr}\left[\left(D_{\mu} \Phi \right)^{\dagger} \left(D^{\mu} \Phi \right) \right] - V_{\Phi}$$

$$SU(2)_A \times SU(2)_B$$

$$\Phi \to U_A \Phi U_B^{\dagger}$$

An oblique Example : W'

$$\mathcal{L}_{\mathrm{UV}} \supset \sum_{\psi} \bar{\psi} i \not{D} \psi - \frac{1}{4} W^{a}_{A\mu\nu} W^{a\mu\nu}_{A} - \frac{1}{4} W^{a}_{B\mu\nu} W^{a\mu\nu}_{B} + \frac{1}{2} \mathrm{tr}\left[\left(D_{\mu} \Phi \right)^{\dagger} \left(D^{\mu} \Phi \right) \right] - V_{\Phi}$$

$$SU(2)_A \times SU(2)_B \quad \rightarrow \quad SU(2)_L$$

$$\Phi \rightarrow U_A \Phi U_B^{\dagger} \qquad \Phi \supset \frac{v_{\Phi}}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

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$$\supset \frac{v_\Phi^2}{8} \left(g_A W^a_{A\mu} - g_B W^a_{B\mu} \right)^2$$

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$$\boxed{D_\mu = \partial_\mu - i g_A W^a_{A\mu} t^a_A - i g_B W^a_{B\mu} t^a_B = \partial_\mu - i g_2 W^a_\mu \left(t^a_A + t^a_B \right) - i W'^a_\mu \left(c_A g_A t^a_A - s_A g_B t^a_B \right)}$$

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$$\mathcal{L}_{\rm UV} \supset \sum_\psi \bar\psi i \not{D} \psi - \frac{1}{4} W^a_{A\mu\nu} W^{a\mu\nu}_A - \frac{1}{4} W^a_{B\mu\nu} W^{a\mu\nu}_B + \frac{1}{2} {\rm tr}\left[\left(D_\mu \Phi\right)^\dagger \left(D^\mu \Phi\right)\right] - V_\Phi$$

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$$\mathcal{L}_{\rm SMEFT} - \mathcal{L}_{\rm SM} = -\frac{2 c_A^4}{v_\Phi^2} J^a_{W\mu} J^{a\mu}_W$$

An oblique Example : W'

$$\mathcal{L}_{\text{UV}} \supset \sum_\psi \bar{\psi} i \not{D} \psi - \frac{1}{4} W^a_{A\mu\nu} W^{a\mu\nu}_A - \frac{1}{4} W^a_{B\mu\nu} W^{a\mu\nu}_B + \frac{1}{2} \text{tr}\left[\left(D_\mu \Phi \right)^\dagger \left(D^\mu \Phi \right) \right] - V_\Phi$$

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$$\mathcal{L}_{\text{SMEFT}} - \mathcal{L}_{\text{SM}} = -\frac{2c_A^4}{v_\Phi^2} J^a_{W\mu} J^{a\mu}_W = \frac{4c_A^4}{v_\Phi^2} \frac{1}{g_2^2} \left[-\frac{1}{2} \left(D^\mu W^a_{\mu\nu} \right)^2 \right]$$

C_{2W} Q_{2W}

$$\Pi_{WW}\left(p^2\right) = \Pi_{33}\left(p^2\right) = -p^4 C_{2W}$$

An oblique Example : W'

$$\mathcal{L}_{\rm UV} \supset \sum_\psi \bar{\psi} i \not{D} \psi - \frac{1}{4} W^a_{A\mu\nu} W_A^{a\mu\nu} - \frac{1}{4} W^a_{B\mu\nu} W_B^{a\mu\nu} + \frac{1}{2} \mathrm{tr}\left[\left(D_\mu \Phi\right)^\dagger \left(D^\mu \Phi\right) \right] - V_\Phi$$

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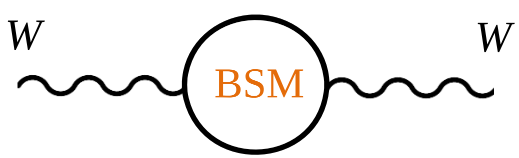
$$\mathcal{L}_{\rm SMEFT} - \mathcal{L}_{\rm SM} = -\frac{2 c_A^4}{v_\Phi^2} J^a_{W\mu} J^{a\mu}_W = \frac{4 c_A^4}{v_\Phi^2} \frac{1}{g_2^2} \left[-\frac{1}{2} \left(D^\mu W^a_{\mu\nu} \right)^2 \right]$$

C_{2W} $\qquad Q_{2W}$



$$\Pi_{WW}\left(p^2\right) = \Pi_{33}\left(p^2\right) = -p^4 C_{2W}$$

$$\alpha T \equiv \frac{\Pi_{WW}(0) - \Pi_{33}(0)}{m_W^2} = 0 \quad , \quad \hat{\rho}_{\rm Veltman} \equiv \frac{\hat{m}_W^2}{\hat{m}_Z^2 c_0^2} = 1 + m_W^2 C_{2W} \neq 1 \quad , \quad \sin^2 2\theta_0 \equiv \frac{2\sqrt{2}\pi \hat{\alpha}_0\left(m_Z^2\right)}{\hat{G}_F \hat{m}_Z^2}$$

$$\hat{\rho}_{\text{Veltman}} \equiv \frac{\hat{m}_W^2}{\hat{m}_W^2 c_0^2} = 1 + \frac{\alpha}{c_{2\theta}}\left(-\frac{1}{2}S + c_\theta^2 T + \frac{c_{2\theta}}{4s_\theta^2}U\right)$$

$$\mathcal{L}_{\text{EFT}} = -\frac{4G_F}{\sqrt{2}}\left\{J_{+\mu}J_-^{\mu} + \rho_*(0)\left[J_3^{\mu} - s_*^2(0)J_{\text{EM}}^{\mu}\right]^2\right\}$$

$$\hat{\rho}_*(0) \sim \frac{\mathcal{M}_{\text{NC}}(0)}{\mathcal{M}_{\text{CC}}(0)} = 1 + \alpha T$$

$$|\alpha T| \equiv \left|\frac{\Pi_{WW}(0) - \Pi_{33}(0)}{m_W^2}\right| \lesssim 0.001$$

$$\mathcal{L}_{\text{UV}} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{BSM}}$$

custodial symmetric?

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$$\boxed{\frac{m_W^2}{m_Z^2} - c_0^2 = \frac{\alpha c^2}{c^2 - s^2}\left[-\frac{1}{2}S + c^2 T + \frac{c^2 - s^2}{4s^2}U\right],}$$

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oblique

custodial symmetric?

$$i\,\Pi_{XY}^{\mu\nu}(p^2) = i\left(\eta^{\mu\nu} - \frac{p^\mu p^\nu}{p^2} \right)\Pi_{XY}(p^2)$$

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$$\boxed{\frac{m_W^2}{m_Z^2} - c_0^2 = \frac{\alpha c^2}{c^2 - s^2}\left[-\frac{1}{2}S + c^2 T + \frac{c^2 - s^2}{4s^2}U \right],}$$

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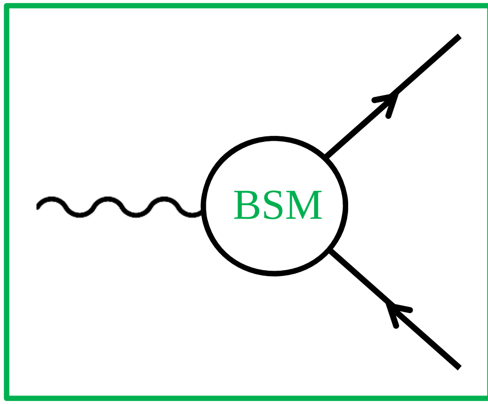
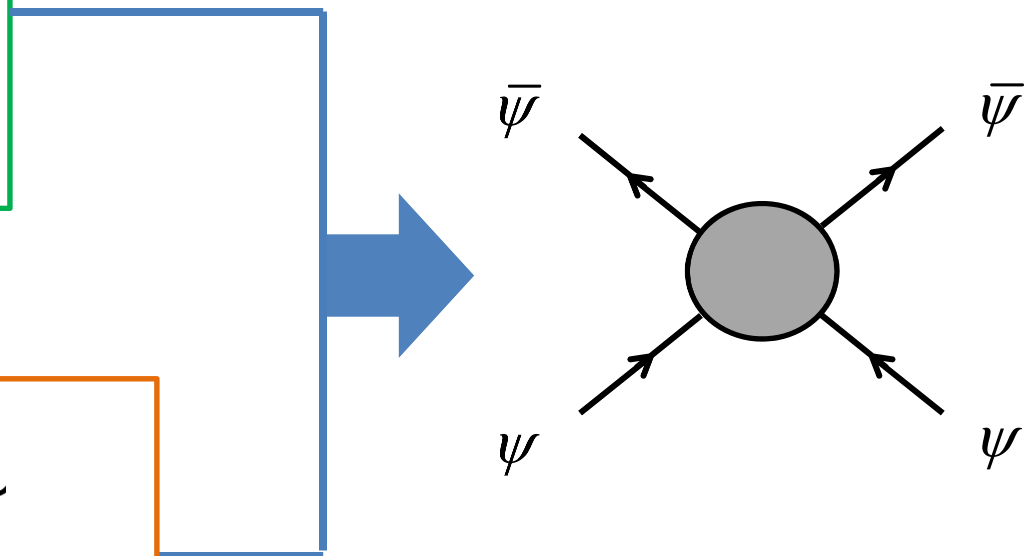
Beyond oblique BSM: *S*, *T*, *U* are no longer observables

non-oblique BSM physics





oblique BSM physics

$$i\,\Pi_{XY}^{\mu\nu}\left(p^2\right)=i\left(\eta^{\mu\nu}-\frac{p^\mu p^\nu}{p^2}\right)\Pi_{XY}\left(p^2\right)$$

Beyond oblique BSM: *S*, *T*, *U* are no longer observables

$$Q_B \equiv g_1 y_H \left(H^\dagger i\overleftrightarrow{D}^\mu H\right)\left(\partial^\nu B_{\mu\nu}\right) = g_1^2 y_H^2 \left(H^\dagger i\overleftrightarrow{D}_\mu H\right)^2 + \mathcal{O}(\psi)$$

$$\alpha S = -4 c_\theta s_\theta \Pi'_{3B}(0)$$

$$X = -\frac{1}{2} m_W^2 \Pi''_{3B}(0)$$

$$\alpha T = \frac{1}{m_W^2}\left[\Pi_{WW}(0) - \Pi_{33}(0)\right]$$

$$\alpha U = 4 s_\theta^2 \left[\Pi'_{WW}(0) - \Pi'_{33}(0)\right]$$

$$V = \frac{1}{2} m_W^2 \left[\Pi''_{WW}(0) - \Pi''_{33}(0)\right]$$

$$W = -\frac{1}{2} m_W^2 \Pi''_{33}(0)$$

$$Y = -\frac{1}{2} m_W^2 \Pi''_{BB}(0)$$

$$-\partial^\mu B_{\mu\nu} = g_1 J_{B\nu} \quad , \quad J_B^\mu \equiv y_H \left(H^\dagger i\overleftrightarrow{D}^\mu H\right) + \sum_\psi y_\psi \bar{\psi}\gamma^\mu \psi \quad , \quad H^\dagger i\overleftrightarrow{D}_\mu H \supset -\frac{g_2}{2c_\theta} v^2 Z_\mu + \mathcal{O}(h)$$

Beyond oblique BSM: *S*, *T*, *U* are no longer observables

$$\alpha S = -4c_\theta s_\theta \Pi'_{3B}(0)$$

$$X = -\frac{1}{2}m_W^2 \Pi''_{3B}(0)$$

$$\alpha T = \frac{1}{m_W^2}\left[\Pi_{WW}(0) - \Pi_{33}(0)\right]$$

$$\alpha U = 4s_\theta^2\left[\Pi'_{WW}(0) - \Pi'_{33}(0)\right]$$

$$V = \frac{1}{2}m_W^2\left[\Pi''_{WW}(0) - \Pi''_{33}(0)\right]$$

$$W = -\frac{1}{2}m_W^2 \Pi''_{33}(0)$$

$$Y = -\frac{1}{2}m_W^2 \Pi''_{BB}(0)$$

$$\boxed{Q_B \equiv g_1 y_H \left(H^\dagger i\overleftrightarrow{D}^\mu H\right)\left(\partial^\nu B_{\mu\nu}\right) = g_1^2 y_H^2 \left(H^\dagger i\overleftrightarrow{D}_\mu H\right)^2 + \mathcal{O}(\psi)}$$

$$\begin{pmatrix} \Pi_{3B} \\ \Pi_{BB} \end{pmatrix} = \begin{pmatrix} -c_\theta s_\theta \\ 2s_\theta^2 \end{pmatrix} m_Z^2 p^2$$

$$\begin{cases} \alpha S = 4c_\theta^2 s_\theta^2 m_Z^2 \\ \alpha T = 0 \end{cases}$$

$$-\partial^\mu B_{\mu\nu} = g_1 J_{B\nu} \quad , \quad J_B^\mu \equiv y_H\left(H^\dagger i\overleftrightarrow{D}^\mu H\right) + \sum_\psi y_\psi \bar{\psi}\gamma^\mu\psi \quad , \quad H^\dagger i\overleftrightarrow{D}_\mu H \supset -\frac{g_2}{2c_\theta}v^2 Z_\mu + \mathcal{O}(h)$$

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$$\begin{pmatrix} \Pi_{3B} \\ \Pi_{BB} \end{pmatrix} = \begin{pmatrix} -c_\theta s_\theta \\ 2s_\theta^2 \end{pmatrix} m_Z^2 p^2 \qquad \begin{pmatrix} \Pi_{33} \\ \Pi_{BB} \\ \Pi_{3B} \end{pmatrix} = \begin{pmatrix} c_\theta^2 \\ s_\theta^2 \\ -c_\theta s_\theta \end{pmatrix} 2s_\theta^2 m_Z^4$$

$$\begin{cases} \alpha S = 4c_\theta^2 s_\theta^2 m_Z^2 \\ \alpha T = 0 \end{cases} \qquad \begin{cases} \alpha S = 0 \\ \alpha T = -2s_\theta^2 m_Z^2 \end{cases}$$

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$$\boxed{Q_B \equiv g_1 y_H \left(H^\dagger i\overleftrightarrow{D}^\mu H\right)\left(\partial^\nu B_{\mu\nu}\right) = g_1^2 y_H^2 \left(H^\dagger i\overleftrightarrow{D}_\mu H\right)^2 + \mathcal{O}(\psi)}$$

$$\begin{pmatrix} \Pi_{3B} \\ \Pi_{BB} \end{pmatrix} = \begin{pmatrix} -c_\theta s_\theta \\ 2s_\theta^2 \end{pmatrix} m_Z^2 p^2 \qquad \begin{pmatrix} \Pi_{33} \\ \Pi_{BB} \\ \Pi_{3B} \end{pmatrix} = \begin{pmatrix} c_\theta^2 \\ s_\theta^2 \\ -c_\theta s_\theta \end{pmatrix} 2s_\theta^2 m_Z^4$$

$$\begin{cases} \alpha S = 4c_\theta^2 s_\theta^2 m_Z^2 \\ \alpha T = 0 \end{cases} \qquad \begin{cases} \alpha S = 0 \\ \alpha T = -2s_\theta^2 m_Z^2 \end{cases}$$

$$\boxed{Q_{2B} \equiv -\frac{1}{2}\left(\partial^\mu B_{\mu\nu}\right)^2 = -\frac{1}{2} g_1^2 y_H^2 \left(H^\dagger i\overleftrightarrow{D}_\mu H\right)^2 + \mathcal{O}(\psi)}$$

$$\Pi_{BB} = -p^4 \qquad \alpha S = \alpha T = 0 \qquad \begin{cases} \alpha S = 0 \\ \alpha T = s_\theta^2 m_Z^2 \end{cases}$$

$$-\partial^\mu B_{\mu\nu} = g_1 J_{B\nu} \quad , \quad J_B^\mu \equiv y_H \left(H^\dagger i\overleftrightarrow{D}^\mu H\right) + \sum_\psi y_\psi \bar{\psi}\gamma^\mu \psi \quad , \quad H^\dagger i\overleftrightarrow{D}_\mu H \supset -\frac{g_2}{2c_\theta} v^2 Z_\mu + \mathcal{O}(h)$$

A new framework: tree-level dim-6 SMEFT

Oblique: low-energy EFT $p^2 \ll v^2$

$$\mathcal{L}_{\text{EFT}} = -\frac{4G_F}{\sqrt{2}}\left\{\underline{J_{+\mu}J_{-}^{\mu} + \rho_*(0)\left[J_3^{\mu} - s_*^2(0)J_{\text{EM}}^{\mu}\right]^2}\right\} \Rightarrow \hat{\rho}_*(0) \sim \frac{\mathcal{M}_{\text{NC}}(0)}{\mathcal{M}_{\text{CC}}(0)} = 1 + \alpha T$$

Custodial symmetric?

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Non-oblique: SMEFT $p^2 \sim v^2$

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \underline{\sum_i C_i Q_i}$$

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Custodial symmetric?

$$\left\{obs_k\left(C_i\right)\right\}$$

Check for special relations!

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$$\mathcal{L}_{\text{EFT}} = -\frac{4G_F}{\sqrt{2}}\left\{ \underline{J_{+\mu} J_{-}^{\mu} + \rho_*(0)\left[J_3^{\mu} - s_*^2(0) J_{\text{EM}}^{\mu} \right]^2} \right\} \Rightarrow \hat{\rho}_*(0) \sim \frac{\mathcal{M}_{\text{NC}}(0)}{\mathcal{M}_{\text{CC}}(0)} = 1 + \alpha T$$

Custodial symmetric?

Non-oblique: SMEFT $p^2 \sim v^2$

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \underline{\sum_i C_i Q_i} = \mathcal{L}_{\text{SM}} + \sum_i a_i O_i$$

$$Q_i \leftrightarrow O_i \qquad C_i \leftrightarrow a_i$$

Custodial symmetric?

$$\{obs_k(C_i)\} \rightarrow \{obs_k(a_i)\}$$ Custodial basis

Check for special relations!

νSMEFT Warsaw basis

$$76 = 42 + (17 + \text{h.c.})$$

$$25 = 7 + (9 + \text{h.c.})$$

G. D. Kribs, XL, A. Martin, and T. Tong, arXiv:2009.10725

$1: X^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$
$Q_{\widetilde{G}}$	$f^{ABC} \widetilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$
Q_W	$\epsilon^{abc} W_\mu^{a\nu} W_\nu^{b\rho} W_\rho^{c\mu}$
$Q_{\widetilde{W}}$	$\epsilon^{abc} \widetilde{W}_\mu^{a\nu} W_\nu^{b\rho} W_\rho^{c\mu}$

$2: H^6$	
Q_H	$\lvert H\rvert^6$

$3: H^4 D^2$	
$Q_{H\Box}$	$-(\partial_\mu \lvert H\rvert^2)(\partial^\mu \lvert H\rvert^2)$
Q_{HD}	$\left[(D_\mu H^\dagger) H\right]\left[H^\dagger (D^\mu H)\right]$

$5: \bar\psi\psi H^3 + \text{h.c.}$	
$Q_{\nu H}$	$\lvert H\rvert^2 (\bar l \widetilde{H} \nu)$
Q_{eH}	$\lvert H\rvert^2 (\bar l H e)$
Q_{uH}	$\lvert H\rvert^2 (\bar q \widetilde{H} u)$
Q_{dH}	$\lvert H\rvert^2 (\bar q H d)$

$4: X^2 H^2$	
Q_{HG}	$\lvert H\rvert^2 G_{\mu\nu}^A G^{A\mu\nu}$
$Q_{H\widetilde{G}}$	$\lvert H\rvert^2 \widetilde{G}_{\mu\nu}^A G^{A\mu\nu}$
Q_{HW}	$\lvert H\rvert^2 W_{\mu\nu}^a W^{a\mu\nu}$
$Q_{H\widetilde{W}}$	$\lvert H\rvert^2 \widetilde{W}_{\mu\nu}^a W^{a\mu\nu}$
Q_{HB}	$\lvert H\rvert^2 B_{\mu\nu} B^{\mu\nu}$
$Q_{H\widetilde{B}}$	$\lvert H\rvert^2 \widetilde{B}_{\mu\nu} B^{\mu\nu}$
Q_{HWB}	$H^\dagger \tau^a H\, W_{\mu\nu}^a B^{\mu\nu}$
$Q_{H\widetilde{W}B}$	$H^\dagger \tau^a H\, \widetilde{W}_{\mu\nu}^a B^{\mu\nu}$

$6: \bar\psi\psi X H + \text{h.c.}$	
$Q_{\nu W}$	$(\bar l \sigma^{\mu\nu} \nu) \tau^a \widetilde{H} W_{\mu\nu}^a$
Q_{eW}	$(\bar l \sigma^{\mu\nu} e) \tau^a H W_{\mu\nu}^a$
$Q_{\nu B}$	$(\bar l \sigma^{\mu\nu} \nu) \widetilde{H} B_{\mu\nu}$
Q_{eB}	$(\bar l \sigma^{\mu\nu} e) H B_{\mu\nu}$
Q_{uG}	$(\bar q \sigma^{\mu\nu} T^A u) \widetilde{H} G_{\mu\nu}^A$
Q_{dG}	$(\bar q \sigma^{\mu\nu} T^A d) H G_{\mu\nu}^A$
Q_{uW}	$(\bar q \sigma^{\mu\nu} u) \tau^a \widetilde{H} W_{\mu\nu}^a$
Q_{dW}	$(\bar q \sigma^{\mu\nu} d) \tau^a H W_{\mu\nu}^a$
Q_{uB}	$(\bar q \sigma^{\mu\nu} u) \widetilde{H} B_{\mu\nu}$
Q_{dB}	$(\bar q \sigma^{\mu\nu} d) H B_{\mu\nu}$

$7: \bar\psi\psi H^2 D$	
$Q_{Hl}^{(1)}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar l \gamma^\mu l)$
$Q_{Hl}^{(3)}$	$(H^\dagger i \overleftrightarrow{D}_\mu^a H)(\bar l \gamma^\mu \tau^a l)$
$Q_{Hq}^{(1)}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar q \gamma^\mu q)$
$Q_{Hq}^{(3)}$	$(H^\dagger i \overleftrightarrow{D}_\mu^a H)(\bar q \gamma^\mu \tau^a q)$
$Q_{H\nu}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar \nu \gamma^\mu \nu)$
Q_{He}	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar e \gamma^\mu e)$
$Q_{H\nu e} + \text{h.c.}$	$(\widetilde{H}^\dagger i D_\mu H)(\bar \nu \gamma^\mu e)$
Q_{Hu}	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar u \gamma^\mu u)$
Q_{Hd}	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar d \gamma^\mu d)$
$Q_{Hud} + \text{h.c.}$	$(\widetilde{H}^\dagger i D_\mu H)(\bar u \gamma^\mu d)$

$8: (\bar L R)(\bar R L) + \text{h.c.}$	
$Q_{l\nu uq}$	$(\bar l^i \nu)(\bar u q^i)$
Q_{ledq}	$(\bar l^i e)(\bar d q^i)$

$8: (\bar L L)(\bar L L)$	
Q_{ll}	$(\bar l \gamma_\mu l)(\bar l \gamma^\mu l)$
$Q_{qq}^{(1)}$	$(\bar q \gamma_\mu q)(\bar q \gamma^\mu q)$
$Q_{qq}^{(3)}$	$(\bar q \gamma_\mu \tau^a q)(\bar q \gamma^\mu \tau^a q)$
$Q_{lq}^{(1)}$	$(\bar l \gamma_\mu l)(\bar q \gamma^\mu q)$
$Q_{lq}^{(3)}$	$(\bar l \gamma_\mu \tau^a l)(\bar q \gamma^\mu \tau^a q)$

$8: (\bar R R)(\bar R R)$	
$Q_{\nu\nu}$	$(\bar \nu \gamma_\mu \nu)(\bar \nu \gamma^\mu \nu)$
Q_{ee}	$(\bar e \gamma_\mu e)(\bar e \gamma^\mu e)$
$Q_{\nu e}$	$(\bar \nu \gamma_\mu \nu)(\bar e \gamma^\mu e)$
Q_{uu}	$(\bar u \gamma_\mu u)(\bar u \gamma^\mu u)$
Q_{dd}	$(\bar d \gamma_\mu d)(\bar d \gamma^\mu d)$
$Q_{ud}^{(1)}$	$(\bar u \gamma_\mu u)(\bar d \gamma^\mu d)$
$Q_{ud}^{(8)}$	$(\bar u \gamma_\mu T^A u)(\bar d \gamma^\mu T^A d)$
$Q_{\nu u}$	$(\bar \nu \gamma_\mu \nu)(\bar u \gamma^\mu u)$
$Q_{\nu d}$	$(\bar \nu \gamma_\mu \nu)(\bar d \gamma^\mu d)$
Q_{eu}	$(\bar e \gamma_\mu e)(\bar u \gamma^\mu u)$
Q_{ed}	$(\bar e \gamma_\mu e)(\bar d \gamma^\mu d)$
$Q_{\nu edu} + \text{h.c.}$	$(\bar \nu \gamma_\mu e)(\bar d \gamma^\mu u)$

$8: (\bar L L)(\bar R R)$	
$Q_{l\nu}$	$(\bar l \gamma_\mu l)(\bar \nu \gamma^\mu \nu)$
Q_{le}	$(\bar l \gamma_\mu l)(\bar e \gamma^\mu e)$
Q_{lu}	$(\bar l \gamma_\mu l)(\bar u \gamma^\mu u)$
Q_{ld}	$(\bar l \gamma_\mu l)(\bar d \gamma^\mu d)$
$Q_{q\nu}$	$(\bar q \gamma_\mu q)(\bar \nu \gamma^\mu \nu)$
Q_{qe}	$(\bar q \gamma_\mu q)(\bar e \gamma^\mu e)$
$Q_{qu}^{(1)}$	$(\bar q \gamma_\mu q)(\bar u \gamma^\mu u)$
$Q_{qd}^{(1)}$	$(\bar q \gamma_\mu q)(\bar d \gamma^\mu d)$
$Q_{qu}^{(8)}$	$(\bar q \gamma_\mu T^A q)(\bar u \gamma^\mu T^A u)$
$Q_{qd}^{(8)}$	$(\bar q \gamma_\mu T^A q)(\bar d \gamma^\mu T^A d)$

$8: (\bar L R)(\bar L R) + \text{h.c.}$	
$Q_{l\nu le}$	$(\bar l^i \nu)\epsilon_{ij}(\bar l^j e)$
$Q_{quqd}^{(1)}$	$(\bar q^i u)\epsilon_{ij}(\bar q^j d)$
$Q_{quqd}^{(8)}$	$(\bar q^i T^A u)\epsilon_{ij}(\bar q^j T^A d)$
$Q_{l\nu qd}^{(1)}$	$(\bar l^i \nu)\epsilon_{ij}(\bar q^j d)$
$Q_{lequ}^{(1)}$	$(\bar l^i e)\epsilon_{ij}(\bar q^j u)$
$Q_{l\nu qd}^{(3)}$	$(\bar l^i \sigma_{\mu\nu} \nu)\epsilon_{ij}(\bar q^j \sigma^{\mu\nu} d)$
$Q_{lequ}^{(3)}$	$(\bar l^i \sigma_{\mu\nu} e)\epsilon_{ij}(\bar q^j \sigma^{\mu\nu} u)$

Construct custodial basis: $\mathcal{L}_{\text{SMEFT}} - \mathcal{L}_{\text{SM}} = \sum_i C_i Q_i = \sum_i a_i O_i$

$$Q_i \leftrightarrow O_i$$
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$$Q_i \leftrightarrow O_i$$
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$$\begin{cases} Q_{uH} = |H|^2 \left(\bar{q}\tilde{H}u\right) \\ Q_{dH} = |H|^2 \left(\bar{q}H d\right) \end{cases}, \quad q_R \equiv \begin{pmatrix} u \\ d \end{pmatrix}, \quad \begin{cases} P_+ \equiv 1 \\ P_- \equiv \tau_R^3 \end{cases}$$

$$O_{qH}^{\pm} \equiv \text{tr}\left(\Sigma^\dagger \Sigma\right)\left(\bar{q}\,\Sigma P_\pm q_R\right) = 2\left(Q_{uH} \pm Q_{dH}\right)$$

Construct custodial basis: $\mathcal{L}_{\text{SMEFT}} - \mathcal{L}_{\text{SM}} = \sum_i C_i Q_i = \sum_i a_i O_i$

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$$O_{qH}^{\pm} \equiv \text{tr}\left(\Sigma^\dagger \Sigma\right)\left(\overline{q}\, \Sigma P_\pm q_R\right) = 2\left(Q_{uH} \pm Q_{dH}\right)$$

$$\text{tr}\left(\Sigma^\dagger i D_\mu \Sigma\right) = i\left(H^\dagger D_\mu H + \text{h.c.}\right) = i\, \partial_\mu |H|^2$$
$$\text{tr}\left(\Sigma^\dagger i D_\mu \Sigma \tau_R^3\right) = -i\left(H^\dagger D_\mu H - \text{h.c.}\right) = -H^\dagger i \overleftrightarrow{D}_\mu H$$

Construct custodial basis: $\mathcal{L}_{\text{SMEFT}} - \mathcal{L}_{\text{SM}} = \sum_i C_i Q_i = \sum_i a_i O_i$

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$$\begin{cases} Q_{uH} = |H|^2 (\bar{q}\tilde{H} u) \\ Q_{dH} = |H|^2 (\bar{q} H d) \end{cases}, \quad q_R \equiv \begin{pmatrix} u \\ d \end{pmatrix}, \quad \begin{cases} P_+ \equiv 1 \\ P_- \equiv \tau_R^3 \end{cases}$$

$$O_{qH}^{\pm} \equiv \text{tr}(\Sigma^\dagger \Sigma)(\bar{q}\,\Sigma P_{\pm} q_R) = 2(Q_{uH} \pm Q_{dH})$$

$$\begin{aligned} \text{tr}(\Sigma^\dagger i D_\mu \Sigma) &= i(H^\dagger D_\mu H + \text{h.c.}) = i\,\partial_\mu |H|^2 \\ \text{tr}(\Sigma^\dagger i D_\mu \Sigma \tau_R^3) &= -i(H^\dagger D_\mu H - \text{h.c.}) = -H^\dagger i \overleftrightarrow{D}_\mu H \end{aligned}$$

$$Q_{H\Box} = -(\partial_\mu |H|^2)^2 = \left[\text{tr}(\Sigma^\dagger i D_\mu \Sigma)\right]^2 \equiv O_{H\Box}$$

$$\begin{aligned} 4Q_{HD} = 4|H^\dagger D_\mu H|^2 &= (H^\dagger D_\mu H + \text{h.c.})^2 - (H^\dagger D_\mu H - \text{h.c.})^2 \\ &= -\left[\text{tr}(\Sigma^\dagger i D_\mu \Sigma)\right]^2 + \left[\text{tr}(\Sigma^\dagger i D_\mu \Sigma \tau_R^3)\right]^2 \\ &= -O_{H\Box} + O_{HD} \end{aligned}$$

Construct custodial basis: $\mathcal{L}_{\text{SMEFT}} - \mathcal{L}_{\text{SM}} = \sum_i C_i Q_i = \sum_i a_i O_i$

$$Q_i \leftrightarrow O_i \qquad C_i \leftrightarrow a_i$$

$$\begin{cases} Q_{uH} = |H|^2 \left(\bar{q}\tilde{H}u\right) \\ Q_{dH} = |H|^2 \left(\bar{q}Hd\right) \end{cases}, \quad q_R \equiv \begin{pmatrix} u \\ d \end{pmatrix}, \quad \begin{cases} P_+ \equiv 1 \\ P_- \equiv \tau_R^3 \end{cases}$$

$$O_{qH}^{\pm} \equiv \text{tr}\left(\Sigma^\dagger \Sigma\right)\left(\bar{q}\,\Sigma P_{\pm} q_R\right) = 2\left(Q_{uH} \pm Q_{dH}\right)$$

$$\text{tr}\left(\Sigma^\dagger i D_\mu \Sigma\right) = i\left(H^\dagger D_\mu H + \text{h.c.}\right) = i\,\partial_\mu |H|^2$$

$$\text{tr}\left(\Sigma^\dagger i D_\mu \Sigma \tau_R^3\right) = -i\left(H^\dagger D_\mu H - \text{h.c.}\right) = -H^\dagger i \overleftrightarrow{D}_\mu H$$

$$Q_{H\square} = -\left(\partial_\mu |H|^2\right)^2 = \left[\text{tr}\left(\Sigma^\dagger i D_\mu \Sigma\right)\right]^2 \equiv O_{H\square}$$

$$\begin{aligned} 4Q_{HD} = 4\left|H^\dagger D_\mu H\right|^2 &= \left(H^\dagger D_\mu H + \text{h.c.}\right)^2 - \left(H^\dagger D_\mu H - \text{h.c.}\right)^2 \\ &= -\left[\text{tr}\left(\Sigma^\dagger i D_\mu \Sigma\right)\right]^2 + \left[\text{tr}\left(\Sigma^\dagger i D_\mu \Sigma \tau_R^3\right)\right]^2 \\ &= -O_{H\square} + O_{HD} \end{aligned}$$

$$\begin{aligned} Q_{Hl}^{(1)} &\equiv \left(H^\dagger i \overleftrightarrow{D}_\mu H\right)\left(\bar{l}\gamma^\mu l\right) \\ &= -\,\text{tr}\left(\Sigma^\dagger i D_\mu \Sigma \tau_R^3\right)\left(\bar{l}\gamma^\mu l\right) \equiv -O_{Hl}^{(1)} \end{aligned}$$

$$\begin{aligned} Q_{Hl}^{(3)} &\equiv \left(H^\dagger i \overleftrightarrow{D}^a_\mu H\right)\left(\bar{l}\gamma^\mu \tau^a l\right) \\ &= \text{tr}\left(\Sigma^\dagger \tau^a i D_\mu \Sigma\right)\left(\bar{l}\gamma^\mu \tau^a l\right) \equiv O_{Hl}^{(3)} \end{aligned}$$

νSMEFT

Custodial basis

G. D. Kribs, XL, A. Martin, and T. Tong, arXiv:2009.10725

$1: X^3$	
O_G	$f^{ABC}G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$
$O_{\widetilde{G}}$	$f^{ABC}\widetilde{G}_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$
O_W	$\epsilon^{abc}W_\mu^{a\nu}W_\nu^{b\rho}W_\rho^{c\mu}$
$O_{\widetilde{W}}$	$\epsilon^{abc}\widetilde{W}_\mu^{a\nu}W_\nu^{b\rho}W_\rho^{c\mu}$

$2: H^6$	
O_H	$\left[\mathrm{tr}\,(\Sigma^\dagger\Sigma)\right]^3$

$3: H^4D^2$	
$O_{H\Box}$	$\left[\mathrm{tr}\,(\Sigma^\dagger iD_\mu\Sigma)\right]^2$
O_{HD}	$\left[\mathrm{tr}\,(\Sigma^\dagger iD_\mu\Sigma\tau^3_R)\right]^2$

$5: \bar\psi\psi H^3 + \mathrm{h.c.}$	
$O^\pm_{lH}$	$\mathrm{tr}(\Sigma^\dagger\Sigma)\left(\bar l\Sigma P_\pm l_R\right)$
$O^\pm_{qH}$	$\mathrm{tr}(\Sigma^\dagger\Sigma)\left(\bar q\Sigma P_\pm q_R\right)$

$4: X^2H^2$	
O_{HG}	$\mathrm{tr}\,(\Sigma^\dagger\Sigma)\,G^A_{\mu\nu}G^{A\mu\nu}$
$O_{H\widetilde{G}}$	$\mathrm{tr}\,(\Sigma^\dagger\Sigma)\,\widetilde{G}^A_{\mu\nu}G^{A\mu\nu}$
O_{HW}	$\mathrm{tr}\,(\Sigma^\dagger\Sigma)\,W^a_{\mu\nu}W^{a\mu\nu}$
$O_{H\widetilde{W}}$	$\mathrm{tr}\,(\Sigma^\dagger\Sigma)\,\widetilde{W}^a_{\mu\nu}W^{a\mu\nu}$
O_{HB}	$\mathrm{tr}\,(\Sigma^\dagger\Sigma)\,B_{\mu\nu}B^{\mu\nu}$
$O_{H\widetilde{B}}$	$\mathrm{tr}\,(\Sigma^\dagger\Sigma)\,\widetilde{B}_{\mu\nu}B^{\mu\nu}$
O_{HWB}	$\mathrm{tr}\,(\Sigma^\dagger\tau^a\Sigma\tau^3_R)\,W^a_{\mu\nu}B^{\mu\nu}$
$O_{H\widetilde{W}B}$	$\mathrm{tr}\,(\Sigma^\dagger\tau^a\Sigma\tau^3_R)\,\widetilde{W}^a_{\mu\nu}B^{\mu\nu}$

$6: \bar\psi\psi XH + \mathrm{h.c.}$	
$O^\pm_{lW}$	$(\bar l\sigma^{\mu\nu}\tau^a\Sigma P_\pm l_R)W^a_{\mu\nu}$
$O^\pm_{lB}$	$(\bar l\sigma^{\mu\nu}\Sigma P_\mp l_R)B_{\mu\nu}$
$O^\pm_{qG}$	$(\bar q\sigma^{\mu\nu}T^A\Sigma P_\pm q_R)G^A_{\mu\nu}$
$O^\pm_{qW}$	$(\bar q\sigma^{\mu\nu}\tau^a\Sigma P_\pm q_R)W^a_{\mu\nu}$
$O^\pm_{qB}$	$(\bar q\sigma^{\mu\nu}\Sigma P_\mp q_R)B_{\mu\nu}$

$7: \bar\psi\psi H^2D$	
$O^{(1)}_{Hl}$	$\mathrm{tr}\,(\Sigma^\dagger iD_\mu\Sigma\tau^3_R)\,(\bar l\gamma^\mu l)$
$O^{(3)}_{Hl}$	$\mathrm{tr}\,(\Sigma^\dagger\tau^a iD_\mu\Sigma)\,(\bar l\gamma^\mu\tau^a l)$
$O^{(1)}_{Hq}$	$\mathrm{tr}\,(\Sigma^\dagger iD_\mu\Sigma\tau^3_R)\,(\bar q\gamma^\mu q)$
$O^{(3)}_{Hq}$	$\mathrm{tr}\,(\Sigma^\dagger\tau^a iD_\mu\Sigma)\,(\bar q\gamma^\mu\tau^a q)$
$O^{(1)\pm}_{Hl_R}$	$\mathrm{tr}\,(\Sigma^\dagger iD_\mu\Sigma\tau^3_R)\left(\bar l_R\gamma^\mu P_\pm l_R\right)$
$O^{(3)\pm}_{Hl_R}$	$\mathrm{tr}\,(\Sigma^\dagger iD_\mu\Sigma\tau^a_R)\left(\bar l_R\gamma^\mu\tau^a_R P_\pm l_R\right)$
$O^{(1)\pm}_{Hq_R}$	$\mathrm{tr}\,(\Sigma^\dagger iD_\mu\Sigma\tau^3_R)\left(\bar q_R\gamma^\mu P_\pm q_R\right)$
$O^{(3)\pm}_{Hq_R}$	$\mathrm{tr}\,(\Sigma^\dagger iD_\mu\Sigma\tau^a_R)\left(\bar q_R\gamma^\mu\tau^a_R P_\pm q_R\right)$

$8: (\bar LR)(\bar RL) + \mathrm{h.c.}$	
$O^\pm_{ll_Rq_Rq}$	$(\bar l^i l^j_R)P^{jk}_\pm(\bar q^k_R q^i)$

$8: (\bar LL)(\bar LL)$	
O_{ll}	$(\bar l\gamma_\mu l)(\bar l\gamma^\mu l)$
$O^{(1)}_{qq}$	$(\bar q\gamma_\mu q)(\bar q\gamma^\mu q)$
$O^{(3)}_{qq}$	$(\bar q\gamma_\mu\tau^a q)(\bar q\gamma^\mu\tau^a q)$
$O^{(1)}_{lq}$	$(\bar l\gamma_\mu l)(\bar q\gamma^\mu q)$
$O^{(3)}_{lq}$	$(\bar l\gamma_\mu\tau^a l)(\bar q\gamma^\mu\tau^a q)$

$8: (\bar RR)(\bar RR)$	
$O^{\pm\pm}_{l_Rl_R}$	$(\bar l_R\gamma_\mu P_\pm l_R)(\bar l_R\gamma^\mu P_\pm l_R)$
$O^{+-}_{l_Rl_R}$	$(\bar l_R\gamma_\mu P_+ l_R)(\bar l_R\gamma^\mu P_- l_R)$
$O^{(1)\pm\pm}_{q_Rq_R}$	$(\bar q_R\gamma_\mu P_\pm q_R)(\bar q_R\gamma^\mu P_\pm q_R)$
$O^{(1)+-}_{q_Rq_R}$	$(\bar q_R\gamma_\mu P_+ q_R)(\bar q_R\gamma^\mu P_- q_R)$
$O^{(3)++}_{q_Rq_R}$	$(\bar q_R\gamma_\mu\tau^a_R q_R)(\bar q_R\gamma^\mu\tau^a_R q_R)$
$O^{(1)\pm\pm}_{l_Rq_R}$	$(\bar l_R\gamma_\mu P_\pm l_R)(\bar q_R\gamma^\mu P_\pm q_R)$
$O^{(1)\pm\mp}_{l_Rq_R}$	$(\bar l_R\gamma_\mu P_\pm l_R)(\bar q_R\gamma^\mu P_\mp q_R)$
$O^{(3)\pm+}_{l_Rq_R}$	$(\bar l_R\gamma_\mu\tau^a_R l_R)(\bar q_R\gamma^\mu\tau^a_R P_\pm q_R)$

$8: (\bar LL)(\bar RR)$	
$O^\pm_{ll_R}$	$(\bar l\gamma_\mu l)(\bar l_R\gamma^\mu P_\pm l_R)$
$O^\pm_{lq_R}$	$(\bar l\gamma_\mu l)(\bar q_R\gamma^\mu P_\pm q_R)$
$O^\pm_{ql_R}$	$(\bar q\gamma_\mu q)(\bar l_R\gamma^\mu P_\pm l_R)$
$O^{(1)\pm}_{qq_R}$	$(\bar q\gamma_\mu q)(\bar q_R\gamma^\mu P_\pm q_R)$
$O^{(8)\pm}_{qq_R}$	$(\bar q\gamma_\mu T^A q)(\bar q_R\gamma^\mu T^A P_\pm q_R)$

$8: (\bar LR)(\bar LR) + \mathrm{h.c.}$	
$O_{ll_Rll_R}$	$(\bar l^i l^k_R)\epsilon_{ij}\epsilon_{kl}(\bar l^j l^l_R)$
$O^{(1)}_{qq_Rqq_R}$	$(\bar q^i q^k_R)\epsilon_{ij}\epsilon_{kl}(\bar q^j q^l_R)$
$O^{(8)}_{qq_Rqq_R}$	$(\bar q^i T^A q^k_R)\epsilon_{ij}\epsilon_{kl}(\bar q^j T^A q^l_R)$
$O^{(1)\pm}_{ll_Rqq_R}$	$(\bar l^i l^k_R)\epsilon_{ij}\left(\epsilon P_\pm\right)_{kl}(\bar q^j q^l_R)$
$O^{(3)\pm}_{ll_Rqq_R}$	$(\bar l^i\sigma_{\mu\nu}l^k_R)\epsilon_{ij}\left(\epsilon P_\pm\right)_{kl}(\bar q^j\sigma^{\mu\nu}q^l_R)$

An example set of precision observables

$$\left\{ \hat{\alpha}\,,\, \hat{G}_F\,,\, \hat{m}_Z^2\,,\, \hat{m}_W^2\,,\, \hat{\Gamma}_{Z\nu_L\bar{\nu}_L}\,,\, \hat{\Gamma}_{Ze_L\bar{e}_L}\,,\, \hat{\Gamma}_{Ze\bar{e}} \right\}$$

$$\mathrm{SM}: \quad g_1\,,\, g_2\,,\, v\,; \qquad \mathrm{BSM}: \quad C_i$$

An example set of precision observables

$$\left\{ \hat{\alpha} \,, \hat{G}_F \,, \hat{m}_Z^2 \,, \hat{m}_W^2 \,, \hat{\Gamma}_{Z\nu_L\bar{\nu}_L} \,, \hat{\Gamma}_{Ze_L\bar{e}_L} \,, \hat{\Gamma}_{Ze\bar{e}} \right\}$$

$$\text{SM}: \quad g_1 \,, g_2 \,, v \,; \qquad \text{BSM}: \quad C_i$$

$$\hat{\rho} \equiv \frac{\hat{m}_W^2}{\hat{m}_Z^2} \frac{2}{\hat{x}} \left(1 - \sqrt{1 - \hat{x}} \right)$$

$$\hat{r}_{Z\nu_L\bar{\nu}_L} \equiv \frac{24\pi}{\sqrt{2}\hat{G}_F \hat{m}_Z^3} \hat{\Gamma}_{Z\nu_L\bar{\nu}_L}$$

$$\hat{r}_{Ze_L\bar{e}_L} \equiv \frac{24\pi}{\sqrt{2}\hat{G}_F \hat{m}_Z^3 \left(1 - \hat{x} \right)} \hat{\Gamma}_{Ze_L\bar{e}_L}$$

$$\hat{r}_{Ze\bar{e}} \equiv \frac{24\pi}{\sqrt{2}\hat{G}_F \hat{m}_Z^3 \left(1 - \sqrt{1 - \hat{x}} \right)^2} \hat{\Gamma}_{Ze\bar{e}}$$

$$\hat{x} \equiv \frac{2\sqrt{2}\pi \hat{\alpha}_0 \left(m_Z^2 \right)}{\hat{G}_F \hat{m}_Z^2} = \sin^2 2\theta_0$$

An example set of precision observables

$$\left\{\hat{\alpha}\,,\hat{G}_F\,,\hat{m}_Z^2\,,\hat{m}_W^2\,,\hat{\Gamma}_{Z\nu_L\bar{\nu}_L}\,,\hat{\Gamma}_{Ze_L\bar{e}_L}\,,\hat{\Gamma}_{Ze\bar{e}}\right\}$$

$$\mathrm{SM}:\quad g_1\,,g_2\,,v\,;\qquad \mathrm{BSM}:\quad C_i$$

$$\hat{\rho}\equiv\frac{\hat{m}_W^2}{\hat{m}_Z^2}\frac{2}{\hat{x}}\left(1-\sqrt{1-\hat{x}}\right)=1+\frac{v^2}{c_{2\theta}}\left[-2s_\theta^2\left(\frac{c_\theta}{s_\theta}C_{HWB}+C_{Hl}^{(3)}\right)+\frac{1}{2}s_\theta^2C_{12}-\frac{1}{2}c_\theta^2C_{HD}\right]$$

$$\hat{r}_{Z\nu_L\bar{\nu}_L}\equiv\frac{24\pi}{\sqrt{2}\hat{G}_F\hat{m}_Z^3}\hat{\Gamma}_{Z\nu_L\bar{\nu}_L}=1+\;v^2\left[\frac{1}{2}C_{12}-\frac{1}{2}C_{HD}-2C_{Hl}^{(1)}\right]$$

$$\hat{r}_{Ze_L\bar{e}_L}\equiv\frac{24\pi}{\sqrt{2}\hat{G}_F\hat{m}_Z^3\left(1-\hat{x}\right)}\hat{\Gamma}_{Ze_L\bar{e}_L}=1+\frac{v^2}{c_{2\theta}^2}\left[-4s_\theta^2\left(\frac{c_\theta}{s_\theta}C_{HWB}+C_{Hl}^{(3)}\right)+\frac{1}{2}C_{12}-\frac{1}{2}C_{HD}+2c_{2\theta}C_{Hl}^{(1)}\right]$$

$$\hat{r}_{Ze\bar{e}}\equiv\frac{24\pi}{\sqrt{2}\hat{G}_F\hat{m}_Z^3\left(1-\sqrt{1-\hat{x}}\right)^2}\hat{\Gamma}_{Ze\bar{e}}=1+\frac{v^2}{c_{2\theta}}\left[2\left(\frac{c_\theta}{s_\theta}C_{HWB}+C_{Hl}^{(3)}\right)-\frac{1}{2}C_{12}+\frac{1}{2}C_{HD}-\frac{c_{2\theta}}{s_\theta^2}C_{He}\right]$$

$$\hat{x}\equiv\frac{2\sqrt{2}\pi\hat{\alpha}_0\left(m_Z^2\right)}{\hat{G}_F\hat{m}_Z^2}=\sin^2 2\theta_0$$

An example set of precision observables

$$\left\{\hat{\alpha}\,,\hat{G}_F\,,\hat{m}_Z^2\,,\hat{m}_W^2\,,\hat{\Gamma}_{Z\nu_L\bar{\nu}_L}\,,\hat{\Gamma}_{Ze_L\bar{e}_L}\,,\hat{\Gamma}_{Ze\bar{e}}\right\}$$

$$\boxed{\mathrm{SM}:\quad g_1\,,g_2\,,v\,;\qquad \mathrm{BSM}:\quad C_i} \longrightarrow a_i$$

$$\hat{\rho}\equiv\frac{\hat{m}_W^2}{\hat{m}_Z^2}\frac{2}{\hat{x}}\left(1-\sqrt{1-\hat{x}}\right)=1+\frac{v^2}{c_{2\theta}}\left[2s_\theta^2\left(\frac{2c_\theta}{s_\theta}a_{HWB}-a_{Hl}^{(3)}\right)+\frac{1}{2}s_\theta^2a_{12}-2c_\theta^2a_{HD}\right]$$

$$\hat{r}_{Z\nu_L\bar{\nu}_L}\equiv\frac{24\pi}{\sqrt{2}\hat{G}_F\hat{m}_Z^3}\hat{\Gamma}_{Z\nu_L\bar{\nu}_L}=1+v^2\left[\frac{1}{2}a_{12}-2a_{HD}+2a_{Hl}^{(1)}\right]$$

$$\hat{r}_{Ze_L\bar{e}_L}\equiv\frac{24\pi}{\sqrt{2}\hat{G}_F\hat{m}_Z^3(1-\hat{x})}\hat{\Gamma}_{Ze_L\bar{e}_L}=1+\frac{v^2}{c_{2\theta}^2}\left[4s_\theta^2\left(\frac{2c_\theta}{s_\theta}a_{HWB}-a_{Hl}^{(3)}\right)+\frac{1}{2}a_{12}-2a_{HD}-2c_{2\theta}a_{Hl}^{(1)}\right]$$

$$\hat{r}_{Ze\bar{e}}\equiv\frac{24\pi}{\sqrt{2}\hat{G}_F\hat{m}_Z^3\left(1-\sqrt{1-\hat{x}}\right)^2}\hat{\Gamma}_{Ze\bar{e}}=1+\frac{v^2}{c_{2\theta}}\left[-2\left(\frac{2c_\theta}{s_\theta}a_{HWB}-a_{Hl}^{(3)}\right)-\frac{1}{2}a_{12}+2a_{HD}\right.$$
$$\left.+\frac{c_{2\theta}}{s_\theta^2}\left(a_{Hl_R}^{(1)+}-a_{Hl_R}^{(1)-}-a_{Hl_R}^{(3)+}+a_{Hl_R}^{(3)-}\right)\right]$$

An example set of precision observables

$$\left\{\hat{\alpha}\,,\hat{G}_F\,,\hat{m}_Z^2\,,\hat{m}_W^2\,,\hat{\Gamma}_{Z\nu_L\bar{\nu}_L}\,,\hat{\Gamma}_{Ze_L\bar{e}_L}\,,\hat{\Gamma}_{Ze\bar{e}}\right\}$$

SM: $g_1\,, g_2\,, v\,;$ BSM: C_i $\longrightarrow$ a_i

$$\hat{\rho}\equiv\frac{\hat{m}_W^2}{\hat{m}_Z^2}\frac{2}{\hat{x}}\left(1-\sqrt{1-\hat{x}}\right)=1+\frac{v^2}{c_{2\theta}}\left[2s_\theta^2\left(\frac{2c_\theta}{s_\theta}a_{HWB}-a_{Hl}^{(3)}\right)+\frac{1}{2}s_\theta^2a_{12}-2c_\theta^2a_{HD}\right]$$

$$\hat{r}_{Z\nu_L\bar{\nu}_L}\equiv\frac{24\pi}{\sqrt{2}\hat{G}_F\hat{m}_Z^3}\hat{\Gamma}_{Z\nu_L\bar{\nu}_L}=1+v^2\left[\frac{1}{2}a_{12}-2a_{HD}+2a_{Hl}^{(1)}\right]$$

$$\hat{r}_{Ze_L\bar{e}_L}\equiv\frac{24\pi}{\sqrt{2}\hat{G}_F\hat{m}_Z^3\left(1-\hat{x}\right)}\hat{\Gamma}_{Ze_L\bar{e}_L}=1+\frac{v^2}{c_{2\theta}^2}\left[4s_\theta^2\left(\frac{2c_\theta}{s_\theta}a_{HWB}-a_{Hl}^{(3)}\right)+\frac{1}{2}a_{12}-2a_{HD}-2c_{2\theta}a_{Hl}^{(1)}\right]$$

$$\hat{r}_{Ze\bar{e}}\equiv\frac{24\pi}{\sqrt{2}\hat{G}_F\hat{m}_Z^3\left(1-\sqrt{1-\hat{x}}\right)^2}\hat{\Gamma}_{Ze\bar{e}}=1+\frac{v^2}{c_{2\theta}}\left[-2\left(\frac{2c_\theta}{s_\theta}a_{HWB}-a_{Hl}^{(3)}\right)-\frac{1}{2}a_{12}+2a_{HD}+\frac{c_{2\theta}}{s_\theta^2}\left(a_{Hl_R}^{(1)+}-a_{Hl_R}^{(1)-}-a_{Hl_R}^{(3)+}+a_{Hl_R}^{(3)-}\right)\right]$$

$$\boxed{\left(\hat{\rho}-1\right)+\frac{1}{2}\left(\hat{r}_{Z\nu_L\bar{\nu}_L}-1\right)-\frac{1}{2}c_{2\theta}\left(\hat{r}_{Ze_L\bar{e}_L}-1\right)\longrightarrow 0}$$

An example set of precision observables

$$\left\{\hat{\alpha}\,,\hat{G}_F\,,\hat{m}_Z^2\,,\hat{m}_W^2\,,\hat{\Gamma}_{Z\nu_L\bar{\nu}_L}\,,\hat{\Gamma}_{Ze_L\bar{e}_L}\,,\hat{\Gamma}_{Ze\bar{e}}\right\}$$

$$\text{SM}:\quad g_1\,,g_2\,,v\,;\qquad \text{BSM}:\quad C_i \longrightarrow a_i$$

$$\hat{\rho}\equiv\frac{\hat{m}_W^2}{\hat{m}_Z^2}\frac{2}{\hat{x}}\left(1-\sqrt{1-\hat{x}}\right)=1+\frac{v^2}{c_{2\theta}}\left[2s_\theta^2\left(\frac{2c_\theta}{s_\theta}a_{HWB}-a_{Hl}^{(3)}\right)+\frac{1}{2}s_\theta^2 a_{12}-2c_\theta^2 a_{HD}\right]$$

$$\hat{r}_{Z\nu_L\bar{\nu}_L}\equiv\frac{24\pi}{\sqrt{2}\hat{G}_F\,\hat{m}_Z^3}\hat{\Gamma}_{Z\nu_L\bar{\nu}_L}=1+\ v^2\left[\frac{1}{2}a_{12}-2a_{HD}+2a_{Hl}^{(1)}\right]$$

$$\hat{r}_{Ze_L\bar{e}_L}\equiv\frac{24\pi}{\sqrt{2}\hat{G}_F\,\hat{m}_Z^3\left(1-\hat{x}\right)}\hat{\Gamma}_{Ze_L\bar{e}_L}=1+\frac{v^2}{c_{2\theta}^2}\left[4s_\theta^2\left(\frac{2c_\theta}{s_\theta}a_{HWB}-a_{Hl}^{(3)}\right)+\frac{1}{2}a_{12}-2a_{HD}-2c_{2\theta}a_{Hl}^{(1)}\right]$$

$$\hat{r}_{Ze\bar{e}}\equiv\frac{24\pi}{\sqrt{2}\hat{G}_F\,\hat{m}_Z^3\left(1-\sqrt{1-\hat{x}}\right)^2}\hat{\Gamma}_{Ze\bar{e}}=1+\frac{v^2}{c_{2\theta}}\left[-2\left(\frac{2c_\theta}{s_\theta}a_{HWB}-a_{Hl}^{(3)}\right)-\frac{1}{2}a_{12}+2a_{HD}\right.$$

$$\left.+\frac{c_{2\theta}}{s_\theta^2}\left(a_{Hl_R}^{(1)+}-a_{Hl_R}^{(1)-}-a_{Hl_R}^{(3)+}+a_{Hl_R}^{(3)-}\right)\right]$$

$$\left(\hat{\rho}-1\right)+\frac{1}{2}\left(\hat{r}_{Z\nu_L\bar{\nu}_L}-1\right)-\frac{1}{2}c_{2\theta}\left(\hat{r}_{Ze_L\bar{e}_L}-1\right)\quad\longrightarrow\quad 0$$

$$=-\frac{1}{2}v^2\left[C_{HD}+4C_{Hl}^{(1)}\right]=-2v^2\left[a_{HD}-a_{Hl}^{(1)}\right]\equiv\alpha\mathscr{T}_l$$

$$Q_{HD}\equiv\left|H^\dagger D_\mu H\right|^2$$

$$Q_{Hl}^{(1)}\equiv\left(H^\dagger i\overleftrightarrow{D}_\mu H\right)\left(\bar{l}\gamma^\mu l\right)$$

$$\left[\frac{2c_\theta}{s_\theta}a_{HWB} - a_{Hl}^{(3)}\right] = -\left[\frac{c_\theta}{s_\theta}C_{HWB} + C_{Hl}^{(3)}\right]$$

$$\hat{\rho} = 1 + \frac{v^2}{c_{2\theta}}\left[-2s_\theta^2\left(\frac{c_\theta}{s_\theta}C_{HWB} + C_{Hl}^{(3)}\right) + \frac{1}{2}s_\theta^2 C_{12} - \frac{1}{2}c_\theta^2 C_{HD}\right]$$

$$\hat{r}_{Z\nu_L\bar{\nu}_L} = 1 + v^2\left[\frac{1}{2}C_{12} - \frac{1}{2}C_{HD} - 2C_{Hl}^{(1)}\right]$$

$$\hat{r}_{Ze_L\bar{e}_L} = 1 + \frac{v^2}{c_{2\theta}^2}\left[-4s_\theta^2\left(\frac{c_\theta}{s_\theta}C_{HWB} + C_{Hl}^{(3)}\right) + \frac{1}{2}C_{12} - \frac{1}{2}C_{HD} + 2c_{2\theta}C_{Hl}^{(1)}\right]$$

$$\hat{r}_{Ze\bar{e}} = 1 + \frac{v^2}{c_{2\theta}}\left[2\left(\frac{c_\theta}{s_\theta}C_{HWB} + C_{Hl}^{(3)}\right) - \frac{1}{2}C_{12} + \frac{1}{2}C_{HD} - \frac{c_{2\theta}}{s_\theta^2}C_{He}\right]$$

$$\left[\frac{2c_\theta}{s_\theta}a_{HWB} - a_{Hl}^{(3)}\right] = -\left[\frac{c_\theta}{s_\theta}C_{HWB} + C_{Hl}^{(3)}\right]$$

$$\hat{\rho} = 1 + \frac{v^2}{c_{2\theta}}\left[-2s_\theta^2\left(\frac{c_\theta}{s_\theta}C_{HWB} + C_{Hl}^{(3)}\right) + \frac{1}{2}s_\theta^2 C_{12} - \frac{1}{2}c_\theta^2 C_{HD}\right]$$

$$\hat{r}_{Z\nu_L\bar{\nu}_L} = 1 + v^2\left[\frac{1}{2}C_{12} - \frac{1}{2}C_{HD} - 2C_{Hl}^{(1)}\right]$$

$$\hat{r}_{Ze_L\bar{e}_L} = 1 + \frac{v^2}{c_{2\theta}^2}\left[-4s_\theta^2\left(\frac{c_\theta}{s_\theta}C_{HWB} + C_{Hl}^{(3)}\right) + \frac{1}{2}C_{12} - \frac{1}{2}C_{HD} + 2c_{2\theta}C_{Hl}^{(1)}\right]$$

$$\hat{r}_{Ze\bar{e}} = 1 + \frac{v^2}{c_{2\theta}}\left[2\left(\frac{c_\theta}{s_\theta}C_{HWB} + C_{Hl}^{(3)}\right) - \frac{1}{2}C_{12} + \frac{1}{2}C_{HD} - \frac{c_{2\theta}}{s_\theta^2}C_{He}\right]$$

SMEFT RPI

I. Brivio and M. Trott, “Scheming in the SMEFT… and a reparameterization invariance!” arXiv: 1701.06424

$$\left[\frac{2c_\theta}{s_\theta}a_{HWB} - a_{Hl}^{(3)}\right] = -\left[\frac{c_\theta}{s_\theta}C_{HWB} + C_{Hl}^{(3)}\right]$$

$$\hat{\rho} = 1 + \frac{v^2}{c_{2\theta}}\left[-2s_\theta^2\left(\frac{c_\theta}{s_\theta}C_{HWB} + C_{Hl}^{(3)}\right) + \frac{1}{2}s_\theta^2 C_{12} - \frac{1}{2}c_\theta^2 C_{HD}\right]$$

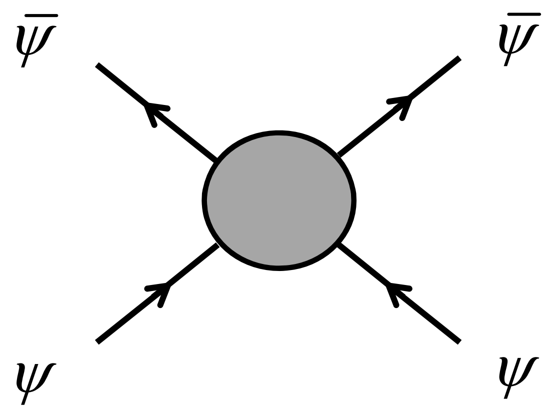
$$\hat{r}_{Z\nu_L\bar{\nu}_L} = 1 + v^2\left[\frac{1}{2}C_{12} - \frac{1}{2}C_{HD} - 2C_{Hl}^{(1)}\right]$$

$$\hat{r}_{Ze_L\bar{e}_L} = 1 + \frac{v^2}{c_{2\theta}^2}\left[-4s_\theta^2\left(\frac{c_\theta}{s_\theta}C_{HWB} + C_{Hl}^{(3)}\right) + \frac{1}{2}C_{12} - \frac{1}{2}C_{HD} + 2c_{2\theta}C_{Hl}^{(1)}\right]$$

$$\hat{r}_{Ze\bar{e}} = 1 + \frac{v^2}{c_{2\theta}}\left[2\left(\frac{c_\theta}{s_\theta}C_{HWB} + C_{Hl}^{(3)}\right) - \frac{1}{2}C_{12} + \frac{1}{2}C_{HD} - \frac{c_{2\theta}}{s_\theta^2}C_{He}\right]$$

SMEFT RPI

I. Brivio and M. Trott, "Scheming in the SMEFT… and a reparameterization invariance!" arXiv: 1701.06424



$$Q_{HHW} \equiv ig_2\left(D^\mu H\right)^\dagger \tau^a \left(D^\nu H\right)W_{\mu\nu}^a$$

$$Q_{HHB} \equiv ig_1\left(D^\mu H\right)^\dagger \left(D^\nu H\right)B_{\mu\nu}$$

$$\left[\frac{2c_\theta}{s_\theta}a_{HWB} - a_{Hl}^{(3)}\right] = -\left[\frac{c_\theta}{s_\theta}C_{HWB} + C_{Hl}^{(3)}\right]$$

$$\hat{\rho} = 1 + \frac{v^2}{c_{2\theta}}\left[-2s_\theta^2\left(\frac{c_\theta}{s_\theta}C_{HWB} + C_{Hl}^{(3)}\right) + \frac{1}{2}s_\theta^2 C_{12} - \frac{1}{2}c_\theta^2 C_{HD}\right]$$

$$\hat{r}_{Z\nu_L\bar{\nu}_L} = 1 + v^2\left[\frac{1}{2}C_{12} - \frac{1}{2}C_{HD} - 2C_{Hl}^{(1)}\right]$$

$$\hat{r}_{Ze_L\bar{e}_L} = 1 + \frac{v^2}{c_{2\theta}^2}\left[-4s_\theta^2\left(\frac{c_\theta}{s_\theta}C_{HWB} + C_{Hl}^{(3)}\right) + \frac{1}{2}C_{12} - \frac{1}{2}C_{HD} + 2c_{2\theta}C_{Hl}^{(1)}\right]$$

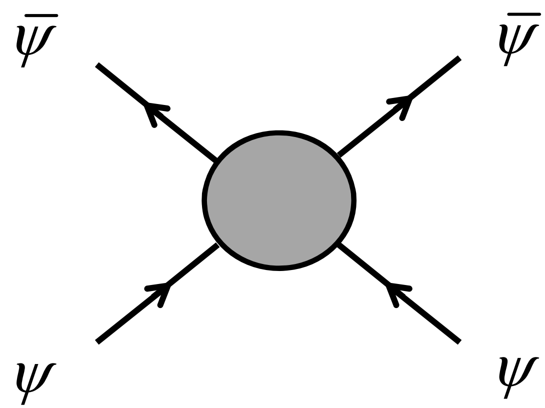
$$\hat{r}_{Ze\bar{e}} = 1 + \frac{v^2}{c_{2\theta}}\left[2\left(\frac{c_\theta}{s_\theta}C_{HWB} + C_{Hl}^{(3)}\right) - \frac{1}{2}C_{12} + \frac{1}{2}C_{HD} - \frac{c_{2\theta}}{s_\theta^2}C_{He}\right]$$

SMEFT RPI

I. Brivio and M. Trott, “Scheming in the SMEFT… and a reparameterization invariance!” arXiv: 1701.06424

$$\epsilon_W Q_{HHW} \quad \Rightarrow \quad \begin{pmatrix} C_{HWB} \\ C_{Hl}^{(3)} \end{pmatrix} \rightarrow \begin{pmatrix} C_{HWB} \\ C_{Hl}^{(3)} \end{pmatrix} + \epsilon_W \begin{pmatrix} -\tan\theta \\ 1 \end{pmatrix}$$

$$\epsilon_B Q_{HHB} \quad \Rightarrow \quad \begin{pmatrix} C_{HWB} \\ C_{HD} \\ C_{Hl}^{(1)} \\ C_{He} \end{pmatrix} \rightarrow \begin{pmatrix} C_{HWB} \\ C_{HD} \\ C_{Hl}^{(1)} \\ C_{He} \end{pmatrix} + \epsilon_B \begin{pmatrix} \cot\theta \\ -4 \\ 1 \\ 2 \end{pmatrix}$$



$$Q_{HHW} \equiv ig_2\left(D^\mu H\right)^\dagger \tau^a \left(D^\nu H\right) W_{\mu\nu}^a$$

$$Q_{HHB} \equiv ig_1\left(D^\mu H\right)^\dagger \left(D^\nu H\right) B_{\mu\nu}$$

Other examples of custodial violation indicators

$$\left(\hat{\rho}-1\right)+\frac{1}{2}\left(\hat{r}_{Z\nu_L\bar{\nu}_L}-1\right)-\frac{1}{2}c_{2\theta}\left(\hat{r}_{Ze_L\bar{e}_L}-1\right)$$
$$=-\frac{1}{2}v^2\left[C_{HD}+4C_{Hl}^{(1)}\right]=-2v^2\left[a_{HD}-a_{Hl}^{(1)}\right]\equiv\alpha\mathscr{T}_l$$

Other examples of custodial violation indicators

$$\left(\hat{\rho}-1\right)+\frac{1}{2}\left(\hat{r}_{Z\nu_L\bar{\nu}_L}-1\right)-\frac{1}{2}c_{2\theta}\left(\hat{r}_{Ze_L\bar{e}_L}-1\right)$$
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Potential complication from EOM redundancies

$$Q_{\text{outside Warsaw}} \quad \rightarrow \quad Q_{\text{Warsaw}}$$

custodial symmetric　　custodial violating

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$$\frac{1}{4}\left[(Y_u + Y_d)O_{qH}^{+} + (Y_u - Y_d)O_{qH}^{-} + (Y_\nu + Y_e)O_{lH}^{+} + (Y_\nu - Y_e)O_{lH}^{-}\right]$$

Application to our non-oblique example

$$\mathcal{L}_{\text{UV}} = \mathcal{L}_{\text{SM}} + \bar{N}\left(i\not{D} - M\right)N + \bar{E}\left(i\not{D} - M\right)E - \left(Y_N\,\bar{l}\,\tilde{H}\,N + Y_E\,\bar{l}\,H\,E + \text{h.c.}\right)$$

If $|Y_N| = |Y_E|$, $U_R \in SU(2)_R$ is a symmetry

$$\Sigma \equiv \begin{pmatrix}\tilde{H} & H\end{pmatrix} \longrightarrow \Sigma U_R^\dagger \quad , \quad \begin{pmatrix}N \\ E\end{pmatrix} \longrightarrow U_R \begin{pmatrix}N \\ E\end{pmatrix}$$

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Summary

- Veltman rho is NOT an indicator of custodial violation; it receives contributions also from custodial preserving BSM physics

- Peskin-Takeuchi T parameter works as an indicator of custodial violation only when the BSM physics is oblique

- Beyond oblique BSM scenario, *S*, *T*, *U*, *etc.* are no longer observables; we need new indicators of custodial violation

- We took dim-6 SMEFT at tree level as a first step, and constructed several working indicators of custodial violation

- The role of SMEFT RPI and EOM redundancies are also investigated