Holographic Transport.

Andreas Karch (University of Washington, Seattle)

(talk at UC Davis, 3/19/15)

Holography = Solvable Toy Model

Solvable models of strong coupling dynamics.

- Study Transport, real time
- Study Finite Density of electrons or quarks
- Study far from equilibrium

Common Theme: Experimentally relevant, calculations challenging.

Gives us qualitative guidance/intuition.

Why toy models?

Strong coupling =
no perturbation theory!

But can't we just do numerical simulations?

Challenge for Computers:

We do have methods for strong coupling:

e.g. Lattice QCD

But: typically relies on importance sampling.

e^{-S} weighting in Euclidean path integral.

Monte-Carlo techniques.

FAILS FOR DYNAMIC PROCESSES OR AT FINITE DENSITY (sign problem)

Holographic Toy models.

Can we at least get a qualitative understanding of what dynamics look like at strong coupling?

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Holographic Theories:

Examples known:

- in d=1, 2, 3, 4, 5, 6 space-time dimensions
- with or without super-symmetry
- conformal or confining
- with or without chiral symmetry breaking
- with finite temperature and density

Holographic Theories:

Holographic toy models have two key properties:

"Large N": theory is essentially classical

"Large λ": large separation of scales in the spectrum

$$m_{\text{spin-2-meson}} \sim \lambda^{1/4}\ m_{\text{spin-1-meson}}$$

QCD: **1275 MeV** **775 MeV**

(note: there are some exotic examples where the same parameter N controls both, classicality and separation of scales in spectrum)

Mathematical Foundations

The "glue":

Find asymptotically hyperbolic solutions to Einstein's equations. Full geometry includes compact internal factor.

Required geometric data found long ago by two mathematicians, Fefferman and Graham.

Mathematical Foundations

The “quarks”:

Find minimal area submanifolds in asymptotically Einstein spaces.

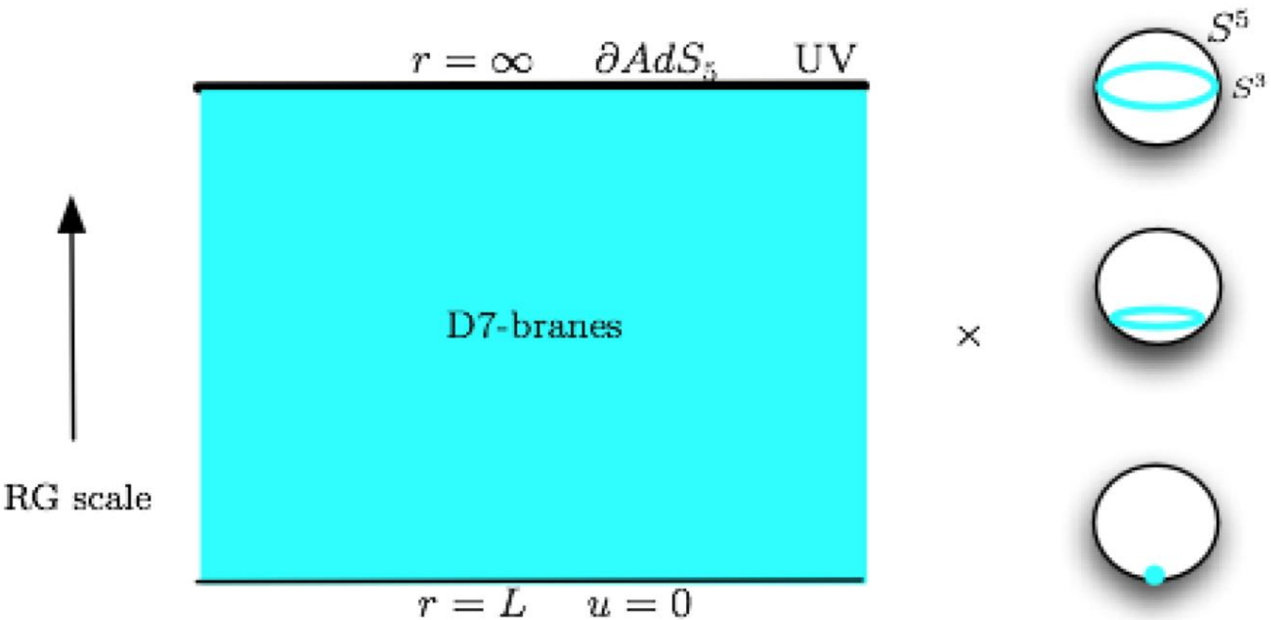
(AK, Katz)

Flavor Branes

Required geometric data for just asympt. Einstein constructed by Graham and Witten; generalized to include internal space by Graham and AK.

A holographic dual:

E.g: Maximally SUSY SU(N) YM with fundamental rep hypermultiplets:



(picture from CLMRW-review, 2011)

Applications to QCD Transport.

"The strong force […] is called the strong force because it is so strong"

(from Lisa Randall's "Warped Passages")

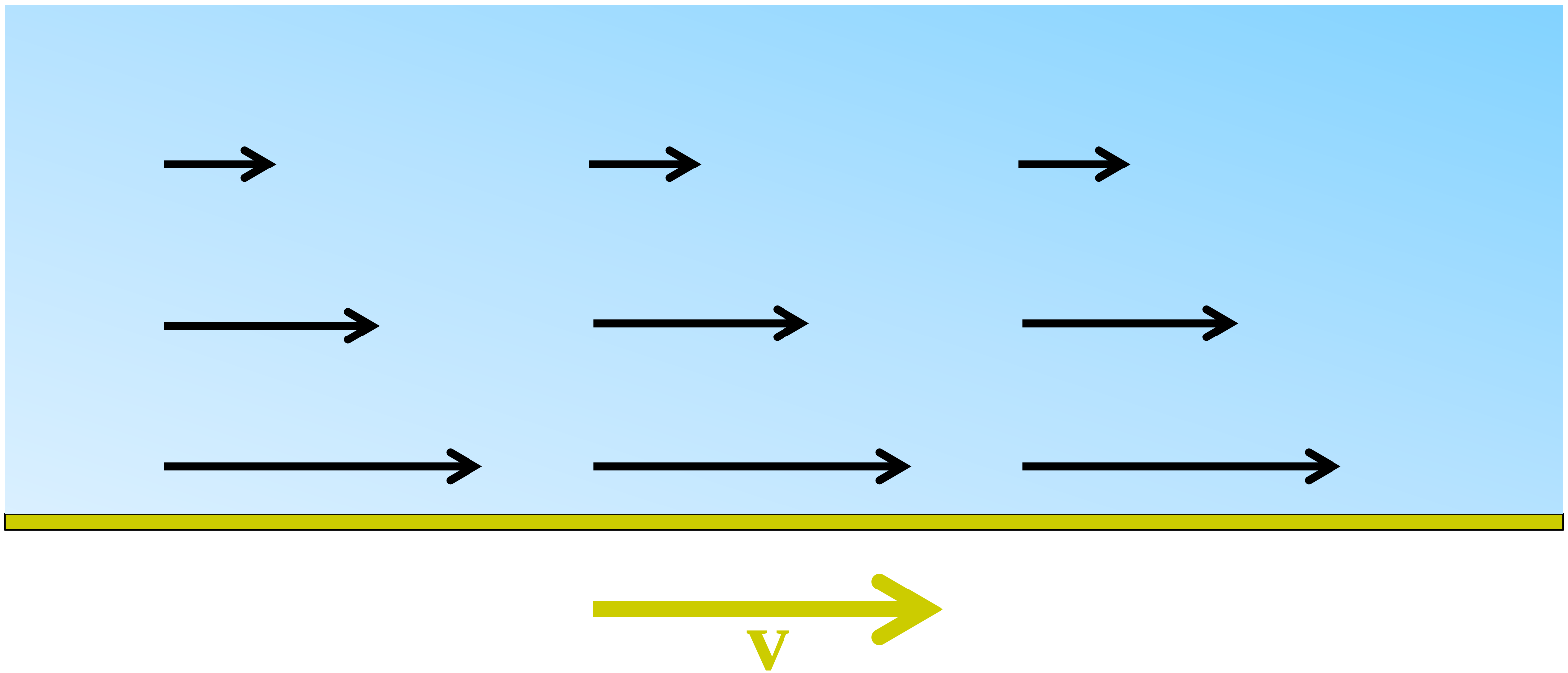
Applications to QCD Transport

(as experimentally probed in Heavy Ion Collisions)

- Viscosity and Hydrodynamics
- Energy Loss

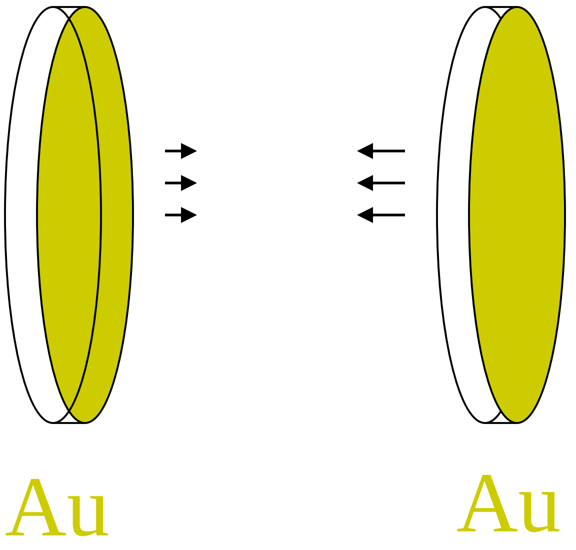
Shear Viscosity

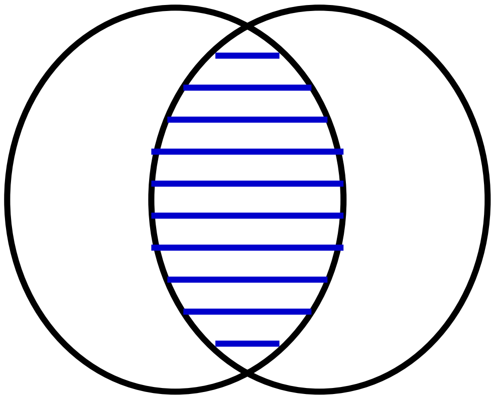
Viscosity = Diffusion constant for momentum

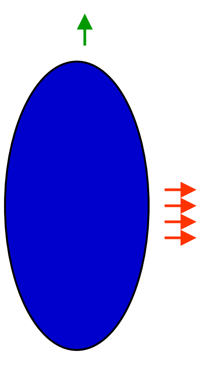


Viscosity = [(force/area)] per unit velocity gradient

Viscosity in Heavy Ions.







How does the almond shaped fluid expand?

Viscosity

Viscosity can be quantified:

water: 1 centipoise (cp)
air: 0.02 cp
honey: 2000-10000 cp

(1 cp = 10^{-2} P = 10^{-3} Pa·s)

Measuring Viscosity - an example

Pitch drop experiment

Started in 1930

8 drops fell so far

but no one has ever witnessed a drop fall

2005 Ig Nobel Prize in Physics

Viscosity of pitch: 230 billions times that of water

(2.3 10^{11}cp)

Measuring Viscosity - an example

Recall: Viscosity of pitch: ~ 2.3 10^{11}cp

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RHIC's measurement of QGP (confirmed by LHC):

$$\eta \sim \frac{\hbar}{4\pi} s \sim \frac{10^{-27}\text{erg}\cdot\text{s}}{(10^{-13}\text{cm})^3} \sim 10^{14}\text{cp}$$

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BNL press release 2005:

"The degree of collective interaction, rapid thermalization, and extremely low viscosity of the matter being form at RHIC makes this the most nearly perfect liquid ever observed."

Viscosity in Holography:

In a large class of systems:

$$\frac{\eta}{s} = \frac{\hbar}{4\pi}$$

(Kovtun, Son, Starinets)

- pinpoints correct observable
- in contrast to QGP, η/s enormous for pitch
- gives ball-park figure
- large at weak coupling: bound?

Viscosity – Recent Developments

Not a bound! (Kats, Petrov, 2007, using flavor branes)

$$\frac{\eta}{s} = \frac{1}{4\pi}\left(1 - \frac{1}{2N}\right)$$

$\mathcal{N} = 2$ Sp(N)
4 fundamental
1 antisymmetric traceless

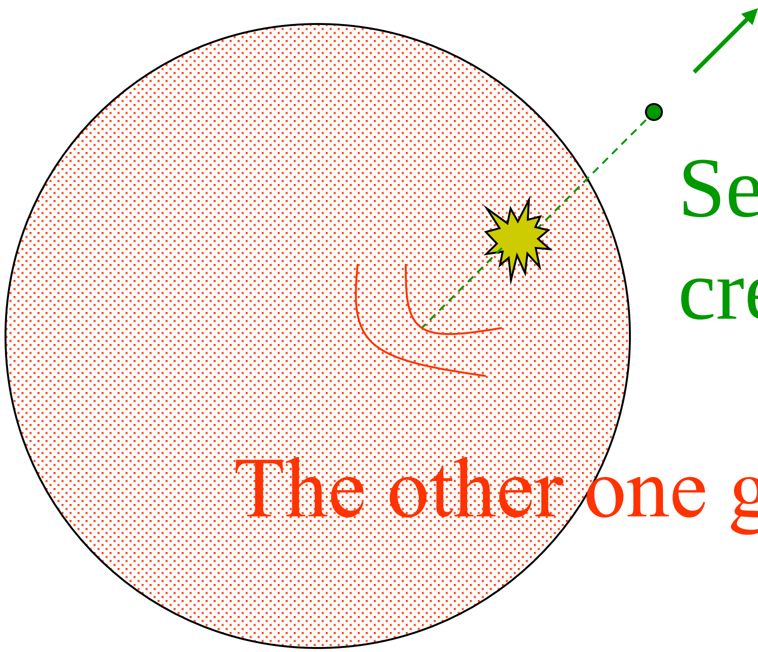
Higher Curvature corrections violate bound.

(Brigante, Liu, Myers, Shenker, Yaida, Buchel, Sinha, ….)

Calculations only reliable if violations are small.

Energy Loss

Energy Loss in Heavy Ions.

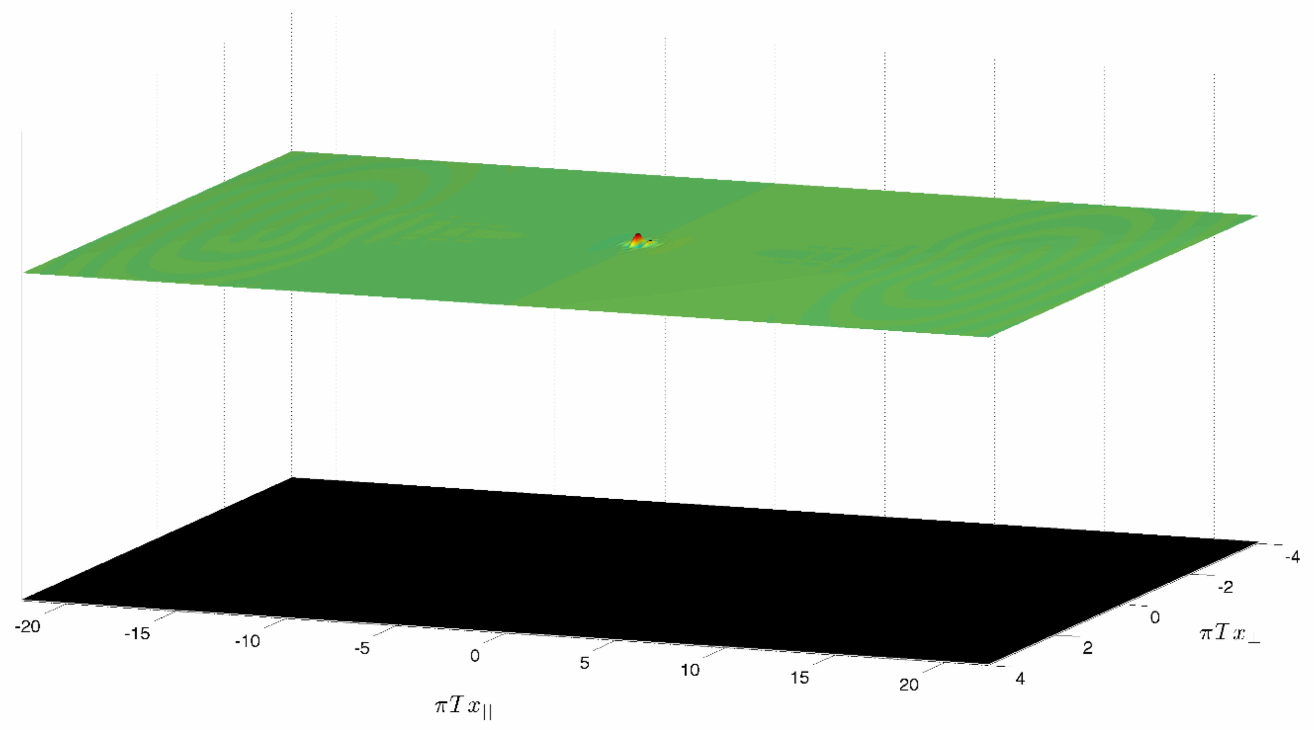


See one of two back-to-back created particles.

The other one got “stuck” in the fireball

Jet quenching is a direct indication of large drag.

Holographic Energy Loss



Observable: Stopping Distance

Perturbative QCD: **$L \sim E^{1/2}$** (BDMPS, …)

Holography:

Maximal Stopping Distance: **$L \sim E^{1/3}$**

(Chesler, Jensen, AK, Yaffe; Gubser, Gulotta, Pufu, Rocha)

Typical Stopping Distance: **$L \sim E^{1/4}$**

(Arnold, Vaman - 2011)

Experiment: **RHIC: holography good**

LHC: holography bad -- weak coupling?

Observable: Stopping Distance

Perturbative QCD: $L \sim E^{1/2}$ (BDMPS, …)

Holography: **Exponents!**

Maximal Stopping Distance: $L \sim E^{1/3}$

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Experiment: **RHIC: holography good**
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Applications to Condensed Matter Physics.

Strong Coupling in CM.

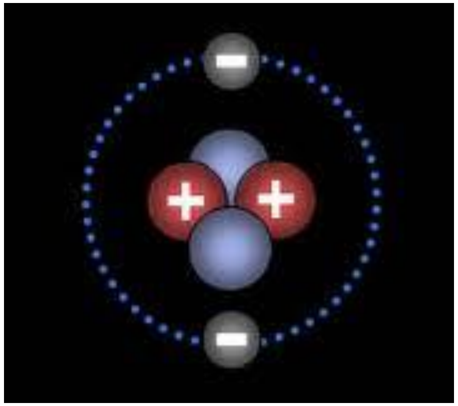
The theory of everything:

$$H = \sum_{Nuclei,A} \frac{P_A{}^2}{m_A} + \sum_{electron,i} \frac{p_i{}^2}{m_e} - \sum_{A,i} \frac{e^2}{|x_i - x_A|} + \sum_{i \neq j} \frac{e^2}{|x_i - x_j|}$$

How hard can it be?

Strong Coupling in CM

Already Helium too difficult to solve analytically.



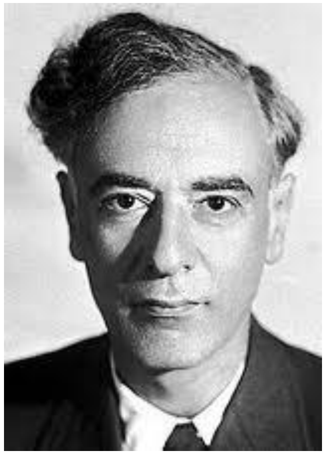
electron/electron Coulomb repulsion not weak!

if it is negligible, we have good theory control:

Band structure! Insulators and conductors.

but what to do when it is not?

Landau's paradigms:



- Identify physical candidates for low energy degrees of freedom.

dominate transport

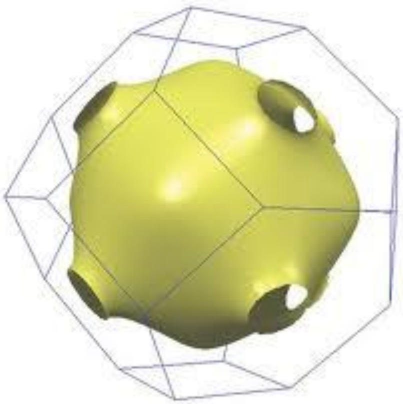
- Write down most general allowed interactions

many interactions "irrelevant" = scale to zero

- See how interactions scale in low energy limit

What could they be?

1) weakly coupled fermions.



Landau Fermi Liquid

- Fermi Surface
- Low energy excitations near Fermi Surface
- Only Cooper Pair Instability survives at low energies, all other interactions scale to zero

at low temperatures resistivity grows as T^2 (applies to the first two points)

← **universal!** (applies to the third point)

What could they be?

1) weakly coupled bosons.

Landau's Theory of Phase Transitions

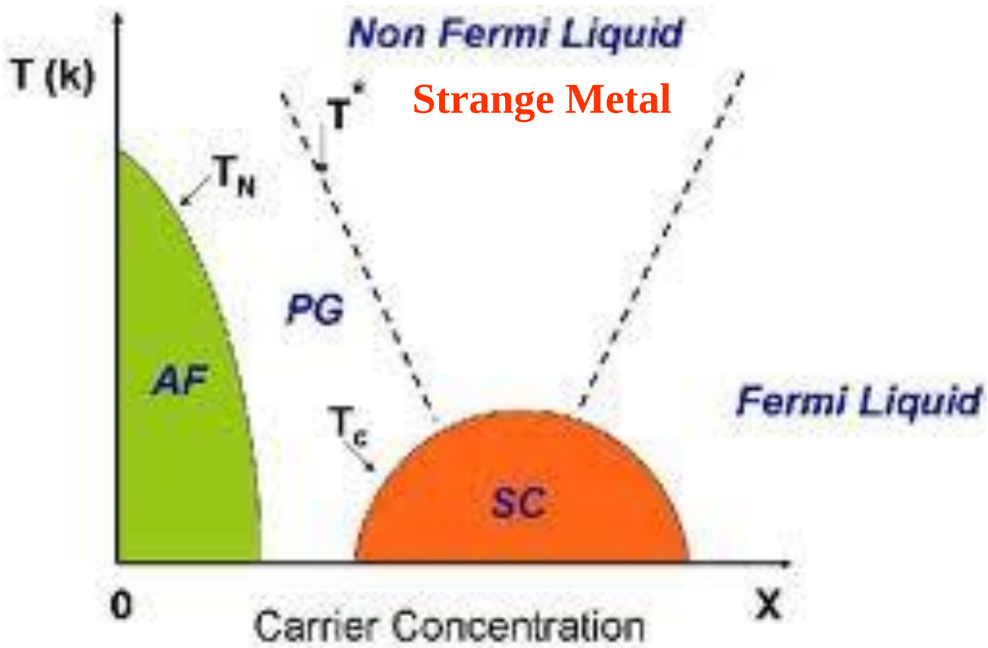
$$G(m,T) = \frac{b_0}{2}(T - T_c)m^2 + \frac{c}{4}m^4 + \frac{d}{6}m^6 + \cdots .$$

free energy

order parameter
= scalar field.

Scalar mass relevant; dominates at low energies.
Can be tuned to zero close to a phase transition.

Is this all?



Degrees of freedom in high Tc superconductors are neither!

Non-Fermi Liquid

at low temperatures resistivity grows as T

What else could it be?

Perfect questions to ask a solvable toy model:

- What are the possible low energy behaviors?
- Are their qualitative new phenomena hiding at strong coupling?

Two Applications

- Far from equilibrium steady states.
- Novel Scaling Exponents.

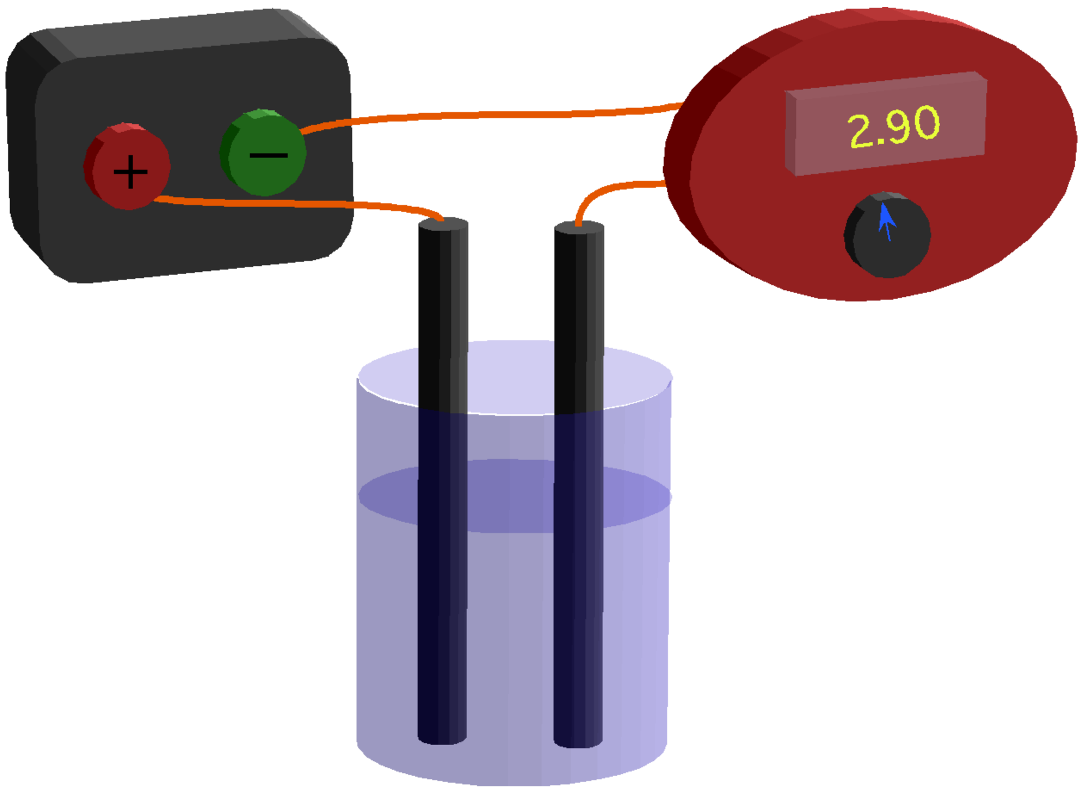
Steady States

Non-equilibrium

Strongly correlated non-equilibrium physics is intrinsically difficult, even in holography.

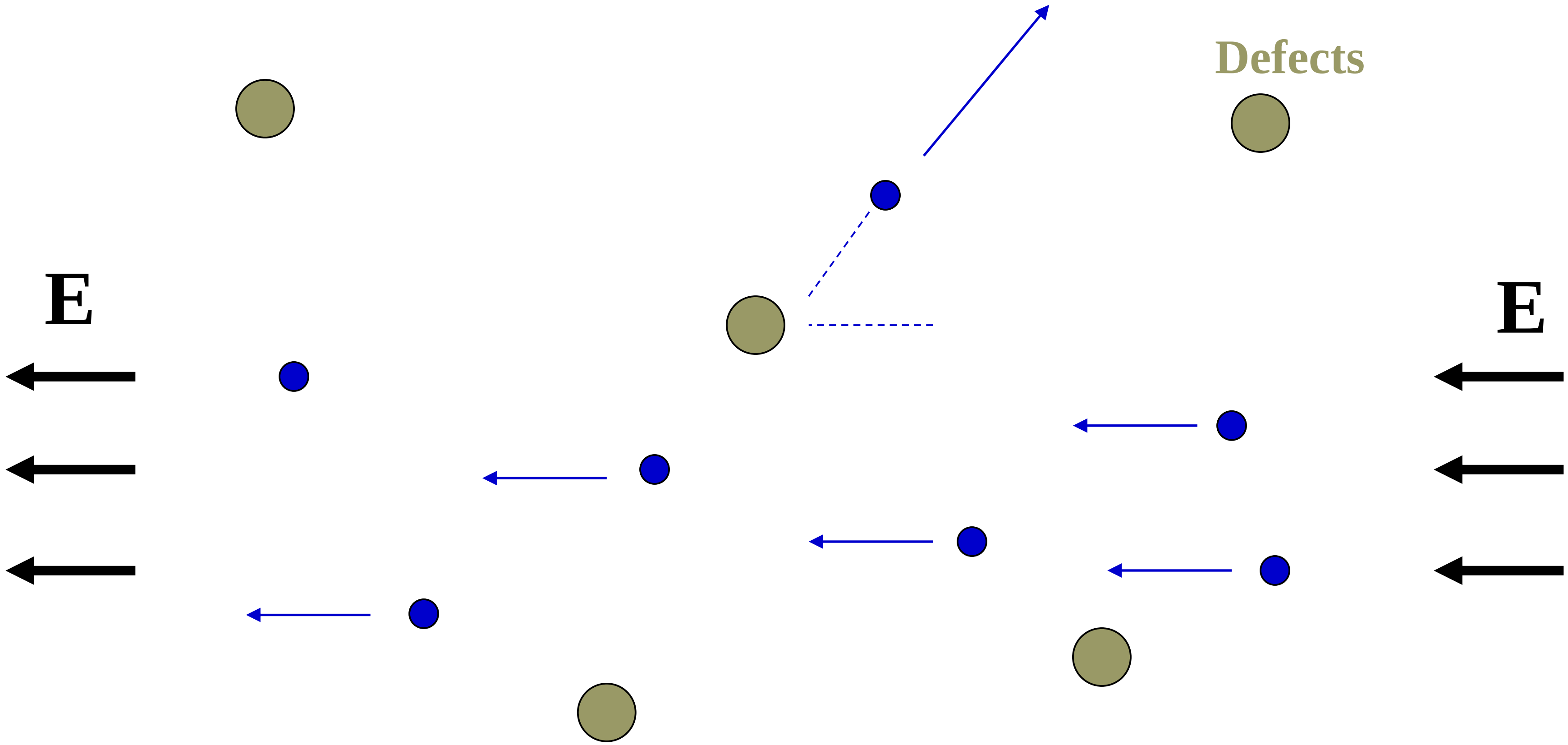
The simplest and most tractable non-equilibrium systems are non-equilibrium steady states.

DC Conductivity/Resistivity



one of the most basic transport properties of any matter/fluid

Steady State is Out of Equilibrium



Dissipation driven Steady States

Acceleration from electric field balanced
by momentum dissipation.

Typically requires broken translation invariance.

Constant Entropy Production. Ohmic Heating.

First Holographic Realization by AK and O'Bannon.

Quantum Critical Transport:

(AK, Shivaji Sondhi).

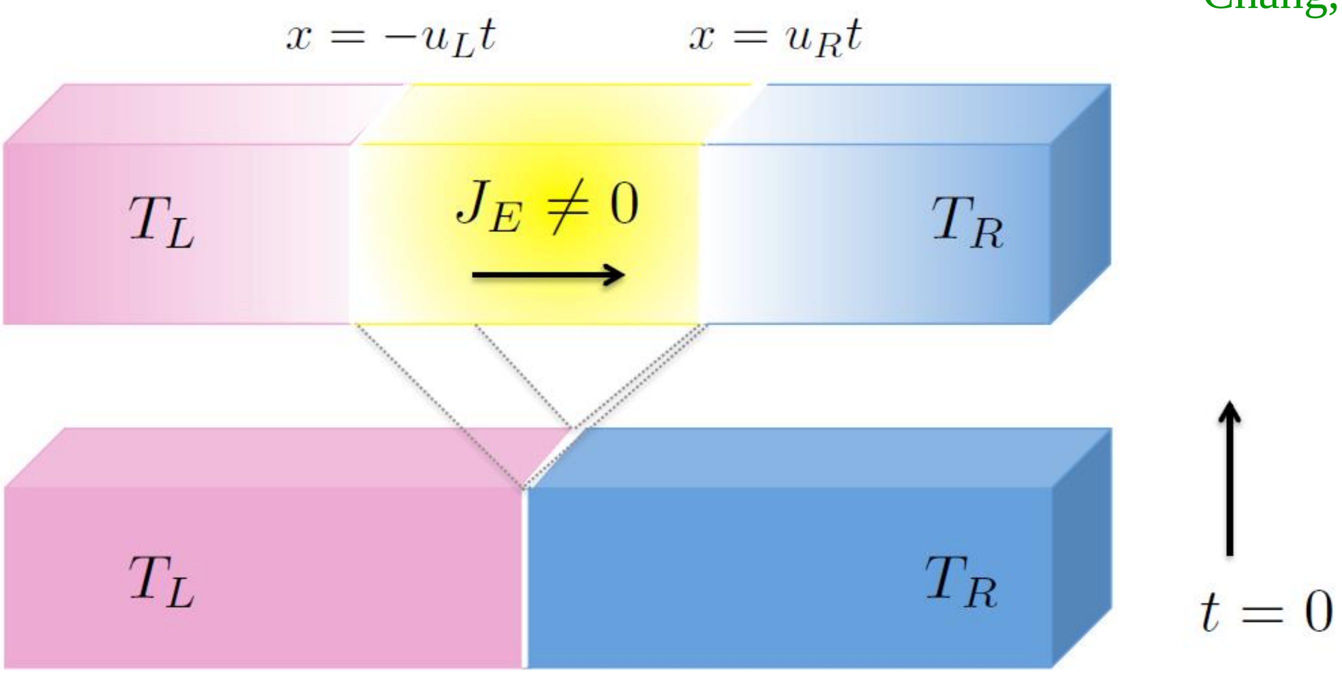
At quantum critical point DC conductivity non-linear!

$$j = \sigma E \propto E^{3/2}$$

Predicted by Greene and Sondhi based on scaling.
Holography provides only known calculable example.

Flow Driven Steady State

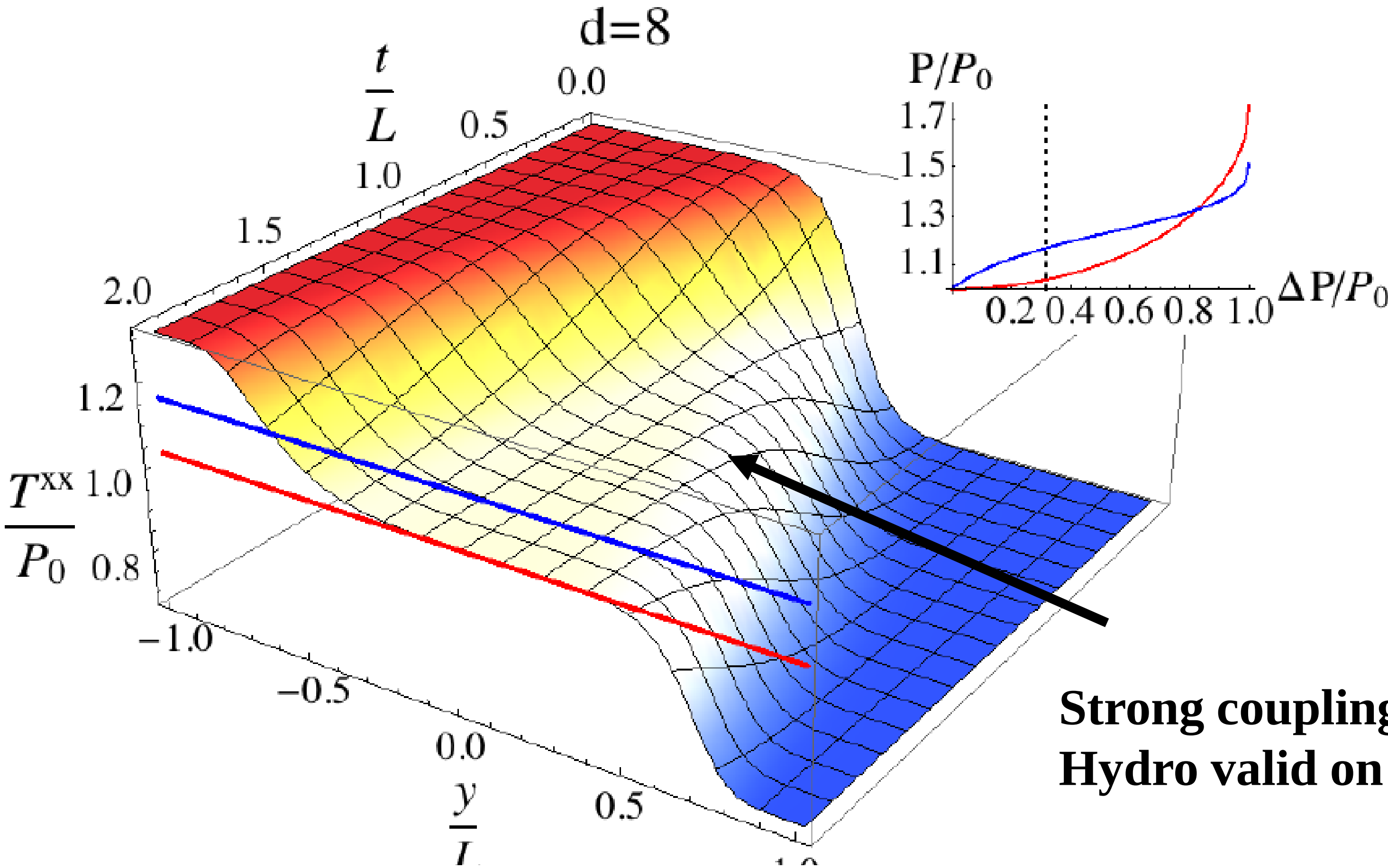
(Bernard, Doyon;
Doyon, Lucas, Schalm, Bhaseen;
Chang, AK, Yarom)



(intermediate time steady state)

(picture from Doyon,Lucas, Schalm, Bhaseen)

Flow Driven Steady State



Strong coupling=
Hydro valid on Plateau

Summary, steady states

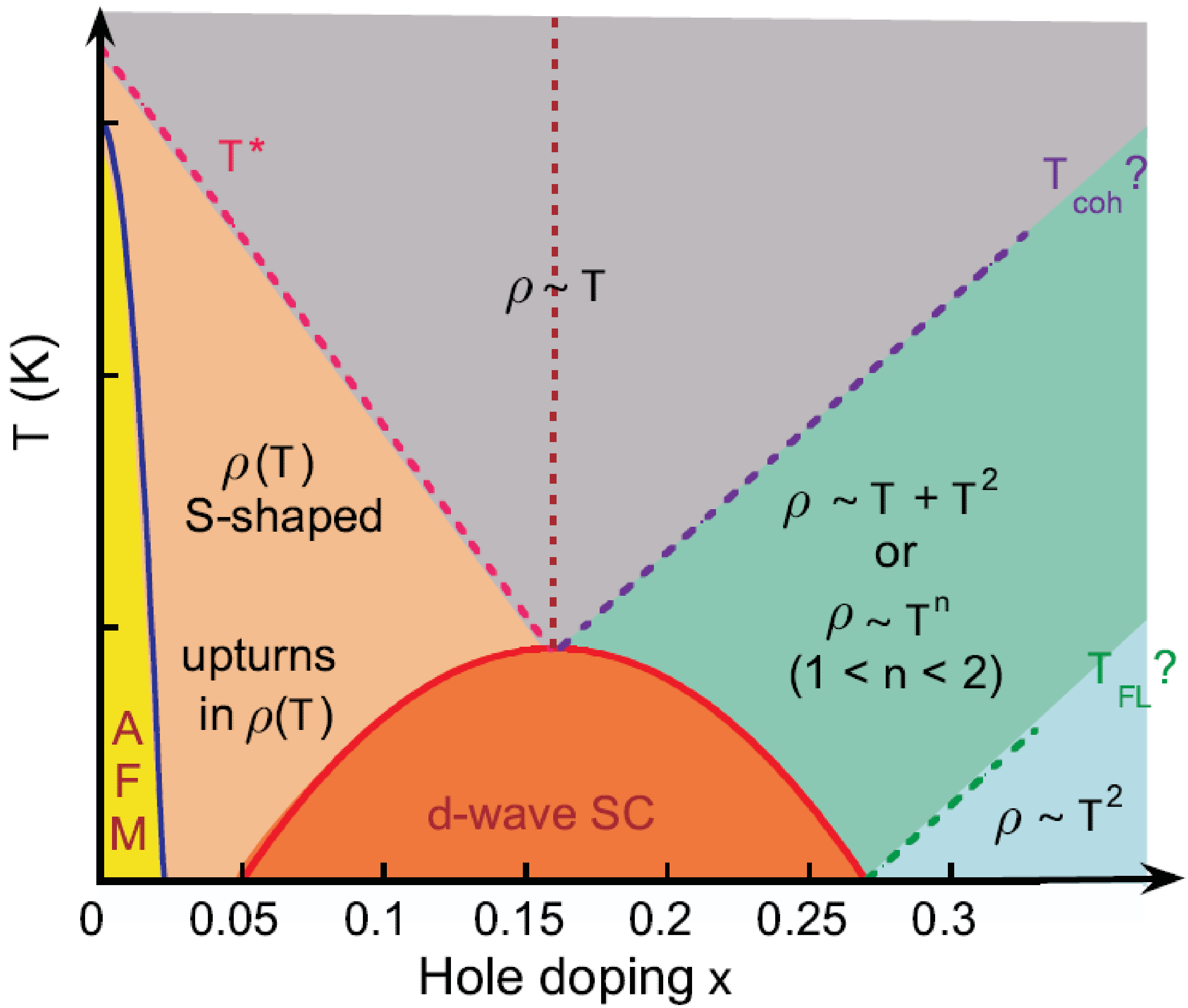
Holography gives solvable realizations of strongly correlated steady states.

- Confirms (theoretical existence) of non-linear transport at quantum critical points
- Points to existence of qualitatively novel (flow driven) steady states at strong coupling.

Novel Scaling Exponents

(recent work with Sean Hartnoll)

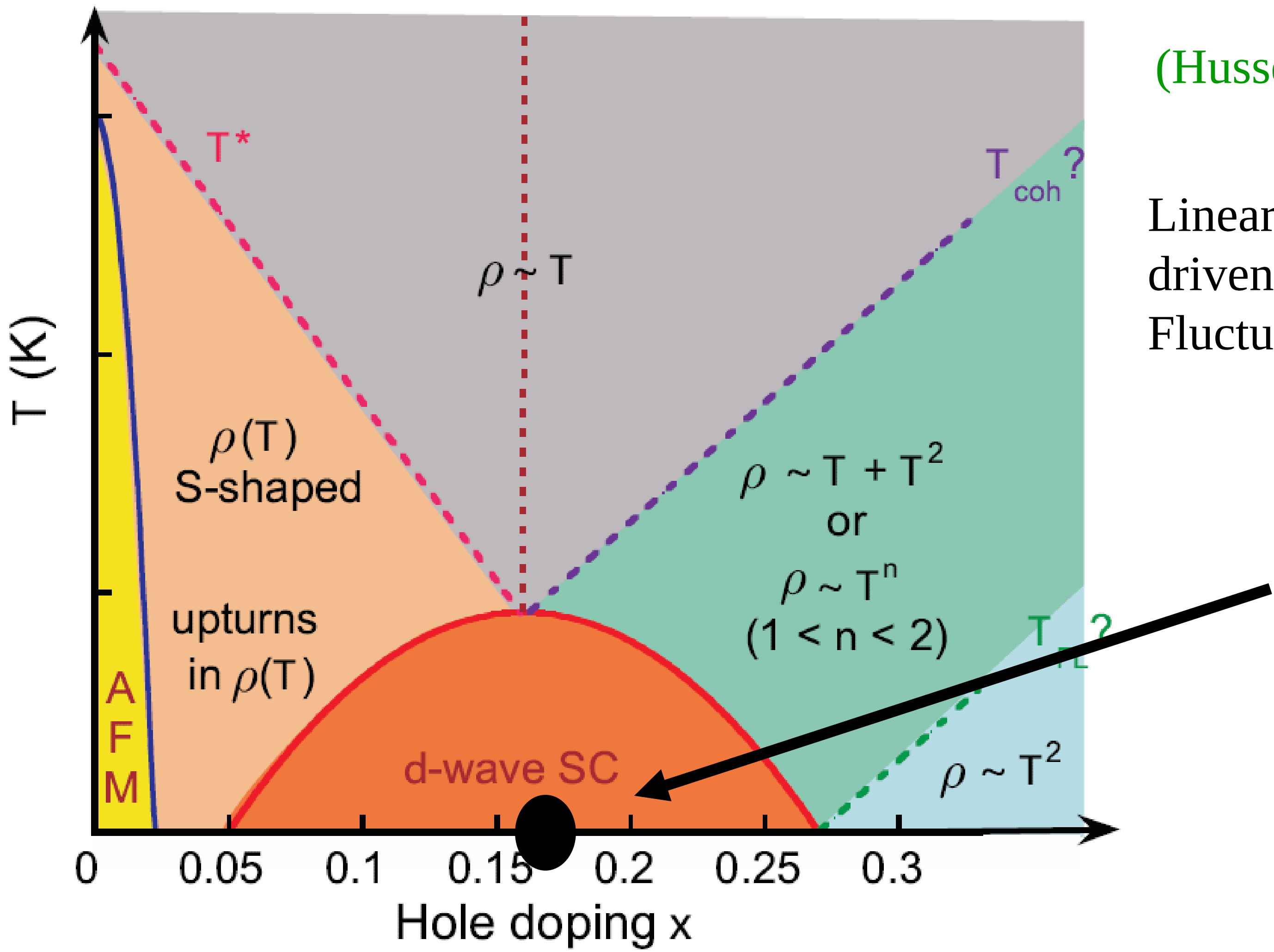
Strange Metal / QCP



(Hussey; Sachdev)

Linear resistivity directly driven by Quantum Critical Fluctuations?

Strange Metal / QCP



(Hussey; Sachdev)

Linear resistivity directly driven by Quantum Critical Fluctuations?

QCP?

Dimensional Analysis at QCP

$$[x] = -1 \qquad [t] = -z$$

Dynamical Critical Exponent.

Dimensional Analysis at QCP

$$[x] = -1 \qquad [t] = -z$$

$$[s] = d - \theta$$

Hyperscaling Violating Exponent.

Dimensional Analysis at QCP

$$[x] = -1 \qquad [t] = -z$$

$$[s] = d - \theta$$

$$[E] = 1 + z - \Phi$$

(AK)

Anomalous Coupling to E&M Fields.

Scaling and the Cuprates.

If we try to explain scaling in the cuprates, is non-zero Φ needed?

Is there a simple physical observable whose dimension is zero unless Φ is non-zero?

$$[\kappa] = d - \theta + z - 2$$ thermal conductivity

$$[\sigma] = d - \theta + 2\phi - 2$$ electric conductivity

$$[L] = \left[\frac{\kappa}{\sigma T}\right] = -2\phi$$ Lorenz ratio

Lorenz Ratio

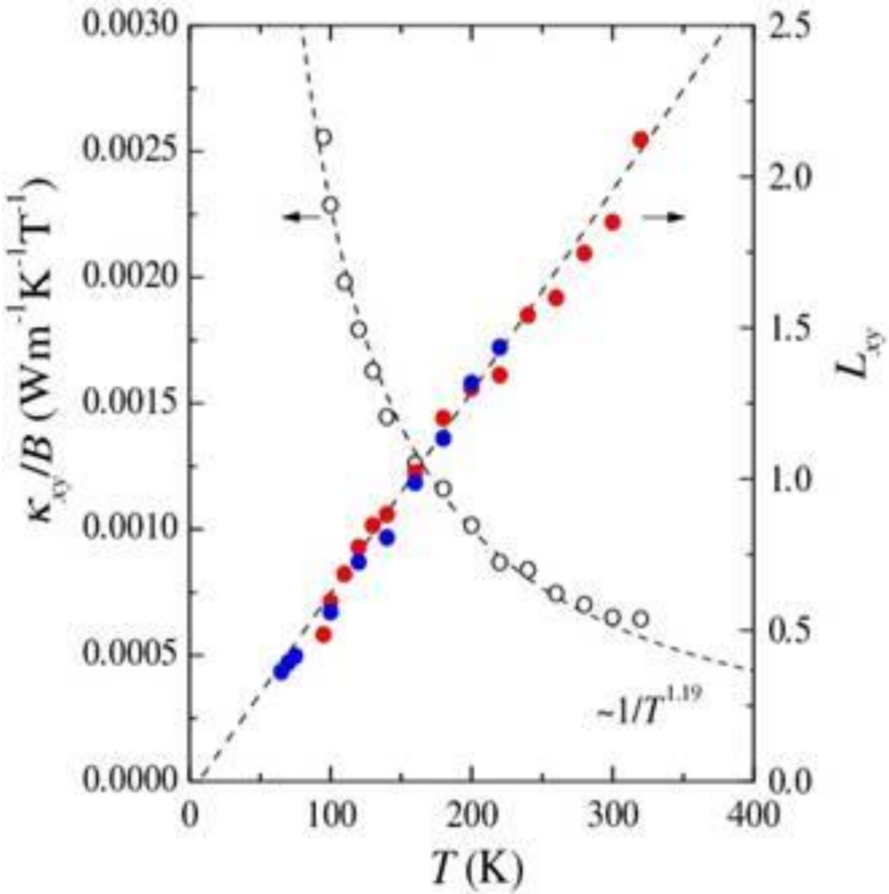
Thermal conductivity receives contributions from all degrees of freedom including phonons.

Expect system to be: QCP + neutral heat bath

(can carry spin, but no charge)

Isolate: Hall Lorenz ratio.

Wiedemann-Franz Law Violation



$$\left[\frac{\kappa_{xy}}{T\,\sigma_{xy}}\right] = -2\phi$$

(Zhang et al)

Scaling analysis of Cuprates

(Sean Hartnoll, AK):

Can a simple scaling analysis based on 3 exponents, z, θ and Φ give an acceptable phenomenology of the normal phase of the cuprates?

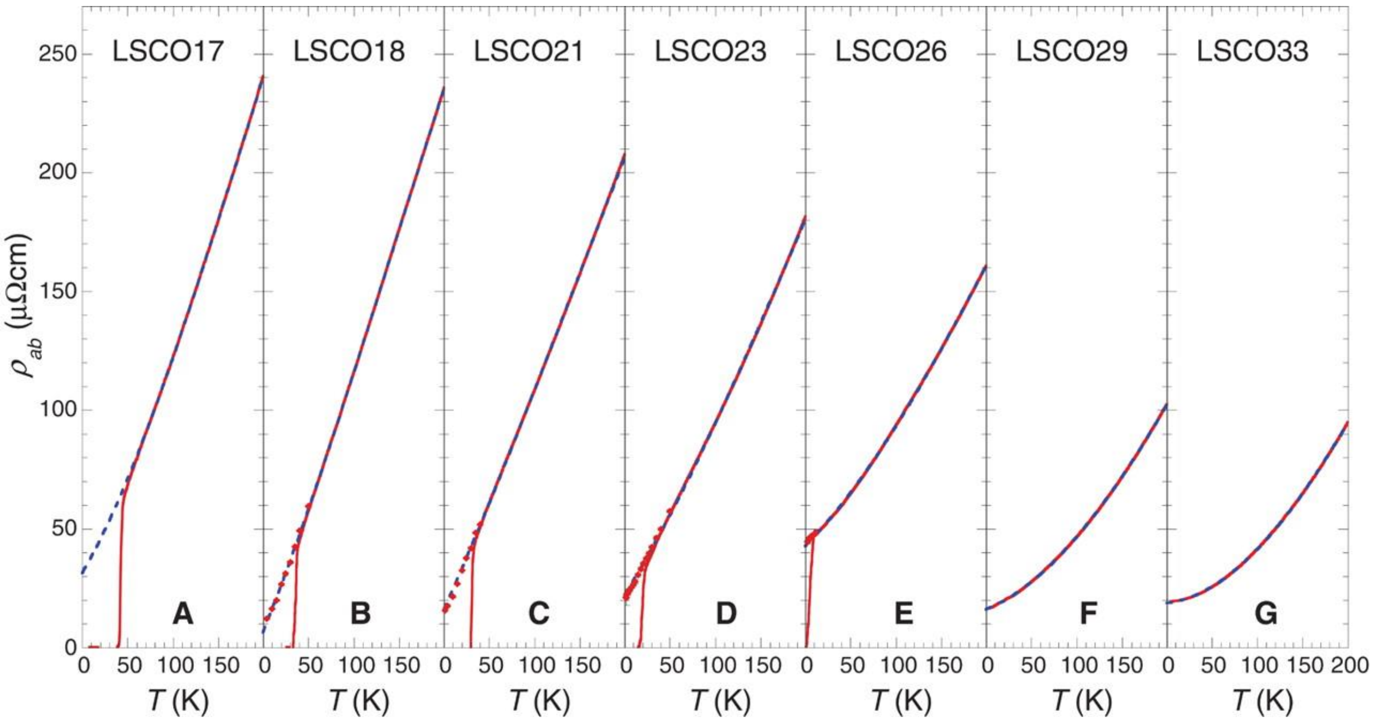
Inputs

Need 3 experimentally well established scalings to pin down the three exponents.

1) Lorenz Ratio linear in T

$$z = -2\phi$$

2) Linear Resistivity

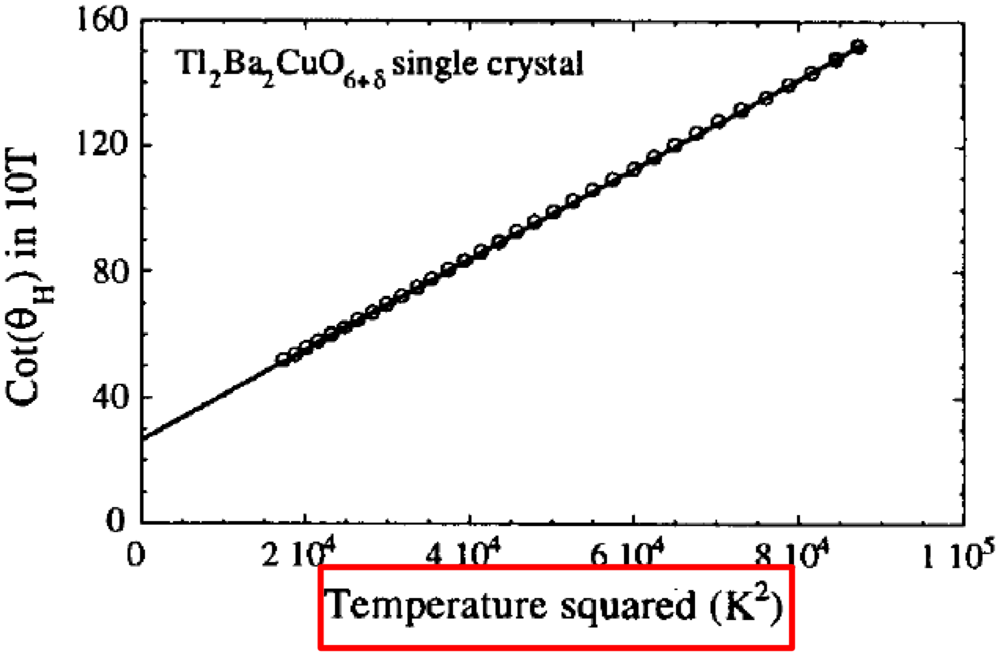


θ = 0

$\sigma_{xx} \sim T^{(d+2\phi-\theta-2)/z}$

Cooper et al, Science (2009)

3) Hall Angle



(Tyler and Mackenzie, 1997)

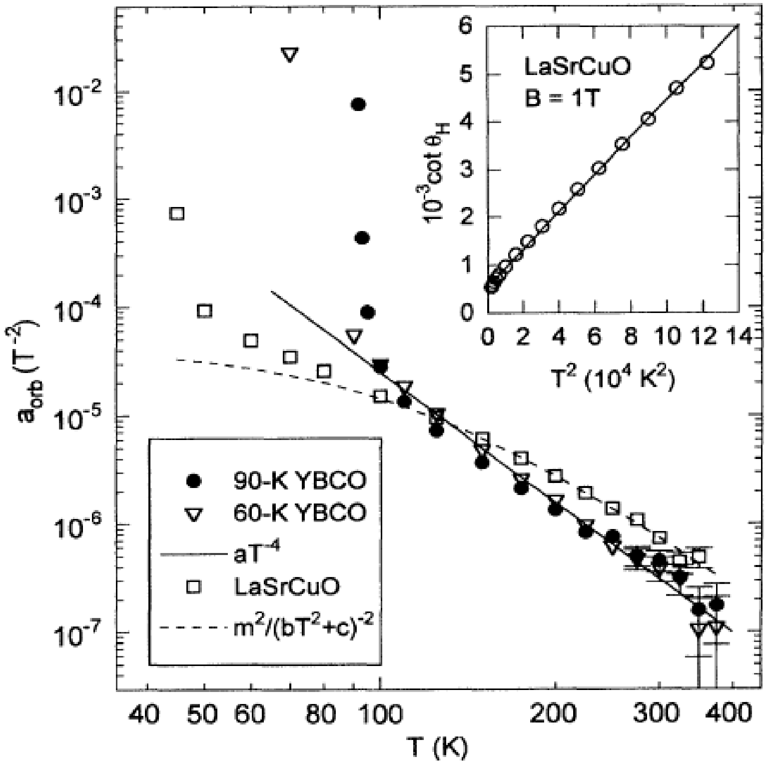
$$\cot(\theta_H) = \frac{\sigma_{xx}}{\sigma_{xy}}$$

$$\sigma_{xy} \sim B T^{(d+3\phi-\theta-4)/z}$$

$$z = \frac{4}{3}$$

$$\phi = -\frac{2}{3}$$

Prediction 1: Magnetoresistance



(Harris et al, 1996))

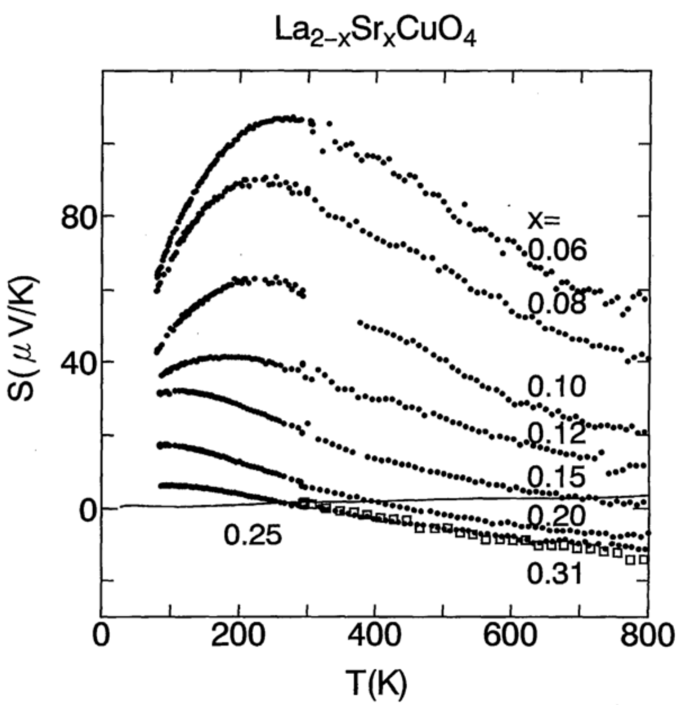
Scaling implies:

$$\frac{\Delta\rho}{\rho_{B=0}} \sim \frac{B^2}{T^4}$$

Perfectly agrees with experimental data!

Prediction 2: Thermoelectric

Typically measured as Seebeck:



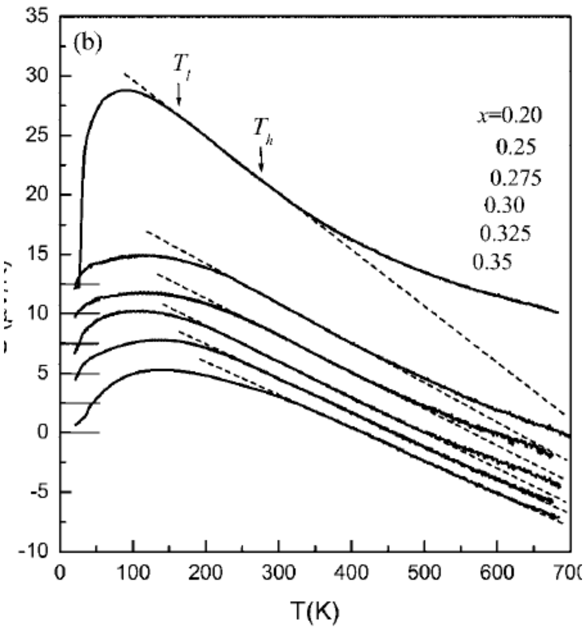
(Nishikawa et al, 1994)

$$S \equiv \frac{\alpha_{xx}}{\sigma_{xx}} \sim -T^{1/2}$$

(find E so that no current flows in response to T-gradient)

No fit to shape of data attempted in early experimental work.

Prediction 2: Thermoelectric



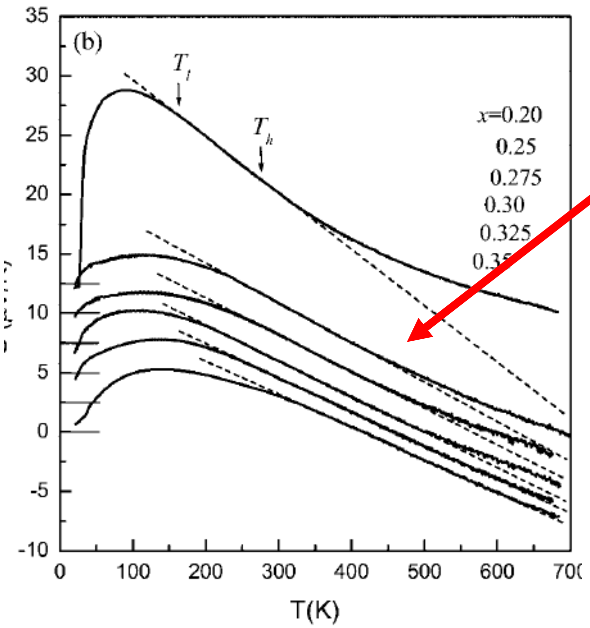
(Kim et al, 2004)

Ten years later data looks much cleaner !

The published linear fit clearly doesn't capture high T.

Does this look like const.-$\sqrt{T}$?

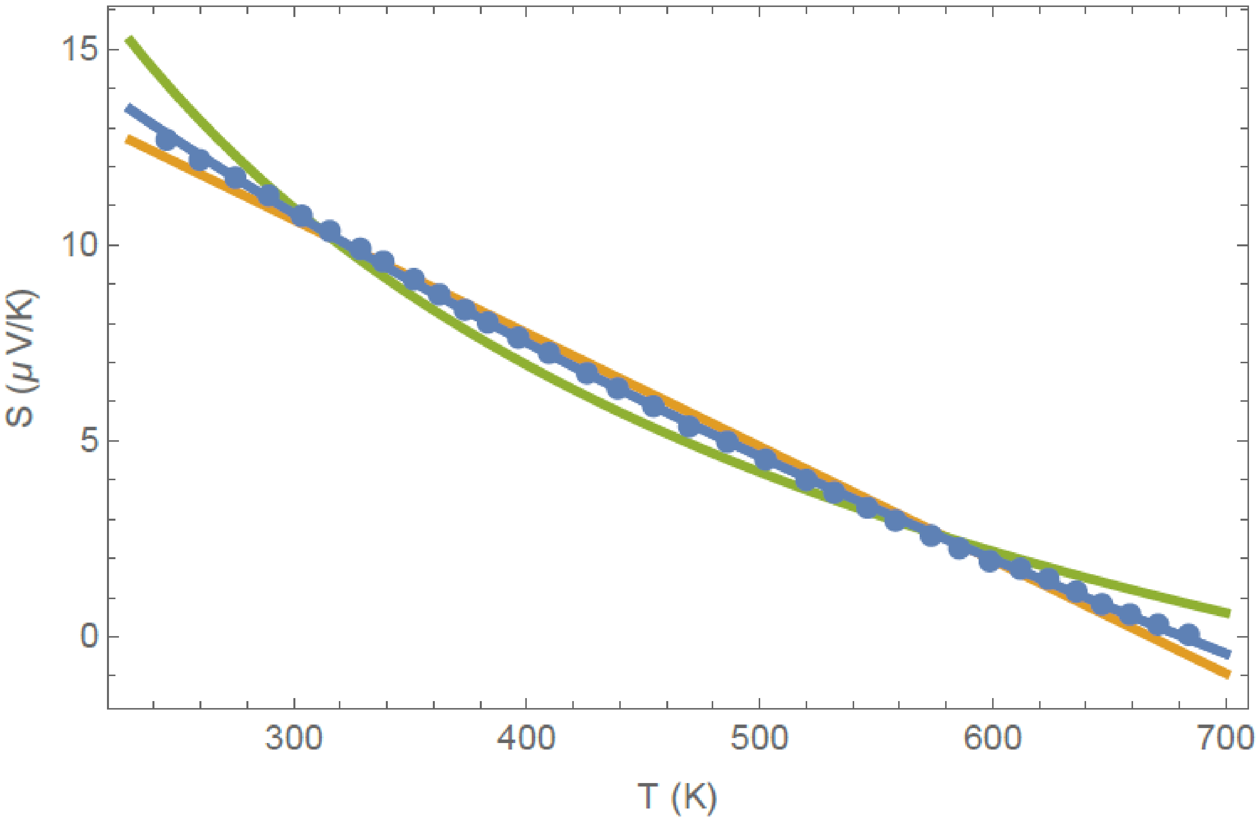
Prediction 2: Thermoelectric



(Kim et al, 2004)

Use Mathematica to pick out points along the x=0.25 curve and attempt our own fit!

Seebeck Coefficient



$a - b\,T^{1/2}$

fits data head on!

$a - b\,T^{1}$ **and**

$a - b\,T^{-1/2}$

don’t.

Summary, scaling

Scaling theory works for transport!

- New exponent Φ needed by Lorenz data
- Other transport (Nernst) consistent but needs more high T data
- Thermo not scaling; extra “conventional” component
- Can be tested in other materials (pnictides)

Summary.

Holography

=

Solvable models of strong coupling dynamics.