The Superconformal Bootstrap Program

Leonardo Rastelli

Yang Institute for Theoretical Physics
Stony Brook

Based on work with
C. Beem, M. Lemos, P. Liendo, W. Peelaers and Balt van Rees

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SuperConformal Field Theories in $d > 2$

Fast-growing body of results:

- Many new models, most with no known Lagrangian description.
- A hodgepodge of techniques:
  localization, integrability, effective actions on moduli space.

  Powerful but with limited scope.
  Conformal symmetry not fully used.

We advocate a more systematic and universal approach:
the conformal bootstrap.

Basic bootstrap philosophy

We'll think of a CFT is an abstract collection of local operators $\{\mathcal{O}_k(x)\}$ and their correlation functions $\langle \mathcal{O}_1(x_1) \ldots \mathcal{O}_n(x_n) \rangle$.

Local operators form an algebra. Operator product expansion,

$$\text{OPE}: \quad \mathcal{O}_1(x)\mathcal{O}_2(0) = \sum_k c_{12k}(x)\mathcal{O}_k(0)\,.$$

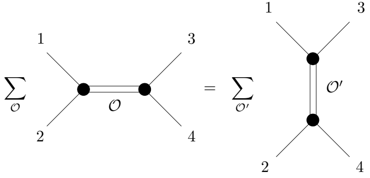
This is a true operator equation. The sum converges.

(This definition does not capture non-local observables (*e.g.* Wilson loops) or constraints from non-trivial geometries.)

Local operators $\mathcal{O}_{\Delta,\ell,f}$ are labeled by a conformal dimension Δ, a Lorentz representation ℓ and possibly a flavor quantum number f.

The CFT data $\{(\Delta_i, \ell_i, f_i)\,, c_{ijk}\}$ completely specify the theory. All correlators can be computed by taking successive operator products till $\langle \mathbf{1} \rangle \equiv 1$.

The CFT data are constrained by the requirement of associativity of the operator algebra, *i.e.*, crossing symmetry of the four-point functions:



The hope of the bootstrap is that supplemented with a finite amount of input, these constraints will determine everything.

Until recently, one real success story was rational CFT is $d = 2$.

Since 2008, there has been a steady flow of results implementing a *numerical* approach to deriving bounds on CFT data in any d from the constraints of crossing and unitarity Rattazzi Rychkov Tonni Vichi

Two sorts of questions for the superconformal bootstrap

What is the space of consistent SCFTs in $d \leqslant 6$?

For maximal susy, well-known list of theories.
Is the list complete?

What is the list with less susy?

Can we bootstrap concrete models?

The bootstrap should be particularly powerful for models uniquely cornered by few discrete data.
Only method presently available for finite N, non-Lagrangian theories, such as the $6d$ (2,0) SCFT.

More technically, not clear how much susy can really help.

A natural question:

> *Do the bootstrap equations in $d > 2$ admit a solvable truncation for superconformal theories?*

The answer is Yes for large classes of theories:

(A) Any $d = 4$, $\mathcal{N} \geqslant 2$ or $d = 6$, $\mathcal{N} = (2, 0)$ SCFT admits a subsector $\cong$ $2d$ chiral algebra.

(B) Any $d = 3$, $\mathcal{N} \geqslant 4$ SCFT admits a subsector $\cong$ $1d$ TQFT.

Beem Lemos Liendo Peelaers LR van Rees

Beem LR van Rees

Bootstrapping in two steps

For $d = 4$, $\mathcal{N} \geqslant 2$ and $d = 6$, $\mathcal{N} = (2,0)$ SCFTs,
the crossing equations

(1) Equations that depend only on the
intermediate BPS operators. Captured by the $2d$ chiral algebra.
"Minibootstrap"

(2) Equations that also include
intermediate non-BPS operators.
"Maxibootstrap"

(1) are tractable and determine an infinite amount of CFT data, given flavor symmetries and central charges.

This is essential input to the full-fledged bootstrap (2),
which can be studied numerically.

Lightning review of superconformal symmetry

Conformal symmetry: P_μ, $M_{\mu\nu}$, D, K_μ.

Supersymmetry: add fermionic generators Q and $\tilde{Q}$ of dimension $\frac{1}{2}$, as "the square root" of P_μ,

$$\{Q, \tilde{Q}\} = P\,.$$

We may take $\mathcal{N}$ copies of this structure: Q^A and $\tilde{Q}_A$, $A = 1, \ldots \mathcal{N}$,

$$\{Q^1, \tilde{Q}_1\} = \{Q^2, \tilde{Q}_2\} = \ldots \{Q^\mathcal{N}, \tilde{Q}_\mathcal{N}\} = P\,.$$

A $U(\mathcal{N})$ rotation of the supercharges (the "R-symmetry", with generators $R^A_{\ B}$) preserves the susy algebra.

Conformal symmetry + supersymmetry: we must also take the square root of K_μ. Extra fermonic generators S_A and $\tilde{S}^A$ of dimension $-\frac{1}{2}$, with

$$\{S^1, \tilde{S}_1\} = \{S^2, \tilde{S}_2\} = \ldots \{S^\mathcal{N}, \tilde{S}_\mathcal{N}\} = K\,.$$

Schematically,

$$\{Q\,, S\} = M + D + R\,, \quad \{\tilde{Q}\,, \tilde{S}\} = M - D - R\,.$$

Warm-up: $\mathcal{N} = 1$ chiral ring

Chiral operators in an $\mathcal{N} = 1$, $d = 4$ QFT

$$\{\tilde{Q}_{\dot{\alpha}}\,, \mathcal{O}(x)] = 0\,, \quad \dot{\alpha} = \dot{+}, \dot{-}\,, \quad \Leftrightarrow \quad \Delta = \frac{3}{2}r$$

Further define cohomology class $[\mathcal{O}(x)]_{\tilde{Q}}$

$$\mathcal{O}(x) \sim \mathcal{O}(x) + \{\tilde{Q}, \dots]\,.$$

From susy algebra $P = \{Q, \tilde{Q}\}$:

$$\frac{\partial}{\partial x}\mathcal{O}(x) = [P, \mathcal{O}(x)] = \{\tilde{Q}, \mathcal{O}'(x)]\,, \quad \mathcal{O}'(x) = \{Q, \mathcal{O}(x)]\,,$$

so $[\mathcal{O}(x)]_{\tilde{Q}}$ is x-independent. Concretely,

$$\langle \mathcal{O}_1(x_1)\,\mathcal{O}_2(x_2)\dots\mathcal{O}_n(x_n)\rangle = \text{constant} \qquad x_i \in \mathbb{R}^4\,.$$

In fact, in an $\mathcal{N} = 1$ superconformal theory, constant $\equiv 0$ since $r_i > 0$.

Meromorphy in $\mathcal{N} = 2$, $d = 4$ SCFTs

Fix a plane $\mathbb{R}^2 \subset \mathbb{R}^4$, parametrized by $(z, \bar{z})$.

Claim : $\exists$ subsector $\mathcal{A}_\chi = \{\mathcal{O}_i(z_i, \bar{z}_i)\}$ with meromorphic

$$\langle \mathcal{O}_1(z_1, \bar{z}_1)\, \mathcal{O}_2(z_2, \bar{z}_2) \ldots \mathcal{O}_n(z_n, \bar{z}_n) \rangle = f(z_i)\,.$$

Rationale: $\mathcal{A}_\chi \equiv$ cohomology of a nilpotent $\mathbb{Q}$,

$$\mathbb{Q} = \mathcal{Q} + \mathcal{S}\,,$$

$\mathcal{Q}$ Poincaré, $\mathcal{S}$ conformal supercharges.

$\bar{z}$ dependence is $\mathbb{Q}$-exact: cohomology classes $[\mathcal{O}(z, \bar{z})]_{\mathbb{Q}} \rightsquigarrow \mathcal{O}(z)$.

Analogous to (but richer than) the $d = 4$, $\mathcal{N} = 1$ chiral ring, where cohomology classes $[\mathcal{O}(x)]_{\tilde{\mathcal{Q}}_{\dot{\alpha}}}$ are x-independent.

Cohomology at the origin: Schur operators

At the origin of $\mathbb{R}^2$, $\mathbb{Q}$-cohomology $\mathcal{A}_\chi$ easy to describe.

$\mathcal{O}(0,0) \in \mathcal{A}_\chi \leftrightarrow \mathcal{O}$ obeys the chirality condition

$$\frac{\Delta - \ell}{2} = R$$

Δ conformal dimension
$\ell = j_1 + j_2$ spin on $\mathbb{R}^2$, with (j_1, j_2) Lorentz spins in $\mathbb{R}^4$
R Cartan generator of $SU(2)_R$ R-symmetry

These are precisely the operators counted by the Schur limit of the superconformal index. Gadde LR Razamat Yan

Killed by 2 real Poincaré supercharges (out of 8), one $\mathcal{Q}$ and one $\tilde{\mathcal{Q}}$, an intrinsically $\mathcal{N} = 2$ condition.

Schur operators

Multiplet	$\mathcal{O}_{\text{Schur}}$	h	Lagrangian "letters"
$\hat{\mathcal{B}}_R$	$\Psi^{11\ldots1}$	R	Q, $\tilde{Q}$
$\mathcal{D}_{R(0,j_2)}$	$\tilde{\mathcal{Q}}^1_{\dot{+}}\,\Psi^{11\ldots1}_{\dot{+}\ldots\dot{+}}$	$R+j_2+1$	Q, $\tilde{Q}$, $\tilde{\lambda}^1_{\dot{+}}$
$\bar{\mathcal{D}}_{R(j_1,0)}$	$\mathcal{Q}^1_{+}\,\Psi^{11\ldots1}_{+\ldots+}$	$R+j_1+1$	Q, $\tilde{Q}$, λ^1_{+}
$\hat{\mathcal{C}}_{R(j_1,j_2)}$	$\mathcal{Q}^1_{+}\,\tilde{\mathcal{Q}}^1_{\dot{+}}\,\Psi^{11\ldots1}_{+\ldots+\,\dot{+}\ldots\dot{+}}$	$R+j_1+j_2+2$	$D_{+\dot{+}}$, Q, $\tilde{Q}$, λ^1_{+}, $\tilde{\lambda}^1_{\dot{+}}$

- $\hat{\mathcal{B}}_R$: Higgs branch chiral ring operators
- $\mathcal{D}_{R(0,j_2)}/\bar{\mathcal{D}}_{R(j_1,0)}$: Additional $\mathcal{N}=1$ (anti-)chiral ring operators. "Hall-Littlewood" chiral ring.
- $\hat{\mathcal{C}}_{R(j_1,j_2)}$: Other less familiar semi-short operators. $\hat{\mathcal{C}}_{0(0,0)}$ is the stress-tensor multiplet, also containing R-symmetry currents.
- Coulomb branch $\frac{1}{2}$ BPS operators (such as $\mathrm{Tr}\,\phi^k$) not Schur.

$$[\mathbb{Q}, \mathfrak{sl}(2)] = 0 \quad \text{but} \quad [\mathbb{Q}, \overline{\mathfrak{sl}(2)}] \neq 0$$

To define $\mathbb{Q}$-closed operators $\mathcal{O}(z, \bar{z})$ away from origin, we twist the right-moving generators by $SU(2)_R$,

$$\widehat{L}_{-1} = \bar{L}_{-1} + \mathcal{R}^{-}\,, \quad \widehat{L}_{0} = \bar{L}_{0} - \mathcal{R}\,, \quad \widehat{L}_{1} = \bar{L}_{1} - \mathcal{R}^{+}\,.$$

$$\widehat{\mathfrak{sl}(2)} = \{\mathbb{Q}, \dots\}$$

$\mathbb{Q}$-closed operators are “twisted-translated”

$$\mathcal{O}(z, \bar{z}) = e^{zL_{-1} + \bar{z}\widehat{L}_{-1}}\, \mathcal{O}(0)\, e^{-zL_{-1} - \bar{z}\widehat{L}_{-1}}\ .$$

$SU(2)_R$ orientation correlated with position on $\mathbb{R}^2$. (Cf. Drukker Plefka)

Chirality condition $\frac{\Delta - \ell}{2} - R = 0 \Leftrightarrow \widehat{L}_0 = 0$

By the usual formal argument, the $\bar{z}$ dependence is exact,

$$[\mathcal{O}(z,\bar{z})]_{\mathbb{Q}} \quad \rightsquigarrow \quad \mathcal{O}(z) \; .$$

Cohomology classes define left-moving $2d$ operators $\mathcal{O}_i(z)$, with conformal weight

$$h = R + \ell .$$

They are closed under OPE,

$$\mathcal{O}_1(z)\mathcal{O}_2(0) = \sum_k \frac{c_{12k}}{z^{h_1+h_2-h_k}}\,\mathcal{O}_k(0)\,.$$

$\mathcal{A}_\chi$ has the structure of a $2d$ chiral algebra

Example: free hypermultiplet

Look for states with $\widehat{L}_0 = \frac{\Delta - j_1 - j_2}{2} - R = 0$.
The complex scalars Q and $\tilde{Q}$ fit the bill, since $\Delta = 1$, $R = \frac{1}{2}$, $j_1 = j_2 = 0$.
They are top components of $SU(2)_R$ doublets,

$$Q^{\mathcal{I}} = \begin{pmatrix} Q \\ \tilde{Q}^* \end{pmatrix} , \qquad \tilde{Q}^{\mathcal{I}} = \begin{pmatrix} \tilde{Q} \\ -Q^* \end{pmatrix} .$$

Away from the origin, consider twisted-translated operators

$$q(z, \bar{z}) := Q(z, \bar{z}) + \bar{z} \tilde{Q}^*(z, \bar{z}), \qquad \tilde{q}(z, \bar{z}) := \tilde{Q}(z, \bar{z}) - \bar{z} Q^*(z, \bar{z}) .$$

Elementary exercise:

$$q(z, \bar{z})\, \tilde{q}(0) \sim \bar{z}\, \tilde{Q}^*(z, \bar{z})\, \tilde{Q}(0) \sim \frac{\bar{z}}{z \bar{z}} = \frac{1}{z} .$$

The cohomology classes $[q(z, \bar{z})]_{\mathbb{Q}}$, $[\tilde{q}(z, \bar{z})]_{\mathbb{Q}}$ define a pair of symplectic bosons of weight $h = \frac{1}{2}$, for which $c_{2d} = -1$.
Normal ordered products of $\partial^n q$ and $\partial^n \tilde{q}$ span the entire chiral algebra associated to the free hypermultiplet.

Example: free vector multiplet

The gauginos λ^1_+ and $\tilde{\lambda}^1_{\dot{+}}$ satisfy $\widehat{L}_0 = 0$.

The twisted-translated operators

$$\lambda(z,\bar{z}) := \lambda^1_+(z,\bar{z}) + \bar{z}\lambda^2_+(z,\bar{z})\,, \quad \tilde{\lambda}(z,\bar{z}) := \tilde{\lambda}^1_{\dot{+}}(z,\bar{z}) + \bar{z}\tilde{\lambda}^2_{\dot{+}}(z,\bar{z})$$

give rise in cohomology to chiral fields $\lambda(z)$, $\tilde{\lambda}(z)$, with

$$\tilde{\lambda}(z)\lambda(0) \sim \frac{1}{z^2}\ , \quad \lambda(z)\tilde{\lambda}(0) \sim -\frac{1}{z^2}\ .$$

Setting

$$\tilde{\lambda}(z) := b(z)\ , \quad \lambda(z) := \partial c(z)\ .$$

we recognize a bc ghost system of weights (1, 0), for which $c_{2d} = -2$.

χ : 4d $\mathcal{N} = 2$ SCFT $\longrightarrow$ 2d Chiral Algebra.

- Virasoro enhancement of $\mathfrak{sl}(2)$, with $T(z)$ arising from a component of the $SU(2)_R$ conserved current, $T(z) := [\mathcal{J}_R(z, \bar{z})]_{\mathbb{Q}}$, with

$$c_{2d} = -12\, c_{4d}\,,$$

where c_{4d} is one of the conformal anomaly coefficient.

- Affine symmetry enhancement of global flavor symmetry, with $J(z)$ arising from the moment map operator, $J(z) := [M(z, \bar{z})]_{\mathbb{Q}}$, with

$$k_{2d} = -\frac{k_{4d}}{2}\,.$$

- Guaranteed generators:
Generators of the $4d$ Higgs branch $\Rightarrow$ generators of the chiral algebra.
Higgs branch relations encoded in null states of the chiral algebra!
(Crucial that k_{2d} takes special negative levels).
More generally, generators of HL ring $\Rightarrow$ generators of the chiral algebra.

Consequences for $4d$ physics

Full-fledged 4pt function of Schur operators

$$\mathcal{A}^{\mathcal{I}_1\mathcal{I}_2\mathcal{I}_3\mathcal{I}_4}(z,\bar{z}) = \langle\, \mathcal{O}^{\mathcal{I}_1}(0)\; \mathcal{O}^{\mathcal{I}_2}(z,\bar{z})\; \mathcal{O}^{\mathcal{I}_3}(1)\; \mathcal{O}^{\mathcal{I}_4}(\infty)\,\rangle$$

$\mathcal{I}$ = index of $SU(2)_R$ irrep.

Associated chiral algebra correlator

$$f(z) = \langle \mathcal{O}(0)\mathcal{O}(z)\mathcal{O}(1)\mathcal{O}(\infty)\rangle\,,\ \mathcal{O}(z) = [u_{\mathcal{I}}(\bar{z})\mathcal{O}^{\mathcal{I}}(z,\bar{z})]_{\mathbb{Q}}\ .$$

Double-OPE expansion

$$\mathcal{A}(z,\bar{z}) = \sum p_i^{\text{short}}\, \mathcal{G}_i(z,\bar{z}) + \sum p_k^{\text{long}}\, \mathcal{G}_k(z,\bar{z})$$

$\mathcal{G}_i$ = superconformal blocks $=\sum_{\text{finite}}$ conformal blocks $G_{\Delta,\ell}$.

The short part can be entirely reconstructed from $f(z)$,
assuming *absence* of HS currents.

Symmetries & central charges $\mathbf{c}$

$\downarrow$

Chiral algebra correlator $f(z; \mathbf{c})$

$\downarrow$

Short spectrum and OPE coefficients $p_i^{\text{short}}(\mathbf{c})$
(unique assuming no higher-spin symmetry)

$\downarrow$

$\mathcal{A}^{\text{short}}(z, \bar{z}; \mathbf{c})$

$\downarrow$

Finally, numerical bootstrap of $\mathcal{A}^{\text{long}}(z, \bar{z})$

Unitarity requires $p_i^{\text{short}}(\mathbf{c}) \geqslant 0 \rightarrow$ novel bounds on central charges.

Bounds from moment-map $4pt$ function

- From non-singlet channels:

G_F		Bound
$\mathrm{SU}(N)$	$N \geqslant 3$	$k_{4d} \geqslant N$
$\mathrm{SO}(N)$	$N = 4, \ldots, 8$	$k_{4d} \geqslant 4$
$\mathrm{SO}(N)$	$N \geqslant 8$	$k_{4d} \geqslant N - 4$
$\mathrm{USp}(2N)$	$N \geqslant 3$	$k_{4d} \geqslant N + 2$
G_2		$k_{4d} \geqslant \frac{10}{3}$
F_4		$k_{4d} \geqslant 5$
E_6		$k_{4d} \geqslant 6$
E_7		$k_{4d} \geqslant 8$
E_8		$k_{4d} \geqslant 12$

Bounds saturated by rank one SCFTs of type $H_1, H_2, D_4, E_6, E_7, E_8$, whose Higgs branches are one-instanton moduli spaces.

$k_{4d} = k_{min} \Rightarrow$ extra nulls in the affine Lie algebra.
$4d$ interpretation: Joseph relations of the moment map

$$(M \otimes M)|_{\mathcal{I}_2} = 0 \,, \qquad \mathrm{Sym}^2(\mathbf{adj}) = (2\,\mathbf{adj}) \oplus \mathcal{I}_2 \,.$$

• In the singlet channel, stress-tensor also contributes:

A universal bound involving conformal and flavor anomalies

$$\frac{\dim G_F}{c_{4d}} \geqslant \frac{24 h^\vee}{k_{4d}} - 12 \ .$$

Gauging prescription

Start with $4d$ SCFT $\mathcal{T}$, with flavor symmetry G_F.
We can generate a new SCFT $\mathcal{T}_G$ by gauging $G \subset G_F$, provided $\beta_G = 0$.

If we already know the chiral algebra $\chi[\mathcal{T}]$, can we find $\chi[\mathcal{T}_G]$?

Extra $4d$ vector multiplet $\Rightarrow$ extra $(b^A c_A)$ ghost system, in the adjoint of G.
We must also restrict to gauge singlets.
This is the correct answer at zero gauge coupling. But at finite coupling, some states are lifted and the chiral algebra must be smaller.

Elegant prescription to find quantum chiral algebra. Pass to the cohomology of

$$Q_{\rm BRST} := \oint \frac{dz}{2\pi i}\, j_{\rm BRST}(z)\,, \quad j_{\rm BRST} := c_A \left[J^A - \frac{1}{2} f^{AB}{}_C\, c_B b^C \right],$$

where J^A is the G affine current of $\chi[\mathcal{T}]$.

$Q^2_{\rm BRST} = 0$ precisely when the $\beta_G = 0$, which amounts to $k_{2d} = -2h^\vee$.

By this prescription, we can in principle find $\chi[\mathcal{T}]$ for any Lagrangian SCFT $\mathcal{T}$.

Some examples

In several interesting cases the chiral algebra is finitely generated:

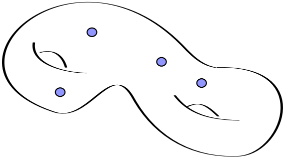
- $SU(2)$ super QCD with $N_f = 4 \Rightarrow \mathfrak{so}(8)_{-2}$ AKM algebra.
- E_6 SCFT $\Rightarrow (\mathfrak{e}_6)_{-3}$ AKM algebra.
- $\mathcal{N} = 4$ SYM $\Rightarrow \mathcal{N} = 4$ super $\mathcal{W}$-algebra, with generators given by chiral primaries of dimensions $\{h_i = \frac{r_i+1}{2}\}$, $\{r_i\} \equiv$ exponents of gauge group G.
- (A_1, A_{2k}) AD theories $\Rightarrow$ Virasoro algebra of $(2, 2k+3)$ model (Tantalizing connection with Cecotti Neitzke Vafa.)

Here, simplest guess (generators from HL ring + stress tensor) works.

There are examples with additional generators.

Chiral algebras of class $\mathcal{S}$

Beem Peelaers LR van Rees



SCFTs of class $\mathcal{S}$: labelled by decorated Riemann surfaces $\mathcal{C}_{g,s}$.

By passing to the chiral algebra, we obtain
a generalized TQFT valued in chiral algebras.

Generalized S-duality ⇔ Associativity of the TQFT.

4d physics ⇔ 2d algebraic structures. For example,
Higgsing flavor symmetry ⇔ Quantum Drinfeld-Sokolov reduction.

χ_6 : 6d (2,0) SCFT $\longrightarrow$ 2d Chiral Algebra.

- Global $\mathfrak{sl}(2) \to$ Virasoro, indeed $T(z) := [\mathcal{O}_{\mathbf{14}}(z, \bar{z})]_{\mathbb{Q}}$, with $\mathcal{O}_{\mathbf{14}}$ the stress-tensor multiplet superprimary,

$$c_{2d} = c_{6d}$$

in normalizations where c_{6d} (free tensor) $\equiv 1$.

- All $\frac{1}{2}$-BPS operators $(\Delta = 2R)$ are in $\mathbb{Q}$ cohomology.

  Generators of the $\frac{1}{2}$-BPS ring $\to$ generators of the chiral algebra.

- Some semi-short multiplets also play a role.

Chiral algebra for $(2,0)$ theory of type A_{N-1}

One $\frac{1}{2}$-BPS generator each of dimension $\Delta = 4, 6, \dots 2N$

$\Downarrow$

One chiral algebra generator each of dimension $h = 2, 3, \dots N$.

Most economical scenario: these are all the generators.
Check: the superconformal index computed by Kim[3] is reproduced.

$$\mathcal{I}(q,s) := \mathrm{Tr}(-1)^F q^{E-R} s^{h_2+h_3} \,.$$

$$\mathcal{I}(q,s;n) = \prod_{k=1}^{n} \prod_{m=0}^{\infty} \frac{1}{1-q^{k+m}} = \mathrm{PE}\left[\frac{q+q^2+\cdots+q^n}{1-q}\right] ,$$

Plausibly a unique solution to crossing for this set of generators.

- The chiral algebra of the A_{N-1} theory is $\mathcal{W}_N$, with

$$c_{2d} = 4N^3 - 3N - 1 \,.$$

(Similar proposals for D and E theories).

Half-BPS 3pt functions of $(2,0)$ SCFT

OPE of $\mathcal{W}_{\mathfrak{g}}$ generators $\Rightarrow$ half-BPS 3pt functions of SCFT.

Let us check the result at large N.

$W_{N\to\infty}$ with $c_{2d} \sim 4N^3$ $\rightarrow$ a *classical* W-algebra.
(Gaberdiel Hartman, Campoleoni Fredenhagen Pfenninger)

We find

$$C(k_1,k_2,k_3) = \frac{2^{2\alpha-2}}{(\pi N)^{\frac{3}{2}}}\Gamma\left(\frac{\alpha}{2}\right)\left(\frac{\Gamma\left(\frac{k_{123}+1}{2}\right)\Gamma\left(\frac{k_{231}+1}{2}\right)\Gamma\left(\frac{k_{312}+1}{2}\right)}{\sqrt{\Gamma(2k_1-1)\Gamma(2k_2-1)\Gamma(2k_3-1)}}\right)$$

$k_{ijk} \equiv k_i + k_j - k_k$, $\alpha \equiv k_1 + k_2 + k_3$,
in precise agreement with calculation in $11d$ sugra on $AdS_7 \times S^4$!
(Corrado Florea McNees, Bastianelli Zucchini)

$1/N$ corrections in W_N OPE $\Rightarrow$ quantum M-theory corrections.

Outlook

- From physical expectations about $4d$ SCFTs, interesting purely mathematical conjectures about chiral algebras.
  Develop cohomological tools to prove them.
- For a given SCFT $\mathcal{T}$, develop systematic tools to characterize $\chi[\mathcal{T}]$ in terms of generators.
- Landscape of $\mathcal{N} = 2$ SCFTs $\Leftrightarrow$ Classification of special chiral algebras
- Add non-local operators.
  Particularly interesting in $d = 6$: a derivation of AGT?
- For Lagrangian theories, relation to localization?
- Holographic chiral algebras.

Outlook: numerical boostrap

Some highlights:

- Stress-tensor 4pt function in $d = 4$, $\mathcal{N} = 4$. Beem LR van Rees
  Recently extended to higher BPS operators. Alday Bissi
- Stress tensor 4pt function in$d = 6$, $(2, 0)$ theory.
  Beem Lemos LR van Rees, to appear
- Exploration of landscape of $\mathcal{N} = 2$ SCFTs,
  especially non-Lagrangian ones.
  Beem Lemos Liendo LR van Rees, to appear