Supersymmetry: Lecture 2: The Supersymmetrized Standard Model

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Part I: The Supersymmetrized SM: motivation and structure

2-3 years ago, all fundamental particles we knew had spin 1 or spin 1/2
but we now have the Higgs: it's spin 0
the world and (QM courses) would have been very different if the particle we know best, the electron, were spin-0
of course spin-0 is the simplest possibility
spin-1 is intuitive too (we all understand vectors)
from a purely theoretical standpoint, supersymmetry would provide an explanation for why we have fermions

Fine tuning:
The Higgs mass is quadratically divergent

$$\delta m^2 \propto \Lambda_{UV} \tag{1}$$

unlike
fermions (protected by chiral symmetry)
gauge bosons (protected by gauge symmetry)
practically: we don't care (can calculate anything in QFT, just put in a counter term)

theoretically: believe Λ_{UV} is a concrete physical scale, eg: mass of new fields, scale of new strong interactions
then

$$m^2(\mu) = m^2(\Lambda_{UV}) + \#\, \Lambda_{UV}^2 \tag{2}$$

$m^2(\Lambda_{UV})$ determined by the full UV theory
$\#$ determined by SM
and the parameters of the 2 theories must be tuned to

$$\frac{\mathrm{TeV}^2}{\Lambda_{UV}^2} \tag{3}$$

with supersymmetry (even softly broken):
only log divergence:

$$m^2(\mu) = m^2(\Lambda_{UV}) \left[1 + \#\, \log\left(\frac{m^2(\Lambda_{UV})}{\Lambda_{UV}^2} \right) \right] \tag{4}$$

just as for fermions (indeed because supersymmetry ties the scalar mass to the fermion mass) so let's supersymmetrize the SM

Field content: gauge

each gauge field is now part of a vector supermultiplet
recall

$$A_\mu^a \longrightarrow (\tilde{\lambda}^a, A_\mu^a, D^a) \tag{5}$$

$$G_\mu^a \longrightarrow (\tilde{g}^a, G_\mu^a, D^a) \tag{6}$$

physical fields: gluon + gluino

$$W_\mu^I \longrightarrow (\tilde{w}^I, W_\mu^I, D^I) \tag{7}$$

physical fields: W + wino

$$B_\mu \longrightarrow (\tilde{b}, B_\mu, D) \tag{8}$$

physical fields: B + bino

Field content: matter

each fermion is now part of a chiral supermultiplet (ϕ, ψ, F)
we take all SM fermions
q, u^c, d^c, l, e^c
to be L-fermions

$$q \to (\tilde{q}, q, F_q) \quad \text{all transforming as} \quad (3,2)_{1/6} \tag{9}$$

physical fields: (doublet) quark q + squark $\tilde{q}$

$$u^c \to (\tilde{u}^c, u^c, F_u) \quad \text{all transforming as} \quad (\bar{3},1)_{-2/3} \tag{10}$$

physical fields: (singlet) up-quark u^c + up squark $\tilde{u}^c$

$$d^c \to (\tilde{d}^c, d^c, F_d) \quad \text{all transforming as} \quad (\bar{3},1)_{1/3} \tag{11}$$

physical fields: (singlet) down-quark d^c + down squark $\tilde{d}^c$

$$l \to (\tilde{l}, l, F_l) \quad \text{all transforming as} \;\; (1,2)_{-1/2} \tag{12}$$

physical fields: (doublet) lepton $l + \tilde{l}_L$

$$e^c \to (\tilde{e}^c, e^c, F_e) \quad \text{all transforming as} \;\; (1,1)_1 \tag{13}$$

physical fields: (singlet) lepton e^c + slepton $\tilde{e}^c$

with EWSB: the doublets split:

$$q = \begin{pmatrix} u \\ d \end{pmatrix} \qquad \tilde{q} = \begin{pmatrix} \tilde{u} \\ \tilde{d} \end{pmatrix} \tag{14}$$

$$l = \begin{pmatrix} \nu \\ l \end{pmatrix} \qquad \tilde{l} = \begin{pmatrix} \tilde{\nu} \\ \tilde{l} \end{pmatrix} \tag{15}$$

Field content: Higgs fields

The SM Higgs is a complex scalar, so it must be part of a chiral supermultiplet

$$H \to (H, \tilde{H}, F_H) \quad \text{all transforming as} \quad (1,2)_{-1/2} \tag{16}$$

we immediately see three problems (3 faces of the same problem):
even considering the scalar Higgs field, there is a problem with a single Higgs scalar
we want the Higgs (and only the Higgs) to get a VEV
but that means a nonzero D term:

$$V = D^I D^I + D_Y^2 \tag{17}$$

where

$$D^I = \langle H^\dagger \rangle T^I \langle H \rangle \quad D_Y = \langle H \rangle^\dagger \frac{1}{2} \langle H \rangle \tag{18}$$

you might think this is good, but it's not (for many reasons)
here's one:
the non-zero D-terms would generate masses for the squarks, sleptons:
consider D_Y for example:

$$D_Y = \frac{1}{2}v^2 + \sum_i Y_i|\tilde{f}|_i \tag{19}$$

where $\tilde{f}$ sums over all squarks, sleptons and Y_i is their hypercharge so some of these will get negative masses-squared of order v^2
this is a disaster: SU(3), EM broken at v!
if we add a second Higgs scalar, *with opposite charges* this can be avoided: the 2 scalars should then get equal VEVs with all $D = 0$

2) $\tilde{H}$ is a Weyl fermion

if this is all there is, we will have a massless fermion around—the Higgsino, which we don't see

in order to get rid of it, we need a second Weyl fermion, with cojugate charges, so together they form a massive fermion

3) in the presence of massless fermions, gauge symmetries can become anomalous

the SM is amazing: the fermion content is such that there are no anomalies

so far we added scalars (squarks and sleptons, known collectively as sfermions) which are harmless

and gauginos: these are fermions, but they are adjoint fermions, and these don't generate any anomalies (adjoint = real rep)

but the Higgsino $\tilde{H}$ is a massless fermion which is a doublet of SU(2) and charged under $U(1)_Y$

the simplest way to cancel the anomaly is to add a second Higgsino in the conjugate rep

so we add a second Higgs field

when we consider interactions, we will see another reason why we must do this so call the SM Higgs H_D and the new Higgs H_U

$$H_D \to (H_D, \tilde{H}_D, F_{HD}) \quad \text{all transforming as} \quad (1,2)_{-1/2} \tag{20}$$

$$H_U \to (H_D, \tilde{H}_D, F_{HU}) \quad \text{all transforming as} \quad (1,2)_{1/2} \tag{21}$$

and in the limit of unbroken supersymmetry

$$\langle H_U \rangle = \langle H_D \rangle \tag{22}$$

Interactions: gauge

nothing to do: completely dictated by gauge symmetry and supersymmetry
we wrote the Lagrangian for a general gauge theory in the previous lecture:

$$\begin{aligned} \mathcal{L} &= \mathcal{L}_{gauge} + \mathcal{D}^{\mu}\phi_i^* \mathcal{D}_{\mu}\phi_i + \psi_i^{\dagger} i\bar{\sigma}^{\mu}\mathcal{D}_{\mu}\psi_i + F_i^* F_i & (23) \\ &- \sqrt{2}(\phi_i^* \lambda^{aT} T^a \varepsilon \psi_i - \psi_i^{\dagger}\varepsilon\lambda^{a*} T^a \phi_i) + gD^a \phi_i^* T^a \phi_i & (24) \end{aligned}$$

here

$$\psi_i = q_i, u_i^c, d_i^c, l_i, e_i^c + \tilde{H}_U, \tilde{H}_D \quad (25)$$

$$\phi_i = \tilde{q}_i, \tilde{u}_i^c, \tilde{d}_i^c, \tilde{l}_i, \tilde{e}_i^c + H_U, H_D \quad (26)$$

the covariant derivatives contain the SU(3), SU(2), U(1) gauge fields

$$
\begin{aligned}
\lambda^a &\rightarrow \tilde{g}^a, \tilde{w}^I, \tilde{b} \qquad (27)\\
D^a &\rightarrow D^a, D^I, D_Y \qquad (28)
\end{aligned}
$$

and there's of course the pure gauge Lagrangian that I haven't written (we saw it in the previous lecture) solving for the D terms we get the scalar potential

$$V = \frac{1}{2} g_3^2 D^a D^a \frac{1}{2} + g_2^2 D^I D^I + \frac{1}{2} g_1^2 D_Y D_Y \tag{29}$$

where

for SU(3): (recall $T_{\bar{3}} = -T_3^*$ and we will write things in terms of the fundamental generators)

$$D^a = \tilde{q}^\dagger T^a \tilde{q} - \tilde{u}^{c\dagger} T^{a*} u^c - \tilde{d}^{c\dagger} T^{a*} u^c \tag{30}$$

similarly for the SU(2) and

$$D_Y = \sum_i Y_i \tilde{f}_i^\dagger \tilde{f}_i + \frac{1}{2}(H_U^\dagger H_u - H_D^\dagger H_D) \tag{31}$$

get: 4 scalar interactions with coupling = gauge couplings
in particular: a Higgs potential! with coupling = g_2, g_Y!
(recall: part of the reason we wanted 2 scalars: no D^I, D_Y VEVs)

but note: no freedom (and no new parameter)

Yukawa couplings

In the SM we have Higgs-fermion-fermion Yukawa couplings consider the down-quark Yukawa first

$$y_D H_D q^T \varepsilon d^c \tag{32}$$

with supersymmetry, this must be accompanied by

$$y_D(\tilde{q}\tilde{H}_D^T \varepsilon d^c + \tilde{d}^c \tilde{H}_D^T \varepsilon q) \tag{33}$$

all coming from the superpotential

$$W_D = y_D H_D q d^c \tag{34}$$

similarly for the lepton Yukawa:

$$W_l = y_l H_D l e^c \rightarrow \tag{35}$$

$$\mathcal{L}_l = y_l(H_D l^T \varepsilon e^c + \tilde{l}\tilde{H}_D^T \varepsilon e^c + \tilde{e}^c \tilde{H}_D^T \varepsilon l + \mathrm{hc}) \tag{36}$$

what about the up Yukawa?
need

$$(\mathrm{Higgs})q^T\varepsilon u^c \tag{37}$$

this coupling must come from a superpotential

$$(\mathrm{Higgs})qu^c \tag{38}$$

in the SM (Higgs)$= H_D^\dagger$
but the superpotential is **holomorphic** : no daggers allowed
this is the 4th reason why we needed a second Higgs field with the opposite charges (but they are all the same reason really)

$$\begin{aligned} W_U &= y_U H_U q u^c \rightarrow & (39)\\ \mathcal{L}_u &= y_U(H_U q^T \varepsilon u^c + \tilde{q}\tilde{H}_U^T\varepsilon u^c + \tilde{u}^c\tilde{H}_U^T\varepsilon q) + \mathrm{hc} & (40) \end{aligned}$$

you can see what's going on:
holomorphy makes a scalar field "behave like a fermion":
in a supersymmetric theory, the interactions of scalar fields are controlled by the superpotential, which is holomorphic
for a fermion to get mass you need an LR coupling
so starting from an L fermion you need an R fermion
or another L fermion with the opposite charge(s)
for a scalar ϕ to get mass in a non-supersymmetric theory:
you don't need anything else (just use ϕ^*)
not so in a susy theory
because you cant use ϕ^*, must have another scalar with the opposite charge(s)

but note: no freedom (and no new parameter)

R-symmetry

Also note: we have a $U(1)_R$ symmetry:
let's take:
gaugino$= -1$
sfermions$= 1$
Higgsinos$= 1$
(all others neutral)

to recap:
we wrote down the Supersymmetric Standard Model
gauge bosons + gauginos (spin 1/2)
fermions + sfermions (spin 0)
2 Higgses + 2 Higgsinos (spin 1/2)
the interactions are all dictated by SM + SUSY:
the new ones are:
gauge-boson - scalar - scalar
gauge-boson - gauge-boson - scalar - scalar
gaugino-sfermion-fermion
gauge-boson Higgsino Higgsino
4-scalar (all gauge invariant contributions)
all these have couplings = gauge couplings
in particular: a 4-Higgs coupling: quartic Higgs potential

Yukawa part:
Higgsino-quark-squark
coupling = SM Yukawa

Implications

no quadratic divergence in Higgs mass
each quark contribution canceled by L, R squarks
similarly: Higgs self coupling (from D term) canceled by Higgsino
each gauge boson contribution canceled by gaugino

Implications

but we now have massless gluinos, a wino degenerate with the W a selectron degenerate with the electron etc

supersymmetry must be broken

it would be nice if the SSM broke it spontanously (after all we have lots of scalars with a complicated potential)

but no such luck

so we must add more fields and interactions that break supersymmetry

these new fields must couple to the SM fields in order to generate masses for the superpartners

The supersymmetrized standard model with supersymmetry-breaking superpartner masses

General structure

SB —— SSM

SB: new fields interactions such that supersymmetry spontaneously broken

as a result: in SB: mass splittings between bosons-fermions of different supermultiplets

—— = some couplings between SM fields and SB fields

as a result: mass splitting between bosons and fermions of various supermultiplets

the couplings —— mediate the breaking

the mediation of the breaking is what determines the supersymmetry-breaking terms in the SSM

the supersymmetry-breaking terms
what do we expect?
remember: any term is allowed unless a symmetry prevents it
now that we broke supersymmetry, new supersymmetry breaking terms are allowed
matter sector: sfermions get mass
(fermions don't: protected by chiral symmetry
easiest to think about this before EWSB: what's happening in SB does not break EW symmetry)
gauge sector: gauginos get mass
(gauge bosons don't: protected by gauge symmetry)
Higgs sector: Higgses get mass
(Higgsinos don't: protected by chiral symmetry
so this isn't so good and we have to do something about it)

in addition: there are trilinear scalar terms that can appear:
Higgs-squark-squark Higgs-slepton-slepton
(allowed by gauge symmetry, and supersymmetry is no longer there to forbid them)

So the supersymmetry-breaking part of the SSM Lagrangian is:

$$
\begin{aligned}
\mathcal{L}_{soft} &= -\frac{1}{2}[\tilde{m}_3\tilde{g}^T\varepsilon\tilde{g} + \tilde{m}_2\tilde{w}^T\varepsilon\tilde{w} + \tilde{m}_1\tilde{b}^T\varepsilon\tilde{b}] \\
&- \tilde{q}^*\tilde{m}_q^2\tilde{q} - \tilde{u}^{c*}\tilde{m}_{uR}^2\tilde{u}^c - \tilde{d}^{c*}\tilde{m}_{dR}^2\tilde{d}^c \\
&- \tilde{l}^*\tilde{m}_l^2\tilde{l} - \tilde{e}^{c*}\tilde{m}_{eR}^2\tilde{e}^c \\
&- H_U^* m_{H_U}^2 H_U - H_D^* m_{H_D}^2 H_U \\
&- H_U\tilde{q}^* A_U\tilde{u}^c - H_D\tilde{q}^* A_U\tilde{d}^c - H_D\tilde{l}^* A_l\tilde{e}^c \\
&- B\mu H_U H_D
\end{aligned}
\qquad (41)
$$

- gauge indices are contracted (δ_i^j, $\epsilon^{\alpha\beta}$)
- the last line: a quadratic term for the Higgs scalars
- the line before last: new trilinear scalar interactions when the Higgses get VEVs these too will turn into sfermion mass terms (mixing L and R scalars)
- m_q^2 etc are 3×3 matrices in generation space so are A_U etc

the values of the (supersymmetry breaking) parameters are determined by the SB theory and (mainly) the mediation
you sometimes hear people criticize supersymmetric extensions of the SM for having a hundred or so new parameters (the parameters of $\mathcal{L}_{soft}$)
but as we said: these are all determined by the SB and the mediation
often: very few new parameters

also remember:
the parameters of $\mathcal{L}_{soft}$ are the only freedom we have
and where all the interesting physics lies:
they determine the spectrum of squarks, sleptons
these in turn determine the way supesymmetry manifests itself
in Nature
$=$ experimental signatures

R-parity

The gaugino masses and A-terms break the $U(1)_R$ symmetry
but there's something left: a Z_2
this is R-parity:
under R-parity: gauginos, sfermions, Higgsinos: odd
all SM fields: even
so: supersymmetrizing the SM (without adding any new interactions)
we have a new parity
$\rightarrow$ **the lightest superpartner is stable**

the mu-term: a supersymmetric Higgs, Higgsino mass

before we go on, let's discuss one remaining problem:
we have 2 massless Higgsinos in the theory
(can't get mass by supersymmetry-breaking)
so must also include a supersymmetric mass term:

$$W = \mu H_U H_D \tag{42}$$

SUSY breaking basics

already saw: SB iff some F and/or D nonzero
global susy: spontaneous breaking → goldstone fermion = "goldstino":

$$Q|0\rangle \tag{43}$$

which is a fermion
what is it concretely? let's look at the susy current

$$J_\mu \sim \sum_\phi \frac{\delta L}{\delta(\partial_\mu \phi)} (\delta\phi)_\alpha \tag{44}$$

the only things that can get a VEV (without breaking Lorentz) are:
in the chiral sfield: $\delta\psi \propto F$
in the vector sfield: $\delta\lambda \propto D$
so

$$J_\mu \sim \sum \frac{\delta L}{\delta(\partial_\mu \psi_i)} < F_i > + \sum \frac{\delta L}{\delta(\partial_\mu \lambda_a)} < D^a > \tag{45}$$

Tree-level breaking: F terms

we already saw the O'Raifeartaigh model in which supersymmetry is broken by F terms

Tree-level breaking: D terms

the simplest example of tree-level D-term breaking is the Fayet-Iliopoulos model

This is a U(1) gauge theory with fields Q, $\bar{Q}$ of charges 1 and -1

in a U(1) theory, the D term is gauge invariant, so one add a D-term tadpole to the Lagrangian

thus take the Kähler potential to be

$$K = Q^\dagger e^V Q + \bar{Q}^\dagger e^V \bar{Q} + \xi_{FI} V \tag{47}$$

and the superpotential

$$W = m\bar{Q}Q \tag{48}$$

so

$$V = \frac{1}{2} g^2 \left[|Q|^2 - |\bar{Q}|^2 + \xi_{FI}^2\right] + m^2 \left[|Q|^2 + |\bar{Q}|^2\right] \tag{49}$$

susy is broken

- $g^2 \xi_{FI}^2 < m^2$: the U(1) is unbroken, $D \neq 0$, $F_i = 0$,

Mediating the breaking

Gauge interactions

gauge interactions are the ones we know best
so gauge mediation gives full, concrete (and often calculable) supersymmetric extensions of the SM

The simplest gauge mediation models: Minimal Gauge Mediation

suppose we have a supersymmetry-breaking model with chiral supermultiplets Q_i and $\bar{Q}_i$, $i = 1, 2, 3$
and we constructed the supersymmetry-breaking model such that the fermions Q_i and $\bar{Q}_i$ combine into a Dirac fermion of mass M and, and the scalars have masses-squared $M^2 \pm F$ $(F < M^2)$
now identify i as an SU(3) color index
so Q is a 3 of SU(3), $\bar{Q}$ is a $\bar{3}$ of SU(3)
the gluino gets mass because of the SU(3) gauge interactions
the squarks get mass because of the SU(3) gauge interactions

so we have
a gluino mass

$$m_{\tilde{g}} = \# \frac{\alpha}{4\pi} \frac{F}{M} + \mathcal{O}(F^2/M^2) \tag{50}$$

a squark mass

$$m_{\tilde{q}} = \# \frac{\alpha^2}{(4\pi)^2} \frac{F^2}{M^2} + \mathcal{O}(F^4/M^6) \tag{51}$$

with similar expressions for the R up-squarks and R down-squark
the numbers are group theory factors
we can infer this very simply:
the masses should vanish as $F \to 0$, and as $M \to 0$

this is very elegant
soft masses are determined by gauge couplings
the squark matrices are flavor-blind ($\propto 1_{3\times 3}$ in flavor space)
gluino masses $\sim$ squark masses
the only new parameter* is F/M (a scale)
if want soft masses around TeV, $F/M \sim 100$ TeV
the new fields Q, $\bar{Q}$ are the *messengers* of susy breaking
* but there's running: the soft masses are generated at the messenger scale$\sim M$
to calculate them at the TeV we need to include RGE effects
so the messenger scale M is also important

the gravitino mass $m_{3/2} = F_{eff} / M_P$
where F_{eff} is the the dominant F term
so

$$m_{3/2} \geq \frac{F}{M_P} \sim \frac{M}{M_P} 100 \text{ TeV} \tag{52}$$

so for a low messenger scale, the gravitino can be very light (eV)

in order to give masses to everything we need messenger field charged under SU(3), SU(2), U(1)
eg, N_5 copies of $(3,1)_{-1/3} + (\bar{3},1)_{1/3}$ and $(1,2)_{-1/2} + (1,2)_{1/2}$ (filling up a $5 + \bar{5}$ of SU(5))
parameters:
N_5 (number of messengers)
F/M (overall scale)
M where soft masses generated (run down from there)

this is just a simple toy model: gauge mediation can in principle have a very different structure
the only defining feature is that the soft masses are generated by the SM gauge interactions
but there are a few generic features:
colored superpartners (gluinos, squarks) are heavier than non-colored (EW gauginos, sleptons..) by a factor

$$\frac{\alpha_3}{\alpha_2} \quad \text{or} \frac{\alpha_3}{\alpha_2} \tag{53}$$

in particular: gaugino masses scale as

$$\alpha_3 : \alpha_2 : \alpha_1 \tag{54}$$

no A terms at M

Gravity Mediation

with gauge mediation, we had to do some real work:
add new fields, make sure they get some supersymmetry-breaking masses
but supersymmetry breaking is one place where we expect a free lunch:
imagine we have, in addition to the SM, some supersymmetry-breaking fields
eg, the O'Raifeartaigh model
since supersymmetry is a space-time symmetry, the SM fields should know this automatically
we would expect soft terms to be generated, suppressed by M_P
this is known as "gravity mediation"
we will discuss first the purest form of gravity mediation: anomaly mediation
and then what's commonly referred to as gravity mediation

Anomaly mediation

so we imagine supersymmtery is broken by some fields that have no coupling to the SM (the hidden sector)
the gravitino gets mass $m_{3/2}$ (a **scale**)
would the SSM "know" about supersymmtery breaking?
yes: at the quantum level, it's not scale-invariant:
all the couplings (gauge, Yukawa) run— the beta functions are nonzero
so **all** the soft terms are generated

gaugino masses:

$$m_{1/2} = b\frac{\alpha}{4\pi}\, m_{3/2} \tag{55}$$

where b, α are the appropriate beta-function coefficient and coupling
[for an SU(N) with N_F flavors of fundamental + antifundamental supermultiplets $b = 3N - N_F$]
so for SU(3) $b = 3$, for SU(2) $b = -1$ and for U(1) $b = -33/5$

sfermions get masses proportional to their anomalous dimensions:

$$m_0^2 \sim \frac{1}{16\pi^2}(y^4 - y^2 g^2 + b g^4)\, m_{3/2}^2 \qquad (56)$$

for the first and second generation sfermions, we can neglect the Yukawas so

$$m_0^2 \sim \frac{g^4}{16\pi^2}\, b\, m_{3/2}^2 \qquad (57)$$

A terms are generated too, proportional to the beta functions of the appropriate Yukawa

this is amazing: these contributions are **always there**
everything determined by SM couplings
one new parameter: the gravitino mass
too good to be true: while SU(3) is ASF $b_3 > 0$, SU(2), U(1) are not: $b_2, b_1 < 0$
so the sleptons are tachyonic
there are various fixes to this

but the gaugino masses are fairly robust:
putting in the numbers:

$$m_{\tilde{w}} : m_{\tilde{b}} : m_{\tilde{g}} : m_{3/2} \sim 1 : 3.3 : 10 : 370 \tag{58}$$

wino(s) are lightest!
the gravitino is roughly a loop factor heavier than the SM superpartners

Gravity mediation: mediation by Planck suppressed operators

return to our basic setup
there are some new fields and interactions that break supersymmetry (the hidden sector)
generically, we would expect some higher-dimension operators (suppressed by M_P) that couple these fields to the SM
some of the hidden sector fields have non-zero F terms (or D terms)
so we expect nonzero soft terms
sfermion masses from

$$\propto \frac{|F|^2}{M_P^2}\tilde{f}^\dagger \tilde{f} \tag{59}$$

gaugino masses from

$$\frac{|F|}{M_P}\lambda^T \varepsilon \lambda \tag{60}$$

you can think of these as mediated by tree-level exchange of

all this is at the high scale (where the soft masses are generated)
running to low scales:

$$\frac{d}{dt} m_{1/2} \propto g^2 m_{1/2} \tag{65}$$

starting from a common gaugino mass at the GUT scale one finds at low energies:
the gaugino masses scale as

$$\alpha_3 : \alpha_2 : \alpha_1 \tag{66}$$

as in gauge mediation
(bino lightest)

scalar masses squared:
schematically:

$$\frac{d}{dt} m_0^2 \sim +\# g^2 m_{1/2}^2 + \# g^2 m_0^2 - \# y^2 m_0^2 \tag{67}$$

where all the # are positive
so:
a positive contribution from the gaugino mass [largest]
a positive contribution from scalar masses (via the gauge coupling)
a negative contribution from scalar masses (via the Yukawa coupling)
so for sfermions: a large universal (=generation independent) contribution from the gaugino mass
so we have near degeneracy at low scales:
even if start with *different* $\tilde{q}$ masses at the high scale, at the low scale splittings are around 15% only
(and same for $\tilde{u}^c$, $\tilde{d}^c$)

the gravitino mass?
of order the superpartner masses

Other possibilities

these are a few possibilities but by no means an exhaustive list
example: Flavored Gauge Mediation:
in minimal gauge mediation: messenger fields
$(1,2)_{1/2}$ and $(1,2)_{-1/2}$
same charges as H_U and H_D
so in principle: superpotential couplings of the messengers to matter fields
new (calculable) contributions to soft terms

Implications

EWSB and the Higgs mass

The SUSY Higgs mechanism: A U(1) toy model

want to break the U(1) gauge symmetry by the Higgs mechanism
need a charged chiral supermultiplet ϕ_+
must add second Higgs fields of opposite charges ϕ_-
SUSY limit:

$$\langle \phi_+ \rangle = \langle \phi_- \rangle \qquad (68)$$

(otherwise anomalous, $D \neq 0$)
double the number in the non-susy case
double the number of would-be Nambu-Goldtone-Bosons ??

resolution:
one combination of ϕ_+, ϕ_- remains massless
and is eaten by photon (gives longitudinal polarization)
so the massive vector multiplet: 3 dof's
susy unbroken: must have a fermion of same mass:
(Higgsino-gaugino) 4 dof's
to balance: need the second combination of ϕ_+, ϕ_-
so the massive photon supermultiplet:
gauge boson (3)
Dirac fermion (4)
real scalar (1)
ex: work out the details of the susy Higgs mechanism in this example. Expand around the vacuum (68)

$$\phi_+(x) = v + i\pi(x) + iA(x) + h(x) + H(x) \tag{69}$$

$$\phi_-(x) = v - i\pi(x) + iA(x) + h(x) - H(x) \tag{70}$$

and find the spectrum.

The MSSM Higgs spectrum

In the SSM: H_U and H_D:

$$\langle H_U \rangle = \begin{pmatrix} v_U \\ 0 \end{pmatrix} \quad \langle H_D \rangle = \begin{pmatrix} 0 \\ v_D \end{pmatrix} \tag{71}$$

count scalars:
8 real dofs
3 eaten by $W^{\pm}$, Z
start with SUSY limit (with $\mu = 0$):

$$D = 0 \quad \rightarrow \quad v_U = v_D \tag{72}$$

3 join the heavy $W^{\pm}$, Z supermultiplets
usually called $H^{\pm}$ and H; with masses M_W, M_Z
2 neutral fields remain:
(2 because must form the complex scalar of a chiral supermultiplet)
h (real part: CP even) and A (imaginary part: CP odd)

NO POTENTIAL for h : not surprising
we haven't added any Higgs superpotential so only quartic is from V_D
but along D-flat direction: physical Higgs is massless
Higgs mass must come from supersymmetry breaking !

EWSB

fortunately (1) supersymmetry is broken—we have soft terms
The Higgs potential comes from the following sources:
quadratic terms:
A. the mu term: $W = \mu H_U H_D$

$$\delta V = |\mu|^2 |H_U|^2 + |\mu|^2 |H_D|^2 \quad (73)$$

B. the Higgs soft masses:

$$\delta V = \tilde{m}^2_{H_U} |H_U|^2 + \tilde{m}^2_{H_D} |H_D|^2 \quad (74)$$

so need $m^2_{H_U} < 0$ and/or $m^2_{H_U} < 0$
C. the $B\mu$ term:

$$\delta V = B\mu H_U H_D + \mathrm{hc} \quad (75)$$

quartic terms:

$$\delta V = \frac{1}{2} g_2^2 D^I D^I + \frac{1}{2} g_1^2 D_Y D_Y \tag{76}$$

where

$$D^I = H_U^\dagger \tau^I H_U - H_D^\dagger \tau^{I*} H_D \tag{77}$$

and

$$D_Y = \sum_i Y_i \tilde{f}_i^\dagger \tilde{f}_i + \frac{1}{2}(H_U^\dagger H_u - H_D^\dagger H_D) \tag{78}$$

parameters: 2 VEVs:
trade for:
1. $\sqrt{v_U^2 + v_D^2}$: determined by W mass to be 246 GeV
2. $\tan\beta \equiv v_U / v_D$

requiring a minimum of the potential determines:

$$B\mu = \frac{1}{2}(m_{H_U}^2 + m_{H_D}^2 + 2\mu^2)\sin 2\beta \quad (79)$$

$$\mu^2 = \frac{m_{H_D}^2 - m_{H_U}^2 \tan^2\beta}{\tan^2\beta - 1} - \frac{M_Z^2}{2} \quad (80)$$

so for given $m_{H_U}^2$, $m_{H_D}^2$: $B\mu$ and μ determined
free parameters: $\tan\beta$, $\sin\mu$

scalar spectrum:

$$\begin{aligned} H^{\pm} &: \quad M_W^2 + M_A^2 \qquad\qquad (\mathrm{SUSY}: M_W^2) \\ H^0 &: \quad \frac{1}{2}(M_Z^2 + M_A^2) + \frac{1}{2}\sqrt{(M_Z^2 + M_A^2)^2 - 4m_A^2 M_Z^2 \cos^2 2\beta} \\ & \qquad\qquad\qquad\qquad (\mathrm{SUSY}: M_Z^2) \\ A^0 &: \quad M_A^2 = B\mu(\cot\beta + \tan\beta) \qquad (\mathrm{SUSY}: 0) \end{aligned} \tag{81}$$

for the light Higgs (SUSY:=0)

$$m_h^2 = \frac{1}{2}(M_Z^2 + M_A^2) - \frac{1}{2}\sqrt{(M_Z^2 + M_A^2)^2 - 4m_A^2 M_Z^2 \cos^2 2\beta} \tag{82}$$

PREDICTION:

$$m_h \leq m_Z |\cos 2\beta| \leq M_Z \tag{83}$$

The measurement of the Higgs mass provides the first quantitaive test of the Minimal Supersymmetric Standard Model

[saturated for $M_A^2 \gg M_Z^2$: the DECOUPLING LIMIT]

does it fail?
the result (82) is at tree-level
there are large radiative corrections from stop masses
(will see why soon)
in the decoupling limit

$$m_h^2 \sim m_Z^2 \cos^2 2\beta + \frac{3m_t^2}{4\pi^2 v^2} \left[\log \frac{M_S^2}{m_t^2} + \frac{X_t^2}{M_S^2} \right] \qquad (84)$$

where

$$\begin{aligned} X_t &= A_t - \mu \cot \beta \quad \text{the LR stop mixing} \\ M_S &= \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}} \quad \text{the average stop mass} \end{aligned}$$

can raise Higgs mass to around 130 GeV

for 126 GeV need:
heavy stops and/or large stop A terms
fine-tuning !
at best, stops at 1.5-2TeV
at worst: minimal gauge mediation: no A-terms at messenger scale
stops around 8-10 GeV (and other squarks close)
so: Higgs mass is a stronger constraint than direct searches
caveat: can easily add a quartic potential for the Higgs through (see next slide)

compare to SM (part I: quartic): not so bad
SM: added a quartic Higgs potential to get the Higgs mass
here we didn't have to: D-terms give a quartic potential
but no new parameter: $\lambda = g$
could add a quartic interaction a la the SM:
must add at least one new field:
a SM singlet S:

$$W = \lambda S H_U H_D \rightarrow \lambda^2(|H_U|^2|H_D|^2 + \ldots) \tag{85}$$

aka the "NMSSM" Next to Minimal SSM

compare to SM (part II: quadratic): much more beautiful
SM: EWSB by hand: put in a negative mass-squared
MSSM: a dynamical origin:
supersymmetry breaking (2):
RGE drives Higgs mass-squared negative!
(through Yukawa coupling to stop)
dynamical origin of EWSB !

EWSB

fortunately (2) supersymmetry is broken—we have soft terms
The Higgs potential comes from the following sources:
quadratic terms:
A. the mu term: $W = \mu H_U H_D$

$$\delta V = |\mu|^2 |H_U|^2 + |\mu|^2 |H_D|^2 \quad (86)$$

B. the Higgs soft masses:

$$\delta V = \tilde{m}^2_{H_U} |H_U|^2 + \tilde{m}^2_{H_D} |H_D|^2 \quad (87)$$

so need $m^2_{H_U} < 0$ and/or $m^2_{H_U} < 0$
C. the $B\mu$ term:

$$\delta V = B\mu H_U H_D + \mathrm{hc} \quad (88)$$

starting with $\tilde{m}^2_{H_U} > 0$ at the supersymmetry breaking scale, RGEs generically drives it negative
reason: large stop contribution:

$$\frac{d}{dt} m^2_{H_U} \sim + \frac{g^2}{16\pi^2}\, m^2_{1/2} - \frac{y_t^2}{16\pi^2}\, \tilde{m}^2_t \tag{89}$$

large because of large Yukawa (compared to SU(2), U(1) coupling)
color factor $= 3$
NOTE: many scalars in MSSM but Higgs is special:
SU(3) singlet: so no large $(+)$ contribution from gluino
does have an order-1 Yukawa (to the colored stop)

Recap: EWSB and Higgs

putting aside the 125 GeV Higgs mass:
supersymmetry gives a very beautiful picture:
the MSSM (SSM + soft terms): only log divergence
quadratic divergence in Higgs mass-squared is cut off at $\tilde{m}$
(tuning $\sim M_Z^2/\tilde{m}^2$)
the hierarchy between the EWSB scale and the Planck/GUT scale is stabilized
furthermore:
starting with $\tilde{m}^2_{H_U} > 0$ in the UV:
the running (stop) drive it negative
electroweak symmetry is broken: proportional to $\tilde{m}$

and finally:
with a SB sector that breaks supersymmetry dynamically:
the supersymmetry breaking scale is exponetially suppressed:
$\tilde{m}$ can naturally be around the TeV
the hierarchy between the EWSB scale and the Planck/GUT scale is generated

with $m_h = 126$ GeV:
Minimal SSM is stretched: need heavy stops: tuning is worse
more practically: discovery becomes more of a challenge

Neutralino spectrum

we have 4 neutral 2-component spinors: two gauginos and 2 Higgsinos

$$\tilde{b}\,, \quad \tilde{W}^0\,, \quad \tilde{H}_D^0\,, \quad \tilde{H}_U^0 \tag{90}$$

with the mass matrix

$$\begin{pmatrix} M_1 & 0 & -g_1 v_D/\sqrt{2} & g_1 v_U/\sqrt{2} \\ 0 & M_2 & g_2 v_D/\sqrt{2} & -g_2 v_U/\sqrt{2} \\ -g_1 v_D/\sqrt{2} & g_2 v_D/\sqrt{2} & 0 & \mu \\ g_1 v_U/\sqrt{2} & -g_2 v_U/\sqrt{2} & \mu & 0 \end{pmatrix} \tag{91}$$

4 neutralinos $\tilde{\chi}^0$ $i = 1, \ldots, 4$
similarly: 2 charginos (charged Higgsino+wino) $\tilde{\chi}_i^{\pm}$ $i = 1, 2$

Sfermion spectrum

consider eg up squarks
6 complex scalars: $\tilde{u}_{Li} \quad \tilde{u}_{Ra}$
6×6 mass-squared matrix:

$$\begin{pmatrix} m^2_{LL} & m^2_{LR} \\ m^{2\dagger}_{LR} & m^2_{RR} \end{pmatrix} \tag{92}$$

consider $m^2_{U,LL}$: gets contributions from:

1. the SSM Yukawa (supersymmetric)
2. the SUSY breaking mass-squared
3. the D-term (because $D \sim v_U^2 - v_D^2 + \tilde{q}^\dagger T q + \cdots$) (supersymmetry breaking)

$$m^2_{U,LL} = m_u^\dagger m_u + \tilde{m}_q^2 + D_U 1_{3\times 3} \tag{93}$$

consider m^2_{LR}: gets contributions from:

1. the A term (susy breaking)
2. the μ term:

$$\left|\frac{\partial W}{\partial H_D}\right|^2 \quad \to \quad \frac{\partial W}{\partial H_D} = \mu H_U + y_U q u^c \tag{94}$$

so

$$m^2_{U,LR} = v_U \left(A_U^* - y_U \mu \cot\beta\right) \tag{95}$$

Flavor structure

so we have: in quark mass basis (up, charm, top):

- up squark mass matrix
- bino - u_{Li} - $\tilde{u}_{Lj}$ interaction
- bino - u_{Ri} - $\tilde{u}_{Rj}$ interaction
- ...

so for a generic up squark mass matrix:
physical parameters: 6 masses + mixings
similarly for 6 down squarks, 6 charged sleptons
(3 sneutrinos: LL only)
physical parameters: 6 masses + mixings

Flavor structure

neglect for simplicity LR: and consider 3 L up squarks:

- up squark mass matrix $m^2_{U,LL}$ $(3 \times 3$
- bino - u_{Li} - $\tilde{u}_{Lj}$ interaction

working in quark mass basis:
$bino - u_{Li} - \tilde{u}_{Li}$ interaction: defines $\tilde{u}_L$, $\tilde{c}_L$, $\tilde{t}_L$
diagonalizing $m^2_{U,LL}$ get 3 mass eigenstates $\tilde{u}_{L,a}$ with $a = 1, 2, 3$
and quark-squark mixings:

$$K_{ia}\, u_{Li}\, \tilde{u}^*_{La} - \text{bino}$$