A Scaleful Theory

Sergei Dubovsky

CCPP, NYU & ICTP, Trieste

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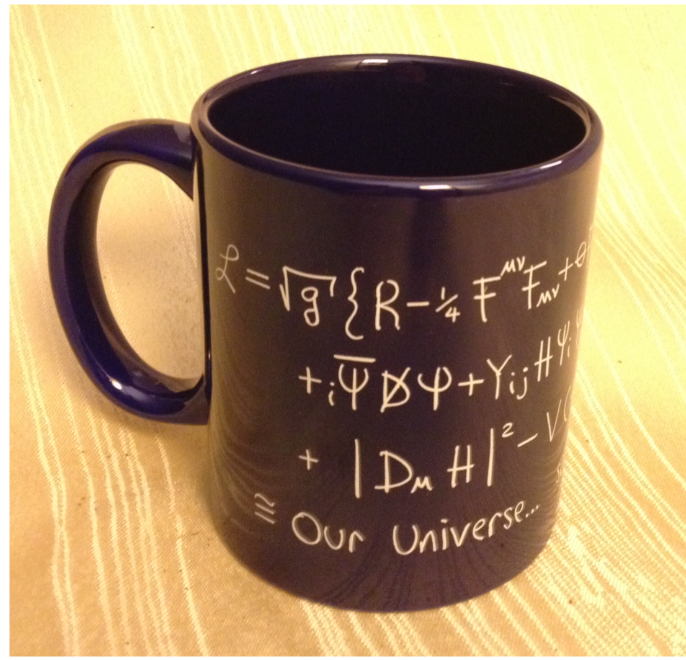
Quantum Field Theories

are ubiquitous and extremely efficient tool to describe Nature:

- ✓Particle physics
- ✓Condensed matter
- ✓Cosmology
- ✓ Hydrodynamics
- ✓...



but we don't really know yet what QFTs really are

one may say QFTs play the same role as numbers used to play

Brief history of numbers:

is the history of asking questions

- ✓Natural — counting
- ✓Integers
- ✓Rational
- ✓ Algebraic
- ✓ π , e — more interesting questions

solving algebraic equations

it's easy to run into infinities on the way, especially if the wrong question is asked

most real numbers won't answer any question

(c.f. generic effective QFT)

we are done with numbers...

What is the space of QFT's?

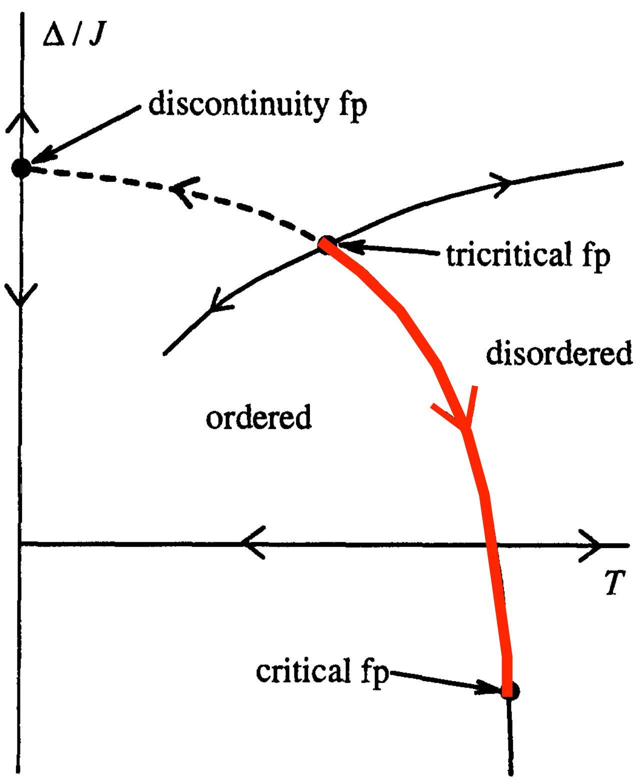
it feels we are only starting to learn how to ask interesting questions here

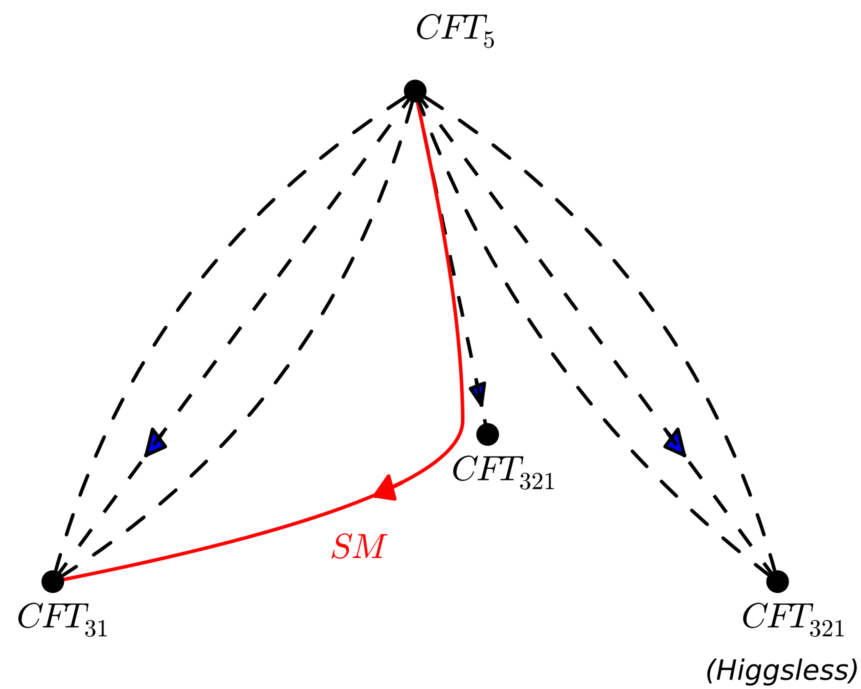
a related outstanding question

What are gravitational theories?

Wilson provided us with a very useful tool to organize our thinking about the space of QFT's:

Wilsonian Renormalization Group Flow



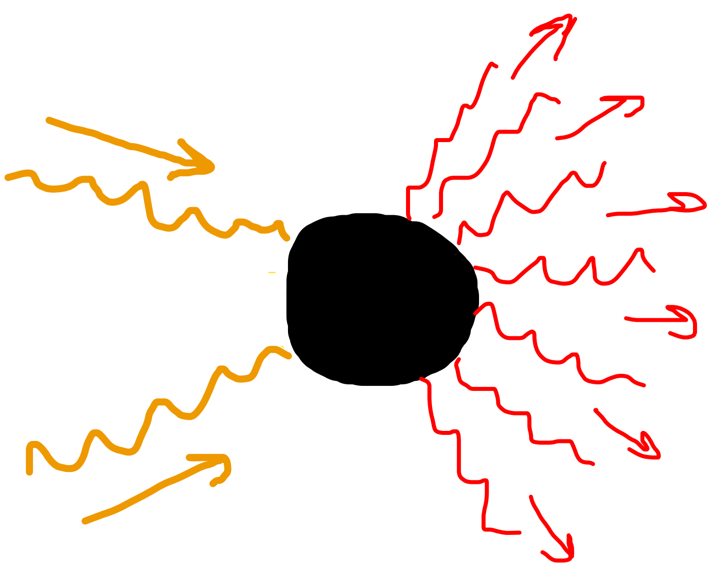


UV Complete QFT =
RG flow from a UV fixed point to an IR fixed point

Fixed points: Scale (conformal) invariant theories

Classic example: Quantum Chromodynamics (an asymptotically free theory)

GRAVITY IS DIFFERENT
Memory of the Planck scale never fades away



$$\sigma = \ell_{pl}^4 E^2$$

$$\Delta t = \ell_{pl}^4 E^3$$

The principal goal of this talk:

Describe a simple class of *scaleful* theories
and
discuss their physics

This is how these theories should have been found:

What a [...] massless

What is the simplest QFT?

Everything is determined by a two-particle phase shift:

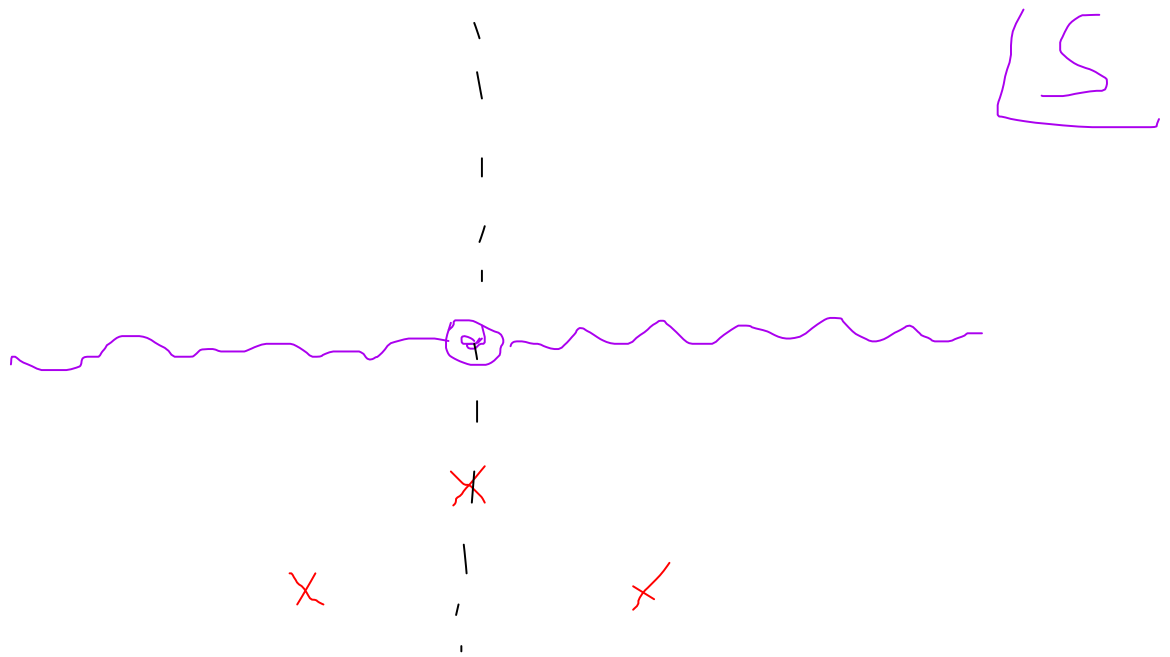
$$S = e^{2i\delta(s)}\mathbf{1}$$

Unitarity+Analyticity+Crossing:

Zamolodchikov '91

$$e^{2i\delta(s)} = \prod_j \frac{\mu_j + s}{\mu_j - s} e^{iP(s)}$$

$\mathrm{Im}\ s > 0$



Expectation from Locality: $P(s) = 0$

A candidate simplest QFT:

$$e^{2i\delta(s)} = \frac{iM^2 - s}{iM^2 + s}$$

This is the "solution" of a theory. It would be nice to recover physics...

A tool:

TBA (Thermodynamic Bethe Anzats):

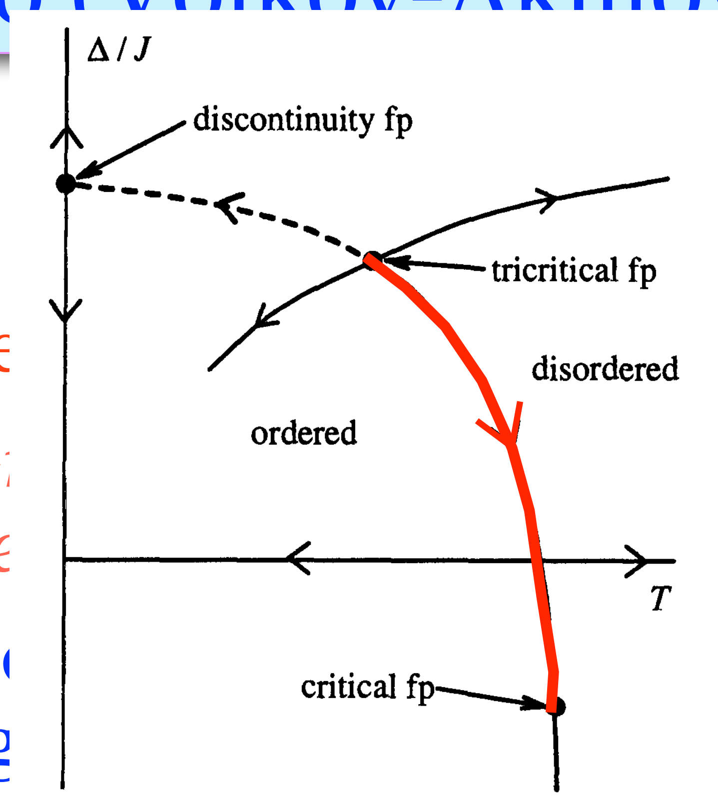
A machine to calculate the finite volume spectrum from the S-matrix

$$f(T) = TE_0(1/T) = \frac{\pi}{6} c_{UV} T^2 \quad \text{as } T \to \infty$$

$$e^{2i\delta(s)} = \frac{iM^2 - s}{iM^2 + s}$$

turns out to be:

Goldstino (Volkov-Akulov) Theory



$\mathcal{L} = \psi\bar{\partial}\psi$ … $\bar{\partial}\bar{\psi}) + \ldots$

A simple e … ic Safety":

naively non- … *flows into a*
strongly couple … *new stuff added*

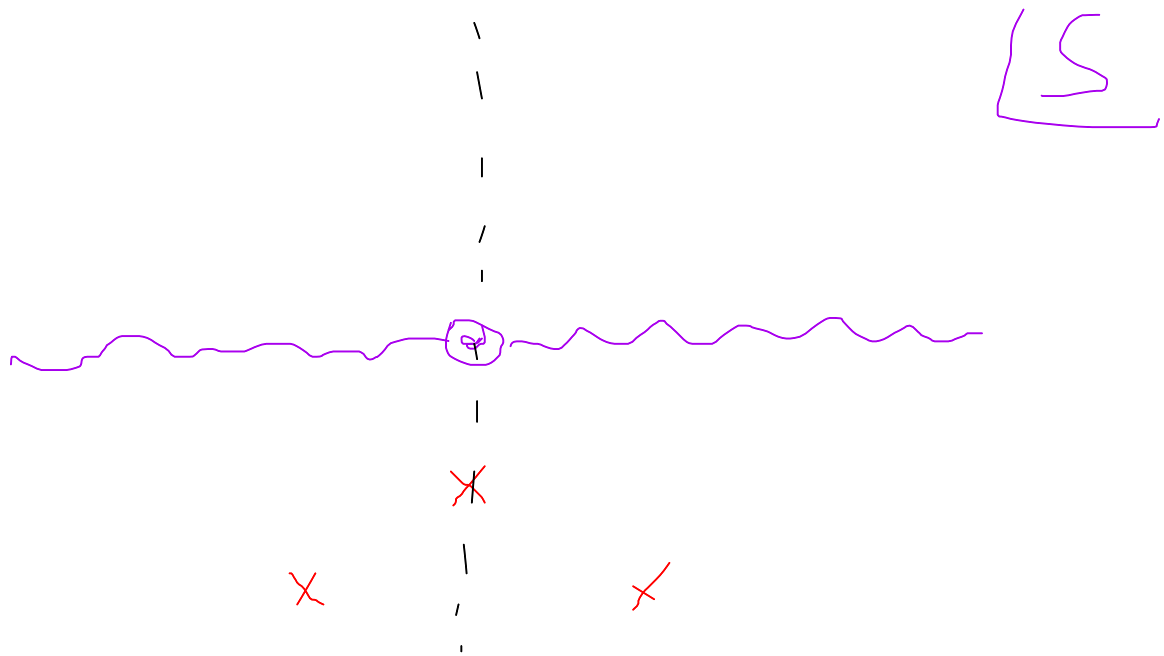
Correspon … w between
tricritical Ising … Ising model in
the IR

(equivalently, N=1 Wess-Zumino model in the UV and free fermion in the IR)

Unitarity+Analyticity+Crossing:

$$e^{2i\delta(s)} = \prod_j \frac{\mu_j + s}{\mu_j - s} e^{iP(s)}$$

$$\text{Im } s > 0$$



Expectation from Locality: $P(s) = 0 + \ell^2 s$

Let us look at (D-2) bosons with

$$e^{2i\delta(s)} = e^{is\ell^2/4}$$

✶Polynomially bounded on the physical sheet

✶No poles anywhere. A cut all the way to infinity with an infinite number of broad resonances

✶*Scale survives all the way to the UV !*

One can reconstruct the entire finite volume spectrum using Thermodynamic Bethe Ansatz

$$E(N,\tilde{N}) = \sqrt{\frac{4\pi^2(N-\tilde{N})^2}{R^2} + \frac{R^2}{\ell^4} + \frac{4\pi}{\ell^2}\left(N+\tilde{N}-\frac{D-2}{12}\right)}$$

This is a light-cone quantized bosonic string *an extension of the Euler formula:*

$$e^{is} = \int \mathcal{D}X e^{-S_{NG}(X)}$$

A new type of RG flow behavior:

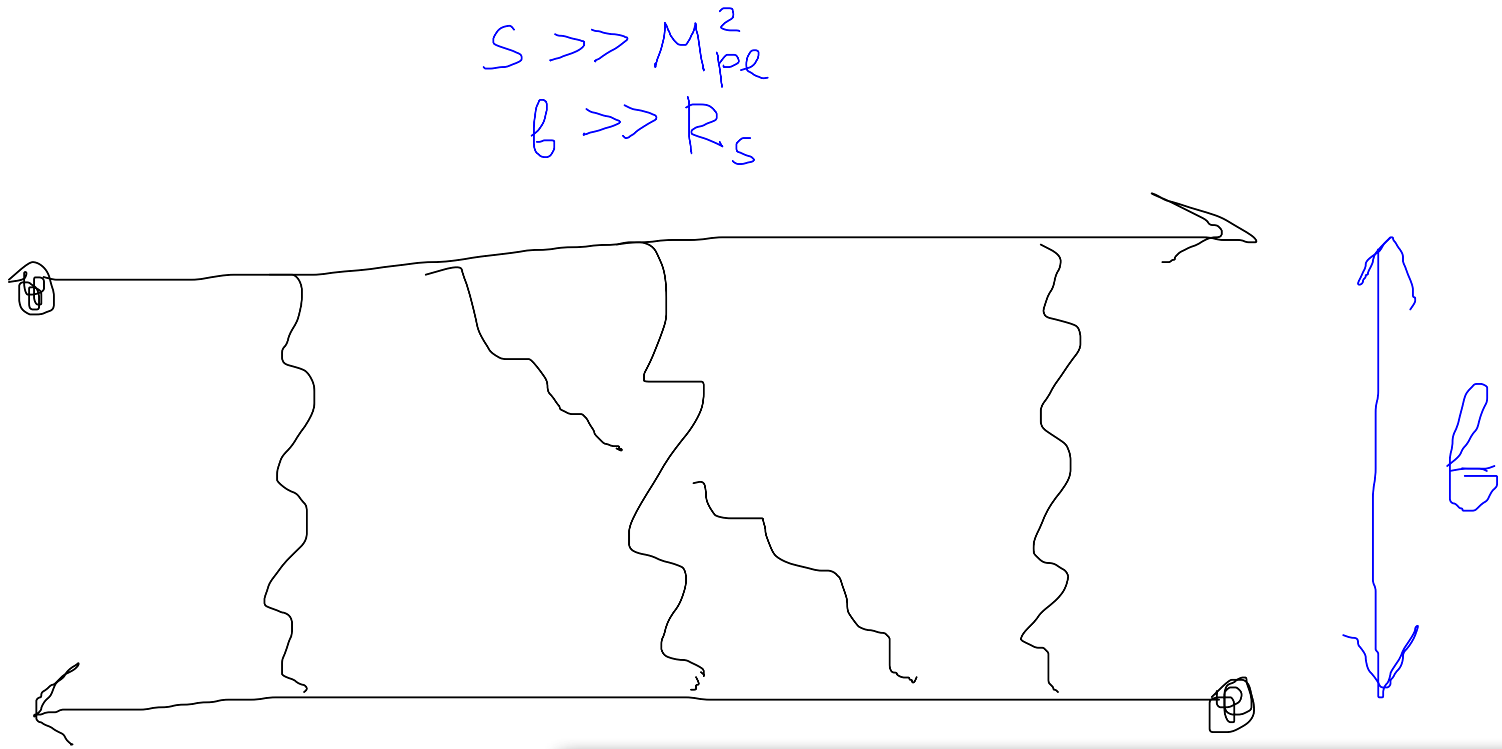
Asymptotic Fragility

Integrable theory of gravity

Integrable QG rather than QFT

Gravitational shock waves:

Dray, 't Hooft '85
Amati, Ciafaloni, Veneziano '88



Eikonal phase shift:

$$e^{i2\delta_{eik}(s)} = e^{i\ell^2 s/4}$$

$$\ell^2 \propto G_N b^{4-d}$$

Some properties of the theory

classical action:

$$S_{NG} = -\ell^2 \int d^2\sigma \sqrt{-\det\left(\eta_{\alpha\beta} + \partial_\alpha X^i \partial_\beta X^i\right)}$$

✶Theory of gravitational shock waves.

✶No UV fixed point and central charge.

✶Maximal achievable (Hagedorn) temperature.

✶Integrable cousins of black holes.

✶Minimal length.

✶No local off-shell observables.

✶Big Bang solutions.

Integrable Black Hole Precursors

Time Delay

$$\Delta t_{cms} = \frac{1}{2}\ell_s^2 E_{cms}$$

c.f. $\Delta t_H = \ell_{Pl}^4 E_{cms}^3$ for Hawking evaporation in 4d

Equivalence Principle at work

Δt is the same for a single hard particle and for a bunch of soft ones

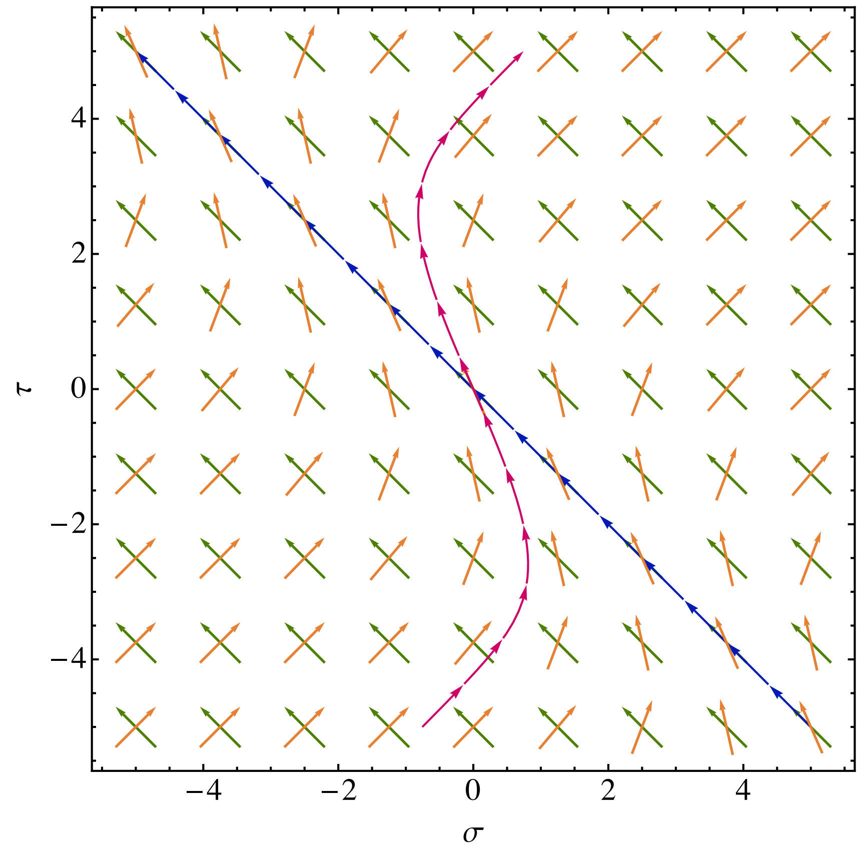
String uncertainty principle

$$\Delta x_L \Delta x_R \geq \ell_s^2$$

for identical packets $\Delta x_{out}^2 = \Delta x_{in}^2 + \frac{\ell_s^4}{\Delta x_{in}^2}$

Classical Origin of the Time Delay

$X^i_{cl}(\tau+\sigma)$ is a solution



$$\Delta t = \int_{-\infty}^{\infty} dz X'^2_{cl} = \ell^2 E$$

exactly reproduces the quantum answer

Hagedorn Temperature

UV central charge from

$$T^{-1} f(T) = E_0(T^{-1}) \simeq \frac{\pi c_{UV}}{6} T \quad \text{at} \quad T \to \infty$$

we have

$$f(T) = \frac{1}{\ell_s^2}\left(\sqrt{1 - T^2/T_H^2} - 1\right) \qquad T_H = \frac{1}{\ell_s}\sqrt{\frac{3}{\pi(D-2)}}$$

$$c_v \simeq (T_H - T)^{-3/2}$$

Chaplygin gas $\quad \rho = \dfrac{p}{1 - \ell_s^2 p} \qquad c_s = \left(\dfrac{\partial \rho}{\partial p}\right)^{-1/2} = 1 - \ell_s^2 p$

Absence of local observables

perturbatively: theory is non-renormalizable

$$\left[\partial_\alpha X^i\right]_{ren} = \partial_\alpha X^i - \frac{\ell_s^2}{8\pi\epsilon}\partial_\alpha\left(\partial_\beta\partial_\gamma X^i \partial^\beta X^j \partial^\gamma X^j\right)$$

non-pe ... orm-
factors

Theory does not want to answer this question

"Asymptotic Fragility"

Riemann-Hilbert problem: $f(\beta) = e^{2i\delta_{NG}(4\ell_s^{-2}e^\beta)} f(\beta + 2\pi i)$

$$f(\beta) = \exp\left(-\beta e^\beta / 2\pi\right)$$

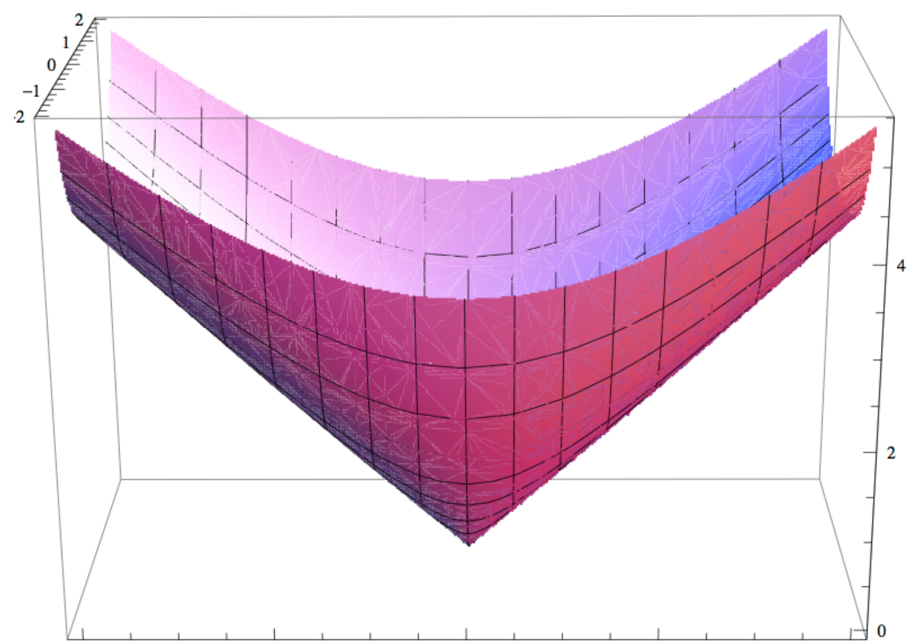
Big Bang Cosmology

$$X_{cl}(\sigma^2 - \tau^2)$$

$$\tau^2 - \sigma^2 - (\sinh X_{cl})^2 = 0$$

induced metric is of the FRW form

$$ds^2 = -dt^2 + (t^2 + 2t)d\lambda^2$$



This was an integrable QG coupled to (D-2) massless bosons.
Is there a generalization to other (non-integrable) theories?

Start with an arbitrary UV complete QFT $\mathcal{L}(\psi, H)$

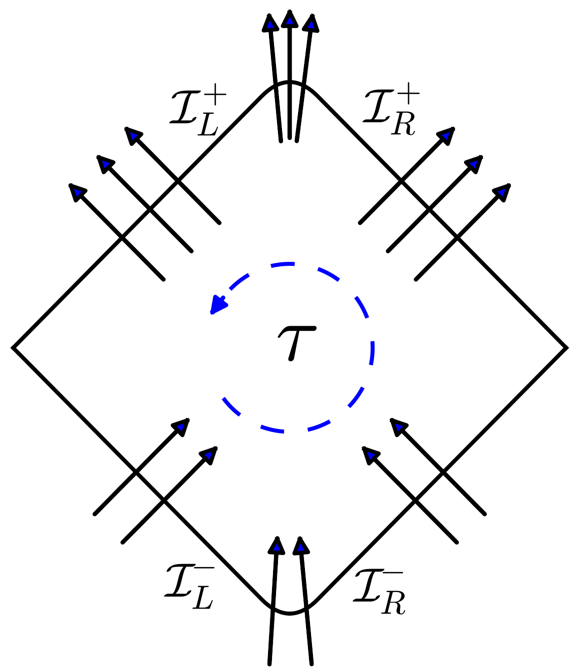
↓

Calculate S-matrix $S_n(p_i)$

↓

Gravitational dressing gives $\hat{S}_n(p_i, \ell)$

Gravitational Dressing



$$\hat{S}_n(p_i) = e^{i\ell^2/4\sum_{i<j} p_i * p_j} S_n(p_i)$$

Properties of gravitational dressing

- ✴Results in a well-to-do S-matrix
- ✴Physical spectrum remains the same
- ✴Low energy EFT description:

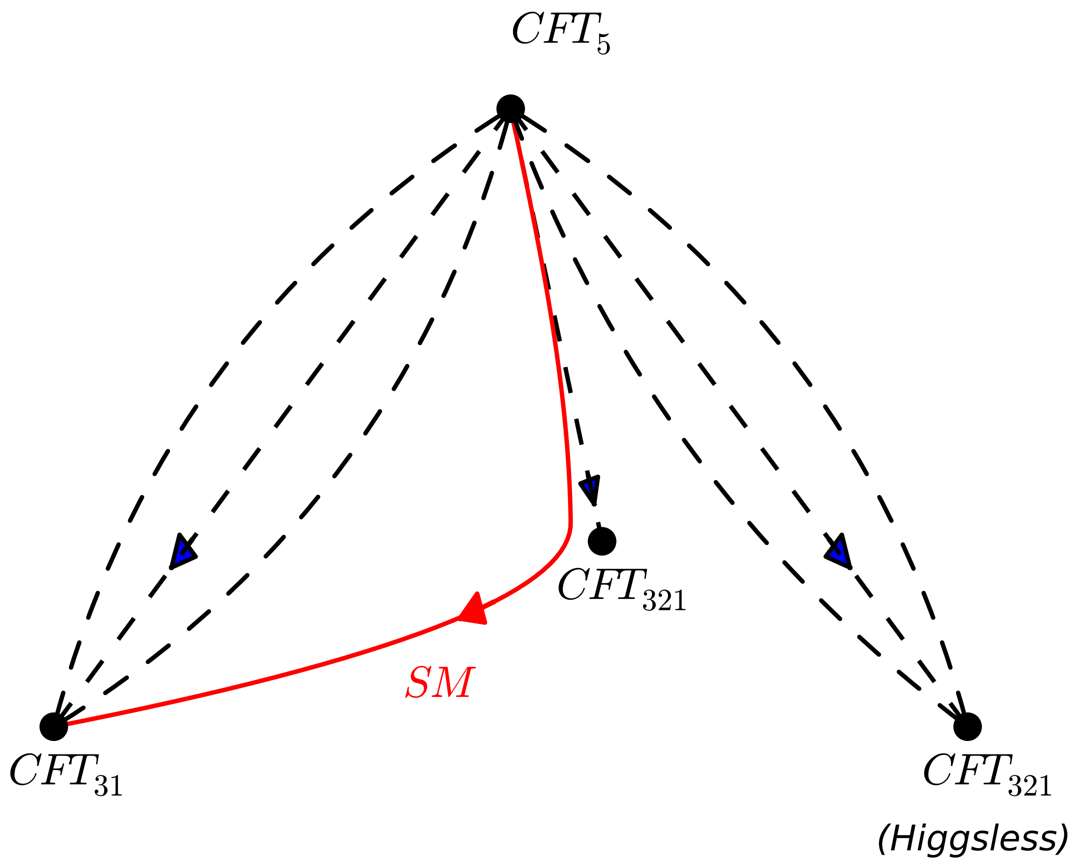
$$\mathcal{L}(\psi, H) + \sum_{\Delta_i > 2} \ell^{\Delta_i - 2} \mathcal{O}_i$$

free massive scalar:

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 + \frac{\ell^2}{8}\left((\partial\phi)^4 - m^4\phi^4\right) + \ldots$$

✴THIS CONSTRUCTION SHOULD NOT

BE POSSIBLE ?!



No picture like that in this example. Energy scale does not correspond to a threshold. No scale invariance and no Wilsonian RG above the scale.

An alternative definition of naturalness?

Every natural QFT is an answer to some *interesting* question.
Perhaps we should learn to ask more questions.

c.f. the following naturalness problem:

314159265358979323846264338327950288419716939937...

is this sequence of digits “natural”?

Instead of Conclusions:

Identifying the simplest QM system—harmonic oscillator—was immensely helpful for developing intuition about (weakly coupled) QFTs

It is very satisfactory that a building block of string theory presents itself a very simple (but non-trivial) gravitational system

Thank you!