

# Radiatively-driven Natural Supersymmetry

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## References:

Radiative natural SUSY with a 125 GeV Higgs boson (with V. Barger, P. Huang, A. Mustafayev and X. Tata), Phys. Rev. Letters 109 161802 (2012).

Post-LHC7 fine-tuning in the mSUGRA/CMSSM model with a 125 GeV Higgs boson (with Barger, Huang, Mickelson, Mustafayev and Tata), Phys. Rev. D87 (2013) 035017.

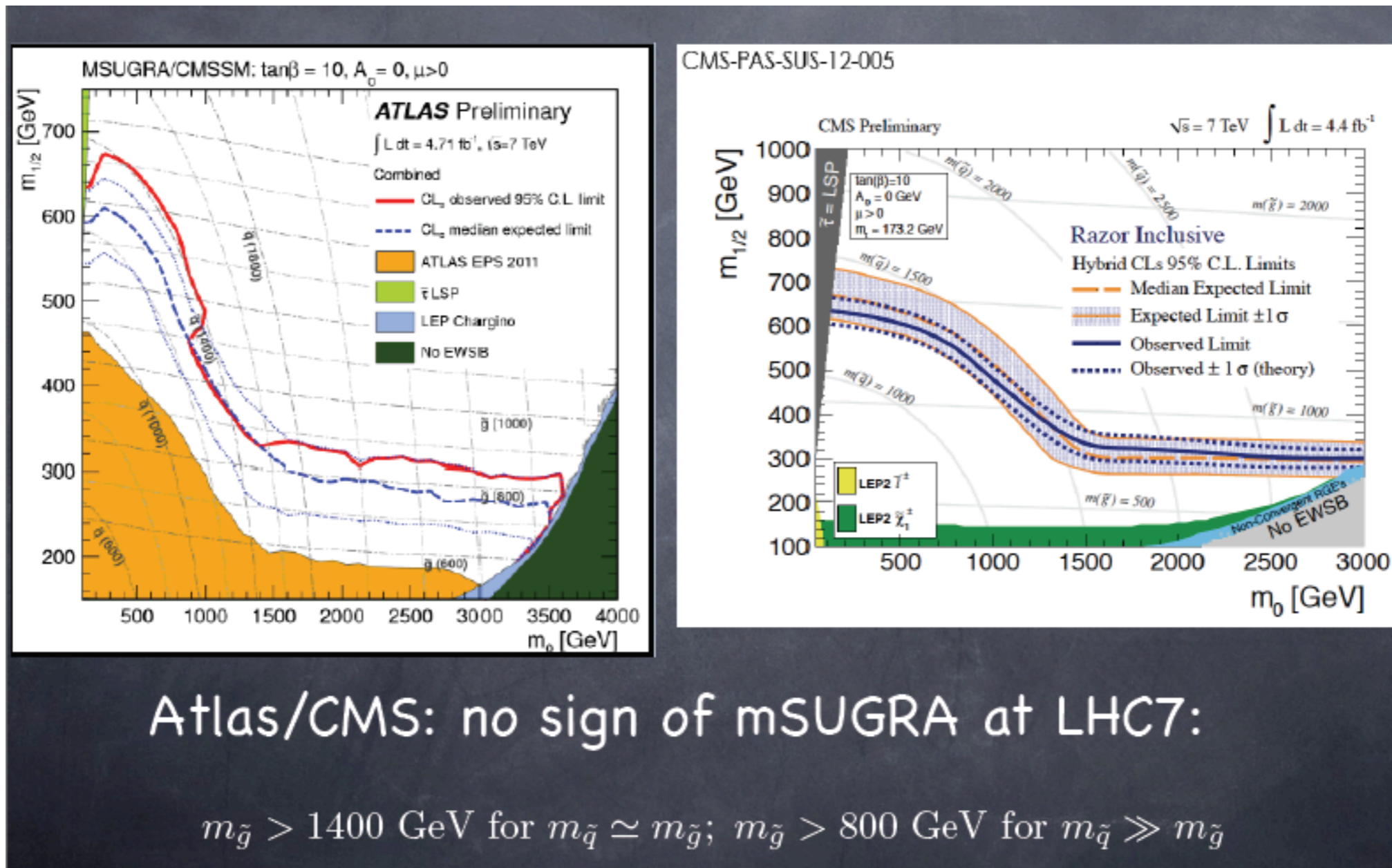
Radiative natural supersymmetry: reconciling electroweak finetuning with the Higgs mass (with Barger, Huang, Mickelson, Mustafayev and Tata), arXiv:1212.2655 (2012).

Same sign diboson signature from supersymmetry models with light higgsinos at the LHC (with V. Barger, P. Huang, D. Mickelson, A. Mustafayev, W. Sreethawong and X. Tata), Phys. Rev. Lett. 110 (2013) 151801.

Dark Radiation Constraints on Mixed Axion/Neutralino Dark Matter (with K. J. Bae and A. Lessa) arXiv:1301.7428 (2013).

Direct and indirect detection of higgsino-like WIMPs: concluding the story of electroweak naturalness (with V. Barger and D. Mickelson) arXiv:1303.3816

# SUSY status post-LHC7



Oft-repeated **story of SUSY electroweak naturalness:**  
 sparticles should be  $< \sim \text{TeV}$ :  
 Exacerbates “Little Hierarchy Problem”:  
 disparity between weak scale and sparticle mass scale

# Natural SUSY

## Incarnation#1: Kitano-Nomura 2005

$$m_h^2 = |\mu|^2 + m_{H_u}^2|_{\text{tree}} + m_{H_u}^2|_{\text{rad}},$$

$$m_{H_u}^2|_{\text{rad}} \simeq -\frac{3y_t^2}{8\pi^2} (m_{Q_3}^2 + m_{U_3}^2 + |A_t|^2) \ln\left(\frac{M_{\text{mess}}}{m_{\tilde{t}}}\right)$$

$$\Delta \equiv \frac{2\delta m_H^2}{m_h^2}$$

$$m_{\tilde{t}}^2 \lesssim \frac{2\pi^2}{3y_t^2} \frac{M_{\text{Higgs}}^2}{\left(1 + \frac{x^2}{2}\right) \Delta^{-1} \ln \frac{M_{\text{mess}}}{m_{\tilde{t}}}} \approx (700 \text{ GeV})^2 \frac{1}{1 + \frac{x^2}{2}} \left(\frac{20\%}{\Delta^{-1}}\right) \left(\frac{3}{\ln \frac{M_{\text{mess}}}{m_{\tilde{t}}}}\right) \left(\frac{M_{\text{Higgs}}}{200 \text{ GeV}}\right)^2$$

\* low  $\mu$

\* light 3rd generation

\* light sub-TeV spectra in pre-LHC era model

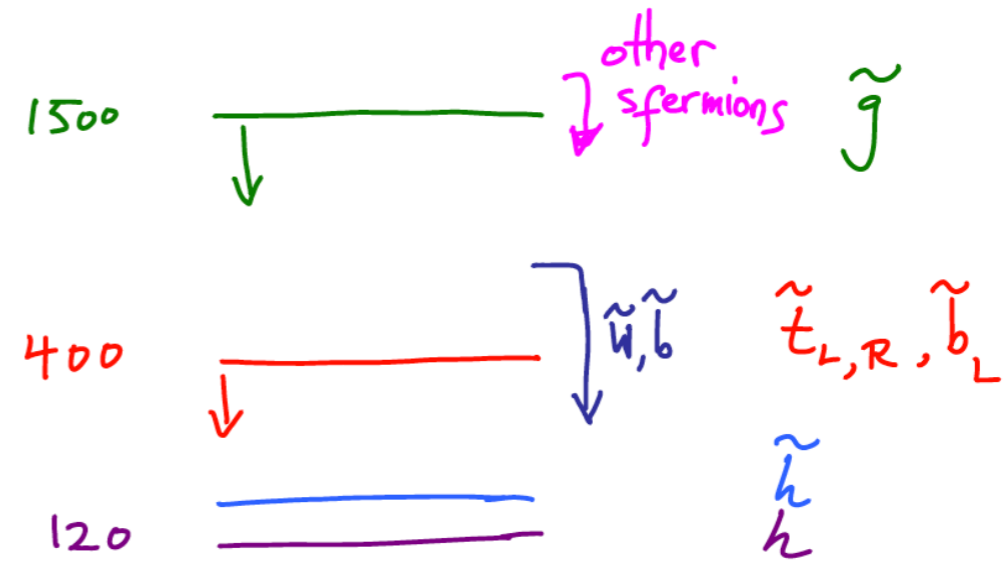
\*  $M_{\text{mess}}$  not too far from TeV; minimize large logs

\* sample spectra now highly excluded from LHC/m(h)

# NS#2: post LHC7 but pre LHC8/Higgs

- Arkani-Hamed 2011
- Arganda et al.
- Papucci et al.
- Brust et al.
- Essig et al.
- HB, Barger, Huang, Tata
- Wymant

*Most exciting, alive + natural SUSY spectrum*

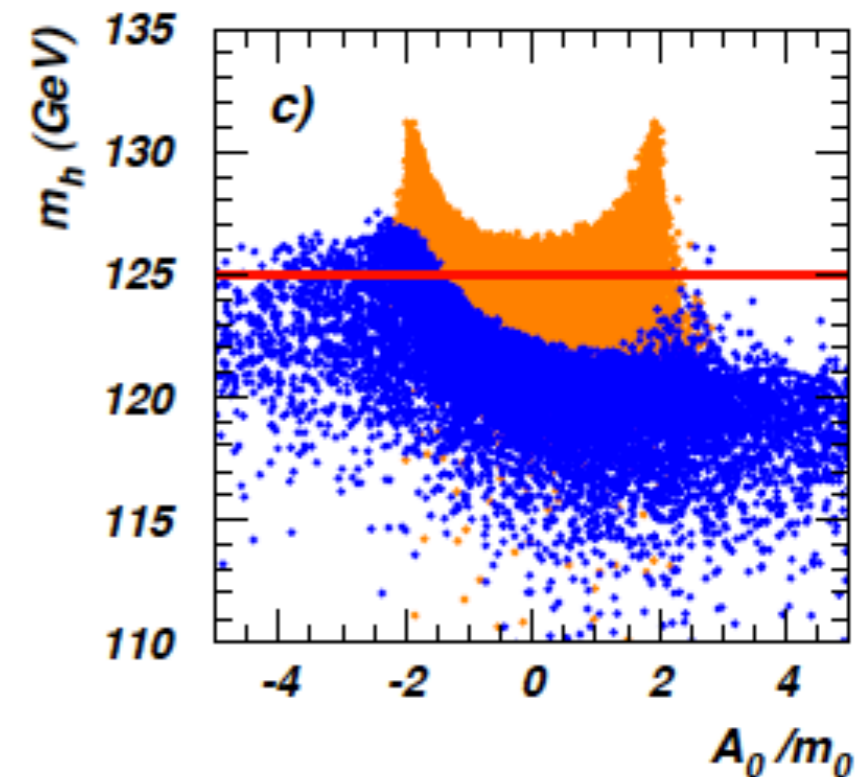


- \*  $\mu \sim 100-250$  GeV
- \*  $m(t_1, t_2, b_1) < \sim 500$  GeV
- \*  $m(\text{gluino}) < 1.5$  TeV
- $m(\text{sq, slep}) \sim 10-20$  TeV

# What else have we learned from LHC8

- Higgs-like resonance at  $\sim 125$  GeV!
- $m(h)$  falls squarely within MSSM window!
- requires:  $m(t1), m(t2) \sim \text{TeV}$  regime
- large mixing
- or else, extra beyond MSSM mass contributions e.g. NMSSM, exotic matter,...

e.g. Hall, Pinner, Ruderman, JHEP1204(2012)131



blue:  $m_0 < 5$  TeV  
orange:  $m_0 < 20$  TeV

HB, Barger, Mustafayev,  
PRD85(2012)075010

## From LHC8:

- No sign of SUSY: in models such as mSUGRA
- $m_{\tilde{q}} \sim m_{\tilde{g}} > 1.4 \text{ TeV}$  or  $m_{\tilde{g}} > \sim 1 \text{ TeV}$  if  $m_{\tilde{g}} \ll m_{\tilde{q}}$
- Squark mass bound and even more  $m(h)$  (which needs  $m(t1, t2) > \text{TeV}$ ) seemingly create even greater tension with naturalness bounds:
- Little Hierarchy Problem more severe?
- These results have prompted many groups to reconsider what weak scale SUSY would look like: is it now unlikely or even excluded?

see e.g. M. Shifman review, arXiv:1211.0004

# Some reactions from community

- Ignore naturalness: e.g. K-L-O or Kane et al. G2MSSM stringy model with moduli stabilization: scalars  $\sim 100$  TeV with AMSB-like gauginos and wino=LSP or live far out in mSUGRA plane (note: Kane et al. claim lower  $\mu \sim .5-1$  TeV so maybe not so bad, but still heavy stops); see also Hall, Nomura et al, Spread SUSY
- natural SUSY ala Kitano-Nomura successor models (Arkani-Hamed, Brust et al., Papucci et al.): these models, couched in MSSM, tend to have  $m(h) < 125$  GeV and large deviations to  $b \rightarrow s \gamma$
- compressed spectra: low energy release from cascade decays to maintain sub-TeV SUSY masses but hide SUSY from LHC
- RPV: similar approach: LSP decays hadronically
- retain naturalness (light stops) but give extra contributions to  $m(h)$ : NMSSM, lambda-SUSY, vector-like or other exotic matter: model builders delight
- accept some finetuning but try to minimize: HB/FP region of mSUGRA, effective SUSY
- re-examine naturalness

# Traditional measure of EW finetuning:

Barbieri-Giudice (even earlier Ellis et al.) introduced the measure:

$$\Delta_{BG} \equiv \max_i \left| \frac{\Delta m_Z^2 / m_Z^2}{\Delta a_i / a_i} \right| = \max_i \left| \frac{\partial \ln m_Z^2}{\partial \ln a_i} \right|$$

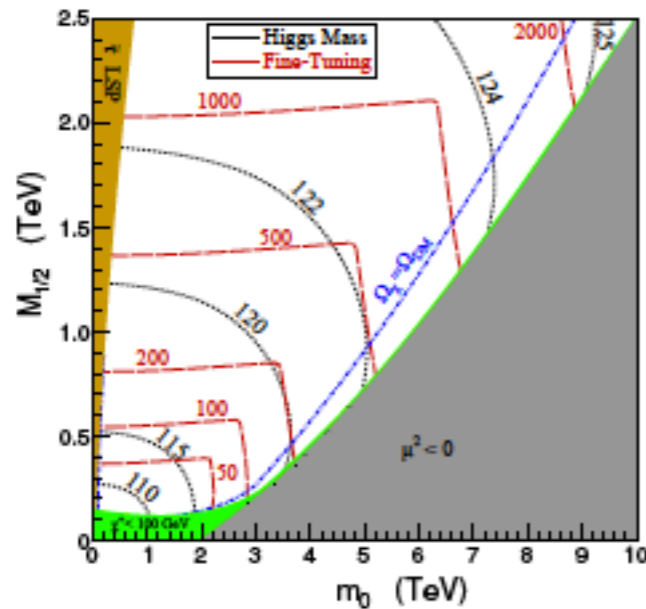
This measures fractional variation in  $m(Z)^2$  due to fractional variation in parameters  $a_i$

This measure was used by BG and DG to show that better than 10% EWFT requires  $m(\text{chargino}) < \sim 100$  GeV;  
SUSY already finetuned post-LEP2?



# Some sample results using $\Delta_{BG}$

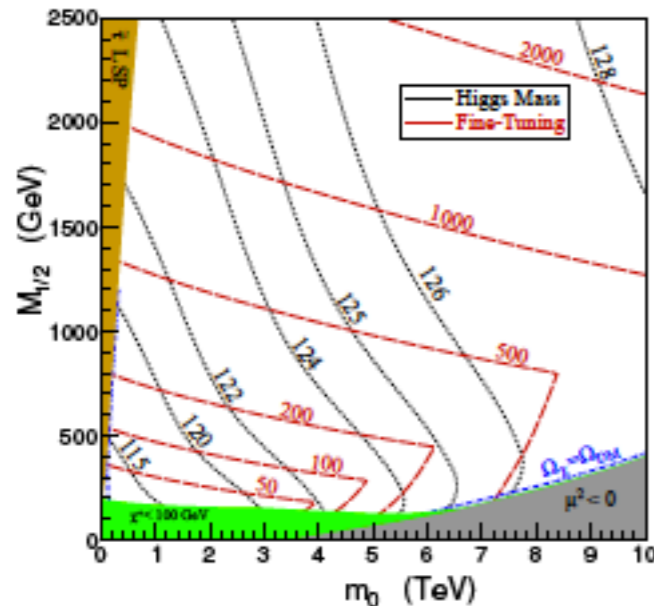
For recent review, see J. L. Feng, arXiv:1302.6587



Feng & Sanford, PRD86 (2012) 055015

$A_0=0$  nearly excluded by  $m(h)\sim 125$  GeV results unless  $\Delta > 2000$

$$a_i \ni \{m_0, m_{1/2}, A_0, B_0, \mu_0\}$$

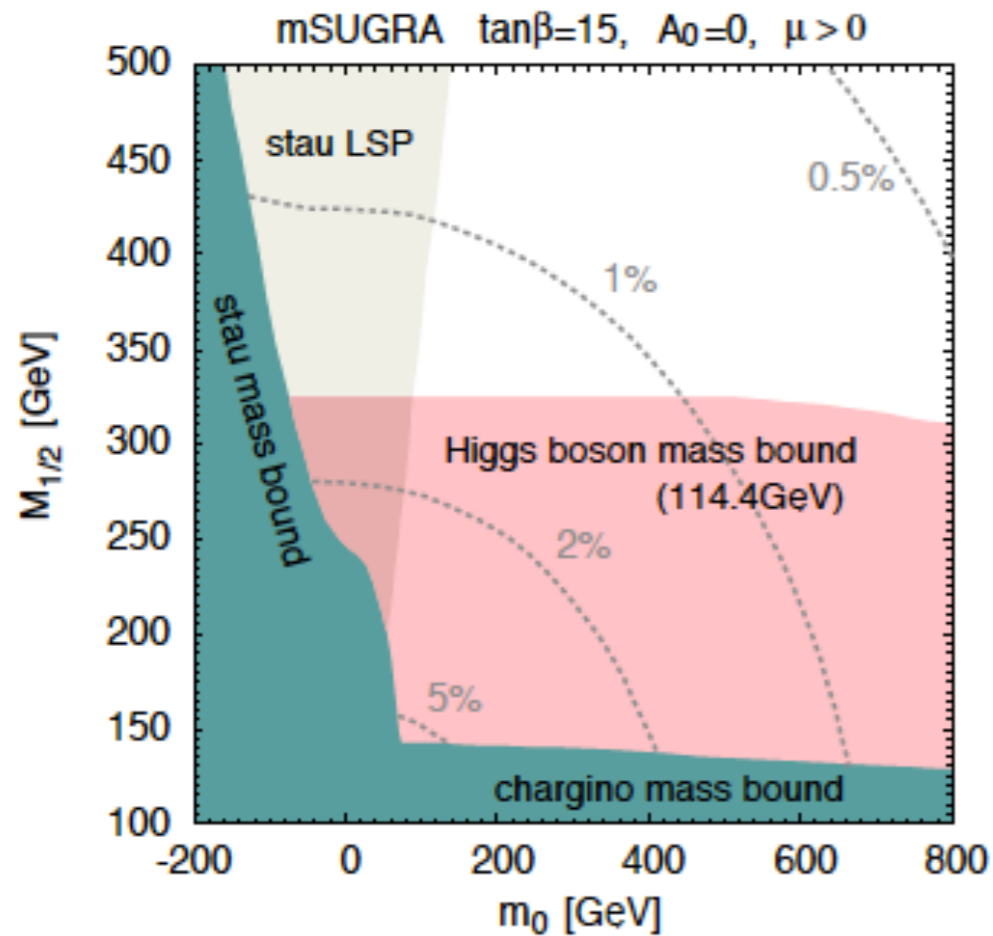


Allow non-universality, but with  $m_{Hu}$  still fixed relative to  $m_0$ ;  
can allow  $A_0 \neq 0$  to raise  $m(h)$ ;  
still,  $\Delta > 200-500$

Hidden top Yukawa dependence since

$$\mathcal{L} \ni a_t = A_0 f_t$$

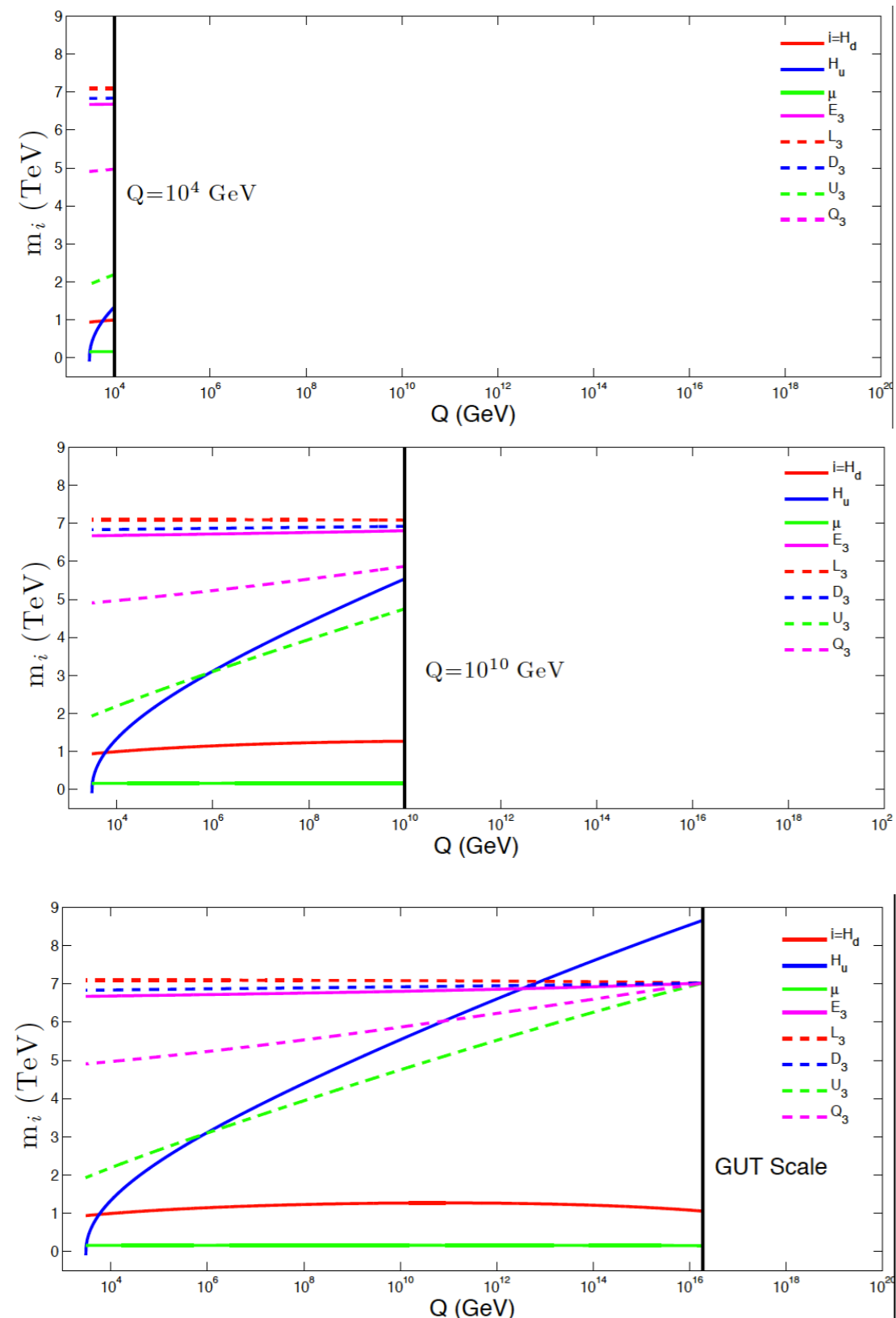
measure depends on highly on which high-scale parameter set one adopts



This plot from Kitano-Nomura PRD73 (2006) 095004 uses  $m(t)$  along with SUSY terms

The behavior is quite different:  
low  $D_{BG}$  favors low  $m_0$ , low  $m_{hf}$

$\Delta_{BG}$  also depends on where the high scale is



These three models have exactly the same weak scale spectra, but very different values of

$$\Delta_{BG}$$

Re-phrase Little Hierarchy problem:

Question: how can it be that

$$m(Z)=91.2 \text{ GeV}$$

while gluino and squark masses  
sit at TeV or even far beyond  
values?

Simple answer:

the parameters that enter the scalar potential and contribute to  $m(Z)$  are all not too far from  $m(Z)$

No large uncorrelated contributions to  $m(Z)$ !

By answering this question, we shall see  
that naturally accommodating  
both  $m(Z)=91.2$  GeV and  $m(h)=125$  GeV  
is enormously constraining:  
SUSY parameter space is not egalitarian  
but instead these criteria are **highly  
selective!**

Furthermore, we will find the results are  
model independent, and deeply rooted in  
data (why is  $m(Z)=91.2$  GeV?)  
and they are highly predictive!

In the MSSM, value of  $m(Z)$  is determined by combinations of parameters which enter into the scalar potential; minimization leads to a relation between  $m(Z)$  and weak scale SUSY parameters:

$$\frac{m_Z^2}{2} = \frac{(m_{H_d}^2 + \Sigma_d^d) - (m_{H_u}^2 + \Sigma_u^u) \tan^2 \beta}{\tan^2 \beta - 1} - \mu^2 \simeq -(m_{H_u}^2 + \Sigma_u^u) - \mu^2$$

The radiative corrections  $\Sigma_u^u, \Sigma_d^d$  contain additional terms

$$\Delta_{EW} \equiv \max(C_i) / (M_Z^2 / 2)$$

$$C_{H_u} \equiv | - m_{H_u}^2 \tan^2 \beta / (\tan^2 \beta - 1) |, C_\mu \equiv | - \mu^2 | \text{ and } C_{H_d} \equiv | m_{H_d}^2 / (\tan^2 \beta - 1) |$$

HB, Barger, Huang, Mustafayev, Tata, PRL 109(2012)161802

# New measure of naturalness:

how can  $m(Z)=91.2$  GeV when sparticles  $\gg$  TeV?

$$\frac{m_Z^2}{2} = \frac{(m_{H_d}^2 + \Sigma_d^d) - (m_{H_u}^2 + \Sigma_u^u) \tan^2 \beta}{\tan^2 \beta - 1} - \mu^2 \simeq -(m_{H_u}^2 + \Sigma_u^u) - \mu^2$$

Each contribution to  $m(Z)$  relation ought be of order  $m(Z)$ !  
i.e. no large cancellations amongst independent contributions to  $m(Z)$

$$\Delta_{EW} \equiv \max(C_i) / (M_Z^2 / 2)$$

- Model independent (impose at weak scale!)
- Conservative (necessary but perhaps not sufficient)
- measureable (reconstruct from weak scale Lagrangian)
- unambiguous (depends on spectra not parameters)
- predictive [ $m(\text{higgsino}) \sim m(\text{higgs})$ ]
- falsifiable (no light higgsinos at 1 TeV ILC then SUSY EW naturalness dead)
- simple to compute (Isajet 7.83)

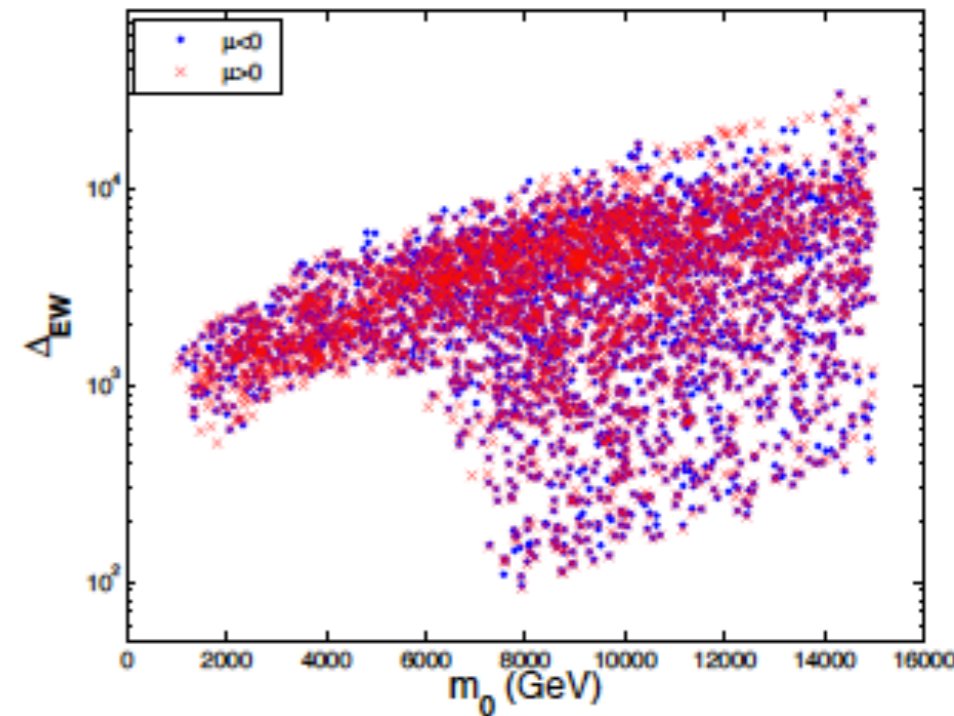
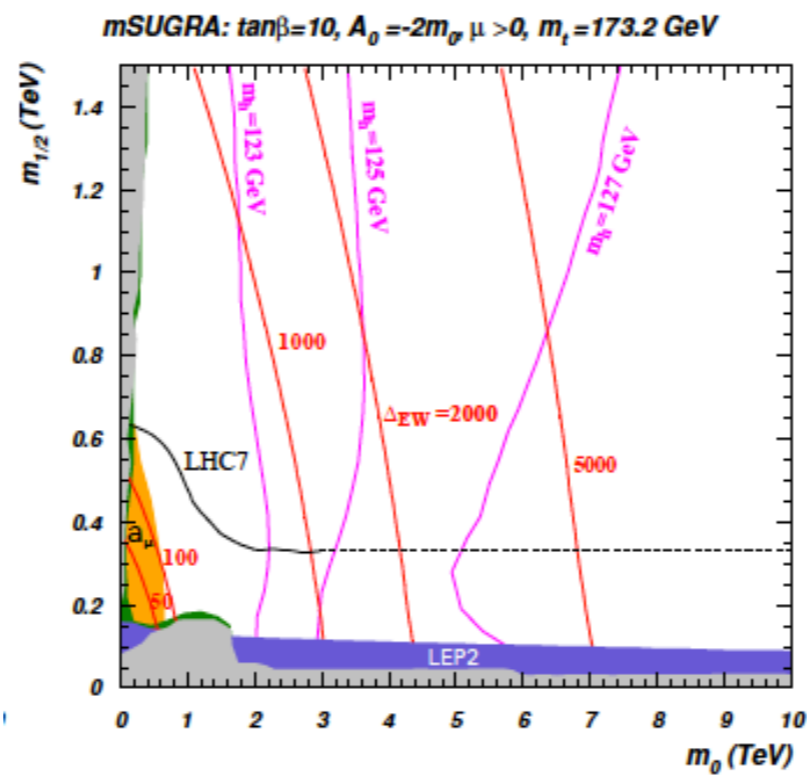
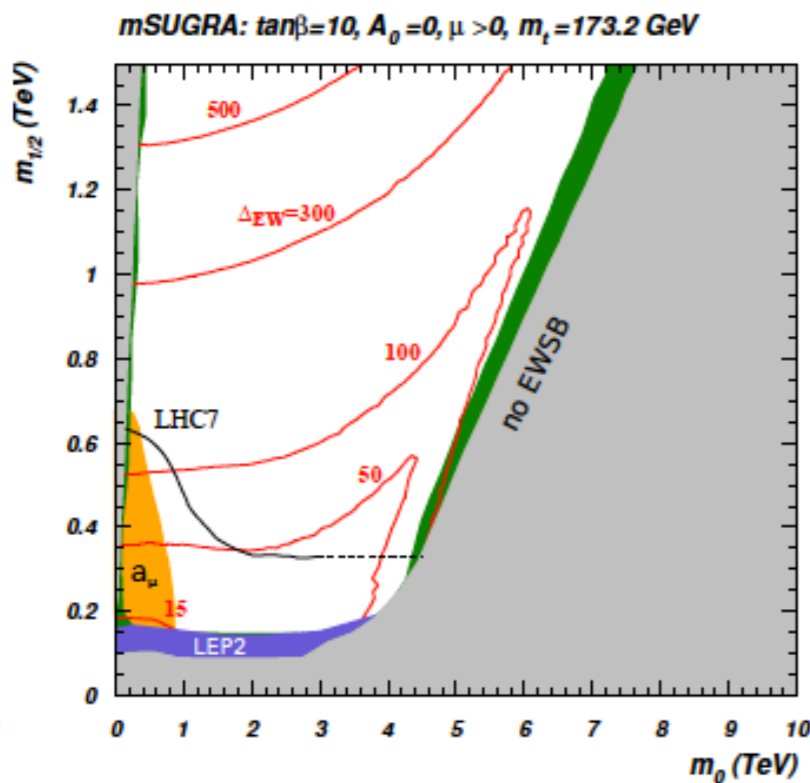


What about high scale parameters?

Maybe only small portion of p-space leads to low  $\Delta_{EW}$ . What if I vary HS parameters and  $\Delta_{EW}$  moves up? Isn't this instability, and hence aren't you really still finetuned?

No. Nature doesn't have any adjustable parameters. We regard the MSSM as an effective theory where the parameters "parametrize" our ignorance of a more fundamental theory where parameters are fixed. The utility of parameters is that: if you find a set which allows for agreement with data, then use those to predict further phenomena. Then devise an experiment to check consistency. If predictions are verified, then model may be a good description of nature.

While  $\Delta_{EW}$  ignores large logs in  $m_{H_u}^2$  running, even making use of these to generate low  $m_{H_u}^2$  at weak scale, it is nonetheless highly constraining: e.g. mSUGRA at best 1% EWFT and usually much worse



Reason: as we increase  $m_0$  into low  $\mu$  region to reduce EWFT,  $m(t_1, t_2)$  are dragged up and increase EWFT: culprit:  $m_{H_u} = m_0$

HB, Barger, Huang, Mickelson, Mustafayev, Tata, arXiv: 1210.3019

Each contribution  $\sim m(Z)$

$$\frac{M_Z^2}{2} = \frac{m_{H_d}^2 + \Sigma_d^d - (m_{H_u}^2 + \Sigma_u^u) \tan^2 \beta}{\tan^2 \beta - 1} - \mu^2$$

Most important:

low  $\Delta_{EW}$  also requires  $\mu^2 \sim M_Z^2/2$ .

In models such as mSUGRA,  $\mu$  is determined by  $m(Z)$  applied as constraint

here,  $\mu$  is its own free parameter: NUHM models

Why should  $\mu$  be so small when  $m(gl, sq)$  are so big?

Plausible: in gravity-mediation  $\mu$  gets its mass differently, e.g. in Giudice-Masiero or Kim-Nilles:

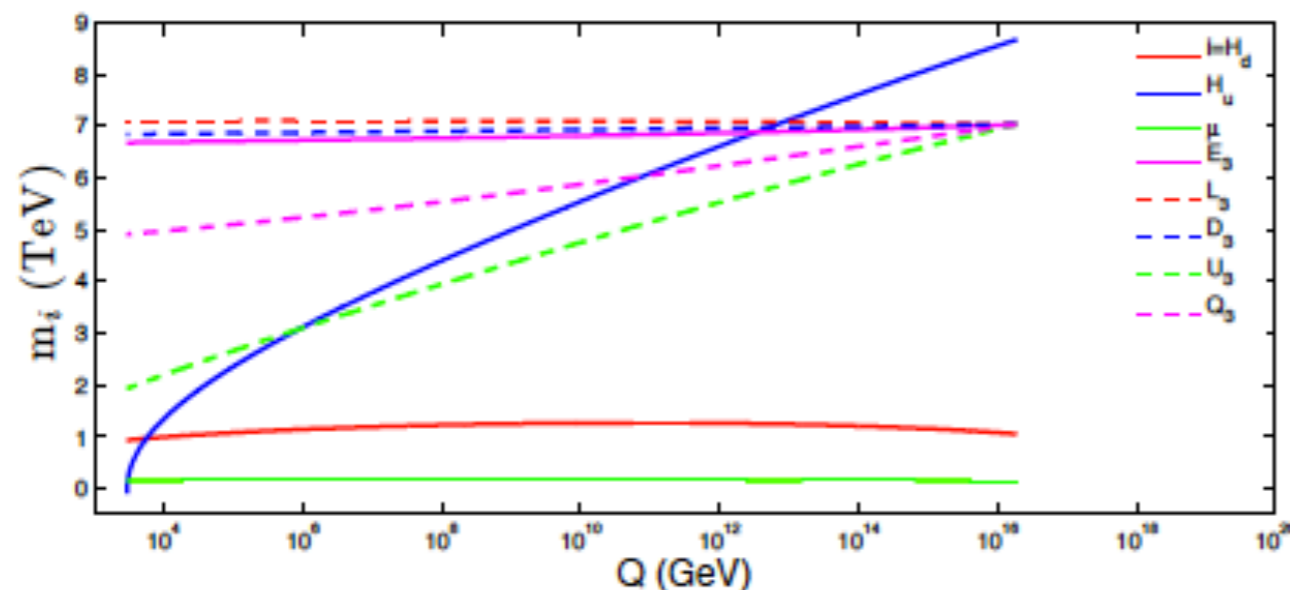
$$\mu \sim \lambda m_{3/2} \quad \text{so that} \quad |\mu| \ll m_{3/2}$$

Next: how can  $-m_{H_u}^2(m_{weak}) \sim m_Z^2/2$ ?

Large top Yukawa radiatively drives  
 $m_{H_u}^2$  to small negative values

$$\frac{dm_{H_u}^2}{dt} = \frac{2}{16\pi^2} \left( -\frac{3}{5}g_1^2 M_1^2 - 3g_2^2 M_2^2 + \frac{3}{10}g_1^2 S + 3f_t^2 X_t \right)$$

$$X_t = m_{Q_3}^2 + m_{t_R}^2 + m_{H_u}^2 + A_t^2$$



Large logs are a feature, not a hindrance; they are large because  $m(t)=173.2$  GeV.

Why is  $m(t)$  so large?  
 I don't know, but I am glad it is.

In mSUGRA, this only happens in HB/FP region where stops also are heavy;  
 in NUHM models, this can occur even if lighter stops

$$m_{H_u}^2(m_{GUT}) \sim (1.3 - 2)m_0^2$$

# Next: radiative corrections

Adopt Coleman-Weinberg eff. pot'l approach:

$$V_{Higgs} = V_{\text{tree}} + \Delta V$$

$$\Delta V = \sum_i \frac{(-1)^{2s_i}}{64\pi^2} (2s_i + 1) c_i m_i^4 \left[ \log \left( \frac{m_i^2}{Q^2} \right) - \frac{3}{2} \right]$$

minimization gives:

$$B\mu v_d = (m_{H_u}^2 + \mu^2 - g_Z^2(v_d^2 - v_u^2)) v_u + \Sigma_u$$

$$B\mu v_u = (m_{H_d}^2 + \mu^2 + g_Z^2(v_d^2 - v_u^2)) v_d + \Sigma_d,$$

$$\Sigma_{u,d} = \left. \frac{\partial \Delta V}{\partial h_{u,d}} \right|_{\text{min}}$$

$$\Sigma_u = \Sigma_u^u v_u + \Sigma_u^d v_d,$$

$$\Sigma_d = \Sigma_d^u v_u + \Sigma_d^d v_d \quad \text{and}$$

$$\Sigma_d^u = \Sigma_u^d$$

$\Sigma_u^d$  terms cancel

$$\Sigma_u^u = \left. \frac{\partial \Delta V}{\partial |h_u|^2} \right|_{\text{min}},$$

$$\Sigma_d^d = \left. \frac{\partial \Delta V}{\partial |h_d|^2} \right|_{\text{min}} \quad \text{and}$$

$$\Sigma_u^d = \left. \frac{\partial \Delta V}{\partial (h_u h_d + \text{c.c.})} \right|_{\text{min}}.$$

$$M_Z^2/2 = \frac{(m_{H_d}^2 + \Sigma_d^d) - (m_{H_u}^2 + \Sigma_u^u) \tan^2 \beta}{\tan^2 \beta - 1} - \mu^2,$$

$$B\mu = \left( (m_{H_u}^2 + \mu^2 + \Sigma_u^u) + (m_{H_d}^2 + \mu^2 + \Sigma_d^d) \right) \sin \beta \cos \beta + \Sigma_u^d.$$

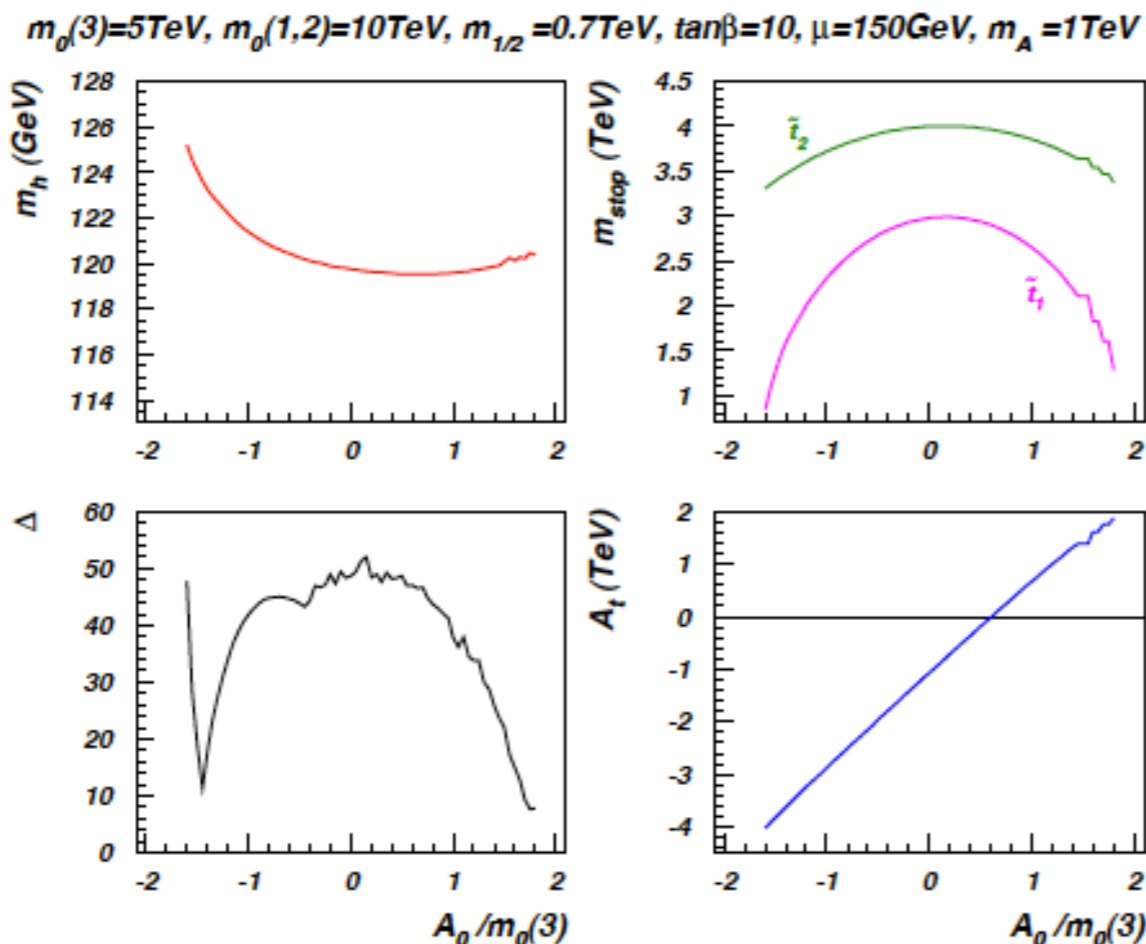
HB, Barger, Huang, Mickelson, Mustafayev, Tata, arXiv:1212.2655

largest contribution usually from stops:

$$\Sigma_u^u(\tilde{t}_{1,2}) = \frac{3}{16\pi^2} F(m_{\tilde{t}_{1,2}}^2) \times \left[ f_t^2 - g_Z^2 \mp \frac{f_t^2 A_t^2 - 8g_Z^2 (\frac{1}{4} - \frac{2}{3}x_W) \Delta_t}{m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2} \right]$$

$$F(m^2) = m^2 (\log(m^2/Q^2) - 1), \text{ with } Q^2 = m_{\tilde{t}_1} m_{\tilde{t}_2}$$

large stop mixing softens both  $t_1$  and  $t_2$   
radiative corrections  
while increasing  $m(h)$  up to 125 GeV!



HB, Barger, Huang, Mustafayev, Tata,  
PRL109(2012)161802

One need not depart too far from mSUGRA/  
CMSSM to find a model which allows low  
Delta\_EW while maintaining  
desirable features of SUSY GUTs:

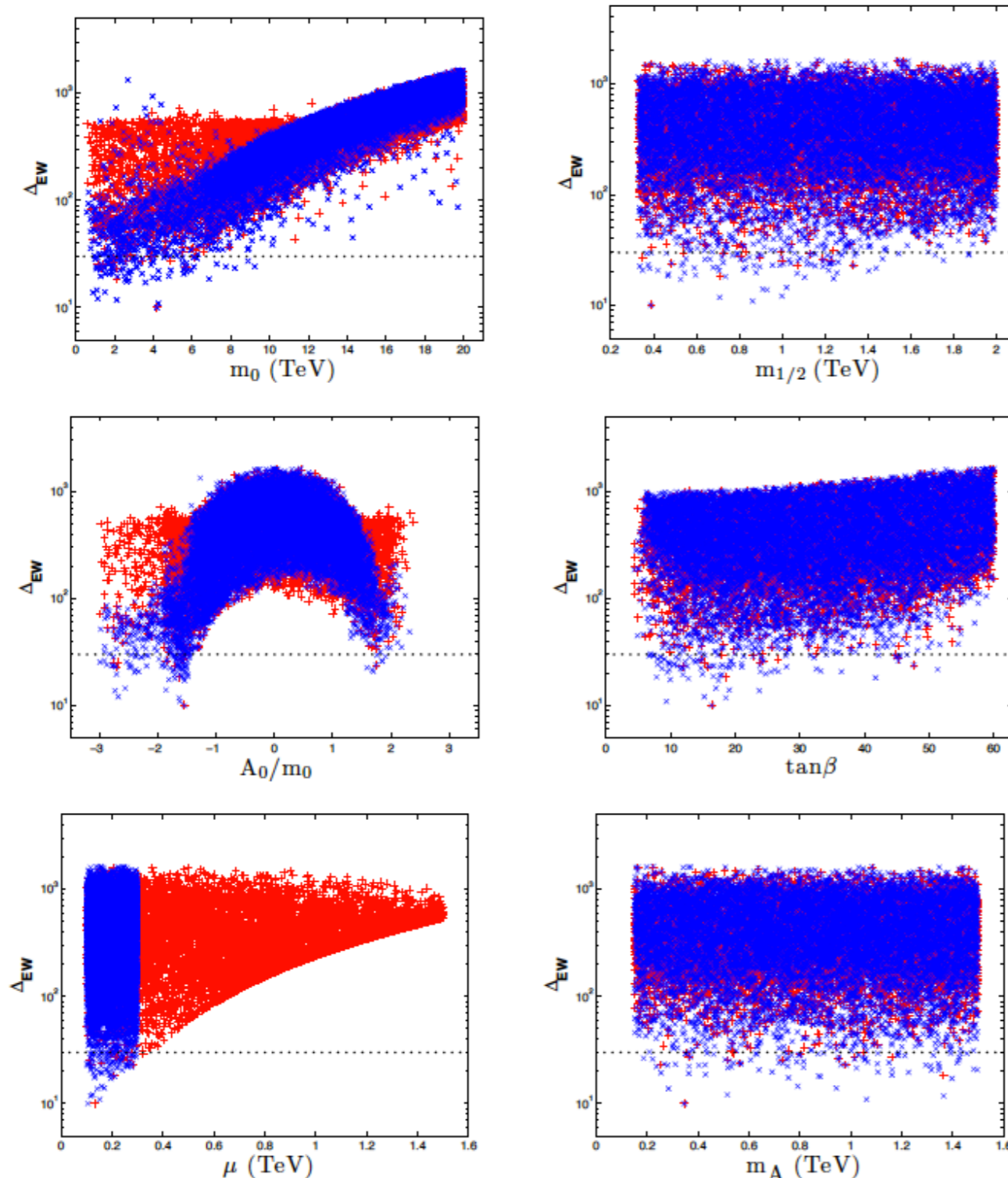
2-extra parameter non-universal Higgs model

$m_0, m_{1/2}, A_0, \tan\beta, \mu$  and  $m_A$ .

Here, we trade  $m_{H_u}^2, m_{H_d}^2 \Rightarrow \mu, m_A$



Which parameter choices lead to low EWFT and how low can  $\Delta_{EW}$  be?



$\Delta_{EW} \sim 10$  or 10% *EWFT*

High-scale models with low  $\Delta_{EW}$ :

Radiatively-driven natural SUSY, or RNS

HB, Barger, Huang, Mickelson, Mustafayev, Tata, arXiv:1212.2655



# Compare RNS to mSUGRA for similar parameters

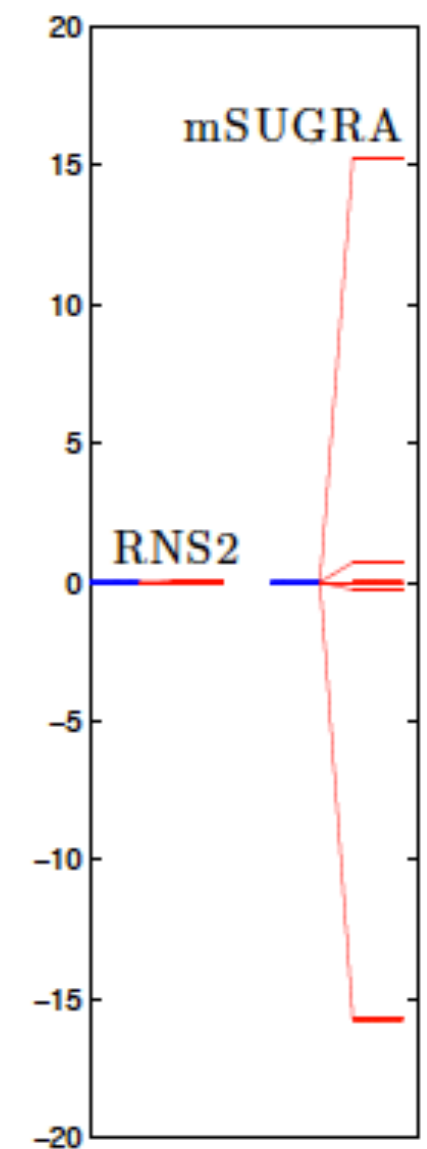
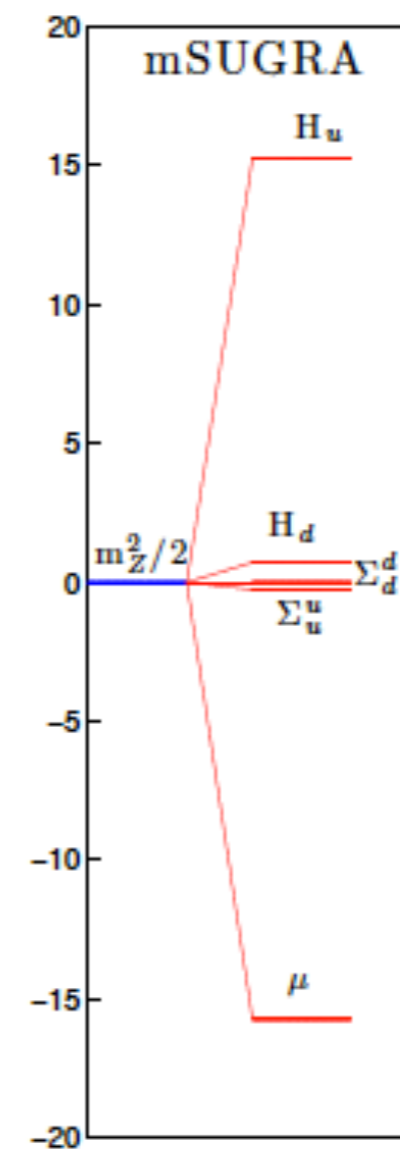
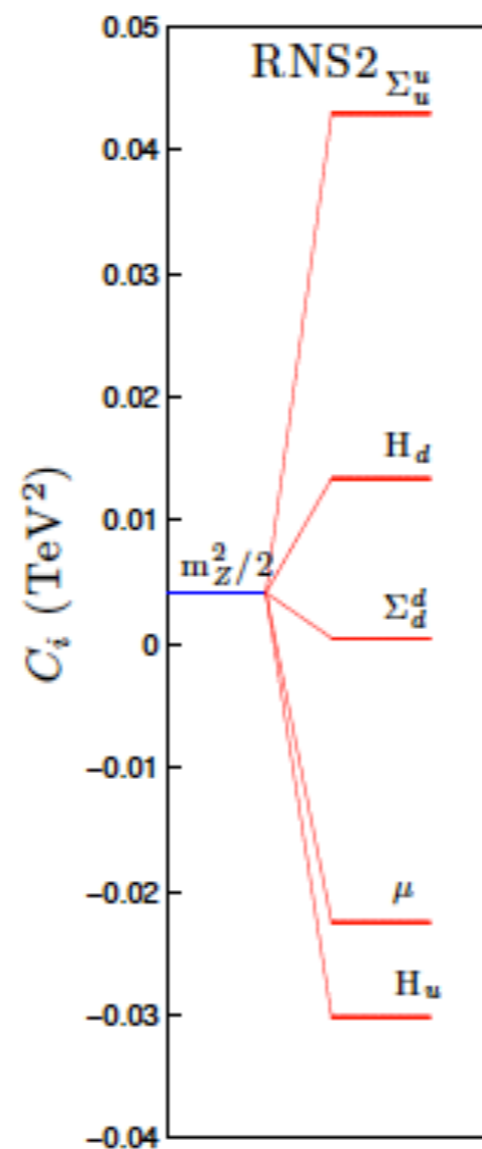
$m_0 = 7025 \text{ GeV}$ ,  $m_{1/2} = 568.3 \text{ GeV}$ ,  $A_0 = -11426.6 \text{ GeV}$ ,  $\tan\beta = 8.55$  with  $\mu = 150 \text{ GeV}$  and  $m_A = 1000 \text{ GeV}$

## RNS

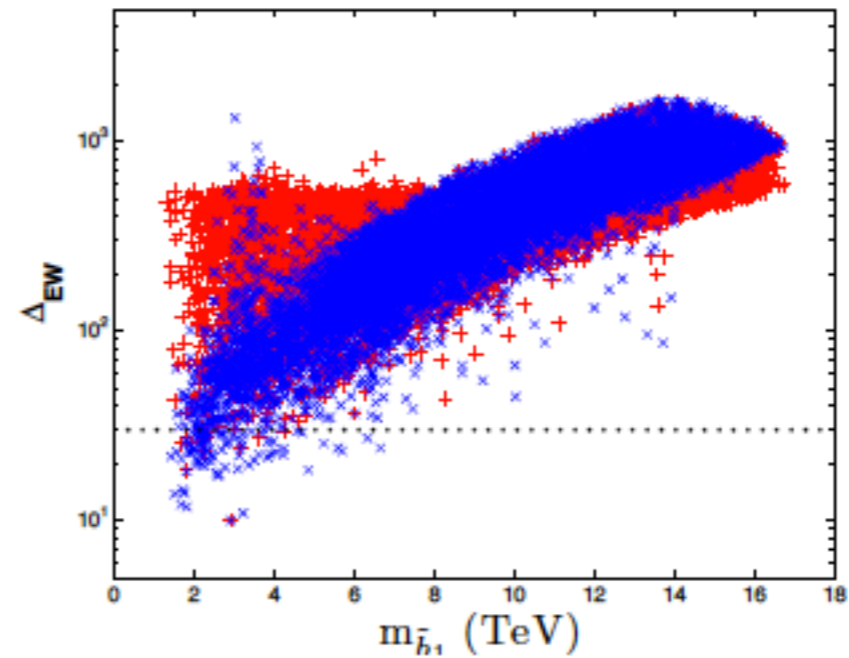
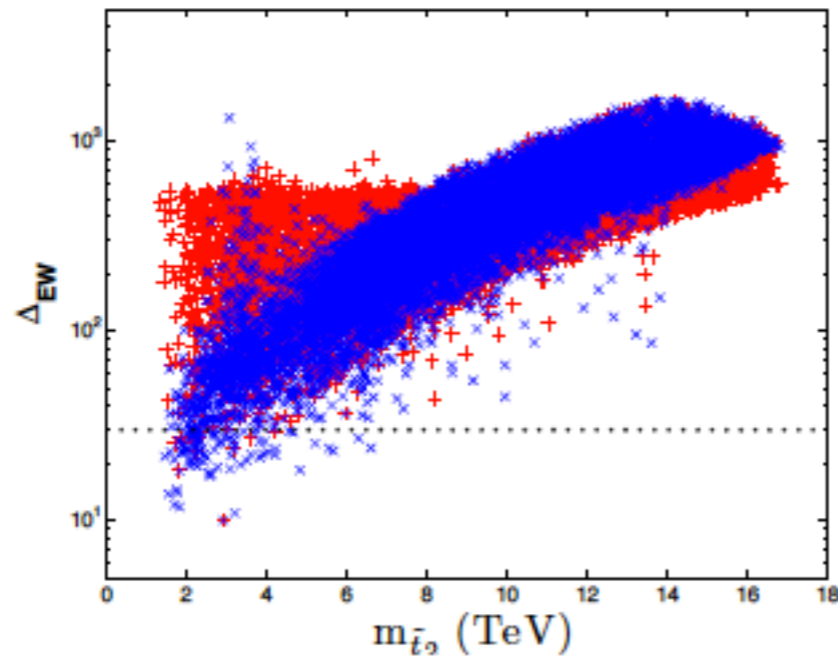
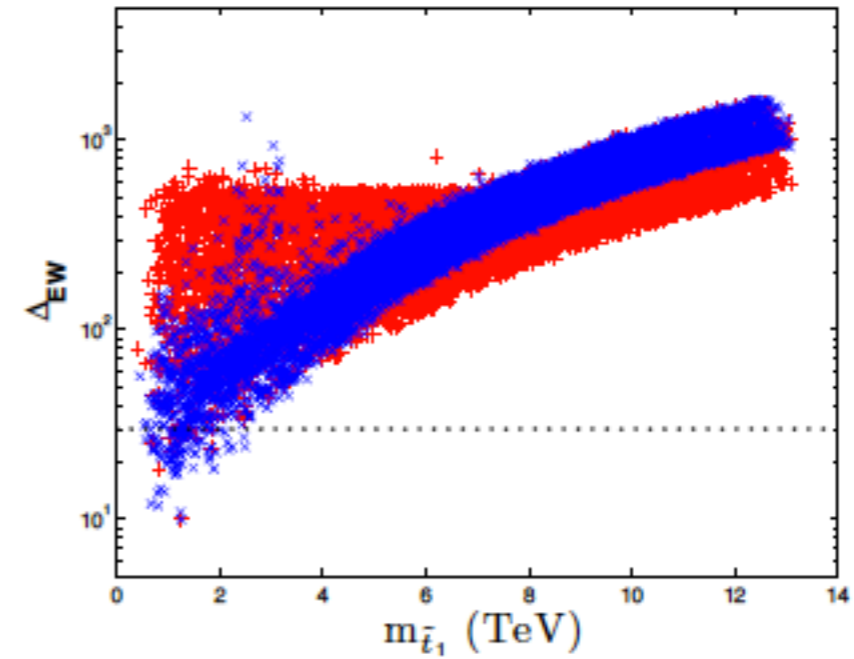
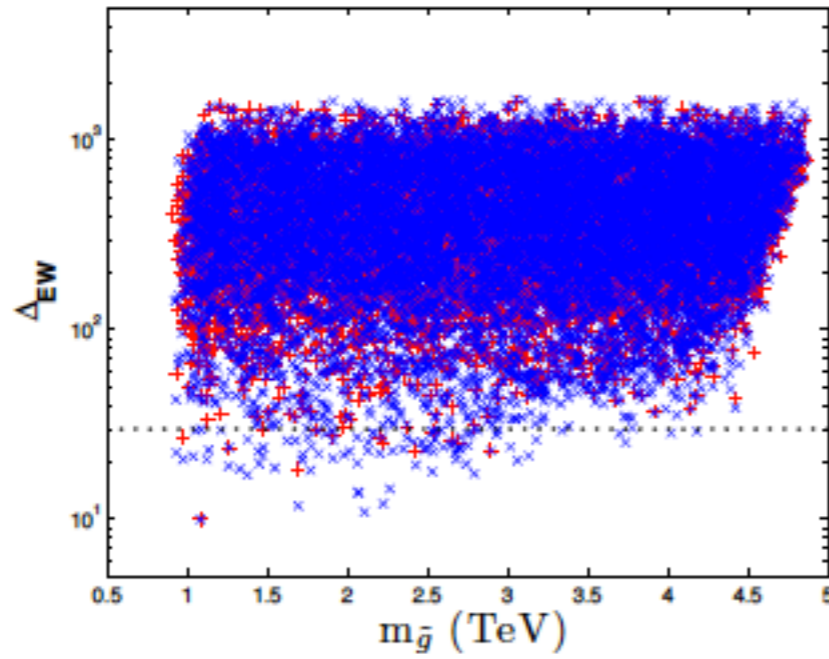
- $C_{\Sigma_u^u} \sim (205 \text{ GeV})^2$
- $C_{H_d} \sim (114 \text{ GeV})^2$
- $C_{\Sigma_d^d} \sim (22 \text{ GeV})^2$
- $C_\mu \sim -(148 \text{ GeV})^2$
- $C_{H_u} \sim -(173 \text{ GeV})^2$
- $m_Z^2/2 \simeq (65 \text{ GeV})^2$

## mSUGRA

- $C_{H_u} \simeq (3.87 \text{ TeV})^2$
- $C_\mu \simeq -(3.93 \text{ TeV})^2$



# Sparticle masses:



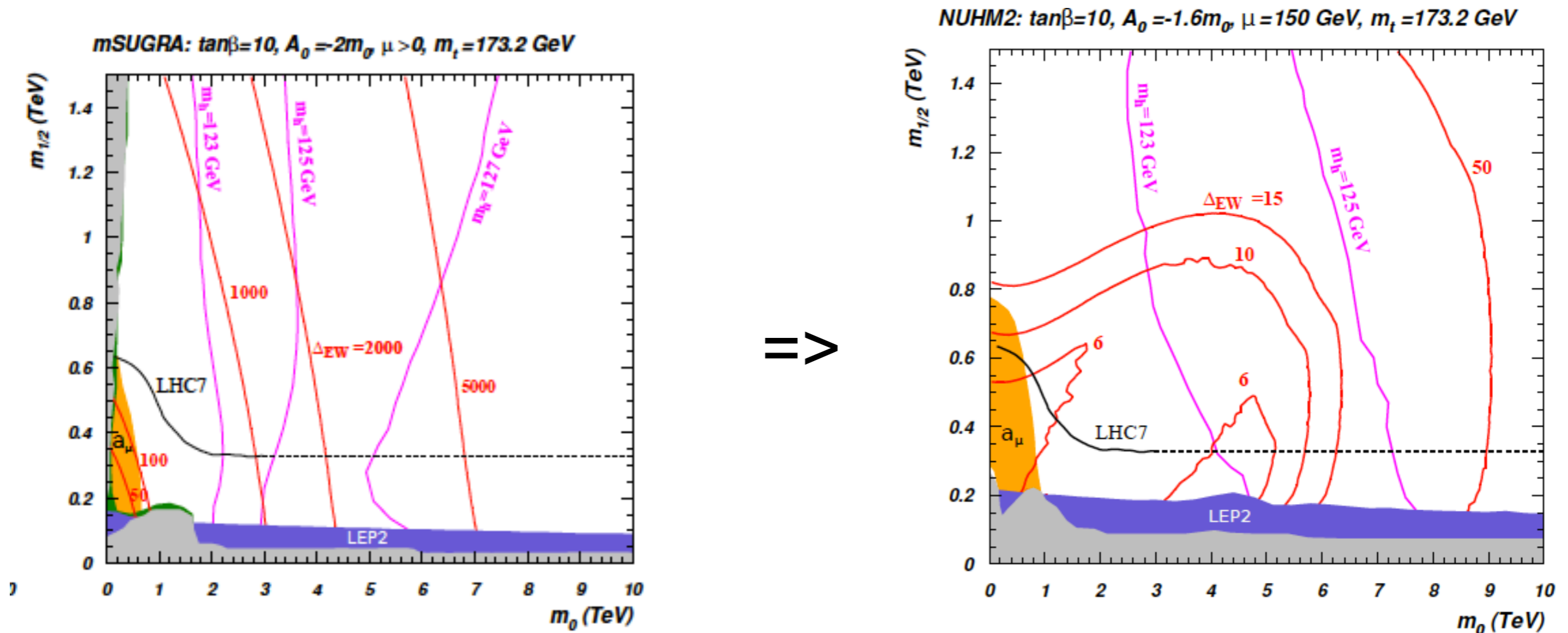
# SUSY spectra from radiatively-driven natural SUSY (RNS)

## scan NUHM2 space:

- light higgsino-like  $\tilde{W}_1$  and  $\tilde{Z}_{1,2}$  with mass  $\sim 100 - 300$  GeV,
- gluinos with mass  $m_{\tilde{g}} \sim 1 - 4$  TeV,
- heavier top squarks than generic NS models:  $m_{\tilde{t}_1} \sim 1 - 2$  TeV and  $m_{\tilde{t}_2} \sim 2 - 5$  TeV,
- first/second generation squarks and sleptons with mass  $m_{\tilde{q},\tilde{\ell}} \sim 1 - 8$  TeV. The  $m_{\tilde{\ell}}$  range can be pushed up to 20-30 TeV if non-universality of generations with  $m_0(1,2) > m_0(3)$  is allowed.

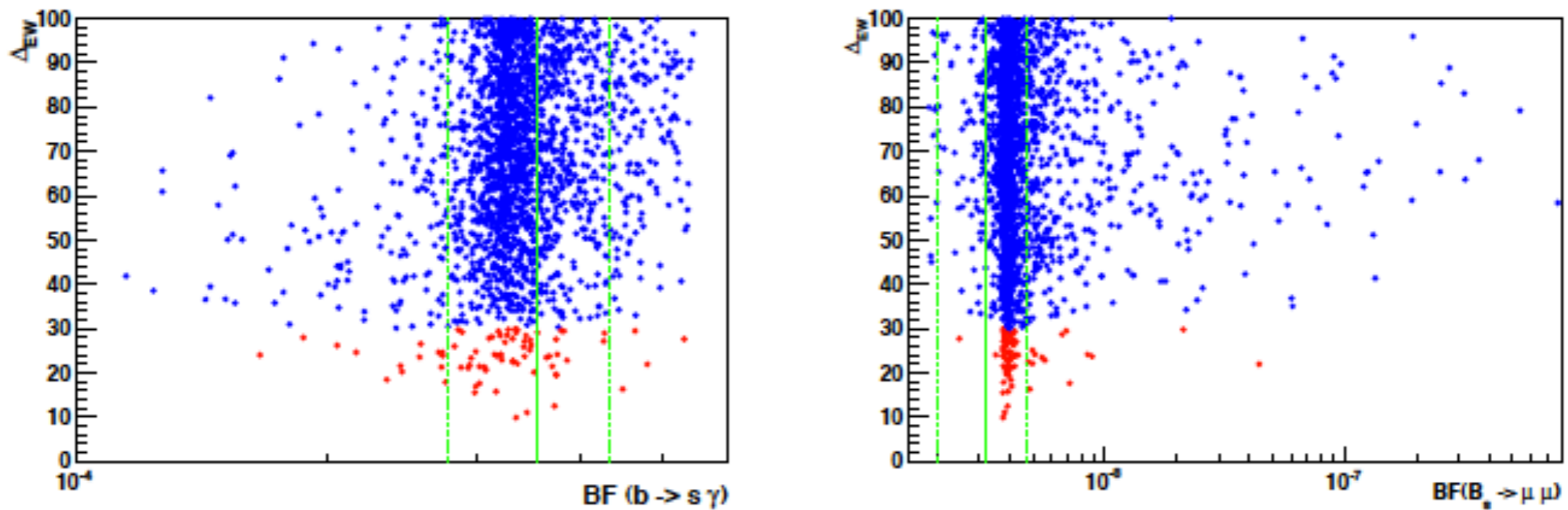
parameter	RNS1	RNS2	NS2
$m_0(1,2)$	10000	7025.0	19542.2
$m_0(3)$	5000	7025.0	2430.6
$m_{1/2}$	700	568.3	1549.3
$A_0$	-7300	-11426.6	873.2
$\tan \beta$	10	8.55	22.1
$\mu$	150	150	150
$m_A$	1000	1000	1652.7
$m_{\tilde{g}}$	1859.0	1562.8	3696.8
$m_{\tilde{u}_L}$	10050.9	7020.9	19736.2
$m_{\tilde{u}_R}$	10141.6	7256.2	19762.6
$m_{\tilde{e}_R}$	9909.9	6755.4	19537.2
$m_{\tilde{t}_1}$	1415.9	1843.4	572.0
$m_{\tilde{t}_2}$	3424.8	4921.4	715.4
$m_{\tilde{b}_1}$	3450.1	4962.6	497.3
$m_{\tilde{b}_2}$	4823.6	6914.9	1723.8
$m_{\tilde{\tau}_1}$	4737.5	6679.4	2084.7
$m_{\tilde{\tau}_2}$	5020.7	7116.9	2189.1
$m_{\tilde{\nu}_\tau}$	5000.1	7128.3	2061.8
$m_{\tilde{W}_2}$	621.3	513.9	1341.2
$m_{\tilde{W}_1}$	154.2	152.7	156.1
$m_{\tilde{Z}_4}$	631.2	525.2	1340.4
$m_{\tilde{Z}_3}$	323.3	268.8	698.8
$m_{\tilde{Z}_2}$	158.5	159.2	156.2
$m_{\tilde{Z}_1}$	140.0	135.4	149.2
$m_h$	123.7	125.0	121.1
$\Omega_{\tilde{Z}_1}^{std} h^2$	0.009	0.01	0.006
$BF(b \rightarrow s\gamma) \times 10^4$	3.3	3.3	3.6
$BF(B_s \rightarrow \mu^+\mu^-) \times 10^9$	3.8	3.8	4.0
$\sigma^{SI}(\tilde{Z}_1 p)$ (pb)	$1.1 \times 10^{-8}$	$1.7 \times 10^{-8}$	$1.8 \times 10^{-9}$
$\Delta$	9.7	11.5	23.7

# What happens to mSUGRA plane?



Little Hierarchy Problem melts away!

What happens to B constraints?  
These are trouble for version#1,2 NS models



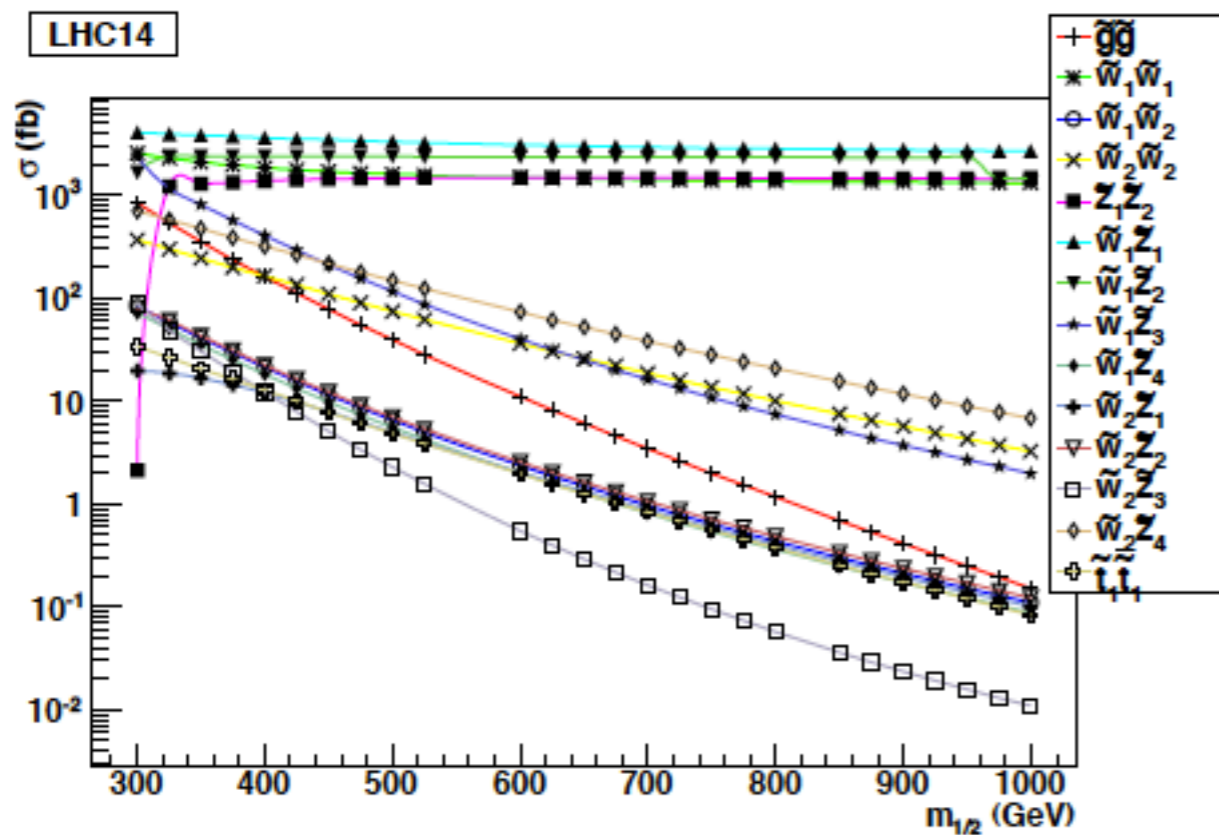
Heavier top squarks ameliorate these



# Prospects for radiatively-driven NS at LHC

## Model line with

$$m_0 = 5 \text{ TeV}, m_{1/2}, A_0 = -1.6m_0, \tan \beta = 15, \mu = 150 \text{ GeV}, m_A = 1 \text{ TeV}$$



$$pp \rightarrow \tilde{g}\tilde{g}X$$

$$\tilde{g} \rightarrow tb\tilde{W}_i, t\bar{t}\tilde{Z}_i$$

$$\tilde{Z}_2 \rightarrow \ell^+\ell^-\tilde{Z}_1$$

$$m_{\tilde{Z}_2} - m_{\tilde{Z}_1} < \sim 10 - 20 \text{ GeV}$$

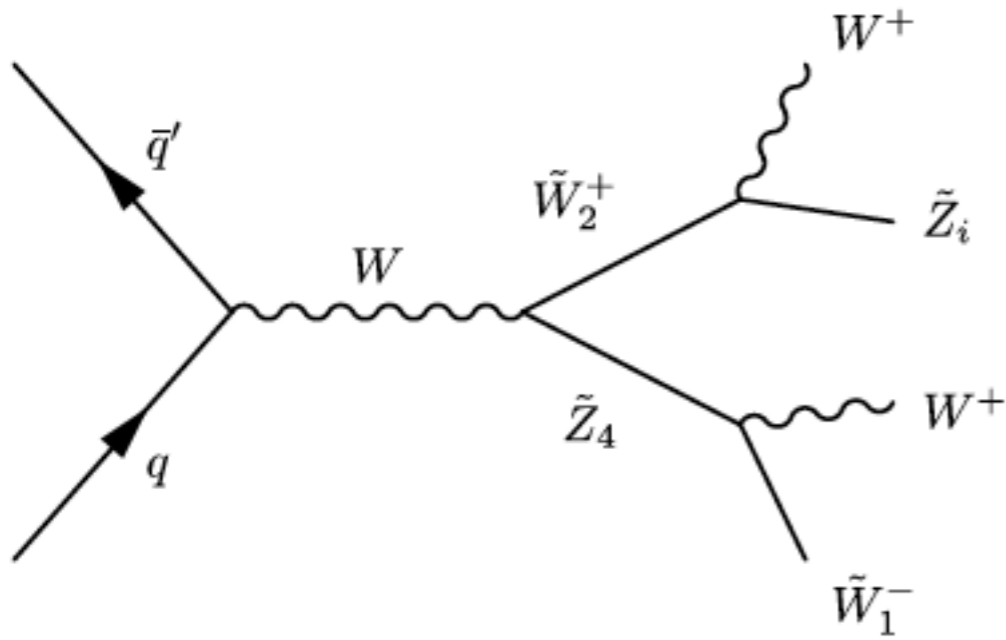
LHC14 reach for gluino pairs:

HB, Barger, Lessa, Tata, PRD86(2012)117701

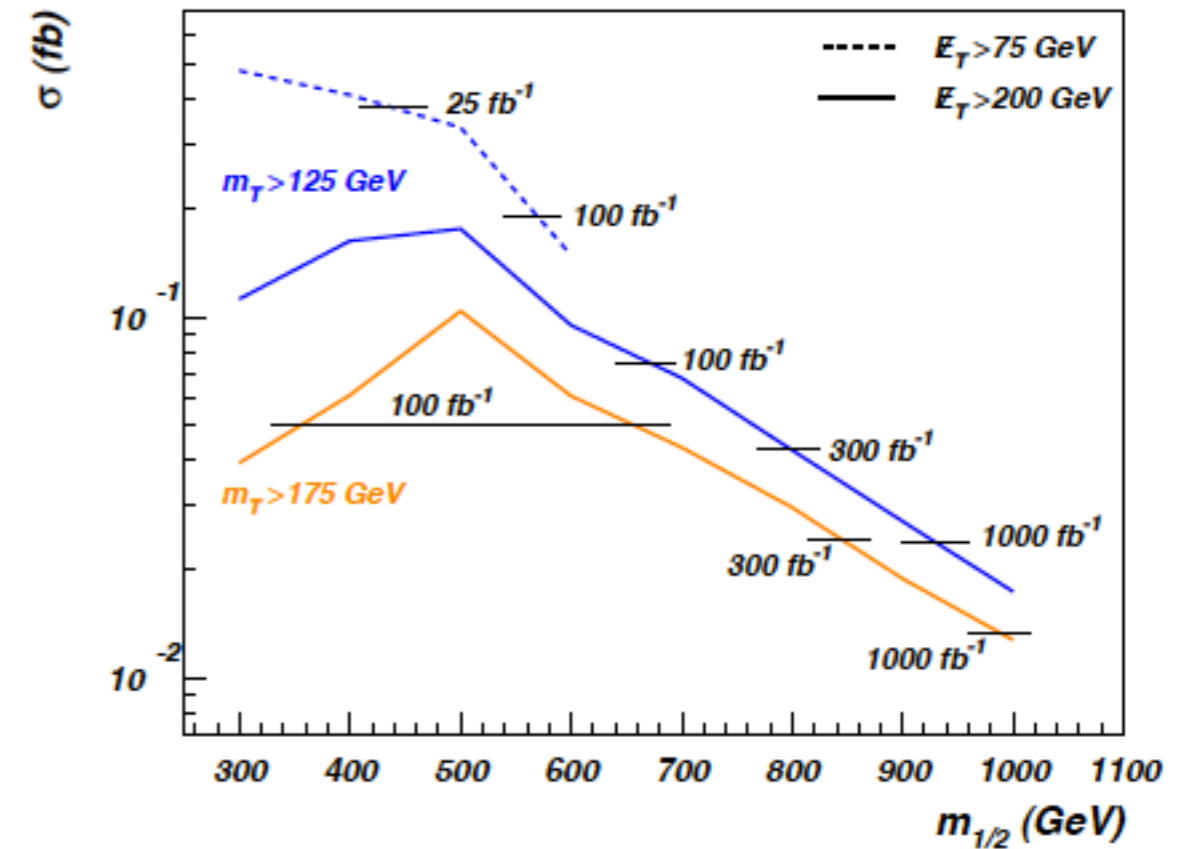
Int. lum. (fb <sup>-1</sup> )	$m_{1/2}$ (GeV)	$m_{\tilde{g}}$ (TeV) [ $[\tilde{g}\tilde{g}]$ ]
10	400	1.4
100	840	1.6
300	920	1.8
1000	1000	2.0

# Distinctive new signature for LHC: same-sign dibosons from models with light higgsinos

NUHM2:  $m_0=5 \text{ TeV}$ ,  $A_0=-1.6m_0$ ,  $\tan\beta=15$ ,  $\mu=150 \text{ GeV}$ ,  $m_A=1 \text{ TeV}$



HB, Barger, Huang, Mickelson, Mustafayev,  
Sreethawong, Tata, arXiv:1302.5816,  
(PRL in press)



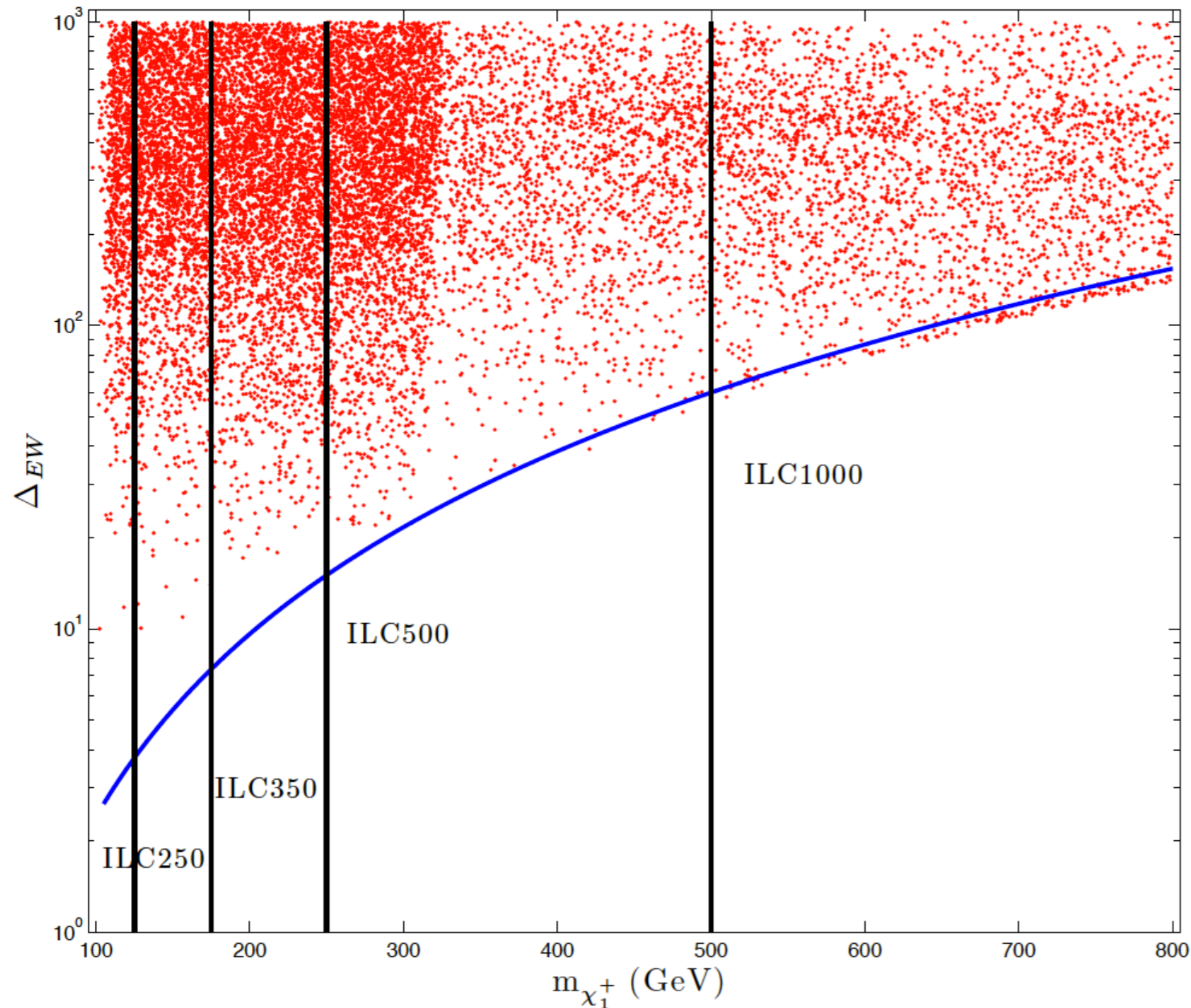
- exactly 2 isolated same-sign leptons with  $p_T(\ell_1) > 20 \text{ GeV}$  and  $p_T(\ell_2) > 10 \text{ GeV}$ ,
- $n(b - jets) = 0$  (to aid in vetoing  $t\bar{t}$  background).
- $m_T^{\min} \equiv \min [m_T(\ell_1, E_T), m_T(\ell_2, E_T)] > 125 \text{ GeV}$   
 $E_T' > 200 \text{ GeV}$

Int. lum. ( $\text{fb}^{-1}$ )	$m_{1/2}$ (GeV)	$m_{\tilde{g}}$ (TeV)	$m_{\tilde{g}}$ (TeV) [ $[\tilde{g}\tilde{g}]$ ]
10	400	0.96	1.4
100	840	2.0	1.6
300	920	2.2	1.8
1000	1000	2.4	2.0

Reach at LHC14 exceeds usual gluino pair search!

# Smoking gun signature: 4 light higgsinos at ILC!

$$e^+e^- \rightarrow \tilde{W}_1^+ \tilde{W}_1^-, \tilde{Z}_1 \tilde{Z}_2$$



$$m_{\tilde{W}_1^\pm}, m_{\tilde{Z}_{1,2}}$$

$$\sqrt{s} \sim \sqrt{2\Delta_{EW}m_Z}$$

ILC/CLIC have capability to measure SUSY parameters and actually reconstruct

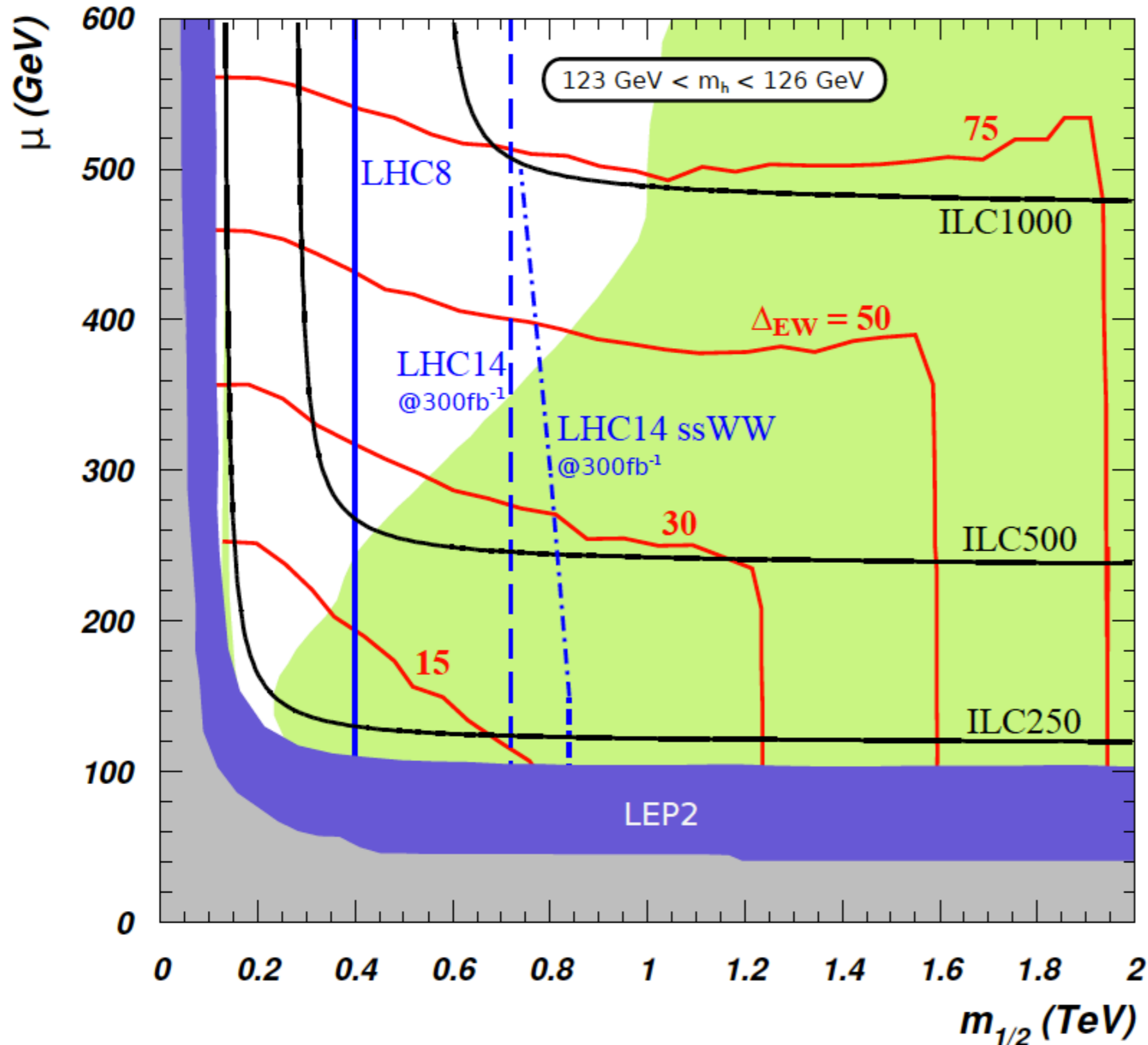
$$\Delta_{EW}$$

measure and check if nature is EWFT'd?



# LHC/ILC complementarity

NUHM2:  $m_0=5\text{ TeV}$ ,  $\tan\beta=15$ ,  $A_0=-1.6m_0$ ,  $m_A=1\text{ TeV}$ ,  $m_t=173.2\text{ GeV}$



While LHC has some capacity, it will require ILC to draw the story of SUSY electroweak naturalness to a conclusion!

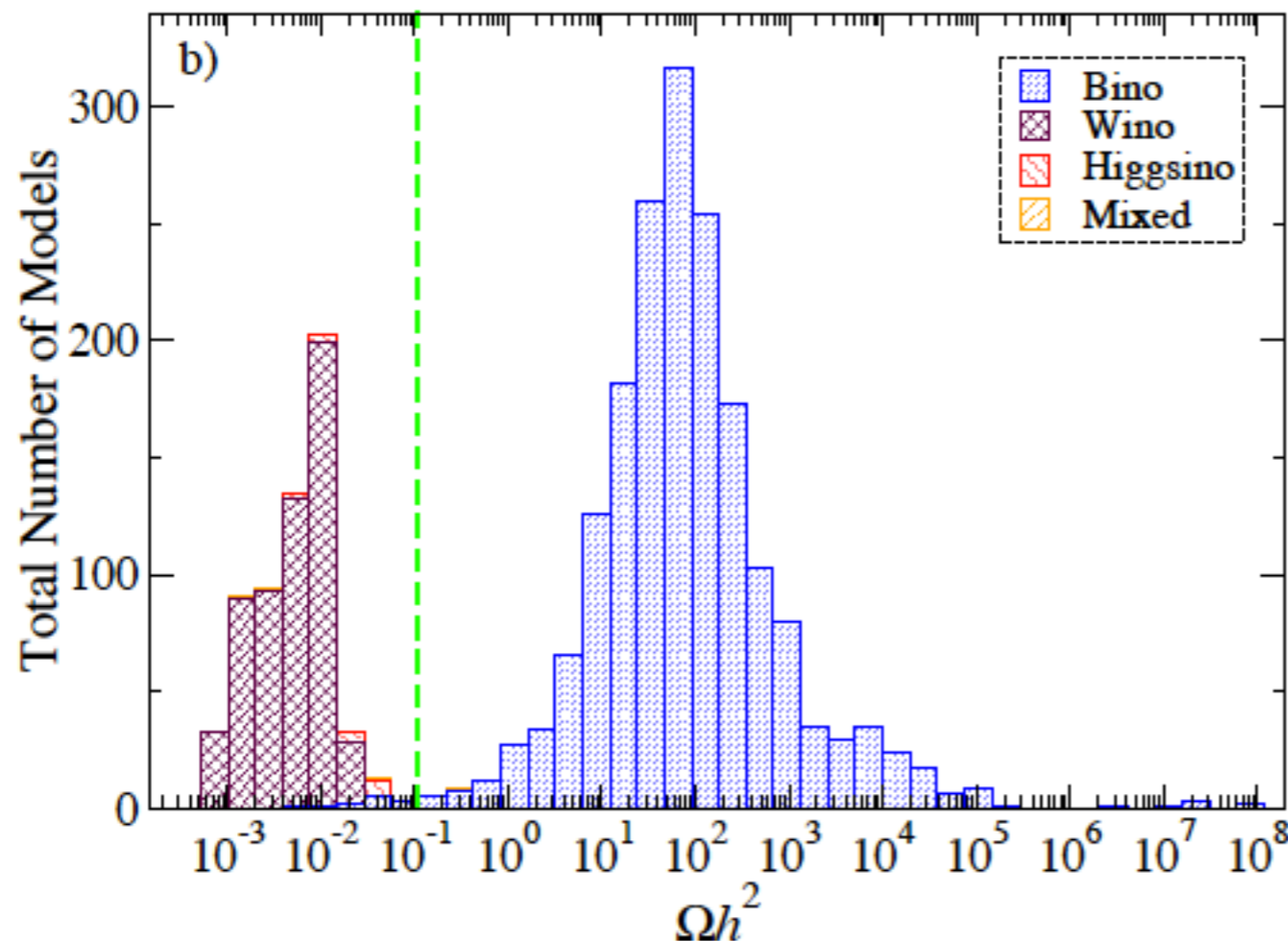
A. Mustafayev plot

# What about DM in RNS?

I heard higgsino-like wimp isn't a good DM candidate?

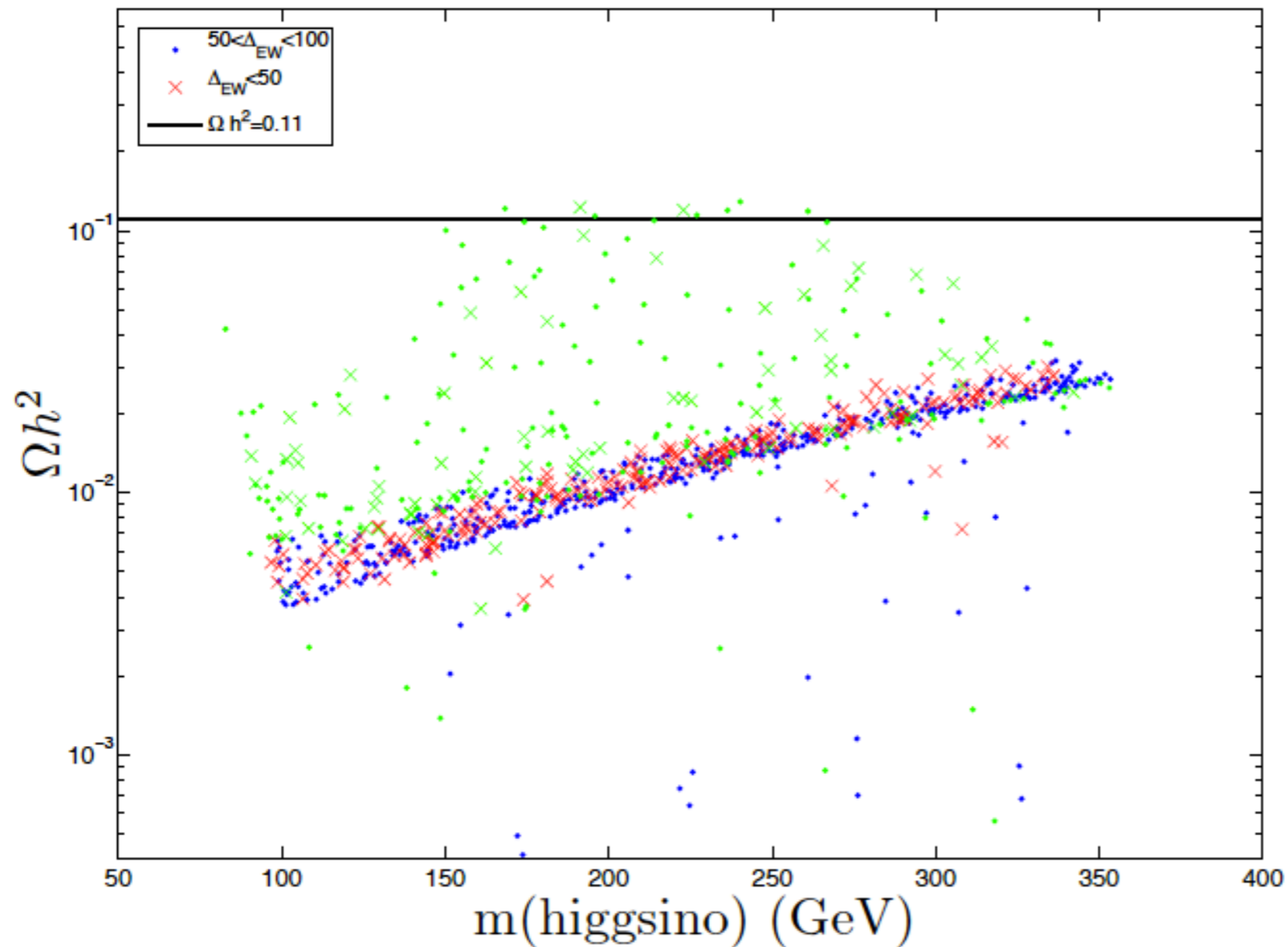
Lightest neutralino all by itself in general not good DM candidate: too much or too little CDM

Scan over 19 parameters:



HB, Box, Summy  
JHEP1010(2010)023

# Standard thermal abundance for RNS model



green: already excluded by WIMP searches

$$\Omega_{\tilde{Z}_1}^{std} h^2 \sim 10 - 15 \text{ low}$$

# Invoke Peccei-Quinn sol'n to strong CP problem with SUSY

## PQMSSM: Axions + SUSY $\Rightarrow$ mixed $a - LSP$ dark matter

- $\hat{a} = \frac{s+ia}{\sqrt{2}} + i\sqrt{2}\bar{\theta}\tilde{a}_L + i\bar{\theta}\theta_L\mathcal{F}_a$  in 4-comp. notation
- Raby, Nilles, Kim; Rajagopal, Wilczek, Turner
- axino is spin- $\frac{1}{2}$  element of axion supermultiplet ( $R$ -odd; possible LSP candidate)
- $m_{\tilde{a}}$  model dependent: keV  $\rightarrow$  TeV, but  $\sim M_{SUSY}$  in gravity mediation
- saxion is spin-0 element:  $R$ -even but gets SUSY breaking mass  $\sim 1$  TeV
- axion is usual QCD axion: gets produced via vacuum mis-alignment/coherent oscillations as usual
- additional PQ parameters:  $(f_a, m_{\tilde{a}}, m_s, \theta_i, \theta_s, )$  and  $T_R$

# Coupled Boltzmann calculation of mixed axion-neutralino abundance

Bae, HB, Lessa, arXiv:1301.7428

Case for dominant  $s \rightarrow aa$  decay:  
contributes to dark radiation

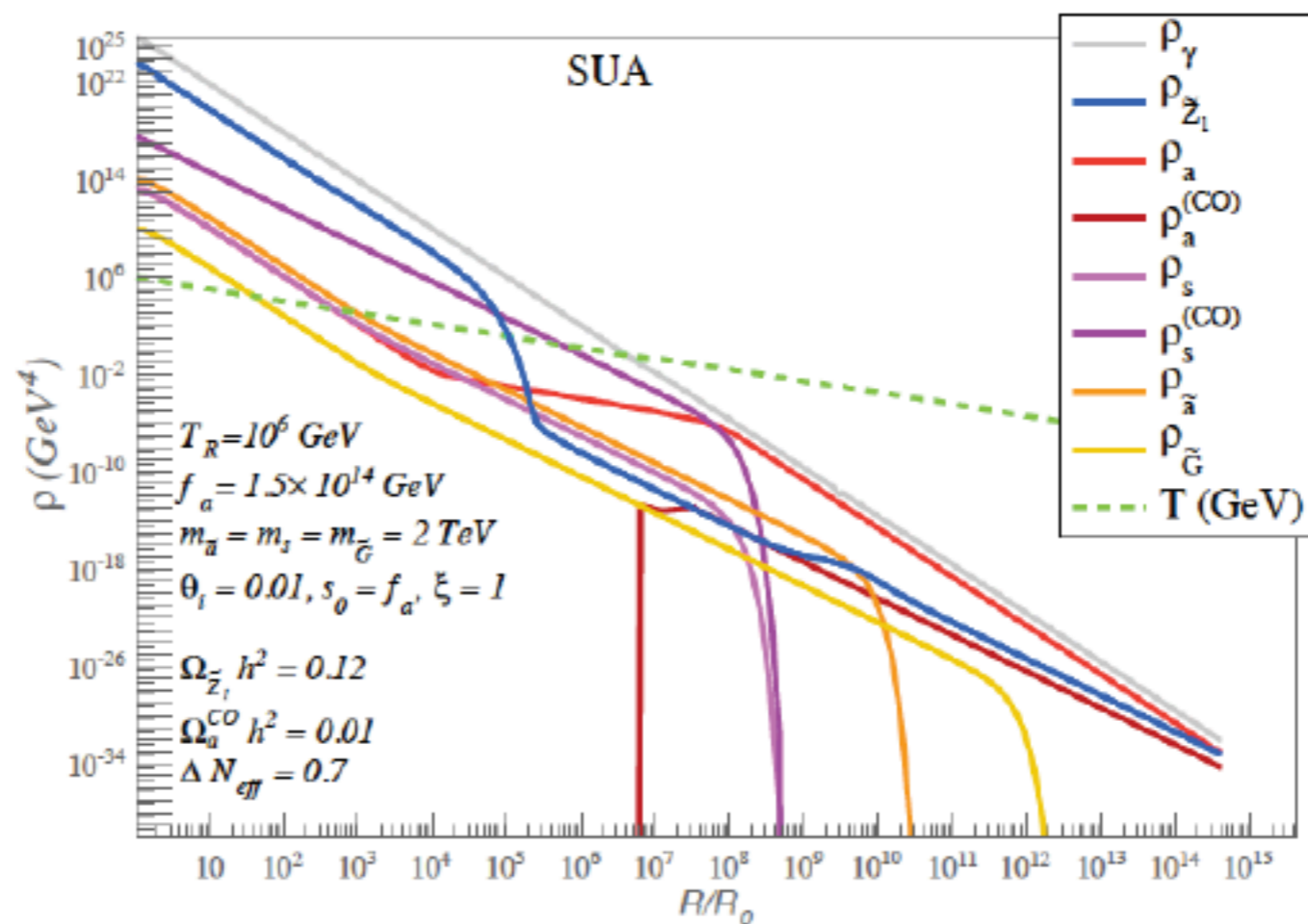
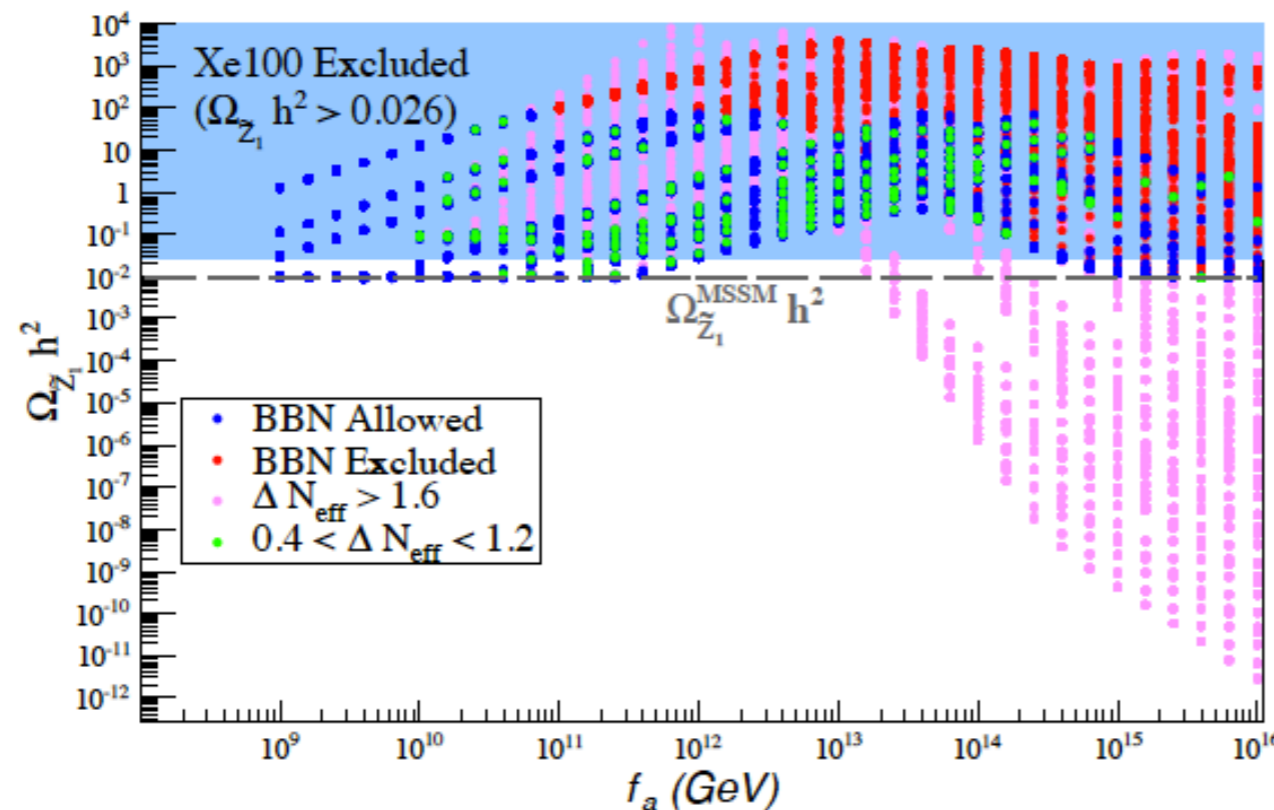


Figure 2: Evolution of various energy densities versus scale parameter  $R/R_0$  for the SUA benchmark.



# Mixed higgsino-axion CDM in radiative natural SUSY



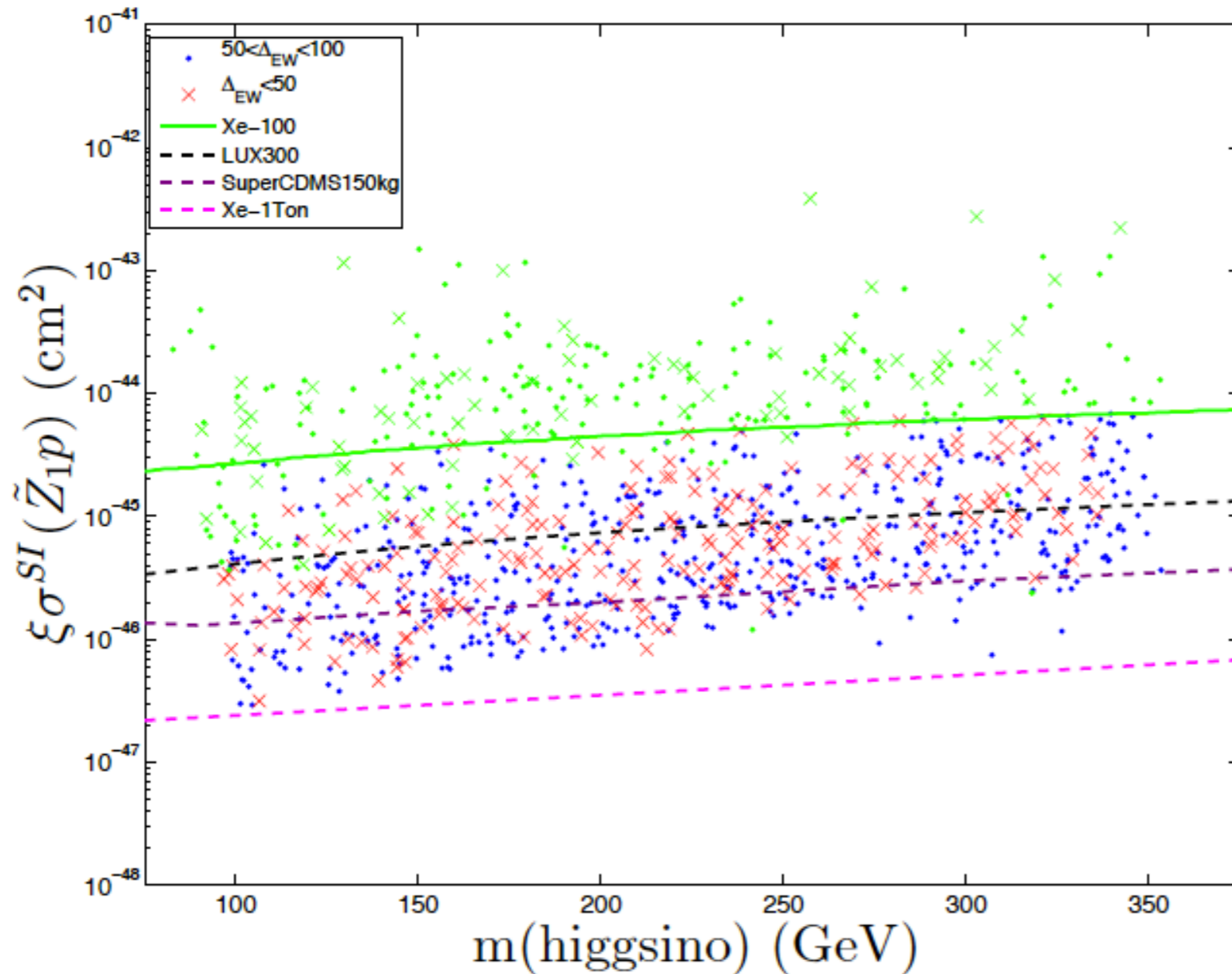
$f_a \sim 10^{14}$  GeV allowed!

(string theorists  
take note)

Abundance of higgsinos is boosted due to thermal production and decay of axinos in early universe: the axion saves the day for WIMP direct detection!

Detection of relic axions also possible

# Direct higgsino detection rescaled for minimal local abundance

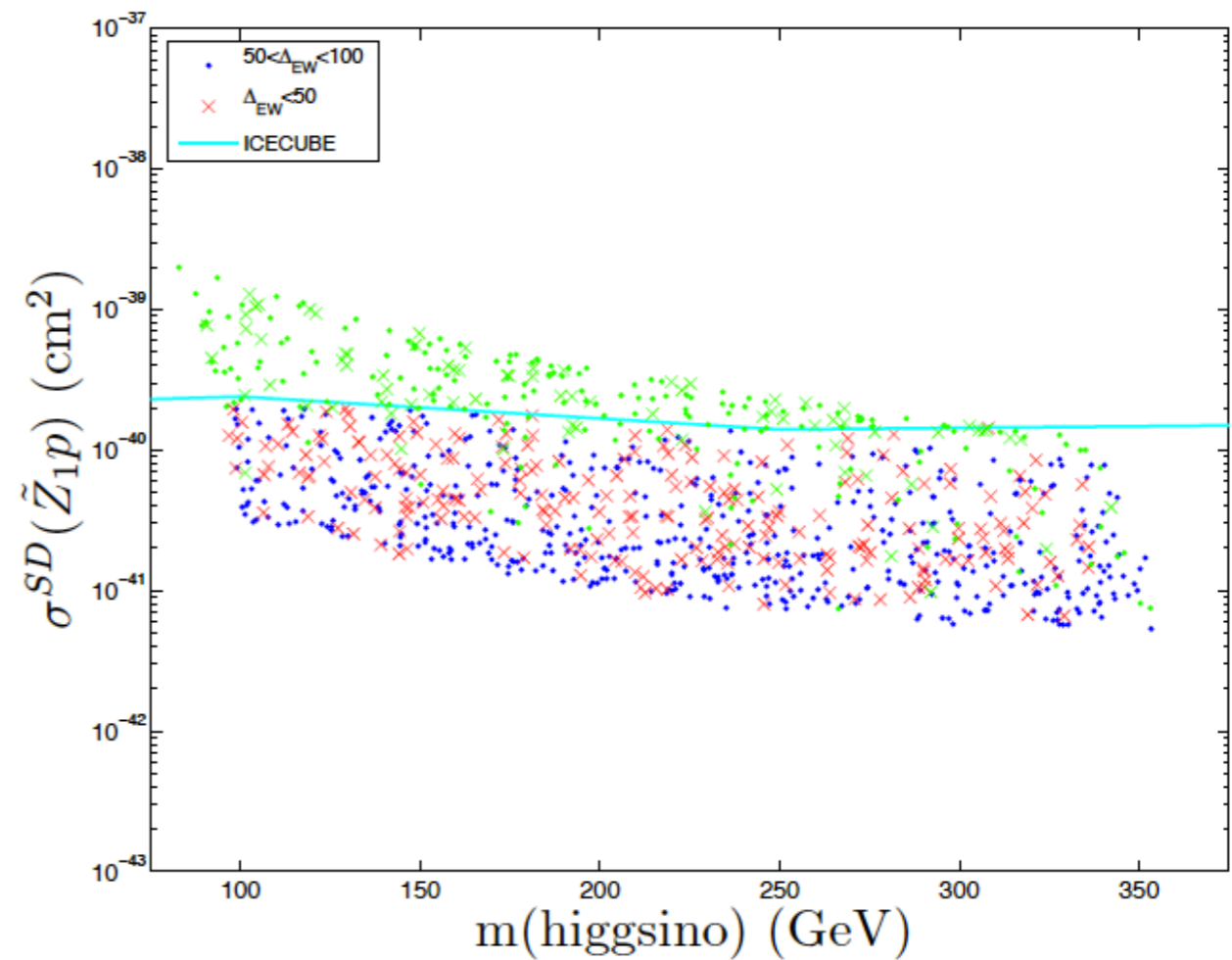
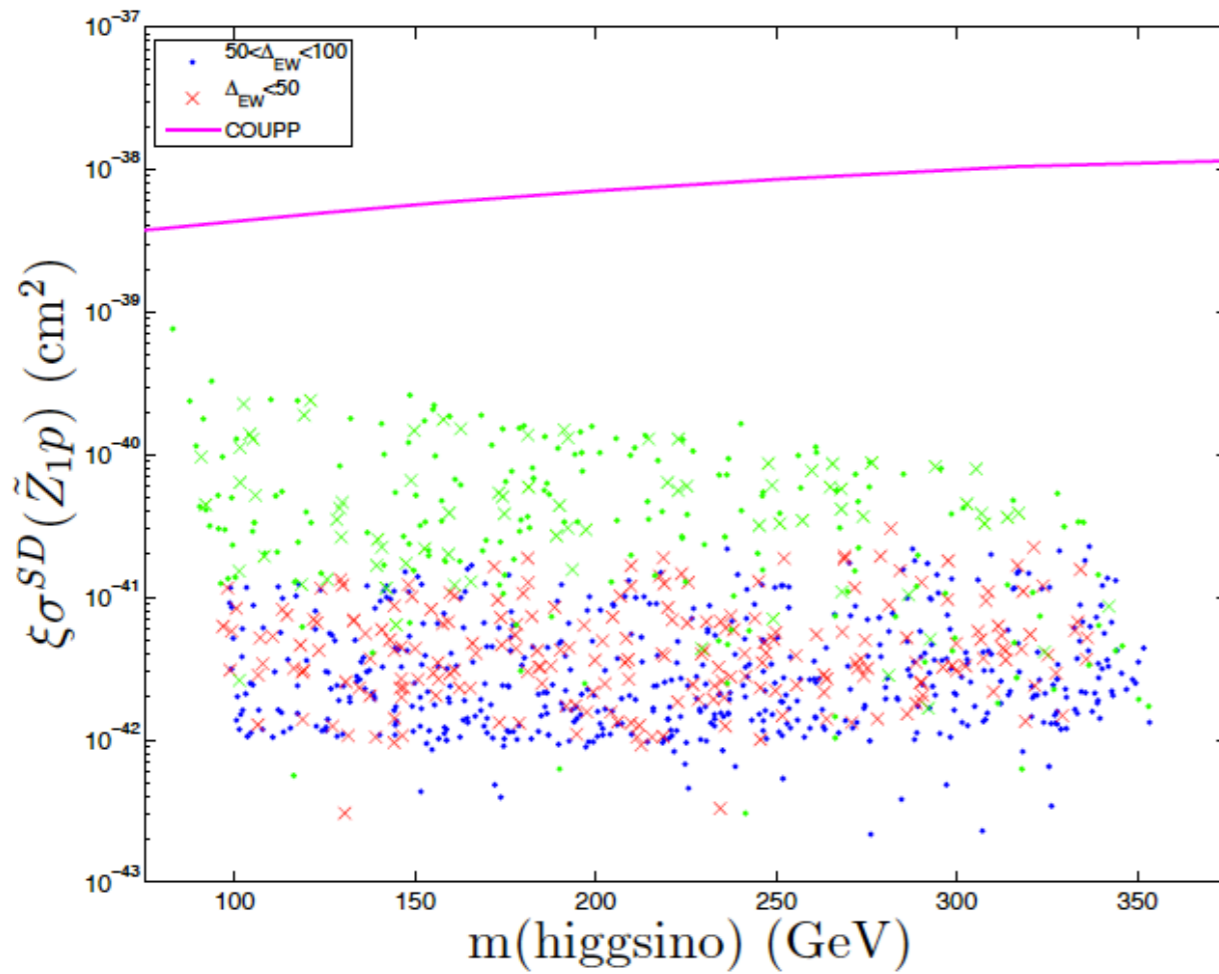


HB, Barger, Mickelson  
arXiv:1303.3816

Deployment of Xe-1ton  
coming soon!

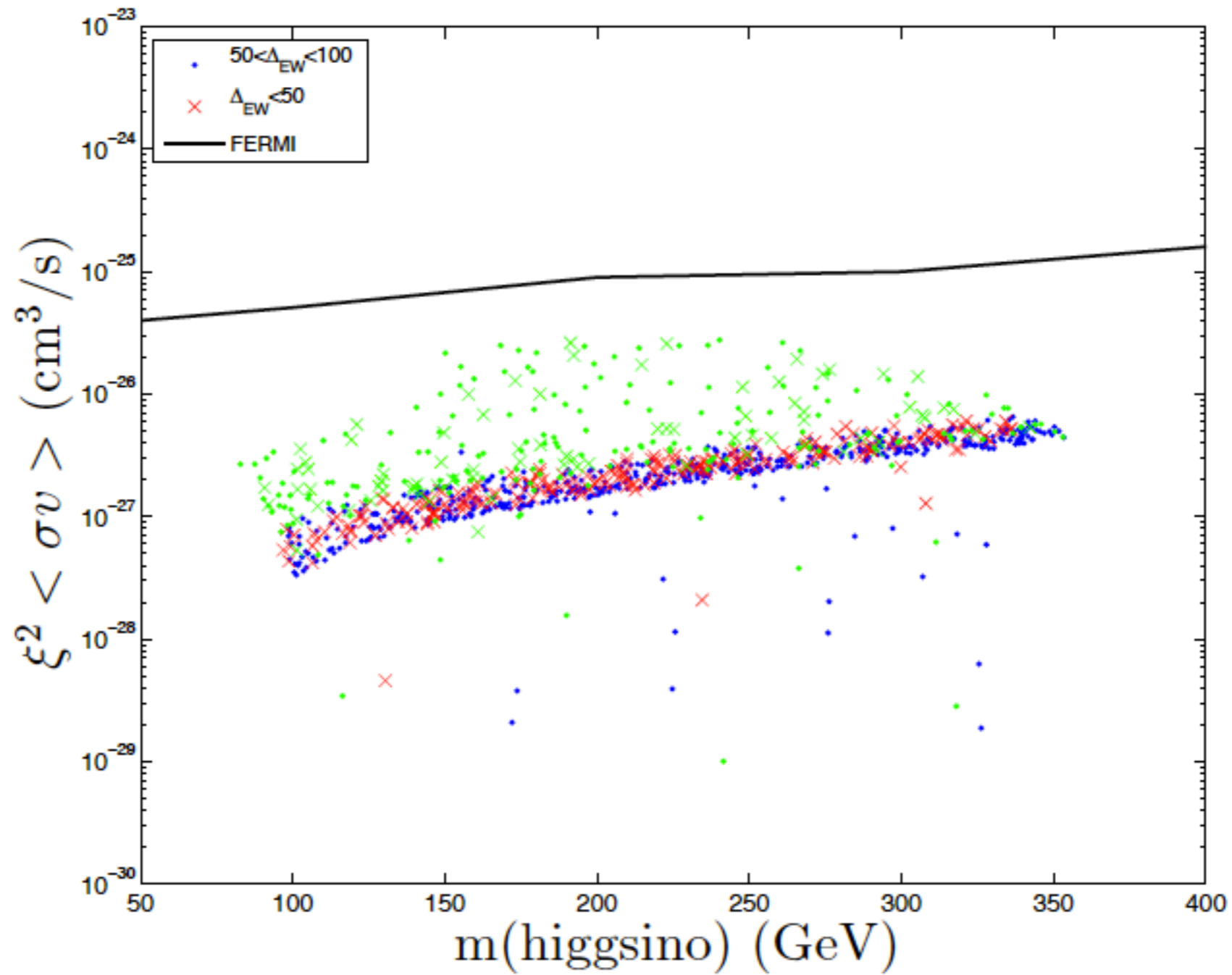
Can test completely with ton scale detector  
or equivalent (subject to minor caveats)

# Spin-dependent higgsino detection:





# Higgsino detection via halo annihilations:



# Conclusions

- $\Delta_{EW}$  is more robust measure of Little Hierarchy problem
- Why are  $m(Z), m(h) \sim 100$  GeV while sparticle masses are  $\gg 1$  TeV?
- $\mu \sim m(Z)$ : light higgsinos (ILC!)
- $m(H_u)$  driven somewhat, not grossly, negative
- large mixing in stop sector

Under these conditions, the Little Hierarchy Problem seems to melt away and the old paradigm of SUSY GUTs remains strong:

but with huge implications for collider/dark matter searches!

- The low lying sparticles (higgsinos) have severely compressed spectra: hard to see at LHC (but new signatures e.g. SS dibosons)
- No large cancellations in  $m(Z)$ ,  $m(h) \Rightarrow$  ILC is the right machine to build!
- Dark matter production more intricate than usual story: here, we suggest mixed axion-higgsino (co-dark-matter)

# Quotes from some practitioners of EWFT arguments

“...naturalness is a notoriously brittle and subjective subject...”  
Feng & Sanford, 2012

In this context, it is natural to wonder whether the continuing absence of sparticles should disconcert advocates of the Minimal Supersymmetric Extension of the Standard Model (MSSM). After all, the only theoretical motivation for the appearance of sparticles at accessible energies is in order to alleviate the fine tuning required to maintain the electroweak hierarchy [5], and sparticles become less effective in this task the heavier their masses. Since the problem of fine-tuning is a subjective one, it is not possible to provide a concise mathematical criterion for deciding whether enough is enough, already. Moreover, the fine tuning can be discussed only in concrete models for the soft supersymmetry breaking terms, and any conclusion refers to the particular model under consideration. The fine-tuning price may also depend on other, optional, theoretical assumptions.

Chankowski, Ellis, Pokorski, 1998

We now return to naturalness and discuss attempts to quantify it in more detail. All such attempts are subject to quantitative ambiguities. However, this fact should not obscure the many qualitative differences that exist in naturalness prescriptions proposed in the literature. In this section, we begin by describing a standard prescription for quantify-

This initial step is absolutely crucial, as all naturalness studies are inescapably model-dependent. In any supersymmetry study, some fundamental framework must be adopted. In studies of other topics, however, there exists, at least in principle, the possibility of a model-independent study, where no correlations among parameters are assumed. This model-independent study is the most general possible, in that all possible results from any other (model-dependent) study are a subset of the model-independent study's results. In studies of naturalness, however, the correlations determine the results, and there is no possibility, even in principle, of a model-independent study in the sense described above.

Feng, 2013 review

**We wish to refute these points of view**



# Some virtues of $\Delta_{EW}$

- *Model independent* (within the context of models which reduce to the MSSM at the weak scale):  $\Delta_{EW}$  is essentially determined by the sparticle spectrum[27], and – unlike  $\Delta_{HS}$  and other measures of fine-tuning – does not depend on the mechanism by which sparticles acquire masses. Since  $\Delta_{EW}$  is determined only from weak scale Lagrangian parameters, the phenomenological consequences which may be derived by requiring low  $\Delta_{EW}$  will apply not only for the NUHM2 model considered here, but also for other possibly more complete (or less complete, such as pMSSM) models which lead to look-alike spectra at the weak scale.
- *Conservative*:  $\Delta_{EW}$  captures the minimal fine-tuning that is necessary for any given sparticle spectrum, and so leads to the *most conservative conclusions* regarding fine-tuning considerations.
- *Measurable*:  $\Delta_{EW}$  is in principle measurable in that it can be evaluated if the underlying weak scale parameters can be extracted from data.
- *Unambiguous*: Fine-tuning measures which depend on high scale parameter choices, such as the Barbieri-Guidice measure  $\Delta_{BG}$  discussed previously, are highly sensitive to exactly which set of model input parameters one adopts: for example, it is well-known that significantly different values of  $\Delta_{BG}$  result depending on whether the high scale top-Yukawa coupling is or is not included as an input parameter[37]. There is no such ambiguity in the fine-tuning sensitivity as measured by both  $\Delta_{EW}$  and  $\Delta_{HS}$ .
- *Predictive*: While  $\Delta_{EW}$  is less restrictive than  $\Delta_{HS}$ , it still remains highly restrictive. The requirement of low  $\Delta_{EW}$  highly disfavors models such as mSUGRA/CMSSM[27], while allowing for very distinct predictions from more general models such as NUHM2.
- *Falsifiable*: The most important prediction from requiring low  $\Delta_{EW}$  is that  $|\mu|$  cannot be too far removed from  $M_Z$ . This implies the existence of light higgsinos  $\sim 100 - 300$  GeV which are hard to see at hadron colliders, but which are easily detected at a linear  $e^+e^-$  collider with  $\sqrt{s} \gtrsim 2|\mu|$ . If no higgsinos appear at ILC1000, then the idea of electroweak naturalness in SUSY models is dead.
- *Simple to calculate*:  $\Delta_{EW}$  is extremely simple to encode in sparticle mass spectrum programs, even if one adopts models with very large numbers of input parameters.

HB, Barger, Huang, Mickelson, Mustafayev, Tata,  
arXiv:1212.2655