Particle masses, spins and couplings at hadron colliders

K.C. Kong

Fermilab

In collaboration with M. Burns, K. Matchev, M. Park:
JHEP10 (2008) 081 (arXiv:0808.2472 [hep-ph])
arXiv:0810.5576 [hep-ph]

Theoretical Physics Seminars
SLAC
November 5, 2008

Outline

- Why do we need to measure the spin?
- Why is it so difficult to measure the spin?
- What is wrong with the previous proposals?
  - Spin measurements
  - Coupling measurements
  - Mixing angle measurements
- Why do you need to read our paper?
- How should you read our paper?
- What does our paper actually say?
  - Analytical approach to measurements of spins and couplings
  - No backgrounds (SM and combinatorial), no simulation
  - Study on the kinematics of signal
  - Present the basic idea of methods
- Examples (SPS1a and top in leptonic decay)
- Mass measurements (using m_{T_2})
- Summary and discussion

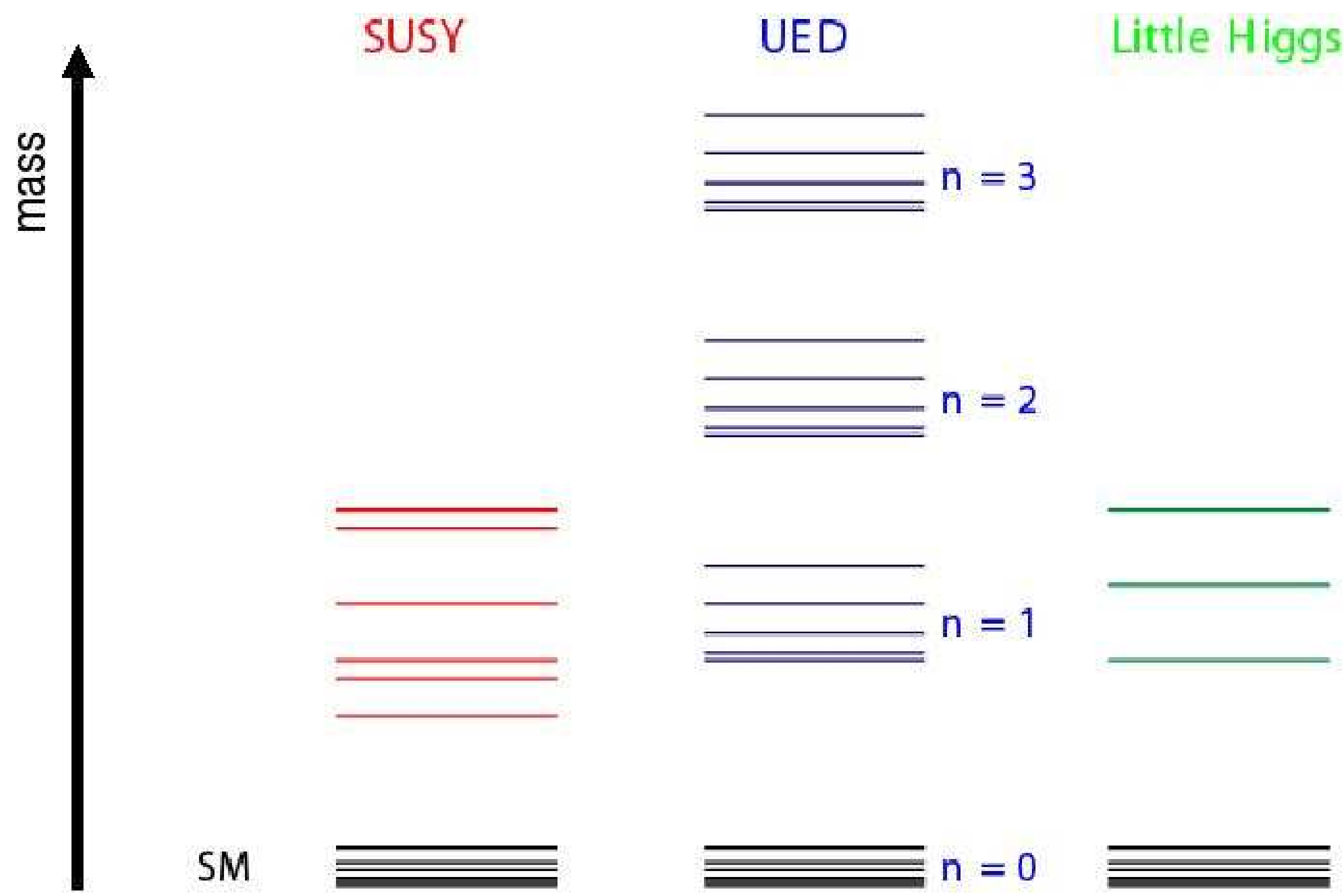
Is measurement of spins and masses important ?

- IPMU Workshop on Masses and Spins - 16-20 March 2009, "Determination of Masses and Spins of New Particles at the LHC"

- Similar workshop will be held at UC Davis (with MC4BSM) 2009

- Many papers $\cdots$

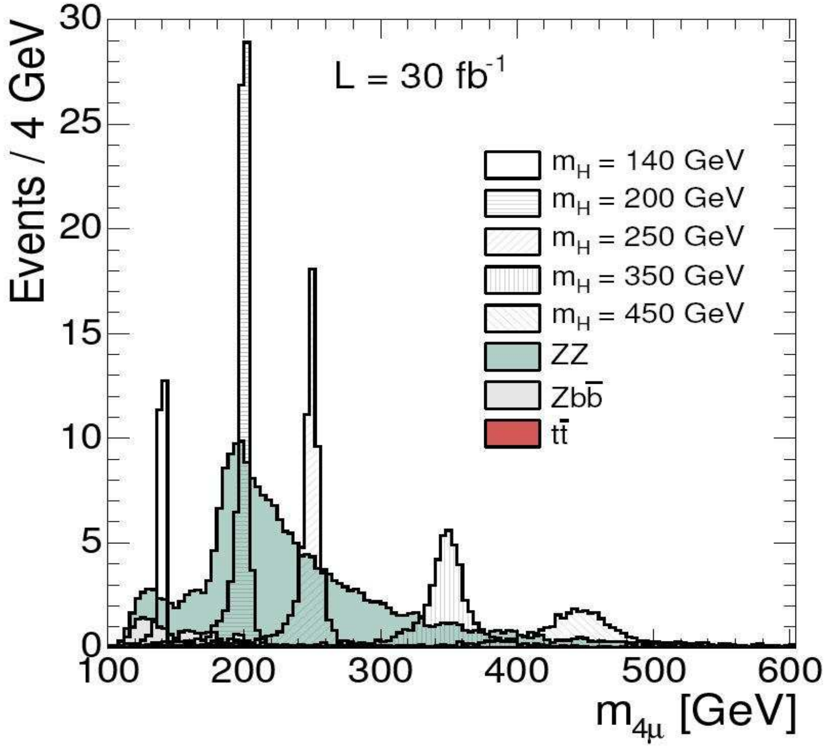
Why do we need to measure the spin?



• spins	differy by 1/2	smae as SM	same as SM
• higher levels	no	yes	no

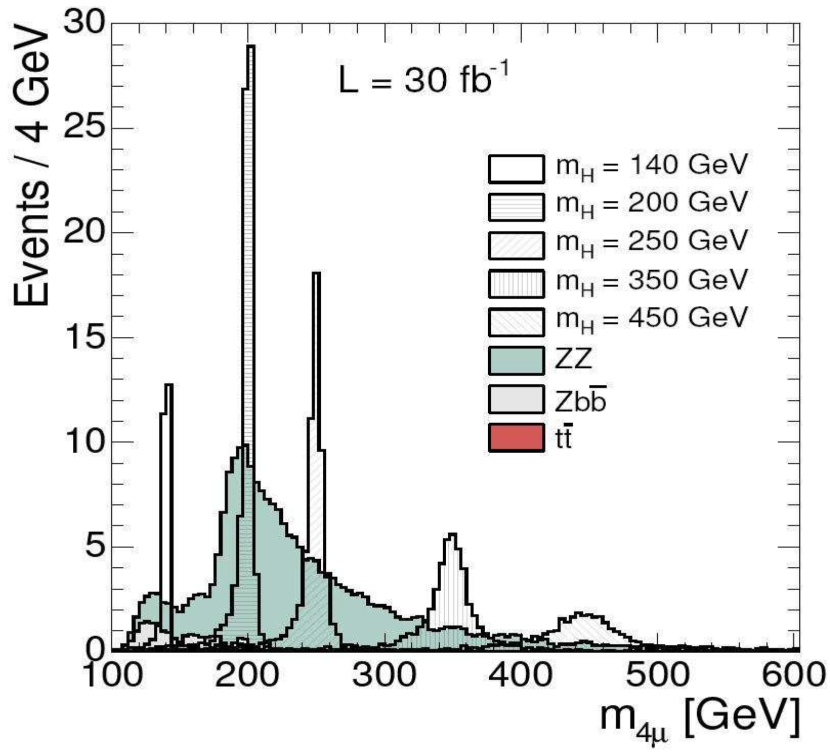
Why is it so difficult to measure the spin?

- How do we measure mass?

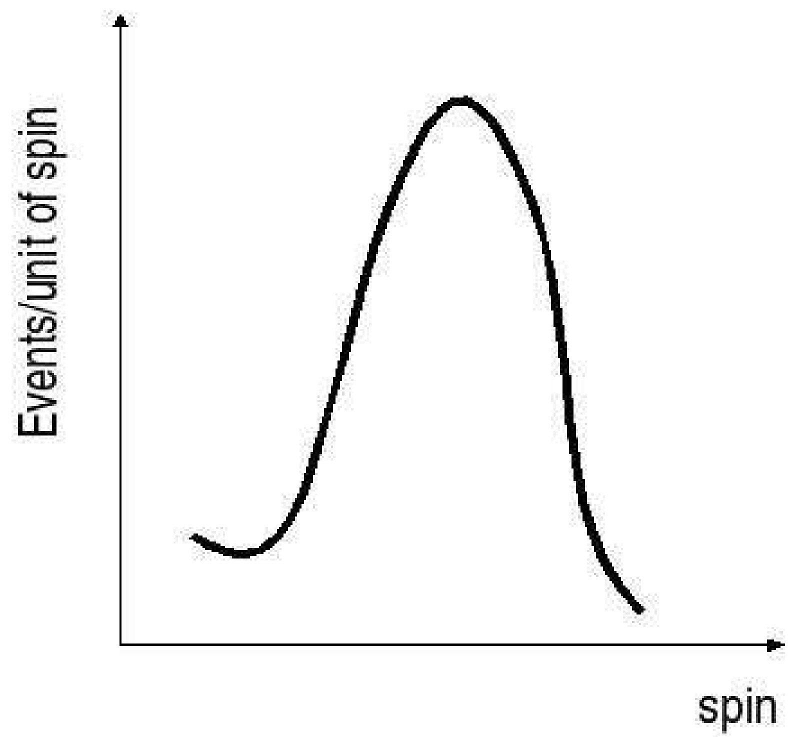


Why is it so difficult to measure the spin?

- How do we measure mass?

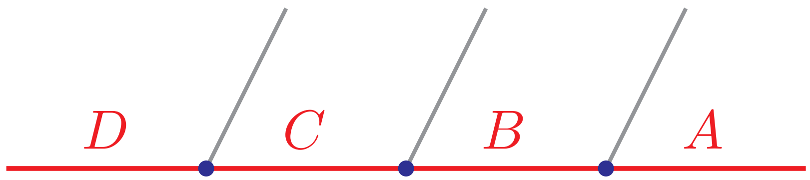


- We cannot measure spin the same way!



Why is it so difficult to measure the spin?

- Missing energy signatures arise from something like:

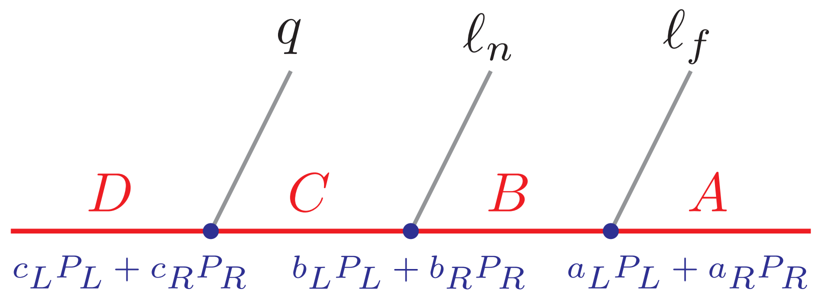


- Several alternative explanations:

S	Spins	D	C	B	A	Example
1	SFSF	Scalar	Fermion	Scalar	Fermion	$\tilde{q} \to \tilde{\chi}_2^0 \to \tilde{\ell} \to \tilde{\chi}_1^0$
2	FSFS	Fermion	Scalar	Fermion	Scalar	$q_1 \to Z_H \to \ell_1 \to \gamma_H$
3	FSFV	Fermion	Scalar	Fermion	Vector	$q_1 \to Z_H \to \ell_1 \to \gamma_1$
4	FVFS	Fermion	Vector	Fermion	Scalar	$q_1 \to Z_1 \to \ell_1 \to \gamma_H$
5	FVFV	Fermion	Vector	Fermion	Vector	$q_1 \to Z_1 \to \ell_1 \to \gamma_1$
6	SFVF	Scalar	Fermion	Vector	Fermion	—

Why is it so difficult to measure the spin?

- Inevitable dependence on other parameters:



$$x = \frac{m_C^2}{m_D^2} \qquad y = \frac{m_B^2}{m_C^2} \qquad z = \frac{m_A^2}{m_B^2}$$

$$\tan\varphi_a = \frac{|a_R|}{|a_L|} \qquad \tan\varphi_b = \frac{|b_R|}{|b_L|} \qquad \tan\varphi_c = \frac{|c_R|}{|c_L|}$$

- Any observable that knows about spins, also knows about
  - Masses (m_A, m_B, m_C, m_D)
  - Couplings and mixing angles (g_L and g_R) : $g_{L,R} \equiv U_F^\dagger g_{L,R}^0 U_B$
  - Particle-antiparticle ($D/\bar{D}$) fraction f ($f + \bar{f} = 1$)
- What is it that you are really measuring?

$$\left(\frac{dN}{d\hat{m}_p^2}\right)_S = \sum_{I=1}^{2}\sum_{J=1}^{2} K_{IJ}^{(p)}(f, \varphi_a, \varphi_b, \varphi_c)\mathcal{F}_{S;IJ}^{(p)}(\hat{m}_p^2; x, y, z) \ ,$$

Most likely chronology

- Measure the SM backgrounds (rediscover SM)
- Find a missing energy signal
- Convince yourself it is a real signal
- Measure the cross-section (times BR)
- Measure the overall mass scale
- Observe structures, measure individual masses
- Measure spins?
- Then perhaps also measure
  - Chirality of the couplings (L versus R)
  - Mixing angles
  - Particle-antiparticle fraction

What is wrong with the previous spin methods ?

- Most previous studies compared two models A and B, which have
- Different spins
  - Identical mass spectrum (OK)
  - Identical couplings and mixing angles (not OK)
  - Identical particle-antiparticle fraction f (not OK)
- If you can see a difference, you have measured the spins
- Not so fast! Problems:
  - It's the wrong chronological order
  - It's not a pure spin measurement

What is wrong with the previous spin methods ?

- We cannot measure the fermion helicity
- Previous work considered only certain helicity combinations (shown in blue)
- We consider all possibilities (both blue and red)

Processes P_{11}		Processes P_{12}	
$\{q_L, \ell^-_L, \ell^+_L\}$ $f\|c_L\|^2\|b_L\|^2\|a_L\|^2$	$\{\bar{q}_L, \ell^+_L, \ell^-_L\}$ $\bar{f}\|c_L\|^2\|b_L\|^2\|a_L\|^2$	$\{q_L, \ell^-_L, \ell^+_R\}$ $f\|c_L\|^2\|b_L\|^2\|a_R\|^2$	$\{\bar{q}_L, \ell^+_L, \ell^-_R\}$ $\bar{f}\|c_L\|^2\|b_L\|^2\|a_R\|^2$
$\{\bar{q}_L, \ell^-_R, \ell^+_R\}$ $\bar{f}\|c_L\|^2\|b_R\|^2\|a_R\|^2$	$\{q_L, \ell^+_R, \ell^-_R\}$ $f\|c_L\|^2\|b_R\|^2\|a_R\|^2$	$\{\bar{q}_L, \ell^-_R, \ell^+_L\}$ $\bar{f}\|c_L\|^2\|b_R\|^2\|a_L\|^2$	$\{q_L, \ell^+_R, \ell^-_L\}$ $f\|c_L\|^2\|b_R\|^2\|a_L\|^2$
$\{q_R, \ell^-_R, \ell^+_R\}$ $f\|c_R\|^2\|b_R\|^2\|a_R\|^2$	$\{\bar{q}_R, \ell^+_R, \ell^-_R\}$ $\bar{f}\|c_R\|^2\|b_R\|^2\|a_R\|^2$	$\{q_R, \ell^-_R, \ell^+_L\}$ $f\|c_R\|^2\|b_R\|^2\|a_L\|^2$	$\{\bar{q}_R, \ell^+_R, \ell^-_L\}$ $\bar{f}\|c_R\|^2\|b_R\|^2\|a_L\|^2$
$\{\bar{q}_R, \ell^-_L, \ell^+_L\}$ $\bar{f}\|c_R\|^2\|b_L\|^2\|a_L\|^2$	$\{q_R, \ell^+_L, \ell^-_L\}$ $f\|c_R\|^2\|b_L\|^2\|a_L\|^2$	$\{\bar{q}_R, \ell^-_L, \ell^+_R\}$ $\bar{f}\|c_R\|^2\|b_L\|^2\|a_R\|^2$	$\{q_R, \ell^+_L, \ell^-_R\}$ $f\|c_R\|^2\|b_L\|^2\|a_R\|^2$
$\{\bar{q}_L, \ell^-_L, \ell^+_L\}$ $\bar{f}\|c_L\|^2\|b_L\|^2\|a_L\|^2$	$\{q_L, \ell^+_L, \ell^-_L\}$ $f\|c_L\|^2\|b_L\|^2\|a_L\|^2$	$\{\bar{q}_L, \ell^-_L, \ell^+_R\}$ $\bar{f}\|c_L\|^2\|b_L\|^2\|a_R\|^2$	$\{q_L, \ell^+_L, \ell^-_R\}$ $f\|c_L\|^2\|b_L\|^2\|a_R\|^2$
$\{q_L, \ell^-_R, \ell^+_R\}$ $f\|c_L\|^2\|b_R\|^2\|a_R\|^2$	$\{\bar{q}_L, \ell^+_R, \ell^-_R\}$ $\bar{f}\|c_L\|^2\|b_R\|^2\|a_R\|^2$	$\{q_L, \ell^-_R, \ell^+_L\}$ $f\|c_L\|^2\|b_R\|^2\|a_L\|^2$	$\{\bar{q}_L, \ell^+_R, \ell^-_L\}$ $\bar{f}\|c_L\|^2\|b_R\|^2\|a_L\|^2$
$\{\bar{q}_R, \ell^-_R, \ell^+_R\}$ $\bar{f}\|c_R\|^2\|b_R\|^2\|a_R\|^2$	$\{q_R, \ell^+_R, \ell^-_R\}$ $f\|c_R\|^2\|b_R\|^2\|a_R\|^2$	$\{\bar{q}_R, \ell^-_R, \ell^+_L\}$ $\bar{f}\|c_R\|^2\|b_R\|^2\|a_L\|^2$	$\{q_R, \ell^+_R, \ell^-_L\}$ $f\|c_R\|^2\|b_R\|^2\|a_L\|^2$
$\{q_R, \ell^-_L, \ell^+_L\}$ $f\|c_R\|^2\|b_L\|^2\|a_L\|^2$	$\{\bar{q}_R, \ell^+_L, \ell^-_L\}$ $\bar{f}\|c_R\|^2\|b_L\|^2\|a_L\|^2$	$\{q_R, \ell^-_L, \ell^+_R\}$ $f\|c_R\|^2\|b_L\|^2\|a_R\|^2$	$\{\bar{q}_R, \ell^+_L, \ell^-_R\}$ $\bar{f}\|c_R\|^2\|b_L\|^2\|a_R\|^2$
Processes P_{21}		Processes P_{22}	

What is the question that we ask?

- Given the data, which of the 6 spin configurations gives a good fit for some choice of the unknown parameters

- We only assume that we know things that would be measured at the time:
  - Mass spectrum

- We allow all remaining unknown parameters to float
  - Chirality of the couplings
  - Superpartner mixing angles
  - Particle-antiparticle fraction

What is wrong with the previous methods for measuring the chirality of SUSY couplings?

What is wrong with the previous methods for measuring the chirality of SUSY couplings?

- What methods for measuring chirality ?

What is wrong with the previous methods for measuring the mixing angles of superpartners?

What is wrong with the previous methods for measuring the mixing angles of superpartners?

- What methods for measuring mixing angles?
- Our method allows to measure not only the spins, but also some combinations of the coupling chiralities and the mixing angles of the new particles

Why do you need to read our paper?

- It is correct.
- You won't need to read the previous literature.
- Our proposed distributions will be studied anyway.
- You will learn a clever trick
- Referee says: The authors have presented a valuable analytic exploration of the spin dependence of the well-studied invariant mass distributions expected in SUSY, UED, and other models. Their approach is completely model independent, unlike all other spin studies to my knowledge, and therefore is far more important than all other spin studies to date. Therefore I recommend publication. These distributions will be used anyway by experimentalists, and their spin properties should be understood and utilized. The creation of new distributions with easily understood properties from the usual invariant masses is quite clever.

How should you read our paper?

- Most common complaint: I would love to read your paper, but it's 67 pages! Come on, do you really expect me to read all that stuff? Who has time for that?

How should you read our paper?

- Most common complaint: I would love to read your paper, but it's 67 pages! Come on, do you really expect me to read all that stuff? Who has time for that?

- Good news: you don't have to read all of it.

- Reading guide:
  - Sec. I: Intro (10 pages, no formulas)
  - Skip Secs. II and III (16 pages, lots of formulas)
  - Sec. IV: The method. (6 pages, 3 basic formulas)
  - Sec. V: Examples. (15 pages, only plots, no formulas)
  - Skip Sec VI: Conclusions. Do not even look at Appendix A, B or C.

What does our paper actually say?

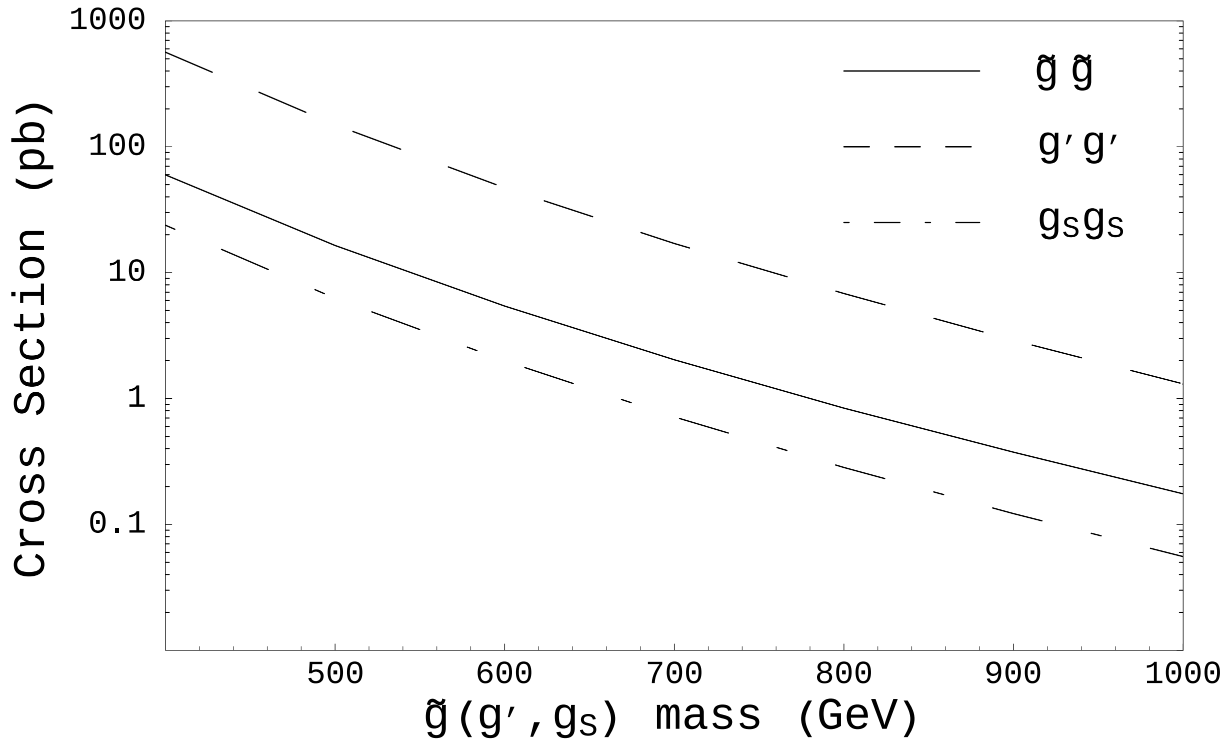
- no simulation
- no background
  - no SM backgrounds
  - no combinatorial backgrounds
    (this depends on the decay of the other side)
  - will include ambiguity between q and $\bar{q}$, and ℓ_n and ℓ_f
- Assume all masses are known
- Present analytical approach to measurement of spins, coupling and mixing angles

Spin determination

- cross section (spin of mother particle + branching fractions)
- azimuthal angle between production and decay planes (spin of intermediate particle)
- production angle (spin of mother particle)
- threshold scan (spin of mother particle)
- asymmetry (exploits shape of invariant mass distributions)
- $\cdots$

Spin determination - production cross-section

- G. Kane et al: "Basically, if the mass and the production cross-section are measured, the spin is then determined" (arXiv:0805.1397)



Spin determination - production cross-section

- Are we really measuring the production cross-section?

$$Rate = \mathcal{L}\left[\sigma(XX) + \sigma(XY)BR(Y \to X) + \sigma(YY)BR^2(Y \to X)\right] BR^2(X \to SM)$$

- How can we be sure that
  - There is no contribution from indirect production of particle Y?
  - Think of W pair production from top quarks
  - The branching fraction B(X $\to$ SM) is 100 %?
- Even if we were able to measure
  - It depends on the mass of X (OK, it will be measured)
  - It depends on the mass of heavy t-channel paticles (?)
  - It depends on the coupling of X to gluons (?)
  - It depends on the coupling of X to quarks (?)
  - It depends on the representation (Number of colors) of X (?)
  - It depends on the spin of X (Yes! That's what we want)
- Conclusion: it is very unlikely that by measuring a single number (such as a cross-section) we will be able to determine the spin. We have to look at distributions.

What is a good distribution to look at?

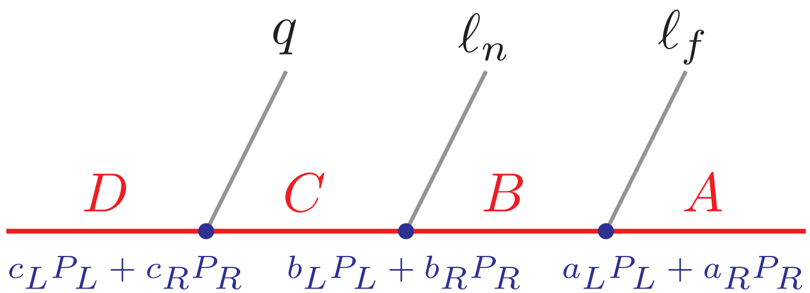
- Invariant mass distributions!
- Advantages: well studied, know about spin.
- For adjacent SM particles

$$\frac{dN}{dm^2} = a_0 + a_2 m^2 + a_4 m^4 + \cdots$$

- Plot versus m^2!
- For an intermediate BSM particle of spin s, the highest order term is m^{4s}
- For non-adjacent BSM particles, there are log terms as well.
- Disadvantage: know about many other things (hidden in the coefficients a), not all of which are measured!
  - Masses m_A, m_B, m_C, m_D (x,y,z)
  - Couplings and mixing angles (g_L and g_R)
  - Particle-antiparticle (D/D*) fraction (f/f*) (f+f*=1)

Model-independent approach

- Most general parameterization of the couplings



$$\mathcal{L}(F, f, \Phi) = \bar{\Psi}_F (g_L P_L + g_R P_R) \Psi_f \Phi + h.c.$$

$$\mathcal{L}(F, f, A_\mu) = \bar{\Psi}_F \gamma^\mu (g_L P_L + g_R P_R) \Psi_f A_\mu + h.c.$$

- Each vertex has 4 real parameters, $|a_L|$, $|a_R|$, 2 phases
  - The phases are unobservable
  - The product $|a_L| \cdot |a_R|$ absorbed in the normalization
  - The ratios $|a_R|/|a_L|$ defined as new parameters

$$\tan \varphi_a = \frac{|a_R|}{|a_L|}\,, \qquad \tan \varphi_b = \frac{|b_R|}{|b_L|}\,, \qquad \tan \varphi_c = \frac{|c_R|}{|c_L|}\,.$$

Helicity combinations

- We cannot measure the fermion helicity
- Previous work considered only certain helicity combinations
- We also consider all remaining possibilities
- Each block gives rise to a specific function $\mathcal{F}^{(p)}_{S;IJ}$
- The functions $\mathcal{F}_{S;IJ}$ provide a model independent basis for the spin analysis

$$\left(\frac{dN}{d\hat{m}_p^2}\right)_S = \sum_{I=1}^{2}\sum_{J=1}^{2} K^{(p)}_{IJ}(f,\varphi_a,\varphi_b,\varphi_c)\mathcal{F}^{(p)}_{S;IJ}(\hat{m}_p^2;x,y,z) \ ,$$

- The index p denotes one of the five possible SM particle pairs: $p = \{j\ell_n^-, j\ell_n^+, j\ell_f^-, j\ell_f^+, \ell^+\ell^-\}$; $\hat{m}_p$ is the unit-normalised invariant mass

$$\hat{m}_p \equiv \frac{m_p}{m_p^{max}} \ , \qquad 0 \le \hat{m}_p \le 1 \ ,$$

  i.e. the invariant mass m_p scaled by the value of the corresponding kinematic endpoint m_p^{max}, which has already been measured from the corresponding m_p distribution.
- K does not depend on masses and spins, only depends on couplings and $f/\bar{f}$.
- $\mathcal{F}$ does not depend on couplings and $f/\bar{f}$, only depends on masses and spins.

Classification of helicity combinations

Processes P_{11}		Processes P_{12}	
$\{q_L, \ell^-_L, \ell^+_L\}$ $f\|c_L\|^2\|b_L\|^2\|a_L\|^2$	$\{\bar{q}_L, \ell^+_L, \ell^-_L\}$ $\bar{f}\|c_L\|^2\|b_L\|^2\|a_L\|^2$	$\{q_L, \ell^-_L, \ell^+_R\}$ $f\|c_L\|^2\|b_L\|^2\|a_R\|^2$	$\{\bar{q}_L, \ell^+_L, \ell^-_R\}$ $\bar{f}\|c_L\|^2\|b_L\|^2\|a_R\|^2$
$\{\bar{q}_L, \ell^-_R, \ell^+_R\}$ $\bar{f}\|c_L\|^2\|b_R\|^2\|a_R\|^2$	$\{q_L, \ell^+_R, \ell^-_R\}$ $f\|c_L\|^2\|b_R\|^2\|a_R\|^2$	$\{\bar{q}_L, \ell^-_R, \ell^+_L\}$ $\bar{f}\|c_L\|^2\|b_R\|^2\|a_L\|^2$	$\{q_L, \ell^+_R, \ell^-_L\}$ $f\|c_L\|^2\|b_R\|^2\|a_L\|^2$
$\{q_R, \ell^-_R, \ell^+_R\}$ $f\|c_R\|^2\|b_R\|^2\|a_R\|^2$	$\{\bar{q}_R, \ell^+_R, \ell^-_R\}$ $\bar{f}\|c_R\|^2\|b_R\|^2\|a_R\|^2$	$\{q_R, \ell^-_R, \ell^+_L\}$ $f\|c_R\|^2\|b_R\|^2\|a_L\|^2$	$\{\bar{q}_R, \ell^+_R, \ell^-_L\}$ $\bar{f}\|c_R\|^2\|b_R\|^2\|a_L\|^2$
$\{\bar{q}_R, \ell^-_L, \ell^+_L\}$ $\bar{f}\|c_R\|^2\|b_L\|^2\|a_L\|^2$	$\{q_R, \ell^+_L, \ell^-_L\}$ $f\|c_R\|^2\|b_L\|^2\|a_L\|^2$	$\{\bar{q}_R, \ell^-_L, \ell^+_R\}$ $\bar{f}\|c_R\|^2\|b_L\|^2\|a_R\|^2$	$\{q_R, \ell^+_L, \ell^-_R\}$ $f\|c_R\|^2\|b_L\|^2\|a_R\|^2$
$\{\bar{q}_L, \ell^-_L, \ell^+_L\}$ $\bar{f}\|c_L\|^2\|b_L\|^2\|a_L\|^2$	$\{q_L, \ell^+_L, \ell^-_L\}$ $f\|c_L\|^2\|b_L\|^2\|a_L\|^2$	$\{\bar{q}_L, \ell^-_L, \ell^+_R\}$ $\bar{f}\|c_L\|^2\|b_L\|^2\|a_R\|^2$	$\{q_L, \ell^+_L, \ell^-_R\}$ $f\|c_L\|^2\|b_L\|^2\|a_R\|^2$
$\{q_L, \ell^-_R, \ell^+_R\}$ $f\|c_L\|^2\|b_R\|^2\|a_R\|^2$	$\{\bar{q}_L, \ell^+_R, \ell^-_R\}$ $\bar{f}\|c_L\|^2\|b_R\|^2\|a_R\|^2$	$\{q_L, \ell^-_R, \ell^+_L\}$ $f\|c_L\|^2\|b_R\|^2\|a_L\|^2$	$\{\bar{q}_L, \ell^+_R, \ell^-_L\}$ $\bar{f}\|c_L\|^2\|b_R\|^2\|a_L\|^2$
$\{\bar{q}_R, \ell^-_R, \ell^+_R\}$ $\bar{f}\|c_R\|^2\|b_R\|^2\|a_R\|^2$	$\{q_R, \ell^+_R, \ell^-_R\}$ $f\|c_R\|^2\|b_R\|^2\|a_R\|^2$	$\{\bar{q}_R, \ell^-_R, \ell^+_L\}$ $\bar{f}\|c_R\|^2\|b_R\|^2\|a_L\|^2$	$\{q_R, \ell^+_R, \ell^-_L\}$ $f\|c_R\|^2\|b_R\|^2\|a_L\|^2$
$\{q_R, \ell^-_L, \ell^+_L\}$ $f\|c_R\|^2\|b_L\|^2\|a_L\|^2$	$\{\bar{q}_R, \ell^+_L, \ell^-_L\}$ $\bar{f}\|c_R\|^2\|b_L\|^2\|a_L\|^2$	$\{q_R, \ell^-_L, \ell^+_R\}$ $f\|c_R\|^2\|b_L\|^2\|a_R\|^2$	$\{\bar{q}_R, \ell^+_L, \ell^-_R\}$ $\bar{f}\|c_R\|^2\|b_L\|^2\|a_R\|^2$
Processes P_{21}		Processes P_{22}	

Invariant masses in the $\{\mathcal{F}^{(p)}_{S;\alpha}, \mathcal{F}^{(p)}_{S;\beta}, \mathcal{F}^{(p)}_{S;\gamma}, \mathcal{F}^{(p)}_{S;\delta}\}$ basis

- New basis

$$\begin{aligned}
\mathcal{F}^{(p)}_{S;\alpha} &= \frac{1}{4}\left\{\mathcal{F}^{(p)}_{S;11} - \mathcal{F}^{(p)}_{S;12} + \mathcal{F}^{(p)}_{S;21} - \mathcal{F}^{(p)}_{S;22}\right\} \ , \\
\mathcal{F}^{(p)}_{S;\beta} &= \frac{1}{4}\left\{\mathcal{F}^{(p)}_{S;11} + \mathcal{F}^{(p)}_{S;12} - \mathcal{F}^{(p)}_{S;21} - \mathcal{F}^{(p)}_{S;22}\right\} \ , \\
\mathcal{F}^{(p)}_{S;\gamma} &= \frac{1}{4}\left\{\mathcal{F}^{(p)}_{S;11} - \mathcal{F}^{(p)}_{S;12} - \mathcal{F}^{(p)}_{S;21} + \mathcal{F}^{(p)}_{S;22}\right\} \ , \\
\mathcal{F}^{(p)}_{S;\delta} &= \frac{1}{4}\left\{\mathcal{F}^{(p)}_{S;11} + \mathcal{F}^{(p)}_{S;12} + \mathcal{F}^{(p)}_{S;21} + \mathcal{F}^{(p)}_{S;22}\right\} \ ,
\end{aligned}$$

- Normalization

$$\int_0^\infty \mathcal{F}^{(p)}_{S;\alpha}(\hat{m}^2; x, y, z)\, d\hat{m}^2 = 0 \ , \quad \int_0^\infty \mathcal{F}^{(p)}_{S;\beta}(\hat{m}^2; x, y, z)\, d\hat{m}^2 = 0 \ ,$$

$$\int_0^\infty \mathcal{F}^{(p)}_{S;\gamma}(\hat{m}^2; x, y, z)\, d\hat{m}^2 = 0 \ , \quad \int_0^\infty \mathcal{F}^{(p)}_{S;\delta}(\hat{m}^2; x, y, z)\, d\hat{m}^2 = 1 \ .$$

- Observable distributions: $q\ell^+$, $q\ell^-$, $\ell^+\ell^-$, $q\ell^+\ell^-$

The method

- Observable distributions

$$
\begin{aligned}
L_S^{+-}(\hat{m}^2_{\ell\ell}; x, y, z, \alpha) &\equiv \left(\frac{dN}{d\hat{m}^2_{\ell\ell}}\right)_S = \mathcal{F}^{(\ell\ell)}_{S;\delta}(\hat{m}^2_{\ell\ell}; x, y, z) + \alpha\,\mathcal{F}^{(\ell\ell)}_{S;\alpha}(\hat{m}^2_{\ell\ell}; x, y, z) \\
S_S^{+-}(\hat{m}^2_{j\ell}; x, y, z, \alpha) &\equiv \left(\frac{dN}{d\hat{m}^2_{j\ell+}}\right)_S + \left(\frac{dN}{d\hat{m}^2_{j\ell-}}\right)_S \\
&= r_n^2\,\mathcal{F}^{(j\ell_n)}_{S;\delta}(r_n^2\hat{m}^2_{j\ell}; x, y, z) + r_f^2\,\mathcal{F}^{(j\ell_f)}_{S;\delta}(r_f^2\hat{m}^2_{j\ell}; x, y, z) \\
&+ \alpha\, r_f^2\,\mathcal{F}^{(j\ell_f)}_{S;\alpha}(r_f^2\hat{m}^2_{j\ell}; x, y, z)\ , \\
D_S^{+-}(\hat{m}^2_{j\ell}; x, y, z, \beta, \gamma) &\equiv \left(\frac{dN}{d\hat{m}^2_{j\ell+}}\right)_S - \left(\frac{dN}{d\hat{m}^2_{j\ell-}}\right)_S \\
&= \gamma\, r_f^2\,\mathcal{F}^{(j\ell_f)}_{S;\gamma}(r_f^2\hat{m}^2_{j\ell}; x, y, z) \\
&+ \beta\, r_f^2\,\mathcal{F}^{(j\ell_f)}_{S;\beta}(r_f^2\hat{m}^2_{j\ell}; x, y, z) - \beta\, r_n^2\,\mathcal{F}^{(j\ell_n)}_{S;\beta}(r_n^2\hat{m}^2_{j\ell}; x, y, z)
\end{aligned}
$$

- $\mathcal{F}$ functions are known in particular cascade decay ($q\ell\ell$ in this case).

- α, β and γ are theory parameters and these can be obtained by fitting experimental distributions with $\mathcal{F}$ functions.
- For $q\ell\ell$, they are

$$\begin{aligned}
\alpha(\varphi_b, \varphi_a) &= \cos 2\varphi_b \cos 2\varphi_a \ , \\
\beta(\tilde{\varphi}_c, \varphi_b) &= \cos 2\tilde{\varphi}_c \cos 2\varphi_b = (f - \bar{f}) \cos 2\varphi_c \cos 2\varphi_b \ , \\
\gamma(\varphi_a, \tilde{\varphi}_c) &= \cos 2\varphi_a \cos 2\tilde{\varphi}_c = (f - \bar{f}) \cos 2\varphi_a \cos 2\varphi_c \ ,
\end{aligned}$$

- The parameters α, β and γ are not completely independent from each other. For any given α, the physically allowed region in the (β, γ) parameter space is described by an envelope which satisfies

$$\begin{aligned}
&\alpha\beta \leq \gamma,\ \beta\gamma \leq \alpha,\ \gamma\alpha \leq \beta\,, &&\text{if } \alpha > 0, \beta > 0 \text{ and } \gamma > 0\,, \\
&\alpha\beta \geq \gamma,\ \beta\gamma \leq \alpha,\ \gamma\alpha \geq \beta\,, &&\text{if } \alpha > 0, \beta < 0 \text{ and } \gamma < 0\,, \\
&\alpha\beta \geq \gamma,\ \beta\gamma \geq \alpha,\ \gamma\alpha \leq \beta\,, &&\text{if } \alpha < 0, \beta > 0 \text{ and } \gamma < 0\,, \\
&\alpha\beta \leq \gamma,\ \beta\gamma \geq \alpha,\ \gamma\alpha \geq \beta\,, &&\text{if } \alpha < 0, \beta < 0 \text{ and } \gamma > 0\,.
\end{aligned}$$

- Couplings (mixing angles) are defined in terms of fitted values

$$
\begin{aligned}
\cos 2\varphi_a &= \pm\frac{1}{\beta}\sqrt{\alpha\beta\gamma}\,, \\
\cos 2\varphi_b &= \pm\frac{1}{\gamma}\sqrt{\alpha\beta\gamma}\,, \\
\cos 2\varphi_c &= \pm\frac{1}{f-\bar{f}}\frac{1}{\alpha}\sqrt{\alpha\beta\gamma}\,,
\end{aligned}
$$

where in all three equations one should take either the "$+$" or the "$-$" sign on the right-hand side.

- The origin of this two-fold ambiguity is easy to understand. Observe that the defining equations (1-1) for α, β and γ are invariant under the simultaneous transformations

$$
\varphi_a \longrightarrow \frac{\pi}{2} - \varphi_a\,, \quad \varphi_b \longrightarrow \frac{\pi}{2} - \varphi_b\,, \quad \varphi_c \longrightarrow \frac{\pi}{2} - \varphi_c\,,
$$

whose effect is precisely to flip the signs in the right-hand sides of definitions of parameters. Given the defining relation, this transformations are equivalent to the chirality exchange.

$$
|a_L| \leftrightarrow |a_R|\,, \quad |b_L| \leftrightarrow |b_R|\,, \quad |c_L| \leftrightarrow |c_R|\,.
$$

- Measurement of couplings (mixing angles)

$$|a_L| = \frac{1}{\sqrt{2}}\left(1 \pm \frac{1}{\beta}\sqrt{\alpha\beta\gamma}\right)^{\frac{1}{2}}, \qquad |a_R| = \frac{1}{\sqrt{2}}\left(1 \mp \frac{1}{\beta}\sqrt{\alpha\beta\gamma}\right)^{\frac{1}{2}},$$

$$|b_L| = \frac{1}{\sqrt{2}}\left(1 \pm \frac{1}{\gamma}\sqrt{\alpha\beta\gamma}\right)^{\frac{1}{2}}, \qquad |b_R| = \frac{1}{\sqrt{2}}\left(1 \mp \frac{1}{\gamma}\sqrt{\alpha\beta\gamma}\right)^{\frac{1}{2}},$$

$$|c_L| = \frac{1}{\sqrt{2}}\left(1 \pm \frac{1}{f-\bar{f}}\frac{1}{\alpha}\sqrt{\alpha\beta\gamma}\right)^{\frac{1}{2}}, |c_R| = \frac{1}{\sqrt{2}}\left(1 \mp \frac{1}{f-\bar{f}}\frac{1}{\alpha}\sqrt{\alpha\beta\gamma}\right)^{\frac{1}{2}},$$

- Measurement of $f/\bar{f}$

$$0 \le f \le \frac{1}{2}\left(1 - \sqrt{\frac{\beta\gamma}{\alpha}}\right) \quad \text{or} \quad \frac{1}{2}\left(1 + \sqrt{\frac{\beta\gamma}{\alpha}}\right) \le f \le 1 \ .$$

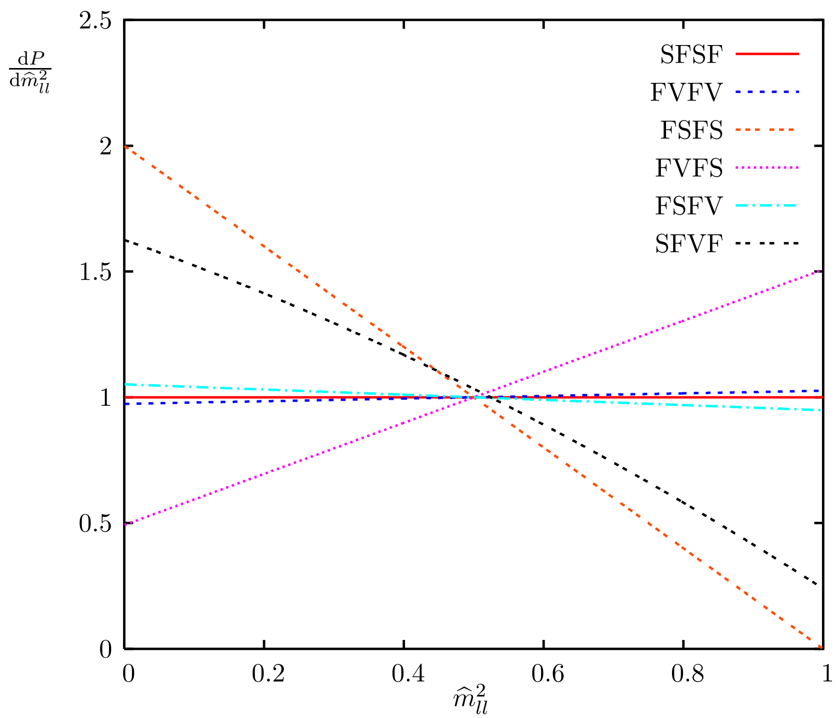
$$\mathrm{LHC}\,(pp) \quad : \quad \frac{1}{2}\left(1 + \sqrt{\frac{\beta\gamma}{\alpha}}\right) \le f \le 1 \ ,$$

$$\mathrm{anti-LHC}\,(\bar{p}\bar{p}) \quad : \quad 0 \le f \le \frac{1}{2}\left(1 - \sqrt{\frac{\beta\gamma}{\alpha}}\right) \ .$$

Does this really make any difference?

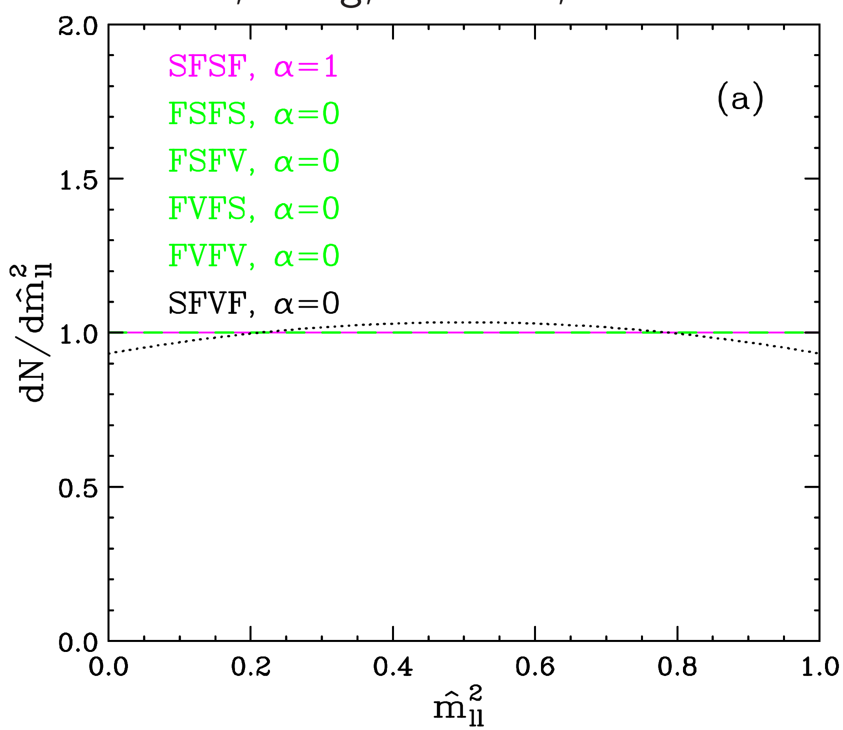
- Yes! Dilepton invariant mass distribution. Data from SPS1a.

Athanasiou, Lester, Smillie, Webber 06



- Spins vary
- Everything else fixed to SPS1a values
- Easy to distinguish!

Burns, Kong, Matchev, Park 08

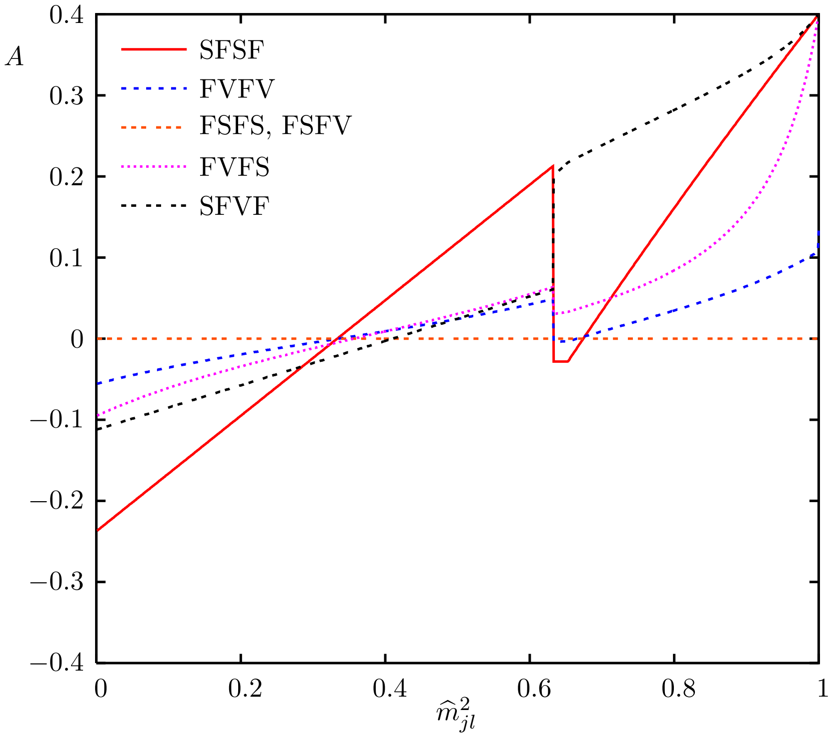


- Mass spectrum fixed to SPS1a values
- Everything else varies
- Difficult to distinguish!

Does this really make any difference?

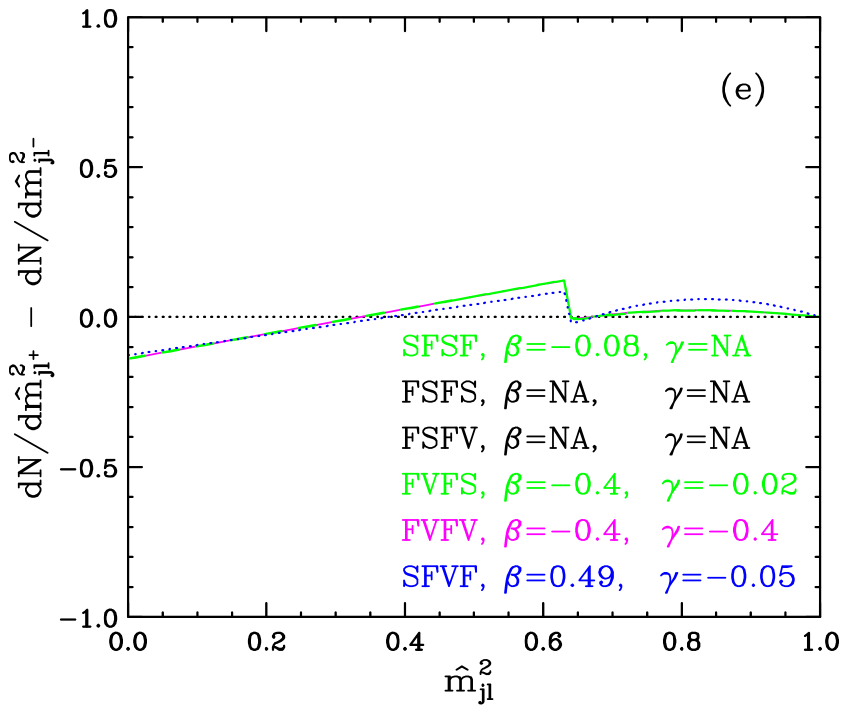
- Yes! Yes! Lepton charge (Barr) asymmetry. Data: "UED" with SPS1a mass spectrum.

Athanasiou, Lester, Smillie, Webber 06



- Spins vary
- Everything else fixed to SPS1a values
- Easy to distinguish!

Burns, Kong, Matchev, Park 08



- Mass spectrum fixed to SPS1a values
- Everything else varies
- Difficult to distinguish!

The twin spin scenario 1: FSFS/FSFV

- $\mathcal{F}$ functions (for any $p \in \{\ell\ell, j\ell_n, j\ell_f\}$) for FSFS and FSFV are related.

$$\begin{aligned} \mathcal{F}_{3;\alpha}^{(p)} &= \mathcal{F}_{2;\alpha}^{(p)} \frac{1-2z}{1+2z} \,, \\ \mathcal{F}_{3;\beta}^{(p)} &= \mathcal{F}_{2;\beta}^{(p)} = 0 \,, \\ \mathcal{F}_{3;\gamma}^{(p)} &= \mathcal{F}_{2;\gamma}^{(p)} = 0 \,, \\ \mathcal{F}_{3;\delta}^{(p)} &= \mathcal{F}_{2;\delta}^{(p)} \end{aligned}$$

- The relation

$$\alpha_2 = \alpha_3 \frac{1-2z}{1+2z}$$

is sufficient to guarantee that *all* invariant mass distributions (L^{+-}, S^{+-} and

D^{+-}) are exactly the same in the case of $S=2$ (FSFS) and $S=3$ (FSFV):

$$L_2^{+-}\left(\hat{m}^2_{\ell\ell};x,y,z,\alpha_3\frac{1-2z}{1+2z}\right) = L_3^{+-}\left(\hat{m}^2_{\ell\ell};x,y,z,\alpha_3\right) ,$$

$$S_2^{+-}\left(\hat{m}^2_{j\ell};x,y,z,\alpha_3\frac{1-2z}{1+2z}\right) = S_3^{+-}\left(\hat{m}^2_{j\ell};x,y,z,\alpha_3\right) ,$$

$$D_2^{+-}\left(\hat{m}^2_{j\ell};x,y,z,\beta_2,\gamma_2\right) = D_3^{+-}\left(\hat{m}^2_{j\ell};x,y,z,\beta_3,\gamma_3\right) .$$

- Our conclusion therefore is that the issue of confusing the two models FSFS and FSFV depends on whether the data comes from FSFV and we are trying to fit it with FSFS, or whether the data comes from FSFS and we are trying to fit it with FSFV. In the former case the two models will always be confused with each other, while in the latter case, the confusion arises only if α_2 happens to satisfy

$$|\alpha_2| \leq \left|\frac{1-2z}{1+2z}\right| .$$

The twin spin scenario 2: FVFS/FVFV

- $\mathcal{F}$ functions (for any $p \in \{\ell\ell, j\ell_n, j\ell_f\}$) for FVFS and FVFV are related.

$$
\begin{aligned}
\mathcal{F}^{(p)}_{5;\alpha} &= \mathcal{F}^{(p)}_{4;\alpha}\,\frac{1-2z}{1+2z}\,, \\
\mathcal{F}^{(p)}_{5;\beta} &= \mathcal{F}^{(p)}_{4;\beta}\,, \\
\mathcal{F}^{(p)}_{5;\gamma} &= \mathcal{F}^{(p)}_{4;\gamma}\,\frac{1-2z}{1+2z}\,, \\
\mathcal{F}^{(p)}_{5;\delta} &= \mathcal{F}^{(p)}_{4;\delta}
\end{aligned}
$$

- Redefine parameters:

$$
\begin{aligned}
\alpha_4 &= \alpha_5\,\frac{1-2z}{1+2z}\,, \\
\beta_4 &= \beta_5\,, \\
\gamma_4 &= \gamma_5\,\frac{1-2z}{1+2z}
\end{aligned}
$$

which would once again guarantee that *all* invariant mass distributions are exactly the same in these two cases:

$$L_4^{+-}\left(\hat{m}^2_{\ell\ell}; x, y, z, \alpha_5\frac{1-2z}{1+2z}\right) = L_5^{+-}\left(\hat{m}^2_{\ell\ell}; x, y, z, \alpha_5\right) ,$$

$$S_4^{+-}\left(\hat{m}^2_{j\ell}; x, y, z, \alpha_5\frac{1-2z}{1+2z}\right) = S_5^{+-}\left(\hat{m}^2_{j\ell}; x, y, z, \alpha_5\right) ,$$

$$D_4^{+-}\left(\hat{m}^2_{j\ell}; x, y, z, \beta_5, \gamma_5\frac{1-2z}{1+2z}\right) = D_5^{+-}\left(\hat{m}^2_{j\ell}; x, y, z, \beta_5, \gamma_5\right) .$$

- Following the same logic as before, we conclude that whenever the data comes from FVFV, the model will always be confused with FVFS. However, if the data comes from FVFS, the confusion arises only if α_4 and γ_4 happen to satisfy

$$|\alpha_4| \leq \left|\frac{1-2z}{1+2z}\right| ,$$

$$|\gamma_4| \leq \left|\frac{1-2z}{1+2z}\right| .$$

In addition to these two equations, the values of α_4, β_4 and γ_4 must also satisfy the domain constraints.

Spin determination at the Tevatron

- At a $p\bar{p}$ collider such as the Tevatron, the symmetry of the initial state implies

$$f = \bar{f} = \frac{1}{2}$$

- On the surface, it may appear that this constraint eliminates only one out of the four model-dependent degrees of freedom (f, φ_a, φ_b and φ_c) that we originally started with. However, the constraint $\cos 2\varphi = (f - \bar{f}) \cos 2\varphi$ in fact completely fixes the $\tilde{\varphi}_c$ parameter

$$\tilde{\varphi}_c = \frac{\pi}{4}$$

and as a result both β and γ vanish identically:

$$\beta = \gamma = 0 \ .$$

- In that case we have

$$D_S^{+-} \equiv 0$$

- A similar result holds for the lepton charge asymmetry

$$A_S^{+-} \equiv 0 \ .$$

Determination of spins and couplings: examples

- For the SPS1a mass spectrum we take the values used in literatures

  $$m_A = 96 \text{ GeV}, \quad m_B = 143 \text{ GeV}, \quad m_C = 177 \text{ GeV}, \quad m_D = 537 \text{ GeV} \ ,$$

  which translate into

  $$x = 0.109, \quad y = 0.653, \quad z = 0.451 \ .$$

- SPS1a is characterised by the following approximate values for the coupling constants

  $$a_L = 0, \quad a_R = 1, \quad b_L = 0, \quad b_R = 1, \quad c_L = 1, \quad c_R = 0,$$

  and particle-antiparticle fractions f and $\bar{f}$ at the LHC

  $$f = 0.7, \quad \bar{f} = 0.3 \ .$$

- The spectrum results in the following kinematic endpoints.

$$
\begin{aligned}
m_{\ell\ell}^{max} &= m_D\sqrt{x(1-y)(1-z)} = 77.31 \text{ GeV} , \\
m_{j\ell_n}^{max} &= m_D\sqrt{(1-x)(1-y)} = 298.77 \text{ GeV} , \\
m_{j\ell_f}^{max} &= m_D\sqrt{(1-x)(1-z)} = 375.76 \text{ GeV} , \\
m_{j\ell\ell}^{max} &= m_D\sqrt{(1-x)(1-yz)} = 425.94 \text{ GeV} .
\end{aligned}
$$

- Since we assume that the spectrum has been measured, the values of these endpoints are also known in advance of the spin measurement. We are therefore still allowed to write the measured invariant mass distributions in terms of the dimensionless invariant masses.
- Substituting the SPS1a parameter choice into the definitions of angles yields the following values of our model-dependent parameters α, β and γ

$$
\alpha = 1, \quad \beta = -0.4, \quad \gamma = -0.4 .
$$

- Fitting procedure

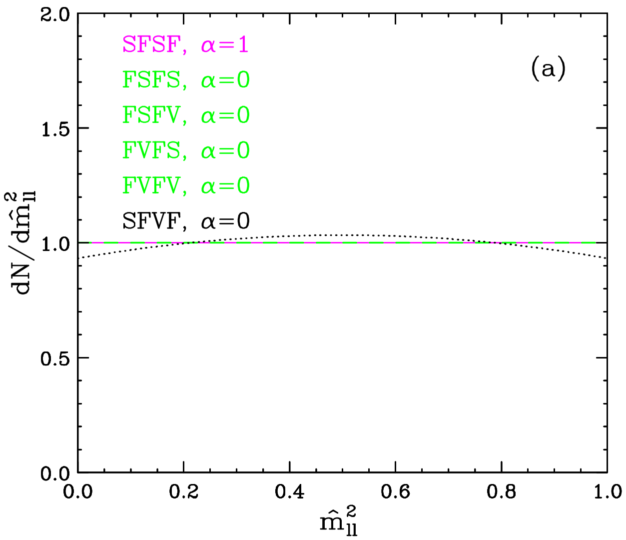
$$
\chi^2(\alpha,\beta,\gamma) \equiv \int_0^1 \Big(f_0(\hat{m}^2,\alpha_0,\beta_0,\gamma_0) - f(\hat{m}^2,\alpha,\beta,\gamma) \Big)^2 d\hat{m}^2 ,
$$

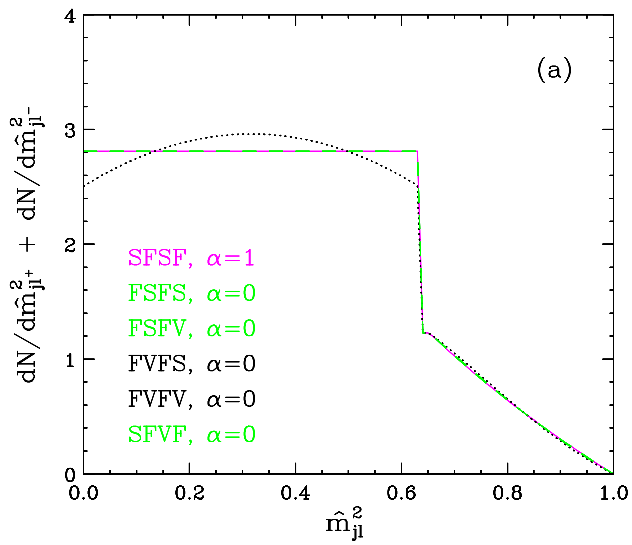
SFSF example ($S = 1$)

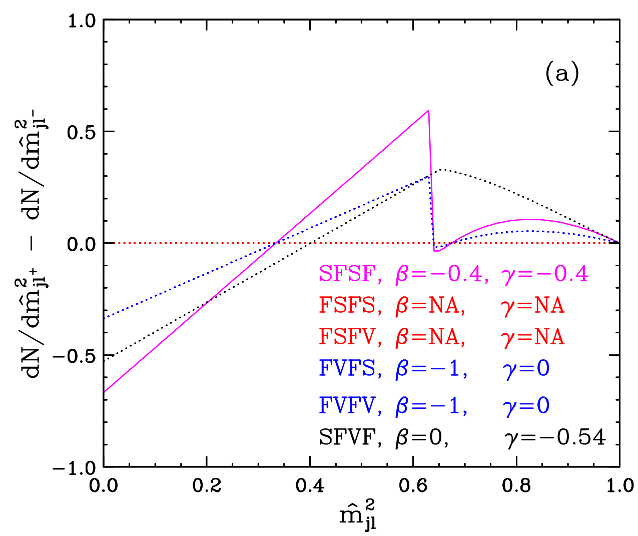
$$L_1^{+-} = 1\,,$$

$$S_1^{+-} = \begin{cases} 2.810 & \hat{m}_{j\ell}^2 \leq 0.632 \\ 1.228 & 0.632 \leq \hat{m}_{j\ell}^2 \leq 0.653 \\ -2.880\ \log \hat{m}_{j\ell}^2 & 0.653 \leq \hat{m}_{j\ell}^2\,, \end{cases}$$

$$D_1^{+-} = \begin{cases} -0.668 + 2.002\,\hat{m}_{j\ell}^2 & \hat{m}_{j\ell}^2 \leq 0.632 \\ -0.035 & 0.632 \leq \hat{m}_{j\ell}^2 \leq 0.653 \\ 6.633 - 6.633\,\hat{m}_{j\ell}^2 + 5.481\ \log \hat{m}_{j\ell}^2 & 0.653 \leq \hat{m}_{j\ell}^2\,. \end{cases}$$





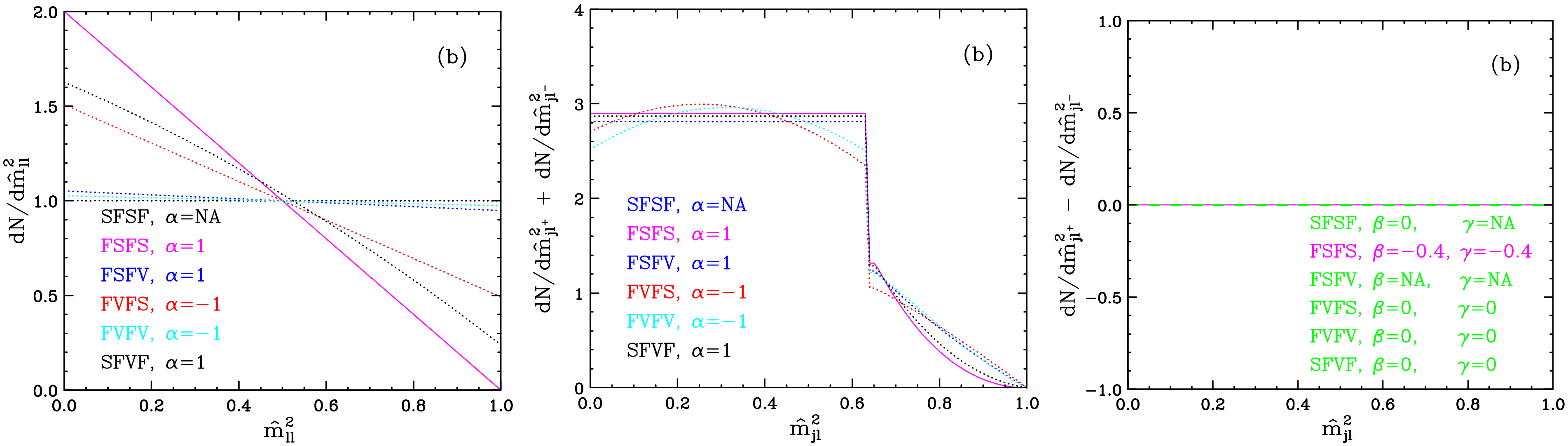


FSFS example ($S = 2$)

$$L_2^{+-} = 2 - 2\hat{m}^2_{\ell\ell}\,,$$

$$S_2^{+-} = \begin{cases} 2.898 & \hat{m}^2_{j\ell} \leq 0.632 \\ 1.316 & 0.632 \leq \hat{m}^2_{j\ell} \leq 0.653 \\ -16.583 + 16.583\,\hat{m}^2_{j\ell} - 16.583\,\log \hat{m}^2_{j\ell} & 0.653 \leq \hat{m}^2_{j\ell}\,, \end{cases}$$

$$D_2^{+-} = 0\,,$$

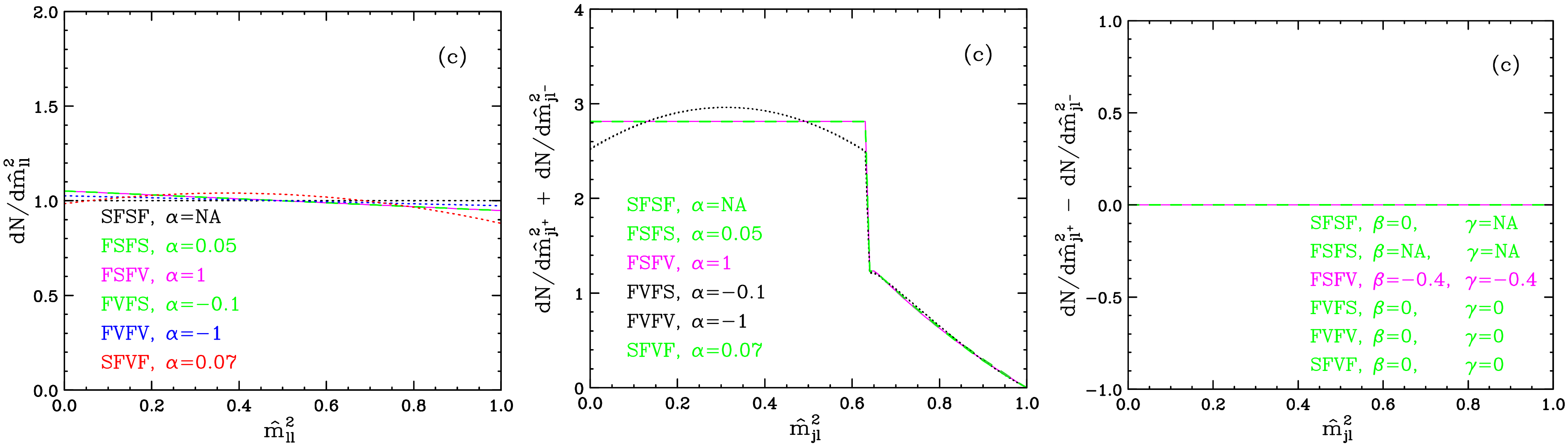


FSFV example ($S = 3$)

$$L_3^{+-} = 1.052 - 0.104\,\hat{m}^2_{\ell\ell}\,,$$

$$S_3^{+-} = \begin{cases} 2.815 & \hat{m}^2_{j\ell} \le 0.632 \\ 1.233 & 0.632 \le \hat{m}^2_{j\ell} \le 0.653 \\ -0.860 + 0.860\,\hat{m}^2_{j\ell} - 3.590\,\log \hat{m}^2_{j\ell} & 0.653 \le \hat{m}^2_{j\ell}\,, \end{cases}$$

$$D_3^{+-} = 0\,,$$

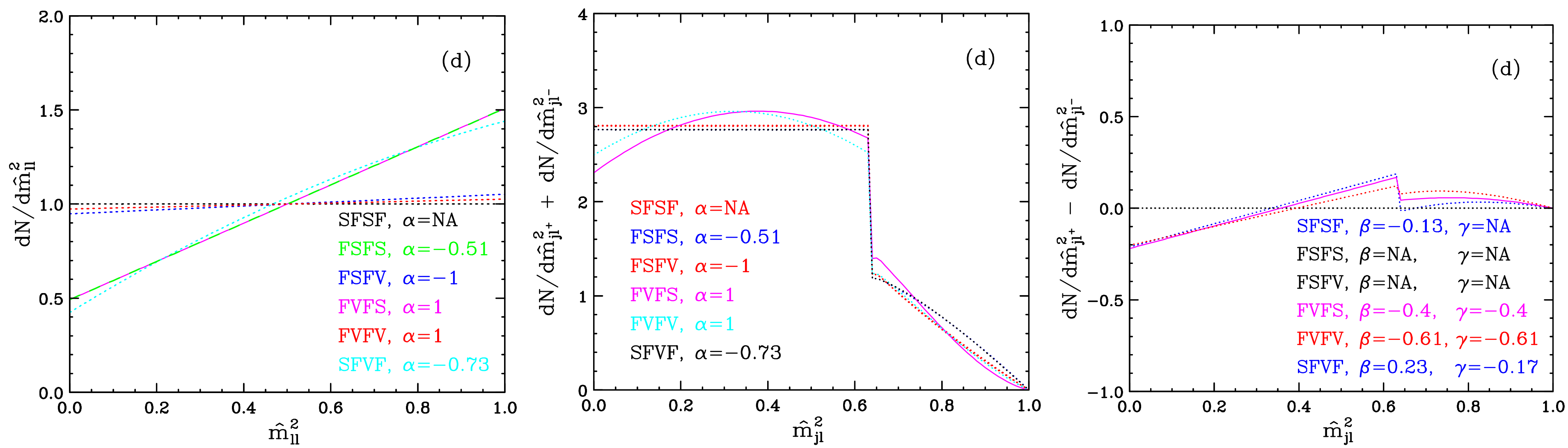


FVFS example ($S = 4$)

$$L_4^{+-} = 0.492 + 1.016\,\hat{m}_{\ell\ell}^2\,,$$

$$S_4^{+-} = \begin{cases} 2.307 + 3.455\,\hat{m}_{j\ell}^2 - 4.553\,\hat{m}_{j\ell}^4 & \hat{m}_{j\ell}^2 \leq 0.632 \\ 1.028 + 0.577\,\hat{m}_{j\ell}^2 & 0.632 \leq \hat{m}_{j\ell}^2 \leq 0.653 \\ -42.563 - 12.368\,\hat{m}_{j\ell}^2 + 54.931\,\hat{m}_{j\ell}^4 & \\ \quad - \left(7.871 + 90.785\,\hat{m}_{j\ell}^2\right)\,\log \hat{m}_{j\ell}^2 & 0.653 \leq \hat{m}_{j\ell}^2\,, \end{cases}$$

$$D_4^{+-} = \begin{cases} -0.22 + 0.616\,\hat{m}_{j\ell}^2 & \hat{m}_{j\ell}^2 \leq 0.632 \\ -0.092 + 0.212\,\hat{m}_{j\ell}^2 & 0.632 \leq \hat{m}_{j\ell}^2 \leq 0.653 \\ -3.087 + 3.087\,\hat{m}_{j\ell}^2 & \\ \quad - \left(0.874 + 2.678\,\hat{m}_{j\ell}^2\right)\,\log \hat{m}_{j\ell}^2 & 0.653 \leq \hat{m}_{j\ell}^2\,, \end{cases}$$

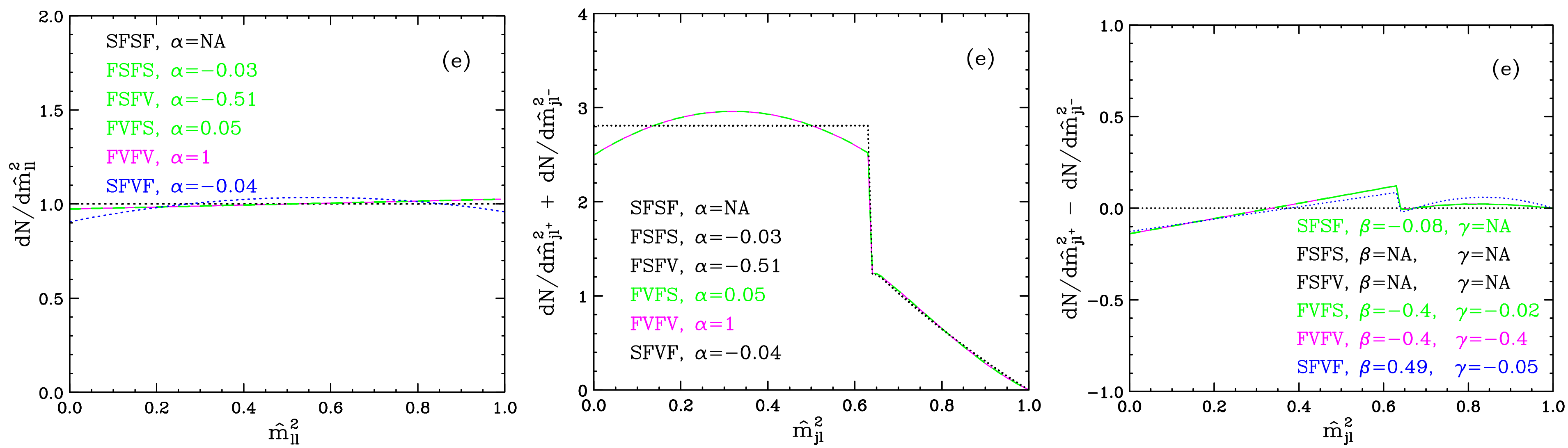


FVFV example ($S = 5$)

$$L_5^{+-} = 0.974 + 0.053\,\hat{m}_{\ell\ell}^2\,,$$

$$S_5^{+-} = \begin{cases} 2.496 + 2.908\,\hat{m}_{j\ell}^2 - 4.553\,\hat{m}_{j\ell}^4 & \hat{m}_{j\ell}^2 \leq 0.632 \\ 1.217 + 0.030\,\hat{m}_{j\ell}^2 & 0.632 \leq \hat{m}_{j\ell}^2 \leq 0.653 \\ 27.809 - 43.679\,\hat{m}_{j\ell}^2 + 15.870\,\hat{m}_{j\ell}^4 & \\ \quad + \left(14.382 - 4.710\,\hat{m}_{j\ell}^2\right)\,\log \hat{m}_{j\ell}^2 & 0.653 \leq \hat{m}_{j\ell}^2\,, \end{cases}$$

$$D_5^{+-} = \begin{cases} -0.139 + 0.415\,\hat{m}_{j\ell}^2 & \hat{m}_{j\ell}^2 \leq 0.632 \\ -0.011 + 0.011\,\hat{m}_{j\ell}^2 & 0.632 \leq \hat{m}_{j\ell}^2 \leq 0.653 \\ 1.109 - 1.109\,\hat{m}_{j\ell}^2 & \\ \quad + \left(1.004 - 0.139\,\hat{m}_{j\ell}^2\right)\,\log \hat{m}_{j\ell}^2 & 0.653 \leq \hat{m}_{j\ell}^2\,, \end{cases}$$

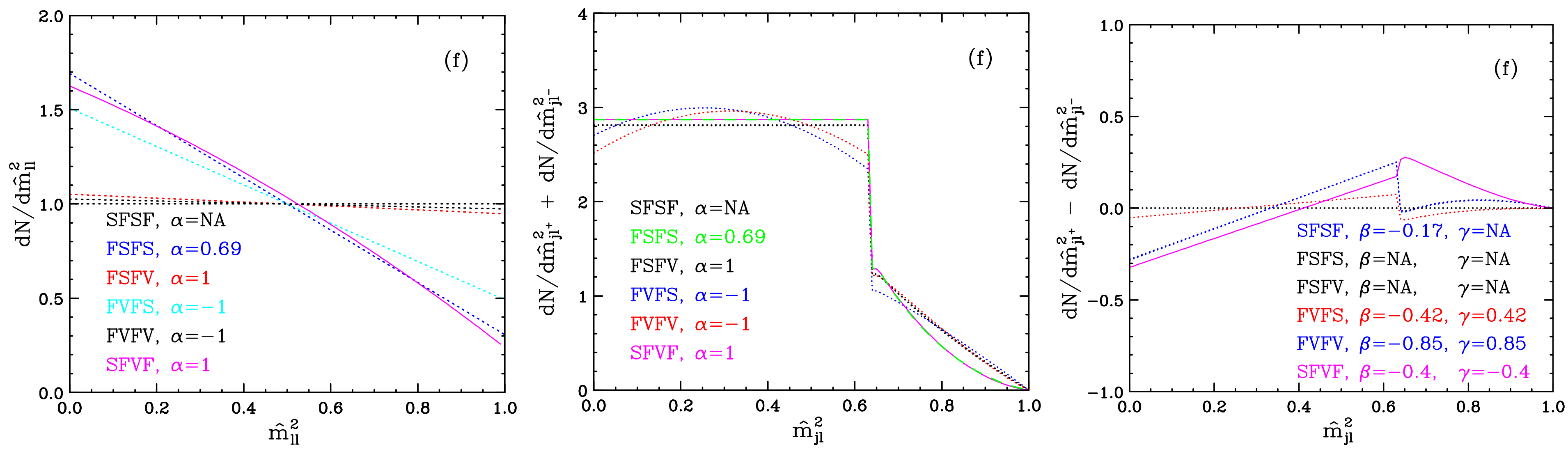


SFVF example ($S = 6$)

$$L_6^{+-} = 1.626 - 0.981\,\hat{m}_{\ell\ell}^2 - 0.405\,\hat{m}_{\ell\ell}^4\,,$$

$$S_6^{+-} = \begin{cases} 2.87 & \hat{m}_{j\ell}^2 \le 0.632 \\ 1.288 & 0.632 \le \hat{m}_{j\ell}^2 \le 0.653 \\ -0.344 - 4.493\,\hat{m}_{j\ell}^2 + 4.837\,\hat{m}_{j\ell}^4 - 5.870\,\log \hat{m}_{j\ell}^2 & 0.653 \le \hat{m}_{j\ell}^2, \end{cases}$$

$$D_6^{+-} = \begin{cases} -0.322 + 0.786\,\hat{m}_{j\ell}^2 & \hat{m}_{j\ell}^2 \le 0.632 \\ -0.406 + 1.051\,\hat{m}_{j\ell}^2 & 0.632 \le \hat{m}_{j\ell}^2 \le 0.653 \\ 5.870 - 11.674\,\hat{m}_{j\ell}^2 + 5.804\,\hat{m}_{j\ell}^4 & \\ \quad + \left(3.384 - 3.595\,\hat{m}_{j\ell}^2\right)\,\log \hat{m}_{j\ell}^2 & 0.653 \le \hat{m}_{j\ell}^2, \end{cases}$$



Determination of spins and couplings: SPS1a

Data from	Can this data be fitted by model					
	SFSF	FSFS	FSFV	FVFS	FVFV	SFVF
SFSF	yes	no	no	no	no	no
FSFS	no	yes	maybe	no	no	no
FSFV	no	yes	yes	no	no	no
FVFS	no	no	no	yes	maybe	no
FVFV	no	no	no	yes	yes	no
SFVF	no	no	no	no	no	yes

- Summary of the results from our spin discrimination analysis
- The two cases labelled "maybe" correspond to the potential confusion of an FSFS (FVFS) chain with an FSFV (FVFV) chain, which occurs only for a certain range of the model-dependent parameters.
- The reverse may also be true, depending on the mass spectrum
- How does simulation change conclusion of these tables ?

Measurements of couplings and mixing angles

$$\frac{dN}{d\hat{m}^2} = \mathcal{F}_{S;\delta} + \alpha \mathcal{F}_{S;\alpha} + \beta \mathcal{F}_{S;\beta} + \gamma \mathcal{F}_{S;\gamma}$$

- Available measurements of the model-dependent parameters α, β and γ for each of the six spin configurations

Spin chain	Parameters measured from distribution			
	L^{+-}	S^{+-}	D^{+-}	$L^{+-} \oplus S^{+-} \oplus D^{+-}$
SFSF	—	—	β	β
FSFS	α	α	—	α
FSFV	α	α	—	α
FVFS	α	α	β, γ	α, β, γ
FVFV	α	α	β, γ	α, β, γ
SFVF	α	α	β, γ	α, β, γ

- Good news for parameter fitters! Typically each distribution requires a 1-parameter fit (at worst, 2-parameter fit).

FVFS, FVFV and SFVF (three measurements)

- If we have correctly determined the spin chain, these values will be simply the starting SPS1a inputs. Substituting those in correct spin scenario, we obtain the two sets of solutions

$$\begin{aligned} |a_L| &= 0, \ |a_R| = 1, \ |b_L| = 0, \ |b_R| = 1, \\ |c_L| &= \sqrt{\frac{1}{2} + \frac{0.2}{2f-1}}, \ |c_R| = \sqrt{\frac{1}{2} - \frac{0.2}{2f-1}}, \end{aligned}$$

and

$$\begin{aligned} |a_L| &= 1, \ |a_R| = 0, \ |b_L| = 1, \ |b_R| = 0, \\ |c_L| &= \sqrt{\frac{1}{2} - \frac{0.2}{2f-1}}, \ |c_R| = \sqrt{\frac{1}{2} + \frac{0.2}{2f-1}}, \end{aligned}$$

- Constraint on f: $0.7 \leq f \leq 1$
- FVFS (S=4) fakes FVFV (S=5): our fitting procedure found

$$\alpha = 0.05, \quad \beta = -0.4, \quad \gamma = -0.02$$

- Corresponding solutions are

$$
\begin{aligned}
|a_L| &= 0.69,\ |a_R| = 0.72,\ |b_L| = 0,\ |b_R| = 1, \\
|c_L| &= \sqrt{\frac{1}{2} + \frac{0.2}{2f-1}},\ |c_R| = \sqrt{\frac{1}{2} - \frac{0.2}{2f-1}},
\end{aligned}
$$

or

$$
\begin{aligned}
|a_L| &= 0.72,\ |a_R| = 0.69,\ |b_L| = 1,\ |b_R| = 0, \\
|c_L| &= \sqrt{\frac{1}{2} - \frac{0.2}{2f-1}},\ |c_R| = \sqrt{\frac{1}{2} + \frac{0.2}{2f-1}}.
\end{aligned}
$$

- For the "wrong" spin chain we also obtain a constraint on the allowed range of the particle-antiparticle fraction f at the LHC:

$$0.7 \le f \le 1 \ .$$

SFSF and FSFS/FSFV

- SFSF: only β is measured

$$\cos 2\varphi_b \cos 2\tilde{\varphi}_c = -0.4 \;\; \text{or} \;\; (2f-1)\cos 2\varphi_b \cos 2\varphi_c = -0.4 \; .$$

- All four parameters φ_a, φ_b, φ_c and f remains completely unknown.
- FSFS and FSFV: α parameter (which gives us a relation between φ_a and φ_b) is measured

$$\alpha = \cos 2\varphi_b \cos 2\varphi_a = 1 \; .$$

- $|a_L|$, $|a_R|$, $|b_L|$ and $|b_R|$, up to the usual $L \leftrightarrow R$ ambiguity:

$$\varphi_a = \varphi_b = \frac{\pi}{2} \implies |a_L| = 0, \; |a_R| = 1, \; |b_L| = 0, \; |b_R| = 1 \; ,$$

or

$$\varphi_a = \varphi_b = 0 \implies |a_L| = 1, \; |a_R| = 0, \; |b_L| = 1, \; |b_R| = 0 \; .$$

- In either case, $|c_L|$, $|c_R|$ and f will remain unconstrained.

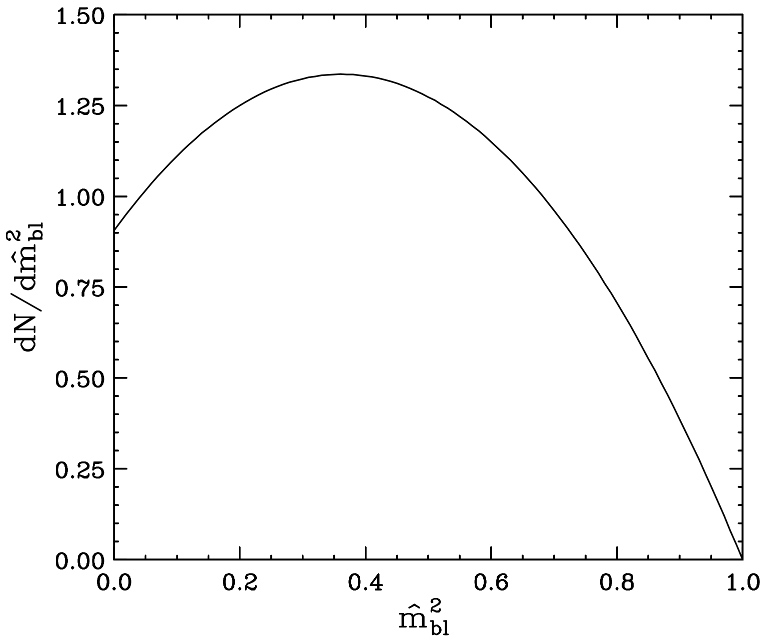
Are twin scenarios really identical ?

- What about $m_{q\ell\ell}$ distribution ?

- Require two identical cascade decays and apply kinematic constraints. In principle we know *all* the momenta of particles including two missing particles. What about invariant distributions involving a missing particle?

- What about other distributions such as p_T and η of visible particles ?

Application to the SM top



- SM predicts $\frac{dN}{d\hat{m}^2_{b\ell}} = 0.91 + 2.34\hat{m}^2 - 3.28\hat{m}^4$
- non-zero $\hat{m}^4$ confirms $W^\pm$ is a vector boson (confirms FVF spin chain)
- The values of coefficients give $\cos 2\phi_{tWb} \cos 2\phi_{W\ell\nu} = 1$, *i.e.*, tWb and $W\ell\nu$ have the same chiral structure (either left-handed or right-handed)
- There are two independent measurements from the shape
- One constraint from the $m_{b\ell}$ end point
- Can we really measure all three masses ? (Yes, can be done analytically)

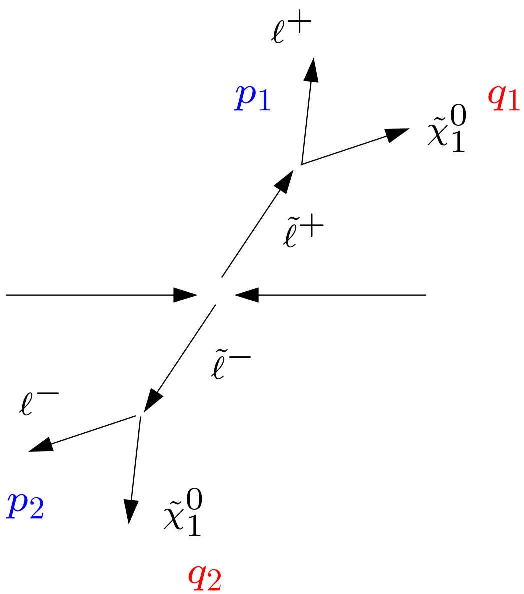
How to measure masses

- Resonance - no missing particles
- Transverse mass - a missing particle is balanced by visible particles
- One side of cascades - invariant masses and kinematic endpoints
- Two identical cascades
  - Kinematic constraints
  - m_{T_2} - measures mass difference only
- $\cdots$

What if there are two missing particles?: m_{T2}

(Barr, Lester, Stephens, hep-ph/0304226, "m(T2): The Truth behind the glamour")

(Lester, Summers, hep-ph/9906349)



- $\not{E}_T = \vec{q}_1 + \vec{q}_2 = -(\vec{p}_1 + \vec{p}_2)$
- If $\vec{q}_1$ and $\vec{q}_2$ are obtainable,

$$m_{\tilde{\ell}}^2 \geq max\{m_T^2(\vec{p}_1, \vec{q}_1), m_T^2(\vec{p}_2, \vec{q}_2)\}$$

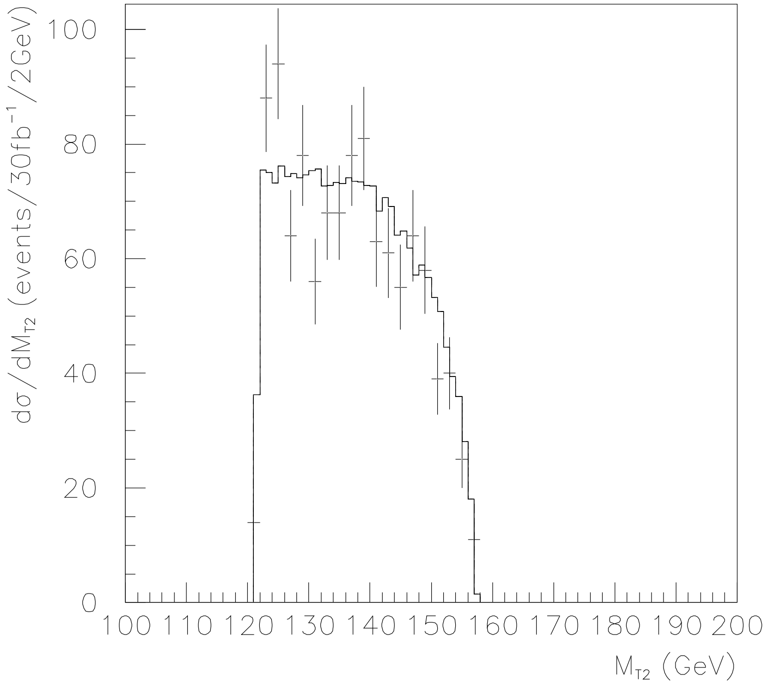
- But $\not{E}_T = \vec{q}_1 + \vec{q}_2 \longrightarrow$ the best we can say is that

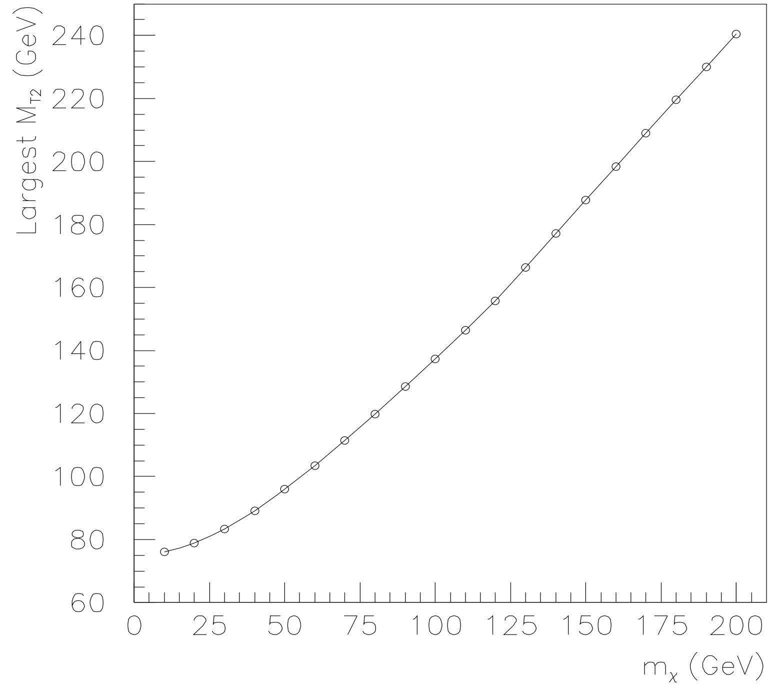
$$m_{\tilde{l}}^2 \geq m_{T2}^2 \equiv \min_{\not{q}_1 + \not{q}_2 = \not{E}_T} \left[\max\{m_T^2(\vec{p}_1, \vec{q}_1), m_T^2(\vec{p}_2, \vec{q}_2)\} \right]$$

What if there are two missing particles?: m_{T2}

(Barr, Lester, Stephens, hep-ph/0304226, "m(T2): The Truth behind the glamour")

(Lester, Summers, hep-ph/9906349)



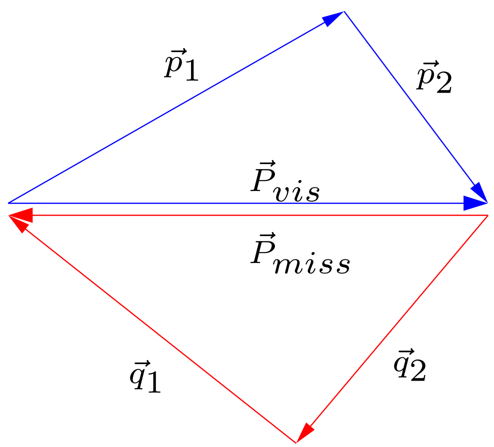


$$m_{\tilde{l}}^2 \geq m_{T2}^2 \equiv \min_{\not{q}_1 + \not{q}_2 = \not{E}_T} \left[\max \left\{ m_T^2(\vec{p}_1, \vec{q}_1), m_T^2(\vec{p}_2, \vec{q}_2) \right\} \right] \geq m_{\tilde{\chi}_1^0}^2$$

- Rely on momentum scan $\longrightarrow$ can be reduced to one dimensional parameter scan
  $\longrightarrow$ can not get analytic differential distribution
- Have to assume $m_{\tilde{\chi}_1^0}$ $\longrightarrow$ correlation between $m_{\tilde{l}}$ and $m_{\tilde{\chi}_1^0}$

The Cambridge m_{T2} Variable Demystified

(Kong, Matchev, 2006)



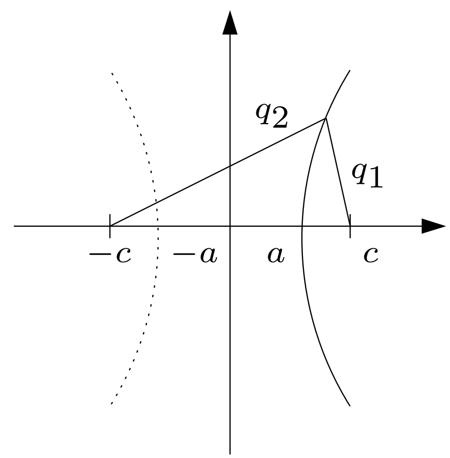
$$m_{T2}^2 \equiv \min_{\not{q}_1 + \not{q}_2 = \not{E}_T} \left[\max \left\{ m_T^2(\vec{p}_1, \vec{q}_1), m_T^2(\vec{p}_2, \vec{q}_2) \right\} \right]$$

Constraint: $m_T^2(\vec{p}_1, \vec{q}_1) = m_T^2(\vec{p}_2, \vec{q}_2)$

$$\rightarrow \sqrt{\vec{q}_2^{\,2} + m^2} - \sqrt{\vec{q}_1^{\,2} + m^2} = |\vec{p}_1| - |\vec{p}_2| > 0$$

- massless case ($m = 0$): WW production, $m_{\tilde{\chi}_1^0} << m_{\tilde{\ell}}$

$$2a \equiv p_1 - p_2 = q_2 - q_1$$
$$2c \equiv \not{E}_T$$
$$e = \frac{c}{a}$$

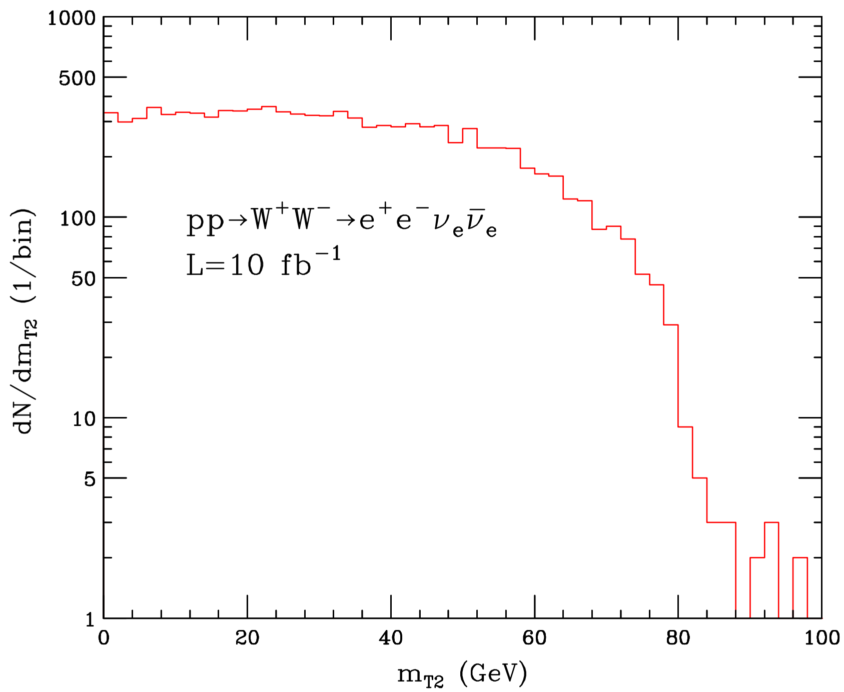


- Solution: $\vec{q}_1 = -\vec{p}_2$ and $\vec{q}_2 = -\vec{p}_1$
- Warning: $\vec{q}_1$ and $\vec{q}_2$ are NOT neutrino momenta

The Cambridge m_{T2} Variable Demystified

(Kong, Matchev, 2006)

- Applications:
  - Mass correlation even if there are two missing particles:
    W and slepton pair production
  - Can be used for background rejection

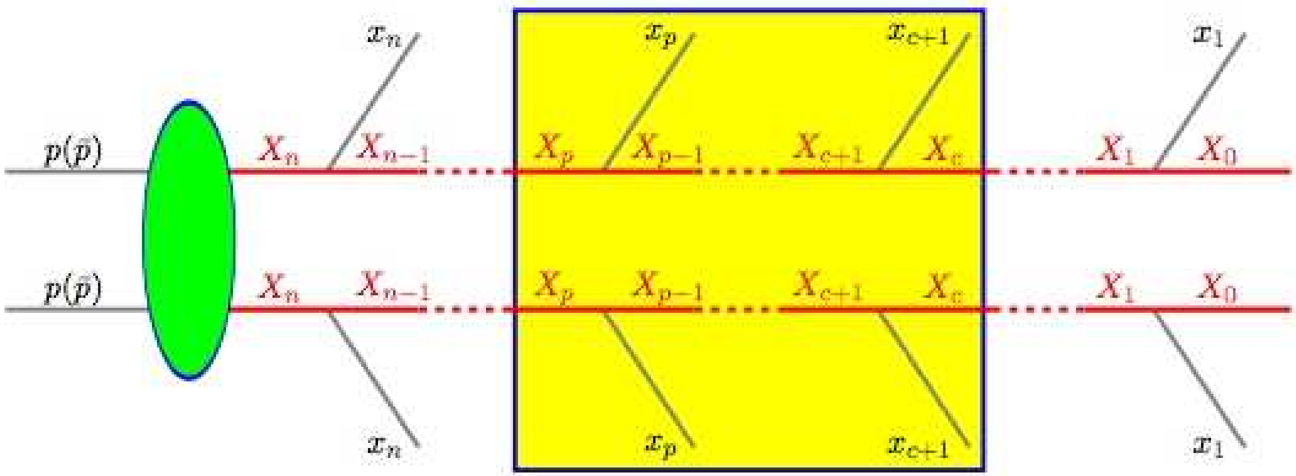


Recent progress on m_{T_2}

- $\cdots$
- unpublished, 2006 by Kong, Matchev
- arXiv:0708.1028 by Lester, Barr
- arXiv:0709.0288 by Cho, Choi, Kim, Park
- arXiv:0709.2740 by Gripaios
- arXiv:0711.4008 by Barr, Gripaios, Lester
- arXiv:0711.4526 by Cho, Choi, Kim, Park
- arXiv:0808.1094 by M. Nojiri, K. Sakurai, Y. Shimizu, M. Takeuchi
- arXiv:0810.4853, Cho, Choi, Kim, Park
- arXiv:0810.5178, Cheng, Han
- arXiv:0810.5576, by Burns, Kong, Matchev, Park
- $\cdots$

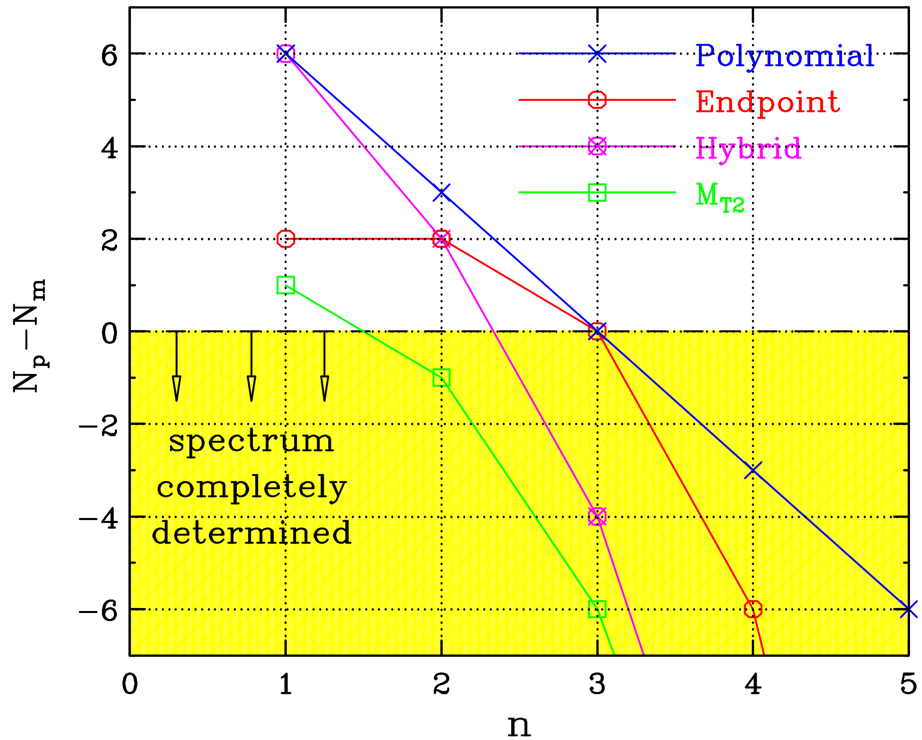
Subsystem m_{T2}

(Burns, Kong, Matchev, Park, arXiv:0810.5576)



- $m_{T2}^{(n,p,c)}$
- # of parameters - # of measurements
  - Endpoint method : $2(n+1) - 2^n$
  - Polynomial method : $9 - 3n \longrightarrow n + 1 - 2(n-2)N_{ev}$
  - m_{T2} method : $\frac{1}{6}(n+1)(6 - 2n - n^2)$

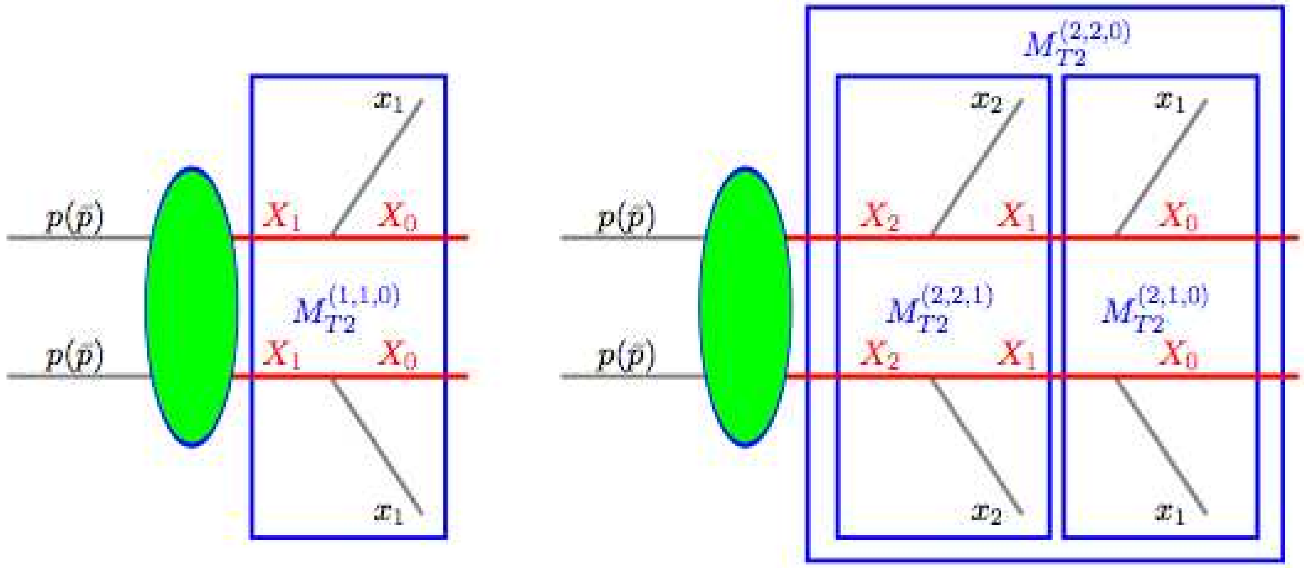
(Burns, Kong, Matchev, Park, arXiv:0810.5576)



- m_{T2} method: the only method that can give analytic solution to masses
- 4 constraints (naive counting) are not independent
- kink structure gives more constraints

Kink in subsystem m_{T2}

(Burns, Kong, Matchev, Park, arXiv:0810.5576)

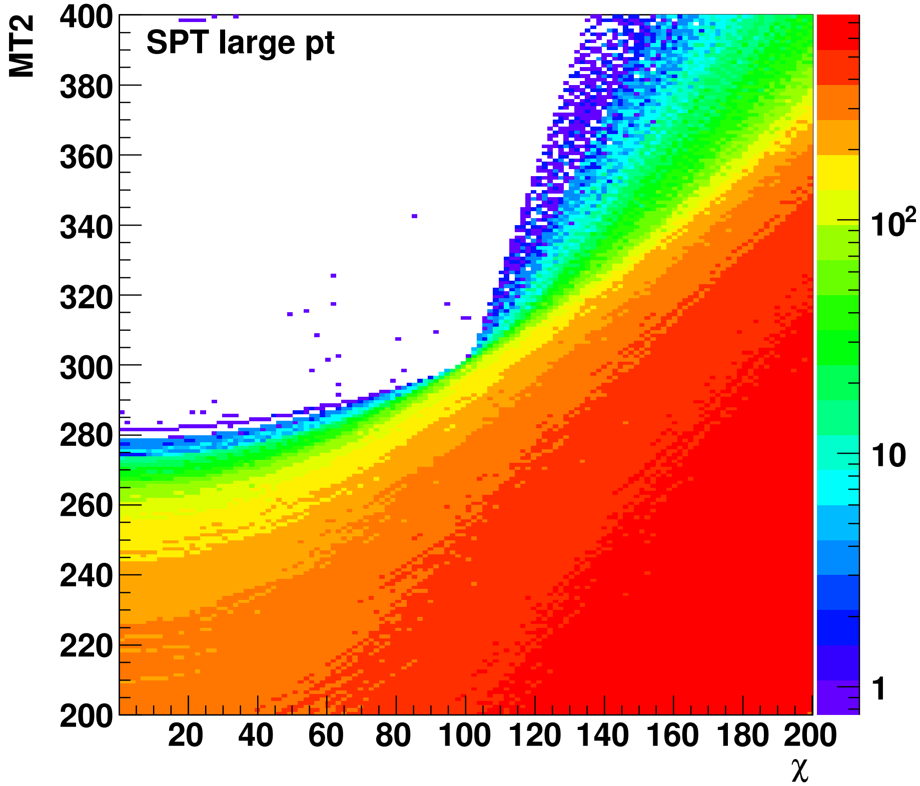


- invariant mass, m_{x1x2}
- $m_{T2,max}$ for $n = 2$:
  - $m_{T2}^{(1,1,0)}$ (kink in the presence of ISR with large p_T)
  - $m_{T2}^{(2,2,0)}$ (kink in the presence of ISR with large p_T)
  - $m_{T2}^{(2,2,1)}$ (kink)
  - $m_{T2}^{(2,1,0)}$ (kink)
- $\mu_{(n,p,c)} = \frac{M_n}{2}\left(1 - \frac{M_c^2}{M_p^2}\right)$

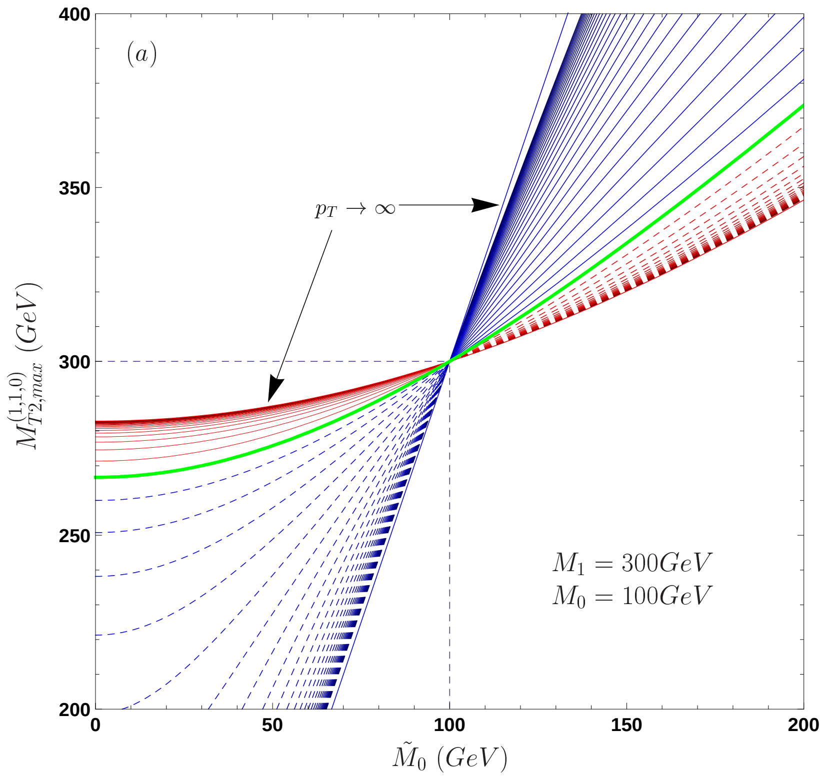
Kink in $m_{T2}^{(1,1,0)}$

- Slepton production with ISR

(Barr, Gripaios, Lester, arXiv:0711.4008)



(Burns, Kong, Matchev, Park, arXiv:0810.5576)

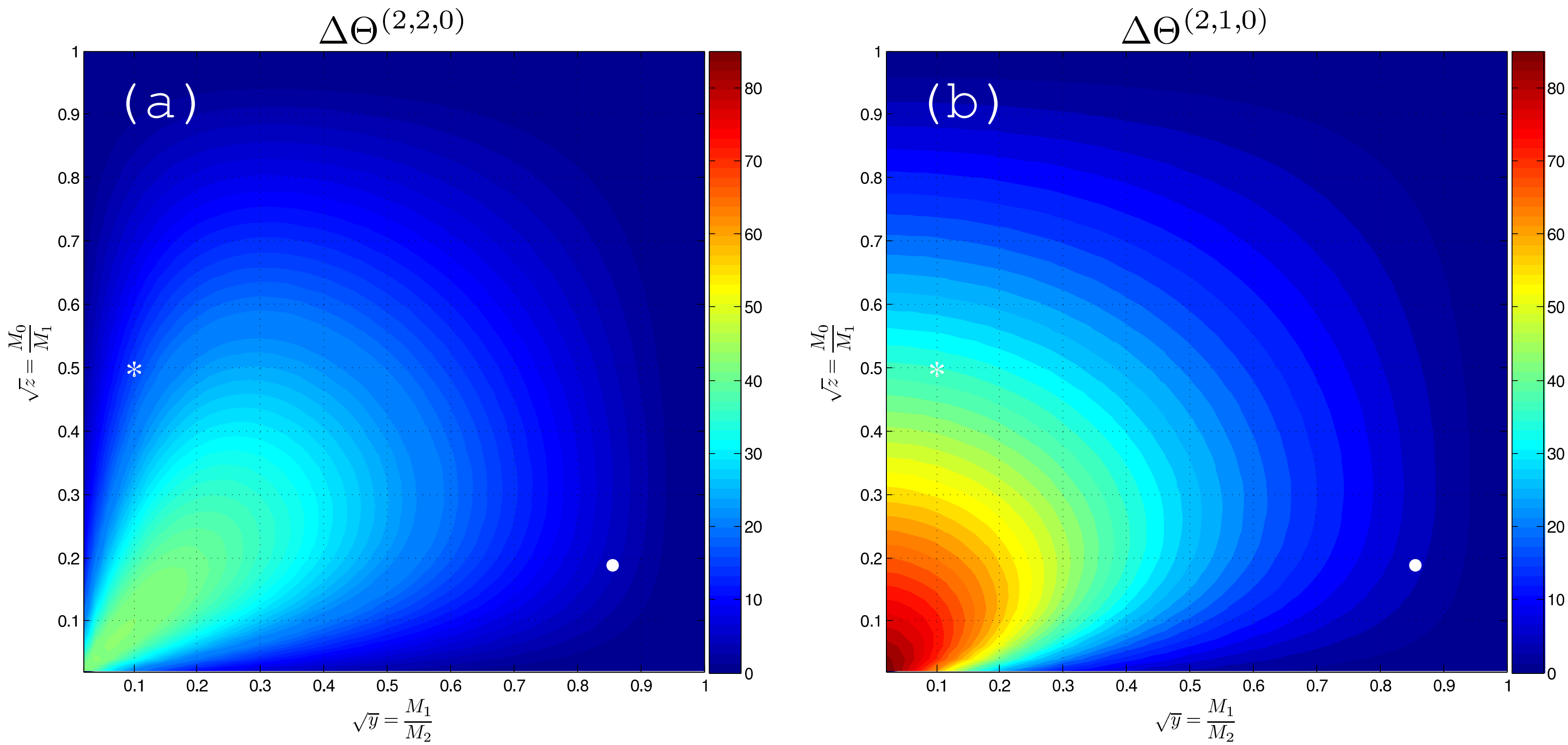


$$F_L^{(1,1,0)}(\tilde{M}_0, P_T) = \left\{ \left[\mu_{(1,1,0)}(P_T) + \sqrt{(\mu_{(1,1,0)}(P_T) + \frac{P_T}{2})^2 + \tilde{M}_0^2} \right]^2 - \frac{P_T^2}{4} \right\}^{\frac{1}{2}}$$

$$\mu_{(1,1,0)}(P_T) = \mu_{(1,1,0)} \left(\sqrt{1 + \left(\frac{P_T}{2M_1}\right)^2} - \frac{P_T}{2M_1} \right) \ , \ \ F_R^{(1,1,0)}(\tilde{M}_0, P_T) = F_L^{(1,1,0)}(\tilde{M}_0, -P_T)$$

Kink in $m_{T2}^{(2,2,0)}$ and $m_{T2}^{(2,1,0)}$

(Burns, Kong, Matchev, Park, arXiv:0810.5576)



$$F_L^{(2,10)}(\tilde{M}_0) = \left\{ \left[\mu_{(2,2,0)} - \mu_{(2,2,1)} + \sqrt{\mu^2_{(2,2,0)} + \tilde{M}_0^2} \right]^2 - \mu^2_{(2,2,1)} \right\}^{\frac{1}{2}}$$

$$F_R^{(2,10)}(\tilde{M}_0) = \left\{ \left[\mu_{(2,1,0)} + \sqrt{\left(\mu^2_{(2,2,1)} - \mu_{(2,1,0)} \right)^2 + \tilde{M}_0^2} \right]^2 - \mu^2_{(2,2,1)} \right\}^{\frac{1}{2}}$$

Summary and discussion

- LHC will find new physics (new particles) if it exists at TeV scale
- The next question is to measure the properties of new particles: masses, spins and couplings
- A model-independent way of measuring spins and couplings (mixing angles)
  - Make plots versus m^2
  - Spins can be measured from invariant mass distributions in a general and model-independent way
  - A side benefit of the method is the measurement of the couplings and mixing angles encoded in the parameters α, β and γ
  - CMS/ATLAS should measure α, β and γ and let the theorists figure out what are the underlying values of the model parameters that correspond to those
- Questions
  - Generalize to (longer, other?) decay chains? Other application ?
  - Backgrounds and detector simulations ? Are the twin scenarios really identical ?
  - Can we really measure all masses ?
- Generalization of m_{T_2} in the case of long cascade decay
  - Complete mass determination using m_{T2} only
  - Analytic solution makes m_{T_2} computation faster
  - Application to $m_{T_{Gen}}$ and better understanding of the kink structure