Lepton and Quark masses from Top loops

Patrick Fox

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Bogdan Dobrescu
to appear...

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Loopy masses for leptons and quarks

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Standard Model Higgs

Responsible for W, Z mass and (charged) fermion masses

Associated hierarchies:

Gauge hierarchy $m_W \ll M_{pl}$

Yukawa hierarchy $y_e \ll y_t$

Yukawa hierarchy

Technically natural but would still like an explanation

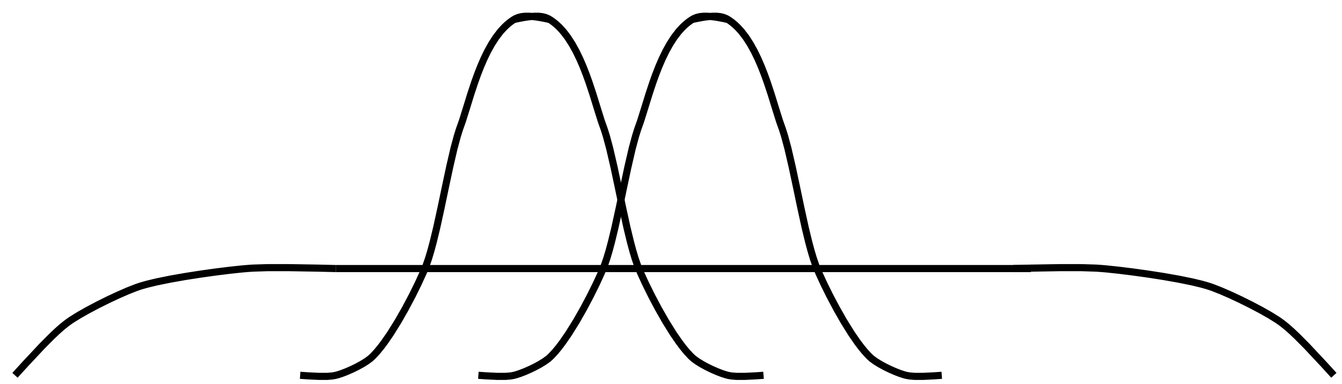
Symmetries (Froggatt Nielsen Models)

$$Y_{ij} \left(\frac{\phi}{M}\right)^{q_i+q_j+q_H} H\bar{\psi}_i\psi_j$$

$$Y_{ij}^{SM} = Y_{ij}\,\epsilon^{q_i+q_j+q_H} \qquad \epsilon = \frac{\langle\phi\rangle}{M}$$

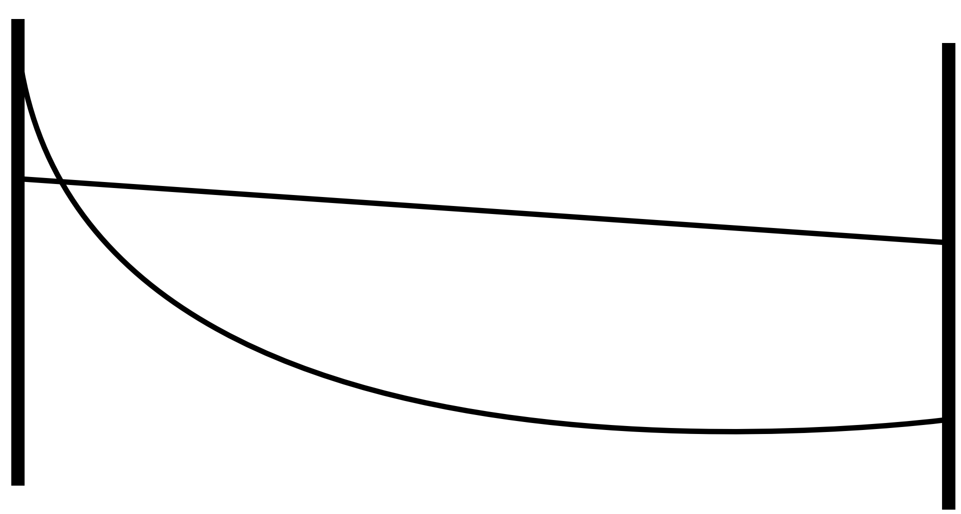
Charge the SM fermions differently

Geography (Extra dimensional models)



Arkani-hamed, Schmaltz

$$Y_{ij}^{SM} = \int dx_5 \, \psi_i(x_5)\psi_j(x_5)h(x_5)$$



Place the SM fermions in different places

Quantum mechanics

Nelson, Strassler

- The SM is coupled to a strongly coupled CFT
- SM fields get large anomalous dimensions
- Enters approximate fixed point at scale μ and leaves at scale μ_0

$$Y_{ij}^{SM}(\mu) = Y_{ij}(\mu_0)\left(\frac{\mu}{\mu_0}\right)^{\frac{1}{2}(\gamma_i+\gamma_j+\gamma_H)}$$

SM fermions have different couplings

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- Many clever mechanisms exist but must treat SM fermions separately.

- Convert small differences to large differences

- Example where SM fermions all charged the same way but get differences in Yukawas?

Quantum mechanics

Masses are generated through quantum effects

Electron mass from muon mass? Georgi and Glashow, `73

Work in the `80's, mainly one and two loop mass generation

Babu and Ma, `89

Quantum mechanics

Masses are generated through quantum effects

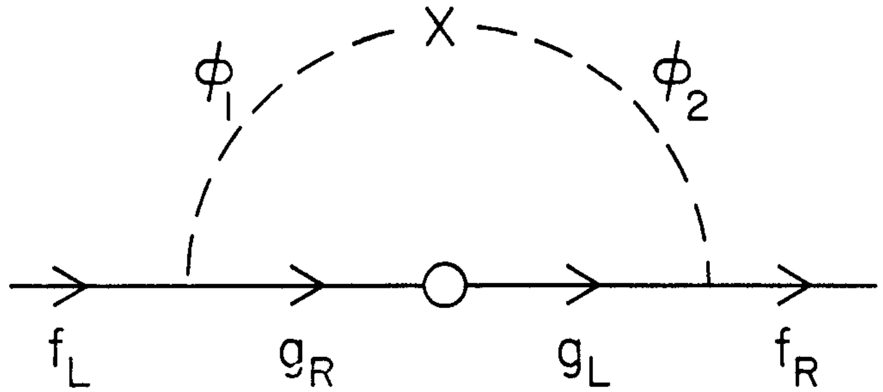
Electron mass from muon mass? Georgi and Glashow, `73

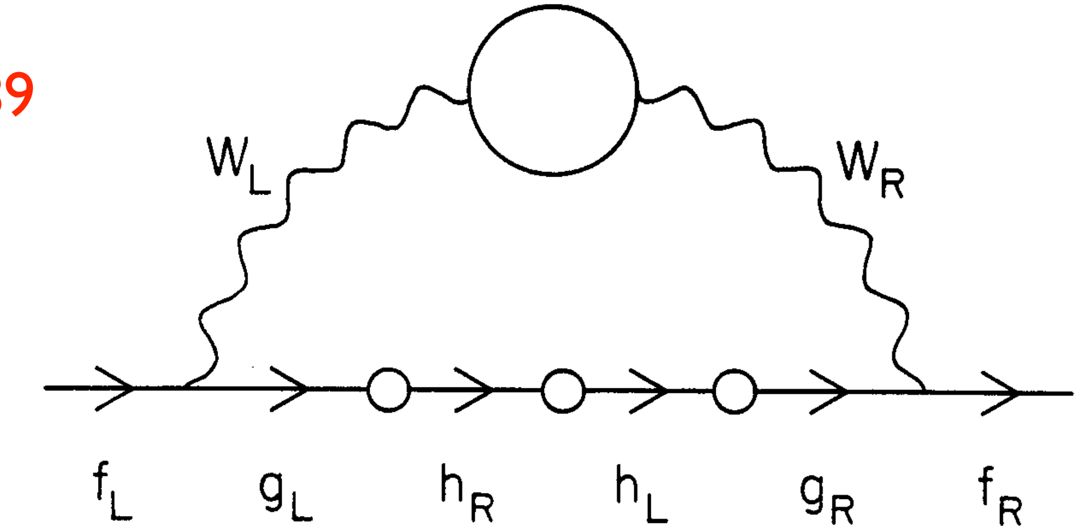
Work in the `80's, mainly one and two loop mass generation

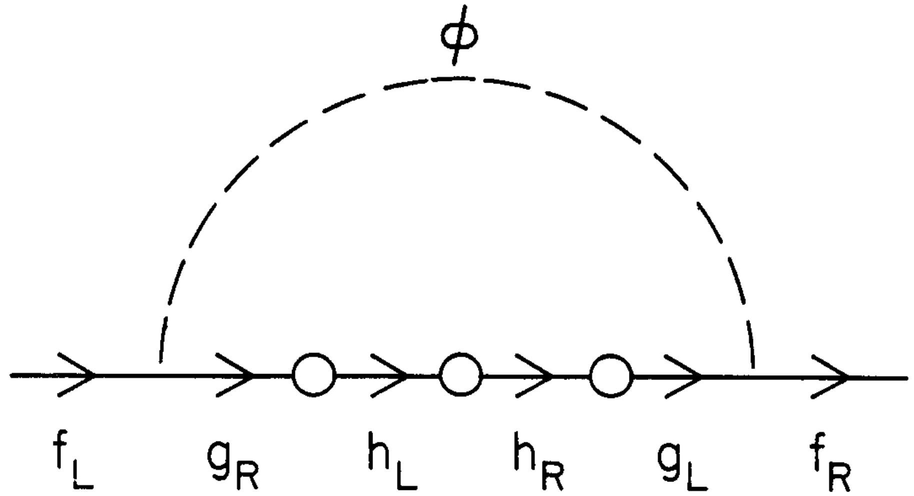
Babu and Ma, `89

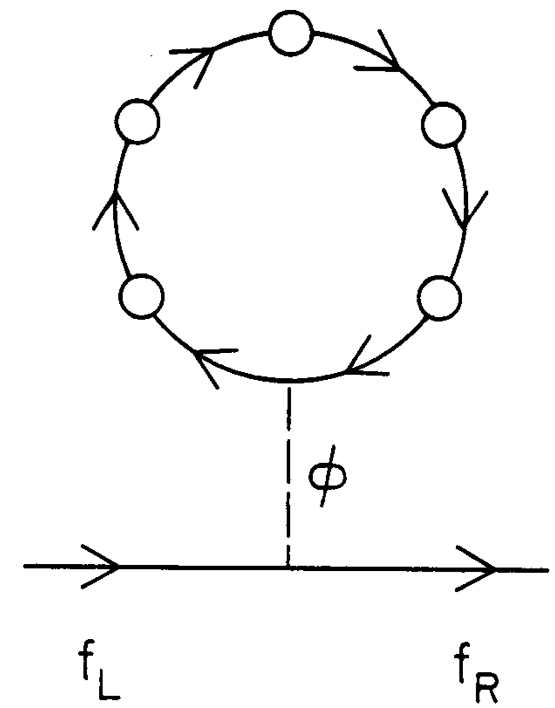
Naively all masses at approximately the same loop order

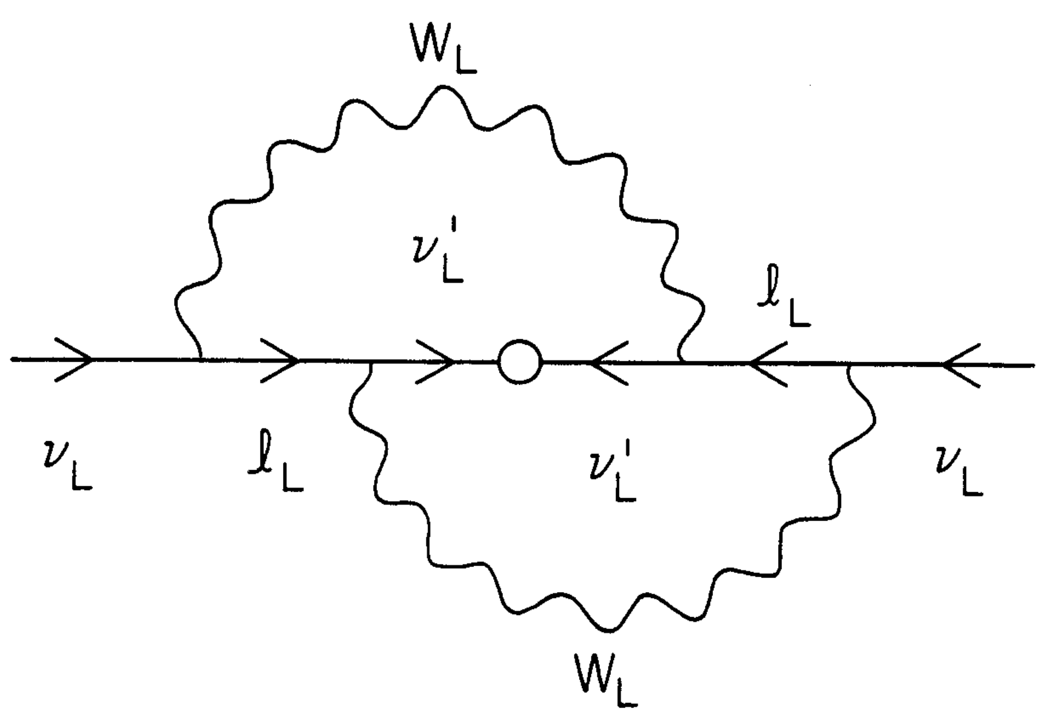
Babu and Ma, `89

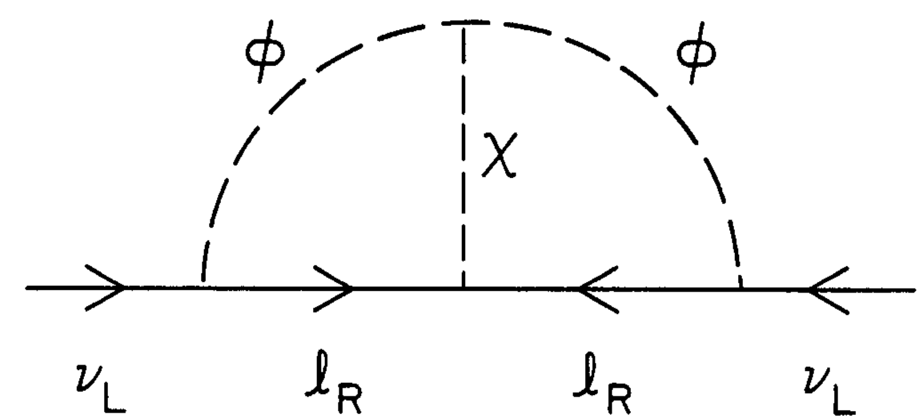






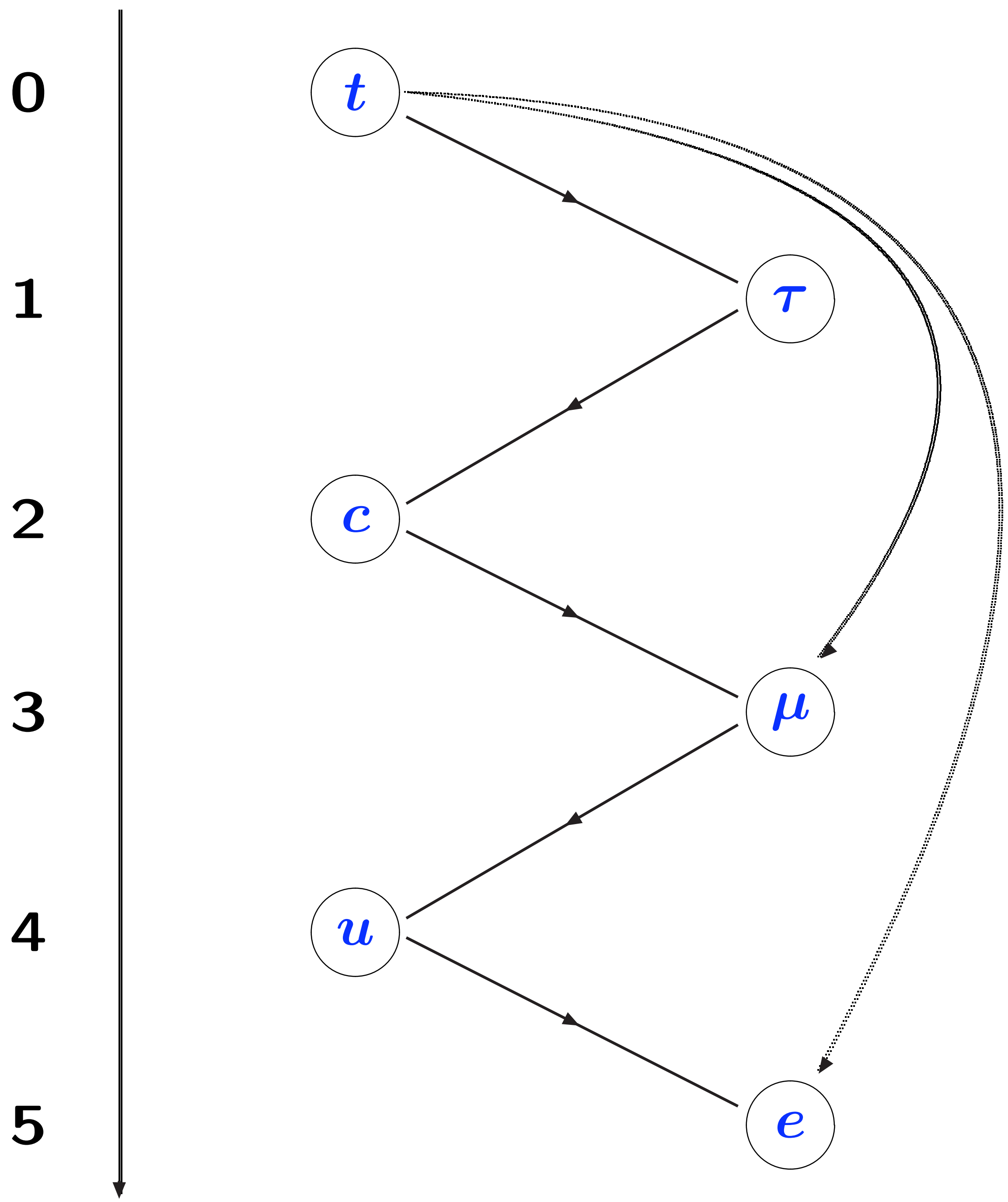






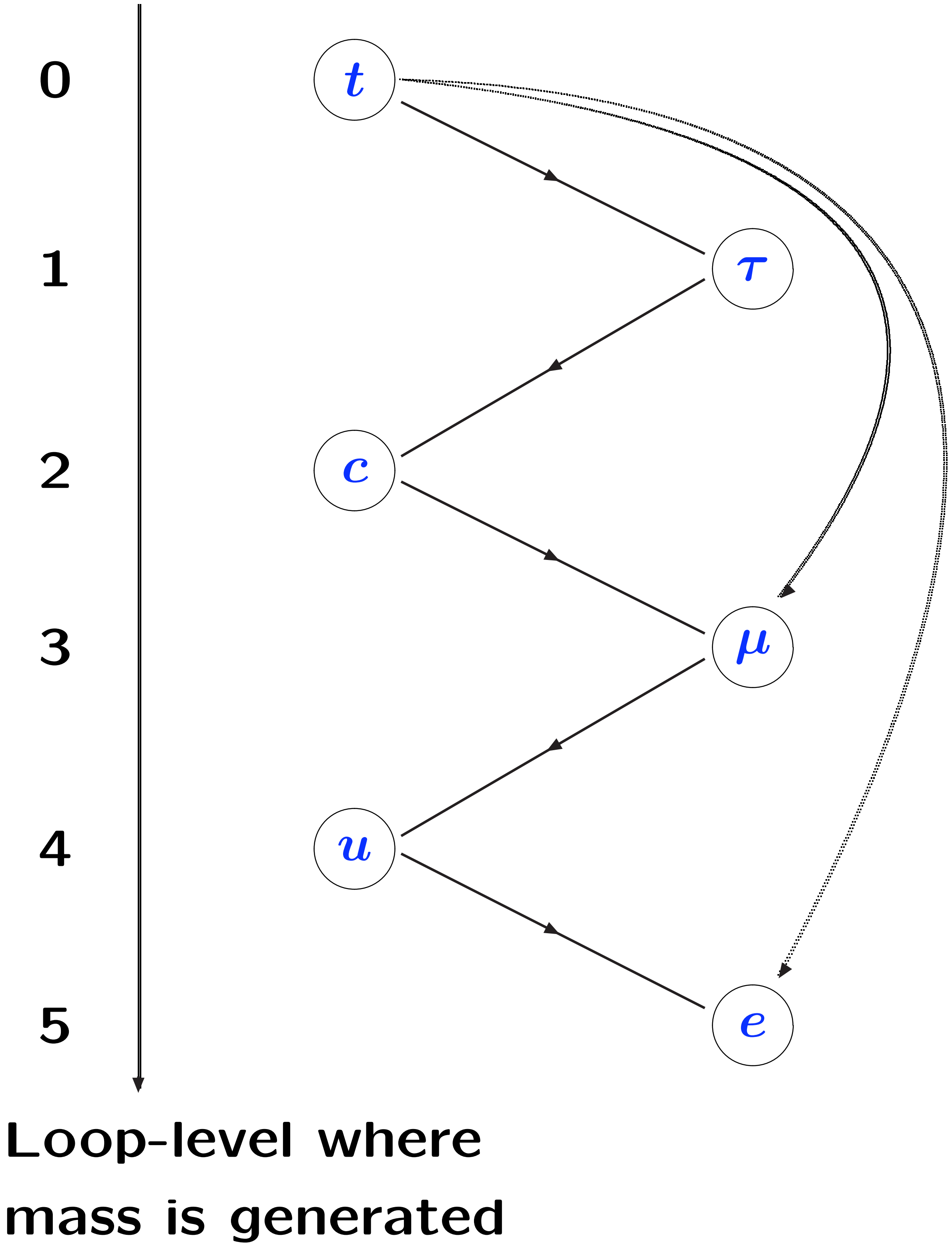
More ambitious attempt

PJF and Dobrescu



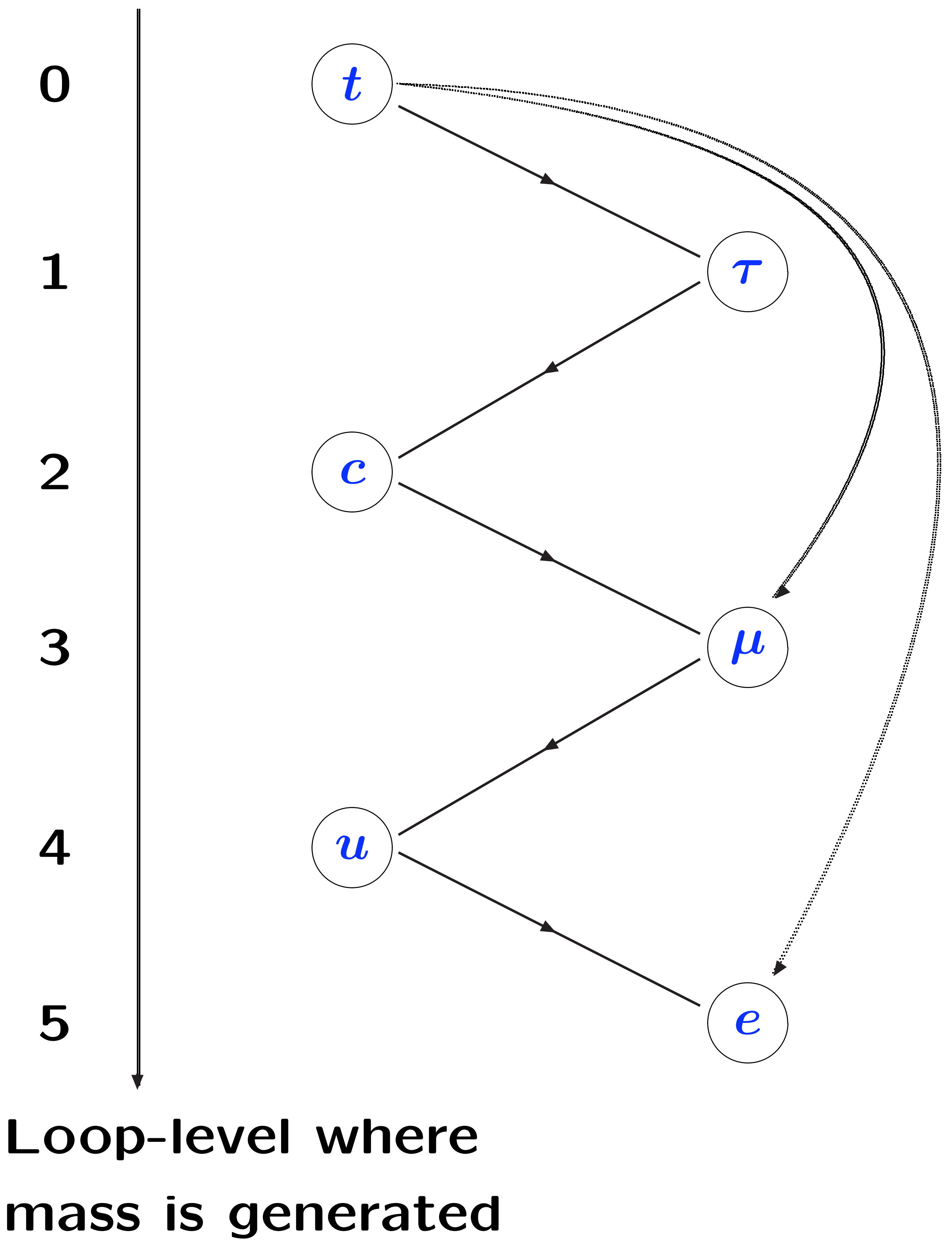
Loop-level where
mass is generated

PJF and Dobrescu



More likely to fail...?

PJF and Dobrescu



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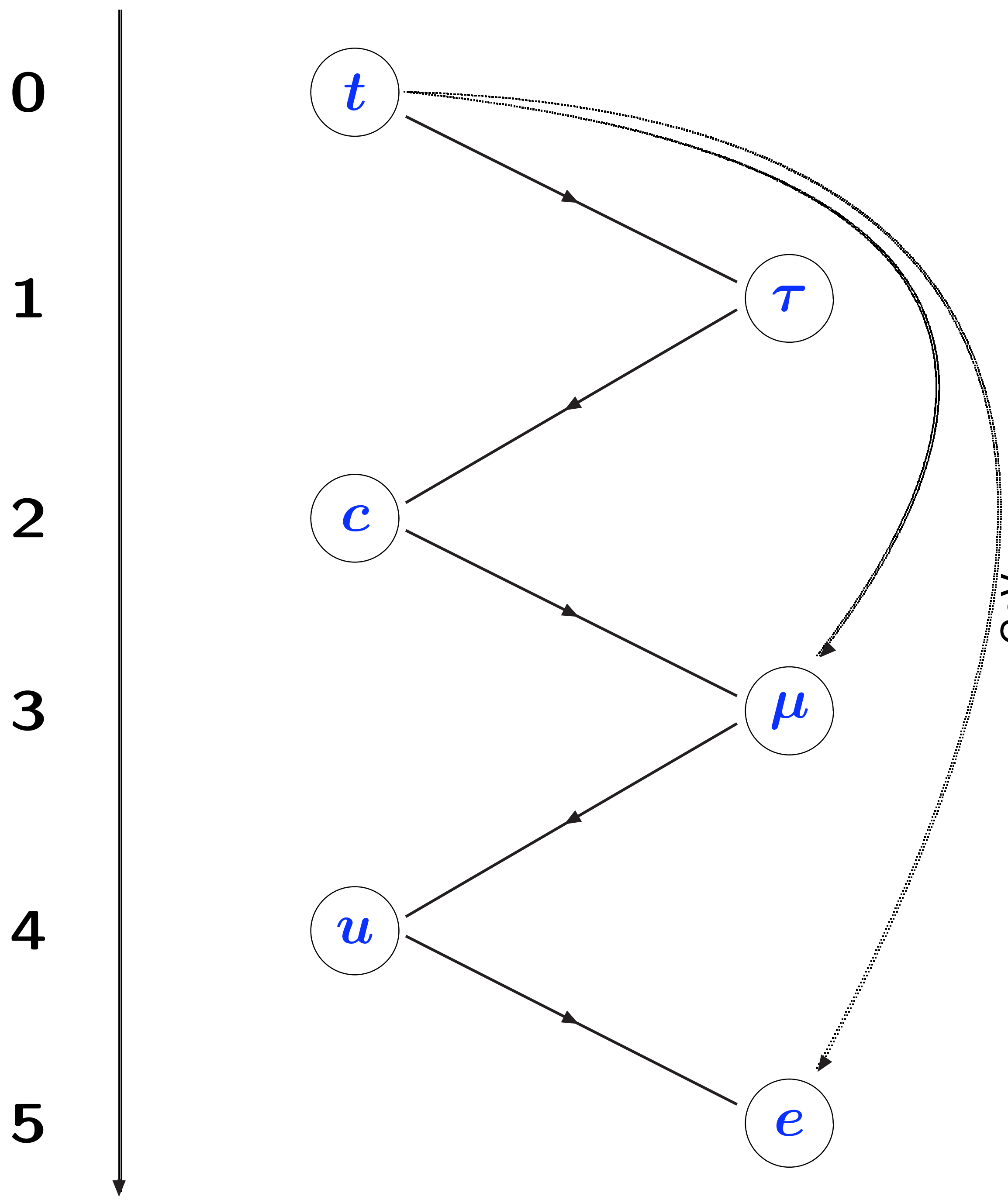
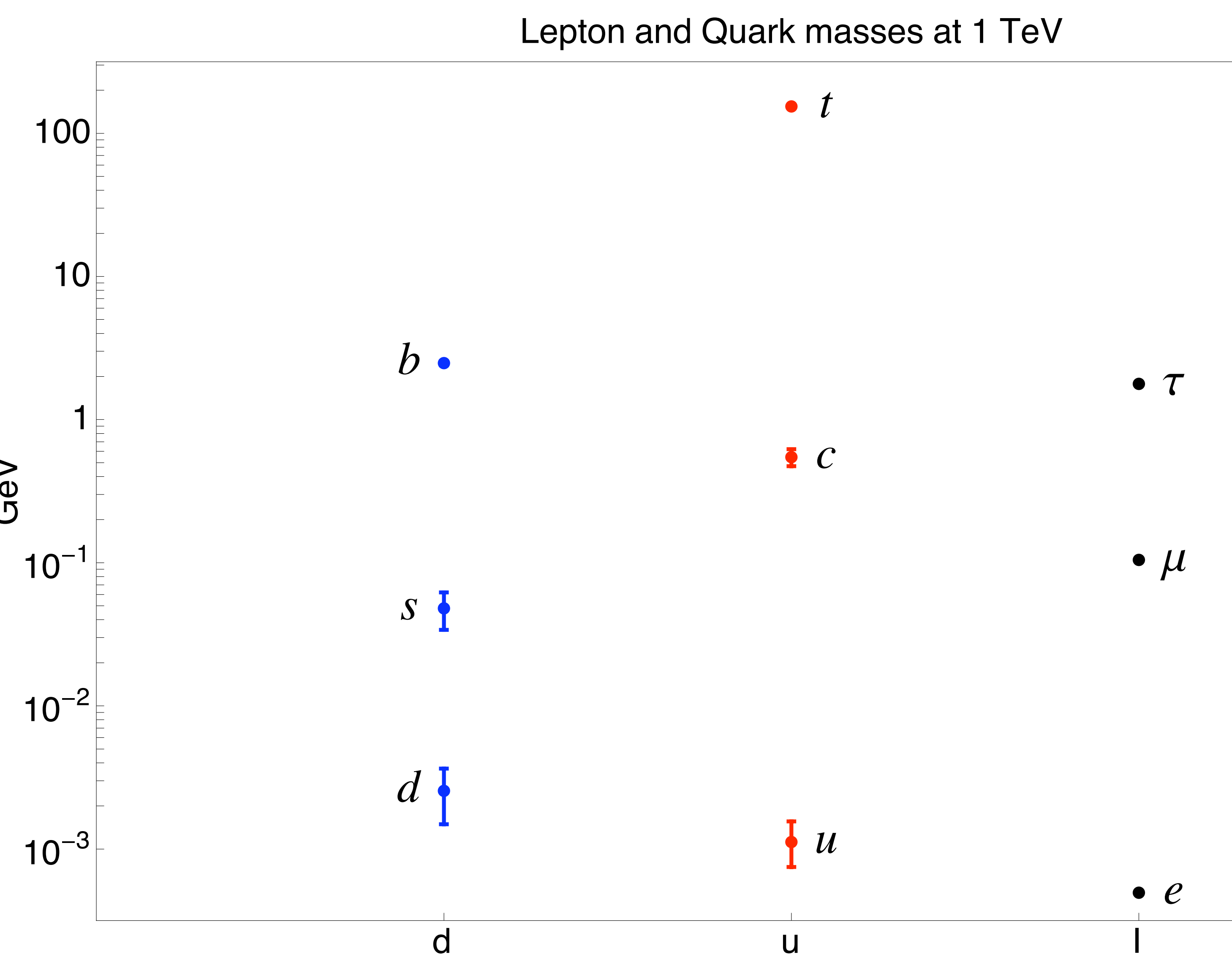


Loop-level where mass is generated



Top is clearly special

So,

assume only the top has a tree level Yukawa

$$y_t H \bar{u}_R^3 Q_L^3$$

Top is clearly special

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Charge the top?

Top is clearly special

So,

assume only the top has a tree level Yukawa

$$y_t H \bar{u}^3_R Q^3_L$$

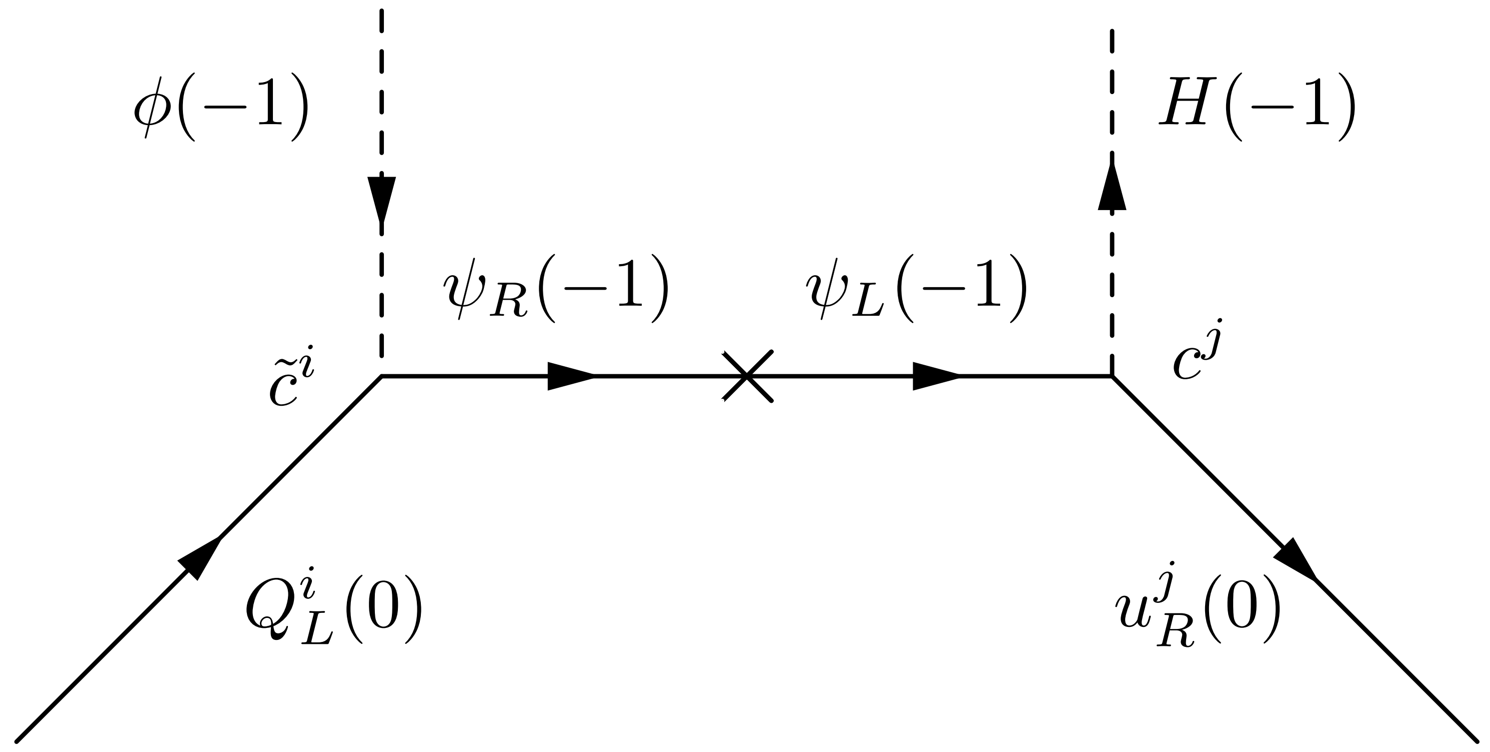
Charge the top?

Instead charge Higgs under an extra $U(1)_H$

$U(1)_H$ broken by the vev of a SM singlet ϕ of charge -1

Introduce a vector like pair of fermions with quantum numbers of left handed quarks, also charged under $U(1)_H$

Yukawas:



$$m_{ij} \propto \tilde{c}_i c_j$$

But lh top and rh top only appear *linearly* in couplings
Redefine couplings so only one lh and one rh couple
Call these the top

Mass matrix is rank 1

Only the top gets a tree level mass

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Chiral symmetries

$$U(3)_Q \times U(3)_u \times U(3)_d \xrightarrow{y_t \neq 0} U(1)_t \times U(2)_Q \times U(2)_u \times U(3)_d$$

Need to break remaining chiral symmetries

Introduce a scalar leptoquark $\boxed{r : \quad (3, 2, +7/6)}$

Chiral symmetries

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Introduce a scalar leptoquark $\boxed{r: \quad (3, 2, +7/6)}$

(charge 0 under extra $U(1)$)

Most general interactions:

$$\lambda_{ij}\, r\, \overline{u}^i_R L^j_L + \lambda'_{ij}\, r\, \overline{Q}^i_L e^j_R + \mathrm{H.c.}$$

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$$U(3)_Q \times U(3)_u \times U(3)_d \xrightarrow{y_t \neq 0} U(1)_t \times U(2)_Q \times U(2)_u \times U(3)_d$$

$$\xrightarrow{\substack{\lambda \neq 0 \\ \lambda' \neq 0}} U(1)_u \times U(3)_d$$

$$U(3)_L \times U(3)_e \xrightarrow{\substack{\lambda \neq 0 \\ \lambda' \neq 0}} U(1)_L$$

With this breaking of chiral symmetries up type quarks and charged leptons can get a mass at some loop order

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With this breaking of chiral symmetries up type quarks and charged leptons can get a mass at some loop order

But what loop order?

$$\underbrace{\lambda_{ij}\, r\, \overline{u}_R^i L_L^j + \lambda'_{ij}\, r\, \overline{Q}_L^i e_R^j}_{\text{Linear couplings}} + \mathrm{H.c.}$$

Redefine fields:

$$\begin{pmatrix} \lambda_{11} & \lambda_{12} & \lambda_{13} \\ \lambda_{21} & \lambda_{22} & \lambda_{23} \\ \lambda_{31} & \lambda_{32} & \lambda_{33} \end{pmatrix}$$

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$$\lambda_{ij}\, r\, \overline{u}^i_R L^j_L + \lambda'_{ij}\, r\, \overline{Q}^i_L e^j_R + \text{H.c.}$$

Linear couplings

Redefine fields:

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Linear couplings

Redefine fields:

- Define L_3 so it only couples only to u_3

$$\begin{pmatrix} \lambda_{11} & \lambda_{12} & \lambda_{13} \\ \lambda_{21} & \lambda_{22} & \lambda_{23} \\ 0 & 0 & \lambda_{33} \end{pmatrix}$$

$$\underbrace{\lambda_{ij}\, r\, \overline{u}_R^i L_L^j + \lambda'_{ij}\, r\, \overline{Q}_L^i e_R^j}_{\text{Linear couplings}} + \text{H.c.}$$

Redefine fields:

- Define L_3 so it only couples only to u_3
- u_2 couples only to L_2 and L_3

$$\begin{pmatrix} \lambda_{11} & \lambda_{12} & \lambda_{13} \\ 0 & \lambda_{22} & \lambda_{23} \\ 0 & 0 & \lambda_{33} \end{pmatrix}$$

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- Rotation of u_1 and u_2

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Redefine fields:

- Define L_3 so it only couples only to u_3
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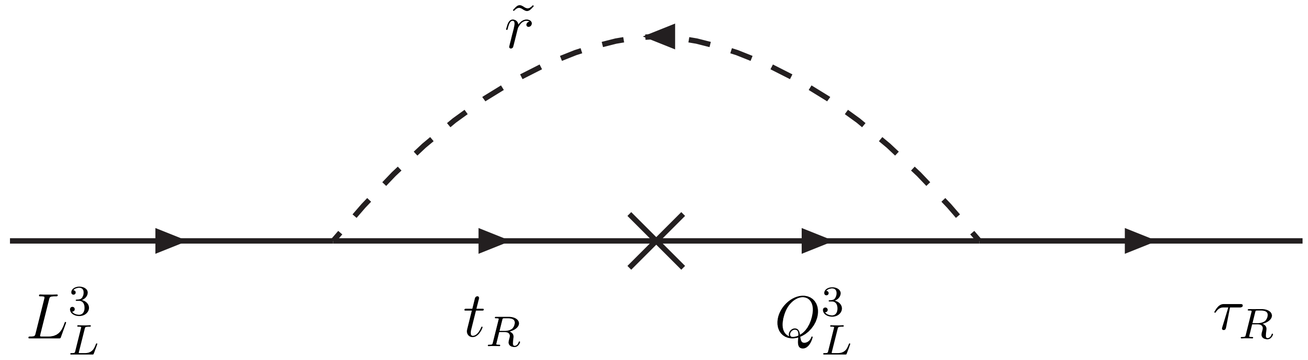
$$\begin{pmatrix} \lambda_{11} & \lambda_{12} & 0 \\ 0 & \lambda_{22} & \lambda_{23} \\ 0 & 0 & \lambda_{33} \end{pmatrix}$$

$\lambda_{ij},\, \lambda'_{ij}$
can be made real
and positive

One loop tau mass

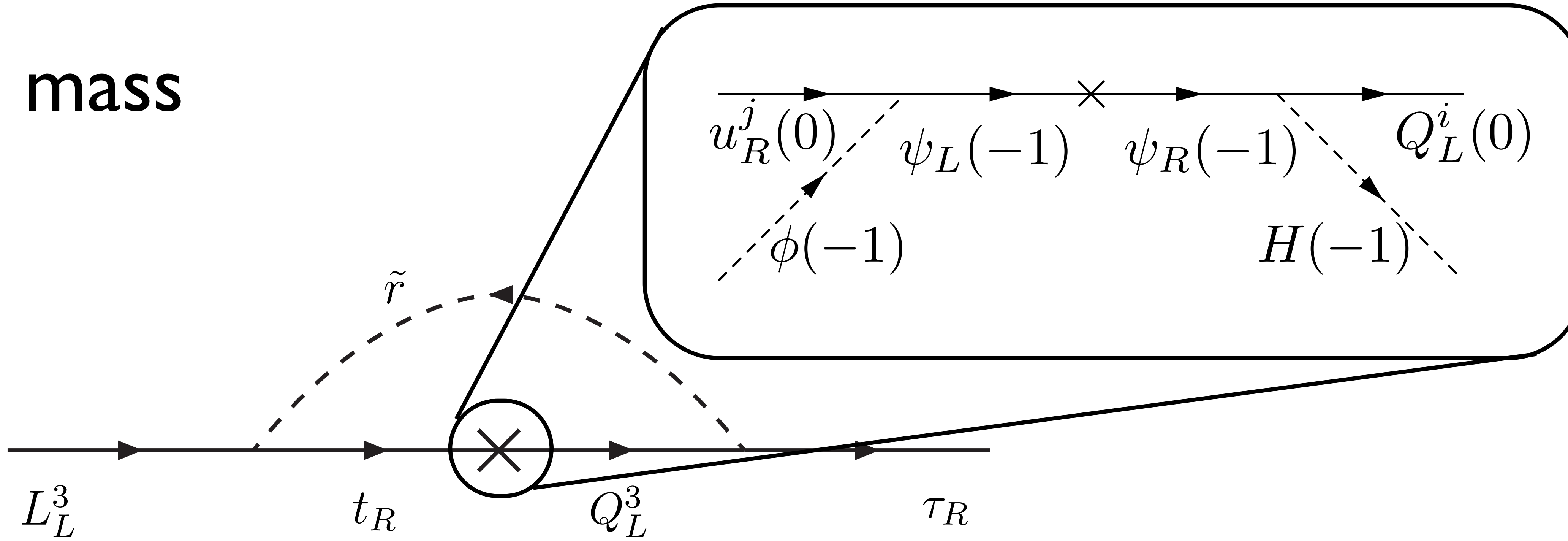


$$m_\tau \simeq \lambda_{33}\lambda'_{33}\, m_t \underbrace{\frac{N_c}{16\pi^2} \ln\left(\frac{\Lambda^2}{M_{\tilde{r}}^2}\right)}$$

≈ 0.09 for $\Lambda \approx 10 M_{\tilde{r}}$

$\lambda_{33}\lambda'_{33} \approx (0.36)^2$ for correct m_τ / m_t ratio

One loop tau mass



Really M_Ψ

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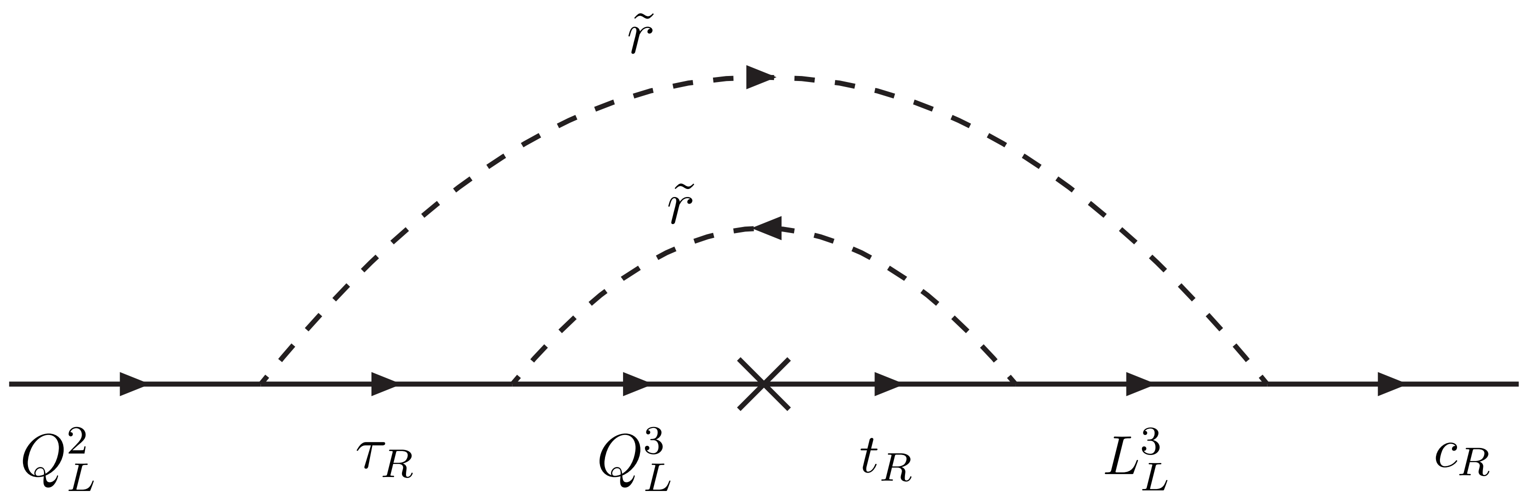
Two loop charm mass - a "rainbow" diagram

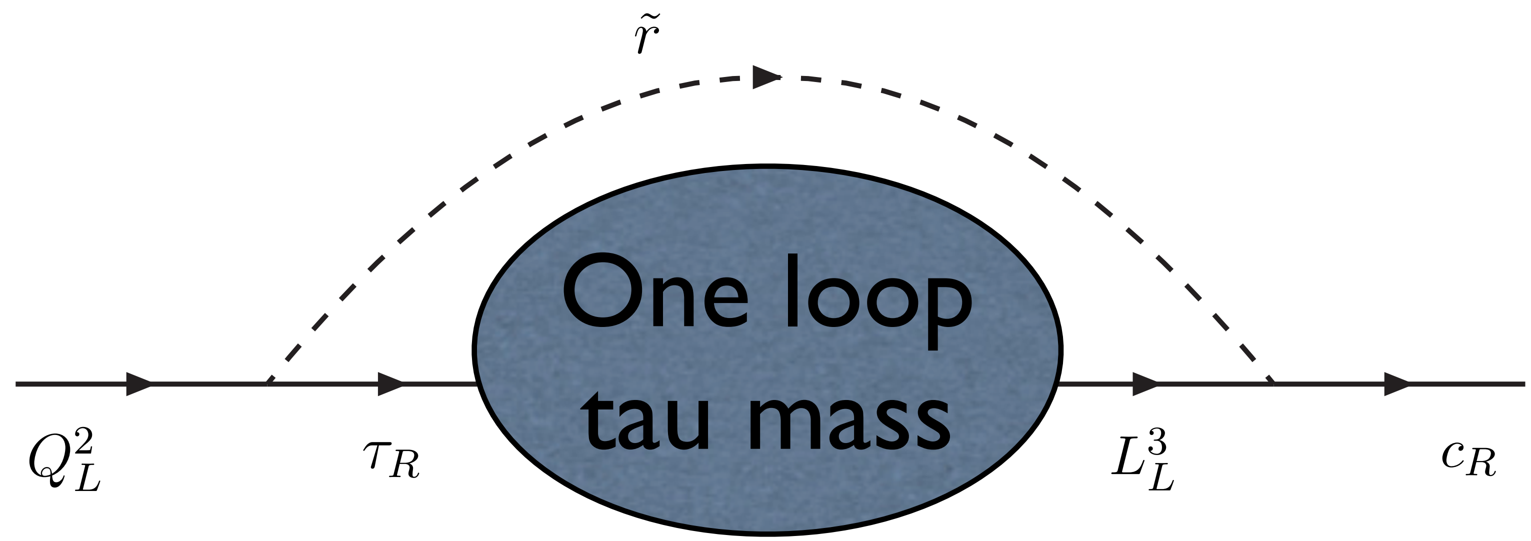


$$M_u[\tilde{r}\tilde{r}] = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \lambda'_{23}\lambda_{23} & \lambda'_{33}\lambda_{23} \\ 0 & \lambda'_{23}\lambda_{33} & \lambda'_{33}\lambda_{33} \end{pmatrix} \lambda'_{33}\lambda_{33}\, m_t\, \epsilon_{\tilde{r}}^{(2)}$$

$$m_c = \lambda'_{23}\lambda_{23} m_\tau \frac{1}{16\pi^2} \log \frac{\Lambda^2}{M_{\tilde{r}}^2}$$

$\lambda_{23}\lambda'_{23} \approx (3.3)^2$ for correct m_c/m_τ ratio

Two loop charm mass - a "rainbow" diagram



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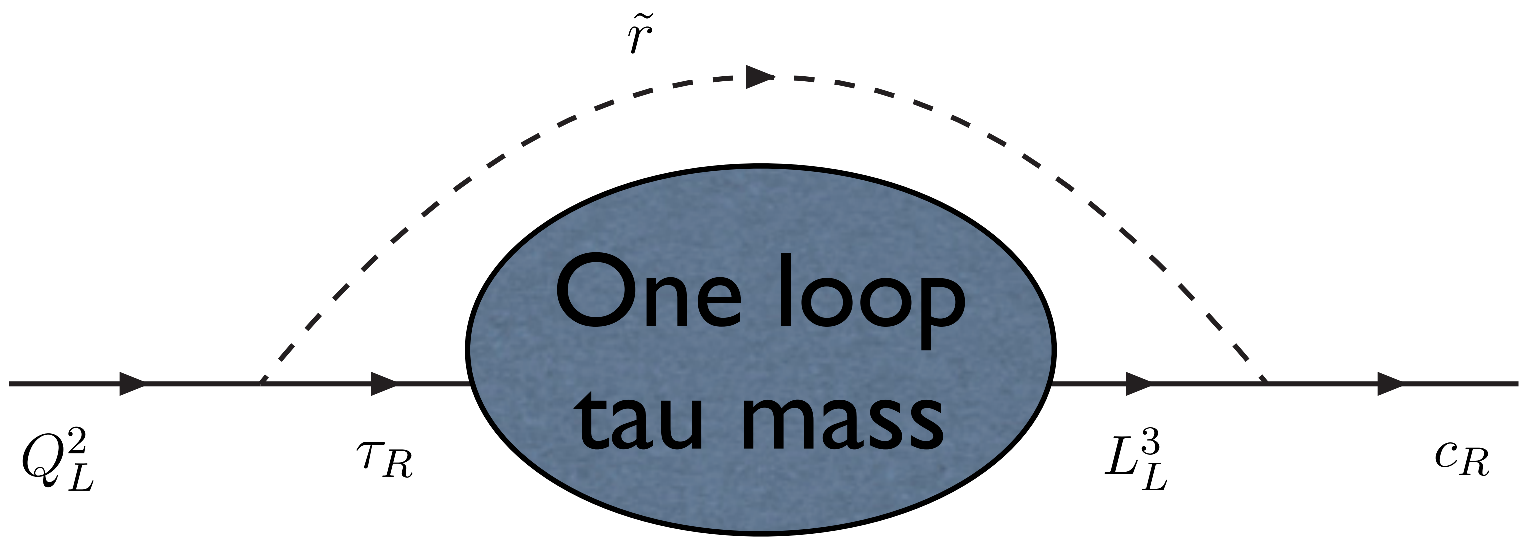

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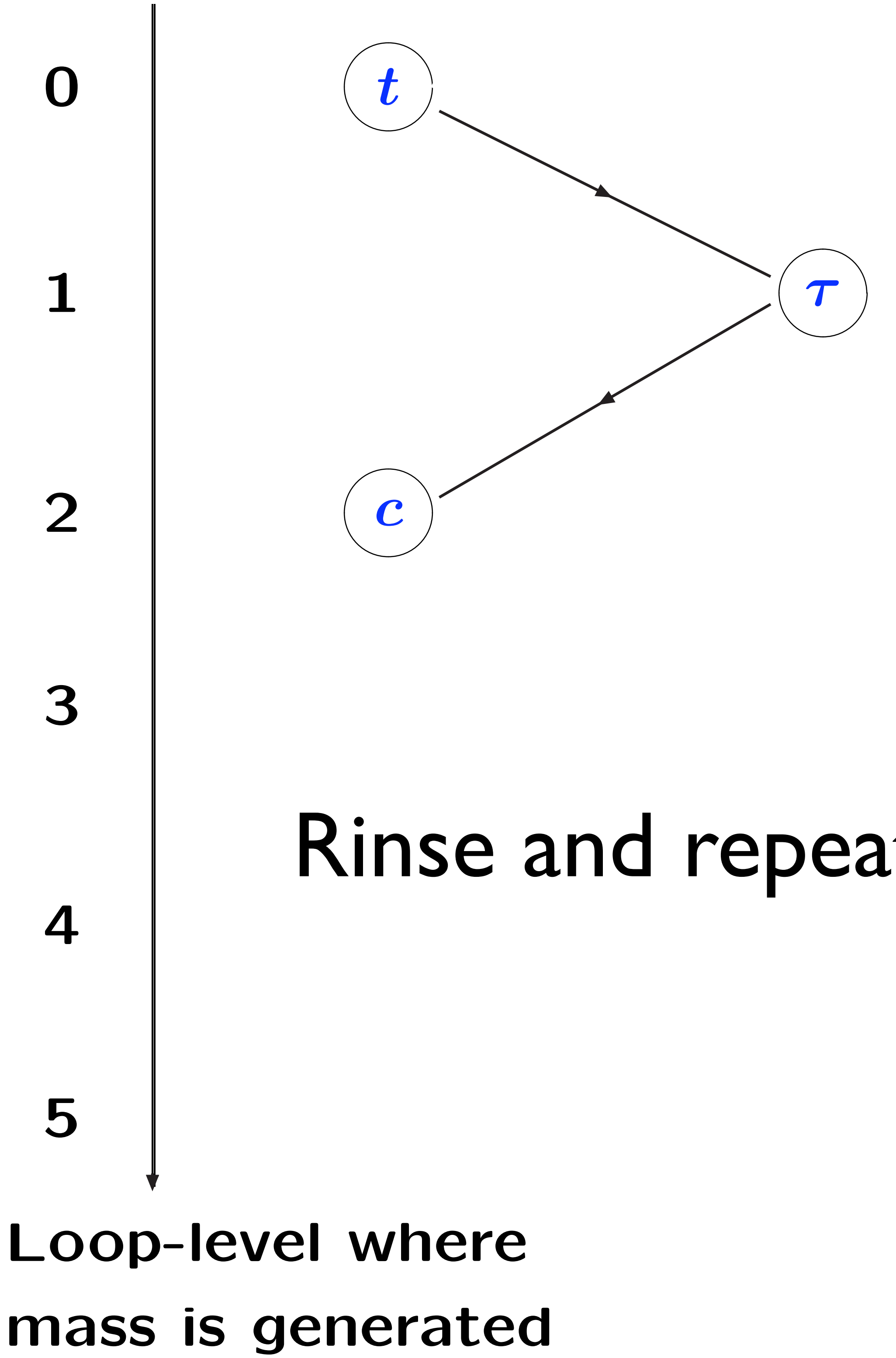
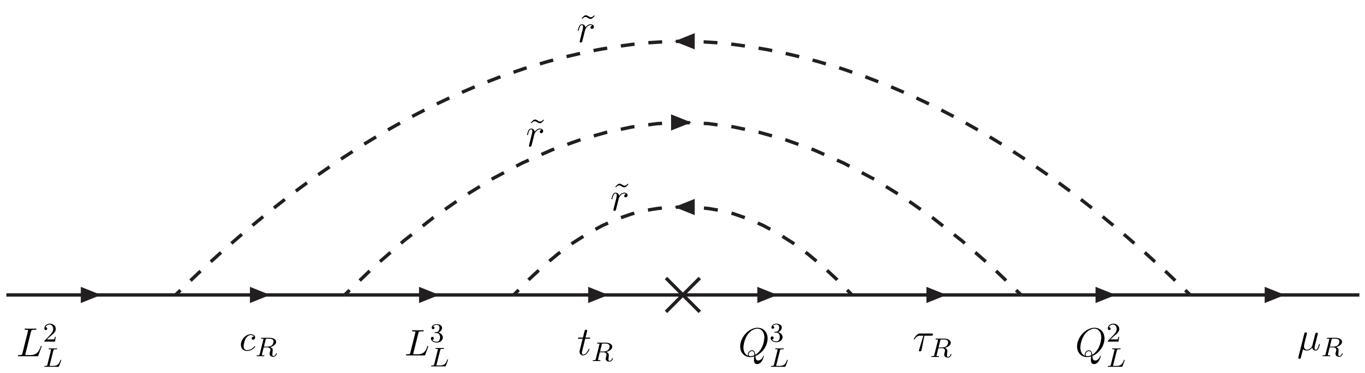
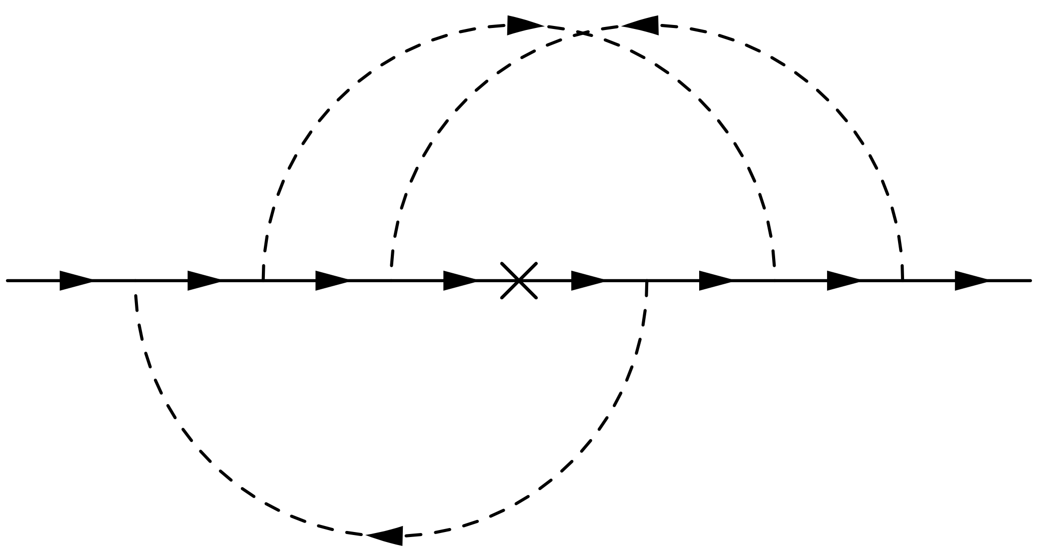


Rinse and repeat?

Three loop muon mass



Rainbow $\sim N_C^2$



The diagram with no name $\sim N_C$

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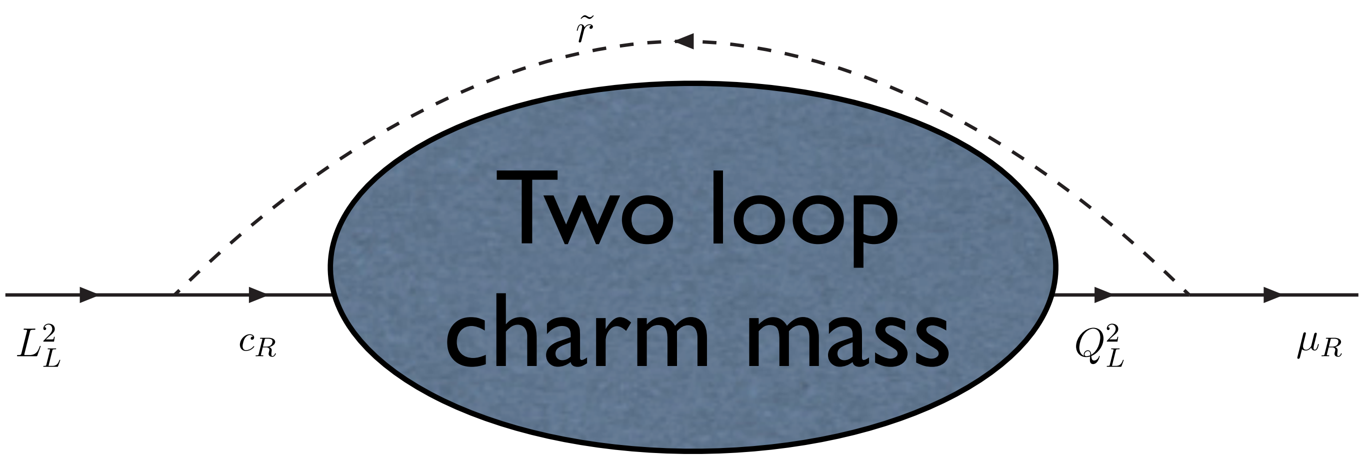

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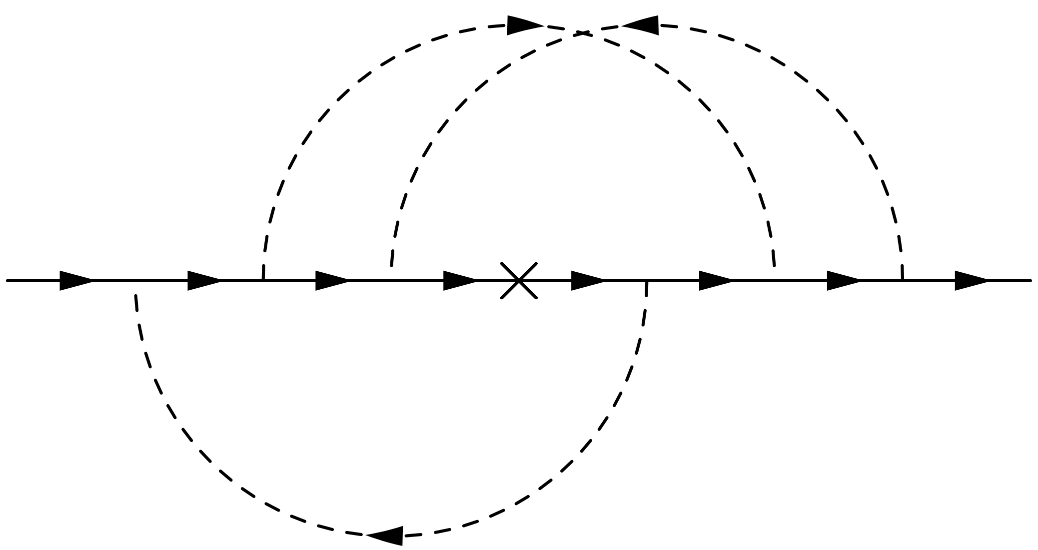

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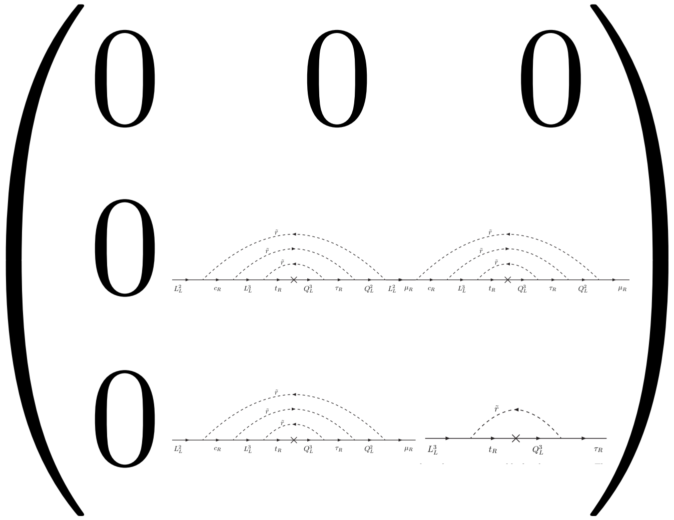

$$m_\mu \approx \lambda'_{22}\lambda_{22} m_c (1+x) \frac{N_c}{16\pi^2} \log \frac{\Lambda^2}{M_{\tilde{r}}^2}$$

$\lambda_{22}\lambda'_{22}(1+x) \approx (1.5)^2$ for correct m_μ/m_c ratio

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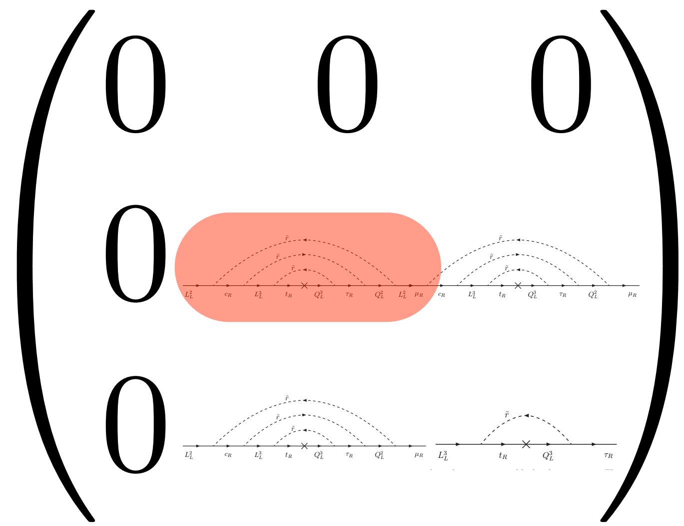
Three loop muon mass



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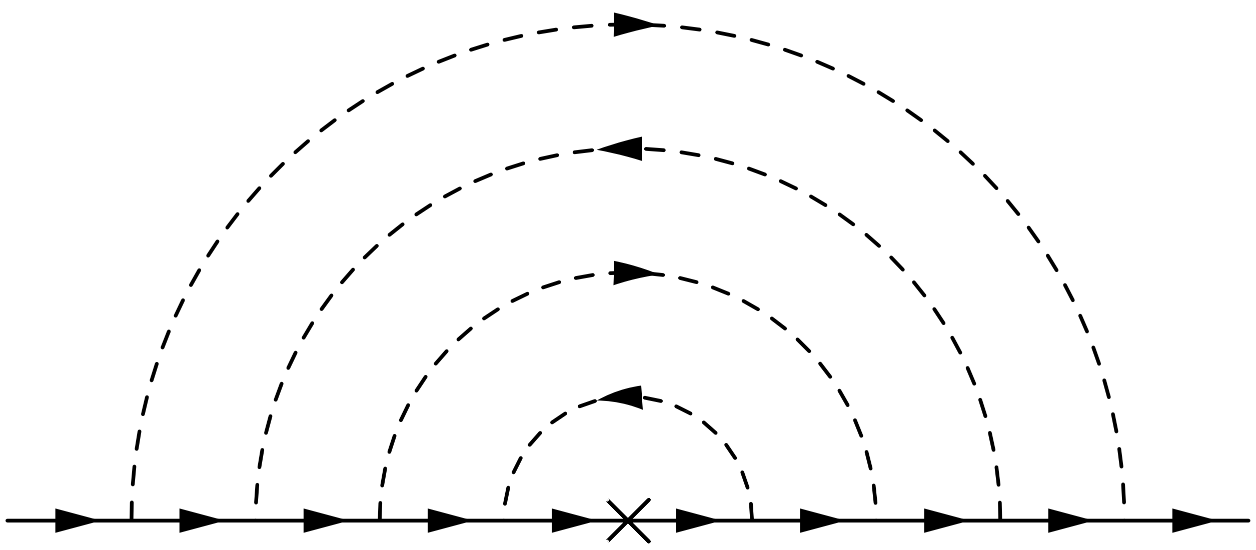
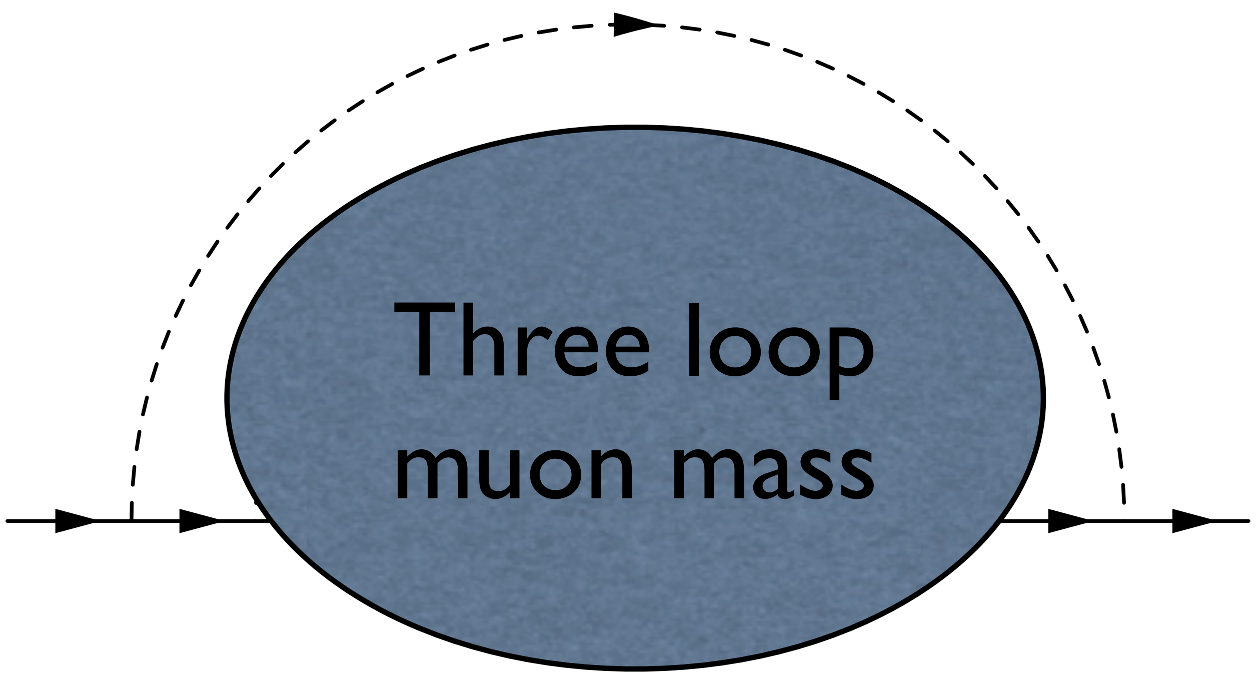
Four loop up quark mass



$+4$ other diagrams

Muon mass implies: $\#\lambda_{12}\lambda'_{12} \approx (0.6)^2$

Four loop up quark mass



$+4$ other diagrams

Muon mass implies: $\# \lambda_{12} \lambda'_{12} \approx (0.6)^2$

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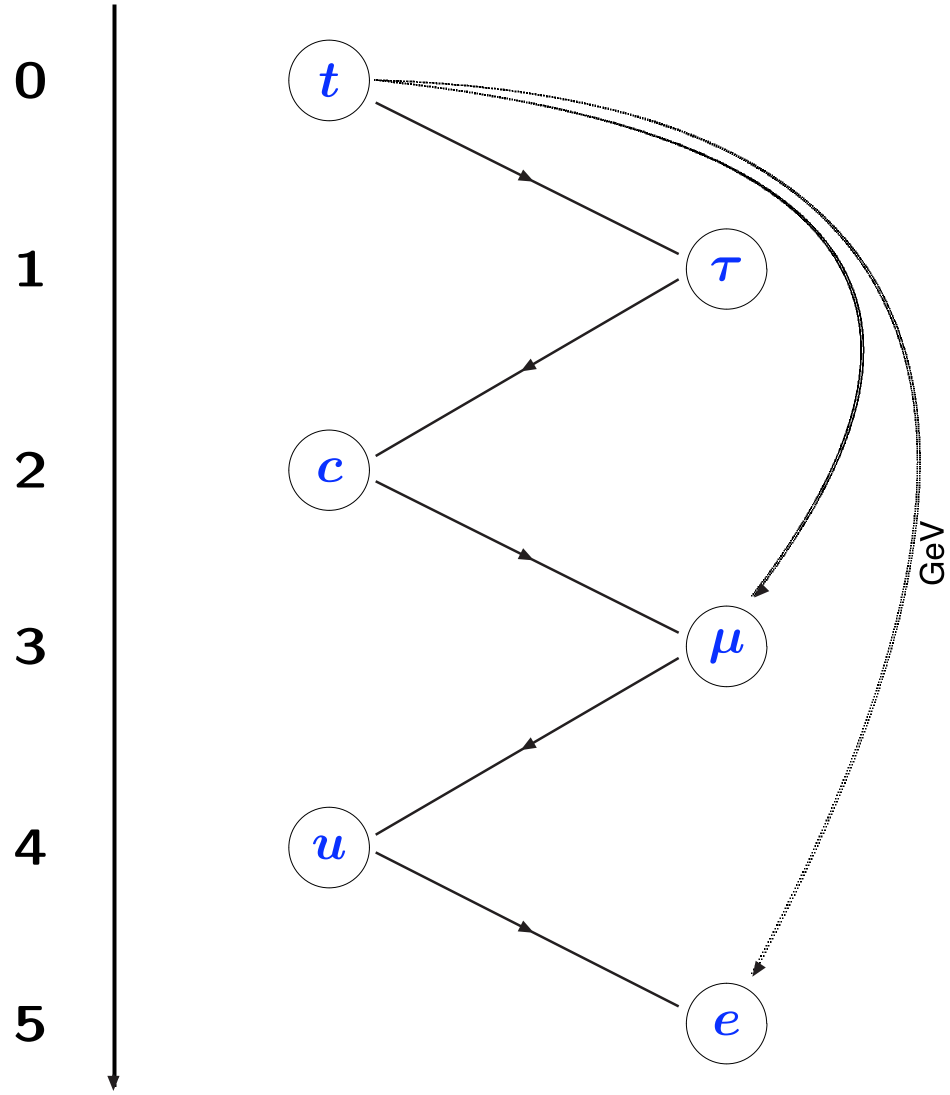
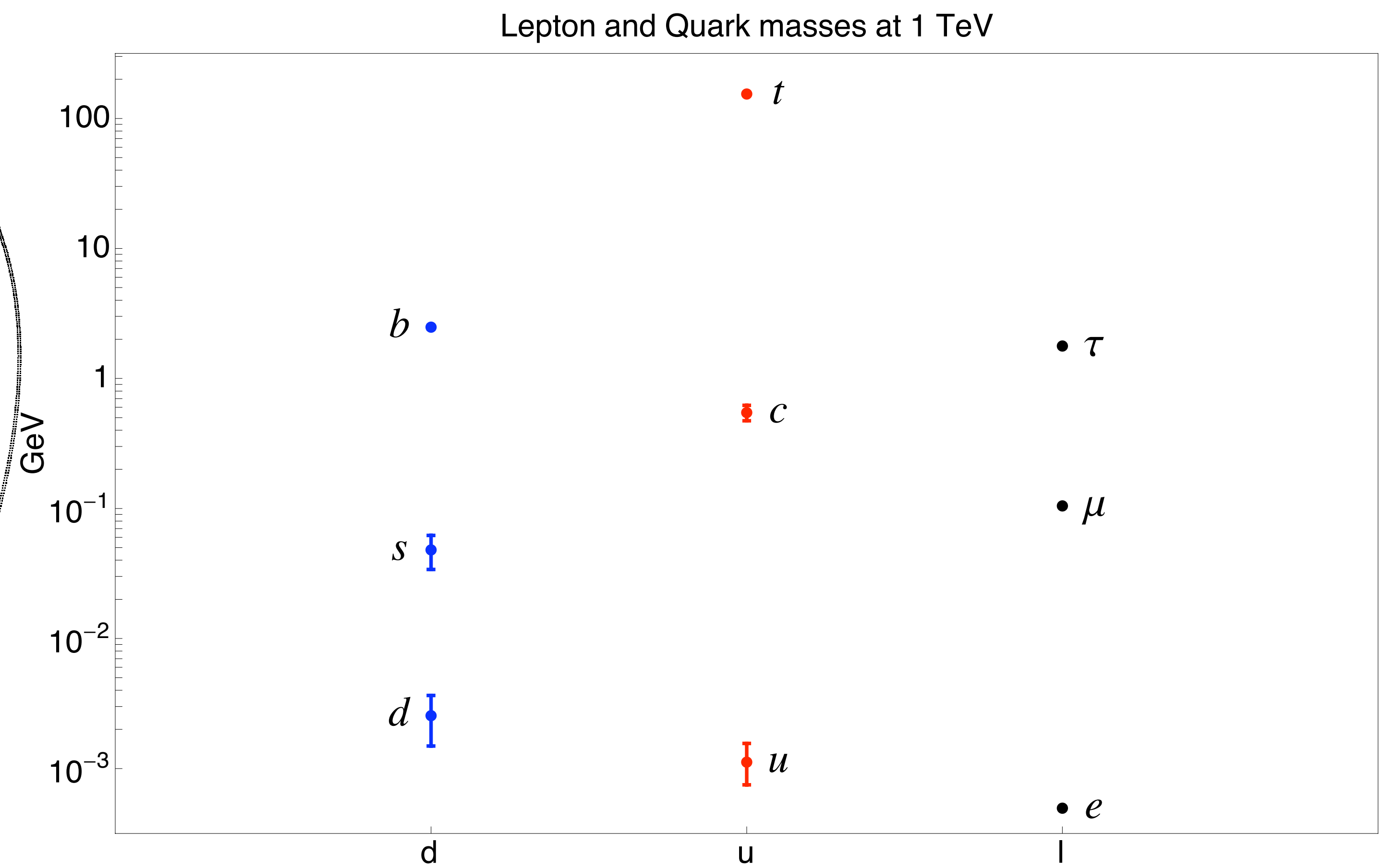
Five loop electron mass

If only source of electron mass will determine $\lambda_{11}\lambda'_{11}$

Only input: $r: \quad (3, 2, +7/6)$ $\qquad \lambda_{ij}\, r\, \overline{u}^i_R L^j_L + \lambda'_{ij}\, r\, \overline{Q}^i_L e^j_R + \text{H.c.}$



Loop-level where mass is generated



Down quark masses

Need to break the remaining chiral symmetries

$$U(3)_d \times U(1)_u \times U(1)_L$$

Have choices diquarks, leptoquarks...

$$H_8 : \quad (8, 2, -1/2)$$

$$\tilde{q} : \quad (3, 2, 1/6)$$

$$\tilde{d}_6 : \quad (\bar{6}, 1, -1/3)$$

$$\tilde{d} : \quad (3, 1, -1/3)$$

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New field content

	ϕ	$\psi_L,\, \psi_R$	H	r	r'	Φ_8	Φ'_8	Φ_3
$SU(3)$	1	3	1	3	3	8	8	$\overline{3}$
$SU(2)$	1	2	2	2	2	2	2	2
$U(1)_Y$	0	1/6	1/2	7/6	7/6	1/2	$-1/2$	$-1/6$
$U(1)_H$	-1	-1	1	0	2	1	1	0

Up quarks and leptons ↑ (columns ϕ to r)

Down quarks ↑ (columns r' to Φ_3)

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Most general couplings

$$\kappa_i \, \Phi_8 \, \overline{u}^i_R \Psi_L + \kappa' \, \Phi'_8 \, \overline{d}^3_R \Psi_L$$

$$\eta_{ij} \, \Phi_3 \, \overline{d}^i_R L^j_L + \text{h.c.}$$

break the remaining chiral symmetries

$$U(3)_d \times U(1)_u \times U(1)_L \to U(1)_L \times U(1)_Q$$

Most general couplings

$$\kappa_i \, \Phi_8 \, \overline{u}^i_R \Psi_L + \kappa' \, \Phi'_8 \, \overline{d}^3_R \Psi_L$$

Only couples to b

$$\eta_{ij} \, \Phi_3 \, \overline{d}^i_R L^j_L + \text{h.c.}$$

break the remaining chiral symmetries

$$U(3)_d \times U(1)_u \times U(1)_L \rightarrow U(1)_L \times U(1)_Q$$

Without altering up type and leptons have the freedom to rotate such that,

$$\eta = \begin{pmatrix} \eta_{11} & \eta_{12} & \eta_{13} \\ \eta_{21} & \eta_{22} & \eta_{23} \\ 0 & \eta_{32} & \eta_{33} \end{pmatrix}$$

$$\kappa = (\kappa_1, \kappa_2, \kappa_3)$$

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Diagonal entries can be made real and positive

$$\kappa = (\kappa_1, \kappa_2, \kappa_3)$$

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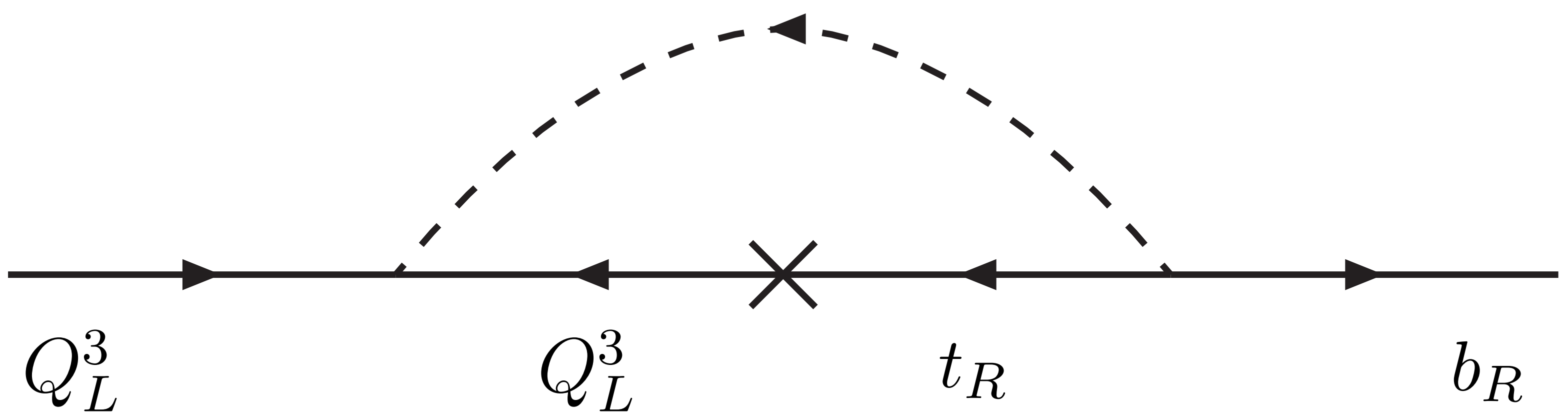
One loop bottom mass



$$m_b \approx N_c \kappa_3 \kappa' c\, m_t \left(\frac{\langle\phi\rangle}{M_\Psi}\right)^2 \frac{1}{16\pi^2} \log\left(\frac{M_\Psi^2}{M_8^2}\right)$$

One loop bottom mass

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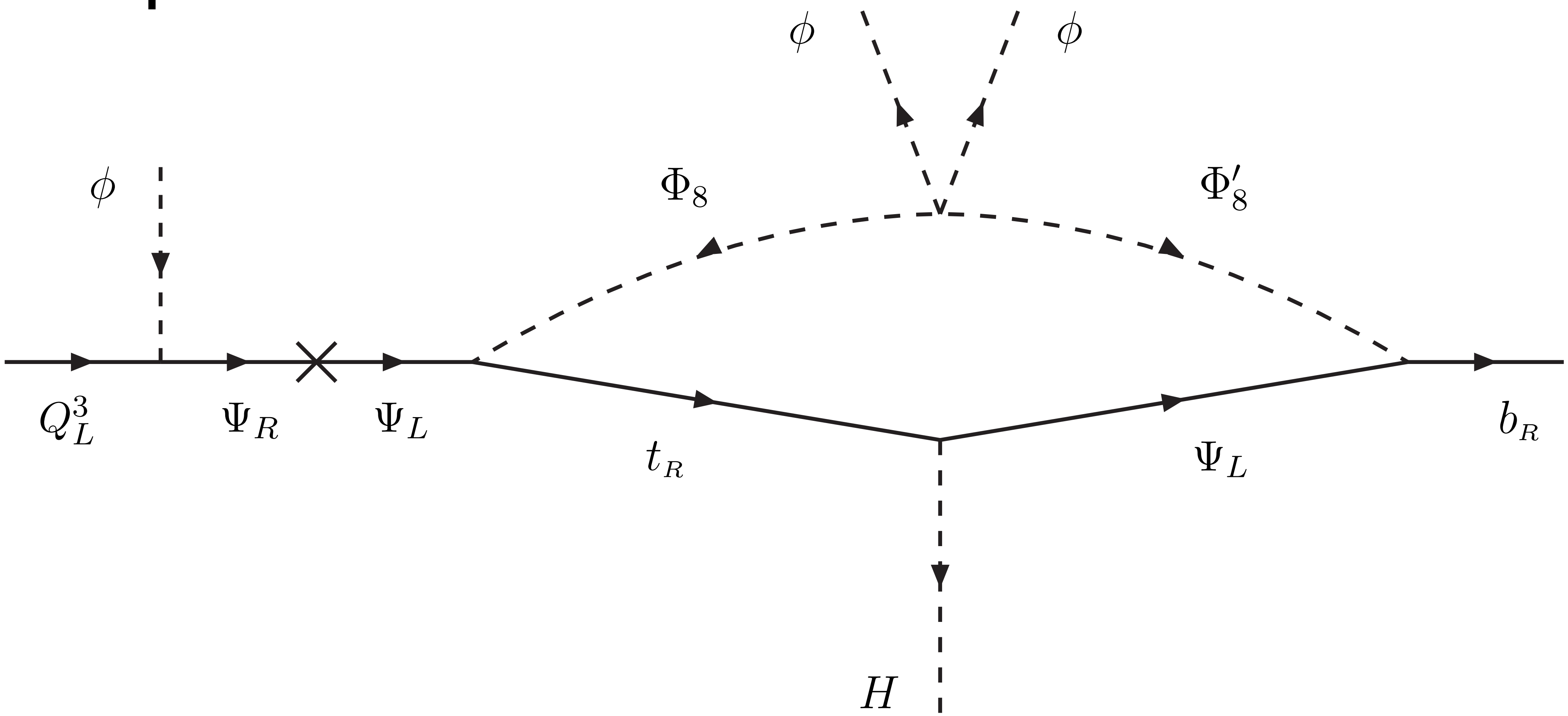
One loop bottom mass



$$m_b \approx N_c \kappa_3 \kappa' c\, m_t \left(\frac{\langle \phi \rangle}{M_\Psi} \right)^2 \frac{1}{16\pi^2} \log \left(\frac{M_\Psi^2}{M_8^2} \right)$$

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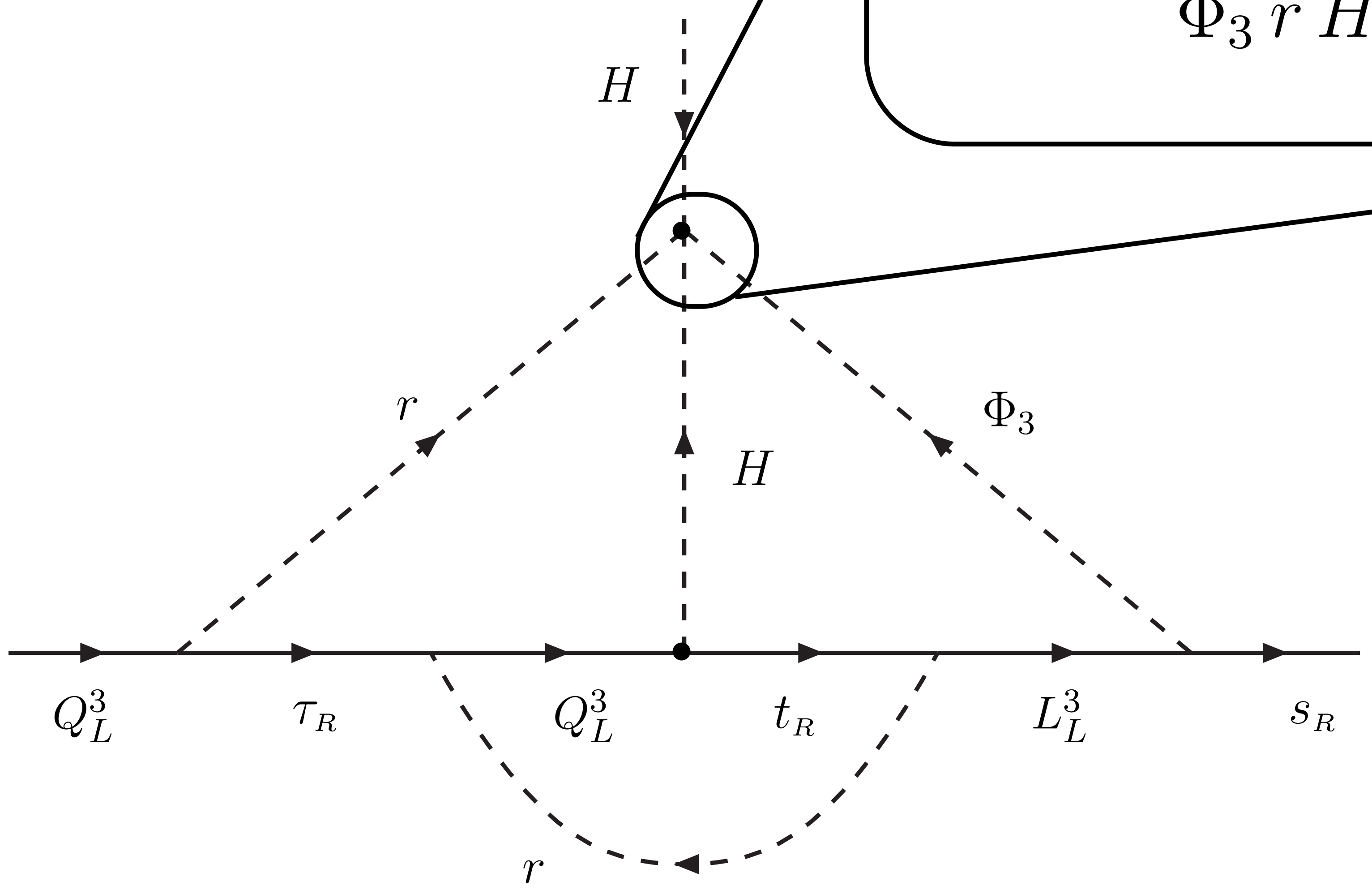
Three loop strange mass

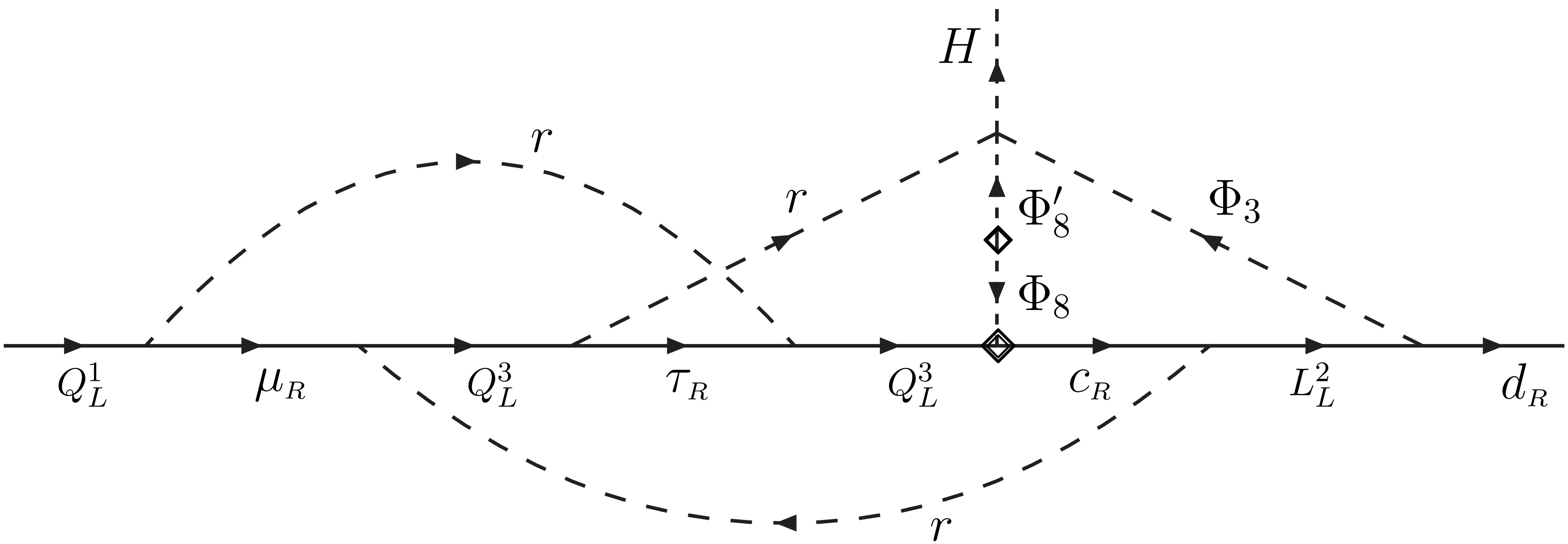
Integrate out r’

$$\Phi_3\, r\, H\, H\, \left(\frac{\phi}{m_{r'}}\right)^2$$



Four loop down masses

The down has a 4 loop mixed diagram
(exercise for reader)


H
r
r
Φ_8'
Φ_3
Φ_8
Q_L^1
μ_R
Q_L^3
τ_R
Q_L^3
c_R
L_L^2
d_R
r

“Cross Talk”

There are also corrections to some of the states that have mass:

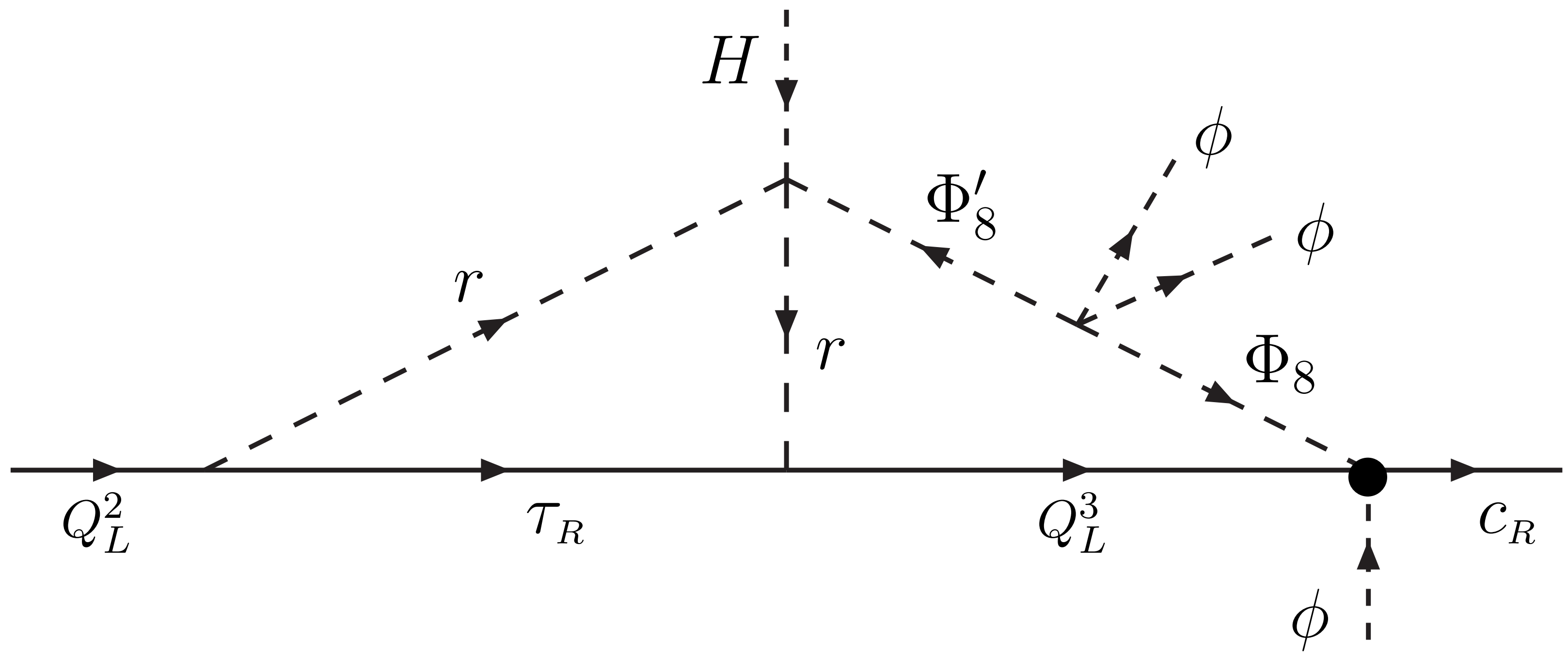
Charm gets a two loop correction

Up gets a four loop correction

Muon gets a three loop correction

Electron gets a five loop correction

Charm gets a two loop correction



- Different parameter dependence
- Different number of logs
- Changes (lowers) certain couplings $\lambda_{23}\lambda'_{23} \approx (3.3)^2$

Doesn't change *loop* counting

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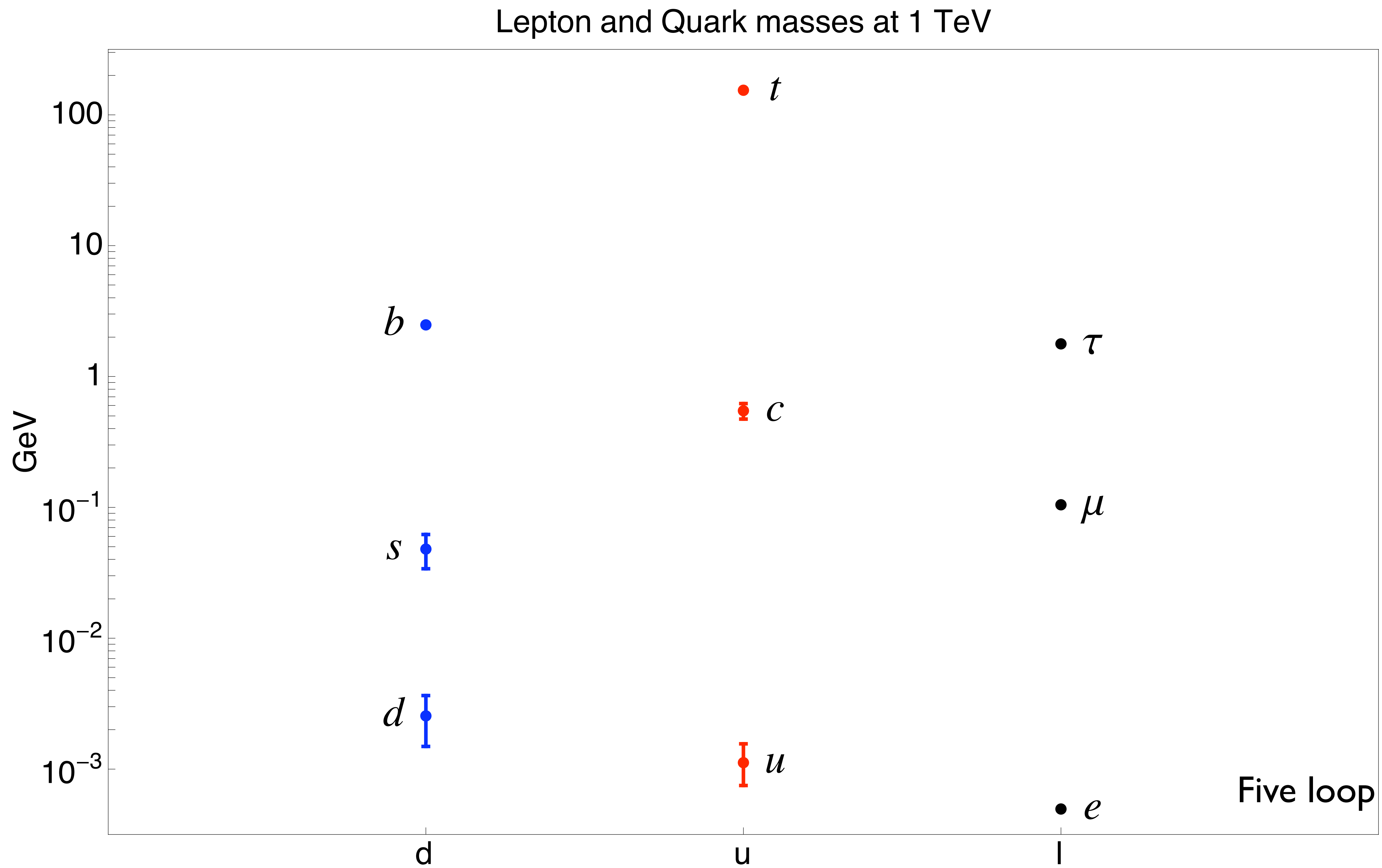
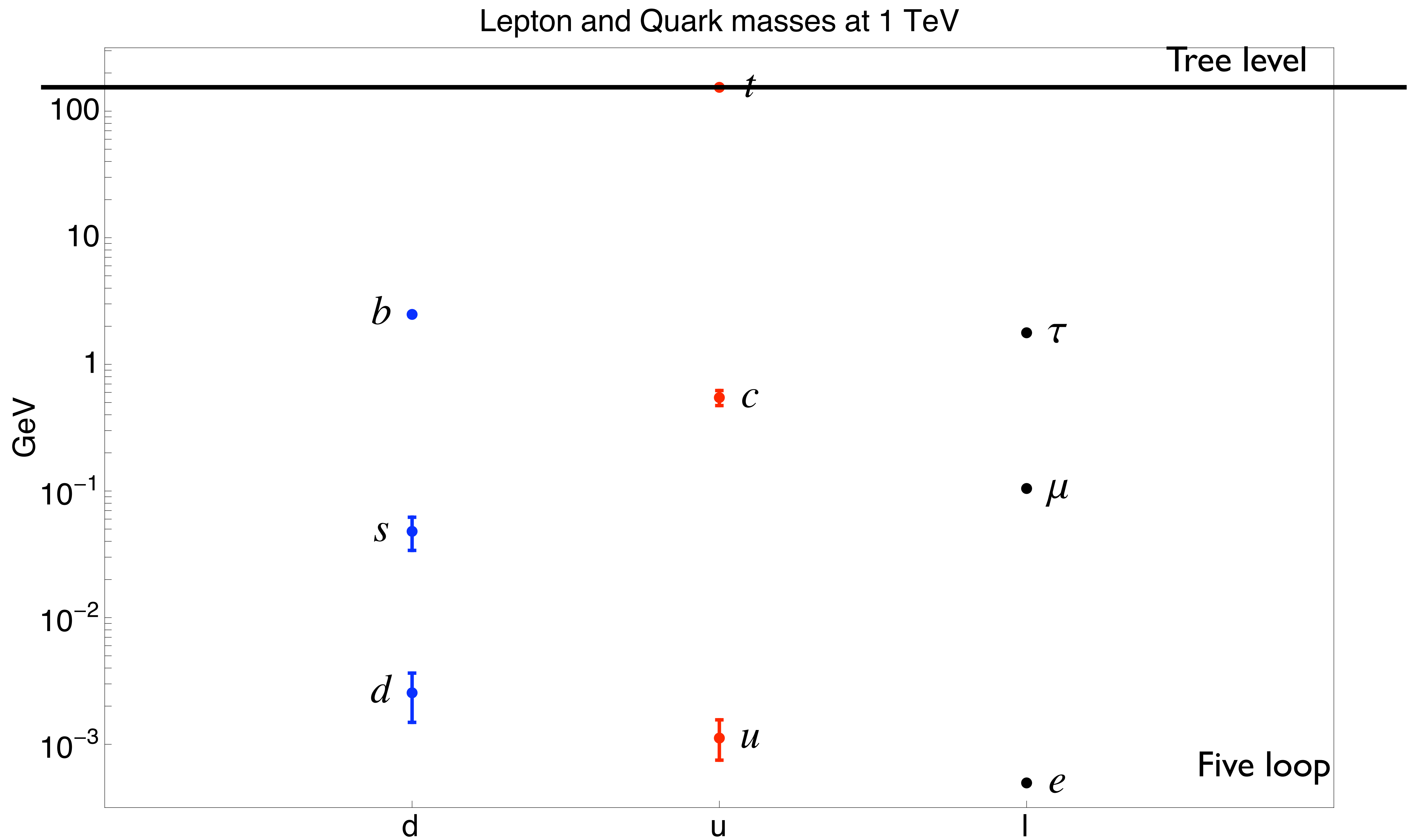


Lepton and Quark masses at 1 TeV
GeV
100
10
1
10−1
10−2
10−3
t
b
τ
c
μ
s
d
u
e
Five loop
d
u
l

Fermilab



Lepton and Quark masses at 1 TeV
Tree level
Five loop
GeV
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10
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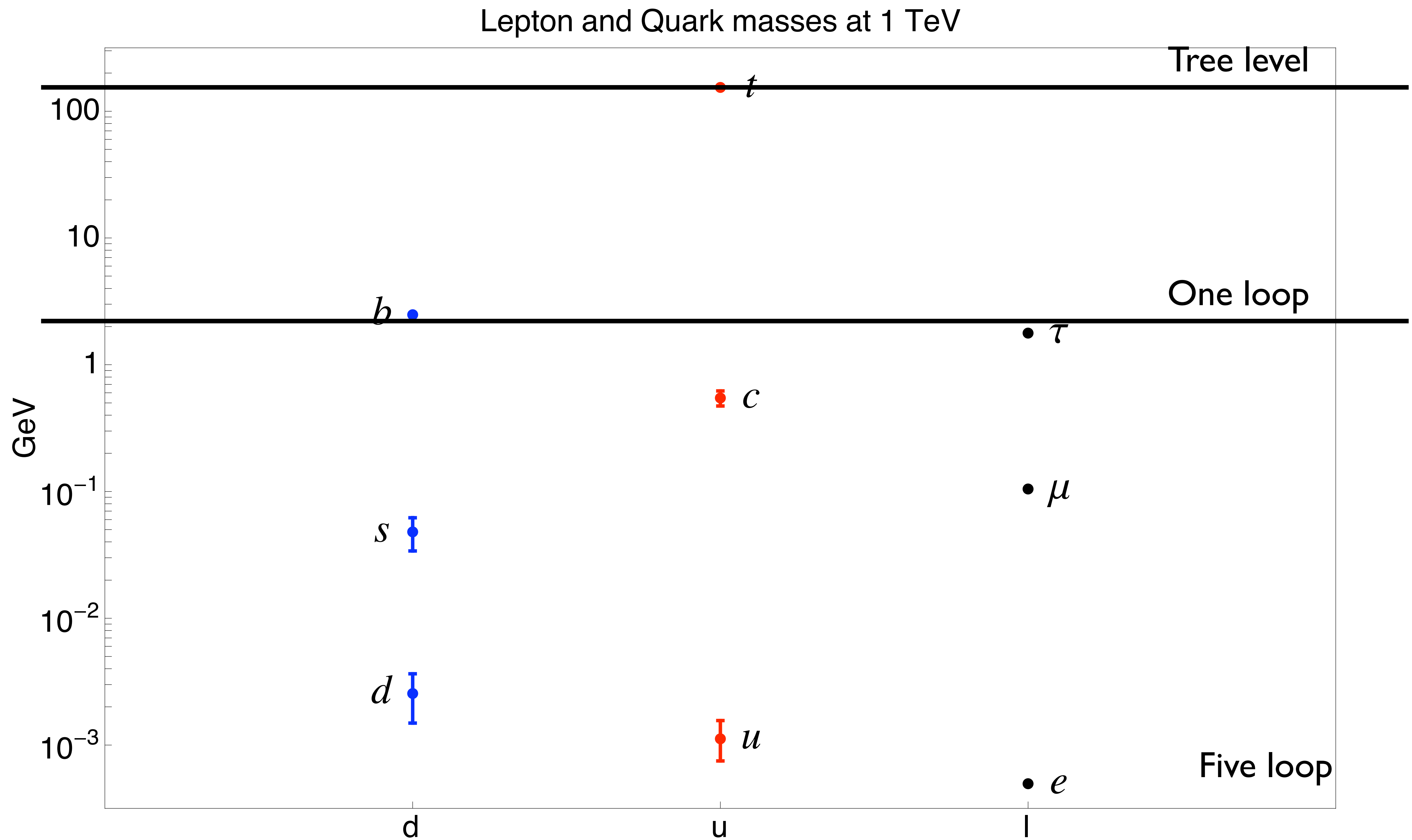
Fermilab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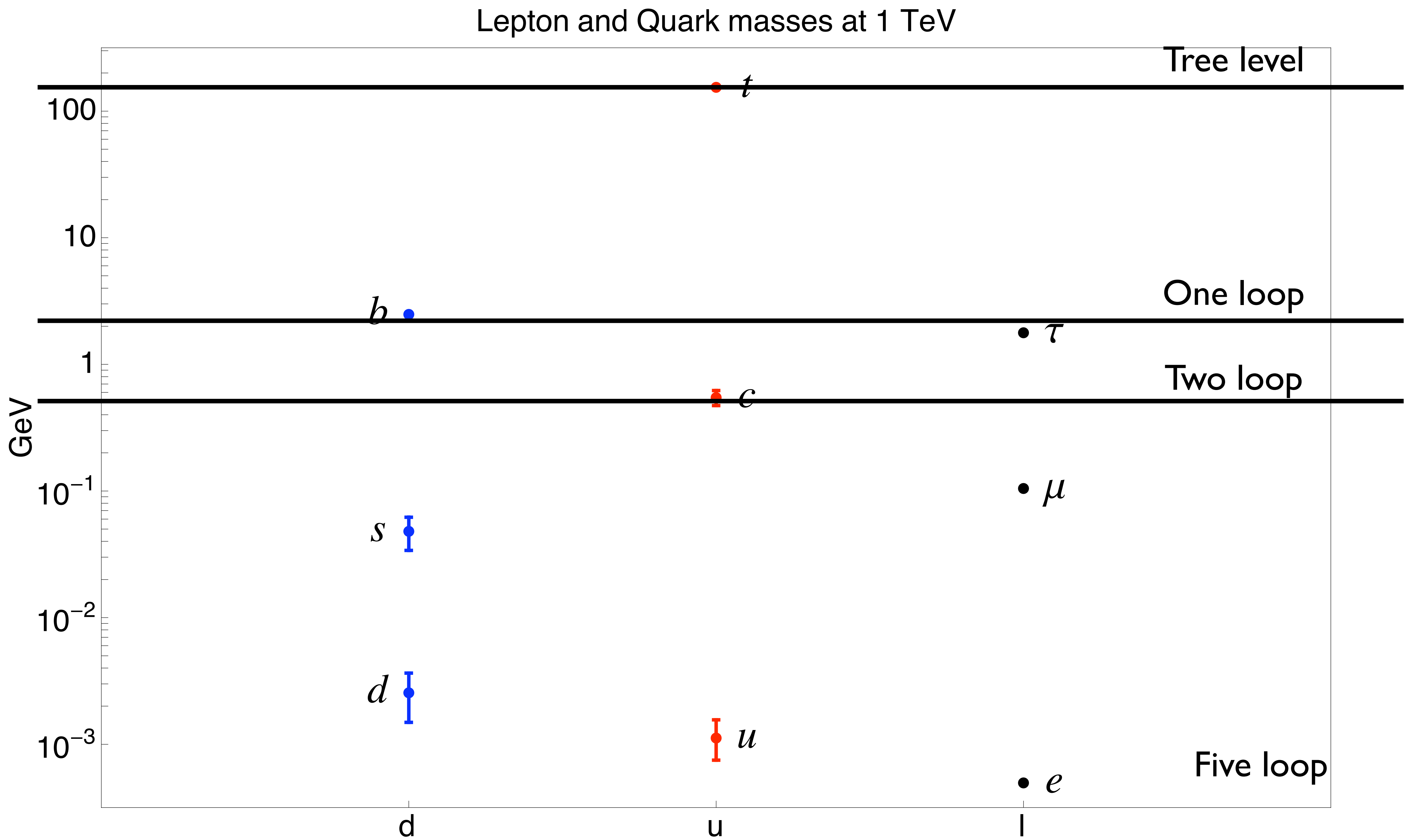
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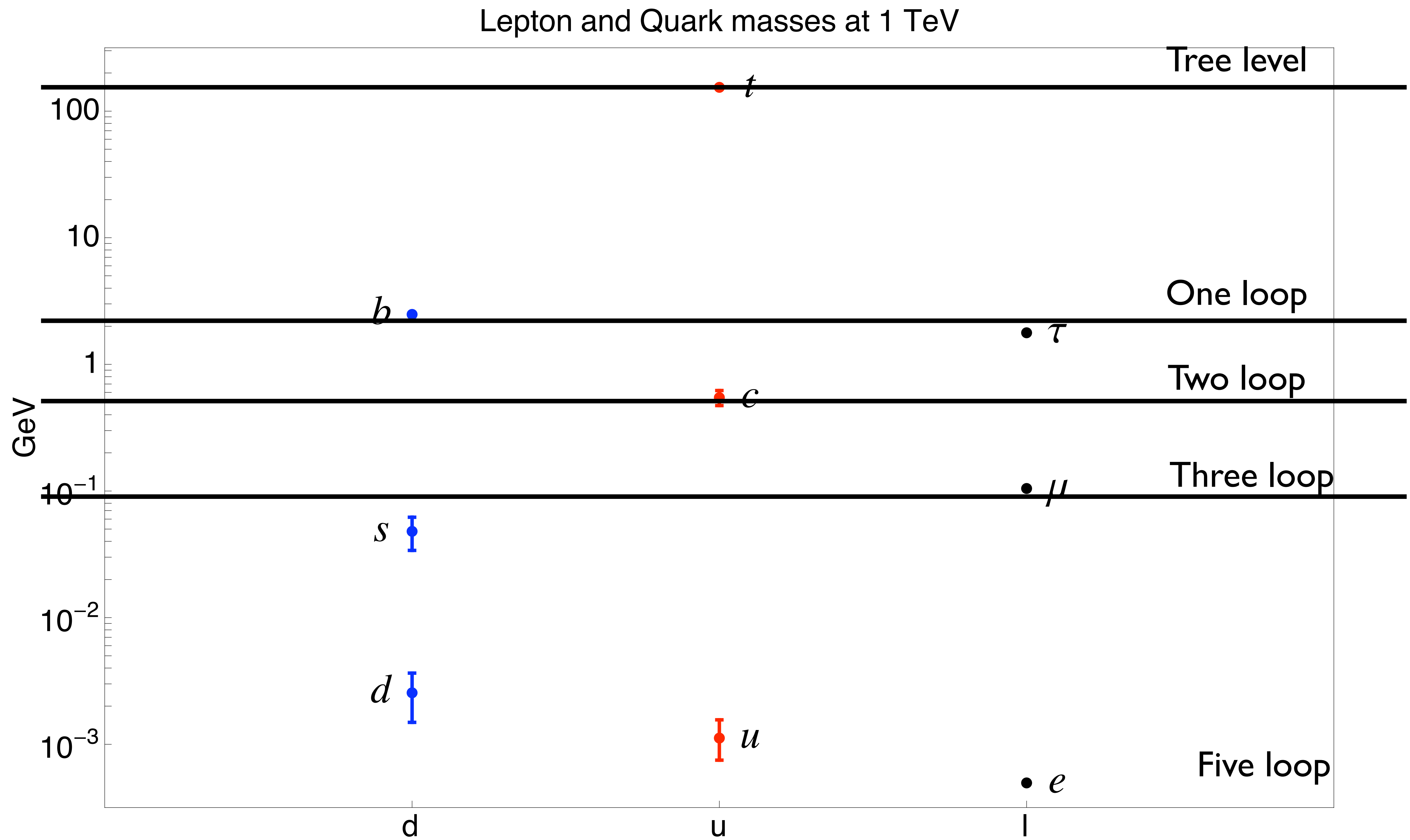
Fermilab


Lepton and Quark masses at 1 TeV
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Three loop
Five loop
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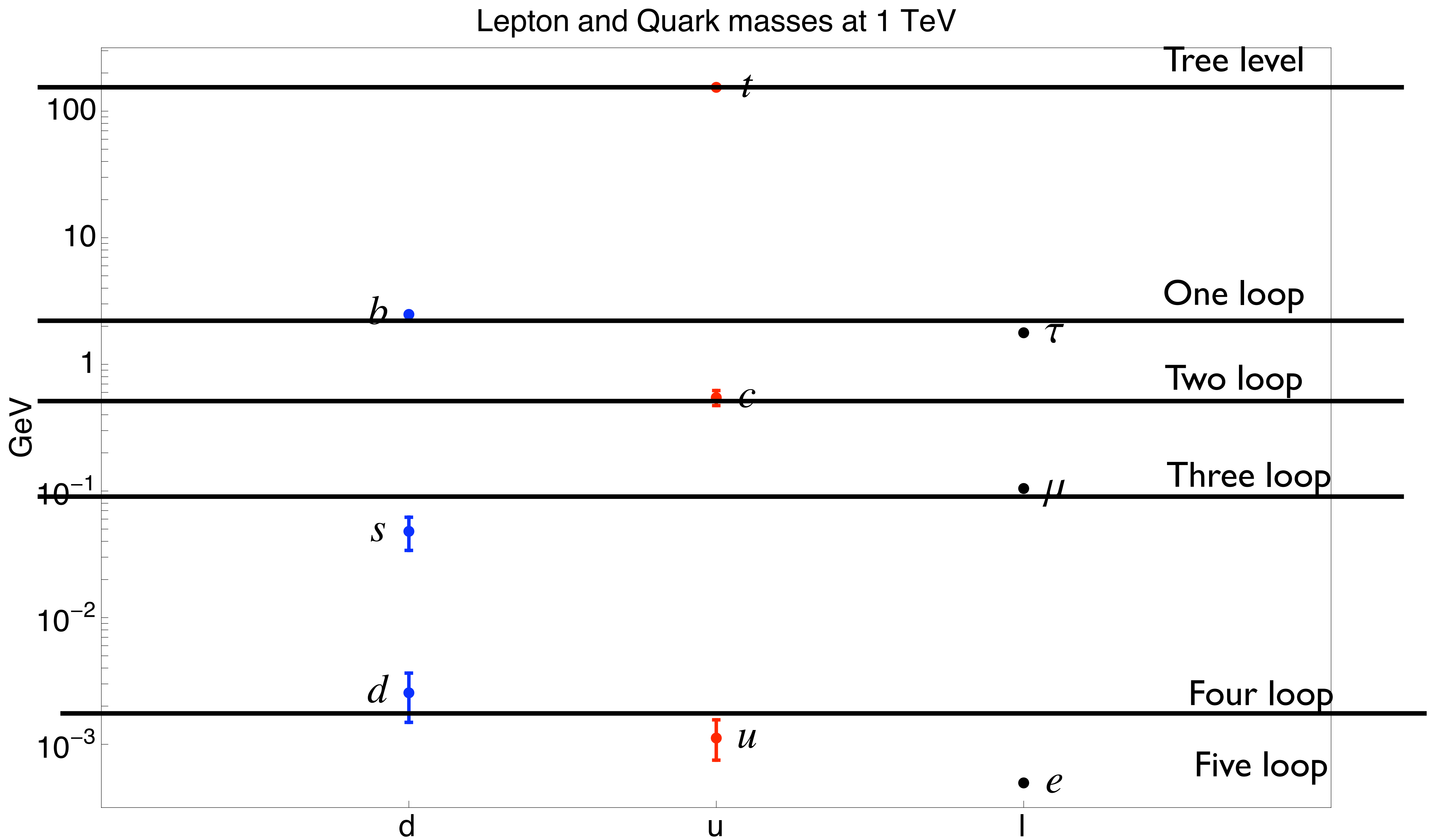
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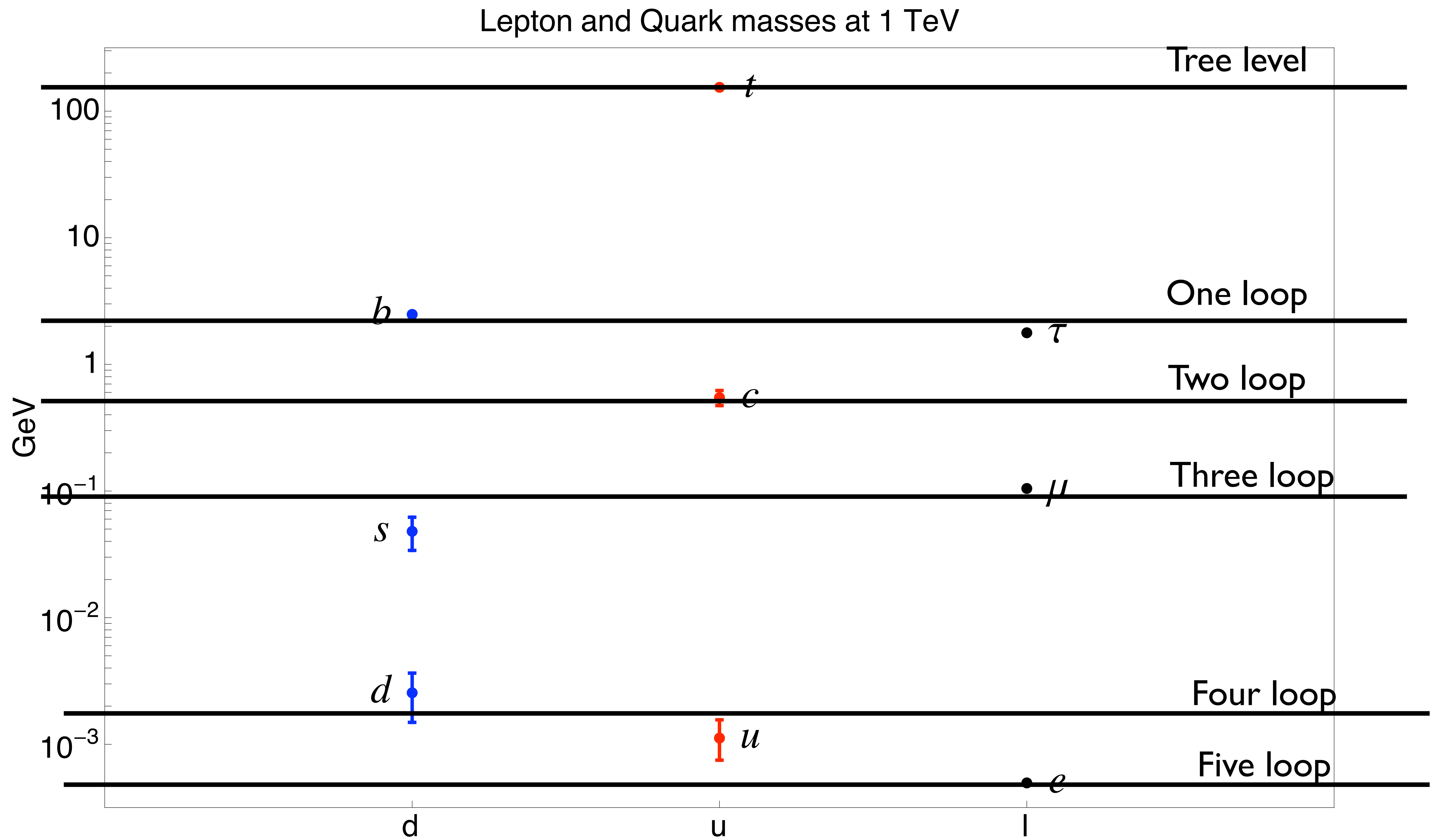
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e
d
u
l

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CKM

$$m_u \approx m_t \begin{pmatrix} \epsilon^4 & \epsilon^2 & \epsilon^2 \\ \epsilon^4 & \epsilon^2 & \epsilon^2 \\ \epsilon^4 & \epsilon^2 & 1 \end{pmatrix} \qquad m_d \approx m_t \begin{pmatrix} \epsilon^4 & \epsilon^4 & \epsilon^4 \\ \epsilon^4 & \epsilon^3 & \epsilon^3 \\ \epsilon^4 & \epsilon^3 & \epsilon \end{pmatrix}$$

Resulting in

$$V_{CKM} \approx \begin{pmatrix} 1-\epsilon^2 & \epsilon & \epsilon^3 \\ -\epsilon & 1-\epsilon^2 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix}$$

Still to think about phases...

The model contains extra fermions and scalar **Leptoquarks**

(Alternative realisation contains diquarks - easier to see at LHC than TeVatron)

Mass scales

$$m_f \approx \text{ parameters } \times m_t \times \left[\frac{1}{16\pi^2}\log\left(\frac{M^2}{M'^2}\right)\right]^n$$

Only determines ratio of masses

Works at all scales, what is the lowest?

Constraints

Tree level exchange of leptoquark can lead to flavour changing processes e.g.

$$K^+ \to \mu^+ e^- \pi^+ \qquad BR < 10^{-11}$$

$$\tau^+ \to K^0 e^+$$

$$\pi^+ \to e^+ \nu \text{ versus } \pi^+ \to \mu^+ \nu$$

$$\mu \to e \text{ conversion}$$

$$\boxed{M \ \gtrsim 5 - 50 \text{ TeV}}$$

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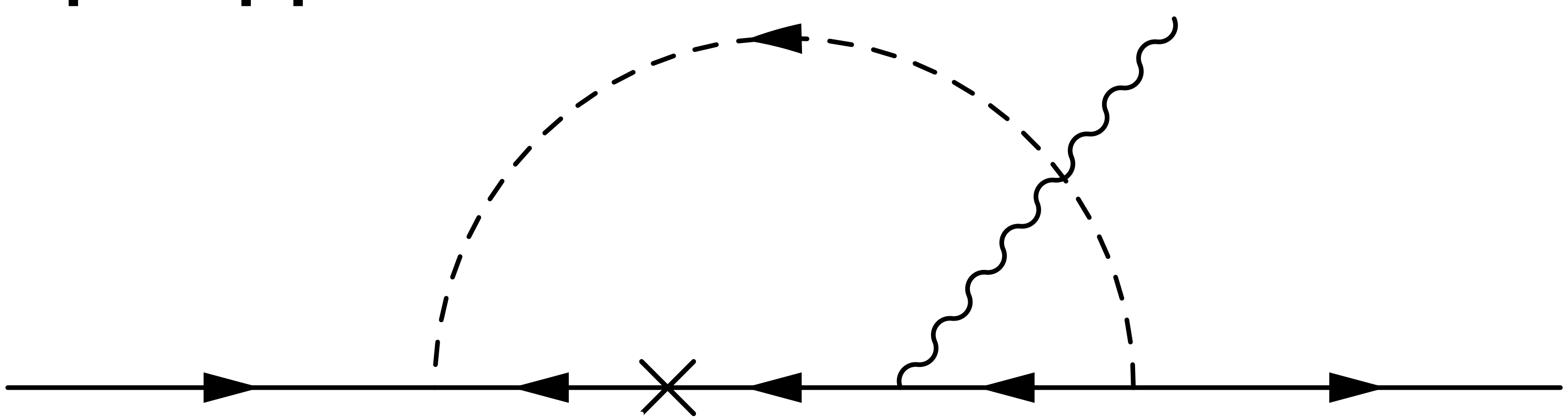
Dipole moments

Usually loop suppressed



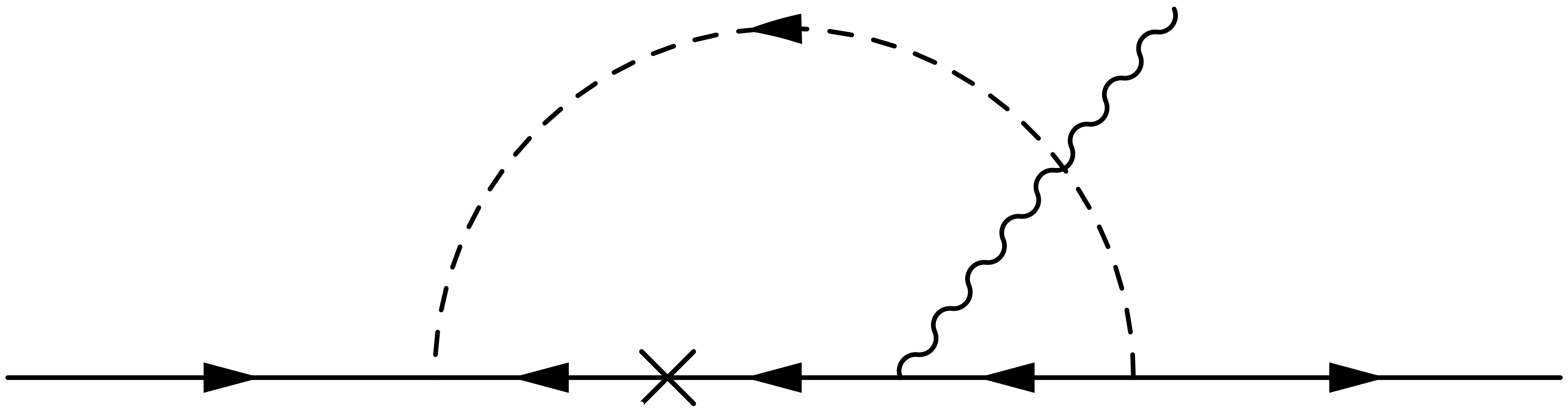
$$\sim \frac{1}{16\pi^2}\frac{m_f}{M^2}e$$

But for us mass is already a loop effect so no additional loop suppression

$$\sim \frac{m_f}{M^2}e$$

Dipole moments

Usually loop suppressed



$$\sim \frac{1}{16\pi^2}\frac{m_f}{M^2}e$$

But for us mass is already a loop effect so no additional loop suppression

$$\sim \frac{m_f}{M^2}e \qquad M > \text{a few TeV}$$

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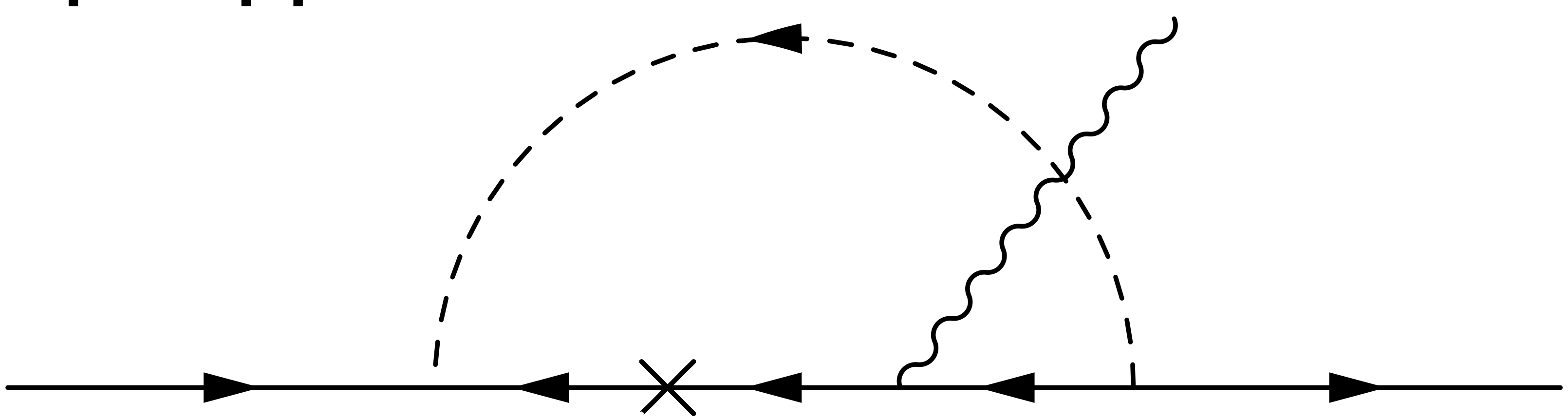
Dipole moments

Usually loop suppressed



$$\sim \frac{1}{16\pi^2}\frac{m_f}{M^2}e$$

But for us mass is already a loop effect so no additional loop suppression

$$\sim \frac{m_f}{M^2}e$$

$M >$ a few TeV

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Conclusions

- Fermions have complicated mass hierarchy
- Many attempts exist to explain it
- Top is probably special, perhaps only top mass has a tree level Yukawa
- With extra scalars coupling to fermions top mass is communicated at loop level
- Interesting structure of fermion mass spectrum arises
- Predicts flavour changing processes

Conclusions

- Fermions have complicated mass hierarchy
- Many attempts exist to explain it
- Top is probably special, perhaps only top mass has a tree level Yukawa
- With extra scalars coupling to fermions top mass is communicated at loop level
- Interesting structure of fermion mass spectrum arises
- Predicts flavour changing processes
- Project X?