# The role of SUSY flat directions in reheating

Marco Peloso, Minnesota

- Reheating after inflation
- Thermalization with SUSY flat directions
- Nonperturbative decay

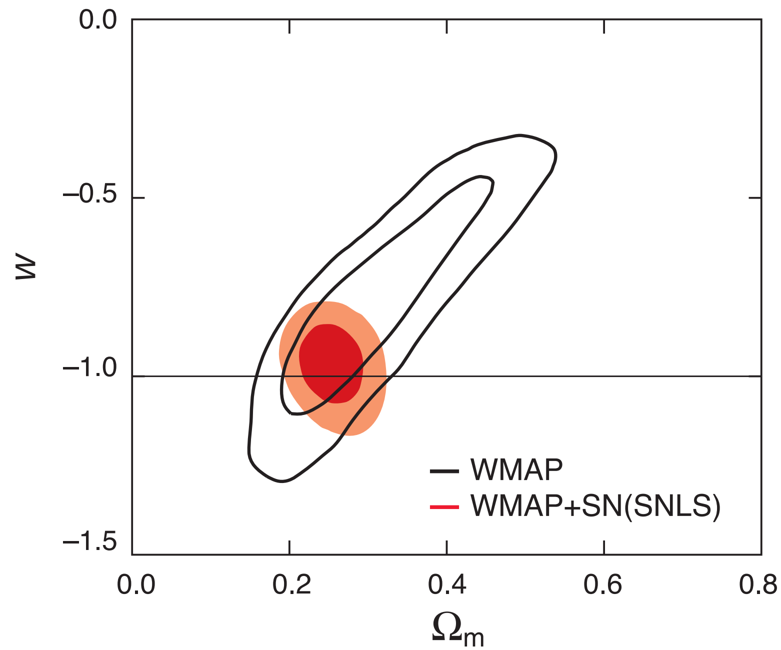
K.A. Olive, MP, PRD 74

A.E. Gumrukcuoglu, K.A. Olive, MP, M. Sexton '08

# History of the Universe

Clear knowledge

from BBN on




WMAP3

$\Omega_m = 0.249^{+0.024}_{-0.031}$ $w = -0.97^{+0.07}_{-0.09}$ $T_\gamma \simeq 2.7K$

Dark energy $z \,\epsilon\, [0,\, 0.4]$

Dark matter $z \,\epsilon\, \left[0.4,\, 10^4\right]$

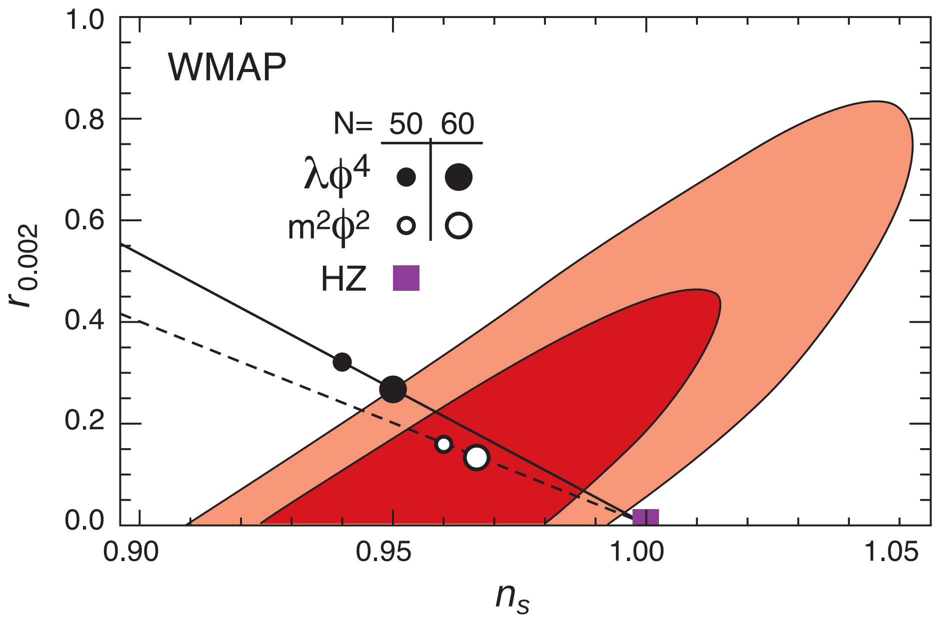
Radiation $z \,\epsilon\, \left[10^4,\, ?\right]$

$z_{\rm BBN} \simeq 10^{10}$

# Good theoretical control & data for inflation

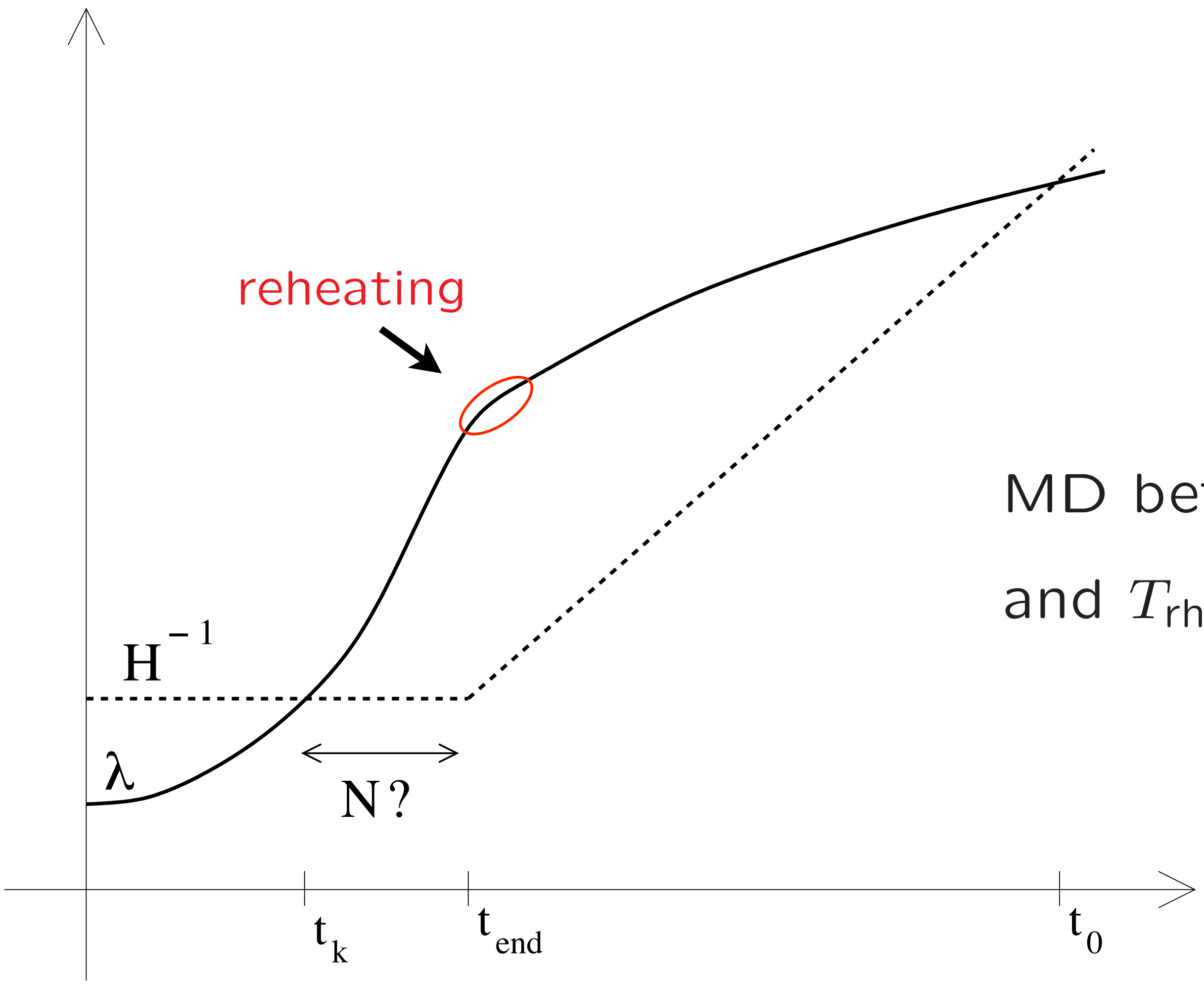
Slow Roll : $\epsilon = \frac{M_p^2}{16\pi}\left(\frac{V'}{V}\right)^2 \;, \;\; \eta = \frac{M_p^2}{8\pi}\frac{V''}{V} \;, \;\ldots$

- COBE normalization $\left(\frac{V}{\epsilon}\right)^{1/4} = 6.7 \cdot 10^{16}\,\mathrm{GeV}$
- Spectral index $n_s - 1 = -6\epsilon + 2\eta$
- Tensor mode $P_T(k) = \frac{2}{M_p^2}\left(\frac{H_k}{2\pi}\right)^2 \;, \quad r = 16\,\epsilon$




WMAP3

# Uncertainty on N




MD between $H_{\mathrm{inf}} = 10^{13}$ GeV

and $T_{\mathrm{rh}} = 10^{9}$ GeV

vs.

immediate decay
at preheating

$\Delta N \sim 4$

Inflation → REHEATING → Hot big-bang cosmology

## Unknowns:

Scale of inflation

Inflaton $\phi$

Coupling to matter

## Require:

$T >$ MeV, for Nucleosynthesis

No gravitinos, $T < 10^9$ GeV

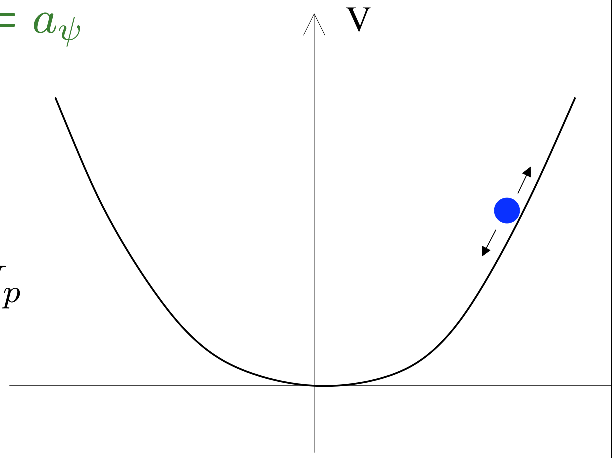
Baryon & dark–matter

Here, assume coupling is small enough $\rightarrow$ no preheating

Gravitational decay $\Gamma \sim m_\psi^3 \,/\, M_p^2$

# Perturbative inflaton decay and thermalization

(order of magnitude estimates)




- Inflaton $\psi$ oscillations start at end of inflation, $a = a_\psi$

$$\rho_\psi = m_\psi^2 \psi^2 = m_\psi^2 M_p^2 \left(a_\psi / a\right)^3$$

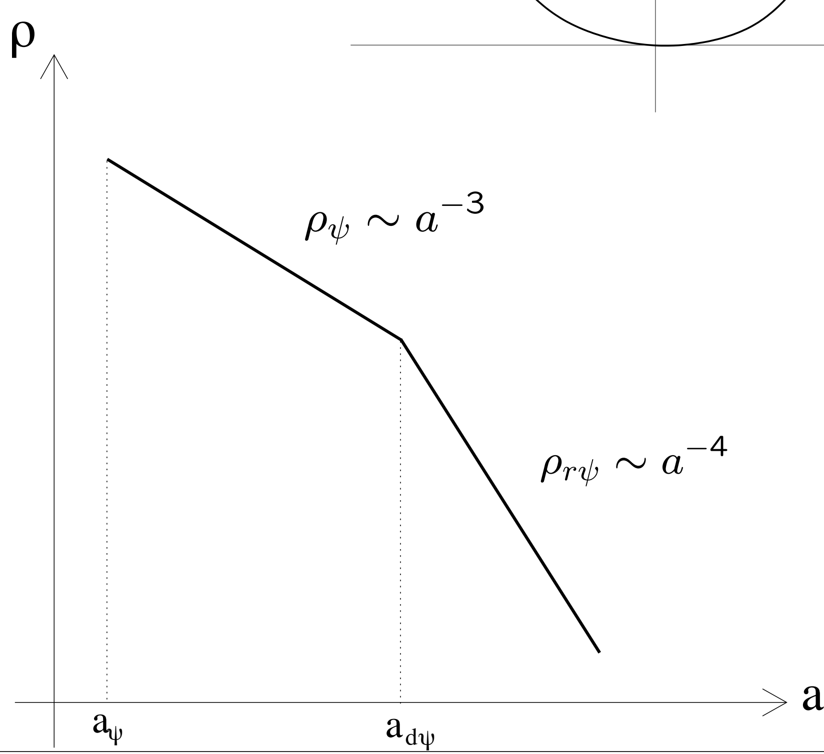
- Decay at $a = a_{d\psi}$, when $\Gamma_\psi \sim m_\psi^3 / M_p^2 = H \sim \rho_\psi^{1/2} / M_p$

$$\rho_{r\psi} = m_\psi^{2/3} M_p^{10/3} \left(a_\psi / a\right)^4$$

- Inflaton $\rightarrow$ relativistic quanta with

$$E \sim m_\psi \quad , \quad N = \frac{\rho}{E} \sim \frac{m_\psi^5}{M_p^2}$$

$E \gg N^{1/3} \quad \rightarrow \quad$ particle dissociation



Assume inflaton decays into particles (fermions) with gauge interactions

Ellis, Enqvist, Nanopoulos, Olive '87

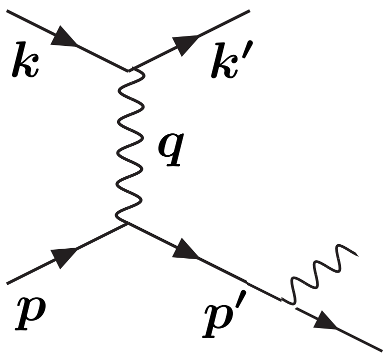
Davidson, Sarkar '00

# $2 \rightarrow 3$ processes

- New particles must be produced to "absorb" the energy loss.

  $2 \rightarrow 2$ lead to kinetic equilibrium, but not to chemical equilibrium

$$\sigma_{inel} \sim \alpha^3 \int \frac{dt}{t^2} \int \frac{dp'^2}{p'^2} \sim \frac{\alpha^3}{\rho_{r\psi}^{1/2}} \ln\left(\frac{m_\psi^2}{\rho_{r\psi^{1/2}}}\right)$$




At inflaton decay , $\frac{1}{\rho_{r\psi}^{1/2}} = \frac{1}{m_\psi^2} \frac{M_p}{m_\psi}$

$$\Gamma_{2\rightarrow 3} \simeq \sigma_{\text{inel}} N > H \sim \frac{\rho_{r\psi}^{1/2}}{M_p}$$

Instantaneous thermalization

# MSSM flat directions

MSSM potential $V = \sum_i |F|^2 + \frac{1}{2}\sum_a g_a^2 D^a D^a$

$$F_i \equiv \frac{\partial W_{MSSM}}{\partial \phi_i}, \quad D^a = \phi^\dagger T^a \phi \qquad W_{MSSM} = \lambda_u Q H_u \bar{u} + \lambda_d Q H_d \bar{d} + \lambda_e L H_d \bar{e} \; + \mu H_u H_d$$

Plethora of flat directions, lifted by

| | $B-L$ | | $B-L$ |
|---|---|---|---|
| $H_u H_d$ | 0 | $L H_u$ | -1 |
| $\bar{u}\bar{d}\bar{d}$ | -1 | $QL\bar{d}$ | -1 |
| $LL\bar{e}$ | -1 | $QQ\bar{u}\bar{d}$ | 0 |
| $QQQL$ | 0 | $QL\bar{u}\bar{e}$ | 0 |
| $\bar{u}\bar{u}\bar{d}\bar{e}$ | 0 | $QQQQ\bar{u}$ | 1 |
| $QQ\bar{u}\bar{u}\bar{e}$ | 1 | $LL\bar{d}\bar{d}\bar{d}$ | -3 |
| $\bar{u}\bar{u}\bar{u}\bar{e}\bar{e}$ | 1 | $QLQL\bar{d}\bar{d}$ | -2 |
| $QQLL\bar{d}\bar{d}$ | -2 | $\bar{u}\bar{u}\bar{d}\bar{d}\bar{d}\bar{d}$ | -2 |
| $QQQQ\bar{d}LL$ | -1 | $QLQLQL\bar{e}$ | -1 |
| $QL\bar{u}QQ\bar{d}\bar{d}$ | -1 | $\bar{u}\bar{u}\bar{u}\bar{d}\bar{d}\bar{d}\bar{e}$ | -1 |

Gherghetta, Kolda, Martin '95

- O (TeV) masses from $\mu-$ and soft $\not{S}$USY- terms
- Nonrenormalizable interactions

$$W = \frac{\lambda}{d}\frac{\Phi^d}{M^{d-3}}$$

- O (H) masses from SUGRA

Dine, Randall, Thomas '95

$$V = \left(\phi_1^2 - \phi_2^2\right)^2 + m_1^2 \phi_1^2 + m_2^2 \phi_2^2 + \ldots$$

# Cosmological evolution of flat directions

Light fields, "build up" in a dS space

- Every $\Delta t \sim H^{-1}$, fluctuations $\delta\phi \sim H$ are generated on each domain $\Delta x \sim H^{-1}$
- Cosmological expansion streches $\phi + \delta\phi$ on super-horizon scales
  $\rightarrow$ new homogeneous background
- New fluctuations add up $\phi \rightarrow \phi + \delta\phi \rightarrow (\phi + \delta\phi) + \delta\phi \rightarrow \ldots$

Random walk, leading to a homogeneous $\langle\phi\rangle \neq 0$

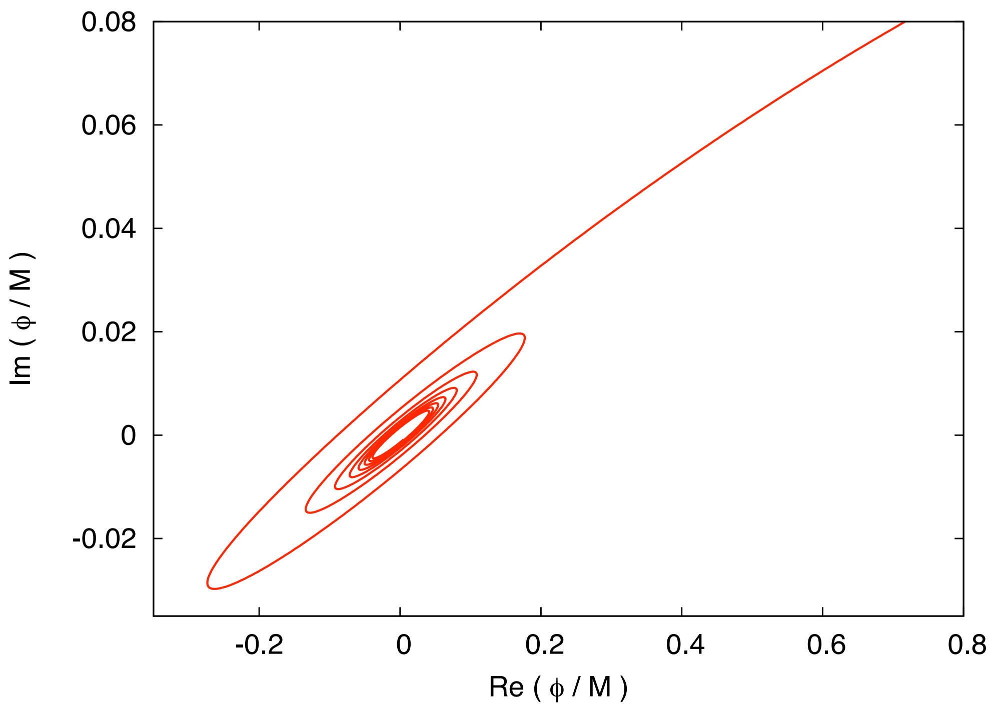
If $m \ll H$ , and if inflation lasts long enough, $\langle\phi^2\rangle = \dfrac{3H^4}{8\pi^2 m^2}$

In practice, flat direction pushed up to $\phi \lesssim M$ , where the nonrenormalizable terms stop the growth

Many flat directions are mutually exclusive; if one is "switched on", many other acquire $\sim |\phi|$ masses. In general, we expect a set of non mutually exclusive flat directions to acquire a large VEV during inflation

- After inflation, flat direction frozen as long as $H > m_\phi$
- As $H$ decreases, spiral motion towards origin

$$V \sim m^2|\phi|^2 + \frac{|\lambda|^2|\phi|^{2d-2}}{M^{2d-6}} + \left( A\,\frac{\lambda\,\phi^d}{d\,M^{d-3}} + \text{h.c.} \right)$$

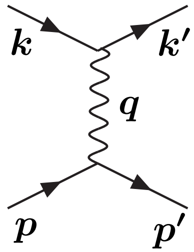



"angular momentum"

from $A$–term

Claim: flat directions delay thermalization by providing a large effective mass to gauge bosons

Allahverdi, Mazumdar '05,'06




$$\frac{\alpha^2}{E^2} = \frac{\alpha^2}{m_\psi^2}\left(\frac{R}{R_{d\psi}}\right)^2 \rightarrow \frac{\alpha^2}{\phi^2}$$

Inflaton decays at $H \lesssim m_\psi^3/M_p^2 \lesssim$ TeV. Flat direction starts evolving shortly before

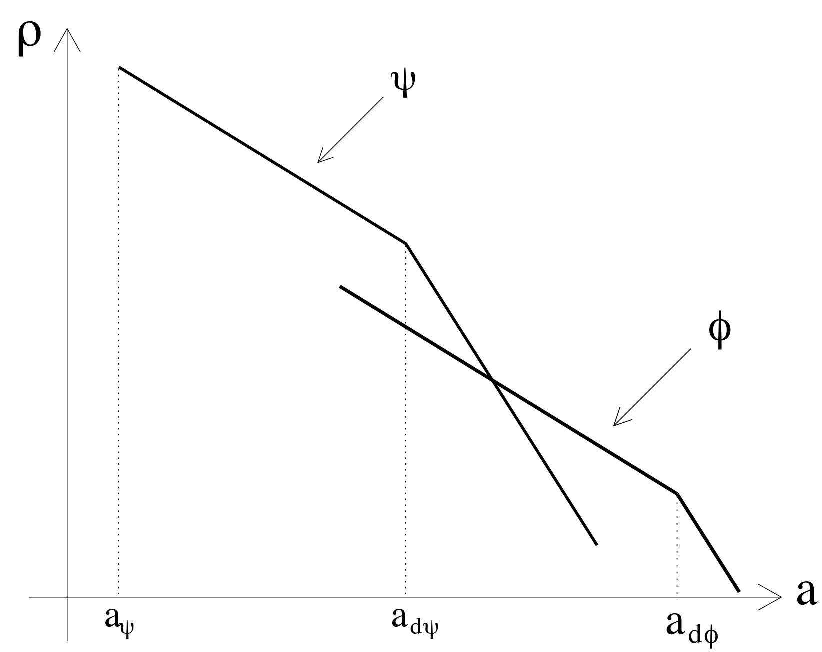
Amplitude: $\phi^2 = \phi_0^2 \frac{m_\psi^2}{m_\phi^2}\left(\frac{a_\psi}{a}\right)^3$ Decay rate $\Gamma \sim m_\phi^3/\phi^2$ Affleck, Dine '84

- Thermalization delayed if

$$\phi_0 \gtrsim \alpha^{3/2}\frac{M_p^{5/2} m_\phi}{m_\psi^{5/2}} \sim 10^{16}\ \text{GeV}$$

- It dominates if $\phi_0 \gtrsim \frac{M_p^{4/3} m_\phi^{5/12}}{m_\psi^{3/4}} \simeq 10^{15}$ GeV

$$\Rightarrow T_{\rm rh} \simeq m_\phi^{5/6} M_p^{1/6} \simeq 10^5\ \text{GeV}$$

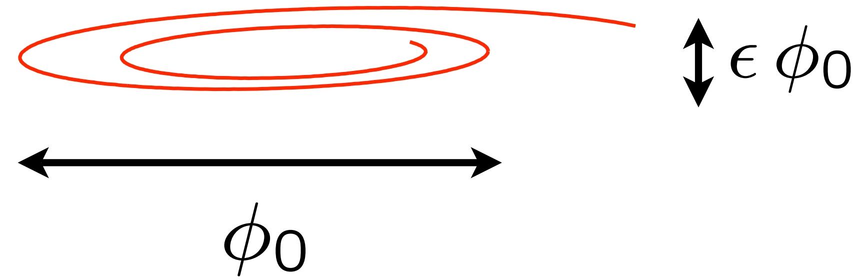


# Nonperturbative decay of flat directions ?

$$V = \frac{1}{2}\, m^2\, |\phi|^2 + \frac{g^2}{2}\, |\phi|^2\, |\chi|^2 \quad , \qquad \omega_\chi^2 = p^2 + g^2\, |\phi|^2 \simeq g^2\, |\phi|^2$$

Nonperturbative production if

$$\omega' \,/\, \omega^2 > 1$$




$$\omega' \simeq g\, \epsilon\, \phi_0\, m_\phi$$

$$\omega^2 \simeq g^2 \epsilon^2 \phi_0^2$$

$$\Rightarrow \quad \frac{\omega'}{\omega^2} \simeq \frac{m_\phi}{g\, \epsilon\, \phi_0} \simeq \frac{10^{-14}}{\epsilon}$$

Typically, $10^{-3} \lesssim \epsilon \lesssim 10^{-1}$ $\quad \Rightarrow$ only perturbative decay

Allahverdi, Shaw, Campbell '99

Postma, Mazumdar '03

Allahverdi, Mazumdar '05

Realistic cases are more interesting

$$H_u = \begin{pmatrix} h_u \\ \phi + \xi_u \end{pmatrix} \qquad H_d = \begin{pmatrix} \phi + \xi_d \\ h_d \end{pmatrix}$$

Quadratic potential in fluctuations:

$$\phi = |\phi|\, \mathrm{e}^{i\sigma}$$

$$V = \frac{\lambda_u^2}{2}|\phi|^2 \left(|Q_u|^2 + |u|^2\right) + \frac{\lambda_d^2}{2}|\phi|^2 \left(|Q_d|^2 + |u|^2\right) + \frac{\lambda_e^2}{2}|\phi|^2 \left(|L_d|^2 + |e|^2\right)$$

$$+\frac{g^2 + g'^2}{16}|\phi|^2 \left(\xi_{u,r} - \xi_{d,r}\ ,\ \xi_{u,i} - \xi_{d,i}\right) \mathcal{M}^2 \begin{pmatrix} \xi_{u,r} - \xi_{d,r} \\ \xi_{u,i} - \xi_{d,i} \end{pmatrix}$$

$$+\frac{g^2}{8}|\phi|^2 \left(h_{u,r} + h_{d,r}\ ,\ h_{u,i} + h_{d,i}\right) \mathcal{M}^2 \begin{pmatrix} h_{u,r} + h_{d,r} \\ h_{u,i} + h_{d,i} \end{pmatrix}$$

$$+\frac{g^2}{8}|\phi|^2 \left(-h_{u,i} + h_{d,i}\ ,\ h_{u,r} - h_{d,r}\right) \mathcal{M}^2 \begin{pmatrix} -h_{u,i} + h_{d,i} \\ h_{u,r} - h_{d,r} \end{pmatrix}$$

(up to TeV masses)

$$\mathcal{M}^2 = \begin{pmatrix} \cos^2\sigma & \cos\sigma\sin\sigma \\ \cos\sigma\sin\sigma & \sin^2\sigma \end{pmatrix}$$

Eigenvalues $\{1,\, 0\}$

Quick $t-$dependence through $\sigma\,(t)$

# Quantized coupled system

Nilles, M.P., Sorbo '01

Non-diagonal mass matrix

$$\phi^T M^2 \phi = \underbrace{\phi^T C}_{\tilde{\phi}^T} \underbrace{C^T M^2 C}_{\mu_d^2} \underbrace{C^T \phi}_{\tilde{\phi}}$$

If $C$ constant ($M$ constant) no physical effect.

Otherwise $\quad \phi'^T \phi' = \tilde{\phi}'^T \tilde{\phi}' + \underbrace{\tilde{\phi}'^T \Gamma \tilde{\phi} + \tilde{\phi}^T \Gamma^T \tilde{\phi}'}_{} + \tilde{\phi}^T C'^T C' \tilde{\phi}$

$\Gamma = C^T C'$ kinetic mixing

$$\tilde{\phi}_i = \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} \left\{ \mathrm{e}^{i\mathbf{kx}} \left[ \underbrace{\frac{\mathrm{e}^{-i\int^t \omega dt}}{\sqrt{2\omega}} A}_{\alpha} + \underbrace{\frac{\mathrm{e}^{i\int^t \omega dt}}{\sqrt{2\omega}} B}_{\beta} \right]_{ij} a_j + \mathrm{e}^{-i\mathbf{kx}} \left[ \ldots \right]_{ij}^* a_j^\dagger \right\}$$

Bogolyubov matrices

$\omega = \sqrt{k^2 + \mu_d^2}$ diagonal

$$\mathcal{H} = \frac{1}{2} \left(a^\dagger,\, a\right) \begin{pmatrix} \alpha^\dagger & \beta^\dagger \\ \beta^T & \alpha^T \end{pmatrix} \begin{pmatrix} \omega & 0 \\ 0 & \omega \end{pmatrix} \underbrace{\begin{pmatrix} \alpha & \beta^* \\ \beta & \alpha^* \end{pmatrix} \begin{pmatrix} a \\ a^\dagger \end{pmatrix}}$$

$t-$dependent annihilation / creation operators $\begin{pmatrix} \hat{a} \\ \hat{a}^\dagger \end{pmatrix}$ $\Rightarrow \; : \mathcal{H} := \omega_i \, \hat{a}_i^\dagger \, \hat{a}_i$

Occupation numbers $N_i(t) = \langle \hat{a}_i^\dagger \, \hat{a}_i \rangle = \left(\beta^* \, \beta^T\right)_{ii}$

Equations of motion:

$$\begin{aligned} \alpha' &= -i\,\omega\,\alpha + \frac{\omega'}{2\omega}\beta - I\,\alpha - J\,\beta \\ \beta' &= i\,\omega\,\beta + \frac{\omega'}{2\omega}\alpha - I\,\beta - J\,\alpha \end{aligned}$$

$$\begin{aligned} I &= \frac{1}{2}\left(\sqrt{\omega}\,\Gamma\,\frac{1}{\sqrt{\omega}} + \frac{1}{\sqrt{\omega}}\,\Gamma\,\sqrt{\omega}\right) \\ J &= \frac{1}{2}\left(\sqrt{\omega}\,\Gamma\,\frac{1}{\sqrt{\omega}} - \frac{1}{\sqrt{\omega}}\,\Gamma\,\sqrt{\omega}\right) \end{aligned}$$

Plane wave ($\omega$ const.)

Standard nonadiabatic production for $\omega' > \omega^2$

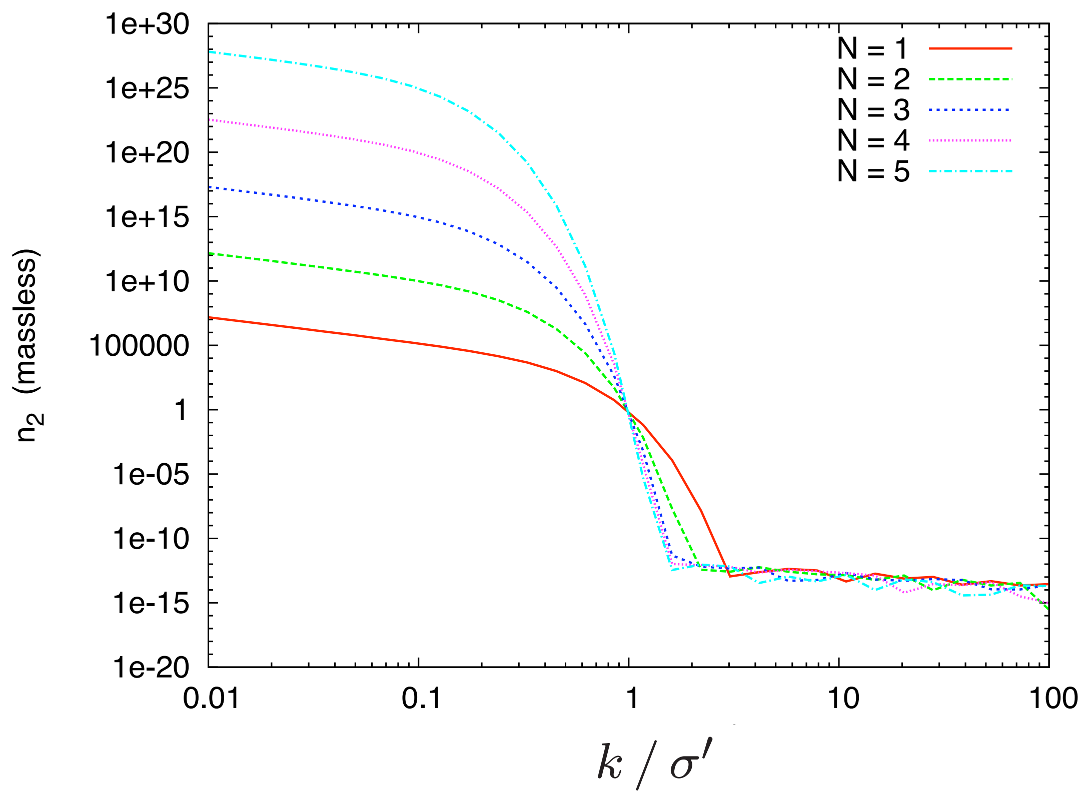
Production from mixing

$$M^2 = 2g^2|\phi|^2 \begin{pmatrix} \cos^2\sigma & \cos\sigma\,\sin\sigma \\ \cos\sigma\,\sin\sigma & \sin^2\sigma \end{pmatrix}$$

Analytic solution if $|\phi|$ and $\sigma' \equiv m_\phi$ constant $\begin{cases} \text{Minkowski space} \\ \text{circular orbit} \end{cases}$

Resonant band

$$k < \sigma'$$



## Toy model $\longrightarrow$ Complete gauge computation

U(1) flat direction $\quad V_D = \frac{e^2}{8}\left(|\Phi_1|^2 - |\Phi_2|^2\right)^2$

$$\Phi_1 = |\phi|\,\mathrm{e}^{i\sigma} + (\xi + \chi)$$
$$\Phi_2 = |\phi|\,\mathrm{e}^{i\sigma} + (\xi - \chi)$$

↑ VEV ↑ fluctuation

$$V_D \rightarrow (\chi_r, \chi_i)\,\mathcal{M}^2 \begin{pmatrix} \chi_r \\ \chi_i \end{pmatrix}$$

Decay (fragmentation) into their own fluctuations

However, light eigenstate $\equiv$ goldstone boson

$\{\Phi_1, \Phi_2, A_\mu\}$

| | | | |
|---|---|---|---|
| 4 + 2 degrees of freedom | $\equiv$ | 1 Massive gauge field | (3) |
| | | 1 Flat direction | (2) |
| | | 1 Higgs | (1) |

# Actual MSSM Flat directions

- If a single flat direction excited, no “rotation” in unitary gauge
- If more flat directions excited, more fields involved in rotation

Eg. LLddd-QQQL
$$\langle \nu_e \rangle = \langle \mu \rangle = \langle d_1^c \rangle = \langle s_2^c \rangle = \langle b_3^c \rangle = \phi\, e^{i\sigma}$$
$$\langle t_2 \rangle = \langle d_3 \rangle = \langle c_1 \rangle = \langle \tau \rangle = \bar{\phi}\, e^{i\bar{\sigma}}$$

54 real fields obtain mass from $D-$terms: 12 goldstone bosons, 12 heavy fields, 22 light fields coupled in the mass matrix, 8 decoupled mass fields

E.g. QLd-udd $\langle s_1 \rangle = \langle \nu_e \rangle = \langle d_1^c \rangle = \phi\, e^{i\sigma}$

$$\langle u_1^c \rangle = \langle s_2^c \rangle = \langle b_3^c \rangle = \bar{\phi}\, e^{i\bar{\sigma}}$$

40 fields . . . 8 coupled light fields

Eg. LLe-QLd-udd

| | $B-L$ | | $B-L$ |
|---|---|---|---|
| $H_uH_d$ | 0 | $LH_u$ | -1 |
| $\bar{u}\bar{d}\bar{d}$ | -1 | $QL\bar{d}$ | -1 |
| $LL\bar{e}$ | -1 | $QQ\bar{u}\bar{d}$ | 0 |
| $QQQL$ | 0 | $QL\bar{u}\bar{e}$ | 0 |
| $\bar{u}\bar{u}\bar{d}\bar{e}$ | 0 | $QQQQ\bar{u}$ | 1 |
| $QQ\bar{u}\bar{u}\bar{e}$ | 1 | $LL\bar{d}\bar{d}\bar{d}$ | -3 |
| $\bar{u}\bar{u}\bar{u}\bar{e}\bar{e}$ | 1 | $QLQL\bar{d}\bar{d}$ | -2 |
| $QQLL\bar{d}\bar{d}$ | -2 | $\bar{u}\bar{u}\bar{d}\bar{d}\bar{d}\bar{d}$ | -2 |
| $QQQQ\bar{d}LL$ | -1 | $QLQLQL\bar{e}$ | -1 |
| $QL\bar{u}QQ\bar{d}\bar{d}$ | -1 | $\bar{u}\bar{u}\bar{u}\bar{d}\bar{d}\bar{d}\bar{e}$ | -1 |

Simplest example: two $U(1)$ flat directions

$$V_D = \left(q|\Phi_1|^2 - q|\Phi_2|^2 + q'|\Phi_3|^2 - q'|\Phi_4|^2\right)^2$$

$$\langle\Phi_1\rangle = \langle\Phi_2\rangle = F\,\mathrm{e}^{i\Sigma/2}$$

$$\langle\Phi_3\rangle = \langle\Phi_4\rangle = G\,\mathrm{e}^{i\tilde{\Sigma}/2}$$

$\{\Phi_i,\, A_\mu\}$

$8+2$ degrees of freedom $\equiv$ 1 Massive gauge field (3)
2 Flat directions (4)
1 Higgs
2 Light fields
} Mixing

Eigenmasses$^2$

$$m_1^2 = e^2\left(F^2+G^2\right)$$

$$m_2^2 = \frac{\left(F^2\,\tilde{m}^2 + G^2\,m^2\right)R^2}{F^2+G^2} + \frac{3\,(F\,G' - F'\,G)}{(F^2+G^2)^2} + \frac{3\,F^2\,G^2\,\left(\Sigma' - \tilde{\Sigma}'\right)^2}{4\,(F^2+G^2)^2}$$

$$m_3^2 = \frac{\left(F^2\,\tilde{m}^2 + G^2\,m^2\right)R^2}{F^2+G^2}$$

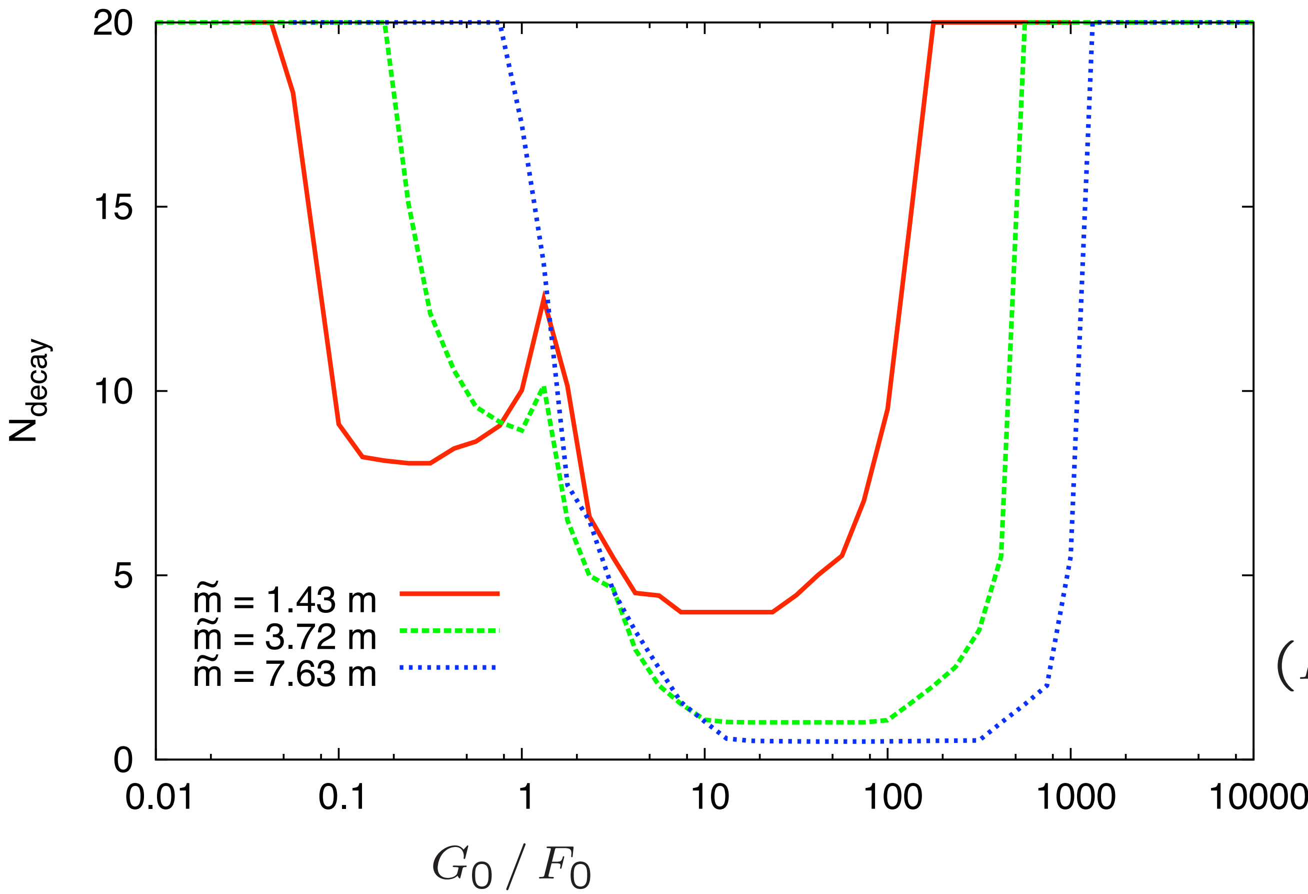
$m_1 \sim M_{\rm GUT} - M_p$ , $m_2,\, m_3 \sim$ TeV

Need to control both scales in simulations.

Fortunately

$$\frac{\rho_{\text{prod}}}{\rho_{\text{flat}}} \simeq \frac{m\,\tilde{m}}{F_0\,G_0} \times A \times 10^{B\,N_{\text{rot}}}$$

$$A,\ B\left[\frac{F_0}{G_0},\ \frac{m}{\tilde{m}}\right]$$

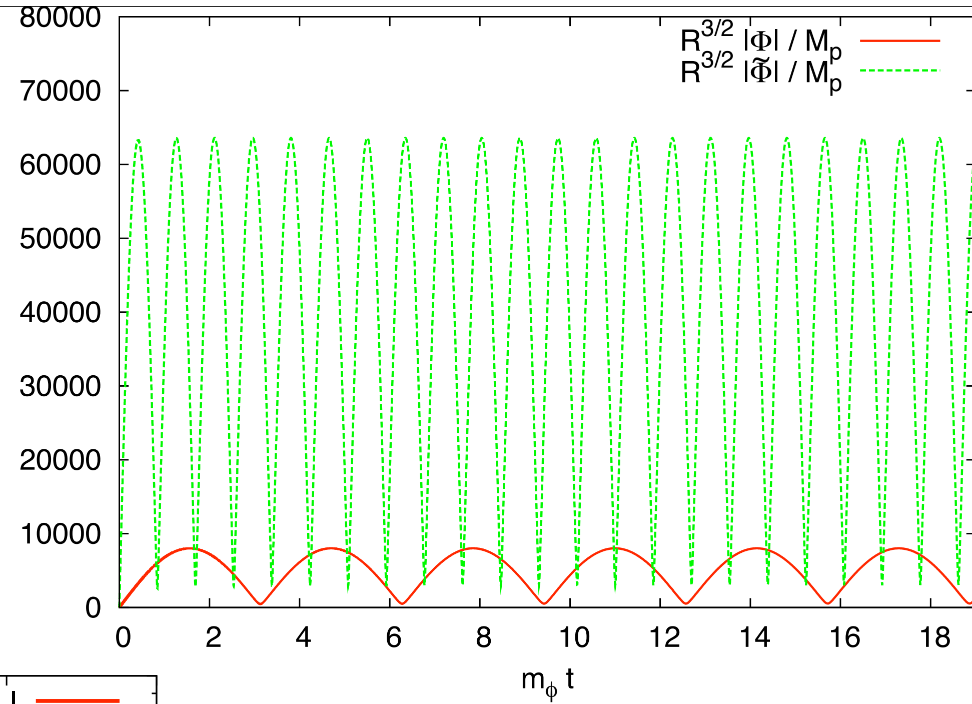



$m = \text{TeV}$

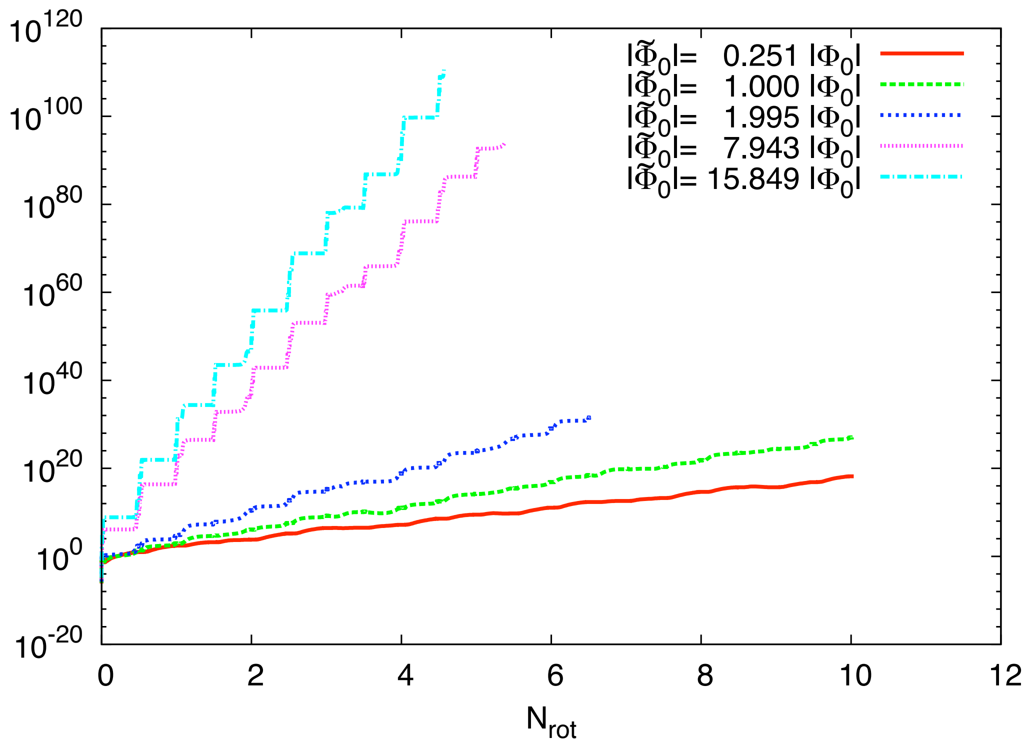
$(F_0\,G_0)^{1/2} = 10^{-2}\,\text{M}_\text{p}$

$$\tilde{m} \sim 3.7\, m$$

$$G_0/F_0 \sim 30$$




$$n_1 + n_2 + n_3\,(k=0)$$




Production whenever

$$|\Phi| \simeq |\tilde{\Phi}|$$

# Nonlinear interactions

After chaotic inflation, for $V = m^2 \phi^2 + g^2 \phi^2 \chi^2 + \lambda \chi^4$

a large quartic term strongly contrasts parametric resonance (large energy in $\lambda \langle \chi^2 \rangle^2$)

Large (gauge) self-interactions for MSSM fields

$$V_D \propto D^a D^a = \left( \phi^* \sum_i c_i^a \, \delta X_i + \text{h.c.} + \sum_{ij} d_{ij}^a \delta \chi_i \, \delta \chi_j \right)^2$$

- Do the large quartic terms prevent preheating, or do we excite combinations of terms for which $D^a$ remains small ?
- Quicker depletion of the zero mode ? (diagrams involving $\phi_0$)
- Combinations of cubic and quartic terms. Do some other fields develop vevs ?

# Conclusions

- Reheating = most unknown stage in cosmology
- Coupled systems $\longrightarrow$ new production mechanism
- Flat directions naturally present in MSSM;

  can affect reheating through their VEVS
- Slow perturbative decay often assumed;

  $\Gamma_\phi \sim m_\phi^3/\phi^2$ gives decay after $10^{11}$ rotations !
- Need to study nonlinear effects (lattice simulations)