TeV Quantum Gravity in 4-Dimensions?

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Outline

- Brief review of models with extra-dimensions and TeV gravity
- TeV quantum gravity in four dimensions?
- Phenomenology of the model
- Brief review of small black hole formation
- Bounds from cosmic rays experiments
- LHC signatures
- Conclusions

Two very different energies!

- Planck mass: 10^{19} GeV

- Fermi scale: 10^{2} GeV

- This difference leads to the hierarchy problem.

$$m_H^2 \approx m_H^{0\,2} + \Lambda^2 F(g, \lambda, \lambda_f)$$

- What is so special about the Planck scale?

$$m_{Planck} = \sqrt{\frac{hc}{G}} = 5.46x10^{-8}\,kg$$

$$\lambda_{Planck} = \sqrt{\frac{Gh}{c^3}} = 4.05x10^{-35}\,m$$

$$t_{Planck} = \sqrt{\frac{Gh}{c^5}} = 1.35x10^{-43}\,\sec$$

- We usually assume that it is the scale at which gravity becomes strong.
- Could the scale for quantum gravity be much lower than expected from naïve dimensional analysis?
- In more than four dimensions, it is well-known that it is the case: ADD, RS models.
- Is it possible in four-dimensions?

TeV gravity extra-dimensions

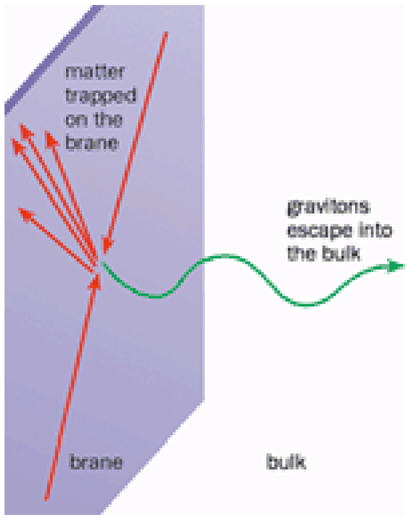
$$\int d^4x\, d^{d-4}x' \sqrt{-g}\ \left(M_*^{d-2}\mathcal{R} + \cdots\ \right)$$

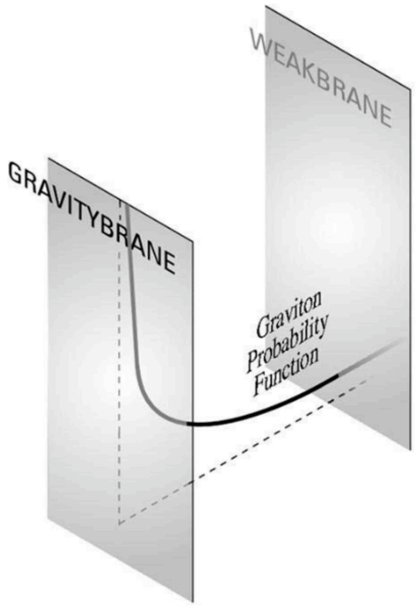
$$M_P^2 = M_*^{d-2} V_{d-4}$$

where M_P is the effective Planck scale in 4-dim

ADD brane world

RS warped extra-dimension





Typical problems of models with TeV Quantum Gravity:

- Light Kaluza-Klein gravitons in ADD: astrophysical constraints
- Higher dimensional operators (proton decay, flavor changing neutral currents, etc): more complicated constructions required
- Dynamical assumptions about extra-dimensions: can these models really be realized in a theory of quantum gravity?

Desirable features:

- Provide solutions to hierarchy problem.
- Rich phenomenology at LHC.
- In particular black holes might be created at LHC.

Is a similar construction possible in 4-dimensions?

- Let us consider a scalar-tensor theory: basically a dilaton-like theory with a potential for the scalar field.

$$\int d^4x \sqrt{-g} \left(\frac{\phi^2}{2}\mathcal{R} + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + V(\phi) + \mathcal{L}_{\rm sm} + \cdots \right)$$

- We pick the following potential:

$$V(\phi) = -M_\star^2\phi^2 + \lambda\phi^4.$$

- Our effective theory should be valid up to cutoff $M_* \sim \text{TeV}$
- We assume that this is the scale of quantum gravity
- Let us assume that ϕ gets a vev

$$M_p = \sqrt{\frac{M_\star^2}{2\lambda}}. \qquad M_p = \sqrt{1/(8\pi G)} = 2.4353 \times 10^{18} \text{ GeV}$$

- Implies: $\lambda = 8 \times 10^{-32}$

- Our model provides a solution to the hierarchy problem modulo some dynamical assumptions to be discussed on the next slides.
- λ is chosen small, but there is not much sensitivity to the cutoff since fundamental scale is at 1 TeV.

$$\lambda = \lambda_0 + F(\lambda_0) \log \Lambda$$

- It is important that F(λ) does not depend on gauge couplings or Yukawa couplings.
- Side remark: This model can be implemented in string theory: little string theory (Antoniadis, Dimopoulos & Giveon 2001):

$$M_P^2 = \frac{1}{g_s^2} M_s^8 V_6$$

- Because the string scale is so low: there is a minimal length of the order of the Planck scale and no real meaning to extra-dimensions which would be fuzzy. Hence we would have different signatures than in ADD.

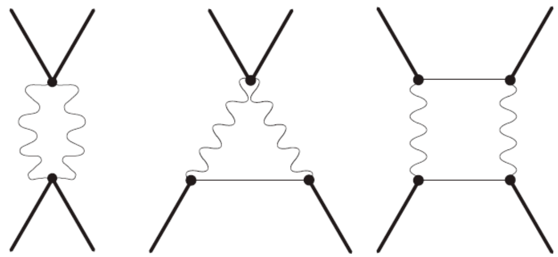
Higher dimensional operators:

- We have a fundamental scale at $M_\star \sim 1$ TeV, naively we can expect the following operators to be generated:

$$\mathcal{O} \sim \frac{1}{M_*^{n-4}}\phi^n \qquad \mathcal{O} \sim \frac{1}{M_*^n}\phi^l \mathcal{O}_{\rm sm} \qquad \mathcal{O} \sim \frac{1}{M_*^n}(\partial^2\phi)\mathcal{O}_{\rm sm}$$

- The first two are dangerous!
- If you insert the vev of ϕ in the first one it is obvious it would destabilize the potential.
- Furthermore the second would lead to flavor changing currents and potentially to proton decay.
- We assume that they are absent: analogous to dynamical assumption about shape/size of extra-dimensions.
- First two operators would be an issue for any model with a $\phi >> M_\star$ such as e.g. chaotic inflation.
- Last operator is not an issue and can lead to interesting pheno.
- Let us check if these operators can be generated by loop corrections.

- Higher dimensional operators:
  - Corrections to the effective potential from self-interactions always involve λ and are thus small.
  - Correction to the effective potential from perturbative quantum gravity:



  - Linearized gravity:

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{\langle\phi\rangle} h_{\mu\nu}$$

canonical normalization of the graviton kinematic term implies $\langle\phi\rangle = M_p$

  - Scale for perturbative quantum gravity is the Planck scale ~ 10^{19} GeV. Note however that the momentum cutoff in loops is low and ~ 1 TeV !

$$\mathcal{O} \sim \frac{1}{M_P^{n-4}} \phi^n \qquad \mathcal{O} \sim \frac{1}{M_P^n} \phi^l \mathcal{O}_{\rm sm} \qquad \mathcal{O} \sim \frac{1}{M_P^n} (\partial^2\phi) \mathcal{O}_{\rm sm}$$

- Coupling to Higgs sector?

$$\alpha\phi H^{\dagger}H \qquad\qquad \phi^{2}H^{\dagger}H$$

  - We assume that these operators are absent. Since ϕ is a singlet these operators will not be generated by gauge interactions and they do not get renormalized. Quantum gravity will generate:

$$\phi^{2}H^{\dagger}H\,\frac{m_{\phi}^{2}m_{H}^{2}}{M_{P}^{4}}\log\frac{\Lambda}{m_{\phi}}$$

which is safe (cutoff $M_{\star}$)

- Higher dimensional operators:
  - Although these operators

$$\mathcal{O} \sim \frac{1}{M_*^{n-4}}\phi^n \qquad \mathcal{O} \sim \frac{1}{M_*^n}\phi^l \mathcal{O}_{\rm sm} \qquad \mathcal{O} \sim \frac{1}{M_*^n}(\partial^2\phi)\mathcal{O}_{\rm sm}$$

will not be generated perturbatively, they could be generated at the nonperturbative level via effects of quantum gravity:

  - Let me stress again: our assumption is that the first two will not be generated.
  - The last one is not dangerous and can be present.

Phenomenology of the model

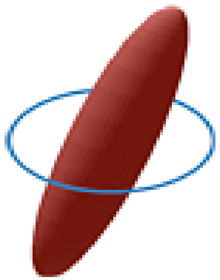
- Very similar to standard model
  - No light Kaluza-Klein states (like in ADD): no bound from astrophysics
  - No sizable modification of 1/R Newton law
  - Graviton emission is suppressed by $M_P=10^{19}$ GeV: no bound from LEP
- New strong dynamics at 1 TeV

$$\mathcal{O} \sim \frac{1}{M_*^n}(\partial^2\phi)\mathcal{O}_{\rm sm}$$

- What are the bounds on this model?
- Will small black holes be produced at colliders?
- First issue is whether BH forms in this model. We need to review arguments in favor of BH formation in ADD and RS.

Black hole formation

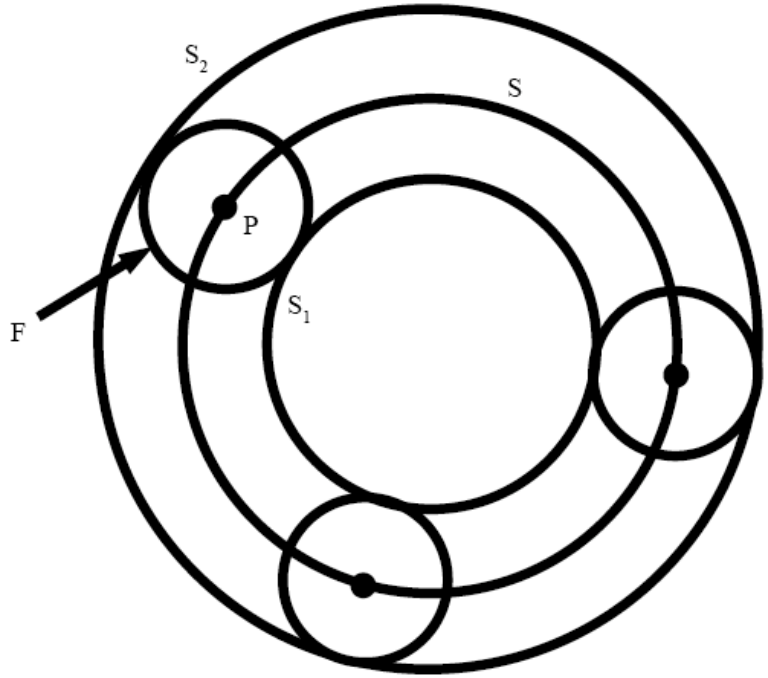
- In trivial situations (spherical distribution of matter), one can solve explicitly Einstein's equations e.g. Schwarzschild metric.
- In more complicated cases one can't solve Einstein equations exactly and one needs some other criteria.
- Hoop conjecture (Kip Thorne): if an amount of energy E is confined to a ball of size R, where R < E, then that region will eventually evolve into a black hole.



- Cross-section:

$$\hat{\sigma} \approx \pi r_s^2 \qquad r_s(M_{\rm BH}) = \frac{1}{M_D}\left[\frac{M_{\rm BH}}{M_D}\right]^{\frac{1}{1+n}}\left[\frac{2^n \pi^{(n-3)/2}\Gamma\left(\frac{n+3}{2}\right)}{n+2}\right]^{\frac{1}{1+n}}$$

- More rigorous: prove the existence of a closed trapped surface (CTS).
- A CTS is a compact spacelike two-surface in space-time such that outgoing null rays perpendicular to the surface are not expanding.



- At some instant, the sphere S emits a flash of light. At a later time, the light from a point P forms a sphere F around P, and the envelopes S_1 and S_2 form the ingoing and outgoing wavefronts respectively. If the areas of both S_1 and S_2 are less than of S, then S is a closed trapped surface.

- Relativists teach us that if a closed trapped surface is formed, there is a singularity in the future evolution of Einstein's equations (Hawking-Penrose theorem)
- Hawking-Penrose theorem assumes some hypotheses on the energy momentum tensor (weak energy condition).
- Cosmic Censorship Conjecture states that there is no naked singularities. All naked singularities are hidden from an observer at infinity by an event horizon. In other words, singularities in the evolution of Einstein's equations imply black hole formation.
- Hawking-Penrose theorem+Cosmic Censorship Conjecture lead to the conclusion that if a closed trapped surface is formed, then a black hole is formed as well.

- Is black hole formation in the collision of two particles with some non-zero impact parameter a semi-classical process? I.e. can the hoop conjecture be used to calculate the cross-section?

- The resolution of the problem came in 2002: Eardley and Giddings (PRD 66, 044011 (2001)) have constructed a closed trapped surface in a region of weak curvature: semi-classical analysis is valid.

- Basic idea of the proof:
  - Consider the Aichelburg-Sexl metric, describing a boosted Schwarzschild metric in the limit of large γ and small mass:

$$ds^2 = -du dv + H_{ij} H_{jk} dx^i dx^j$$

$$H_{ij} = \delta_{ij} + \frac{1}{2} \nabla_i \nabla_j \phi(\mathbf{x}) u \theta(u)$$

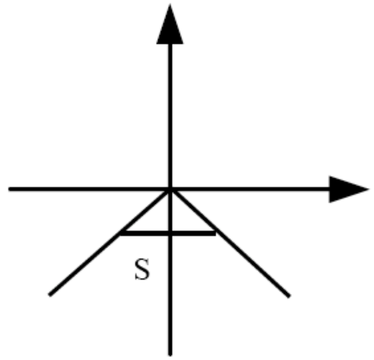
  - u and v are lightcone coordinates t $\pm$z and

$$\nabla^2 \phi = -16\pi R_s \, \delta^{2+d}(\mathbf{x})$$

- Note that ϕ(x) is singular near |x|=0, it thus describes a shock wave.
- Furthermore R_S is the Schwarzschild radius and involves the G_{newton} which appears in Einstein's equations.

- Now superimpose two Aichelburg-Sexl metrics: one coming from -z, the other from z

S in that picture denotes the closed trapped surface



- Eardley and Giddings are able to construct a closed trapped surface in a part of the spacetime diagram where the two shockwaves have not yet had time to interact.
- In four dimensions, their result is valid as long as the impact parameter fulfills:

$$b < 1.6 R_s$$

where R_S is the Schwarzschild radius.

- It is a semi-classical process (S. Hsu, hep-th/0203154).

- The cross-section for black hole production is affected by this result (analytical result in 4-dimensions):

$$\sigma = 0.64 \times \pi r_S^2$$

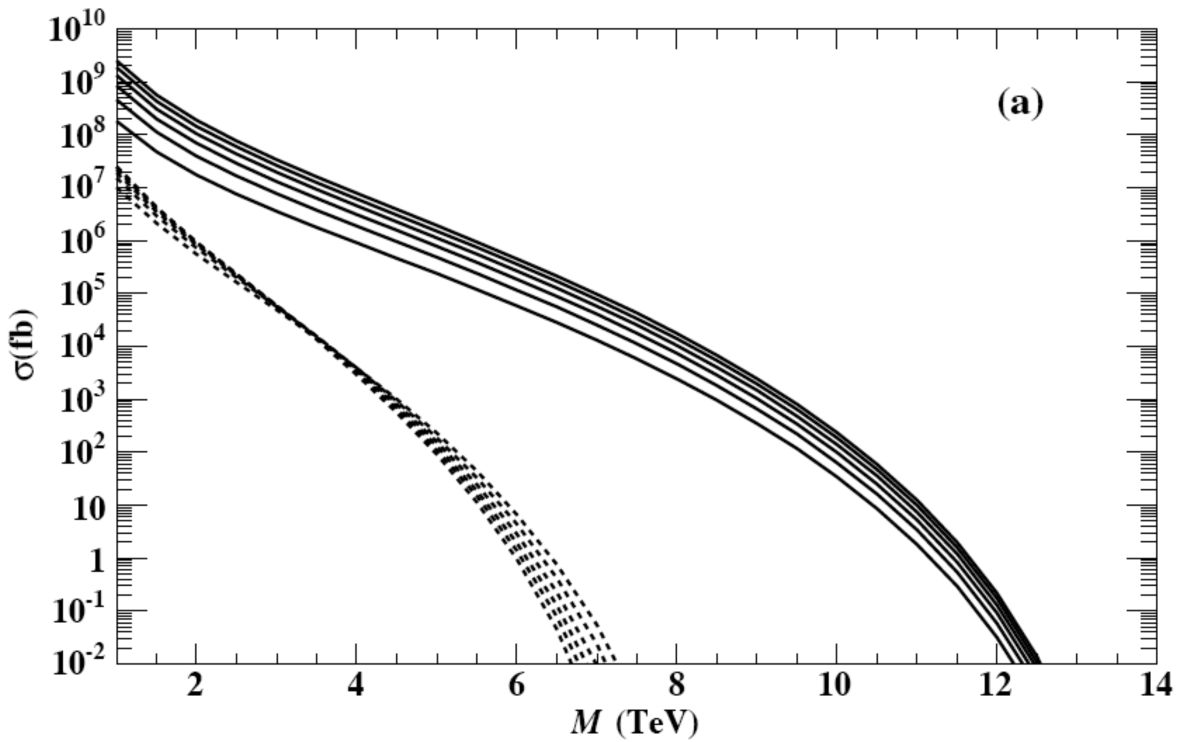
- Note their result is valid for E_{CM}->infinity: it is only trustworthy for large black hole masses compared to the scale of quantum gravity.

- Following Eardley and Giddings, Yoshino and Nambu have calculated numerically the coefficient compensating for b $\neq 0$ in the case of d>4.

- Summary: semi-classical description of BH production is valid (see also nice argument recently given by Kaloper & Terning arXiv:0705.0408).

- Crux: note that M_P appears in the Eardley&Giddings construction for our model but not $M_\star$!

Comparison with extra-dim BHs
(Gingrich, hep-ph/0609055)



$$\sigma \sim \frac{1}{M_D}\left(\frac{M_{BH}}{M_D}\right)^{\left(\frac{2}{1+n}\right)}$$

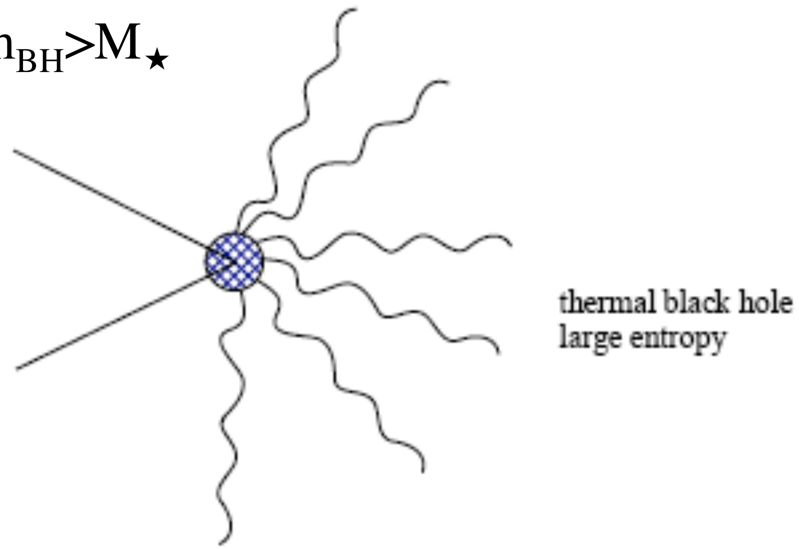
For partons, σ increases with energy but note that PDFs go so fast to zero that they dominate. In other words quantum black holes dominate!

σ(pp->BH+X), $M_\star$=1 TeV

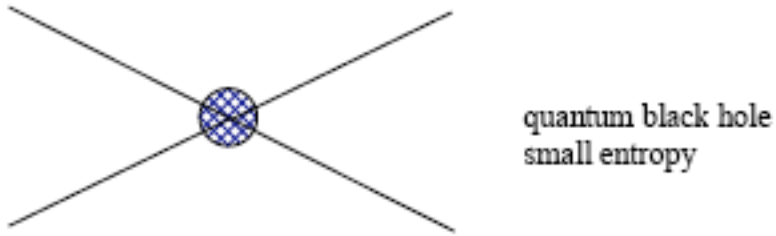
This shows the significance of the inelasticity in BH production

Semi-classical (thermal) versus quantum black hole: calculate the entropy!

$m_{BH} > M_\star$



$m_{BH} \sim M_\star$



$$S = \frac{1+n}{2+n}\frac{M_{BH}}{T_{BH}}$$

$$\langle N \rangle \propto \left(\frac{M_{BH}}{M_\star} \right)^{\frac{n+2}{n+1}}$$

Keep in mind that E-G constructions only works for $m_{BH} \gg M_\star$

No small Black Hole in TeV scale 4-dim quantum gravity

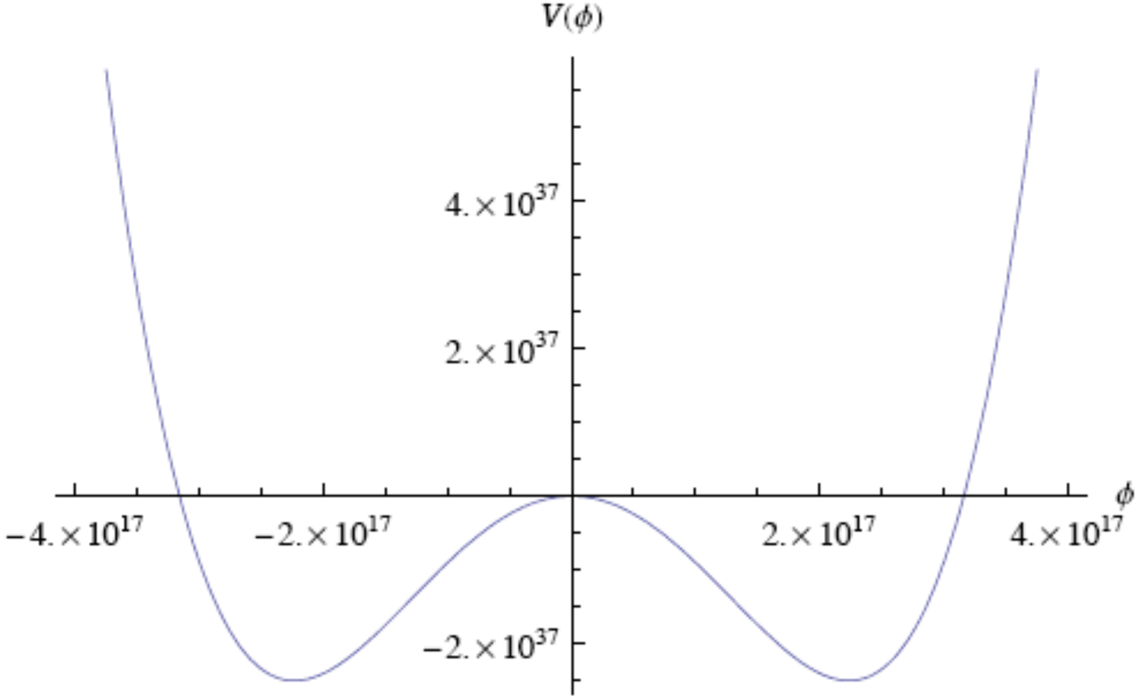
- Let us first look at our modified Einstein's equations:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = -8\pi G(S_{\mu\nu} + T_{\mu\nu})$$

$$S_{\mu\nu} = \left(\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}\partial_\rho\phi\partial^\rho\phi - \phi^2 R_{\mu\nu} + \frac{1}{2}g_{\mu\nu}\phi^2 R - (g_{\mu\nu}g^{\alpha\beta}\phi^2_{;\alpha\beta} - \phi^2_{;\mu\nu})\right)$$

- Important fact: G is small! In extra-dimensions G would be much bigger!
- In Eardley Giddings construction we have to use G~1/(10^{36} GeV^2)
- Semi-classical black holes thus can't be produced at a collider.
- Note that this is very unlikely at LHC in extra-dimensional models as well (see Meade and Randall arXiv:0708:3017)

- Furthermore, the energy density to restore the "unbroken" phase is huge: $M_P^2 M_\star^2$



- Although we expect strong scattering at 1 TeV, higher dimensional operators such as

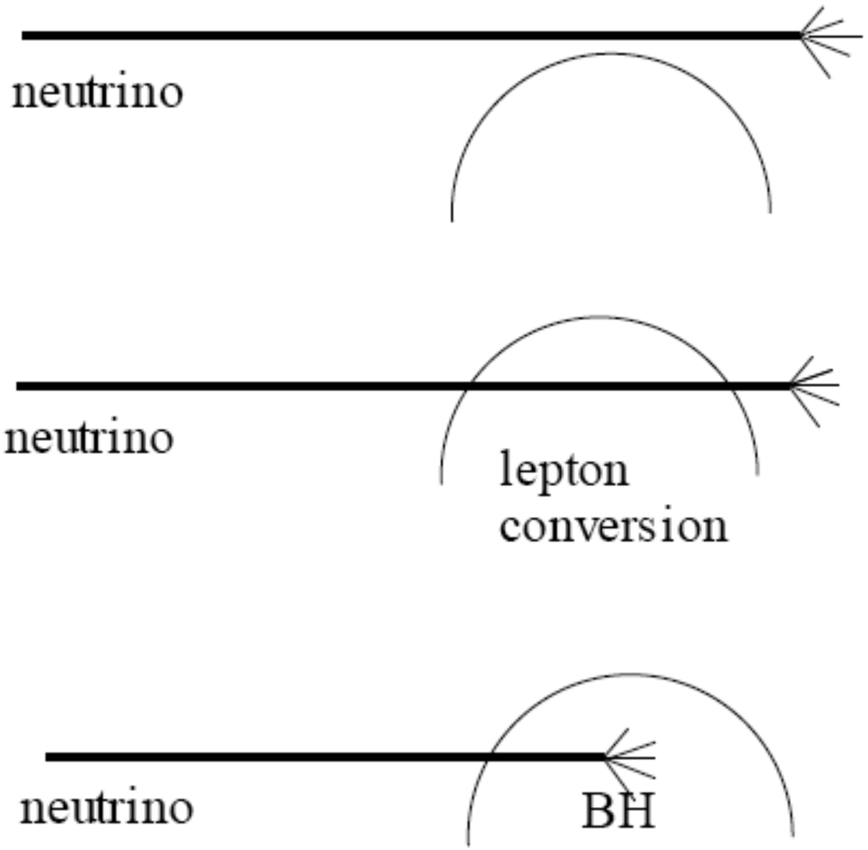
$$\mathcal{O} \sim \frac{1}{M_*^n}(\partial^2\phi)\mathcal{O}_{\rm sm}$$

can't restore strong gravity phase.

Cosmic ray experiments can probe very high energy physics!

- Cosmic rays (Feng et al., Ringwald et al. 2001) can produce quantum BH in Earth atmosphere.
- Their bound may apply to our model. The BHs which are produced in their case are quantum BH which is analogous to strong scattering. Only a few particles are being created.
- Cosmic rays experiments: in particular AGASA can be used to bound the fundamental scale of gravity.
- AGASA is stands for Akeno Giant Air Shower Array and is located in Japan.
- AGASA covers an area of about 100 km^2 and consists of 111 detectors on the ground (surface detectors) and 27 detectors under absorbers (muon detectors).

Compare SM to BH creation scenario

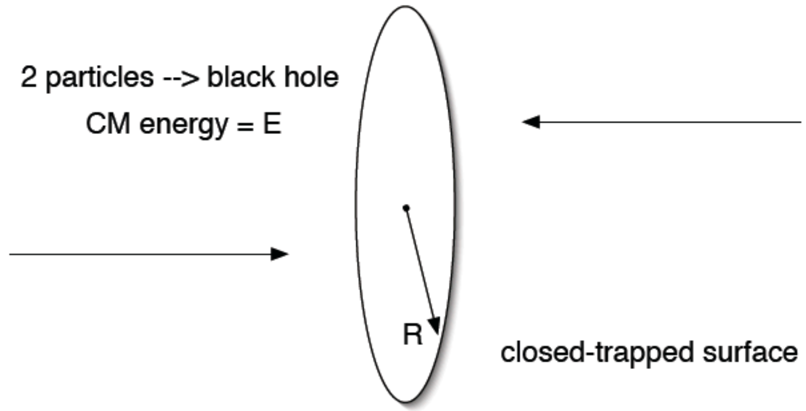


Shower is detected both by ground arrays and by fluorescence detectors

Because of BH production: there are less showers due to earth-skimming neutrinos

- It has been pointed out by Feng et al., Ringwald et al. 2001 that cosmic rays (in particular neutrinos) can produce BH in the Earth atmosphere.
- One needs the cross-section σ(ν N -> BH):

$$\hat{\sigma} \approx \pi r_s^2$$



$$r_s(M_{\rm BH}) = \frac{1}{M_D}\left[\frac{M_{\rm BH}}{M_D}\right]^{\frac{1}{1+n}}\left[\frac{2^n\pi^{(n-3)/2}\Gamma(\frac{n+3}{2})}{n+2}\right]^{\frac{1}{1+n}}$$

- Cross-section for BH production:

$$\sigma(\nu N \to \mathrm{BH}) = \sum_i \int_{(M_{\mathrm{BH}}^{\mathrm{min}})^2/s}^{1} dx\, \hat{\sigma}_i(\sqrt{xs})\, f_i(x, Q)$$

- Important observation:

$$\sigma \sim \frac{1}{M_D} \left(\frac{M_{BH}}{M_D} \right)^{\left(\frac{2}{1+n}\right)}$$

- Number of BHs seen by AGASA

$$N = \int dE_\nu\, N_A\, \frac{d\Phi}{dE_\nu}\, \sigma(E_\nu)\, A(E_\nu)\, T$$

$A(E_\nu)$ is the experiment's acceptance in cm^{3}we sr. water equivalent steradians

$N_A = 6.022 \times 10^{23}$ is Avogadro's number

$d\Phi/dE_\nu$ is the source flux of neutrinos

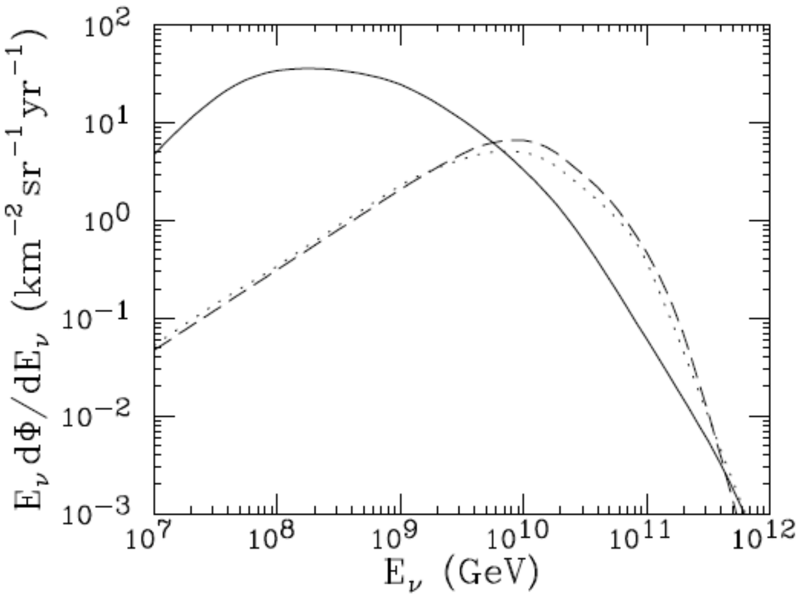
T is the running time of the detector

- Acceptance:

$$A(E) = S \int_{\theta_{\min}}^{\theta_{\max}} 2\pi \sin\theta \, d\theta \int_0^{h_{\max}} \frac{\rho_0}{\rho_{\text{water}}} e^{-h/H} \mathcal{P}(E,\theta,h) \, dh$$

- ρ is the density of the atmosphere at ground level
- $\mathcal{P}(E,\theta,h)$ is the probability of finding a shower with Energy E and zenith angle θ and starting at an altitude h.
- You also need the neutrino flux, however this needs to be modeled:

$$\frac{d\Phi}{dE_\nu}$$



- So it's a complicated business with potentially large uncertainties!
- Here is the result of Feng et al. using AGASA data:

$$n = 4 : \quad M_D > 1.3 - 1.5 \text{ TeV}$$
$$n = 7 : \quad M_D > 1.6 - 1.8 \text{ TeV}$$

Note that Feng et al. have updated their results to take

inelasticity into account: no major impact on bounds.

- Remember that

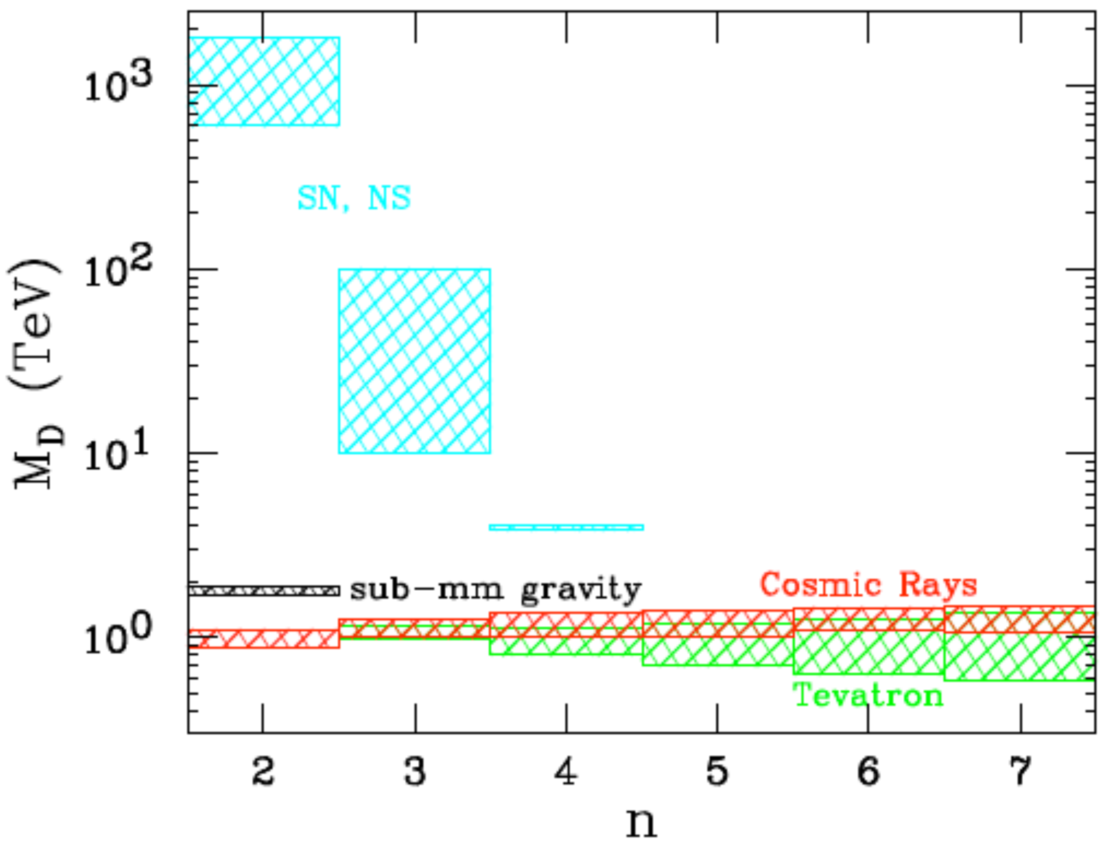
$$\sigma \sim \frac{1}{M_D} \left(\frac{M_{BH}}{M_D} \right)^{\left(\frac{2}{1+n}\right)}$$

- Using the result of Feng et al. we deduce

$$\sigma^{(exp)} < 0.5327 \frac{1}{\text{TeV}^2}$$

- And thus $\begin{matrix} M_D > 1.17 \text{ TeV} \\ M_P > 6 \text{ TeV} \end{matrix}$ for the 4-dim scales assuming $\sigma = \dfrac{1}{M_*^2}$

- As a side remark: AGASA provides the toughest bound on TeV extra-dimensions with n>5 (Anchordoqui et al. hep-ph/0307228)



Cross-sections in TeV d=4 quantum gravity @ LHC

- Bounds from AGASA on the new physics cross-section are in the 0.5 TeV^{-2} region. This corresponds to a new physics scale ~1 TeV.
- At the LHC we could get up to σ(pp -> new physics + X) ~ 10^7 fb for a new physics scale ~ 1 TeV. For a luminosity of 100 fb^{-1}, we expect 10^9 events at the LHC.
- Not quite clear what will be produced, but the cross-sections can be large! It may resemble compositeness as pointed out by Meade & Randall (arXiv 0708.3017)
- Simple to calculate, note that CTEQ has an "unofficial" mathematica package for PDFs: http://www.phys.psu.edu/~cteq/#PDFs

Conclusions

- There are different options for TeV quantum gravity.
- We don't know much about the scale of gravity.
- Quantum gravity could be around the corner: this is really an experimental question
- If this is the case, strong dynamics will be observable at the LHC, but perhaps not black holes.
- Fascinating unification of gravity and weak physics: both scales would be generated via a Higgs effect.