Some results from full 2+1 flavor simulations of QCD

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Collaborators

MILC Collaboration (Jan 2004): E. Gregory, C. Aubin, R. Sugar, UMH, J. Hetrick, S. Gottlieb, C. Bernard, C. DeTar, J. Osborn, D. Toussaint

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$+$ **Fermilab, HPQCD & UKQCD Collaborations:**

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Outline

- Simulation Choices & Ensemble of Configurations
- Result Highlights/Summary
- Pseudoscalar decay constants, quark masses, etc.
- Baryons
- Heavy-light decay constants
- Semileptonic B/D decays
- Summery and Outlook

Simulation Choices

To carry out a simulation we must select certain physical parameters:

- lattice spacing (a) or gauge coupling (β)
- grid size ($N_s^3 \times N_t$)
- quark masses ($m_{u,d} = m_l$, m_s)

To control systematic error we must

- take continuum limit, $a \to 0$
- take infinite volume limit
- extrapolate to physical light quark mass;
  we can work at physical s quark mass, or interpolate to it

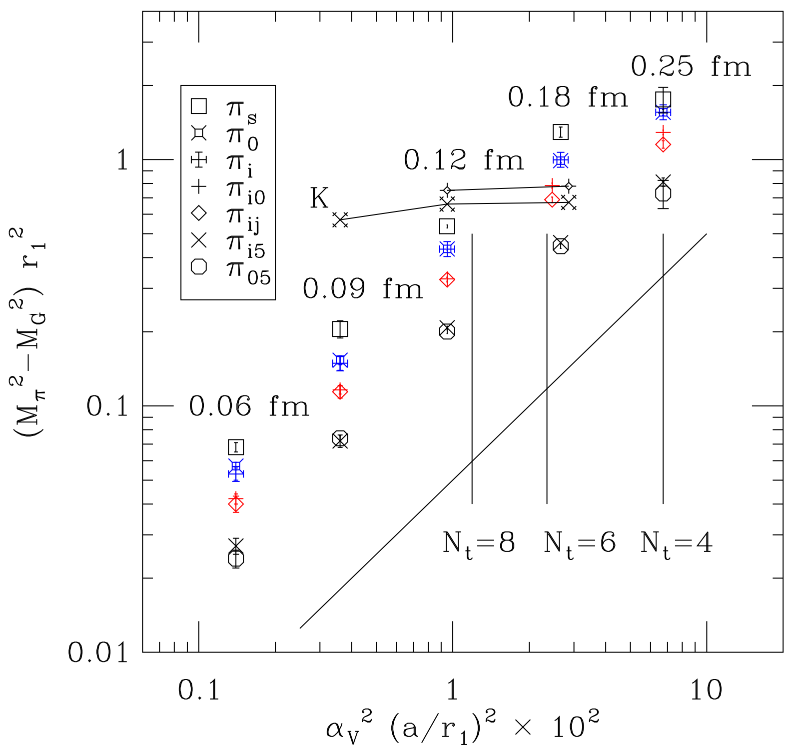
Simulation Choices

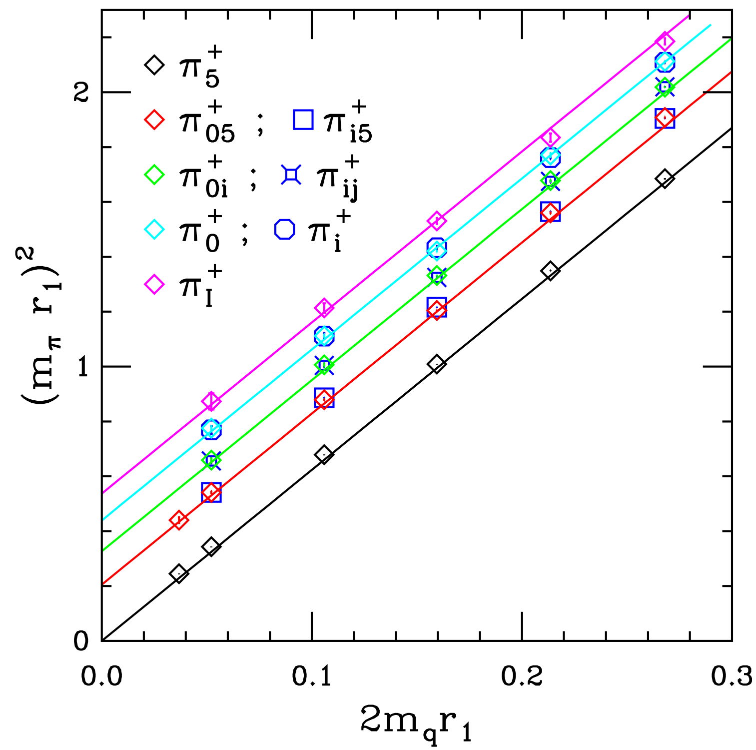
We also must choose an action and a simulation algorithm.

- The gauge action is a 1-loop improved Lüscher-Weisz action, with $\mathcal{O}(\alpha_s^2 a^2)$ discretization errors.
- The fermion action is a tree-level improved staggered action with a “fat” link to suppress taste violations of the staggered fermions. It has $\mathcal{O}(\alpha_s a^2)$ discretization errors.
- The algorithm is the Hybrid Molecular Dynamics R-algorithm, with the $det^{1/4}$ trick to eliminate the extra tastes.
- Recently, switched to exact RHMC algorithm

Whether the $det^{1/4}$ trick induces non-localities in the interacting theory is an open question. Our results, so far, show no sign of a problem.

Taste violations





The taste splittings are independent of m_q, and vanish in the continuum limit, as expected, *i.e.* as $\alpha_s^2 a^2$.

This is consistent with $\det D_{stag} \to (\det D_{1f})^4$ as $a \to 0$.

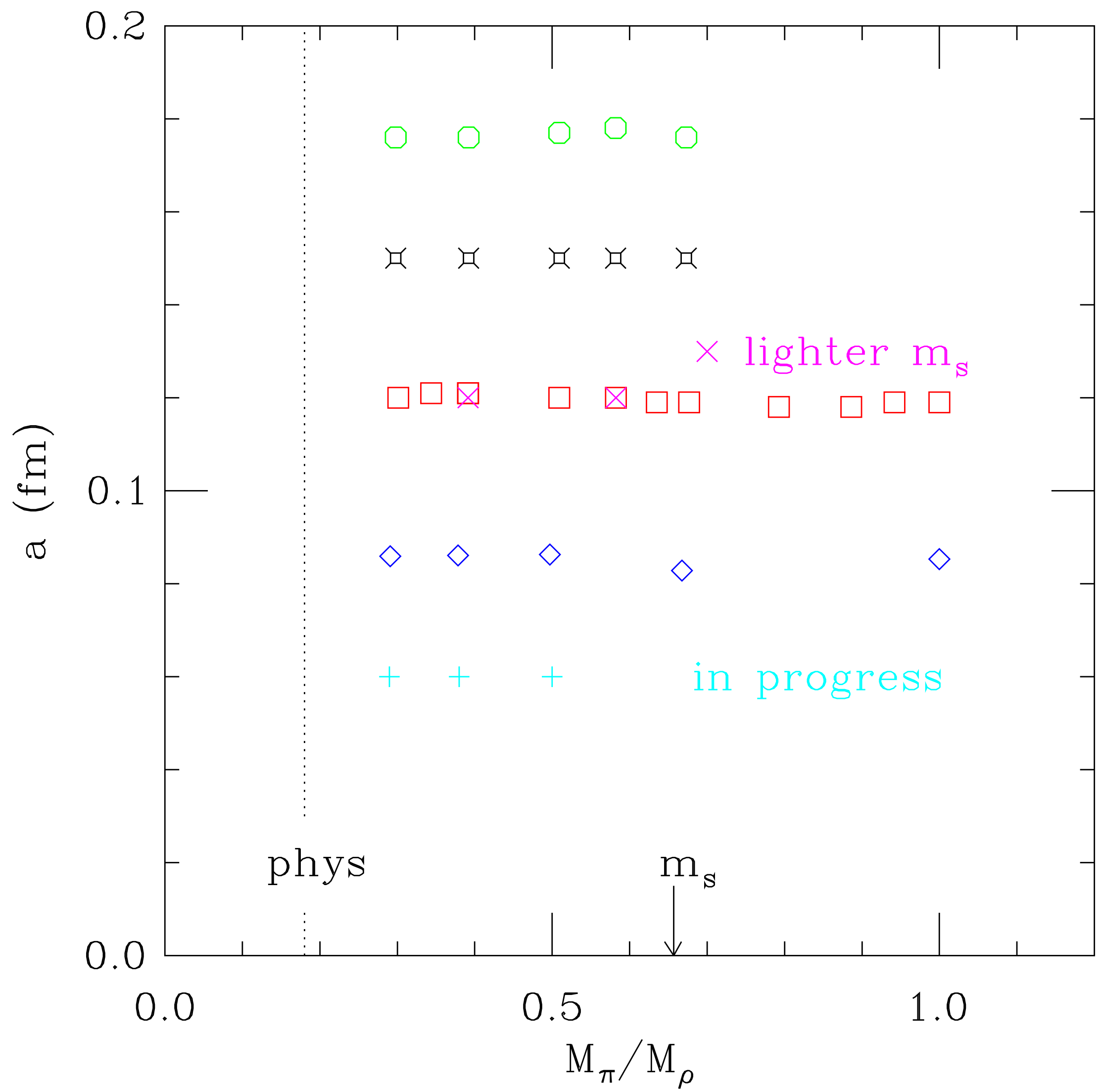
Ensemble of Configurations

MILC has been generating three flavor configurations to allow control of these errors:

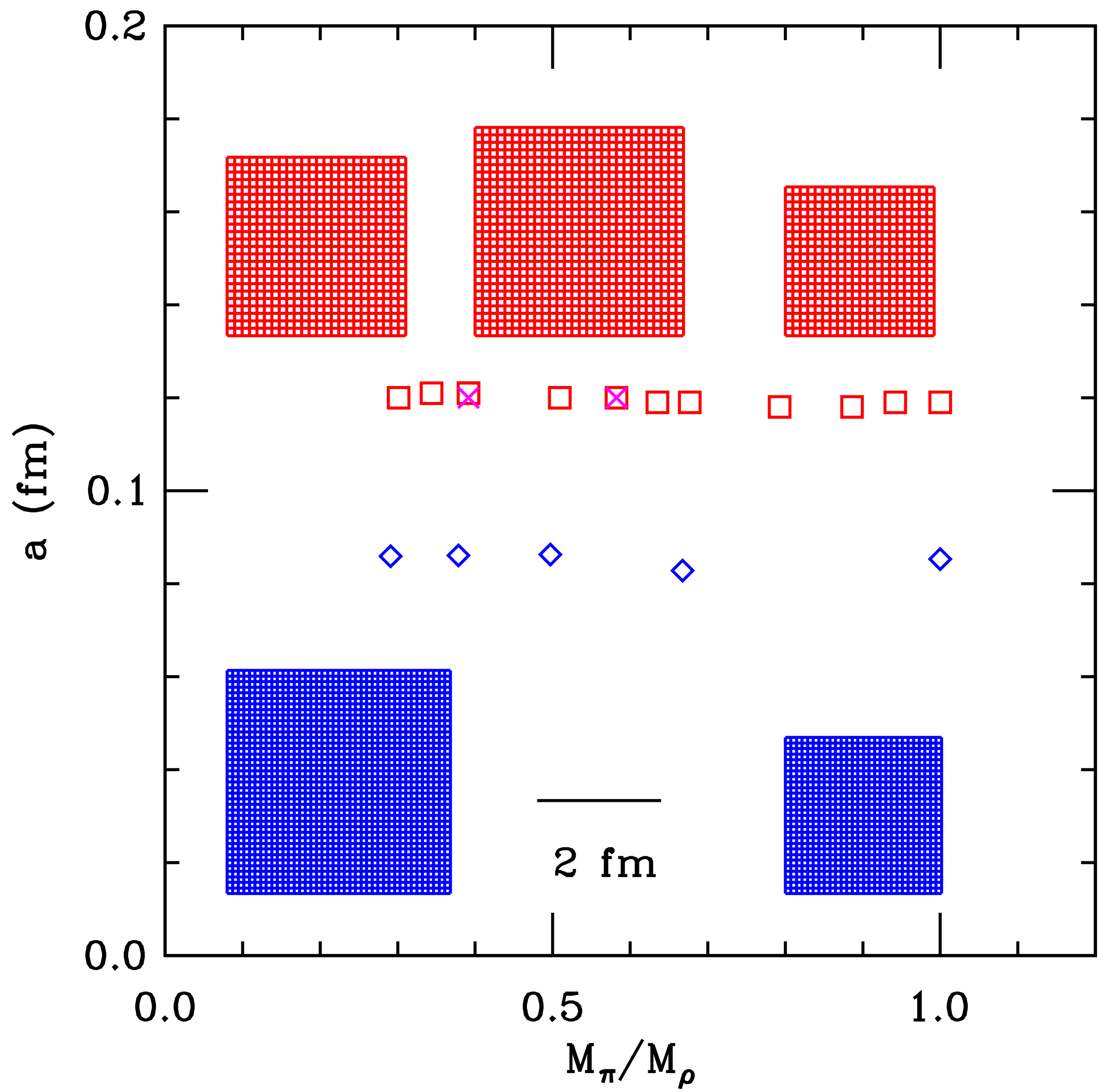
- most results are based on three lattice spacings, 0.121 fm, 0.086 fm, and 0.06 fm, kept fixed for different quark masses; some use also 0.176 fm and ~ 0.15 fm.
- mostly $V_s \sim (2.4 \text{ fm})^3$, with one $\sim (3.4 \text{ fm})^3$, to check for finite volume effects; except 1 set: $m_\pi L > 4$.
- several $m_{u,d} = \hat{m}'$ to extrapolate to physical light quarks; use two m_s' (at $a = 0.12$ fm) to interpolate to physical strange quark mass

The ensembles of configurations are available to others through the NERSC Gauge Connection.
Some new configurations generated with USQCD resources.

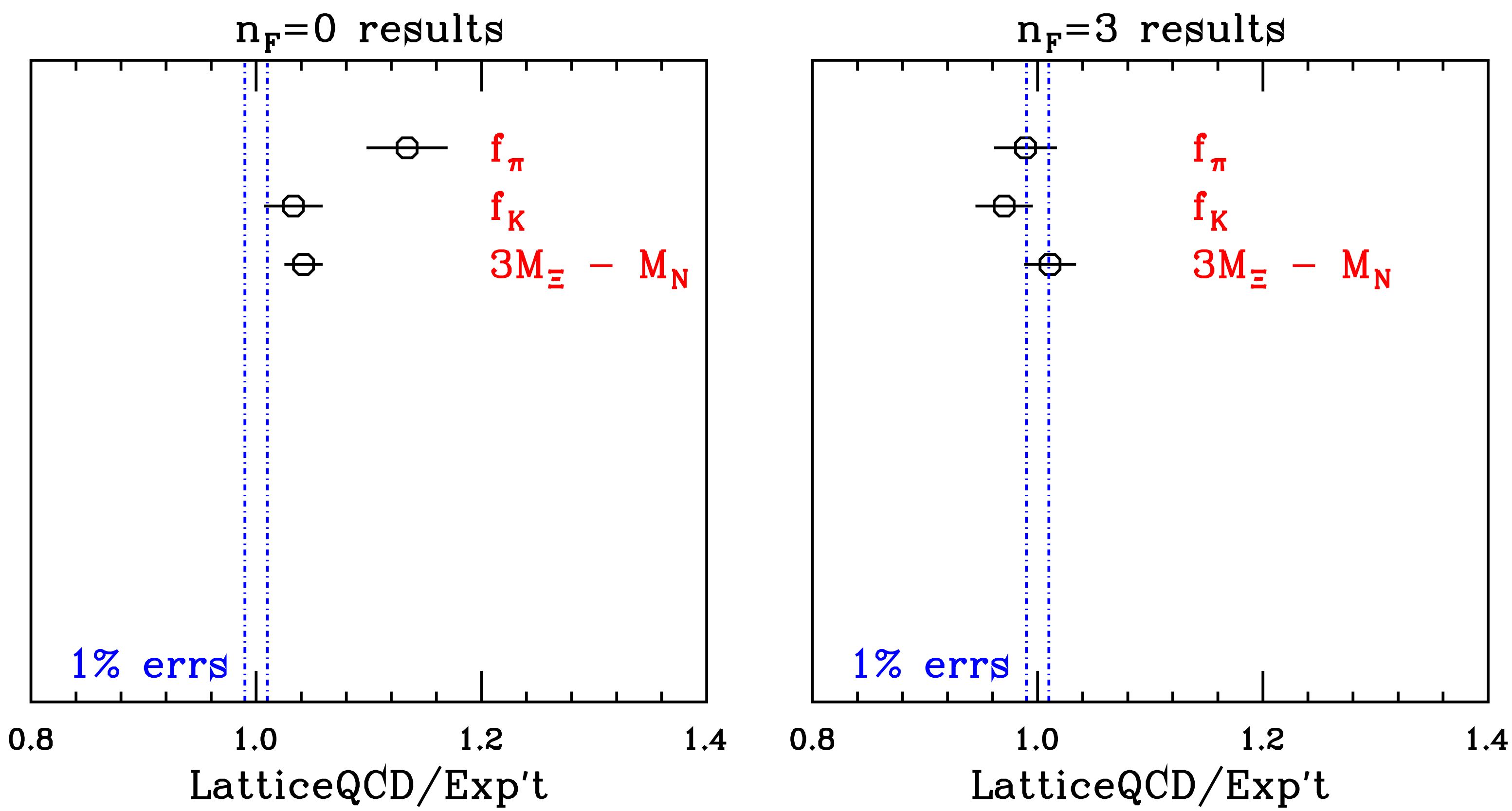
MILC Ensembles



MILC Ensembles

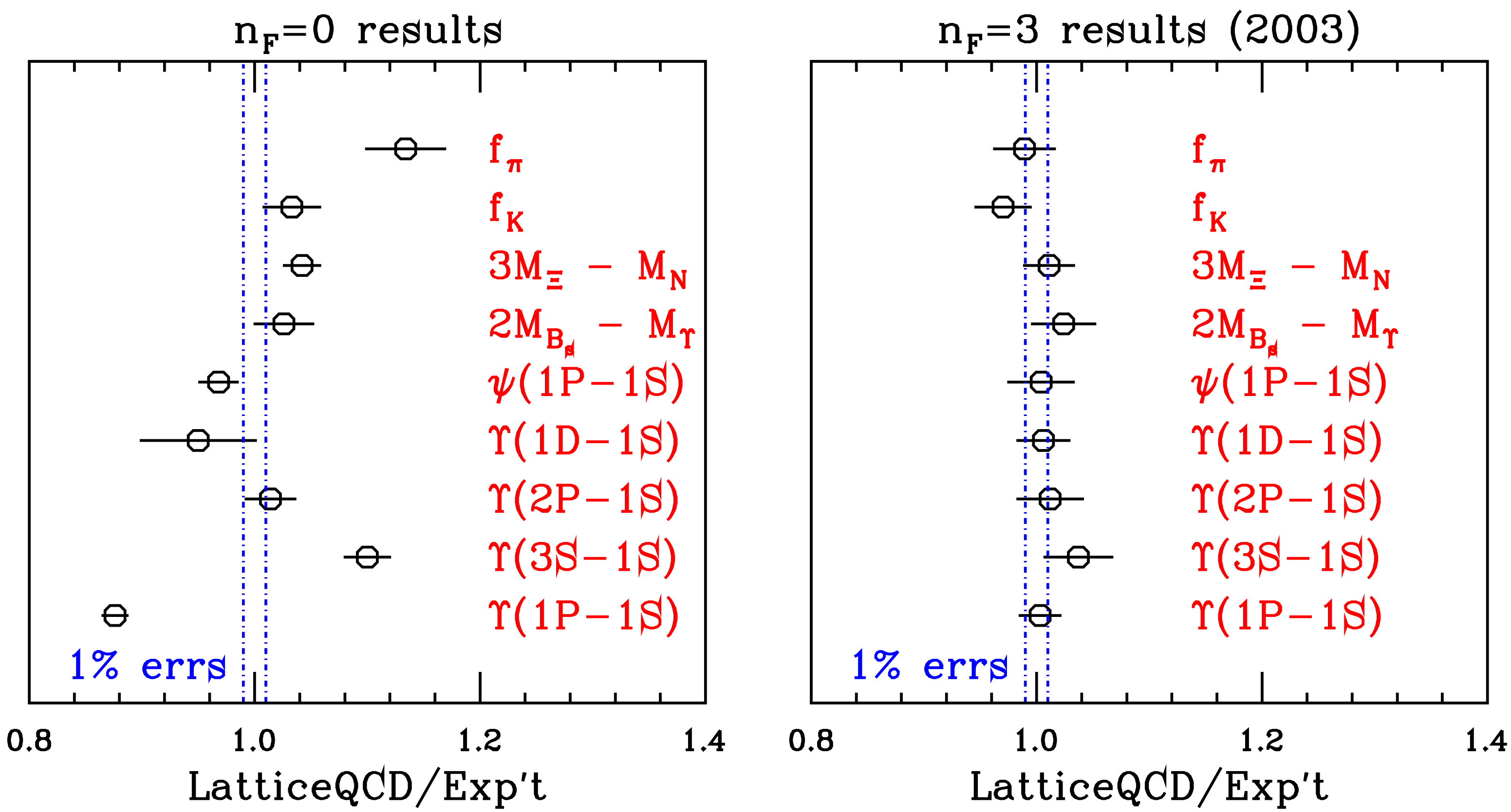


Ratio Plot



n_F=0 results
f_π
f_K
$3M_\Xi - M_N$
1% errs
0.8
1.0
1.2
1.4
LatticeQCD/Exp't
n_F=3 results
f_π
f_K
$3M_\Xi - M_N$
1% errs
0.8
1.0
1.2
1.4
LatticeQCD/Exp't

Ratio Plot



By sharing with FNAL, HPQCD and UKQCD:
C.T.H. Davies *et al.*, PRL 92 (2004) 022001 [hep-lat/0304004]

Chiral perturbation theory (χPT)

For extrapolation in quark mass use chiral perturbation theory (χPT), an effective field theory based on symmetry considerations – and its breaking:

$$\mathcal{L}_{\chi PT} = \frac{f^2}{8}\mathrm{Tr}\left(\partial_\mu \Sigma \partial_\mu \Sigma^\dagger\right) - \frac{1}{4}\mu f^2 \mathrm{Tr}\left(\mathcal{M}\Sigma + \mathcal{M}\Sigma^\dagger\right) + \mathcal{O}(p^4, mp^2, m^2) \, ,$$

where $\Sigma = \exp(i\phi_a t_a/f)$ ($\phi_a = \pi^0, \pi^\pm, K^0, \overline{K}^0, K^\pm, \eta$). One finds,

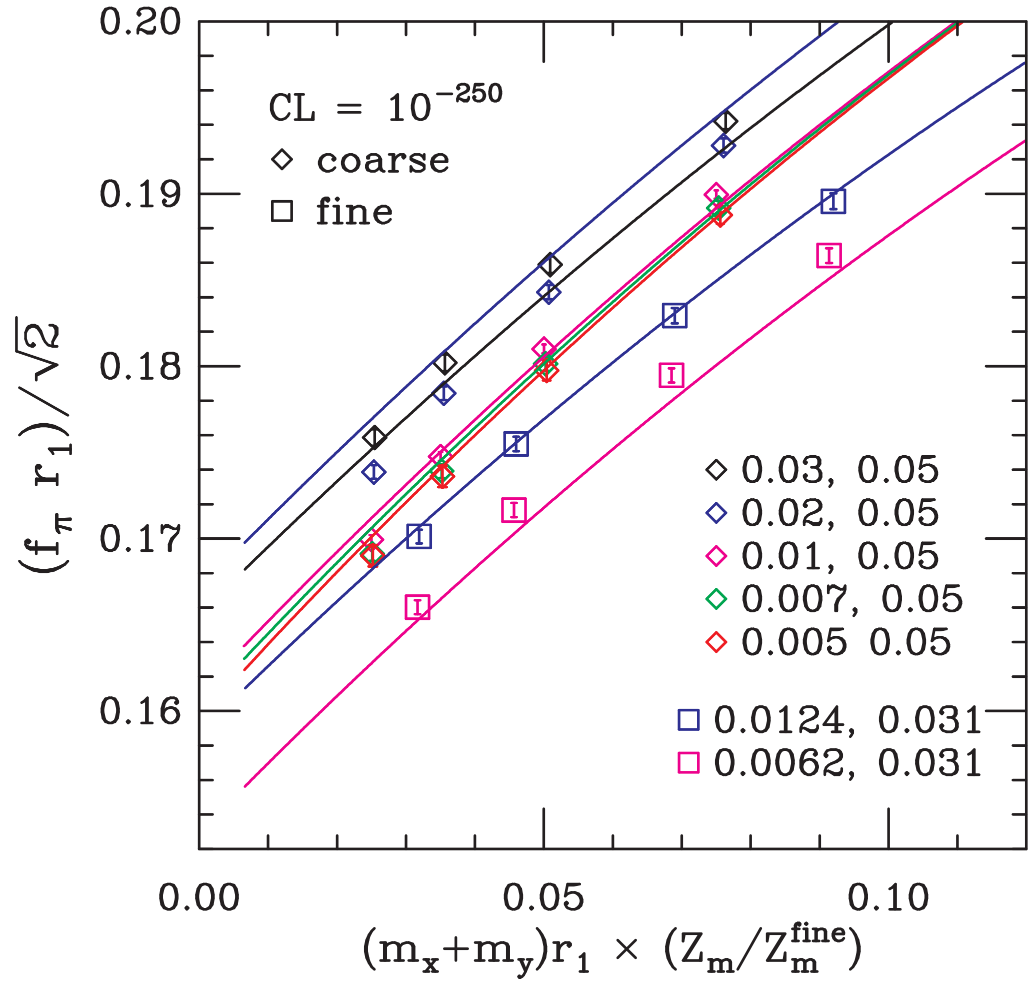
$$\begin{aligned} \frac{m_\pi^2}{2m_q} &= \mu \left\{ 1 + \frac{1}{16\pi^2 f^2}\left[\frac{2}{3}(2\mu m_q)\log\left(\frac{2\mu m_q}{\Lambda^2}\right) + \text{"}p^4 - terms\text{"}\right]\right\} \, , \\ f_\pi &= f \left\{ 1 + \frac{1}{16\pi^2 f^2}\left[-3(2\mu m_q)\log\left(\frac{2\mu m_q}{\Lambda^2}\right) + \text{"}p^4 - terms\text{"}\right]\right\} \, , \end{aligned}$$

for 3 degenerate flavors. The low energy parameters (f, μ, ...) are to be determined from the lattice simulations.

Light pseudoscalar sector

Have precise measurements for mass and decay constants

- Continuum χPT fit to both f_π and m_π at $a = 0.12$ and 0.09 fm simultaneously
- Does not work: CL = 10^{-250}
- Could first extrapolate to continuum and then make χPT fit; but loose information



Staggered chiral perturbation theory (SχPT)

Extend the effective field theory concept to include symmetry breaking terms particular to the staggered lattice regularization:

$$\mathcal{L}_{S\chi PT} = \mathcal{L}_{\chi PT} + a^2 \mathcal{V} \ .$$

Because of the four-fold doubling, each meson field has now 15 partners, *e.g.* $\pi^+ \to \pi_A^+$, $A = 1, \ldots, 16$. At tree level, the new term parametrizes the taste symmetry breaking,

$$m_{\pi_A}^{(0)2} = 2\mu m_q + a^2 \Delta_A \ .$$

At 1-loop, this softens the chiral logs, generically as

$$2\mu m_q \log(2\mu m_q/\Lambda^2) \to \frac{1}{16} \sum_A m_{\pi_A}^{(0)2} \log(m_{\pi_A}^{(0)2}/\Lambda^2) \ .$$

Improved fits

We can do better:

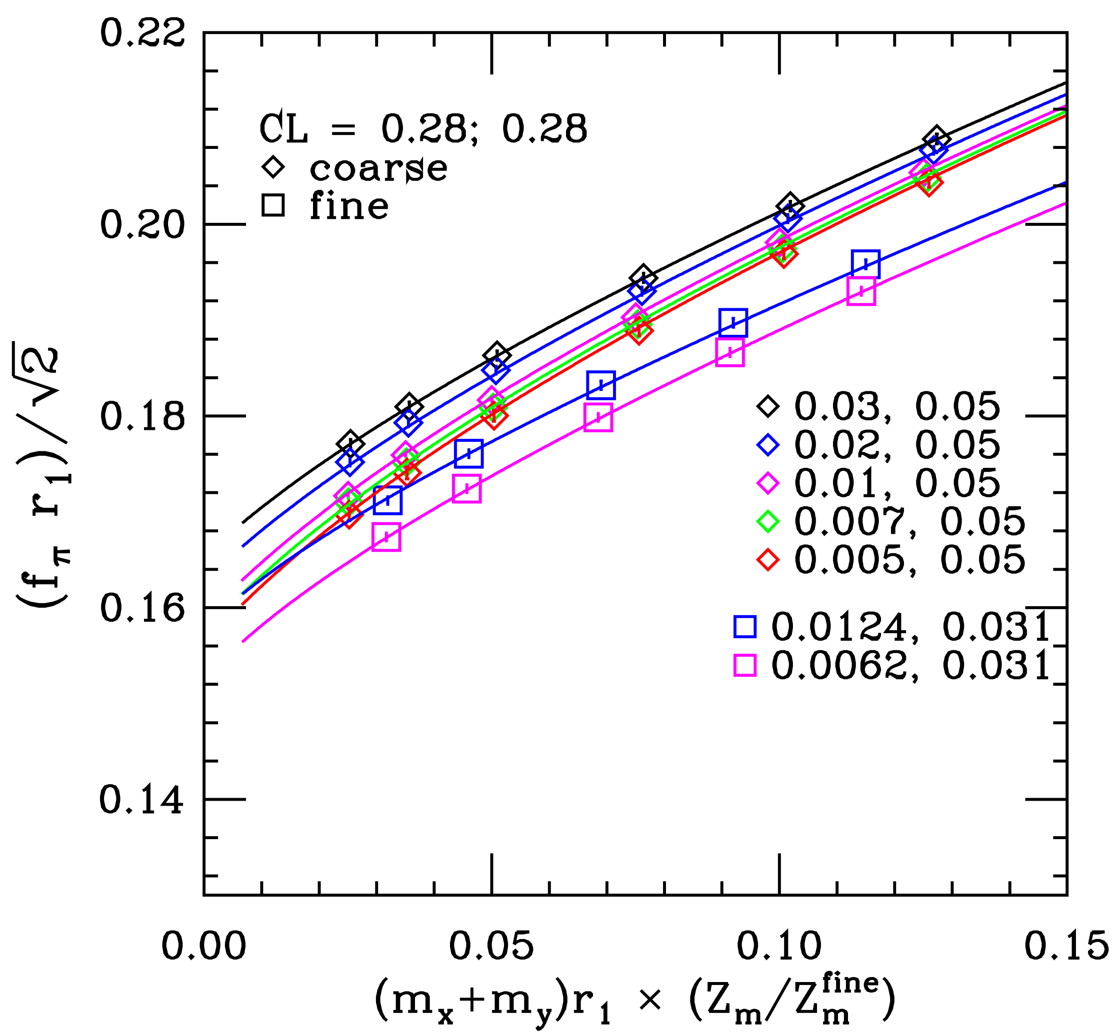
- use SχPT (Aubin & Bernard), *i.e.* with taste violation $\mathcal{O}(a^2)$ effects; include NNLO and NNNLO corrections
- fit coarse and fine lattices together: allow $\mathcal{O}(a^2)$ corrections to physical χPT parameters
- apply finite volume corrections from finite volume χPT

After fit, we:

- extrapolate fit parameters to continuum
- show difference between m_s' (simulation strange mass) and m_s (correct value)
- see C. Aubin *et al.*, PRD 70 (2004) 114501 [hep-lat/0407028]
- major differences: (i) Second m_s' on coarse lattice. (ii) Lower m_l on fine lattice. (iii) Finer lattice with $a = 0.06$ fm.

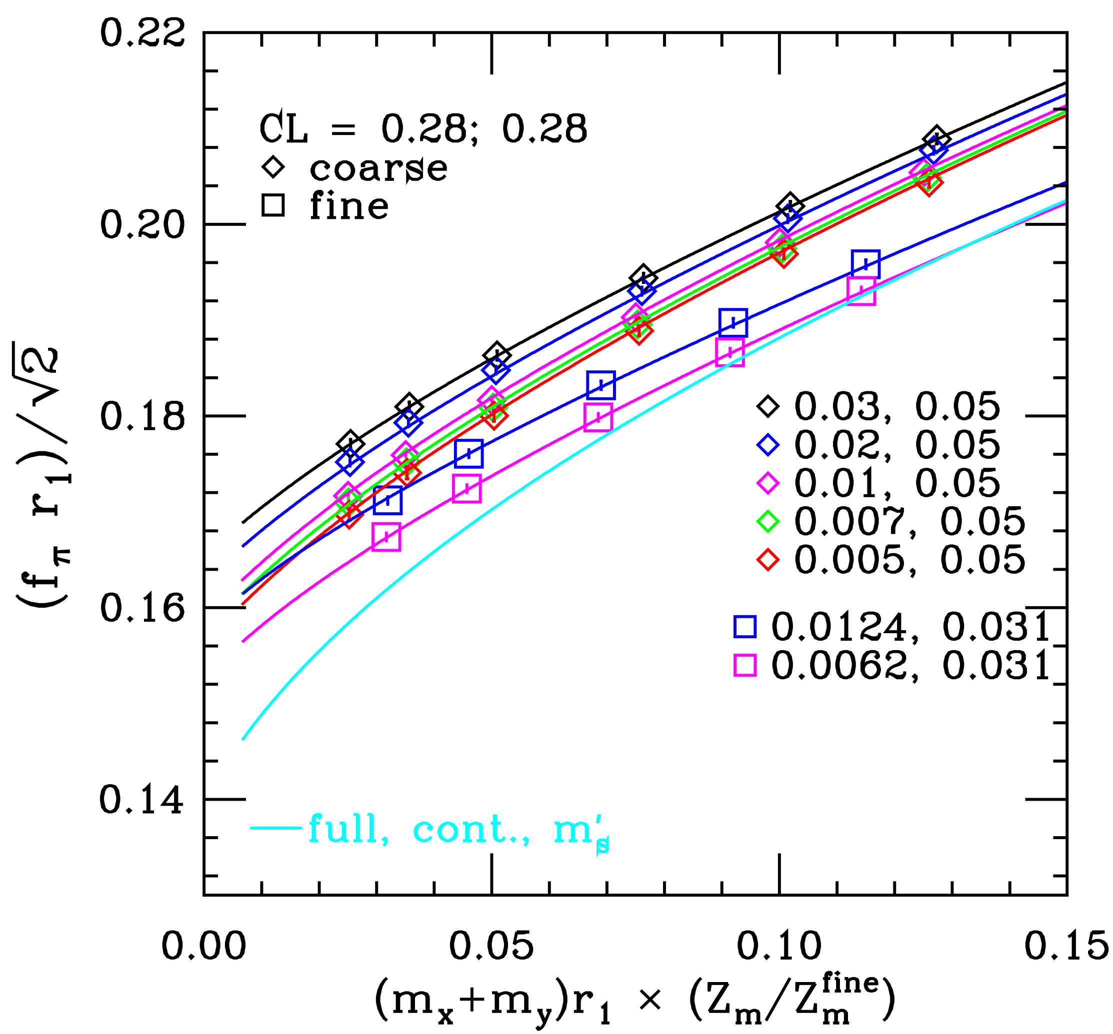
Fit of f_π

- Fit partially quenched f_π (and, simultaneously, m_π) with taste violation terms and $\mathcal{O}(a^2)$ corrections to physical χPT parameters



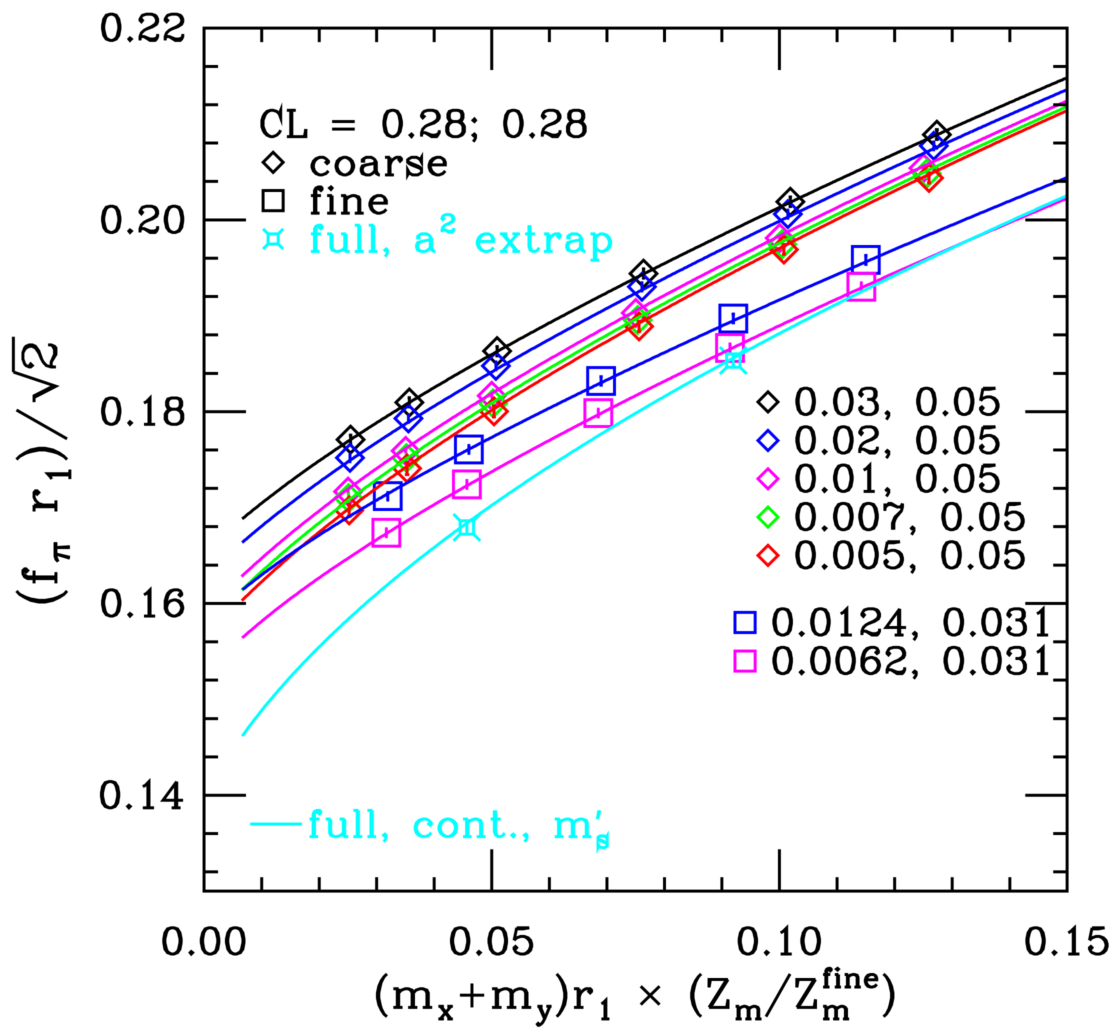
Fit of f_π

- Extrapolate fit params to continuum, *i.e.*, set $\mathcal{O}(a^2)$ terms to zero
- Go to "full QCD:" Set $\hat{m}'_{sea} = \hat{m}'_{val}$ and plot as function of $\hat{m}'_{val}$:



Fit of f_π

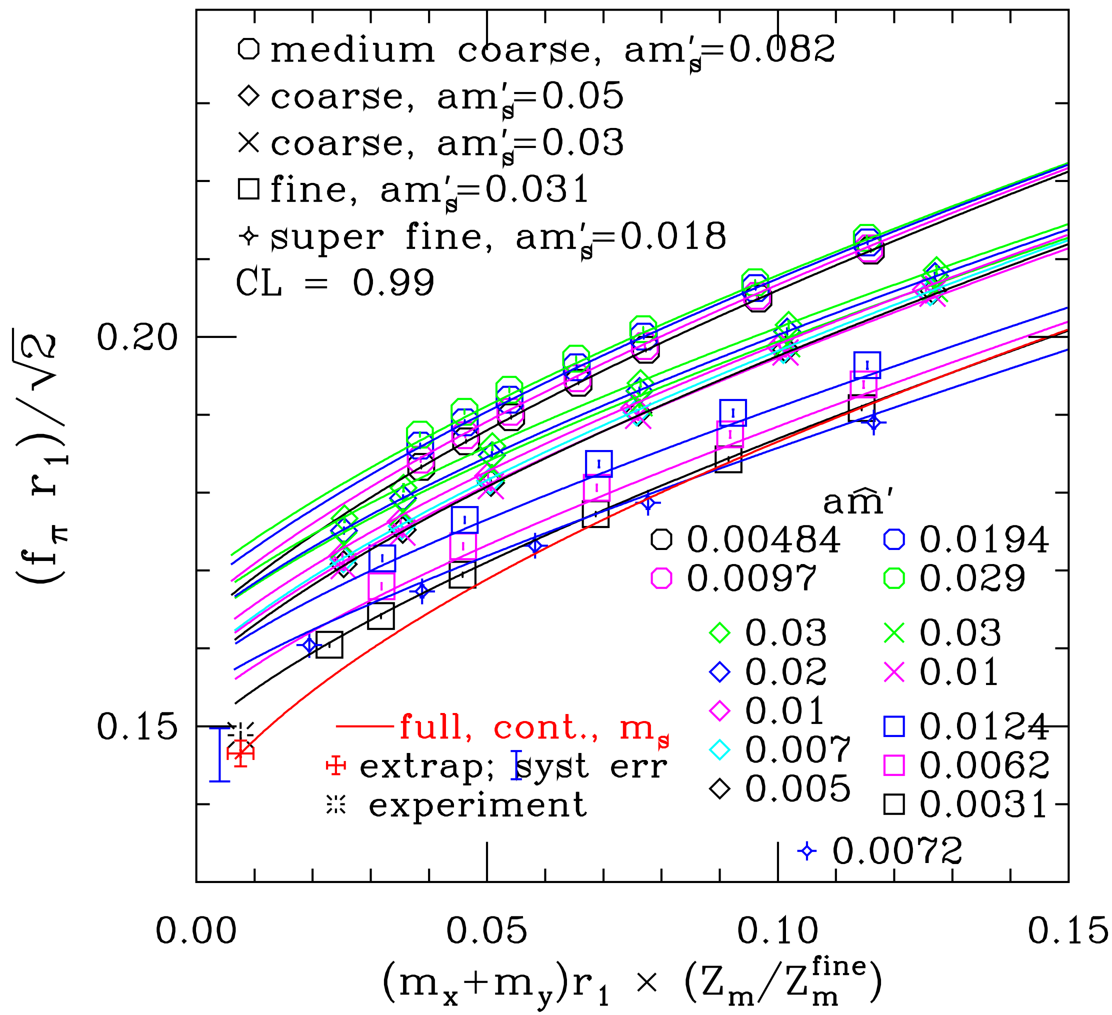
- Consistency check: extrapolate points with sea masses = valence masses to continuum at fixed quark mass



Fit of f_π

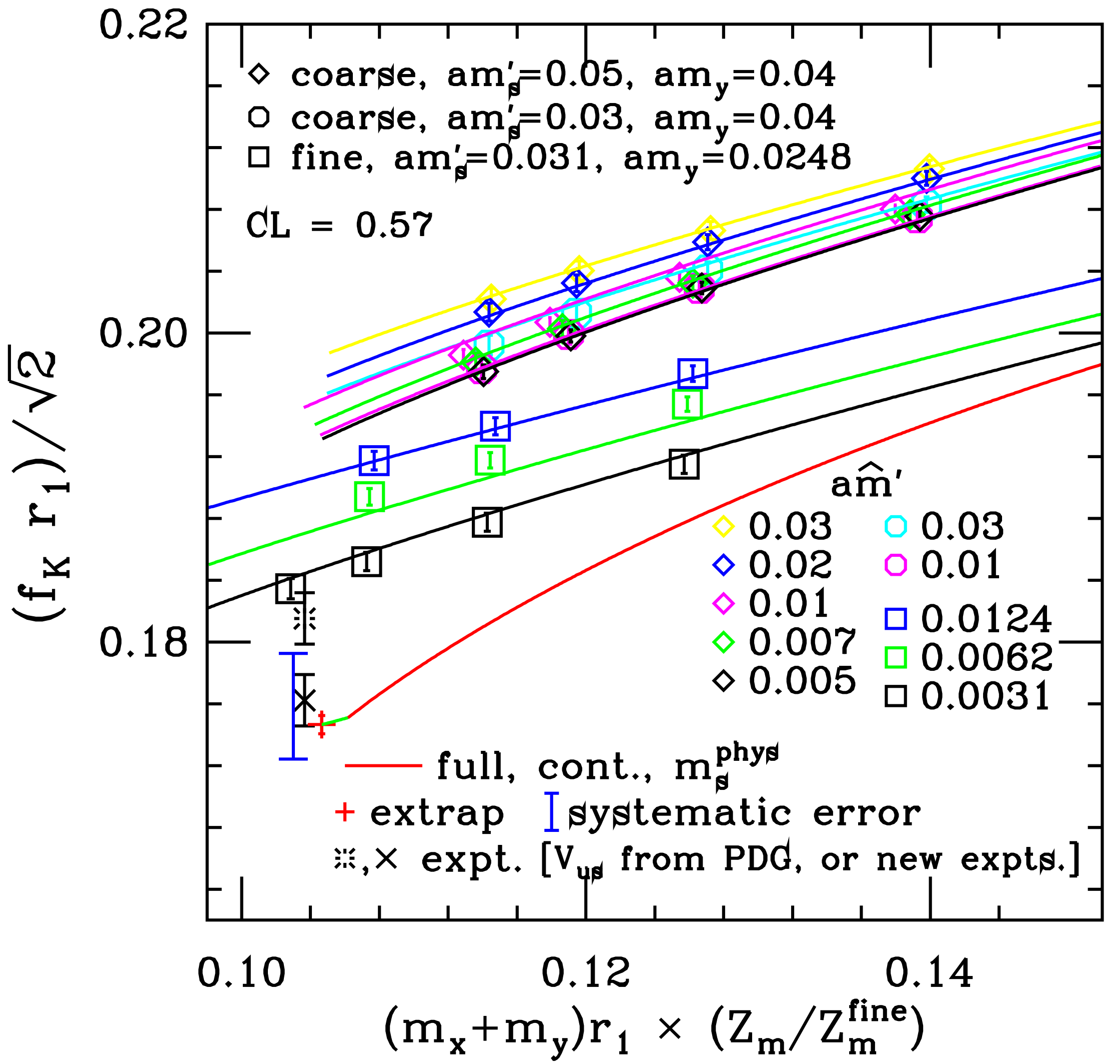
- Correct/interpolate from simulation strange mass, m'_s, to physical value, m_s

In this last plot, data from second m'_s at $a = 0.12$ fm, all data from $a = 0.09$ fm, and first data from $a = 0.06$ fm where included.

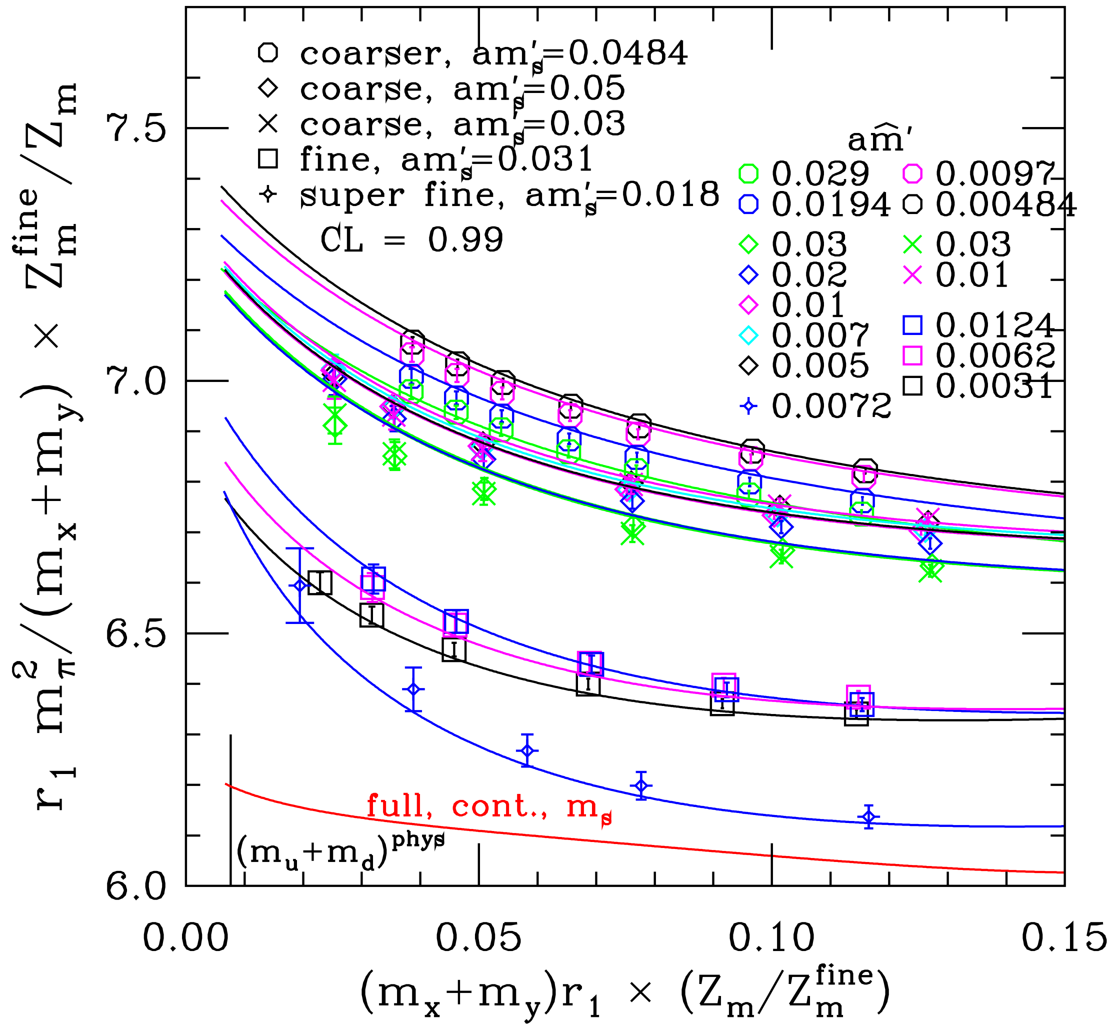


Fit of f_K

- Similar procedure for f_K.
- But note that f_K is the decay constant of K^+
- Here we need to extrapolate light valence quark to m_u, but light sea quark to $\hat{m}$



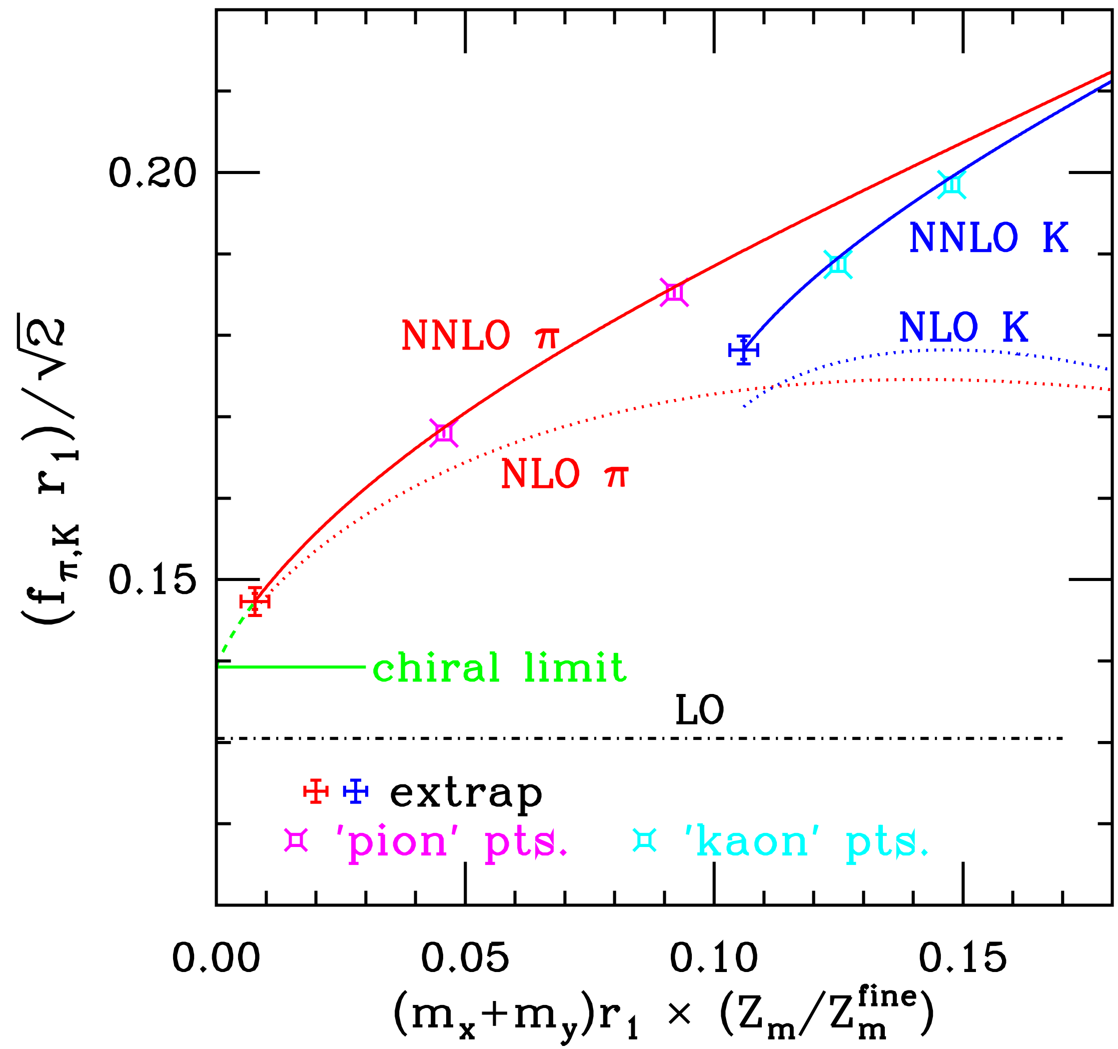
Fit of $m_\pi^2/(m_x + m_y)$



Convergence of $SU(3)_L \times SU(3)_R$ χPT

2004 fit:

i.e. no NNNLO terms



Light Quark Masses

To find quark masses, must extrapolate to the physical meson masses. Electromagnetic and isospin-violating effects are important

- Experimental masses:
  $m_{\pi^0}^{\text{expt}}, m_{\pi^+}^{\text{expt}}, m_{K^0}^{\text{expt}}, m_{K^+}^{\text{expt}}$
- Masses with EM effects turned off:
  $m_{\pi^0}^{\text{QCD}}, m_{\pi^+}^{\text{QCD}}, m_{K^0}^{\text{QCD}}, m_{K^+}^{\text{QCD}}$
- Masses with EM effects turned off and $m_u = m_d = \hat{m}$:
  $m_{\hat{\pi}}, m_{\hat{K}}$

EM & Isospin Violation

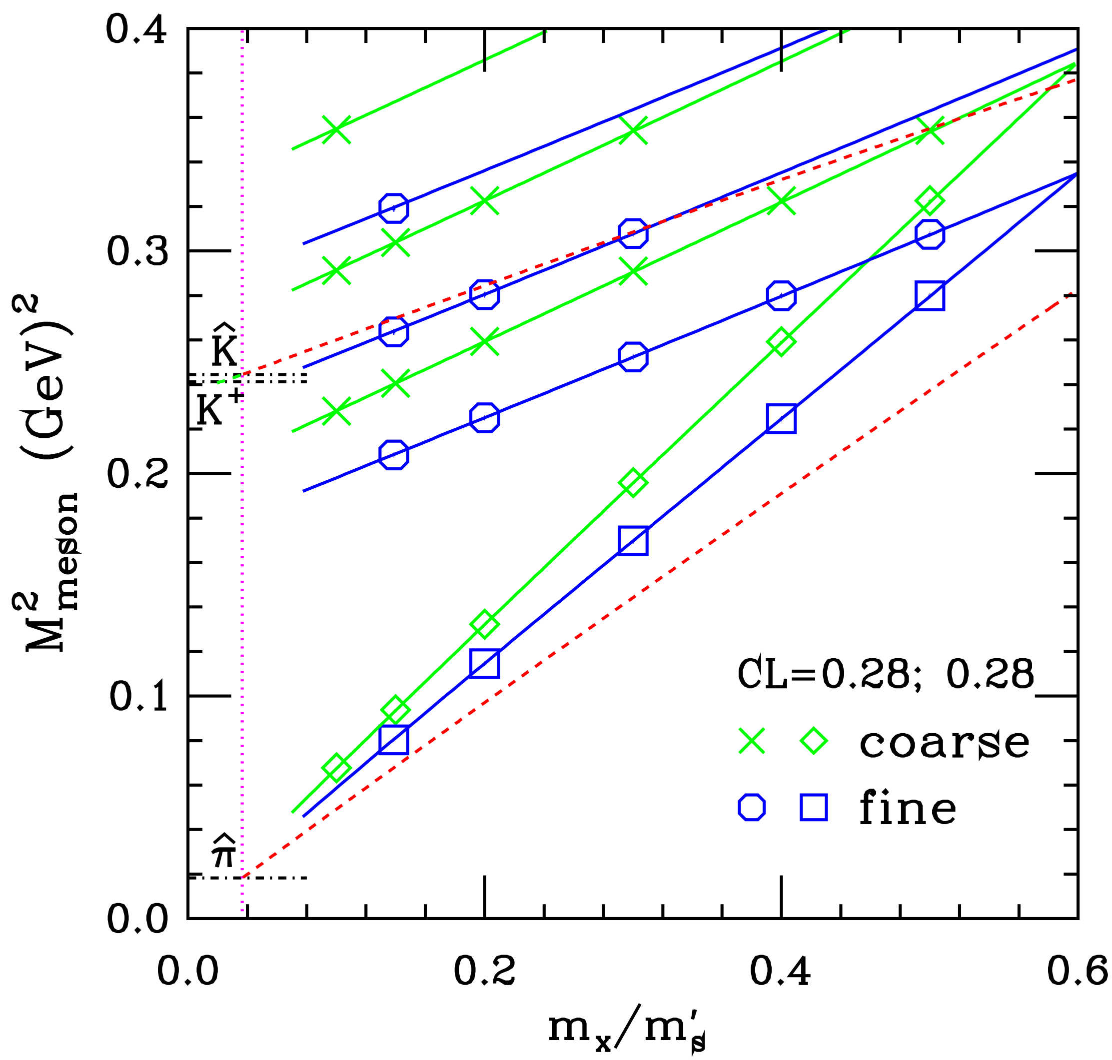
$$
\begin{aligned}
m_{\hat{\pi}}^2 &\approx (m_{\pi^0}^{\rm QCD})^2 \approx (m_{\pi^0}^{\rm expt})^2 \\
m_{\hat{K}}^2 &\approx \frac{(m_{K^0}^{\rm QCD})^2 + (m_{K^+}^{\rm QCD})^2}{2} \\
(m_{K^0}^{\rm QCD})^2 &\approx (m_{K^0}^{\rm expt})^2 \\
(m_{K^+}^{\rm QCD})^2 &\approx (m_{K^+}^{\rm expt})^2 - (1+\Delta_E)\left((m_{\pi^+}^{\rm expt})^2 - (m_{\pi^0}^{\rm expt})^2\right)
\end{aligned}
$$

- $\Delta_E = 0$ is "Dashen's theorem."
- Continuum suggests: $\Delta_E \approx 1$.
- We use $0 < \Delta_E < 2$

Finding $\hat{m}$, m_s

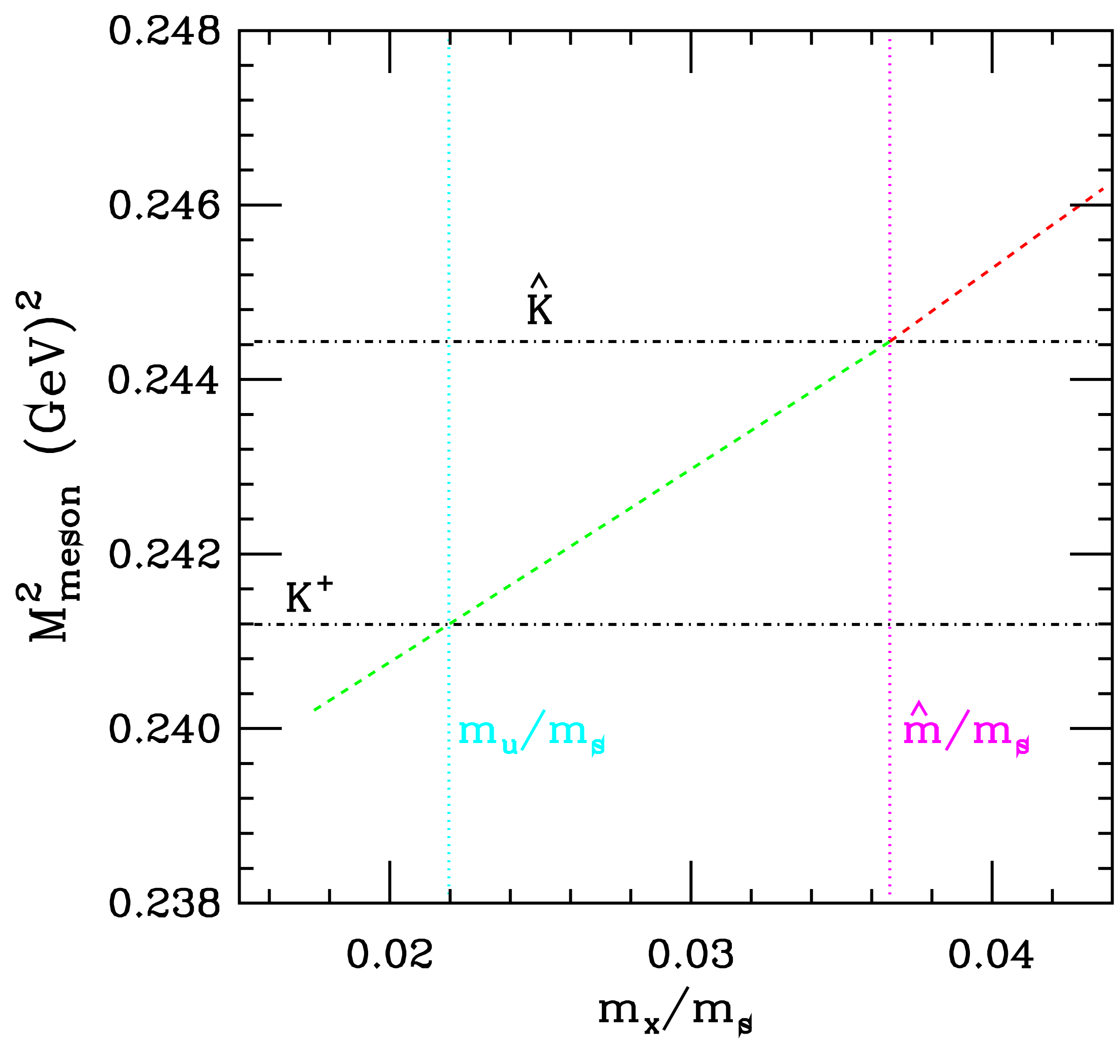
From 2004 fits.

- Subset of data with fits
- Red lines are continuum extrapolated full QCD fits with m_s adjusted so that both $\hat{\pi}$ and $\hat{K}$ are fit



Finding m_u

- Next estimate m_u by extrapolating in quark mass to K^+ mass
- Below $\hat{m}$ only valence mass changes
- There is a small isospin violation because for sea quarks $m_u = m_d = \hat{m}$



Quark mass results

We find

$$m_u/m_d = 0.42(0)(1)(4) \ ,$$

where the errors are statistical (rounded down to 0), lattice systematics, and a conservative estimate of EM effects.

Using instead a phenomenological result of Bijnens and Prades, NPB 490 (1997) 239 [hep-ph/9610360], $\Delta_E = 0.84 \pm 0.25$, we would obtain

$$m_u/m_d = 0.43(0)(1)(2) \ .$$

Quark mass results

Using a recent 2-loop mass renormalization constant (HPQCD collaboration, Q.Mason *et al.*, PRD 73 (2006) 114501 [hep-ph/0511160]) we obtain

$$m_s^{\overline{\mathsf{MS}}} = 90(0)(5)(4)(0)\ \mathrm{MeV}\ ,$$
$$\hat{m}^{\overline{\mathsf{MS}}} = 3.3(0)(2)(2)(0)\ \mathrm{MeV}\ ,$$
$$m_s/\hat{m} = 27.2(0)(4)(0)(0)\ ,$$

where the errors are from statistics, simulation systematics, perturbation theory ($2\alpha^3$), and electromagnetic effects, respectively. The renormalization scale of the masses is $2\,\mathrm{GeV}$. With m_u/m_d from above, then:

$$m_u^{\overline{\mathsf{MS}}} = 2.0(0)(1)(1)(1)\ \mathrm{MeV}\ ,$$
$$m_d^{\overline{\mathsf{MS}}} = 4.6(0)(2)(2)(1)\ \mathrm{MeV}\ .$$

Results for light decay constants

We find (from preliminary 2005 fits):

$$\begin{aligned} f_\pi &= 128.6 \pm 0.4 \pm 3.0 \text{ MeV} \; , \\ f_K &= 155.3 \pm 0.4 \pm 3.1 \text{ MeV} \; , \\ f_K/f_\pi &= 1.208(2)(^{+\ 7}_{-14}) \; . \end{aligned}$$

Experiments:
$f_\pi = 130.7 \pm 0.4 \, \text{MeV}$, $f_K = 159.8 \pm 1.5 \, \text{MeV}$, $f_K/f_\pi = 1.223(12)$.

- Using our f_K/f_π, the experimental $B(K \to \ell\nu)/B(\pi \to \ell\nu)$ and the well known Cabbibo angle V_{ud}: $\Rightarrow$ $V_{us} = 0.2223(^{+26}_{-14})$
- PDG (2006) value: $V_{us} = 0.2257(21)$

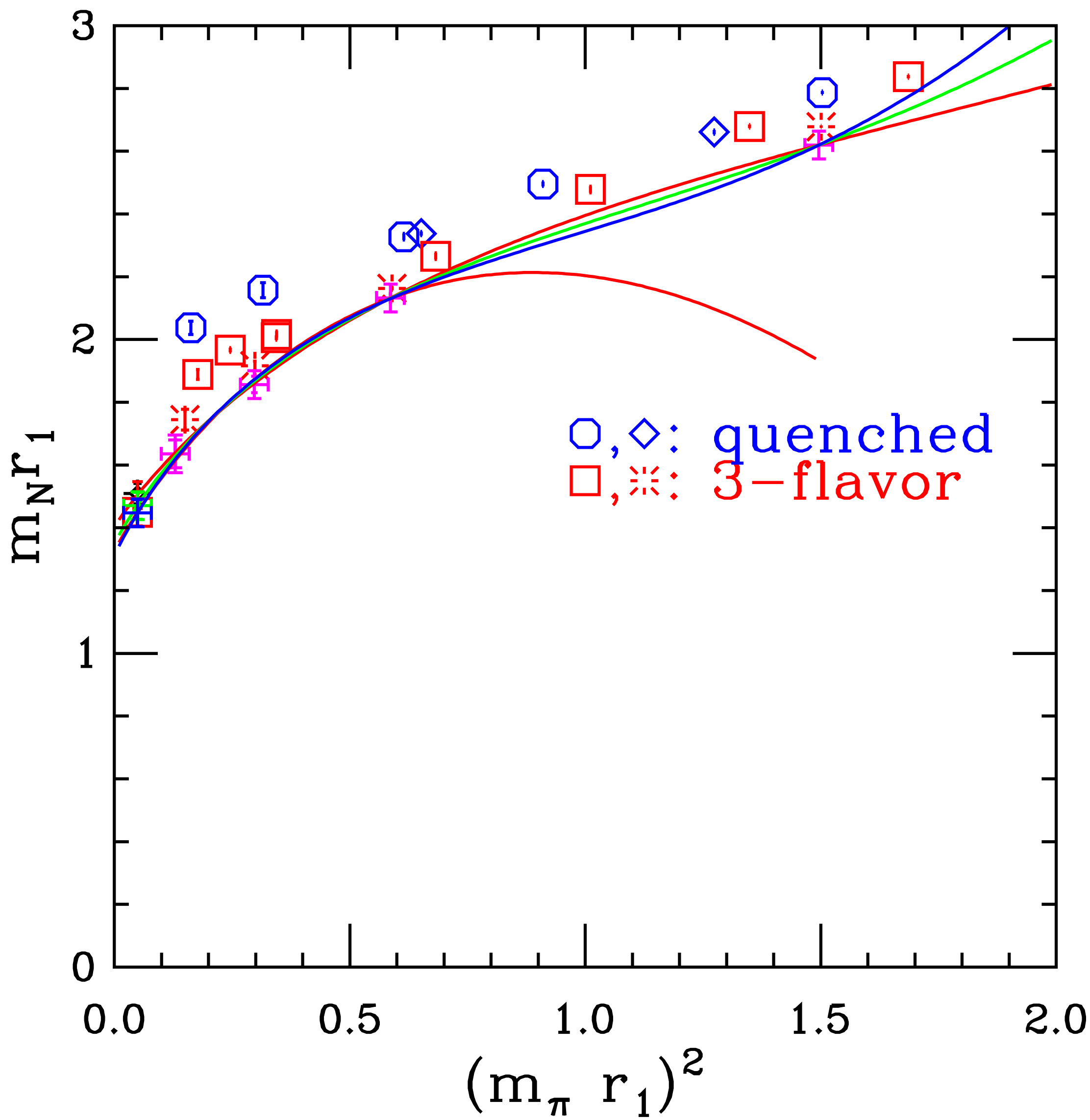
Results: Low Energy Constants

Also get (in units of 10^{-3}, at chiral scale m_η; 2004 fits):

$$
\begin{aligned}
2L_6 - L_4 &= 0.5(1)(2)\ , \\
2L_8 - L_5 &= -0.1(1)(1)\ , \\
L_4 &= 0.1(2)(2)\ , \\
L_5 &= 2.0(3)(2)\ .
\end{aligned}
$$

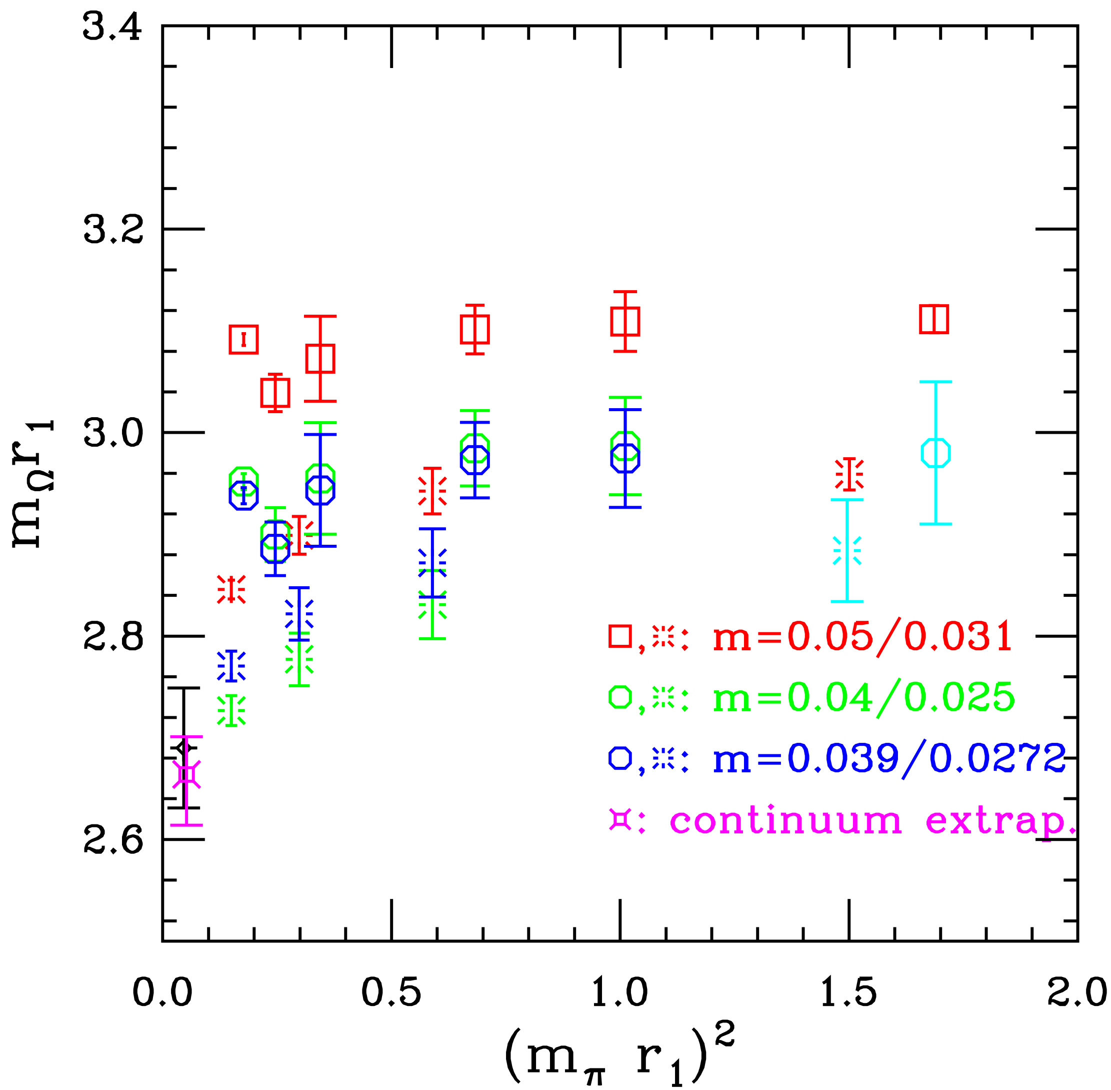
- Consistent with "conventional results" summarized, *e.g.*, in Cohen, Kaplan, & Nelson, JHEP 9911, 027 (1999): $L_5 = 2.2(5)$, $L_6 = 0.0(3)$, $L_4 = 0.0(5)$.
- Our result for $2L_8 - L_5$ is far from range that would allow $m_u = 0$, $-3.4 \leq 2L_8 - L_5 \leq -1.8$ (Kaplan & Manohar; Cohen, Kaplan & Nelson)
- Consistent with (but not independent of) direct determination of m_u.

Nucleon masses



The fancy plusses are continuum extrapolations at fixed $m_\pi r_1$. The curves are different continumm chiral extrapolations (with m_π^3 term [3 lowest points], additional analytic m_π^4 term [all 4 points, red curves]; $m_\Delta - m_N$ included, see V. Bernard *et al.* PLB 622 (2005) 141 [hep-lat/0503022] [green curve]; power series in m_π^2 and $\log(m_\pi^2)$ [blue curve]).

Ω^- baryon



Electroweak interaction and the CKM matrix

The charged electroweak current couples the fermions of the Standard Model to the W-bosons

$$\mathcal{L}_{CC} = -\frac{g_w}{\sqrt{2}} J^{\mu}_{CC} W^{+}_{\mu} + h.c.$$

At low energy, $E << m_W$, this leads to an effective current-current (4-Fermi) interaction

$$\mathcal{L}_{eff} = -2\sqrt{2} G_F J^{\mu}_{CC} J^{\dagger}_{CC,\mu}$$

with

$$G_F = \frac{g_w^2}{4\sqrt{2} M_W^2} = 1.16639(1) \times 10^{-5} GeV^{-2}$$

the Fermi constant.

Electroweak interaction and the CKM matrix

The charged weak current is given by

$$J^{\mu}_{CC} = (\bar{\nu}_e, \bar{\nu}_\mu, \bar{\nu}_\tau) \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix} + (\bar{u}_L, \bar{c}_L, \bar{t}_L) V_{CKM} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} .$$

The Cabbibo-Kobayashi-Maskawa (CKM) matrix shows that the quark families mix in the weak interactions:

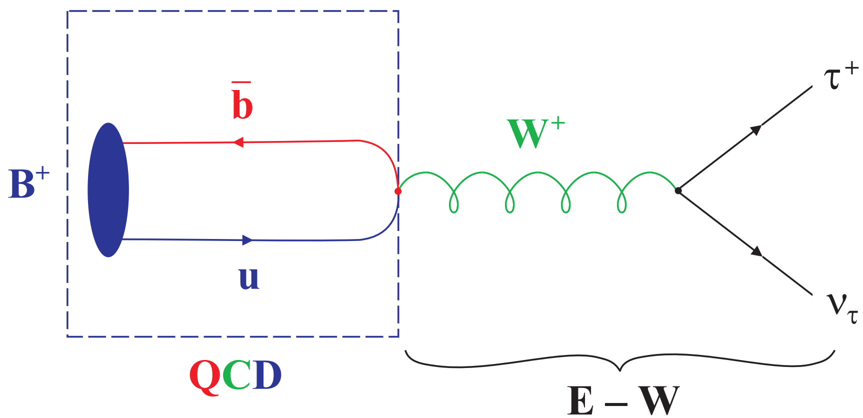
$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} .$$

The elements of the CKM matrix are parameters in the Standard Model that need to be determined from experiment. Theoretical input, from lattice QCD, is needed for this.

Heavy-light decay constants

The leptonic decay branching ratio of a B-meson, and similarly a D-meson (or K^+ or π^+ mesons), goes like

$$B(B \to \ell\nu_\ell) = |V_{ub}|^2 f_B^2 m_\ell \left(1 - \frac{m_\ell^2}{m_B^2}\right) \times \text{known factors} .$$



So computing the QCD matrix element f_B,

$$\langle 0|\bar{b}\gamma_5\gamma_\mu u|B(p)\rangle = i p_\mu f_B \ ,$$

is needed to extract the CKM matrix element V_{ub}.

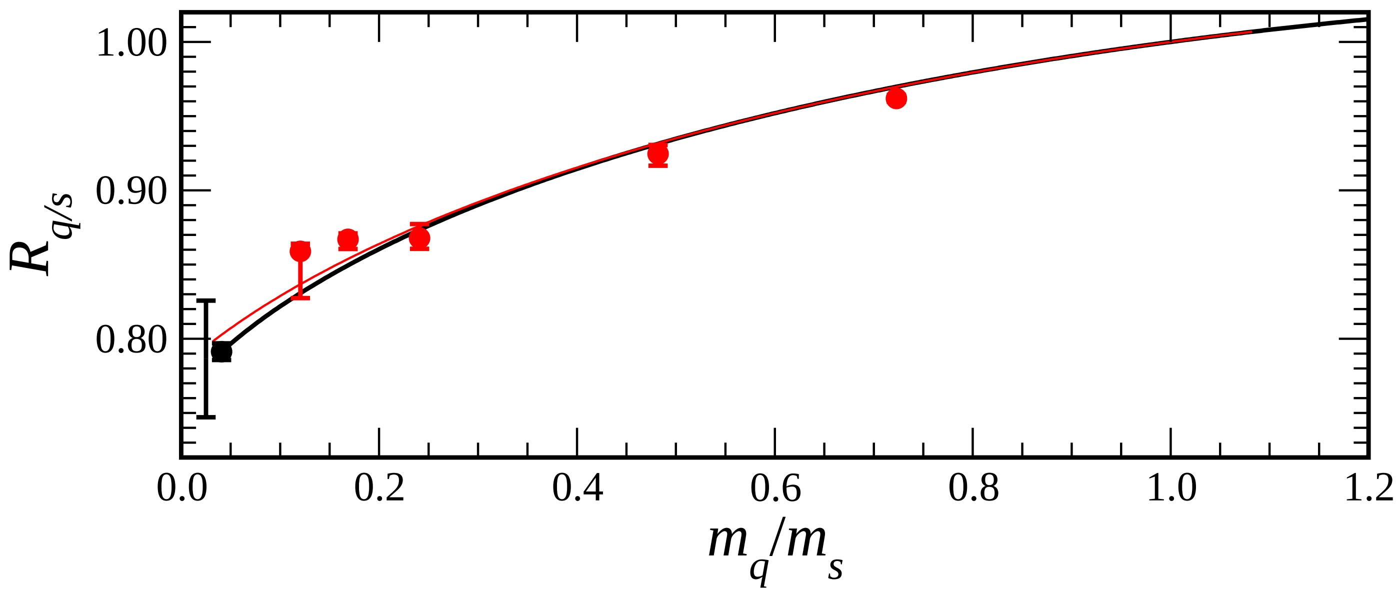
Heavy-light decay constants

With the Fermilab and HPQCD collaborations, we are computing the decay constant with improved staggered light and heavy clover (Fermilab) quarks. Advantages are:

- Can go to lower light valence quarks
- Use SχPT (Aubin & Bernard) for chiral extrapolation to m_d
- Have Z-factors, written as $Z_V^{Qq} = \rho_V (Z_V^{QQ} Z_V^{qq})^{1/2}$, with Z_V^{QQ} and Z_V^{qq} from charge normalization, non-perturbatively, and $\rho_V \approx 1$ to one-loop.

Heavy-light decay constants

Use of SχPT is illustrated in the fit of $R_{q/s} = f_D\sqrt{m_D}/f_{D_s}\sqrt{m_{D_s}}$. The red line and extrapolated point are obtained after removing the $\mathcal{O}(a^2)$ effects from the fit.



Details in C. Aubin *et al.*, PRL 95 (2005) 122002 [hep-lat/0506030]

Heavy-light decay constants

We find:

$$
\begin{aligned}
f_{D_s} &= 249 \pm 3 \pm 16 \text{ MeV} , \\
f_D &= 201 \pm 3 \pm 17 \text{ MeV} , \\
\frac{f_{D_s}\sqrt{m_{D_s}}}{f_D\sqrt{m_D}} &= 1.27 \pm 0.06 \pm 0.06 .
\end{aligned}
$$

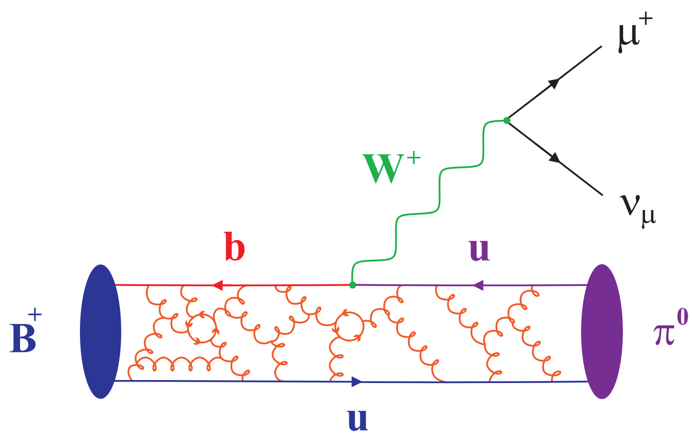
The computations for B-mesons are in progress.

Experimentally measured, from leptonic decays

$$
\begin{aligned}
f_{D_s^+} &= 274 \pm 13 \pm 7 \text{ MeV} \quad \text{(CLEO)}, \\
f_{D_s^+} &= 283 \pm 17 \pm 7 \pm 14 \text{ MeV} \quad \text{(BABAR)}, \\
f_{D^+} &= 222.6 \pm 16.7^{+2.8}_{-3.4} \text{ MeV} \quad \text{(CLEO)}, \\
\frac{f_{D_s^+}}{f_{D^+}} &= 1.23 \pm 0.11 \pm 0.04 .
\end{aligned}
$$

Semileptonic B/D decays

With the Fermilab and HPQCD collaborations, we are computing also form factors for semileptonic $D \to \pi/K$ and $B \to \pi/D$ decays.



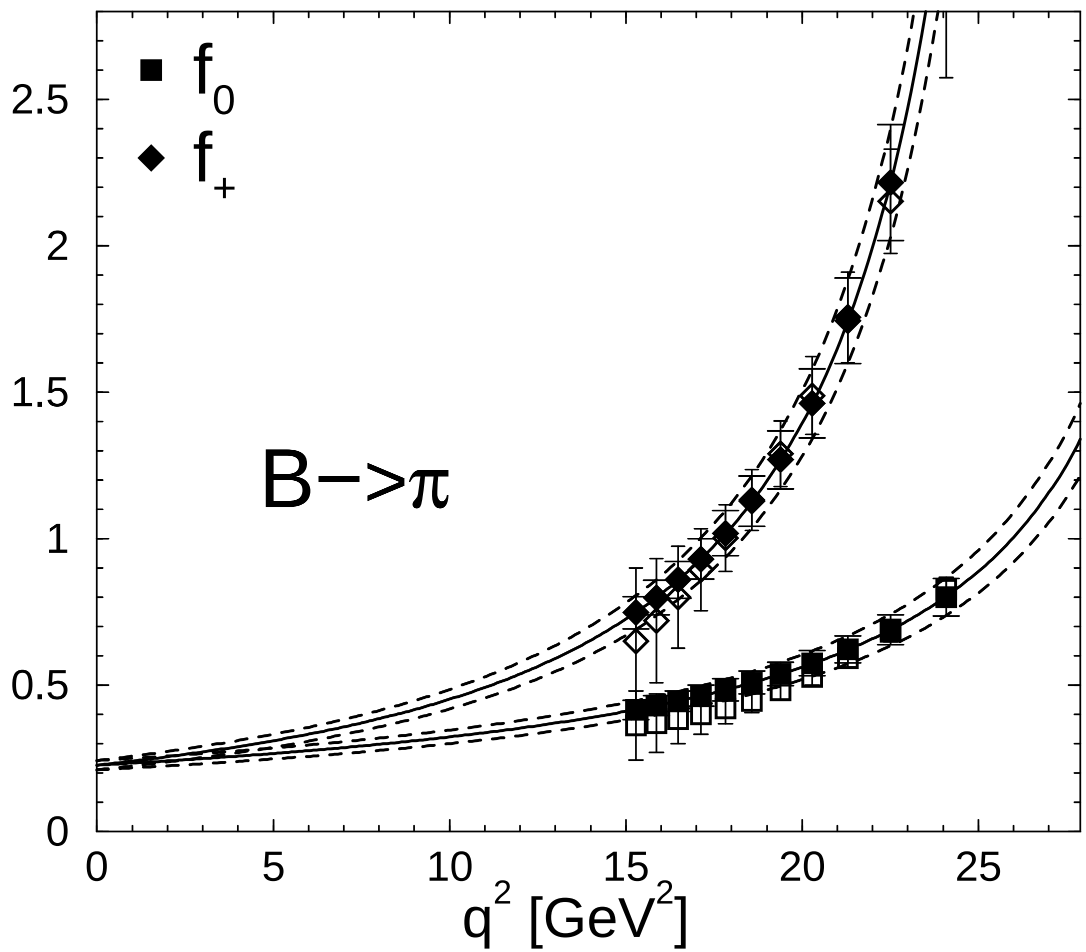
The heavy-to-light decay amplitudes, non-perturbative QCD quantities, are parametrized as

$$\langle P|V^\mu|H\rangle = f_+(q^2)(p_H + p_P - \Delta)^\mu + f_0(q^2)\Delta^\mu \ ,$$

where $\Delta^\mu = (m_H^2 - m_P^2)q^\mu/q^2$.

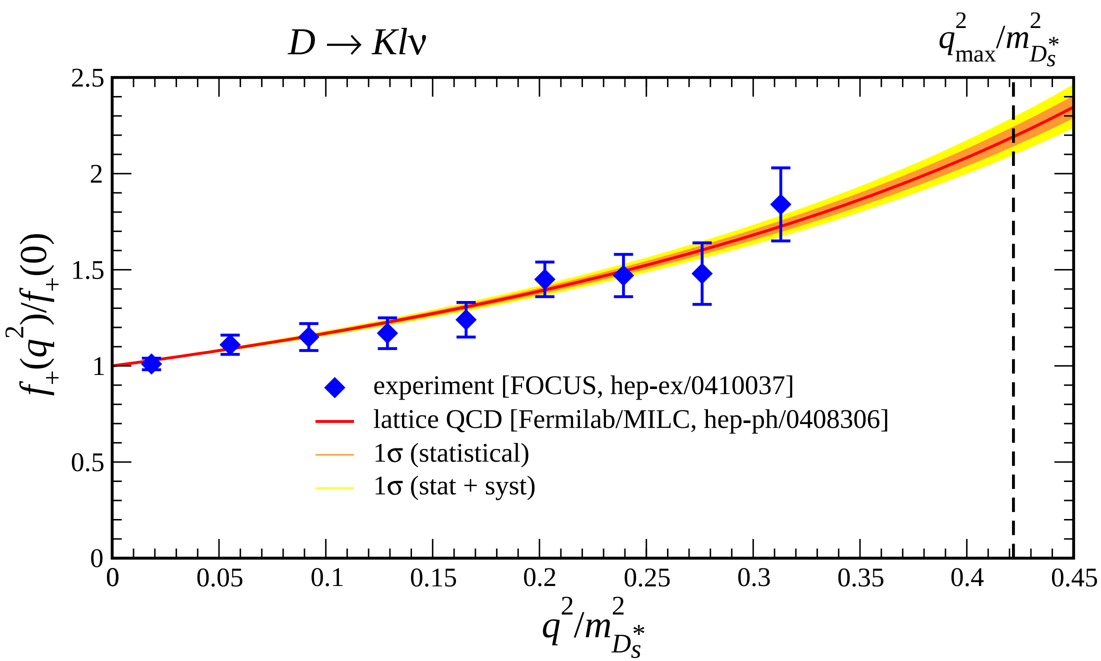
Semileptonic B/D decays

As an example we show the $B \to \pi$ form factors f_0 and f_+, see M. Okamoto *et al.*, hep-ph/0409116 (Lattice 2004):



Semileptonic B/D decays

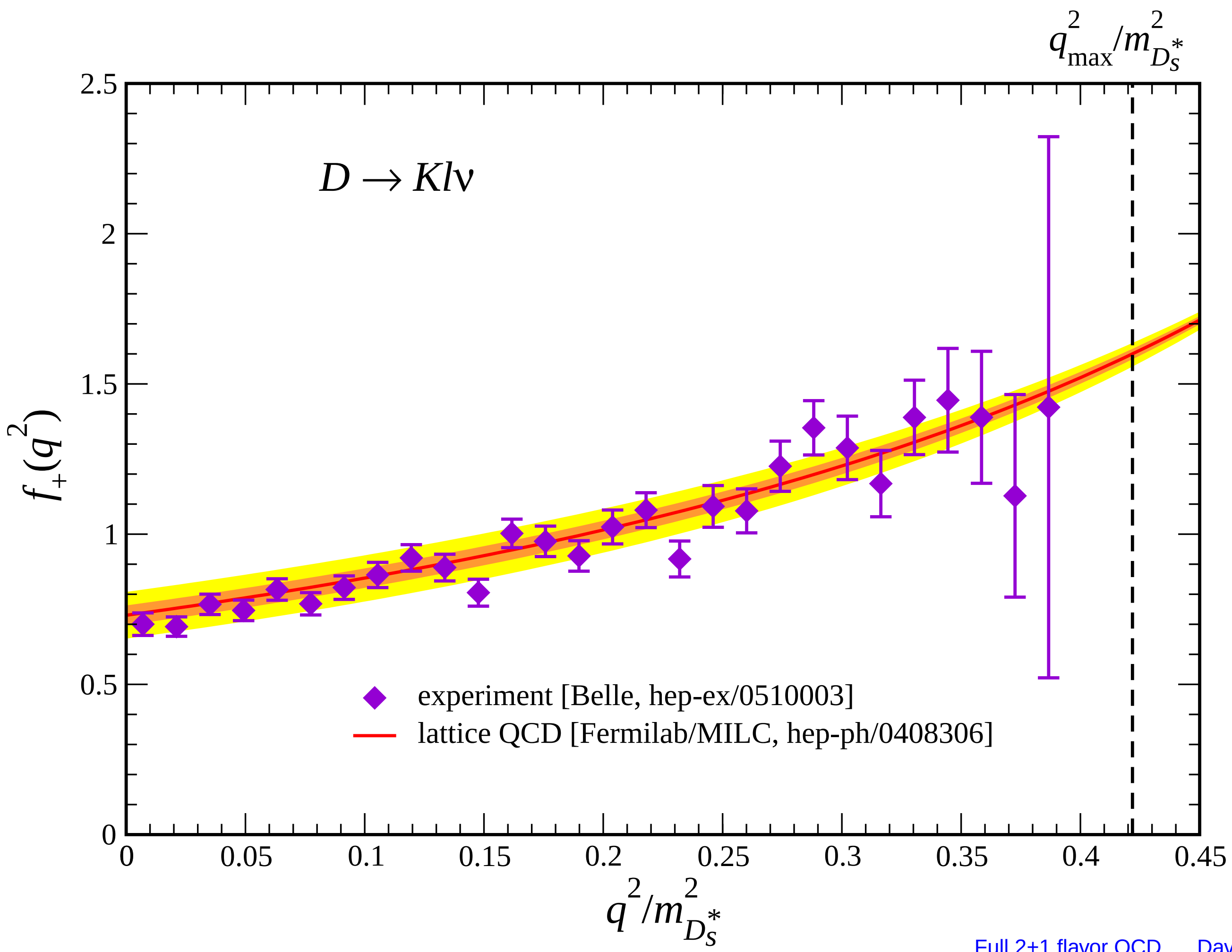
Our calculations has been compared to the experimental form factor $f_+^{(K)}(q^2)$ for the process $D^0 \to K^- \mu^+ \nu$ by the FOCUS collaboration (Phys. Lett. B607 (2005) 233 [hep-ex/0410037]).



see C. Aubin *et al.*, PRL 94 (2005) 011601 [hep-ph/0408306]

Semileptonic B/D decays

Or, we can compare with the recent measurement by Belle, including the normalization (modulo assumptions on $|V_{cs}|$):



Semileptonic B/D decays

The differential semileptonic decay rate is given by

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 {p'}^3}{24\pi^3} |V_{\rm CKM}|^2 \; |f_+(q^2)|^2 \; .$$

p': 3-mom of final-state meson in restframe of inital-state meson.

Knowing $f_+(q^2)$ allows us to extract CKM matrix elements from experiment. We find:

$$|V_{ub}| = 3.78(30)(42)(25) \times 10^{-3} \; , \;\; |V_{cd}| = 0.239(10)(24)(20) \; ,$$
$$|V_{cs}| = 0.969(39)(94)(24) \; , \;\; |V_{cb}| = 3.91(09)(34) \times 10^{-2} \; ,$$

with $\quad (|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2)^{1/2} = 1.00(10) \; .$

Summary and Outlook

Simulations at two (and, in progress, more) lattice spacings and several sea quark masses in full 2+1 flavor QCD lead to precision results in the pseudoscalar sector, including decay constants, V_{us} and quark masses.

Many other "gold-plated" observables also show good agreement with experiment.

The configurations are used for predictions for heavy-light meson decay constants and semileptonic form factors, needed for extraction of CKM matrix elements from experiments.

Improvements will include:

- Simulations with a smaller strange sea quark mass (done for $a = 0.12$ fm, planned for $a = 0.09$ fm)
- Full analysis of coarser ensembles, 0.18 and 0.15 fm
- More light quark masses, and more statistics, for the smaller lattice spacing: $a \sim 0.06$ fm

Spectrum summary