Minimal Dark Matter

- 1) Thinks that everybody knows
- 2) Things that nobody wants to know
- 3) Minimal Dark Matter

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From hep-ph/0512090 (NPB...), with Marco Cirelli, Nicolao Fornengo. www.cern.ch/astrumia/MDM.pdf

Cosmic inventory

Total density = critical density

Present composition:

Inflation explains $\rho = \rho_{\rm Cr}$. Big-bang explains $n_e = n_p$, $n_{4\rm He}/n_p \approx 0.25/4$, $n_{\rm D}/n_p \approx 3 \ 10^{-5}/2$, $n_{\nu_i} + n_{\bar{\nu}_i} = 3n_\gamma/11$,..., Could also explain DM and n_B/n_γ .

Dark matter as thermal relic

What happens to a stable particle at T < m? Scatterings try to give thermal equilibrium

 $n_{\text{DM}} \propto \exp(-m/T)$.

But at $T \leq m$ they become too slow:

 $\Gamma \sim \langle n_{\rm DM} \sigma \rangle \lesssim H \sim T^2 / M_{\rm Pl}$

Out-of-equilibrium relic abundancy:



i.e.

$$16\pi \langle \sigma_{\text{DM DM}} v \rangle = 0.21/\text{TeV}^2$$

LHC and DM searches should clarify.

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Disclaimer

The views expressed herein do not reflect the majority of the community

and are confused because reflect the present unclear situation.

DM candidates from EWSB theories...

 $...m_{\rm DM} \sim {\rm TeV}.$

Solutions to the hierarchy problem employ new physics at the electroweak scale. DM usually studied as a byproduct

* Bino/wino/higgsino/(sneutrino)/gravitino from SUSY. Stable thanks to R or matter parity: Z_2 , Z_3 , Z_6 ,...

* Z' in little-Higgs. Stable thanks to T-parity.

KK of photon or neutrino from would-be-universal extra dimensions.
 Stable thanks to KK parity.

...and their unsatisfactory aspects

- DM stability imposed ad hoc (justify with discrete gauge symmetries?). Known stable particles (ν , e, p) are stable for better reasons.
- These solutions employ embarrassingly rich phenomenology; and nothing seen so far: simplest models **survive by fine-tuning** their free parameters; their motivation is explaining $v \ll M_{\text{Pl}}$ without fine-tuning.
- DM **phenomenology obscured** by many unknown parameters. Different signals in different regions of the parameter space (scatter plots).

Sic transit gloria mundi

Around 1970 theorists anticipated the SM thanks to gauge invariance. To proceed further with BSM a different guideline was adopted: naturalness.

But, recently

1) $V \sim (10^{-3} \,\mathrm{eV})^4 \neq 0$, no natural solution known.

2) The problem of the Higgs mass hierarchy problem.

3) Anthropic considerations can justify unnaturalness: $V^{1/4}, v \ll M_{\text{Pl}}$.

4) Realized in 10^{500} string models?

Waiting for LHC results, focus on DM ignoring naturalness

If the SM holds up to large energies, conservation of B (i.e. stability of the proton), L, L_e, L_μ, L_τ , flavored violation of CP, etc, automatically follows. SUSY & co ruin these features that must be reimposed by ad hoc engineering.

Try to preserve the successes of the SM and extend to DM stability.

Minimal approach to DM

Add to the SM extra particles $\mathcal{X} + h.c.$ Search for assignement of quantum numbers (gauge charges, spin) that make a as-good-as-possible DM candidate:

- 1. Cosmologically stable
- 2. Only one parameter: M
- 3. Lightest component is neutral.
- 4. Allowed

 $\mathscr{L} = \mathscr{L}_{\mathsf{SM}} + c \begin{cases} \bar{\mathcal{X}}(i \not\!\!D + M) \mathcal{X} & \text{when } \mathcal{X} \text{ is a spin } 1/2 \text{ fermionic multiplet} \\ |D_{\mu} \mathcal{X}|^2 - M^2 |\mathcal{X}|^2 & \text{when } \mathcal{X} \text{ is a spin } 0 \text{ bosonic multiplet} \end{cases}$

Can other terms be automatically forbidden by Lorentz+gauge invariance?
 More modest goal: assume that extra terms are 0 and study phenomenology.

EWSB induces a well-defined and non-trivial phenomenology. M fixed by Ω_{DM} .

Recapitulation

Results follow from the following predictive and plausible assumptions:

1) DM is a thermal relic.

2) DM only has gauge interactions.

E.g. axions and right-handed ν are not in this category.

Neutral?

n-tuplets of $SU(2)_L$ containing a neutral components: $Q = T_3 + Y = 0$ for

0 n = 1: Y = 0.

• n = 2: |Y| = 1/2: $\mathcal{X} = (\mathcal{X}^+, \mathcal{X}^0)$ (e.g. Higgsino, lepton or Higgs doublet). All \mathcal{X} components are complex (Dirac) fermions or complex scalars.

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$$n = 3$$
:

- |Y| = 0: $\mathcal{X} = (\mathcal{X}^+, \mathcal{X}^0, \mathcal{X}^-)$ (e.g. wino, fermion triplet of see-saw) \mathcal{X}^0 is real (Majorana) fermion or a real scalar.
- |Y| = 1: $\mathcal{X} = (\mathcal{X}^{++}, \mathcal{X}^{+}, \mathcal{X}^{0})$ (scalar triplet of see-saw or little-Higgs) All \mathcal{X} components are complex (Dirac) fermions or complex scalars.

•
$$n = 4$$
:
 $-|Y| = 1/2$: $\mathcal{X} = (\mathcal{X}^{++}, \mathcal{X}^{+}, \mathcal{X}^{0}, \mathcal{X}^{-})$
 $-|Y| = 3/2$: $\mathcal{X} = (\mathcal{X}^{+++}, \mathcal{X}^{++}, \mathcal{X}^{0})$
• $n = 5$: $|Y| = 0$: $\mathcal{X} = (\mathcal{X}^{++}, \mathcal{X}^{+}, \mathcal{X}^{0}, \mathcal{X}^{-}, \mathcal{X}^{--})$ or $|Y| = \{1, 2\}$.

etc.

Stable?

A coupling with coefficient $1/\Lambda^p$ produces $\tau \sim (\Lambda \text{TeV})^{2p}/\text{TeV}$: renormalizable and dimension-5 couplings with $\Lambda \leq M_{\text{Pl}}$ are dangerous.

E.g. a scalar 5-plet can couple as $\mathcal{X}HHH^*H^*/\Lambda$: bad. Dimension 6 operators are fine, as well known from *p*-decay.

The first automatically stable MDM candidates are:

fermion 5-plets and scalar 7-plets

These also are the last MDM candidates.

Upper limit $n \leq 8$ for scalars and $n \leq 5$ for fermions by demanding

$$\alpha_2^{-1}(E) = \alpha_2^{-1}(M) + \frac{19/6 - \mathcal{O}(n^3)}{2\pi} \ln \frac{E}{M} > 0$$

MDM candidates

Quantum numbers			DM can	DM mass	$m_{DM^\pm} - m_{DM}$	Events at LHC	$\sigma_{ m SI}$ in	Ra-
$SU(2)_L$	$U(1)_Y$	Spin	decay into	in TeV	in MeV	$\int \mathcal{L} dt = 100/\text{fb}$	$10^{-45}{ m cm}^2$	ting
1	0	0	HH^*	?	—	0	?	$\sqrt{\times}$
1	0	1/2	LH^*	—	—	0	0	$\times \times$
2	1/2	0	EL	0.54	350	320÷510	0.3	$\times \times$
2	1/2	1/2	EH	1.2	341	$150 \div 300$	0.3	$\times \times$
3	0	0	HH^*	2.0	166	$0.2 \div 1.0$	1.3	$\sqrt{\times}$
3	0	1/2	LH	2.5	166	0.7 ÷ 3.5	1.3	$\sqrt{\times}$
3	1	0	HH, LL	1.6	540	$3.0 \div 10$	2.5	$\times \times$
3	1	1/2	LH	1.9	526	25 ÷ 80	2.5	$\times \times$
4	1/2	0	HHH^*	2.4	353	$0.1 \div 0.6$	1.9	××
4	1/2	1/2	(LHH^*)	2.4	347	4.8÷23	1.9	$\times \times$
4	3/2	0	HHH	2.9	729	$0.01 \div 0.09$	10	$\times \times$
4	3/2	1/2	(LHH)	2.6	712	$1.5 \div 8.5$	10	$\times \times$
5	0	0	(HHH^*H^*)	5.0	166	≪ 1	12	$\sqrt{\times}$
5	0	1/2	—	4.4	166	$\ll 1$	12	$\sqrt{}$
7	0	0	—	8.5	166	≪ 1	46	$\sqrt{}$

Rating = { allowed without tricks , stable without tricks }

MDM candidates

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Rating = { automatically allowed , automatically stable }

The intra-multiplet mass splitting

Scalar MDM can have non-minimal renomalizable couplings

 $\mathscr{L}_{\text{non minimal}} = \mathscr{L} - \lambda_H (\mathcal{X}^* T^a_{\mathcal{X}} \mathcal{X}) (H^* T^a_H H) - \lambda'_H |\mathcal{X}|^2 |H|^2$

producing a mass splitting suppressed by M

$$\Delta M = \frac{\lambda_H v^2 |\Delta T_{\mathcal{X}}^3|}{4M} = \lambda_H \cdot 7.6 \text{ GeV} \frac{\text{TeV}}{M}$$

One loop corrections generate:

$$M_Q - M_0 = \frac{\alpha_2 M}{4\pi} \left\{ Q^2 s_W^2 f(\frac{M_Z}{M}) + Q(Q - 2Y) \left[f(\frac{M_W}{M}) - f(\frac{M_Z}{M}) \right] \right\}$$

$$M_Q - M_0 = \frac{\alpha_2 M}{4\pi} \left\{ Q^2 s_W^2 f(\frac{M_Z}{M}) + Q(Q - 2Y) \left[f(\frac{M_W}{M}) - f(\frac{M_Z}{M}) \right] \right\}$$

 $f(r) \stackrel{r \to 0}{\simeq} -2\pi r$ for **both fermionic and scalar** multiplets if $M \gg M_Z$: $M_Q - M_{Q=0} \stackrel{M \gg M_Z}{\simeq} Q(Q + \frac{2Y}{\cos \theta_W})\Delta M$ $\Delta M = \alpha_2 M_W \sin^2 \frac{\theta_W}{2} = (166 \pm 1) \text{ MeV}$ The lightest component is neutral

Intuitive explanation

The mass difference corresponds to the *classical* non-abelian Coulomb energy

$$\delta M = \int d^3 r \left[\frac{1}{2} (\vec{\nabla} \varphi)^2 + \frac{M_V}{2} \varphi^2 \right] = \frac{\alpha}{2} M_V + \infty \qquad \varphi(r) = \frac{g e^{-M_V r/\hbar}}{4\pi r}$$



Same physics responsible for very-low-energy scatterings: $\sigma(M, s) \sim 1/M_W^2$.

The DM abundancy



Assume $\sqrt{s} \simeq 2M \gg M_Z$: compute in SU(2)_L-symmetric non-relativistic limit:

Tr
$$T^{a}T^{b}T^{a}T^{b} = \frac{n}{16}(n^{2} - 5)(n^{2} - 1)$$

automatically sums over all co-annihilations. Scalar and fermion DM annihilates into AA with the same σ . Fermions also annihilate in quarks, leptons, higgses.

$$\langle \sigma_A v \rangle \simeq \begin{cases} \frac{g_2^4 (3 - 4n^2 + n^4) + \mathcal{O}(g_2^2 g_Y^2, g_Y^4)}{64\pi \ M^2 \ \text{dof}_{\mathcal{X}}} & \text{if } \mathcal{X} \text{ is a scalar} \\ \frac{g_2^4 (n^4 + 9n^2 - 10) + \mathcal{O}(g_2^2 g_Y^2, g_Y^4)}{64\pi \ M^2 \ \text{dof}_{\mathcal{X}}} & \text{if } \mathcal{X} \text{ is a fermion.} \end{cases}$$

A posteriori $M \gg M_Z$ is a good approximation (sic!). We missed non-perturbative effects, relevant if $M \ge M_Z/\alpha_2 \sim$ few TeV.

DM atoms

Scatterings are non-perturbative at very low energies, when bound states form.

Consider two body systems: (DM^0, DM^+) , and (DM^0DM^0, DM^+DM^-) , ...

$$H = \frac{p^2}{M} - V(r) + 2i\Gamma\delta(\vec{r})$$

• $V \sim \pm \alpha_2 e^{-M_W r}/r$ is the non-abelian Coulomb-like potential.

 \bullet Γ is the perturbative annihilation rate. Both are matrices.

 \boldsymbol{V} is sizable (i.e. bound states form) if

 $M \gtrsim M_W/\alpha_2.$

Wavefunctions $\psi(0)$ distorted at energy

 $H = E \lesssim \alpha_2 M \sim M/30$

Relevant for freeze-out at $T_f \sim M/26$.

[Sommerfeld — Hisano et al. —



Allowed?

DM must be neutral under the γ, g and almost neutral under the Z

MDM candidates with $Y \neq 0$ are already excluded by Z-exchange scattering:



Candidates with $Y \neq 0$ can be resurrected by coupling to the Higgs:

$$\mathscr{L}_{non\ minimal} = \mathscr{L} + \begin{cases} (\mathcal{X}H)^2 + h.c. & \text{if } \mathcal{X} \text{ is a scalar} \\ \mathcal{X}HS + mS^2 & \text{if } \mathcal{X} \text{ is a fermion} \end{cases}$$

This splits the neutral components into two real eigenstates that couple to the Z as $\mathcal{X}_0 \mathcal{X}'_0 Z$. NC scattering kinematically forbidden if $M_{\mathcal{X}'_0} - M_{\mathcal{X}_0} > M\beta^2/2 \lesssim \text{MeV}$. This happens in the MSSM for the Higgsino, with S = bino

For both healthy Y = 0 candidates, and for ill $Y \neq 0$ candidates...

Direct DM searches

The NC signal arises at one loop:



$$\mathscr{L}_{\mathsf{eff}}^W = (n^2 - 1) \frac{\pi \alpha_2^2}{16M_W} \sum_q \left[(\frac{1}{M_W^2} + \frac{1}{m_h^2}) [\bar{\mathcal{X}}\mathcal{X}] m_q [\bar{q}q] - \frac{2}{3M} [\bar{\mathcal{X}}\gamma_\mu\gamma_5\mathcal{X}] [\bar{q}\gamma_\mu\gamma_5q] \right]$$

The SI-contribution is not suppressed by M and does not depend on DM spin. (Disagreement with analogous computations for higgsinos and winos)

Actually, to compute nuclear matrix elements one should leave quarks off-shell obtaining different operators. But $\bar{q}i\partial q$ not yet studied, so for simplicity:

$$\langle N|\sum_{q}m_{q}\bar{q}q|N\rangle \equiv fm_{N} \qquad f = \{0.4, 1.2, ?\} \quad \rightsquigarrow \quad 1/3$$

Predictions for $\sigma_{SI}(DM N)$



Uncertainties: matrix elements, astrophysics ρ_{local}

Indirect DM searches

DM DM annihilations in the Sun, Earth $(\rightarrow \nu)$ or in the Galaxy $(\rightarrow \bar{e}, \gamma, \bar{p}, \bar{d})$ Too small NR cross sections at apparently-dominant order



Resonant non-relativistic enhancement if $M(DM^0DM^0) = M(DM^+DM^-)$: mass difference $\Delta M \sim \alpha M_W$ compensated by binding energy of the two-body state $\Delta E_{\text{bind}} \sim \alpha^2 M$ if $M = M_* \sim M_W/\alpha$ enhancing σ by $\mathcal{O}(1 - M/M_*)^{-2}$

Signal possible if $M \sim M_*$ [Hisano et al.]: relevant for n = 3.

Astrophysical uncertainties and $M - M_*$ make rates significantly uncertain.

CC DM searches?

The quasi elastic $\hat{\sigma}(DMq \rightarrow q'DM^+)$ is 10 orders of magnitude higher than σ_{NC}

$$\hat{\sigma} = \sigma_0 \frac{n^2 - 1}{4} \left[1 - \frac{\ln(1 + 4E^2/M_W^2)}{4E^2/M_W^2} \right], \quad \sigma_0 = \frac{G_F^2 M_W^2}{\pi} = 1.1 \, 10^{-34} \, \text{cm}^2$$

but in our Galaxy is kinematically forbidden, and off-shell becomes negligible.



Can one accelerate and store a unfocused intense p or nuclear beam?

$$\frac{dN}{dt} = \varepsilon N_p \sigma \frac{\rho_{\text{DM}}}{M} = \varepsilon \frac{10}{\text{year}} \frac{N_p}{10^{20}} \frac{\rho_{\text{DM}}}{0.3 \,\text{GeV/cm}^3} \frac{\text{TeV}}{M} \frac{\sigma}{3\sigma_0}$$

The problem is the beam-related backgrounds. If DM^+ had a clean signature...

DM[±] phenomenology

Since 166 > 139 its life-time is $\tau = 44 \text{ cm}/(n^2 - 1)$ rather than $\tau \sim 10 \text{ m}$:

$$\mathsf{DM}^{\pm} \to \mathsf{DM}^{0} \pi^{\pm} \qquad : \ \ \mathsf{\Gamma}_{\pi} = (n^{2} - 1) \frac{G_{\mathsf{F}}^{2} V_{ud}^{2} \Delta M^{3} f_{\pi}^{2}}{4\pi} \sqrt{1 - \frac{m_{\pi}^{2}}{\Delta M^{2}}}, \ \ \mathsf{BR}_{\pi} = 97.7\%$$

$$\mathsf{DM}^{\pm} \to \mathsf{DM}^{0} e^{\pm} (\overline{\nu}_{e}^{)} : \Gamma_{e} = (n^{2} - 1) \frac{G_{\mathsf{F}}^{2} \Delta M^{5}}{60\pi^{3}}$$
 $\mathsf{BR}_{e} = 2.05\%$

 $\mathsf{DM}^{\pm} \to \mathsf{DM}^0 \mu^{\pm} (\overline{\nu}^0_{\mu} : \Gamma_{\mu} = 0.12 \Gamma_e$ $\mathsf{BR}_{\mu} = 0.25\%$



Detection of neutralino DM?

Another experiment allowed by an intense e beam. See hep-ph/0504068 Once that m_N and $m_{\tilde{e}}$ are known:

electron(
$$E = \frac{m_{\tilde{e}}^2 - m_N^2}{2m_N}$$
) + neutralino $\stackrel{\text{resonant}}{\rightarrow}$ selectron

• $E \sim (10 \div 100) \text{ GeV}$, $\sigma \approx \pi/E^2$: can have 1 event/10m/year for $i \approx 100 \text{ A}$.

- Needs a high-intensity beam, such as those for ν -factory, super-B factory...
- Signal is $electron(E', \theta)$.
- Backgrounds from
 - beam;
 - matter in the beam pipe;
 - synchrotron radiation.

Realistic? Better to use muons?

High energy signals

Indirect signals

Corrections to precision data: small constraints and signals

$$\hat{S} = \hat{T} = 0, \qquad W = c \, \operatorname{dof}_{\mathcal{X}} \frac{\alpha_2}{60\pi} \frac{M_W^2}{M^2} \frac{n^2 - 1}{12}, \qquad Y = c \, \operatorname{dof}_{\mathcal{X}} Y^2 \frac{\alpha_Y}{60\pi} \frac{M_W^2}{M^2}$$

 $(c = 1/4 \div 1)$. No flavour effects.

Direct signals at LHC

(SUSY production: dominantly from gluino decays. Signal: $\not\!\!E_T$ + jets, μ). MDM signal: would need dedicated trigger (no) or dedicated small detector:



MDM production for Y = 0 (heavy scalars are *p*-wave suppressed)

 $\hat{\sigma}_{u\bar{d}} = \hat{\sigma}_{d\bar{u}} = 2\hat{\sigma}_{u\bar{u}} = 2\hat{\sigma}_{d\bar{d}} = \frac{\operatorname{dof}_{\mathcal{X}}g_2^4(n^2 - 1)}{13824 \ \pi \hat{s}}\beta \cdot \begin{cases} \beta^2 & \text{if } \mathcal{X} \text{ is a scalar} \\ 3 - \beta^2 & \text{if } \mathcal{X} \text{ is a fermion} \end{cases}$ $E_{\text{beam}} = 2(4) \times E_{\text{LHC}} \text{ needed to test all fermionic (scalar) MDM candidates.}$

Astrophysics



Suppose that some cosmic rays with UHE energy contain some DM...

The DM⁺ life-time is large enough that it behaves in an unusual way: it crosses the earth loosing $E \sim 10^{16} \,\text{eV}$ and doing about half of the trip as a DM⁺.



IceCUBE would see something that looks like an up-ward going muon with TeV-scale energy, expect that it looses energy without stopping in a few km. Maybe IceCUBE cannot tell the difference; a bigger vertical volume is needed.

Non minimal Minimal Dark Matter

Putting multiple multiplets, all of them are lighter than in the minimal case. • E.g. $\sqrt{3}$ times lighter if 3 equal generations, since $\Omega_{DM} \propto M^2$.

Extra quartic interactions of scalar MDM with the Higgs increase MDM mass.

- E.g. by 20% (0.5%) for the scalar triplet (eptaplet) if $\lambda'_H = 1$. At one loop $\lambda'_H = 0$ is radiatively stable for Y = 0.
- Detection cross sections can be strongly enhanced.
- The scalar singlet becomes allowed: $M = 2.2 \text{ TeV} |\lambda'_H|$ if $\gg M_Z$.

Conclusions

Whatever keeps DM stable, tends to prevent its couplings. DM might be a single multiplet with only gauge couplings? We classified this predictive limit.

- Multiplets with Y = 0 are allowed, $Y \neq 0$ need non minimality.
- Fermions are fully predictive, scalars can couple to the Higgs.
- Famous candidates are not automatically stable, others are.

A fermion 5-plet with Y = 0 is automatically stable, allowed, fully predictive.

Broken gauge interactions induce a well-defined non trivial phenomenology. Fixing O(2) factors was hard and crucial: e.g. 166 > 139 > 166/2.

Direct DM searches under planning can probe MDM candidates with higher n (and, if multiple multiplets are present, those that dominate Ω_{DM}). LHC can probe those with lower n (and sub-dominant contributions to Ω_{DM}).