Domain Walls as Probes of Gravity

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hep-th/0612016

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February 5th 2007

What: Gravitational effect of Domain Walls in IR-modified gravity

Why: Gravitating DW solutions in DGP can be found ***exactly***.
Useful information about intrinsically non-linear theory.

Main Results

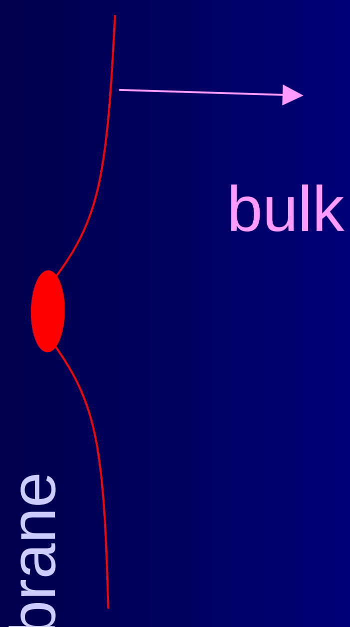
1) Screening of the tension.
Analog to screening in Schwarzchild-like solutions.

2) Domain Walls are short-distance probes of large-distance modified gravity.

Dvali Gabadadze Porrati (DGP)

$$S = \int d^5x \sqrt{-g}\, \frac{M_*^3}{2}\, R_5 + \int d^4x \sqrt{-h}\, \frac{m_P^2}{2}\, R_4$$

Effectively 4D for distances less than $$r_c = \frac{m_P^2}{2M_*^3}$$



$$M_*^3 G^{(5)}_{\mu\nu} = \left[T_{\mu\nu} - m_P^2 G^{(4)}_{\mu\nu} \right] \delta(y)$$

$$M_*^3 \left(K_{\mu\nu} - K\, h_{\mu\nu} \right) = T_{\mu\nu} - m_P^2\, G^{(4)}_{\mu\nu}$$

Generic IR-modified gravity

Linearized Einstein's eqs:

$$\mathcal{E}^{\alpha\beta}_{\mu\nu} h_{\alpha\beta} - m^2(\Box)\,(h_{\mu\nu} - \eta_{\mu\nu} h) = -16\pi G_N\, T_{\mu\nu}$$

$m^2(\Box) = \sqrt{\Box}/r_c$ DGP

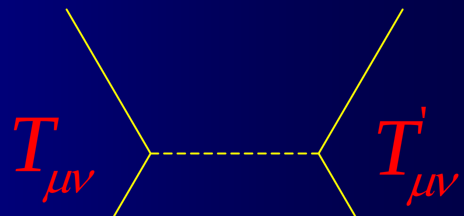
$m^2(\Box) = \text{const}$ Fierz–Pauli Massive gravity

vDVZ discontinuity

IR modified

(more d.o.f.'s)

$$\mathcal{A} \propto G_N \frac{T_{\mu\nu}T'^{\mu\nu} - \frac{1}{3}TT'}{\Box - m^2(\Box)}$$



GR

$$\mathcal{A} \propto G_N \frac{T_{\mu\nu}T'^{\mu\nu} - \frac{1}{2}TT'}{\Box}$$

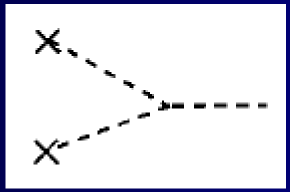
way out: higher orders in Perturbation Theory become important at short distances

Non-perturbative continuity

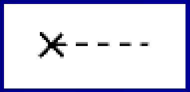
(DGP)

$r_g = 2G_N M$ $\quad$ $r_* = \left(r_g r_c^2 \right)^{1/3}$ $\quad$ r_c

Non-linearities:



$\gg$



Weak field expansion breaks down.

Large r_c expansion is OK.

Non-perturbative continuity

(DGP)

r_g $\quad$ r_* $\quad$ r_c

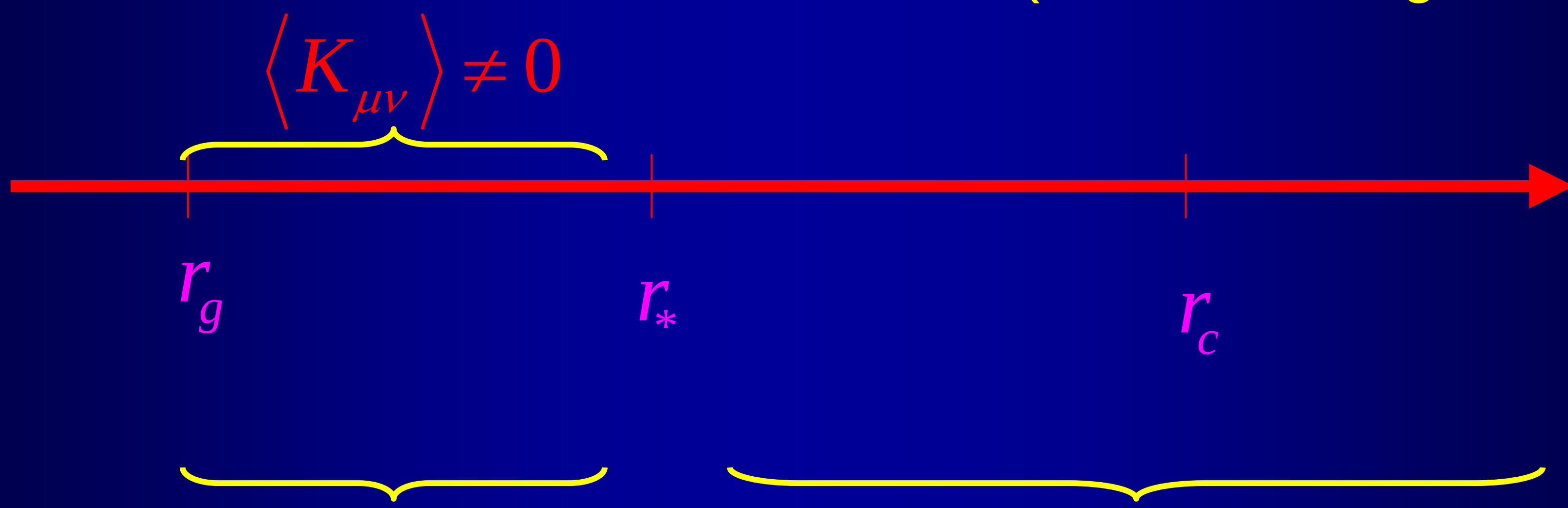
4D GR $\quad$ 4D scalar-tensor $\quad$ 5D

$$\phi_N \simeq -\frac{r_g}{r} + \frac{r^2}{r_c^2}\left[\frac{r_*}{r}\right]^{3/2} \qquad -A\frac{r_g}{r} \qquad -B\frac{2G_5 M}{r^2}$$

Schwarzchild-AdS-like

‘Schwarzchild-DGP’

(Gabadadze-Iglesias)



$$\phi_N \simeq -\frac{r_g}{r} + \frac{r^2}{r_c^2}\left[\frac{r_*}{r}\right]^{3/2-\beta}$$

$$\beta \simeq 0.04$$

$$\phi_N \simeq -\frac{r_* r_g}{r^2}$$

$$A = 0!! \qquad B \simeq \frac{r_*}{r_c}!!$$

‘Schwarzchild-DGP’

(Gabadadze-Iglesias)

$$\phi_N \simeq -\frac{2G_4 M^{(4D)}}{r} - \frac{2G_5 M^{(5D)}}{r^2}$$

Screening:

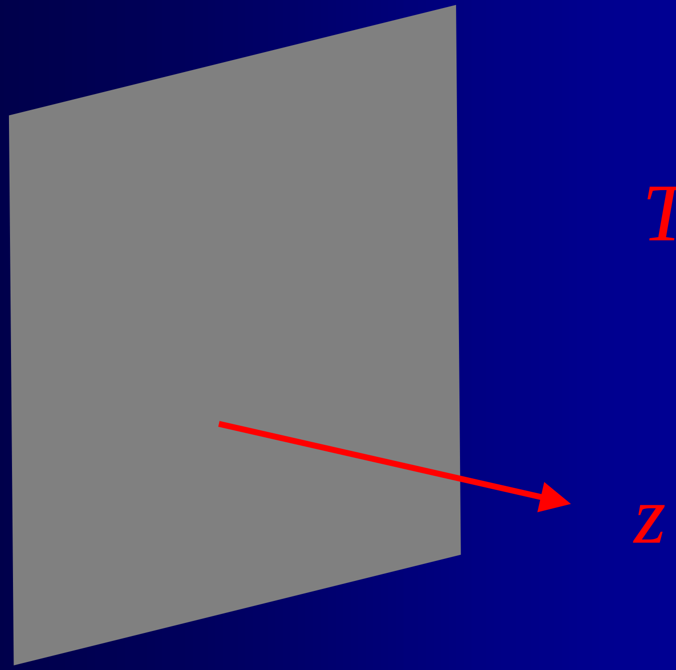
branch	$M^{(4D)}/M$	$M^{(5D)}/M$
CB	0	$0.56\,\frac{r_*}{r_c}$
SAB	0	$-0.45\,\frac{r_*}{r_c}$

OUTLINE

- Domain Walls in GR
- Strings in GR (sub-critical, super-critical)
- Domain Walls on a DGP brane
  - Conventional Branch
  - Self-Accelerated Branch
- Screening of the tension (both 4D and 5D)
- Extension to other IR-modified gravity

Domain Walls in GR
(co-dimension=1)

Domain Walls in GR
(co-dimension=1)



$$T_{\mu\nu} = \sigma \ \delta(z) \ \mathrm{diag}\left(1,-1,-1,0\right)_{\mu\nu}$$

Domain Walls in GR (co-dimension=1)

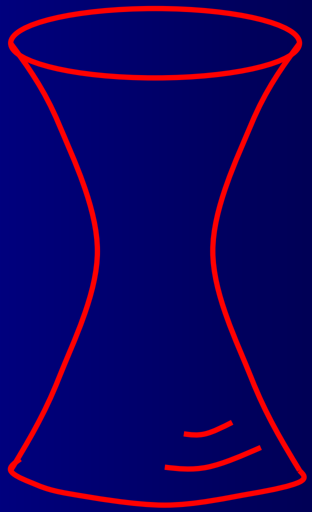
$$ds^2 = (1 - H|z|)^2 \left(-dt^2 + e^{2Ht} \left[dx^2 + dy^2 \right] \right) + dz^2$$

$$H = \frac{\sigma}{4m_P^2}$$

DW inflates.

Hubble rate= H

Horizon at $z = 1/H$



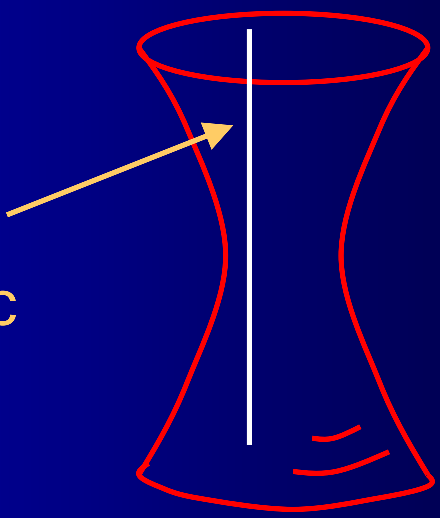
(Vilenkin-Ipser-Sikivie)

Domain Walls in GR
(co-dimension=1)

$$ds^2 = \left(1 - H\,|z|\right)^2 \left(-dt^2 + e^{2H\,t}\left[dx^2 + dy^2\right]\right) + dz^2$$

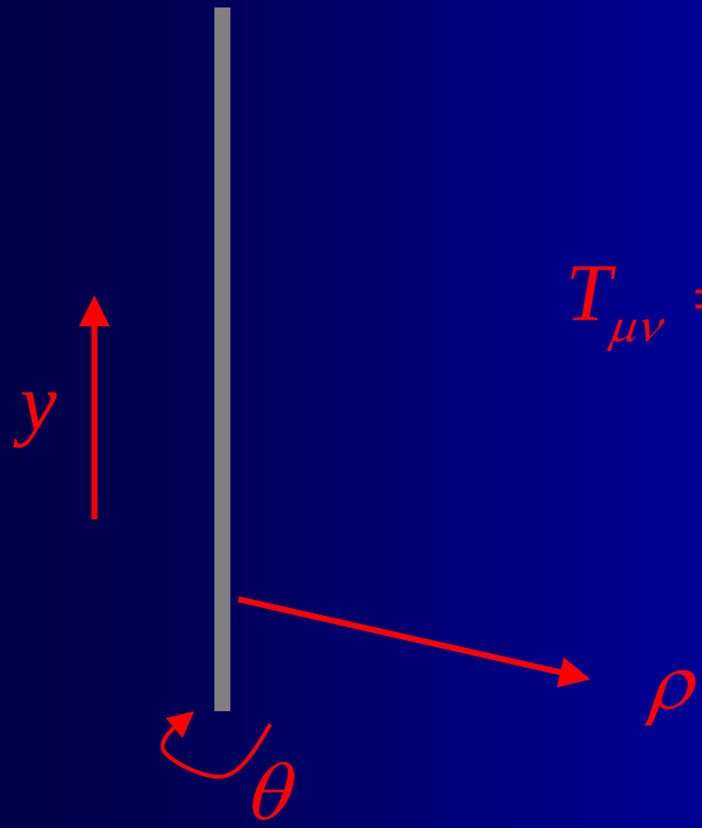
$$H = \frac{\sigma}{4m_P^2}$$

Repulsive force ⟸ geodesic



$$\phi_N = -H\,|z|$$

Strings in GR
(co-dimension=2)



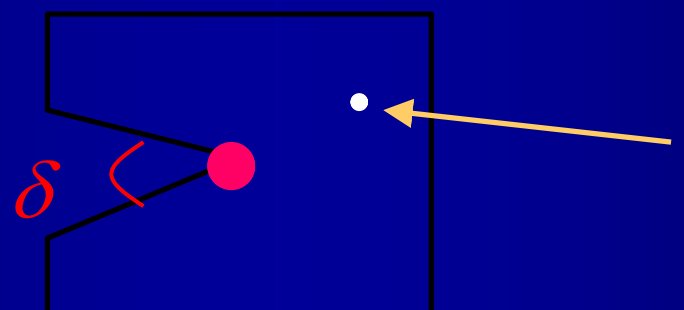
$$T_{\mu\nu} = \mu\ \delta^2(\rho)\ \mathrm{diag}\left(1,-1,0,0\right)_{\mu\nu}$$

Strings in GR
(co-dimension=2)

$$ds^2 = \left(-dt^2 + dy^2\right) + d\rho^2 + \left(1 - \frac{\mu}{m_P^2}\right)^2 \rho^2 d\theta^2$$

$$\delta = 2\pi \frac{\mu}{m_P^2}$$ Deficit angle

No force
(on static particles)



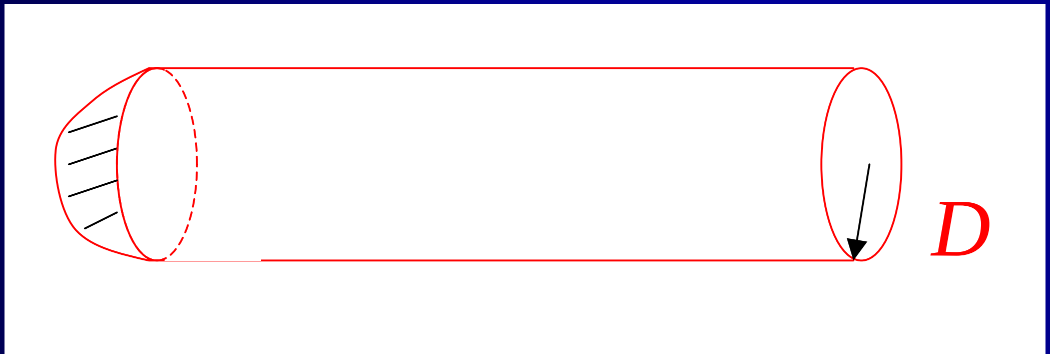
geodesic

Strings in GR
(co-dimension=2)

$\mu = 2\pi \ m_P^2$ 'critical string' $\delta = 2\pi$

$$ds^2 = \left(-dt^2 + dx^2\right) + d\rho^2 + D^2 d\theta^2 \qquad D = \text{thickness}$$

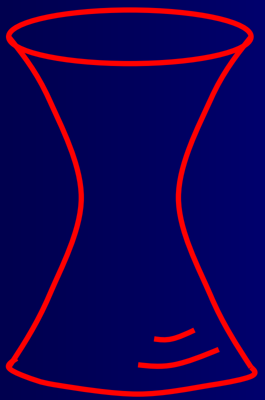
Transverse space compactified.



(flat worldsheet)

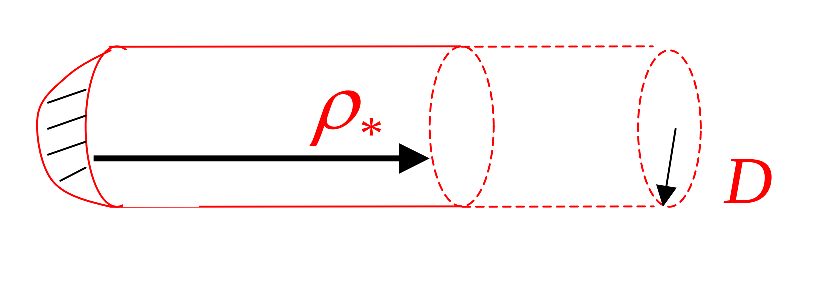
Strings in GR
(co-dimension=2)

$\mu > 2\pi\ m_P^2$ 'super-critical string' $\delta = 2\pi$



(Transverse space compactified)

Topological Inflation $H_* \simeq \dfrac{\mu - 2\pi m_P^2}{4D\ m_P^2}$



Horizon at $\rho_* = 1/H_*$

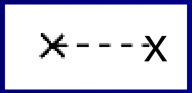
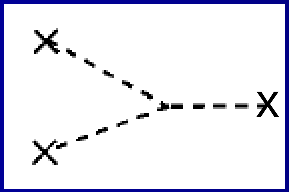
$$\frac{r_c}{R_0} \sim \frac{\sigma}{M_*^3}$$

Critical tension $\sigma_c \sim M_*^3$

Deficit angle $\delta \sim 2\pi$

Domain Walls in DGP: Perturbative arguments

What is r_* for a DW??

 $\approx$  $= 0!!$

$(\partial\chi)^2$

$(\partial\chi)^2 r_c^2 \partial^2 \chi$

$(z \gg r_*)$

$$\Box\chi = \frac{T}{m_P} = \frac{\sigma}{m_P}\delta(z)$$

$\chi \propto |z|$

$r_* =$ thickness

screening inside the wall

Domain Walls in DGP: Exact Solutions

maximally symmetric Domain wall → Flat $\kappa = 0$; (2+1) De Sitter $\kappa = 1$

Birkhoff Theorem:

$$ds^2 = f(R)dZ^2 + \frac{dR^2}{f(R)} + R^2 ds_\kappa^2$$

$$f(R) = \kappa - C/R^2$$

Non-singular

Non-compact bulk

⇒ $C = 0$ 5D Minkowski bulk

Domain Walls in DGP: Exact Solutions

$$ds^2 = dZ^2 + dR^2 + R^{2\kappa} ds_\kappa^2$$

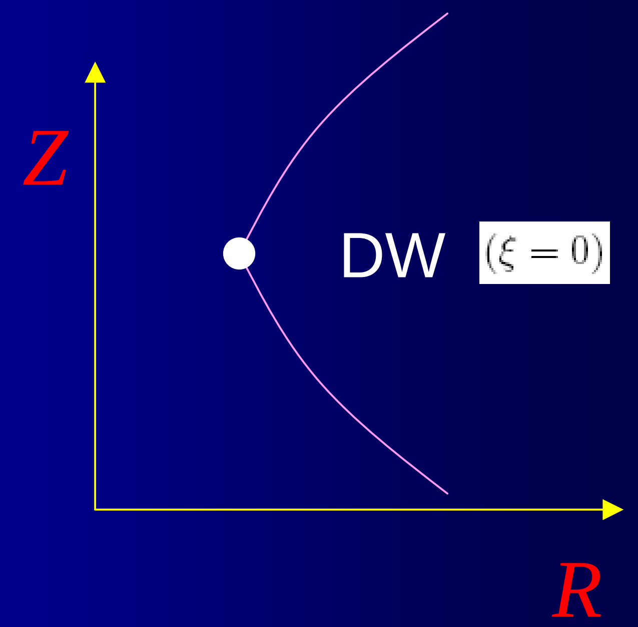
Cartesian $\kappa = 0$

Rindler $\kappa = 1$

Brane parametrization (embedding):

$$(R(\xi), Z(\xi))$$

$$Z'^2 + R'^2 = 1$$



Induced metric on the brane:

$$ds_4^2 = d\xi^2 + R^{2\kappa}(\xi)\, ds_\kappa^2$$

Domain Walls in DGP: Exact Solutions

Equations for the embedding:
Israel junction conditions

$$M_*^3 \left(K_{\mu\nu} - K\, h_{\mu\nu} \right) = T_{\mu\nu} - m_P^2\, G_{\mu\nu}^{(4)}$$

A)
$$\epsilon\, \kappa\, \frac{\sqrt{1 - R'^2}}{r_c\, R} = -\kappa\, \frac{1 - R'^2}{R^2} + \frac{\tau}{3 m_P^2}$$

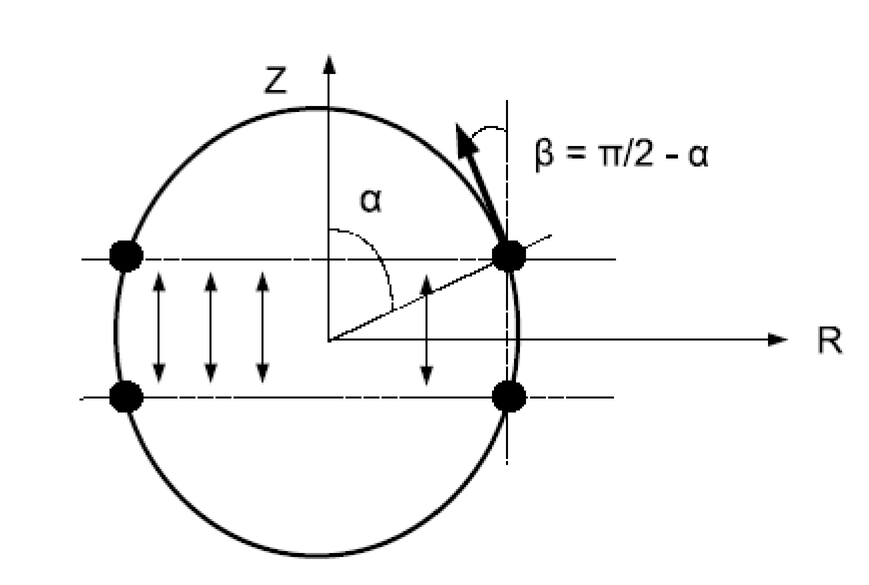
B)
$$-\epsilon \frac{R''}{2 r_c \sqrt{1 - R'^2}} = \kappa\, \frac{R''}{R} + \frac{\sigma}{2 m_P^2} \delta(\xi)$$

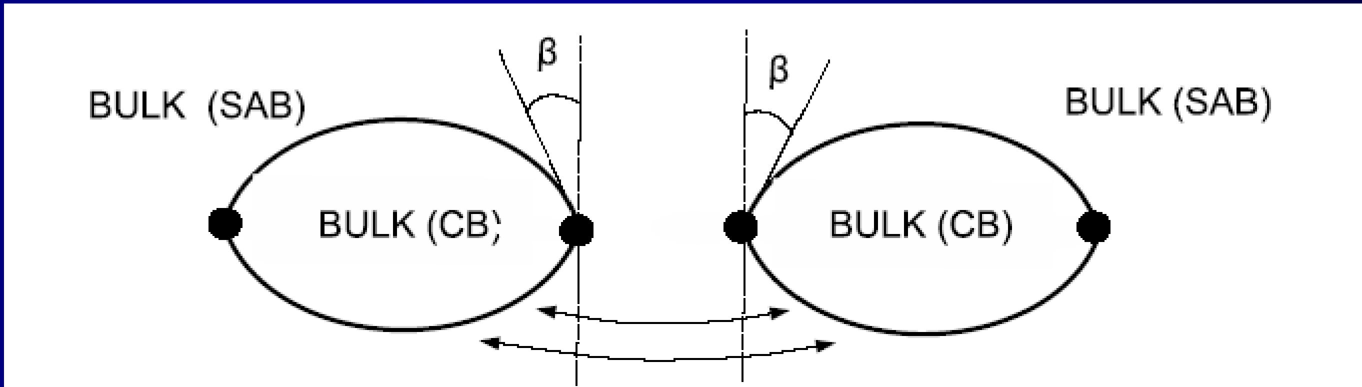
Domain Walls in DGP: Exact Solutions

A) (Outside the DW)

$$R(\xi) = H^{-1} \sin\left[H(\xi_0 - |\xi|)\right]$$

$$\epsilon\ \kappa\ \frac{H}{r_c} = -\kappa\ H^2 + \frac{\tau}{3m_P^2}$$





Domain Walls in DGP: Exact Solutions

B) Junction condition on the DW

$$-\kappa \, \frac{\Delta R_0'}{R_0} + \frac{\delta}{4 r_c} = \frac{\sigma}{2 m_P^2}$$

$$2\kappa \, \epsilon \, H r_c \tan\frac{\delta}{4} + \frac{\delta}{4} = \frac{\sigma}{4 M_*^3}$$

Domain Walls in DGP: Exact Solutions

Subcritical Wall (Conventional Branch)

$$\sigma < 2\pi M_*^3$$

Domain Walls in DGP: Exact Solutions

Subcritical Wall (Conventional Branch)

$$\sigma < 2\pi M_*^3$$

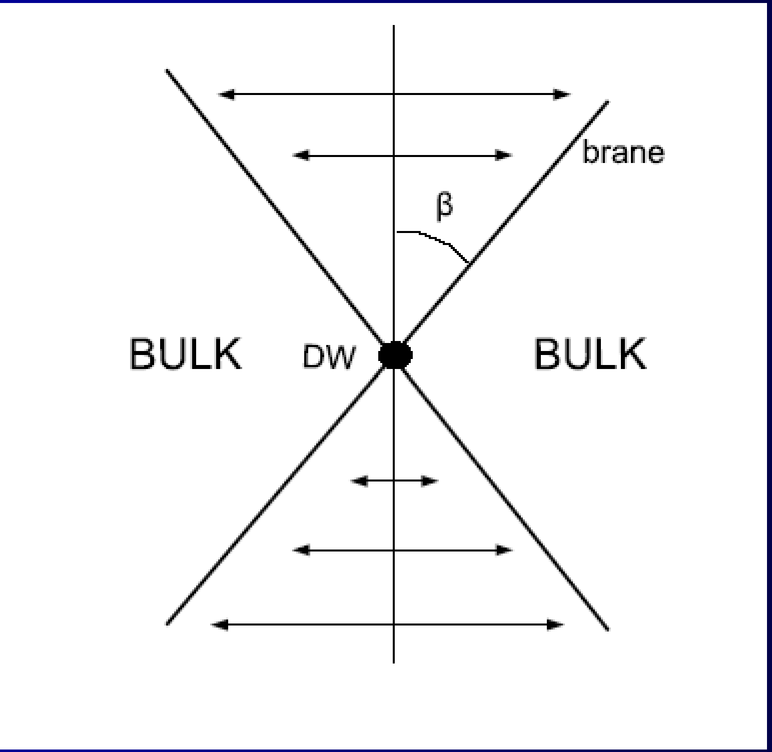
$$\kappa = 0$$

$$R(\xi) = |\xi| \ \sin\beta$$

$$Z(\xi) = \xi \ \cos\beta$$

$$4\beta = \delta = \sigma / M_*^3$$

'brane bending'



Domain Walls in DGP: Exact Solutions

Subcritical Wall (Conventional Branch)

$$\sigma < 2\pi M_*^3$$

$$\kappa = 0$$

5D: $$\sigma_{\rm eff}^{(5D)} \equiv \sigma + \kappa\, m_P^2 \frac{\Delta R_0'}{R_0}$$

No 5D-screening

4D: $$\sigma_{\rm eff}^{(4D)} \equiv \sigma - \delta\, M_*^3 = 0!!$$

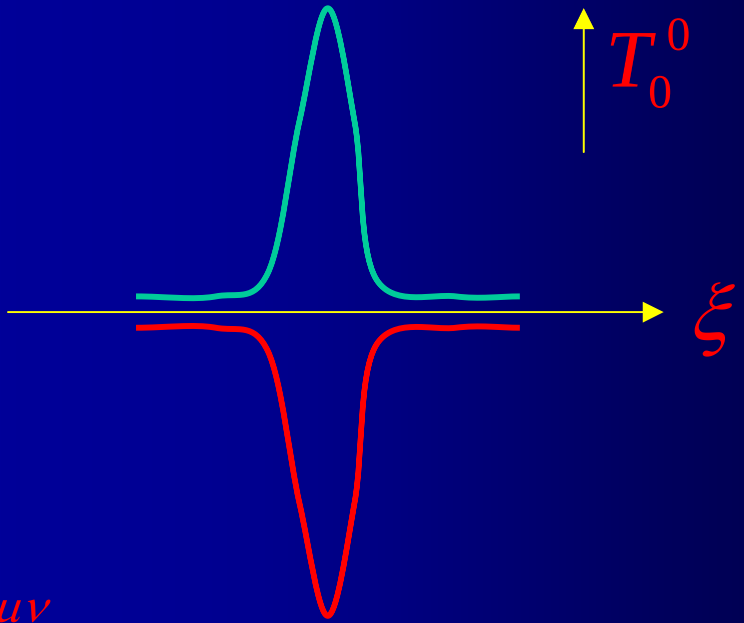
Complete 4D-screening

Domain Walls in DGP: Exact Solutions

Subcritical Wall (Conventional Branch)

$$\sigma < 2\pi M_*^3$$

Domain wall



'Mirror'
Domain wall
produced by $K_{\mu\nu}$

Domain Walls in DGP: Exact Solutions

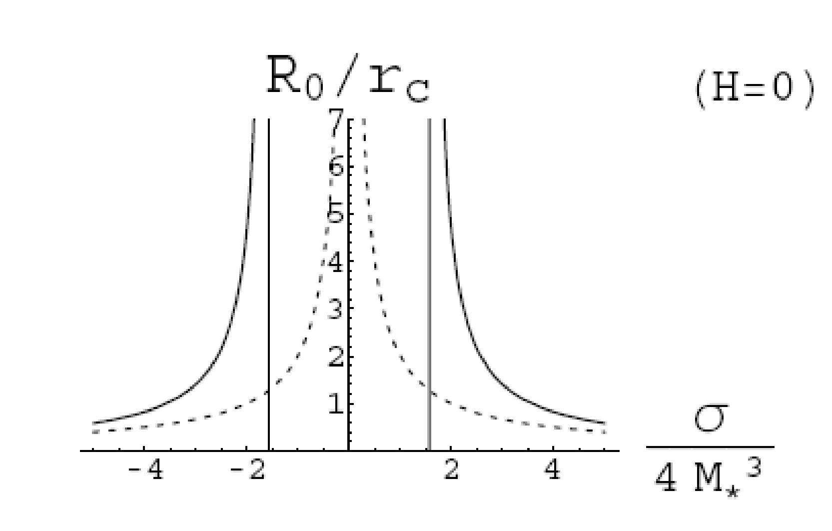
Super-critical Wall (Conventional Branch)

$$\sigma > 2\pi M_*^3$$

$$\kappa = 1$$

$$\frac{1}{R_0} + \frac{\pi}{4r_c} = \frac{\sigma}{4m_P^2}$$

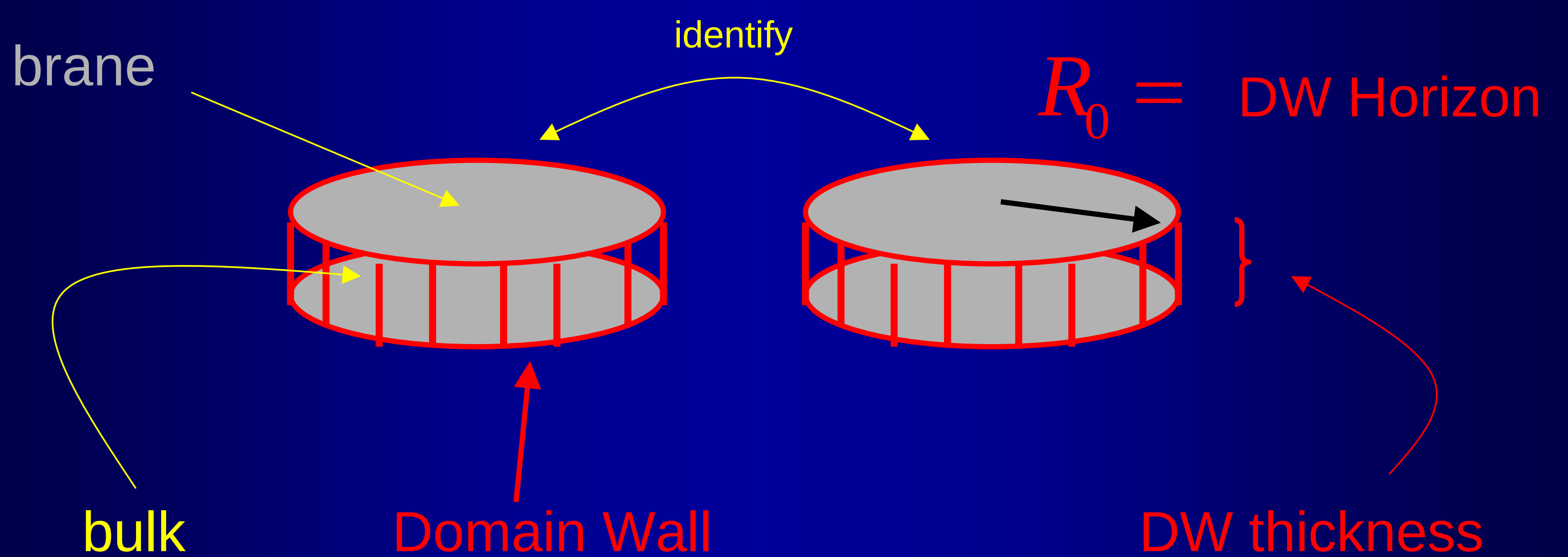
Partial 4D Screening



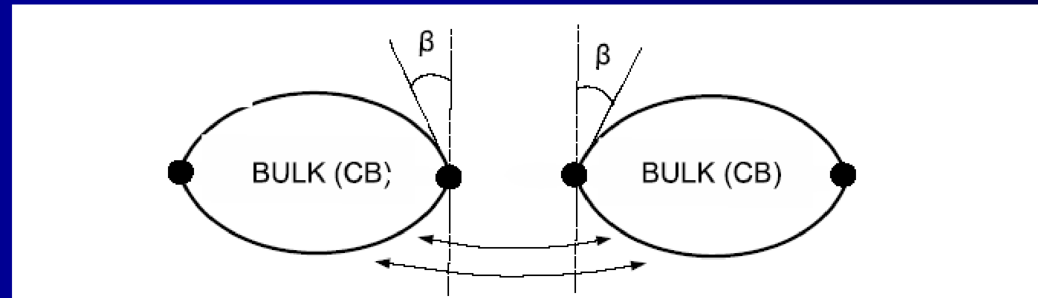
Domain Walls in DGP: Exact Solutions

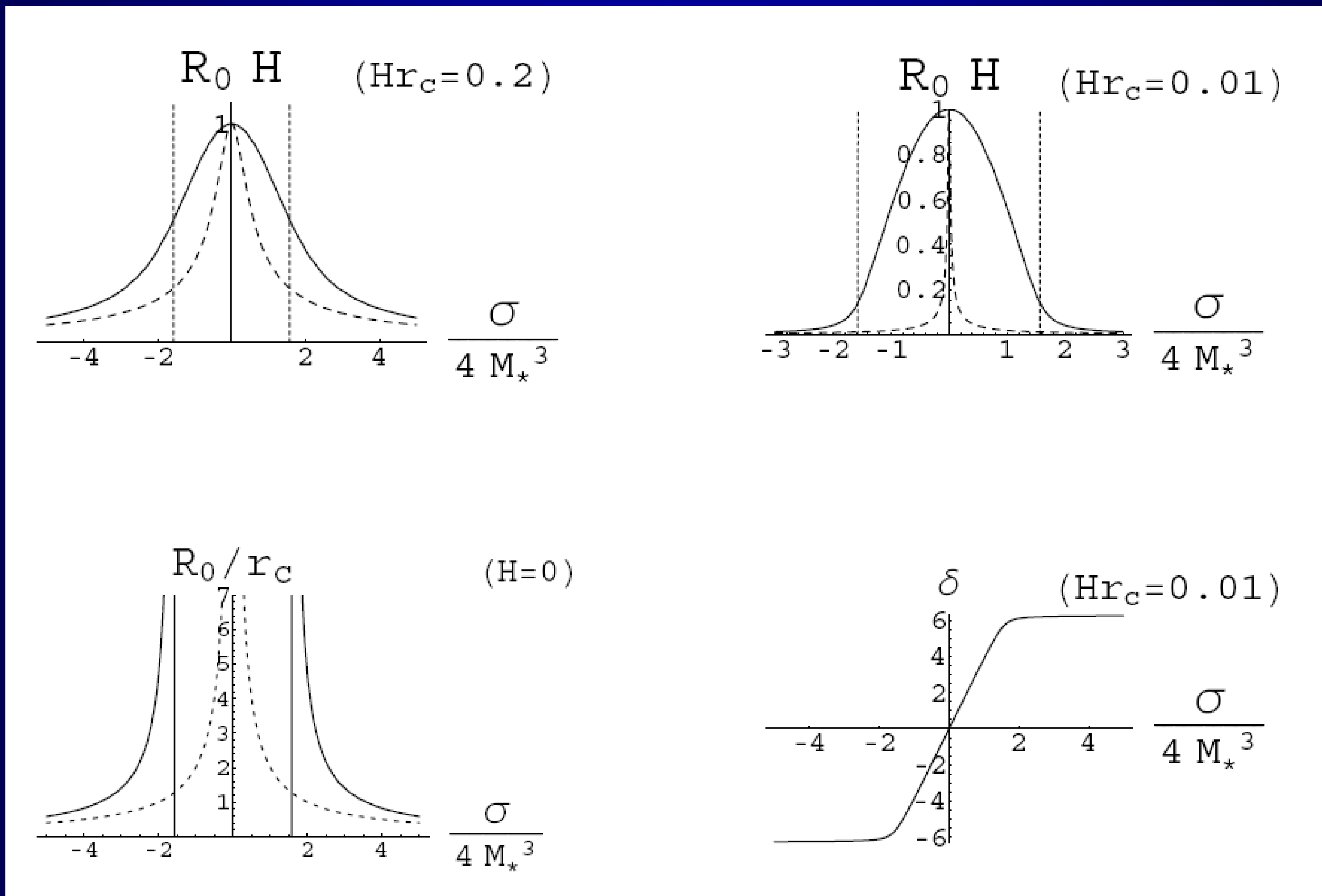
Super-critical Wall (Conventional Branch)

$$\sigma > 2\pi M_*^3$$



Non-zero
brane tension
$H \neq 0$





Domain Walls in DGP: Exact Solutions

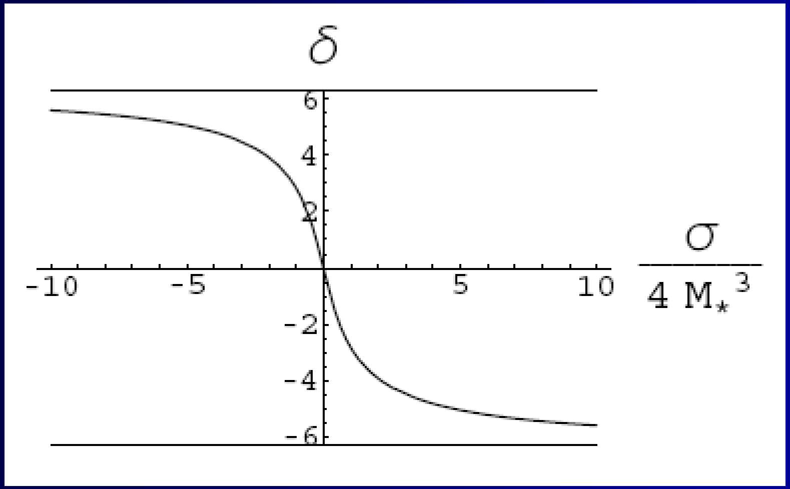
Self-Accelerated Branch $(\kappa = 1)$

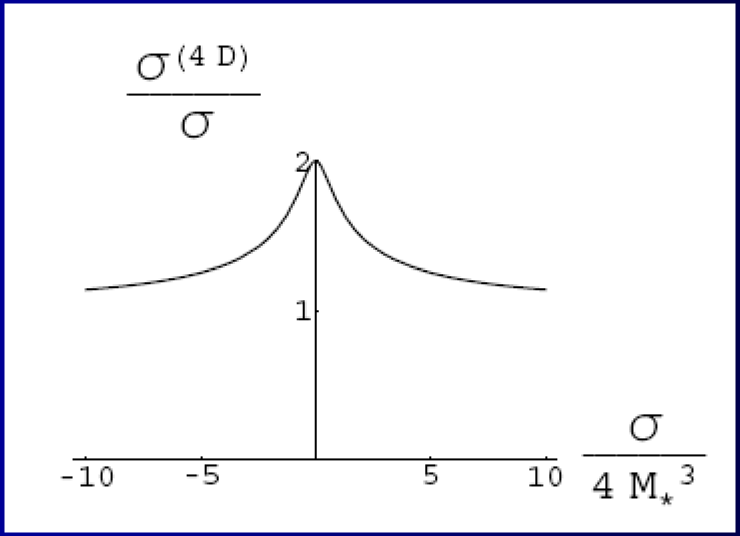
$\delta < 0!!$

5D ‘over-screening’

$\sigma_{eff}^{(4D)} > \sigma!!$

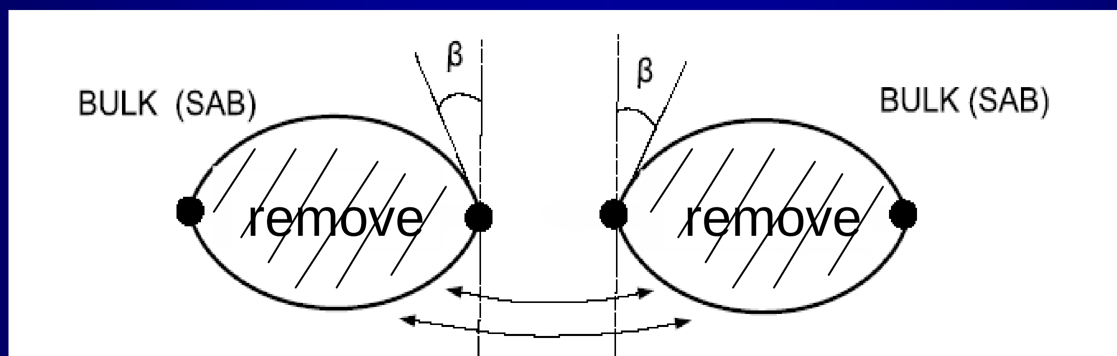
4D ‘anti-screening’



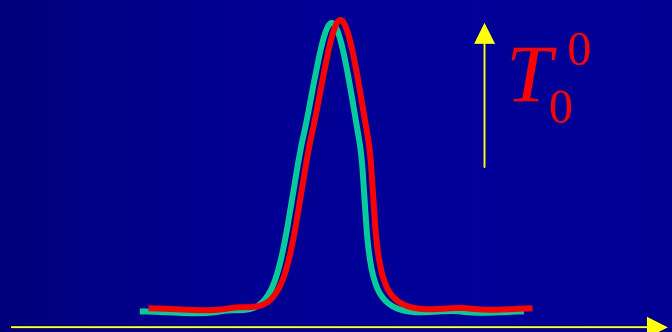


Domain Walls in DGP: Exact Solutions

Self-Accelerated Branch



Domain wall



'Replica' Domain wall produced by $K_{\mu\nu}$

summary

branch	DW tension	$\sigma_{\rm eff}^{(4D)}/\sigma$	$\sigma_{\rm eff}^{(5D)}/\sigma$
CB	$\sigma < 2\pi M_*^3$	0	1
	$\sigma > 2\pi M_*^3$	$1 - \dfrac{2\pi M_*^3}{\sigma}$	??
SAB	$\sigma \ll 2\pi M_*^3$	$2\left(1 - \left(\sigma/M_*^3\right)^2/48 + \dots\right)$	$-\left(1 - \left(\sigma/M_*^3\right)^2/24 + \dots\right)$
	$\sigma \gg 2\pi M_*^3$	$1 + \dfrac{2\pi M_*^3}{\sigma} + \dots$	??

Schwarzchild-DGP

branch	$M^{(4D)}/M$	$M^{(5D)}/M$
CB	0	$0.56\,\dfrac{r_*}{r_c}$
SAB	0	$-0.45\,\dfrac{r_*}{r_c}$

Super-critical walls and 5D-screening

Super-critical walls and 5D-screening

With No DGP 'induced gravity' term
(supermassive codimension-2 brane)

$$\sigma > 2\pi M_*^3 \quad \Rightarrow \quad \begin{cases} D \neq 0 & \text{thickness} \\ P \neq 0 & \text{pressure along normal direction} \end{cases}$$

$$T_\mu^\nu = \mathrm{diag}(-\rho, -\rho, -\rho, P)$$

$$2\pi M_*^3 = \int d\xi \, (\rho + 2P/3) \quad \Rightarrow \quad -\frac{2}{3} P_0 \, d = \sigma - 2\pi M_*^3$$

$$-P_0 = \frac{6M_*^3}{R_0} \quad \Rightarrow \quad \frac{1}{R_0} = \frac{1}{4dM_*^3} \left(\sigma - 2\pi M_*^3\right)$$

Super-critical walls and 5D-screening

With No DGP 'induced gravity' term

(supermassive codimension-2 brane)

$$\sigma \;>\; 2\pi M_*^3 \quad \Rightarrow \quad \frac{1}{R_0} = \frac{1}{4dM_*^3}\left(\sigma - 2\pi M_*^3\right)$$

Super-critical walls and 5D-screening

With No DGP 'induced gravity' term
(supermassive codimension-2 brane)

$$\sigma > 2\pi M_*^3$$ ⇒ $$\frac{1}{R_0} = \frac{1}{4dM_*^3}\left(\sigma - 2\pi M_*^3\right)$$

With induced gravity term,

$$\sigma > 2\pi M_*^3$$ ⇒ $$\frac{1}{R_0} = \frac{1}{4m_P^2}\left(\sigma - 2\pi M_*^3\right)$$

$$\frac{1/R_0^{(DGP)}}{1/R_0^{(5D)}} = \frac{d}{2r_c}$$

The SAME suppression factor!!

$$r_* = d$$

DW

branch	DW tension	$\sigma_{\text{eff}}^{(4D)}/\sigma$	$\sigma_{\text{eff}}^{(5D)}/\sigma$
CB	$\sigma < 2\pi M_*^3$	0	1
	$\sigma > 2\pi M_*^3$	$1 - \frac{2\pi M_*^3}{\sigma}$	$\frac{r_*}{2r_c}$
SAB	$\sigma \ll 2\pi M_*^3$	$2\left(1 - \left(\sigma/M_*^3\right)^2/48 + \dots\right)$	$-\left(1 - \left(\sigma/M_*^3\right)^2/24 + \dots\right)$
	$\sigma \gg 2\pi M_*^3$	$1 + \frac{2\pi M_*^3}{\sigma} + \dots$	$-\frac{r_*}{2r_c}$

Schwarzchild-DGP

branch	$M^{(4D)}/M$	$M^{(5D)}/M$
CB	0	$0.56\ \frac{r_*}{r_c}$
SAB	0	$-0.45\ \frac{r_*}{r_c}$

Generic IR-modified gravity

Linearized Einstein's eqs:

$$\mathcal{E}_{\mu\nu}^{\alpha\beta} h_{\alpha\beta} \; - \; m^2(\Box)\,(h_{\mu\nu} \; - \; \eta_{\mu\nu} h) \; = \; -16\pi G_N \, T_{\mu\nu}$$

$$m^2(\Box) = r_c^{-2(1-\alpha)} \Box^\alpha$$

$$0 \leq \alpha < 1$$

$\alpha = 1/2$ DGP

$\alpha = 0$ Fierz–Pauli Massive gravity

Generic IR-modified gravity

Couples to conserved source as

$$h_{\mu\nu} = -16\pi G_N \frac{1}{\Box - m^2(\Box)} \left\{ T_{\mu\nu} - \frac{1}{3}\left(\eta_{\mu\nu} - \frac{1}{m^2(\Box)}\partial_\mu\partial_\nu\right) T \right\}$$

'pure gauge' form

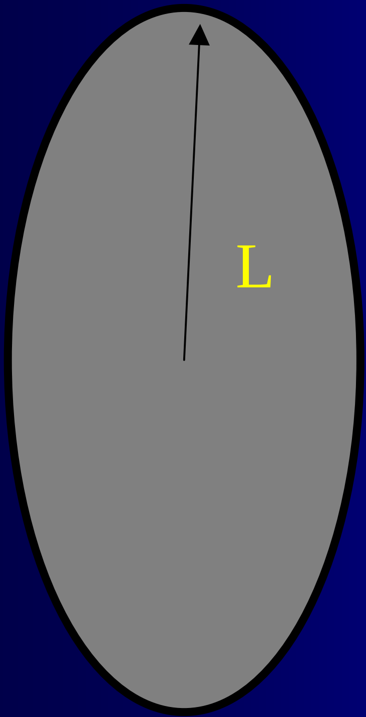
For DWs, $T_{\mu\nu} - (1/3)T\eta_{\mu\nu} \propto \delta^z_\mu \delta^z_\nu$ $\propto \mathrm{diag}(1, \quad 0, \quad 0, \quad 0)$

Lowest order

Tree-level amplitude

$$\mathcal{A} \propto G_N \int d^4x \, \frac{T_{\mu\nu}T'^{\mu\nu} - \frac{1}{3}TT'}{\Box - m^2(\Box)} = 0$$

Finite size DWs



$$M = \pi L^2 \sigma$$

$$r_* \sim \left(\delta L^2 r_c \right)^{1/3}$$

$$\delta \equiv \sigma / M_*^3$$

Non-linearities do not appear if $r_* \ll L$

$$L \gg \delta r_c$$

DW probes large distances in longitudinal directions

CONCLUSIONS

- Screening mechanism in DGP -> Extrinsic curvature
- In DGP, sub-critical DWs do not gravitate

  => short distance probe of gravity
- expect the same in other IR-modified gravity theories

  (non-linearities do not contribute for DWs)
- Almost identical screening pattern as in Schw-DGP

  (Gabadadze&Iglesias)
- Super-massive codimension-2 branes
- Screening of the 5D-tension