Nongaussianity from Tachyonic Preheating in Hybrid Inflation

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Outline

1. Inflation, Fluctuations and Gaussianity
2. Hybrid Inflation and Tachyonic Preheating
3. Cosmological Perturbation Theory
4. Nongaussianity and Constraints
5. Implications for Brane Inflation

Introduction

- ⋆ Hybrid inflation[a] (and Inverted Hybrid Inflation[b]) models are attractive from particle physics perspective.
- ⋆ Appear easy to embed into SUSY, string theory (F-, D-, P-term inflation, KKLMMT)
- ⋆ Inflation ends with nonperturbative amplification of fluctuations called tachyonic preheating.
- ⋆ Preheating may generate large scale curvature perturbations without violation of causality.[c]
- ⋆ Nonadiabatic pressures at second order may give rise to large nongaussianity.[d]
- ⋆ Nongaussianity can be a powerful tool to discriminate between (or constrain) models of inflation.

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[a] Linde, Phys. Rev. D **49**, 748 (1994).

[b] Lyth & Stewart, Phys. Rev. D **54**, 7186 (1996).

[c] Brandenberger & Finelli, Phys. Rev. Lett. **82**, 1362 (1999).

[d] Enqvist et al., Phys. Rev. Lett. **94**, 161301 (2005).

Part 1: Inflation, Fluctuations and Gaussianity

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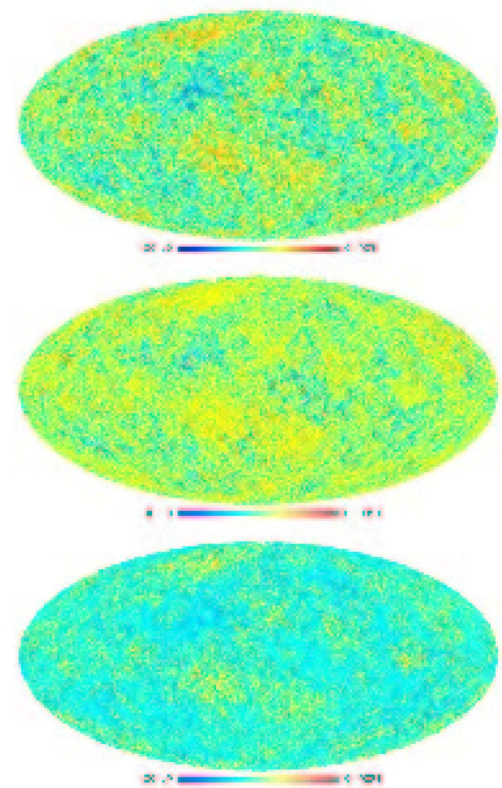


A.Riotto, hep-ph/0210162.

Background and Fluctuations

$$\varphi(t,\vec{x}) = \varphi_0(t) + \delta\varphi(t,\vec{x})$$

Classical Background: φ_0

- ⋆ Slow Roll: $\dot{\varphi}_0 \ll H\varphi_0$

$$\Rightarrow ds^2 \cong -dt^2 + e^{2Ht} d\vec{x}^2$$

- ⋆ Requires flat potentials:

$$\epsilon \cong \frac{M_p^2}{2}\left(\frac{V'}{V}\right)^2 \ll 1$$

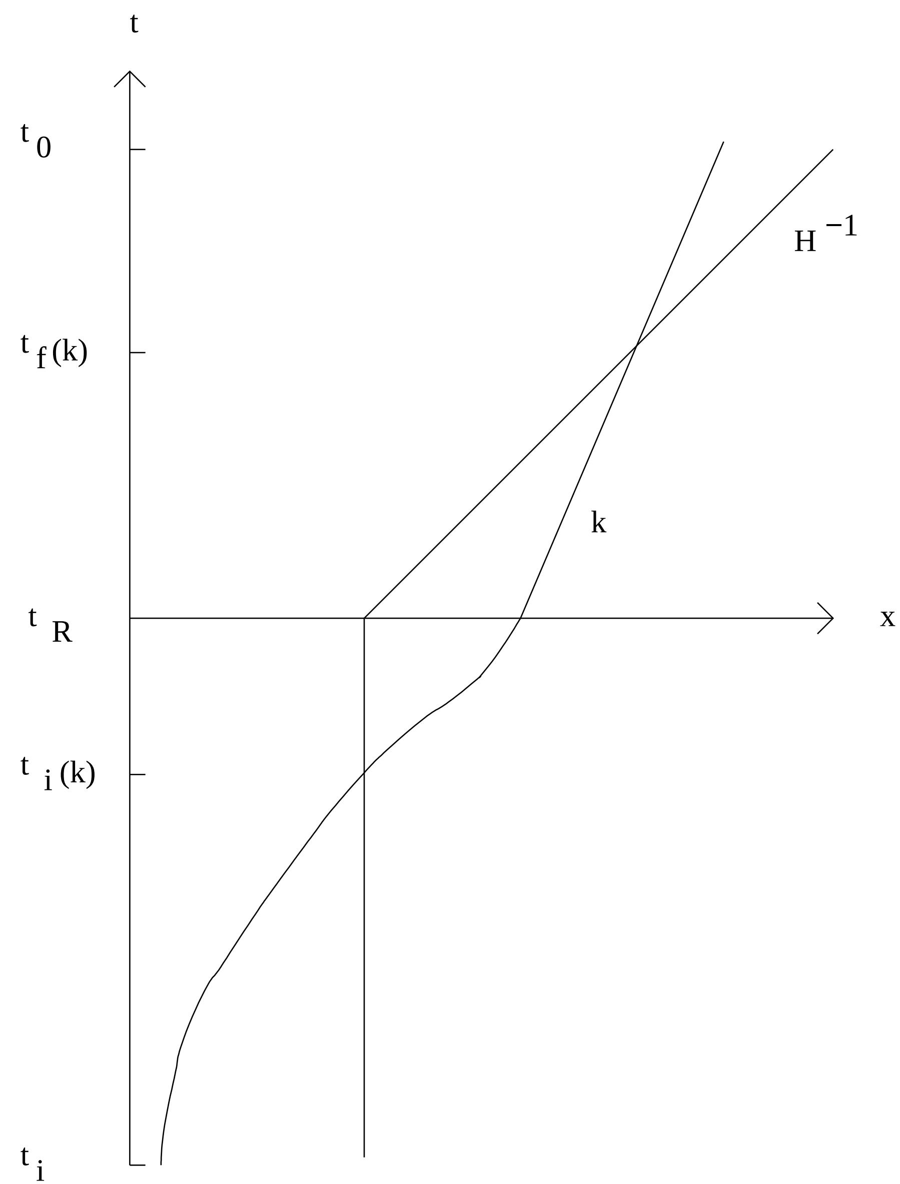
$$\eta \cong M_p^2 \frac{V''}{V} = \frac{m_\varphi^2}{3H^2} \ll 1$$

- ⋆ We require $\epsilon, \eta \ll 1$ for $Ht \cong 60$ e-folds.

Quantum Fluctuations: $\delta\varphi$

- ⋆ Vacuum fluctuations of $\delta\varphi$ generated on small scales $k \gg aH$.
- ⋆ Redshifted by the expansion $k_{\rm phys} = k/a \sim k e^{-Ht}$.
- ⋆ Become classical at horizon crossing $k = aH$.
- ⋆ Fluctuations re-enter horizon after reheating.

Evolution of Scales During Inflation



R.Brandenberger, Lect. Notes Phys. **646**, 127 (2004).

Quantum Fields in deSitter

$$
\begin{aligned}
ds^2 &= -dt^2 + e^{2Ht} dx^2 \\
\chi(t,x) &= \int \frac{d^3k}{(2\pi)^{3/2}} \left[a_k \, \chi_k(t) \, e^{ikx} + a_k^\dagger \, \chi_k^\star(t) \, e^{-ikx} \right]
\end{aligned}
$$

- ⋆ Mode functions satisfy KG equation:

$$
\ddot{\chi}_k + 3H \dot{\chi}_k + \left[\frac{k^2}{a^2} + m^2 \right] \chi_k = 0
$$

- ⋆ Initial data fixes the vacuum $a_k|0\rangle = 0$.
- ⋆ Bunch-Davies vacuum choice corresponds to small scale Minkowski space fluctuations:

$$
\chi_k \cong e^{-ikt}/\sqrt{2k} \quad \text{for} \quad k \gg aH
$$

- ⋆ Large scale behaviour depends crucially on m/H.

Heavy and Light Fields in deSitter

- ⋆ On large scales $k \ll aH$ have:

$$|\chi_k(t)| \cong \begin{cases} \frac{H}{\sqrt{2k}} & \text{if } m \ll H; \\ \frac{a^{-3/2}}{\sqrt{m}} & \text{if } m \gg H. \end{cases}$$

- ⋆ Inflaton fluctuations are light ($\eta \ll 1$) so have scale invariant large scale fluctuations:

$$\langle(\delta\varphi)^2\rangle = \int \frac{d^3k}{(2\pi)^3}|\chi_k(t)|^2 \cong \int d\ln k \underbrace{\left(\frac{H}{2\pi}\right)^2}_{=P_\varphi(k)}$$

- ⋆ Heavy fields have exponentially damped ($\sim e^{-3Ht/2}$) large scale fluctuations.

Curvature Perturbation

- ⋆ Quantum matter fluctuations induce metric fluctuation:

$$\varphi(t,\vec{x}) = \varphi_0(t) + \delta\varphi(t,\vec{x})$$
$$\Rightarrow \quad g_{\mu\nu}(t,\vec{x}) = g^{(0)}_{\mu\nu}(t) + \delta g_{\mu\nu}(t,\vec{x}).$$

- ⋆ Can induce fictitious metric fluctuations by performing small coordinate transformations: $x^\mu \to x^\mu + \xi^\mu$.
- ⋆ Physical observables must be gauge invariant.
- ⋆ Introduce the curvature perturbation ($\phi = \delta g_{00}$):

$$\zeta \cong -\phi - \frac{H}{\dot{\varphi}_0}\delta\varphi$$

- ⋆ The basic observables are the correlators: $\langle \zeta\zeta\cdots\zeta\rangle$.

Spectrum and Gaussianity

- ⋆ The spectrum (two-point function) is almost scale invariant on large scales $k \ll aH$:

$$\langle \zeta_k \zeta_{k'} \rangle = \frac{1}{2\epsilon} \left(\frac{H}{M_p} \right)^2 \frac{1}{2k^3} \left(\frac{k}{aH} \right)^{n-1} \delta^3(k + k')$$

- ⋆ Spectral index: $n - 1 = 2\eta - 6\epsilon \ll 1$.
- ⋆ To linear order ζ contains only one $a_k, a_k^\dagger$:

$$\begin{aligned} \langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle &= 0 \\ \langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \zeta_{k_4} \rangle &= \langle \zeta_{k_1} \zeta_{k_2} \rangle \langle \zeta_{k_3} \zeta_{k_4} \rangle + \text{perms} \\ &\cdots \end{aligned}$$

- ⋆ Two-point correlator is the only independent statistics.

Nongaussianity

- $\star$ Gaussian fluctuations: the connected part of the n-point functions vanishes for $n \geq 3$.
- $\star$ At linear order in perturbation theory the fluctuations are exactly gaussian.
- $\star$ Nongaussianity is expected due to nonlinearities in the KG and gravity equations.
- $\star$ The three-point function (bispectrum) is the lowest order statistics which can discriminate between gaussianity and nongaussianity:

$$\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle = (2\pi)^{-3/2} \, B(k_i) \, \delta^3(k_1 + k_2 + k_3)$$

Nonlinearity Parameter

- ⋆ Usually nongaussianity is parametrized in terms of the nonlinearity parameter f_{NL} as

$$\zeta = \zeta_g - \frac{3}{5} f_{NL} \left(\zeta_g^2 - \langle \zeta_g^2 \rangle \right)$$

- ⋆ Yields a nontrivial bispectrum:

$$\begin{aligned} B(k_i) &\cong -\frac{6}{5} f_{NL} \left[P(k_1) P(k_2) + \mathrm{perms} \right] \\ \langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle &= (2\pi)^{-3/2} B(k_i)\, \delta^3(k_1 + k_2 + k_3) \\ \langle \zeta_{k_1} \zeta_{k_2} \rangle &= P(k_i)\, \delta^3(k_1 + k_2) \end{aligned}$$

- ⋆ WMAP analysis constrains $|f_{NL}| \lesssim 100$. [a]

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[a] Komatsu et al., Astrophys. J. Suppl. **148**, 119 (2003).

Nonlinearity Parameter

- Various scenarios for generating ζ give distinct predictions for f_{NL}.
- Measurement of f_{NL} can discriminate between different models.
- Expect $f_{NL} \sim n - 1$ for the simplest models, which is unlikely to ever be detectable.
- Can get observably large nongaussianity from:
  - Curvaton mechanism.[a]
  - Single field models with small inflaton sound speed.[b] (For example the D-celleration model.[c])
  - Preheating.[d]
  - . . .

[a] Lyth et al., Phys. Rev. D **67**, 023503 (2003)

[b] Chen et al.,arXiv:hep-th/0605045

[c] Silverstein & Tong, Phys. Rev. D **70**, 103505 (2004)

[d] Enqvist et al., Phys. Rev. Lett. **94**, 161301 (2005); NB & Cline Phys. Rev. D **73**, 106012 (2006).

Part 2: Hybrid Inflation and Tachyonic Preheating

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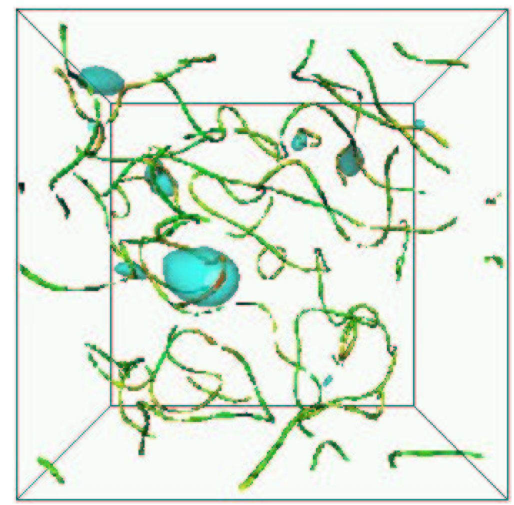
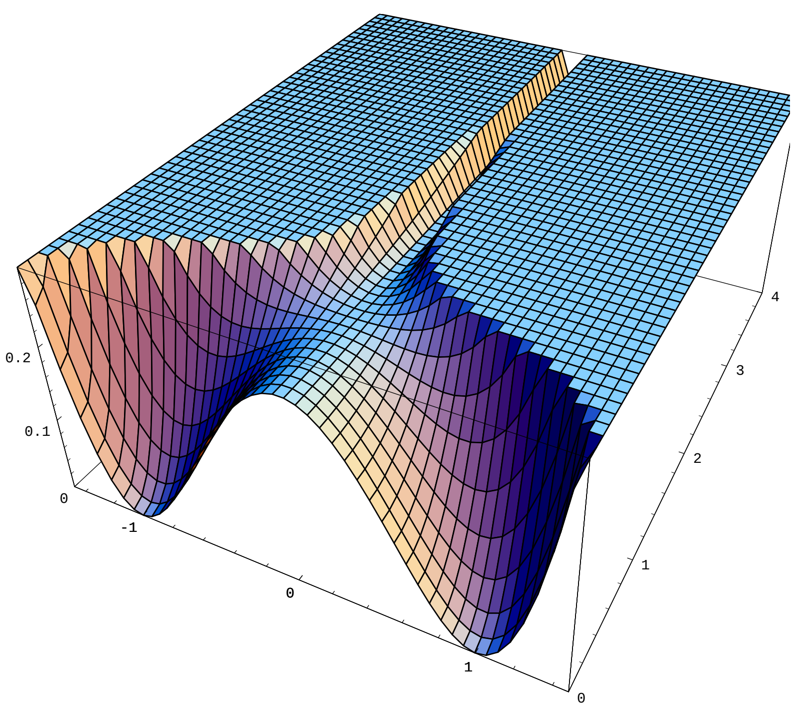


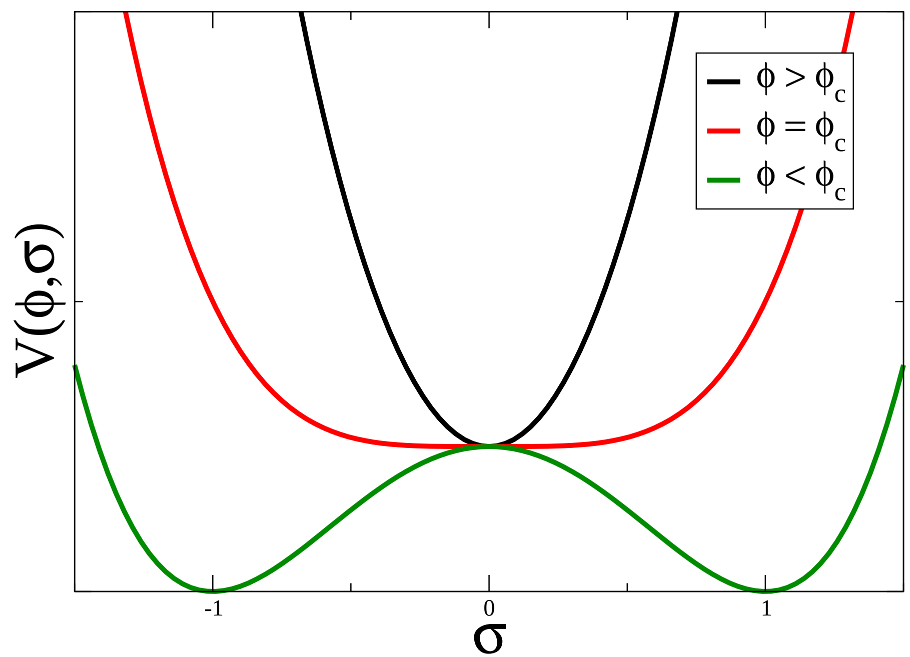
Copeland et al., Phys. Rev. D **65**, 103517 (2002).

Hybrid Inflation: Potential

$$V(\varphi, \sigma) = \frac{\lambda}{4}(\sigma^2 - v^2)^2 + \frac{g^2}{2}\varphi^2\sigma^2 + \frac{m_\varphi^2}{2}\varphi^2$$

$$= \frac{\lambda v^4}{4} + \frac{1}{2}\underbrace{\left(g^2\varphi^2 - \lambda v^2\right)}_{= m_\sigma^2}\sigma^2 + \frac{\lambda}{4}\sigma^4 + \frac{m_\varphi^2}{2}\varphi^2$$





Herdeiro et al., JHEP **0112**, 027 (2001).

Inflationary Dynamics

- ⋆ The tachyon is trapped in the false vacuum: $\langle\sigma\rangle \equiv \sigma_0 = 0$.
- ⋆ Potential along the inflationary trajectory:

$$V_{\rm inf} = \frac{\lambda v^4}{4} + \frac{1}{2} m_\varphi^2 \varphi^2 \cong \frac{\lambda v^4}{4}$$

- ⋆ Slow roll solutions:

$$\begin{aligned} \langle\varphi\rangle \equiv \varphi_0(t) &\cong \frac{\lambda^{1/2} v}{g} \left(\frac{a(t_c)}{a(t)}\right)^\eta \\ 3H^2 &\cong \lambda v^4/(4M_p^2) \end{aligned}$$

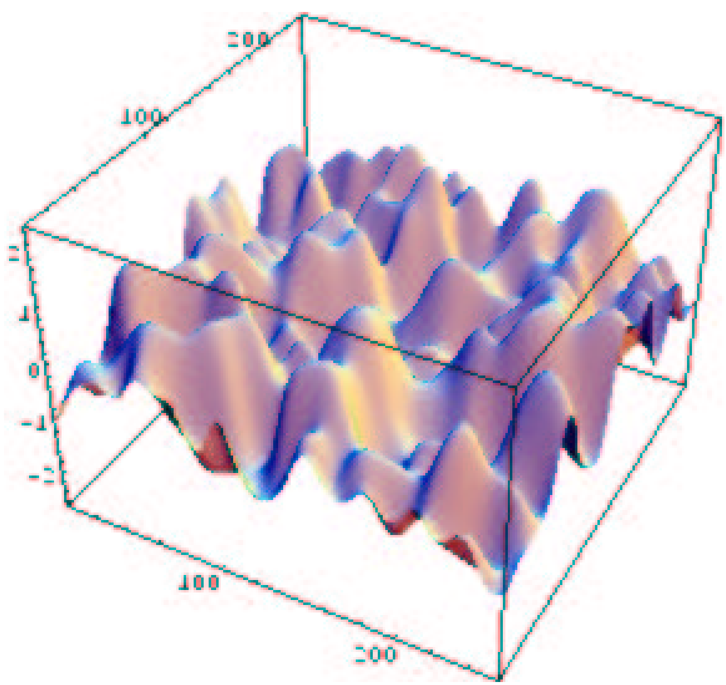
- ⋆ Slow roll parameters never get large: $\dot\epsilon < 0$, $\eta = \text{const}$.

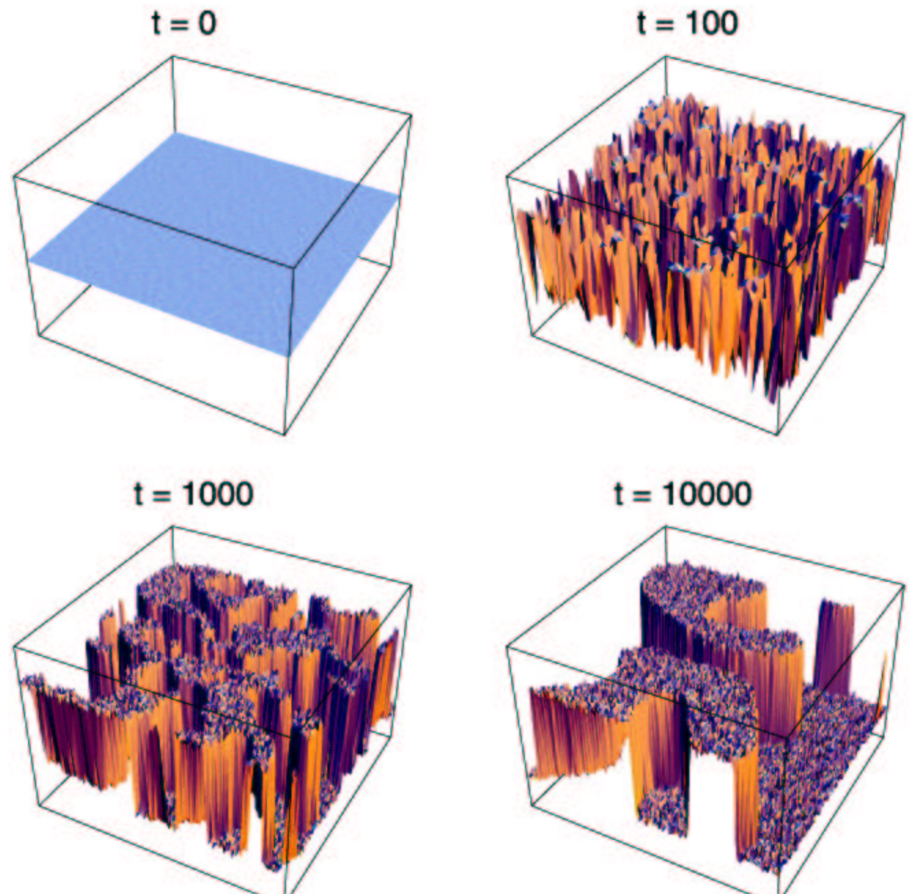
Tachyonic Preheating

- ⋆ Tachyon mass-squared:

$$m_\sigma^2 = g^2\varphi_0^2 - \lambda v^2 \cong -2\lambda v^2 \eta\, H(t - t_c) \equiv -cH^2 N$$

- ⋆ Tachyonic preheating: transfer of energy from the false vacuum $\lambda v^4/4$ to the fluctuations $\delta^{(1)}\sigma_k$.

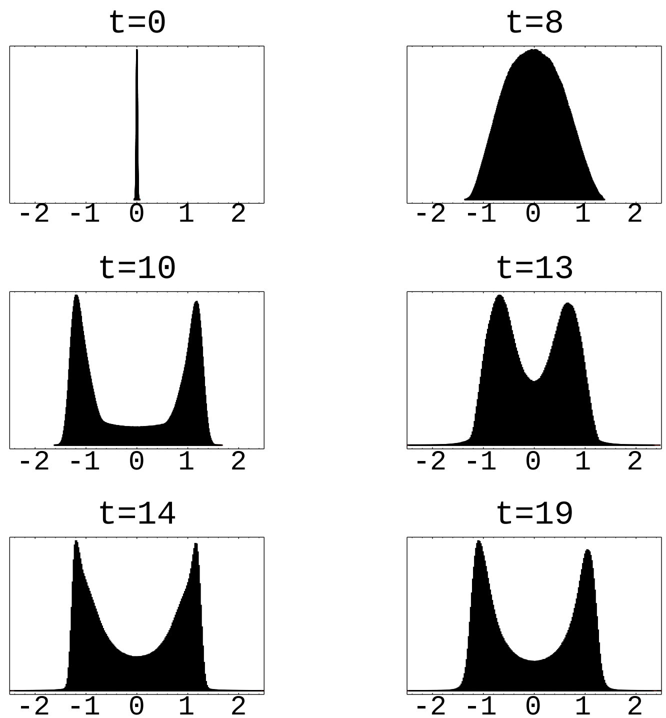




Felder et al., Phys. Rev. Lett. **87**, 011601 (2001).

Domain Walls

- ⋆ Symmetry breaking leads to domain walls.
- ⋆ At late times universe consists of many domains with $\sigma \sim \pm v$.
- ⋆ Even at late times the tachyon averages to zero $\langle \sigma \rangle = \sigma_0 = 0$ over many domains.



Felder et al., Phys. Rev. Lett. **87**, 011601 (2001).

- ⋆ Domain walls will overclose the universe so one should add symmetry breaking terms or consider a complex tachyon which gives cosmic strings...

Tachyon Dynamics

- ⋆ The tachyon mass-squared varies linearly with the number of e-foldings:

$$m_\sigma^2 \cong -cH^2N$$

- ⋆ At early times $m_\sigma^2 > 0$:
  - – Large scale tachyon fluctuations get damped as $a^{-3/2}$ during any e-foldings where $m_\sigma^2 > H^2$.
  - – Scale invariant large scale fluctuations for $m_\sigma^2 < H^2$.
- ⋆ At late times $m_\sigma^2 < 0$:
  - – Large scale tachyon fluctuations are exponentially amplified.
  - – Within a time $t_\star$ the energy from the false vacuum is transfered into the large scale fluctuations $\delta\sigma_k$.

Tachyon Fluctuations

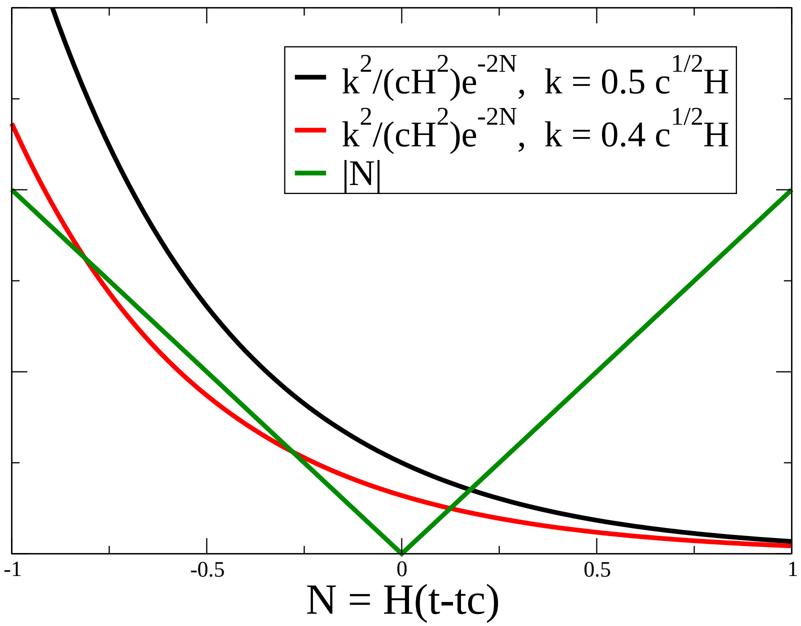
- ⋆ During both inflation and the (early) instability phase the tachyon mode functions obey:

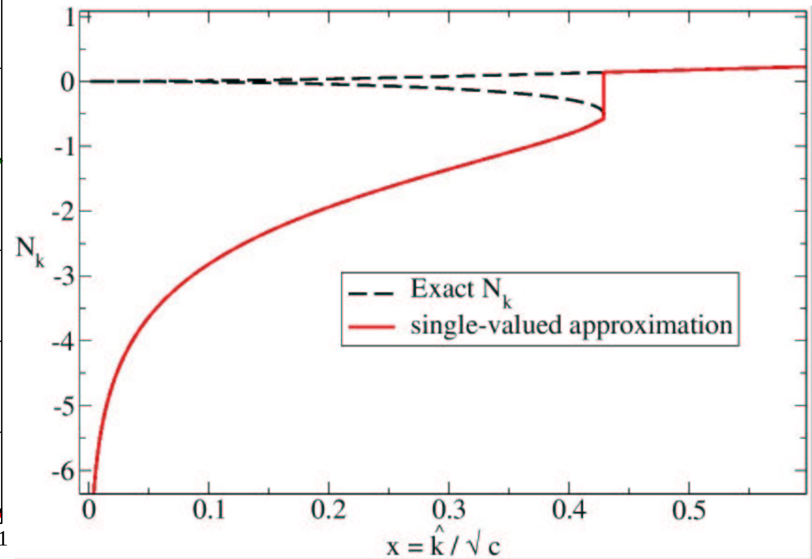
$$\frac{d^2}{dN^2}\delta^{(1)}\sigma_k + 3\frac{d}{dN}\delta^{(1)}\sigma_k + \left[\frac{k^2}{H^2}e^{-2N} - cN\right]\delta^{(1)}\sigma_k = 0$$

where $N = H(t - t_c)$, $c = 2\eta\lambda v^2/H^2$.

- ⋆ In the far UV where the k^2/a^2 term dominates ($k^2 H^{-2} e^{-2N} \gg c|N|$) have Minkowski space modes.
- ⋆ In the far IR where the m_σ^2 term dominates ($k^2 H^{-2} e^{-2N} \ll c|N|$) are exponentially damped if $m_\sigma^2 > 0$ or amplified if $m_\sigma^2 < 0$.
- ⋆ Match solutions at $N = N_k$ defined by: $\frac{k^2}{cH^2}e^{-2N_k} = |N_k|$.

Matching Conditions



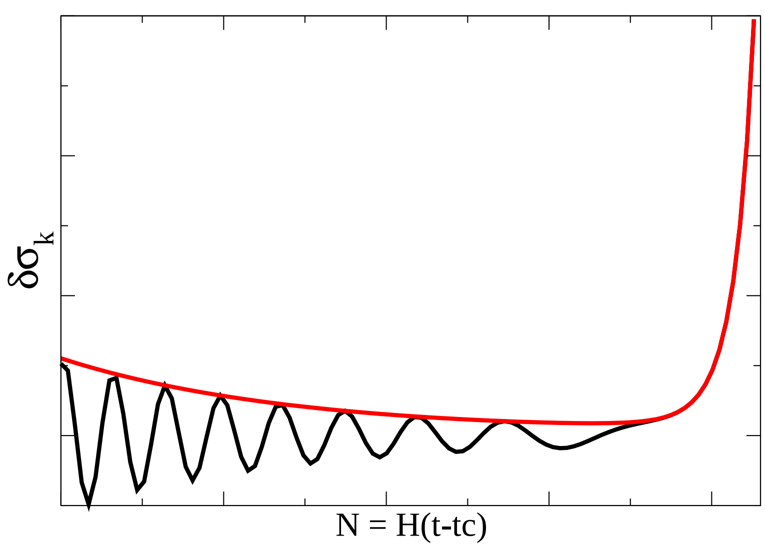


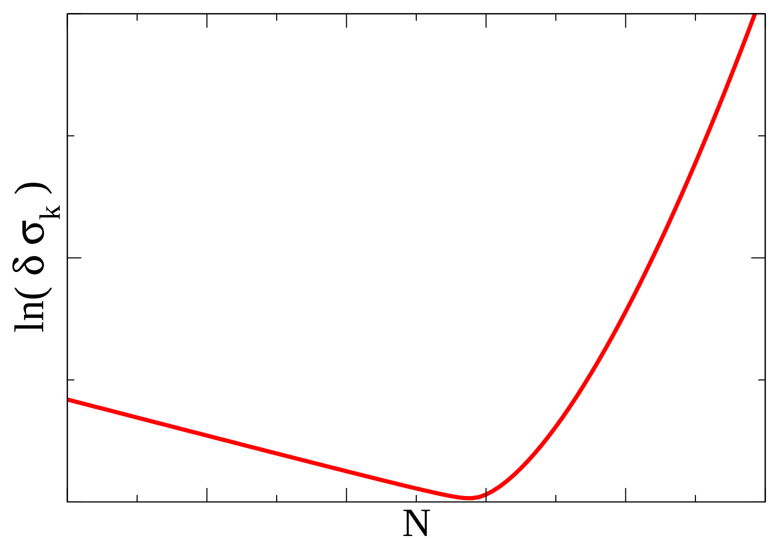
- ⋆ Modes which cross $|m_\sigma|$ while $m_\sigma^2 > 0$ are damped exponentially as $a^{-3/2}$ before the instability sets in.
- ⋆ Modes which cross $|m_\sigma|$ when $m_\sigma^2 < 0$ were light throughout inflation and experience no damping.

Tachyon Mode Functions

- ⋆ Modes in the UV $N < N_k$ ($k \gg a|m_\sigma|$) feel only Minkowski space: $a\,\delta^{(1)}\sigma_k \sim e^{-ik\tau}/\sqrt{2k}$.
- ⋆ These are red-shifted into the IR region $N > N_k$ ($k \ll a|m_\sigma|$) where the mass term becomes important:

$$|\delta^{(1)}\sigma_k(N)| \sim |b_k|\ \exp\left[-\frac{3}{2}N + \frac{9}{4c}\left(1+\frac{4}{9}cN\right)^{3/2}\right]$$





The End of Exponential Growth

- $\star$ Once the tachyon fluctuations become sufficiently large, the exponential growth is replaced by oscillations about the minima $\pm v$.
- $\star$ Our condition for the end of tachyonic growth:

$$\langle (\delta^{(1)}\sigma)^2 \rangle^{1/2} \Big|_{N=N_\star} = \frac{v}{2}$$

- $\star$ Numerical solutions of this equation agree with previous authors.[a]
- $\star$ NOTE: For a very slowly rolling inflaton can have $N_\star \gtrsim 1$.

[a] Garcia-Bellido et al., Phys. Rev. D **67**, 103501 (2003).

Part 3: Cosmological Perturbation Theory

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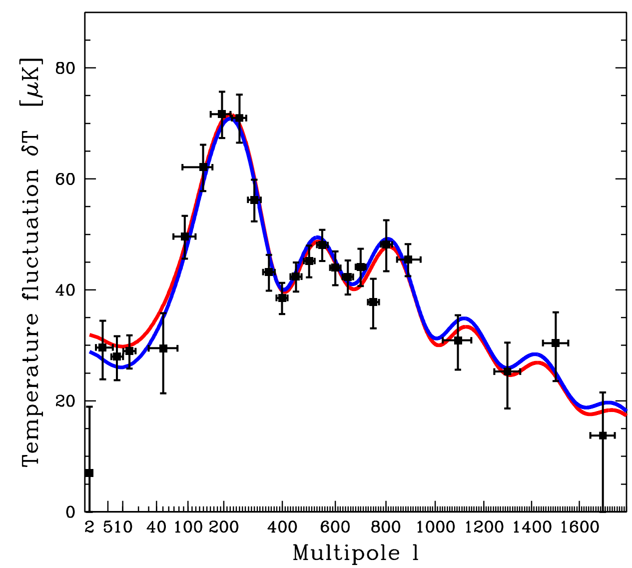

Cosmological Perturbations

- ⋆ Expand the metric in longitudinal gauge as:

$$
\begin{aligned}
g_{00} &= -a(\tau)^2 \left[1 + 2\phi^{(1)} + \phi^{(2)} \right] \\
g_{0i} &= 0 \\
g_{ij} &= a(\tau)^2 \left[\left(1 - 2\psi^{(1)} - \psi^{(2)} \right) \delta_{ij} \right. \\
&+ \left. \frac{1}{2} \left(\partial_i \chi_j^{(2)} + \partial_j \chi_i^{(2)} + \chi_{ij}^{(2)} \right) \right]
\end{aligned}
$$

- ⋆ Expand the matter fields as:

$$
\begin{aligned}
\varphi(\tau, \vec{x}) &= \varphi_0(\tau) + \delta^{(1)}\varphi(\tau, \vec{x}) + \frac{1}{2}\delta^{(2)}\varphi(\tau, \vec{x}) \\
\sigma(\tau, \vec{x}) &= \delta^{(1)}\sigma(\tau, \vec{x}) + \frac{1}{2}\delta^{(2)}\sigma(\tau, \vec{x})
\end{aligned}
$$

- ⋆ Neglect vectors, tensors at first order.
- ⋆ Recall that $\langle \sigma \rangle \equiv \sigma_0 = 0$.

First Order Einstein Equations

- ⋆ At first order there are two independent scalar degrees of freedom: $\phi^{(1)}$, $\delta^{(1)}\sigma$.
- ⋆ Can write a master equation for $\phi^{(1)}$:

$$\phi_k''^{(1)} - \frac{2}{\tau}(\eta - \epsilon)\phi_k'^{(1)} + \left[\frac{2}{\tau^2}(\eta - 2\epsilon) + k^2\right]\phi_k^{(1)} = 0$$

- ⋆ Tachyon fluctuation does not couple to the metric fluctuations:

$$\delta^{(1)}\ddot{\sigma}_k + 3H\delta^{(1)}\dot{\sigma}_k + \left[\frac{k^2}{a^2} + m_\sigma^2\right]\delta^{(1)}\sigma_k = 0$$

- ⋆ Constraint equations fix $\delta^{(1)}\varphi$, $\psi^{(1)}$.

First Order Curvature Perturbation

- ⋆ Physical quantity of interest is the curvature perturbation

$$\zeta = \zeta^{(1)} + \frac{1}{2}\zeta^{(2)}$$

defined so that $\langle\zeta\rangle = 0$.

- ⋆ For $\sigma_0 = 0$ the first order piece depends only on the inflaton:

$$\zeta^{(1)} \cong -\phi^{(1)} - \frac{\mathcal{H}}{\varphi_0'}\delta^{(1)}\varphi$$

- ⋆ First order curvature perturbation is conserved on large scales:

$$\frac{\partial}{\partial\tau}\zeta_k^{(1)} \cong 0 \quad \text{for} \quad k \ll aH$$

- ⋆ Have the usual scale invariant spectrum from $\langle\zeta_{k_1}\zeta_{k_2}\rangle$.

Second Order Einstein Equations

- ⋆ Second order fluctuations are sourced by first order fluctuations.
- ⋆ Two independent scalar fluctuations at second order: $\phi^{(2)}$, $\delta^{(2)}\sigma$.
- ⋆ Can write a master equation for $\phi^{(2)}$:

$$\phi_k''^{(2)} - \frac{2}{\tau}(\eta - \epsilon)\phi_k'^{(2)} + \left[\frac{2}{\tau^2}(\eta - 2\epsilon) + k^2\right]\phi_k^{(2)} = J_k(\tau)$$

  where the source J is constructed from $\delta^{(1)}\sigma$, $\delta^{(1)}\varphi$, $\phi^{(1)}$.
- ⋆ Can solve for the other second order fluctuations using constraints.
- ⋆ Curvature perturbation does not depend on $\delta^{(2)}\sigma$ up to second order.

Second Order Curvature Perturbation

- ⋆ Split the curvature perturbation into inflaton and tachyon contributions:

$$\zeta^{(2)} = \zeta_\varphi^{(2)} + \zeta_\sigma^{(2)}$$

- ⋆ The inflaton part has already been studied and yields negligible nongaussianity [a]

$$\zeta_\varphi^{(2)} \cong \frac{1}{4}(2\eta - 6\epsilon)\left(\zeta^{(1)}\right)^2 \cong \mathrm{const} \quad \text{for} \quad k \ll aH$$

- ⋆ Non-adiabatic pressures at second order will amplify large scale $\zeta_\sigma^{(2)}$ during the instability phase so that $\zeta^{(2)} \cong \zeta_\sigma^{(2)}$ after preheating.

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[a] Maldacena, JHEP **0305**, 013 (2003).

Calculation of $\zeta_\sigma^{(2)}$

$$
\begin{aligned}
\zeta^{(2)} &\ni -\frac{\phi'^{(2)}}{\epsilon\mathcal{H}} - \left(\frac{1}{\epsilon}+1\right)\phi^{(2)} + \frac{1}{3-\epsilon}\frac{\partial^k\partial_k\phi^{(2)}}{\epsilon\mathcal{H}^2} \\
&+ \frac{1}{\epsilon\mathcal{H}}\triangle^{-1}\gamma' + \triangle^{-1}\gamma - \frac{1}{3-\epsilon}\frac{1}{\epsilon\mathcal{H}^2}\gamma \\
&+ \frac{1}{3-\epsilon}\frac{1}{(\varphi_0')^2}\left[\left(\delta^{(1)}\sigma'\right)^2 + a^2 m_\sigma^2\left(\delta^{(1)}\sigma\right)^2\right] \\
&+ \cdots \\
\phi_k^{(2)}(\tau) &= \int d\tau' G_k(\tau,\tau')(-\tau')^{2(\epsilon-\eta)}J_k(\tau') \\
G_k(\tau,\tau') &= \frac{\pi}{2}\Theta(\tau-\tau')(\tau\tau')^{1/2+\eta-\epsilon} \\
&\times \left[J_\nu(-k\tau)Y_\nu(-k\tau') - J_\nu(-k\tau')Y_\nu(-k\tau)\right] \\
\nu &\cong 1/2 + 3\epsilon - \eta
\end{aligned}
$$

Calculation of $\zeta_\sigma^{(2)}$

$$
\begin{aligned}
J(\tau,\vec{x}) &= a^2\kappa^2 m_\sigma^2\left(\delta^{(1)}\sigma\right)^2 - 2\kappa^2\left(\delta^{(1)}\sigma'\right)^2 \\
&+ 2\kappa^2\mathcal{H}(1+\eta-\epsilon)\triangle^{-1}\partial_i\left(\delta^{(1)}\sigma'\partial^i\delta^{(1)}\sigma\right) \\
&+ 4\kappa^2\triangle^{-1}\partial_\tau\partial_i\left(\delta^{(1)}\sigma'\partial^i\delta^{(1)}\sigma\right) \\
&- \mathcal{H}(1+2\epsilon-2\eta)\triangle^{-1}\gamma' + \triangle^{-1}\gamma'' \\
&+ \text{inflaton contributions}
\end{aligned}
$$

$$
\begin{aligned}
\gamma &= -3\kappa^2\triangle^{-1}\partial_i\left(\partial^k\partial_k\delta^{(1)}\sigma\partial^i\delta^{(1)}\sigma\right) \\
&- \frac{\kappa^2}{2}\left(\partial_i\delta^{(1)}\sigma\partial^i\delta^{(1)}\sigma\right) + \cdots .
\end{aligned}
$$

Large Scale $\zeta_\sigma^{(2)}$

- The leading contribution to $\zeta_\sigma^{(2)}$ on large scales:

$$\zeta_\sigma^{(2)} \cong \frac{\kappa^2}{\epsilon}\int_{-1/a_i H}^{\tau} d\tau' \left[\frac{\left(\delta^{(1)}\sigma'\right)^2}{\mathcal{H}(\tau')} \right.$$

$$\left. - \frac{\mathcal{H}(\tau')^2}{\mathcal{H}(\tau)^3}\left(\left(\delta^{(1)}\sigma'\right)^2 - a^2 m_\sigma^2 \left(\delta^{(1)}\sigma\right)^2\right)\right]$$

- The result is manifestly local consistent with the results of other authors.[a]
- We explicitly identify the error in previous calculations which leads to a nonlocal result.

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[a] Malik, JCAP **0511**, 005 (2005); Lyth & Rodriguez, Phys. Rev. Lett. **95**, 121302 (2005); Jokinen & Mazumdar JCAP **0604**, 003 (2006).

Part 4: Nongaussianity and Constraints

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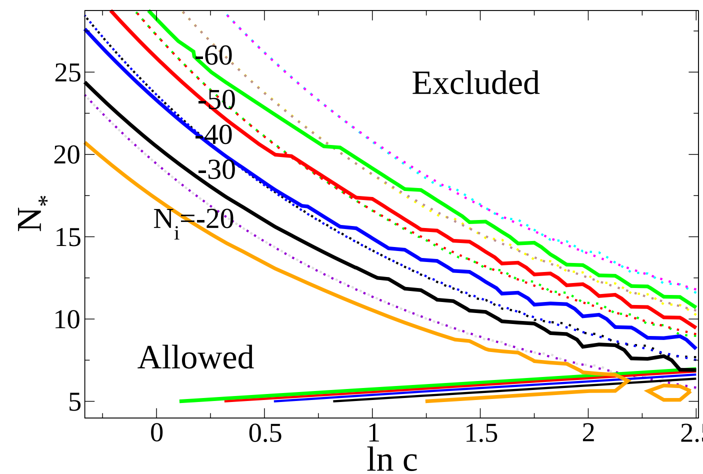


Generating ζ

- The gauge invariant curvature perturbation:

$$\zeta = \zeta^{(1)} + \frac{1}{2}\zeta^{(2)} \sim \zeta^{(1)} + \frac{3}{5} f_{NL} (\zeta^{(1)})^2$$

- The first order curvature perturbation $\zeta^{(1)}$ is the usual scale invariant and conserved quantity.
- The second order curvature perturbation is split into

$$\zeta^{(2)} = \underbrace{\zeta_{\varphi}^{(2)}}_{\propto (2\eta - 6\epsilon)(\zeta^{(1)})^2} + \underbrace{\zeta_{\sigma}^{(2)}}_{\text{amplified by instability}}$$

- After the symmetry breaking completes only one field is dynamical so ζ is conserved on large scales for $t > t_{\star}$.

Tachyon Bispectrum

- ⋆ The bispectrum is dominated by the tachyon part of ζ:

$$\begin{aligned}\langle \zeta_{k_1}\zeta_{k_2}\zeta_{k_3}\rangle &\cong \frac{1}{2^3}\langle \zeta^{(2)}_{\sigma,k_1}\zeta^{(2)}_{\sigma,k_2}\zeta^{(2)}_{\sigma,k_3}\rangle \\ &\equiv (2\pi)^{-3/2}\, B(k_i)\, \delta^3(\vec{k}_1+\vec{k}_2+\vec{k}_3)\end{aligned}$$

- ⋆ Should be compared to the usual inflationary spectrum:

$$\langle \zeta^{(1)}_{k_1}\zeta^{(1)}_{k_2}\rangle = P_\varphi(k_i)\, \delta^3(\vec{k}_1+\vec{k}_2)$$

where $P_\varphi^{1/2} \sim (2\pi)10^{-5}k^{-3/2}$.

- ⋆ Nonlinearity parameter f_{NL}:

$$B(k_i) \equiv -\frac{6}{5} f_{NL}\,[\, P_\varphi(k_1)P_\varphi(k_2) + \mathrm{perms}\,]$$

- ⋆ Demand that $|f_{NL}| < 100$.

The Linearity Parameter

- The two-point function also gets contributions from the tachyon:

$$\langle \zeta_{k_1} \zeta_{k_2} \rangle \cong \langle \zeta^{(1)}_{k_1} \zeta^{(1)}_{k_2} \rangle + \frac{1}{2^2} \langle \zeta^{(2)}_{\sigma,k_1} \zeta^{(2)}_{\sigma,k_2} \rangle$$

- Should compare the second order tachyon spectrum to the first order inflaton spectrum:

$$\frac{1}{2^2} \langle \zeta^{(2)}_{\sigma,k_1} \zeta^{(2)}_{\sigma,k_2} \rangle \equiv S(k_i)\, \delta^3(\vec{k}_1 + \vec{k}_2)$$

- Define the linearity parameter:

$$f_L \equiv \frac{S(k_i)}{P_\varphi(k_i)}$$

- Demand that $|f_L| < 1$ so that the spectrum is due to the inflaton.

(non)Scale-Invariant Fluctuations

If m_σ^2 varies slowly:

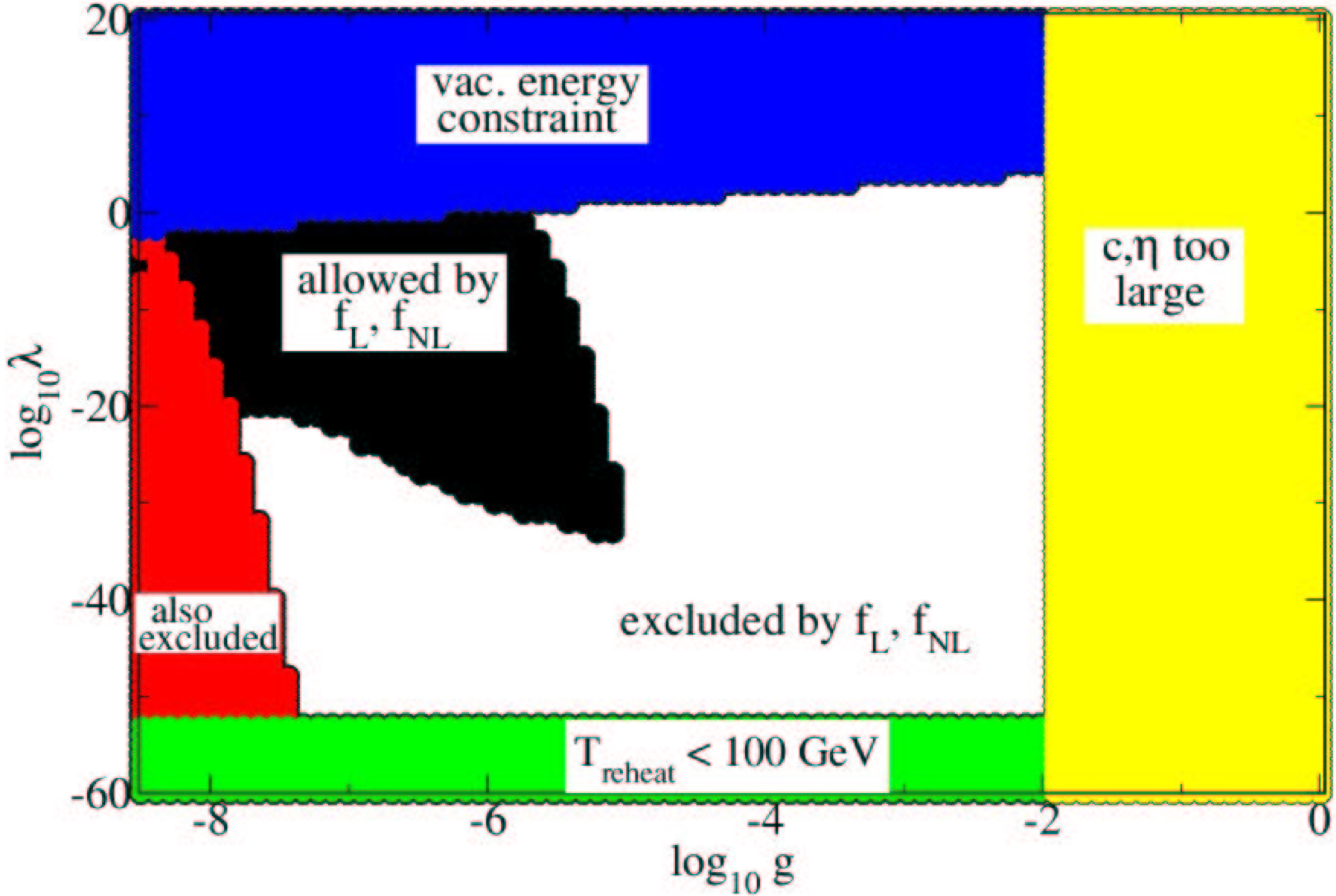
- $\star$ Tachyon is almost massless throughout inflation.
- $\star$ Instability sets in very slowly.
- $\star$ Have $N_\star \gg 1$.
- $\star$ Tachyon fluctuations, bispectrum are scale invariant and can have $|f_{NL}| > 1$.

If m_σ^2 varies quickly:

- $\star$ Tachyon curvature perturbation is blue ($n = 4$).
- $\star$ $\zeta_\sigma^{(2)}$ gets contributions from all tachyon modes in the instablity band.
- $\star$ Preheating distorts the power spectrum on small scales: strongest constraint from $|f_L| < 1$.

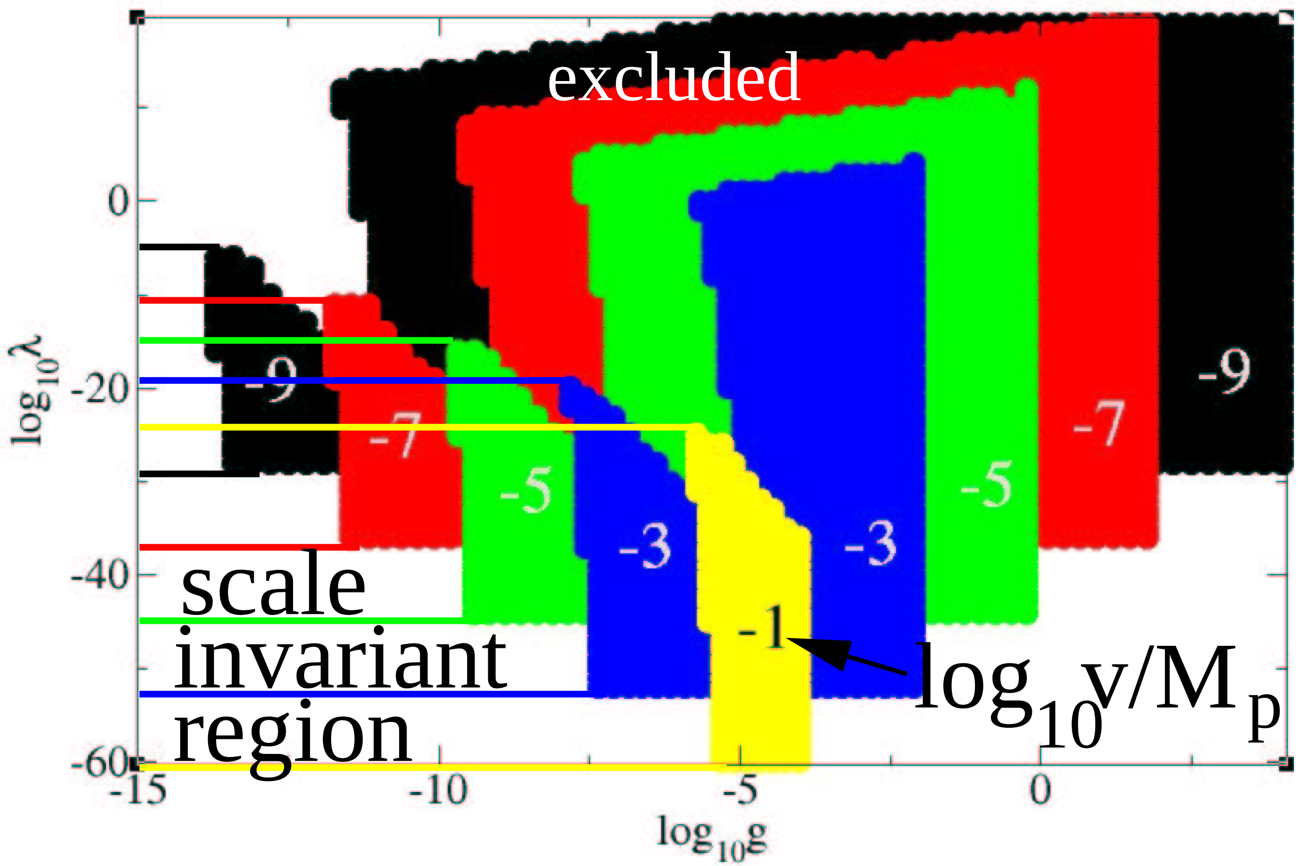
Hybrid Inflation: $v/M_p = 10^{-3}$

⋆ Spectral index: $n - 1 \sim 1.84g$.



Hybrid Inflation

- ⋆ The size of the excluded region depends sensitively on v/M_p.
- ⋆ Larger effect for smaller v/M_p since the amplification goes like $v/H \sim M_p/v$.



Inverted Hybrid Inflation

- ⋆ Simple modification of hybrid inflation which gives spectral index $n < 1$.
- ⋆ SUSY, string theory embeddings of hybrid inflation are more similar to inverted model.
- ⋆ Inverted hybrid inflation potential:

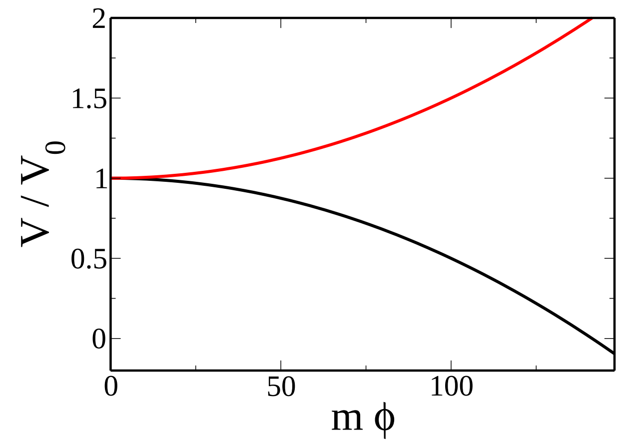
$$V(\varphi, \sigma) = \frac{\lambda}{4}(\sigma^2 + v^2)^2 - \frac{g^2}{2}\varphi^2\sigma^2 - \frac{m_\varphi^2}{2}\varphi^2$$

- ⋆ Obtained from hybrid inflation by flipping the sign of m_φ^2, v^2, g^2.
- ⋆ Potential is unbounded from below without the addition of a $\tilde{\lambda}\varphi^4$ term...

Inverted vs. non-Inverted Model

⋆ Potential along inflationary trajectory:

$$V_{\text{inf}} = \frac{\lambda v^4}{4} \pm \frac{1}{2} m_\varphi^2 \varphi^2$$



Hybrid Inflation

⋆ $n > 1$

⋆ Inflaton rolls towards the flat point $\varphi = 0$.

⋆ Still have slow roll as $\varphi \to \varphi_c$.

⋆ Possible to have a light tachyon, scale invariant fluctuations.

Inverted Hybrid Inflation

⋆ $n < 1$

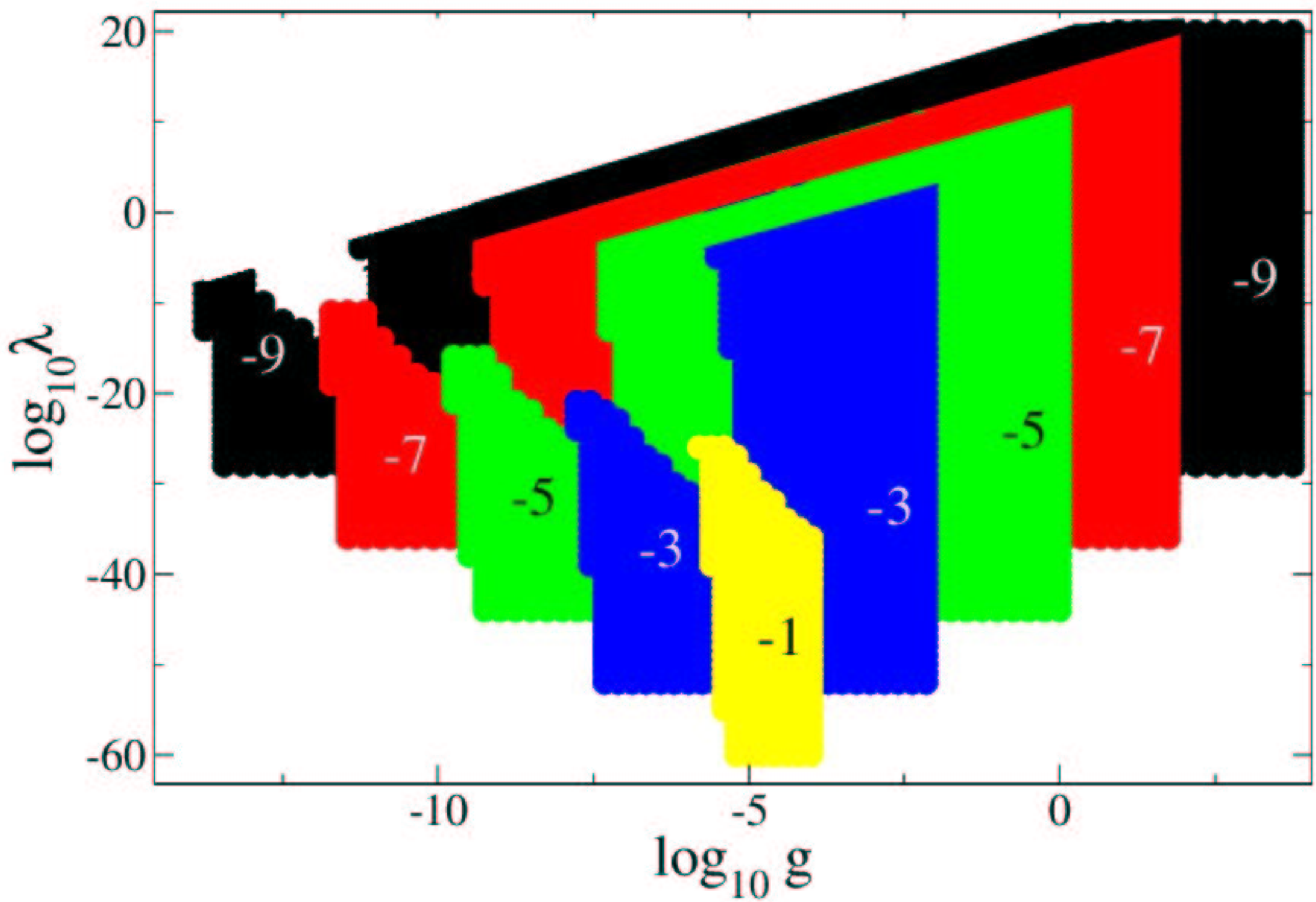
⋆ Inflaton rolls away from the flat point $\varphi = 0$.

⋆ As $\varphi \to \varphi_c$ the inflaton need not be slowly rolling.

⋆ Requires more tuning to keep the tachyon light.

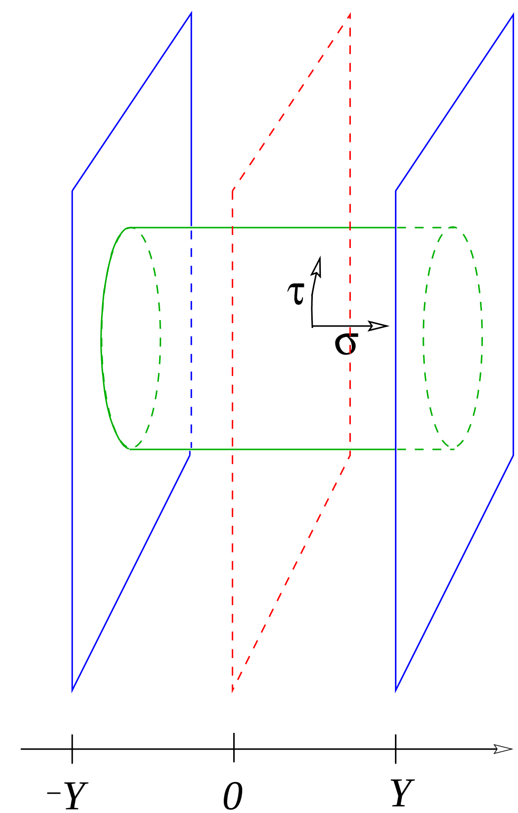
Inverted Hybrid Inflation

- ⋆ Constraints weakened: new allow regions correspond to fast roll through the instability point $\varphi = \varphi_c$.



Part 5: Implications for Brane Inflation

1. Inflation, Fluctuations and Gaussianity
2. Hybrid Inflation and Tachyonic Preheating
3. Cosmological Perturbation Theory
4. Nongaussianity and Constraints
5. Implications for Brane Inflation

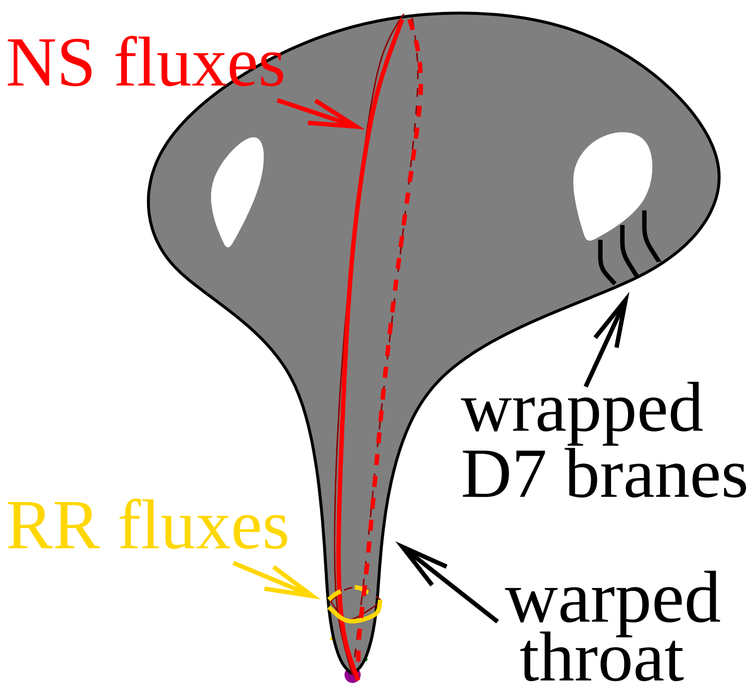


Brane Inflation

- ⋆ Brane inflation is a particularly appealing embedding of hybrid inflation into string theory.
- ⋆ Inflation is driven by potential between D3/D$\bar{3}$ which are parallel to our 3 large dimensions and separated in the extra dimensions.
- ⋆ Inter-brane separation, y, plays the role of the inflation.
- ⋆ Lightest stretched string mode between the branes becomes tachyonic at $y \sim l_s$ (open string tachyon).
- ⋆ Open string tachyon plays the role of the waterfall field σ.
- ⋆ Tachyon in the spectrum signals instability of the system to annihilate.

Flux Compactifications

- Realistic models of brane inflation are embedded in GKP-KKLT flux vacua.
- Complex structure moduli and dilaton are fixed by addition of fluxes of the NS-NS and R-R gauge fields.
- Kahler modulus fixed by nonperturbative effects (eg - gaugino condensation).



- Compactification has warped throat regions where exponentially large hierarchy can be generated from a small hierarchy in the ratio of fluxes.

Brane Inflation in Flux Vacua

- ⋆ In the throat the geometry is locally $AdS_5 \times S^5$:

$$ds^2 \cong e^{-2k|y|}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2 + y^2 d\Omega_5^2.$$

- ⋆ Geometry is identical to Randall-Sundrum I.
- ⋆ Set-up: mobile D3 falling down the throat from the UV ($y = 0$) end towards a fixed D$\bar{3}$ in the IR ($y = y_i > 0$) end.
- ⋆ Exchange of massless gauge fields gives rise to a Coulomb potential between the branes.
- ⋆ The large warping $a_i = e^{-ky_i} \ll 1$ flattens inter-brane potential.

The η-problem

- ⋆ Coulomb potential between the branes:

$$V = a_i^4 \tau_3 \left[1 - \frac{1}{N} \left(\frac{\varphi_0}{\varphi} \right)^4 \right]$$

  is extremely flat.

- ⋆ Unfortunately consistent introduction of volume stabilization introduces an $\mathcal{O}(H)$ contribution to the inflaton mass.
- ⋆ Can salvage inflation to adding some corrections to the superpotential which cancel the large inflaton mass coming from volume stabilization.
- ⋆ Inflation is fine tuned in this scenario.[a]

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[a] Kachru et al., JCAP **0310**, 013 (2003); Burgess et al., JHEP **0409**, 033 (2004).

The End of Inflation

- ⋆ Lightest stretched string mode between the branes has mass:

$$M_T^2 = \frac{M_s^2}{2}\left[\frac{(M_s y)^2}{(2\pi)^2} - \frac{1}{2}\right]$$

which becomes tachyonic at $y \lesssim l_s$.

- ⋆ Brane annihilation is described by the tachyon condensation:[a]
  - Field theory about the tachyon false vacuum $T = 0$ describes the coincident brane-antibrane system
  - The tachyon rolls to $|T| = \infty$ and field theory about this point describes the vacuum with no brane-antibrane

[a] Sen, Phys. Rev. D **68**, 066008 (2003).

Brane Inflation and Nongaussianity

- ⋆ Brane inflation is similar to inverted hybrid inflation:

$$V_{\rm inf} = V_0 \left[1 - \frac{1}{N} \left(\frac{\varphi_0}{\varphi} \right)^4 - \frac{\beta}{3} \left(\frac{\varphi}{M_p} \right)^2 \right]$$

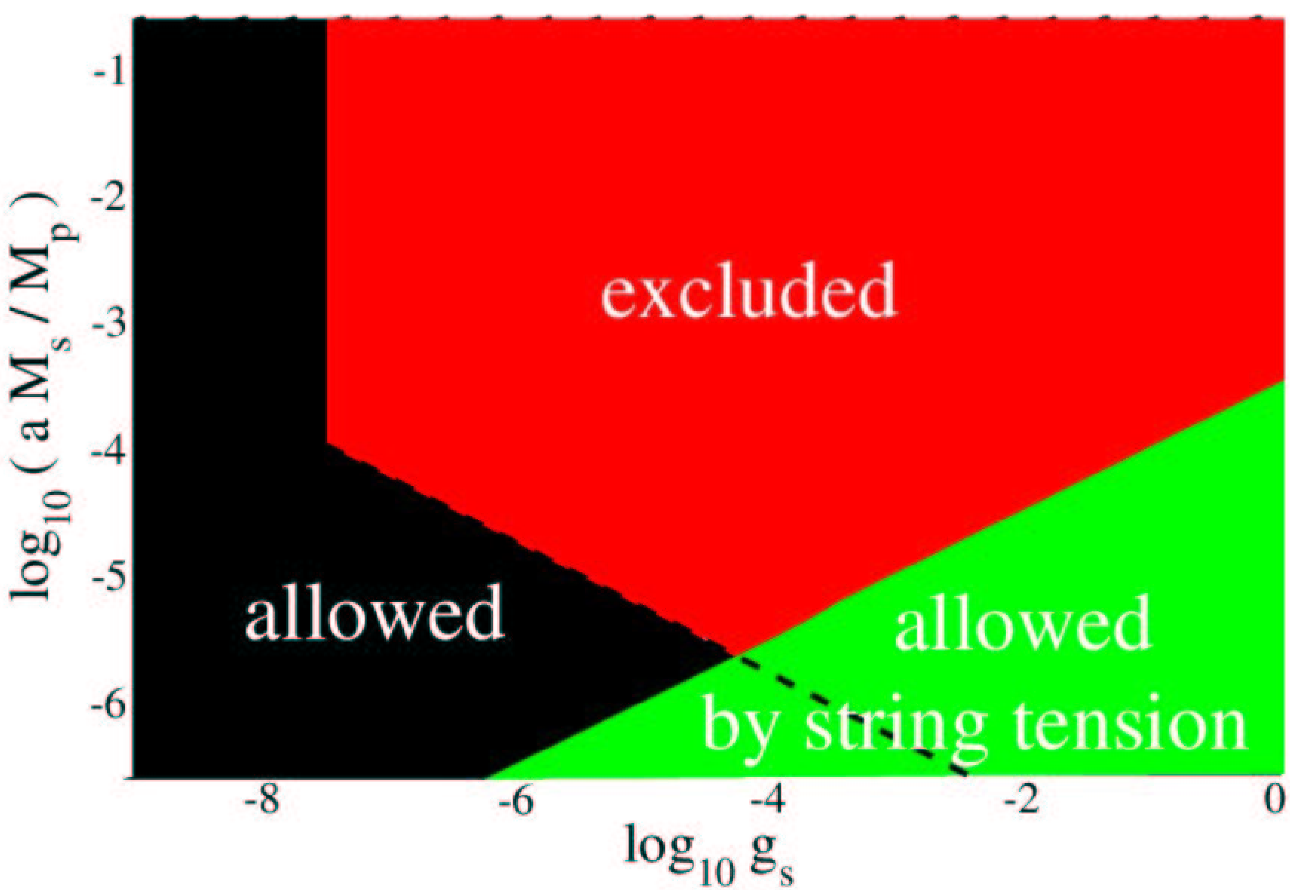
- ⋆ Some differences:
  - tachyon field is complex
  - the tachyon potential is minimized at $T = \pm\infty$: $V(T, y = 0) = \tau_3 \, e^{-|T|^2}$
  - tachyon DBI action: [a]

$$\mathcal{L}_{\rm tac} = -V(T, y) \sqrt{1 + M_s^{-2} |\partial_\mu T|^2}$$

[a] Sen, Int. J. Mod. Phys. A **20**, 5513 (2005).

Excluded Regions

- ⋆ Dimensionally reduce the DBI action on AdS_5 and expand to quadratic order in fields.
- ⋆ Match reduced action to inverted hybrid inflation to estimate g, λ, v in terms of stringy quantities g_s, M_s, a_i.



Conclusions

- ⋆ Variation of the second order curvature perturbation from tachyonic preheating puts interesting constraints on hybrid inflation.
- ⋆ Strongest constraints for small symmetry breaking scale $v/M_p \ll 1$.
- ⋆ Constraints on inverted hybrid inflation are weaker since it is harder to keep the tachyon light during inflation.
- ⋆ Nontrivial constraints on KKLMMT.

Future Directions

- ⋆ This model leads to domain walls which will overclose the universe.
- ⋆ Generalization to D-term inflation, D3/D7, ...
- ⋆ Inveresting hint of excess power on small scales in the CBI and ACBAR CMB data...