Electroweak Constraints on Effective Theories

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Outline

- **The hierarchy problem in the standard model and TeV scale new physics**
- **Electroweak constraints on TeV scale physics**
- **Effective theory analysis of electroweak data**
- **Applications to TeV scale models**
- **Conclusion**

The hierarchy problem of the standard model

- $$\left.\begin{array}{c}\text{Planck scale: } M_{Pl} \sim 10^{19} GeV \\ \text{EWSB scale: } v = 246 GeV\end{array}\right\} \Longrightarrow \text{Huge gap in between.}$$

- EWSB scale unstable—radiative corrections to the Higgs mass.

$$M_h^2 = M_h^2(\text{tree}) + \text{radiative corrections}$$

  - One loop quadratically divergent corrections to the Higgs mass:

$$\begin{array}{rl} \text{top loop:} & -\frac{3}{8\pi^2}\lambda_t^2 \Lambda^2 \\ SU(2) \text{ gauge boson loop:} & \frac{9}{64\pi^2} g^2 \Lambda^2 \\ \text{Higgs loop:} & \frac{1}{16\pi^2}\lambda^2 \Lambda^2 \end{array}$$

Λ : cutoff; λ_t : top Yukawa coupling; λ : Higgs quartic coupling

TeV scale new physics

- Fine-tuning less than 10% $\Longrightarrow \Lambda \lesssim \mathcal{O}(1)$ TeV:

$$\frac{3}{8\pi^2}\lambda_t^2\Lambda^2 < 10 \times (200\text{GeV})^2 \quad \Longrightarrow \quad \Lambda \lesssim 3\text{TeV}$$

- Cutoff $\sim$ TeV $\Longrightarrow$ TeV scale extensions of the SM.

SUSY, technicolor, extra-dimensions, little Higgs, . . .

- Predict heavy particles $\longrightarrow$ awaiting direct probe at Tevatron, LHC, (ILC), . . .

- Indirect information from electroweak precision tests (EWPTs).

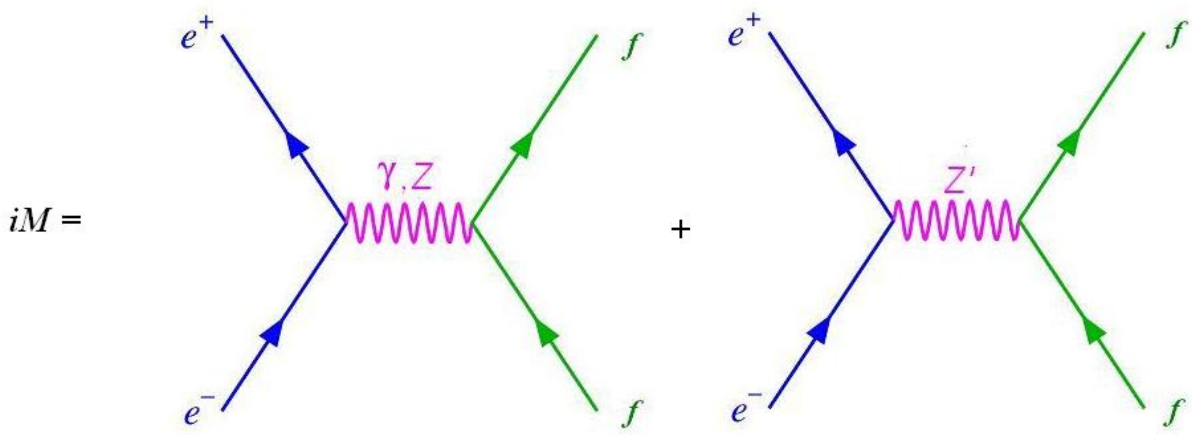
Electroweak precision tests

- Established the SM of electroweak physics.
- Good precision: 1% level or better.
- Include observables from:
  - Atomic parity violation experiments;
  - Deep inelastic scattering: neutrino-nucleon, neutrino-electron scattering;
  - $e^+e^- \to \bar{f}f$, $e^+e^- \to W^+W^-$ scattering;
  - W boson mass;

  $\ldots$

Constraining new physics

- An example: Z' gauge boson.
  - Affect $e^+e^- \to \bar{f}f$ scattering:



  - No significant deviation from the SM prediction.
    $\Longrightarrow$ Constraints on Z' mass or Z'-fermion couplings.

Model independent analysis

- Generally, no significant deviations from the SM $\Longrightarrow$ Constraints reduce the number of models, allow us to focus on more promising ones.
- Model-independent method
  - Global constraints—using all relevant data.
  - Avoid repeating the calculations.
  - Example: oblique parameters $S, T, U, \ldots$
    $\longrightarrow$ Not enough, non-oblique corrections are common. (e.x. Z').
    $\longrightarrow$ Model-dependent corrections to observables calculated from time to time in the literature.

The observables

	Standard Notation	Measurement
Atomic parity violation	$Q_W(Cs)$ $Q_W(Tl)$	Weak charge in Cs Weak charge in Tl
DIS	g_L^2, g_R^2 R^ν κ $g_V^{\nu e}, g_A^{\nu e}$	ν_μ-nucleon scattering from NuTeV ν_μ-nucleon scattering from CDHS and CHARM ν_μ-nucleon scattering from CCFR ν-e scattering from CHARM II
Z-pole	Γ_Z σ_h^0 $R_f^0(f = e, \mu, \tau, b, c)$ $A_{FB}^{0,f}(f = e, \mu, \tau, b, c)$ $\sin^2 \theta_{eff}^{lept}(Q_{FB})$ $A_f(f = e, \mu, \tau, b, c)$	Total Z width e^+e^- hadronic cross section at Z pole Ratios of decay rates Forward-backward asymmetries Hadronic charge asymmetry Polarized asymmetries
Fermion pair production at LEP2	$\sigma_f(f = q, \mu, \tau)$ $A_{FB}^f(f = \mu, \tau)$ $d\sigma_e/d\cos\theta$	Total cross sections for $e^+e^- \to f\overline{f}$ Forward-backward asymmetries for $e^+e^- \to f\overline{f}$ Differential cross section for $e^+e^- \to e^+e^-$
W pair	$d\sigma_W/d\cos\theta$	Differential cross section for $e^+e^- \to W^+W^-$
	M_W	W mass

Table 1: Relevant measurements

The effective theory approach

- Below the cutoff Λ, after integrating out the heavy particles:

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{1}{\Lambda^2} \sum_i c_i O_i + \frac{1}{\Lambda^4}(\ldots) + \ldots$$

- Only the SM fields appear in the operators O_i.
- The operators O_i conserve the SM gauge symmetry. $\Longrightarrow$ The number of O_i is finite to a given order in Λ.
- The coefficients c_i record the effects of the heavy particles. If taken arbitrary $\longrightarrow$ model-independent analysis.

Procedure

- Identify relevant operators,add them to the SM Lagrangian

$$\mathcal{L} = \mathcal{L}_{SM} + \sum_i a_i O_i$$

.

- Calculate deviations to observables from the SM predictions, assuming arbitrary a_i.
- Compare with data, obtain constraints on a_i.
- For a given model, translate constraints on a_i to constraints on model parameters (mass, coupling...).

Reduce the number of operators

- Given the current experimental precision, enough to focus on dimension-6 operators, since higher order operators are suppressed by more powers of Λ^2.
- Keep only independent operators
  - Equations of motion
  - Integration by part
- Keep operators that are relevant to TeV scales: imposing symmetries on the operators. "over-constrained"
- Remove operators not tightly constrained by EWPTs. "poorly constrained"

Symmetries of the operators

Remove "over-constrained" operators by imposing symmetries:

- Baryon and lepton number conservation.
- CP invariance.
- flavor conservation, $U(3)^5$ symmetry.

Flavor symmetry

- Avoid operators like $\frac{1}{\Lambda^2}\bar{s}d\bar{s}d, \quad \Lambda > 1000$ TeV.
- $U(1)$ symmetry for each flavor? $\frac{1}{\Lambda^2}(c_1\bar{d}d\bar{d}d + c_2\bar{s}s\bar{s}s + c_3\bar{b}b\bar{b}b)$? In what basis?
- Simpler solution—flavor universality, $U(3)^5$ symmetry.
  - One $U(3)$ for each SM fermion representation: q, l, u, d, e.
    q, l : left-handed doublets; u, d, e : right handed singlet.
  - Omit flavor indices. $(\bar{q}_i q_i \bar{u}_j u_j \longrightarrow \bar{q}q\bar{u}u)$.
- Processes involving the third generation are not constrained as well $\longrightarrow$ flavor symmetry relaxed to $[U(2) \times U(1)]^5$ later.

Remove operators not tightly constrained

- Remove not or "poorly" constrained operators.
  Example:
  - Not constrained
  $$\partial_\mu (h^\dagger h) \partial^\mu (h^\dagger h)$$
  - Poorly constrained
  $$(\bar{q}\gamma^\mu q)(\bar{q}\gamma_\mu q)$$
- 21 operators are left.

The 21 operators

- Operators modifying gauge boson propagators:

$$O_{WB} = (h^\dagger \sigma^a h) W^a_{\mu\nu} B^{\mu\nu}, \quad O_h = |h^\dagger D_\mu h|^2;$$

  These two operators correspond respectively to the S and T parameters.

- Four-fermion operators:

$$\begin{aligned}
&O^s_{ll} = \frac{1}{2}(\bar{l}\gamma^\mu l)(\bar{l}\gamma_\mu l), \quad O^t_{ll} = \frac{1}{2}(\bar{l}\gamma^\mu \sigma^a l)(\bar{l}\gamma_\mu \sigma^a l),\\
&O^s_{lq} = (\bar{l}\gamma^\mu l)(\bar{q}\gamma_\mu q), \quad O^t_{lq} = (\bar{l}\gamma^\mu \sigma^a l)(\bar{q}\gamma_\mu \sigma^a q),\\
&O_{le} = (\bar{l}\gamma^\mu l)(\bar{e}\gamma_\mu e), \quad O_{qe} = (\bar{q}\gamma^\mu q)(\bar{e}\gamma_\mu e),\\
&O_{lu} = (\bar{l}\gamma^\mu l)(\bar{u}\gamma_\mu u), \quad O_{ld} = (\bar{l}\gamma^\mu l)(\bar{d}\gamma_\mu d),\\
&O_{ee} = \frac{1}{2}(\bar{e}\gamma^\mu e)(\bar{e}\gamma_\mu e), \quad O_{eu} = (\bar{e}\gamma^\mu e)(\bar{u}\gamma_\mu u), \quad O_{ed} = (\bar{e}\gamma^\mu e)(\bar{d}\gamma_\mu d);
\end{aligned}$$

The 21 operators

- Operators modifying gauge-fermion couplings:

$$O^s_{hl} = i(h^\dagger D^\mu h)(\bar{l}\gamma_\mu l) + \text{h.c.}, \quad O^t_{hl} = i(h^\dagger \sigma^a D^\mu h)(\bar{l}\gamma_\mu \sigma^a l) + \text{h.c.},$$
$$O^s_{hq} = i(h^\dagger D^\mu h)(\bar{q}\gamma_\mu q) + \text{h.c.}, \quad O^t_{hq} = i(h^\dagger \sigma^a D^\mu h)(\bar{q}\gamma_\mu \sigma^a q) + \text{h.c.},$$
$$O_{hu} = i(h^\dagger D^\mu h)(\bar{u}\gamma_\mu u) + \text{h.c.}, \quad O_{hd} = i(h^\dagger D^\mu h)(\bar{d}\gamma_\mu d) + \text{h.c.},$$
$$O_{he} = i(h^\dagger D^\mu h)(\bar{e}\gamma_\mu e) + \text{h.c.}\,;$$

- Operator modifying the triple-gauge couplings:

$$O_W = \epsilon^{abc}\, W^{a\nu}_{\mu} W^{b\lambda}_{\nu} W^{c\mu}_{\lambda}.$$

The calculation

- $$\mathcal{L} = \mathcal{L}_{SM} + a_{WB} O_{WB} + a_h O_h + \ldots a_W O_W$$
$$a_i \text{ dimension } (-2) \quad \sim \frac{1}{\Lambda^2}$$
.

- Calculate to linear order in a_i the corrections to the observables—only consider the interference between the SM and the new physics contribution:

Tree level calculation, amplitude $\mathcal{M}_{NP}$ linear in a_i

$$X_{th}(a_i) \sim |\mathcal{M}|^2 = |\mathcal{M}_{SM} + \mathcal{M}_{NP}|^2 = |\mathcal{M}_{SM}|^2 + 2Re(\mathcal{M}_{SM}\mathcal{M}^*_{NP})$$

$X_{th}(a_i)$: theoretical prediction for an observable X, linear in a_i.

$$X_{th}(a_i) = X_{SM} + \sum_i a_i \Delta X_i.$$

The observables

M_Z, α, G_F: input parameters $\rightarrow g, g', v$.

	Standard Notation	Measurement
Atomic parity violation	$Q_W(Cs)$ $Q_W(Tl)$	Weak charge in Cs Weak charge in Tl
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Z-pole	Γ_Z σ_h^0 $R_f^0(f = e, \mu, \tau, b, c)$ $A_{FB}^{0,f}(f = e, \mu, \tau, b, c)$ $\sin^2\theta_{eff}^{lept}(Q_{FB})$ $A_f(f = e, \mu, \tau, b, c)$	Total Z width e^+e^- hadronic cross section at Z pole Ratios of decay rates Forward-backward asymmetries Hadronic charge asymmetry Polarized asymmetries
Fermion pair production at LEP2	$\sigma_f(f = q, \mu, \tau)$ $A_{FB}^f(f = \mu, \tau)$ $d\sigma_e/d\cos\theta$	Total cross sections for $e^+e^- \rightarrow f\overline{f}$ Forward-backward asymmetries for $e^+e^- \rightarrow f\overline{f}$ Differential cross section for $e^+e^- \rightarrow e^+e^-$
W pair	$d\sigma_W/d\cos\theta$	Differential cross section for $e^+e^- \rightarrow W^+W^-$
	M_W	W mass

Table 2: Relevant measurements

g-2?

- High precision.
- Operator $\frac{1}{\Lambda^2}\bar{l}\sigma^{\mu\nu}ehB_{\mu\nu}$ contributes at tree level, but violates flavor symmetry.
- Other operators contribute at loop level, but not constrained as well as using all other measurements.
- For simplicity, ignore $g-2$.

- Constraints on individual operators not useful, because corrections are correlated.
- Compare with experiments and calculate the χ^2 distribution in terms of a_i:

$$\chi^2(a_i) = \sum_X \frac{(X_{th}(a_i) - X_{exp})^2}{\sigma_X^2} = \chi^2_{SM} + a_i \hat{v}_i + a_i M_{ij} a_j.$$

M_{ij}, v_i: our results

M_{ij} : 21 by 21 symmetric matrix; v_i : 21-vector

- Diagonal elements M_{ii}, tell us how well the operators are constrained. $\Lambda \sim M_{ii}^{\frac{1}{4}} = 1.3 \sim 17$ TeV.
- Constraints can be obtained from the χ^2 for arbitrary linear combinations of the operators. $\rightarrow$ Constrain generic models.

Numerical results

a_{WB}	9.1e4																				
a_h	2.4e4	7.9e3																			
a_{ll}^s	−78.	−51.	5.8e2																		
a_{ll}^t	−3.9e4	−1.2e4	6.7e2	2.2e4																	
a_{lq}^s	−1.4e3	−1.6e2	0.	1.5e2	2.7e3																
a_{lq}^t	−5.5e2	−1.4e2	0.	5.9e2	4.6e2	2.9e3															
a_{le}	−56.	−9.7	2.8e2	3.0e2	0.	0.	1.3e3														
a_{qe}	1.3e3	72.	0.	−1.4e2	−2.7e3	−7.4e2	0.	2.8e3													
a_{lu}	−4.0e2	3.8	0.	−1.1e2	1.2e3	−2.5e2	0.	−1.2e3	7.1e2												
a_{ld}	−6.9e2	−6.9	0.	66.	1.4e3	3.3e2	0.	−1.4e3	5.8e2	7.8e2											
a_{ee}	−59.	−42.	5.3e2	6.1e2	0.	0.	2.6e2	0.	0.	0.	4.8e2										
a_{eu}	7.8e2	1.1e2	0.	−2.1e2	−1.3e3	−9.1e2	0.	1.4e3	−4.8e2	−7.3e2	0.	8.4e2									
a_{ed}	4.2e2	−83.	0.	1.7e2	−1.3e3	5.5e2	0.	1.3e3	−7.3e2	−6.8e2	0.	4.7e2	8.8e2								
a_{hl}^s	−1.7e4	−4.1e3	1.5e2	9.7e3	−5.9e2	8.3e2	17.	3.7e2	−3.9e2	−1.6e2	1.3e2	66.	3.8e2	5.5e4							
a_{hl}^t	5.9e4	1.7e4	−43.	−3.0e4	−7.1e2	−6.6e2	−31.	6.6e2	−82.	−3.4e2	−32.	4.9e2	47.	1.5e4	6.3e4						
a_{hq}^s	−1.9e3	−1.4e3	0.	2.7e3	−2.6e3	−72.	0.	2.6e3	−1.2e3	−1.4e3	0.	1.2e3	1.4e3	−6.6e3	−8.7e3	6.0e3					
a_{hq}^t	−9.3e3	−4.5e3	0.	8.7e3	−49.	3.5e2	0.	56.	−1.4e2	−36.	0.	−64.	1.8e2	−2.4e4	−3.1e4	7.7e3	2.6e4				
a_{hu}	−6.1e2	−6.6e2	0.	1.2e3	−1.2e3	−4.	0.	1.2e3	−5.1e2	−6.9e2	0.	5.7e2	6.7e2	−3.7e3	−4.4e3	2.2e3	4.1e3	1.4e3			
a_{hd}	1.2e3	4.3e2	0.	−8.1e2	−1.4e3	−1.3e2	0.	1.4e3	−6.9e2	−7.2e2	0.	6.7e2	7.3e2	3.3e3	3.6e3	4.2e2	−2.9e3	1.6e2	1.1e3		
a_{he}	−2.8e4	−4.6e3	−1.1e2	9.0e3	4.6e2	−1.6e2	23.	−4.5e2	2.5e2	2.4e2	−96.	−1.7e2	−3.0e2	−2.5e4	−3.2e4	4.5e3	1.7e4	2.3e3	−2.1e3	3.2e4	
a_W	7.7	4.5	0.	−4.2	0.	0.	0.	0.	0.	0.	0.	0.	0.	6.3	−1.7	0.	0.8	0.	0.	1.4	2.6
	a_{WB}	a_h	a_{ll}^s	a_{ll}^t	a_{lq}^s	a_{lq}^t	a_{le}	a_{qe}	a_{lu}	a_{ld}	a_{ee}	a_{eu}	a_{ed}	a_{hl}^s	a_{hl}^t	a_{hq}^s	a_{hq}^t	a_{hu}	a_{hd}	a_{he}	a_W

Table 3: The elements of the matrix $\mathcal{M}$. Since it is a symmetric matrix we do not list the redundant elements. The matrix is equal to the numbers listed above times $10^{12}(\mathrm{GeV})^4$. We abbreviate the powers 10^n as en to save space.

$$\begin{aligned}\hat{v}_i \;&=\; \{1.5, 10^2, -23., 49., 76., -1.1, 10^2, -2.4, 10^2, 29., 1.4, 10^2, -36., -68., 44.,\\ &\quad 1.0, 10^2, 15., -6.4, 10^2, -88., 1.0, 10^2, 1.7, 10^2, 71., 63., 1.8, 10^2, 1.0\}\end{aligned}$$

the S and T fit

- $$S = \frac{4scv^2 a_{WB}}{\alpha}, \quad T = -\frac{v^2}{2\alpha} a_h.$$
- Setting all a_i, but a_{WB} and a_h, to zero.

$$\begin{aligned} \chi^2 &= \chi_0^2 + (a_{WB} \quad a_h) \begin{pmatrix} 9.1\,10^{16} & 2.4\,10^{16} \\ 2.4\,10^{16} & 7.9\,10^{15} \end{pmatrix} \begin{pmatrix} a_{WB} \\ a_h \end{pmatrix} + 1.5\,10^8 a_{WB} - 2.3\,10^7 a_h \\ &= \chi_0^2 + (S \quad T) \begin{pmatrix} 5.4\,10^2 & -4.8\,10^2 \\ -4.8\,10^2 & 5.3\,10^2 \end{pmatrix} \begin{pmatrix} S \\ T \end{pmatrix} + 12.\,S + 5.9\,T. \end{aligned}$$

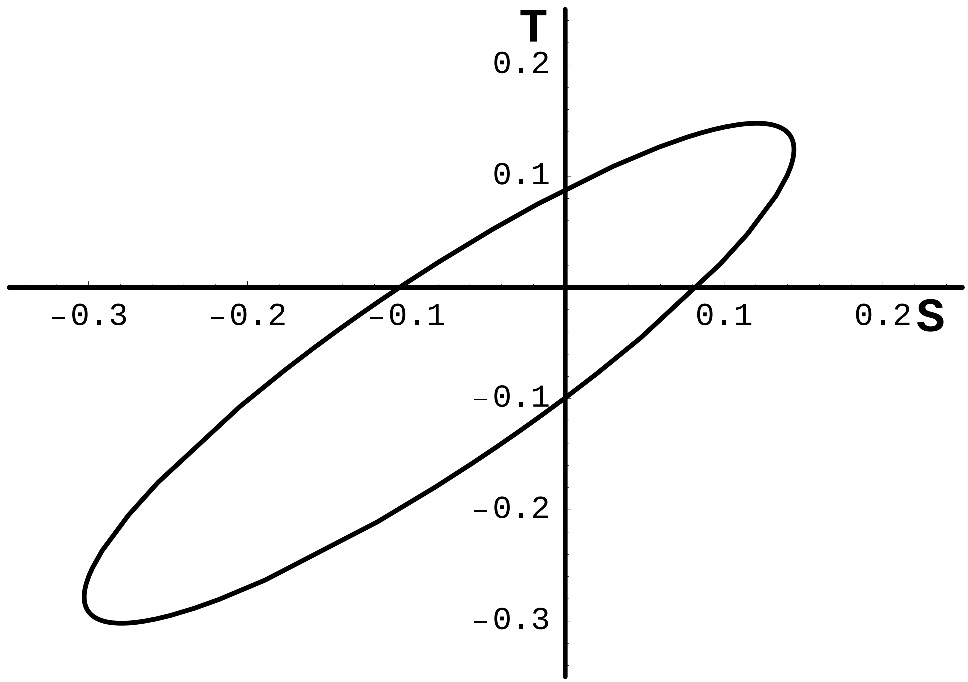
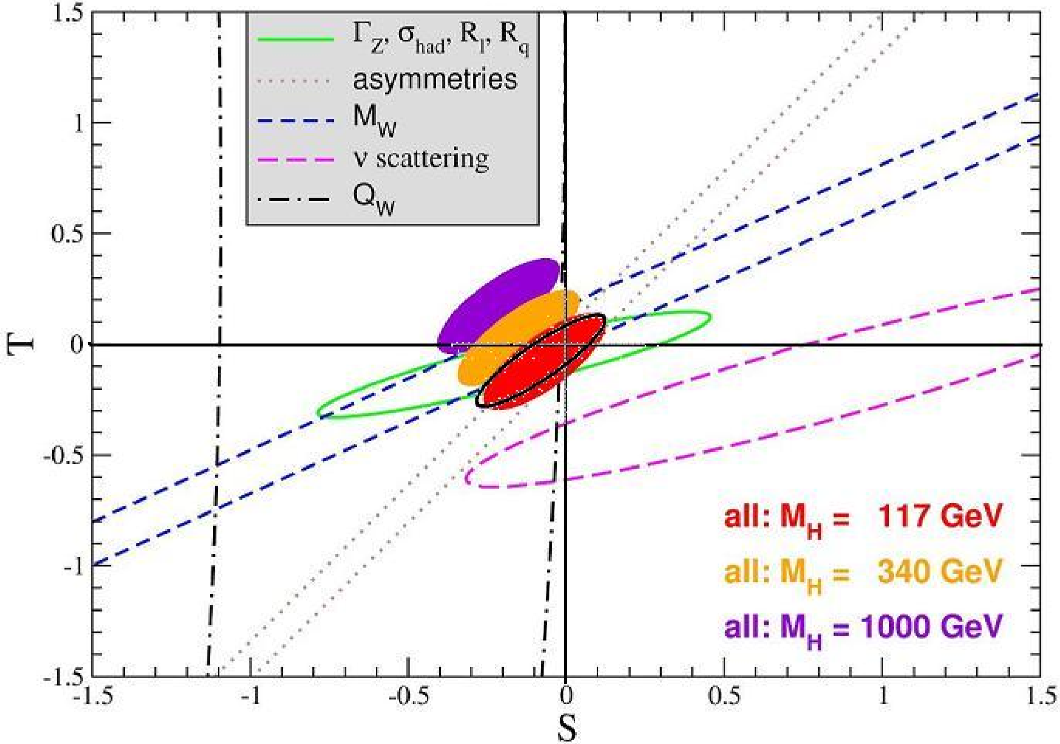


Figure 1: Allowed region for S and T at 90% confidence level.

Oblique Parameters

constraints on gauge boson self-energies



Relax the flavor symmetry

- TeV scale flavor physics involving the third generation still allowed.
- Treat the third generation differently: $U(3) \to U(2) \times U(1)$.

$$\text{Example: } \frac{1}{\Lambda^2}\bar{e}e\bar{b}b.$$

- 16 more operators.
- Do not add flavor-changing experiments.

Old operators

- Sum over only the first two generations.

$$
\begin{aligned}
&O_{WB} = (h^\dagger \sigma^a h) W^a_{\mu\nu} B^{\mu\nu}, \quad O_h = |h^\dagger D_\mu h|^2; \\
&O^s_{ll} = \frac{1}{2}(\bar{l}\gamma^\mu l)(\bar{l}\gamma_\mu l), \quad O^t_{ll} = \frac{1}{2}(\bar{l}\gamma^\mu \sigma^a l)(\bar{l}\gamma_\mu \sigma^a l), \\
&O^s_{lq} = (\bar{l}\gamma^\mu l)(\bar{q}\gamma_\mu q), \quad O^t_{lq} = (\bar{l}\gamma^\mu \sigma^a l)(\bar{q}\gamma_\mu \sigma^a q), \\
&O_{le} = (\bar{l}\gamma^\mu l)(\bar{e}\gamma_\mu e), \quad O_{qe} = (\bar{q}\gamma^\mu q)(\bar{e}\gamma_\mu e), \\
&O_{lu} = (\bar{l}\gamma^\mu l)(\bar{u}\gamma_\mu u), \quad O_{ld} = (\bar{l}\gamma^\mu l)(\bar{d}\gamma_\mu d), \\
&O_{ee} = \frac{1}{2}(\bar{e}\gamma^\mu e)(\bar{e}\gamma_\mu e), \quad O_{eu} = (\bar{e}\gamma^\mu e)(\bar{u}\gamma_\mu u), \quad O_{ed} = (\bar{e}\gamma^\mu e)(\bar{d}\gamma_\mu d); \\
&O^s_{hl} = i(h^\dagger D^\mu h)(\bar{l}\gamma_\mu l) + \text{h.c.}, \quad O^t_{hl} = i(h^\dagger \sigma^a D^\mu h)(\bar{l}\gamma_\mu \sigma^a l) + \text{h.c.}, \\
&O^s_{hq} = i(h^\dagger D^\mu h)(\bar{q}\gamma_\mu q) + \text{h.c.}, \quad O^t_{hq} = i(h^\dagger \sigma^a D^\mu h)(\bar{q}\gamma_\mu \sigma^a q) + \text{h.c.}, \\
&O_{hu} = i(h^\dagger D^\mu h)(\bar{u}\gamma_\mu u) + \text{h.c.}, \quad O_{hd} = i(h^\dagger D^\mu h)(\bar{d}\gamma_\mu d) + \text{h.c.}, \\
&O_{he} = i(h^\dagger D^\mu h)(\bar{e}\gamma_\mu e) + \text{h.c.}\,; \\
&O_W = \epsilon^{abc} W^{a\nu}_\mu W^{b\lambda}_\nu W^{c\mu}_\lambda .
\end{aligned}
$$

New operators

- Q, L, b, t, τ : the third generation fermions.
- Four-fermion operators:

$$
\begin{aligned}
O_{lL}^{s} &= (\bar{l}\gamma^{\mu}l)(\overline{L}\gamma_{\mu}L), \quad O_{lL}^{t} = (\bar{l}\gamma^{\mu}\sigma^{a}l)(\overline{L}\gamma_{\mu}\sigma^{a}L),\\
O_{lQ}^{s} &= (\bar{l}\gamma^{\mu}l)(\overline{Q}\gamma_{\mu}Q), \quad O_{lQ}^{t} = (\bar{l}\gamma^{\mu}\sigma^{a}l)(\overline{Q}\gamma_{\mu}\sigma^{a}Q),\\
O_{Le} &= (\overline{L}\gamma^{\mu}L)(\bar{e}\gamma_{\mu}e), \quad O_{l\tau} = (\bar{l}\gamma^{\mu}l)(\bar{\tau}\gamma_{\mu}\tau),\\
O_{Qe} &= (\overline{Q}\gamma^{\mu}Q)(\bar{e}\gamma_{\mu}e), \quad O_{lb} = (\bar{l}\gamma^{\mu}l)(\bar{b}\gamma_{\mu}b),\\
O_{e\tau} &= (\bar{e}\gamma^{\mu}e)(\bar{\tau}\gamma_{\mu}\tau), \quad O_{eb} = (\bar{e}\gamma^{\mu}e)(\bar{b}\gamma_{\mu}b);
\end{aligned}
$$

- Operators modifying gauge-fermion couplings:

$$
\begin{aligned}
O_{hL}^{s} &= i(h^{\dagger}D^{\mu}h)(\overline{L}\gamma_{\mu}L) + \text{h.c.}, \quad O_{hL}^{t} = i(h^{\dagger}\sigma^{a}D^{\mu}h)(\overline{L}\gamma_{\mu}\sigma^{a}L) + \text{h.c.},\\
O_{hQ}^{s} &= i(h^{\dagger}D^{\mu}h)(\overline{Q}\gamma_{\mu}Q) + \text{h.c.}, \quad O_{hQ}^{t} = i(h^{\dagger}\sigma^{a}D^{\mu}h)(\overline{Q}\gamma_{\mu}\sigma^{a}Q) + \text{h.c.},\\
O_{h\tau} &= i(h^{\dagger}D^{\mu}h)(\bar{\tau}\gamma_{\mu}\tau) + \text{h.c.}, \quad O_{hb} = i(h^{\dagger}D^{\mu}h)(\bar{b}\gamma_{\mu}b) + \text{h.c.}.
\end{aligned}
$$

$\rightarrow$ **16 more operators, but the same method.**

Applications

- General procedure
  - Integrating out the heavy particles.
  - Obtain operator coefficients a_i as functions of the parameters in the model.
    $$a_i = a_i(m, g, \ldots)$$
  - Substitute the coefficients in the χ^2 distribution.
  - Calculate bounds, draw plots, ...

Little Higgs models

- One-loop quadratic divergence from top, gauge boson and Higgs loops canceled by particles of same spin.
- Cutoff pushed up to $\gtrsim 10$ TeV.
- Heavy fermions, gauge bosons, scalars $\longrightarrow$ to be integrated out.

The $SU(6)/SP(6)$ Higgs model *I. Low, W. Skiba, D. Smith*

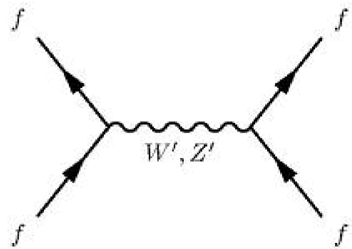
- Gauge group enlarged: $SU(2)_1 \times SU(2)_2 \times U(1)_1 \times U(1)_2 \longrightarrow SU(2)_W \times U(1)_Y$.
  - heavy gauge bosons W', Z' of TeV scale mass;
  - gauge coupling g_1, g_2, g_1', g_2': $g = \frac{g_1 g_2}{\sqrt{g_1^2+g_2^2}}$, $g' = \frac{g_1' g_2'}{\sqrt{g'^2_1+g'^2_2}}$;

$$\text{Define: } g = g_1 s = g_2 c, \quad g' = g_1' s' = g_2' c'.$$

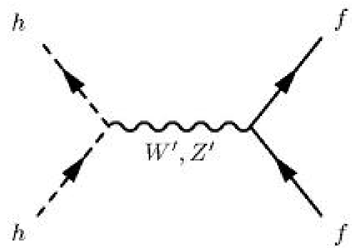
  - $Y = Y_1 + Y_2$.
  - $M_{W'} = \frac{gF}{2sc}, \quad M_{Z'} = \frac{g'F}{\sqrt{8}s'c'}$.

Integrating out heavy particles

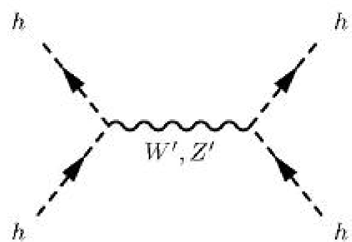
- Heavy gauge bosons introduce O_h, O_{hf}, O_{ff}.
- Choose the heavy fermions to mix with the top quark, but not the bottom quark–does not affect EWPT at tree level.



$\sim O_{ff}$



$\sim O_{hf}$



$\sim O_h$

The operators

$$
\begin{aligned}
a_h &= -\frac{1}{F^2}[(c'^2 - s'^2)^2 + \frac{1}{2}\cos^2(2\beta)], \\
a^t_{hq} &= a^t_{hl} = -\frac{1}{2F^2}(c^2 - s^2)c^2, \\
a^s_{hf} &= \frac{2s'c'(c'^2 - s'^2)}{F^2}\left(Y^f_2\frac{s'}{c'} - Y^f_1\frac{c'}{s'}\right), \\
a^t_{lq} &= a^t_{ll} = -\frac{c^4}{F^2}, \\
a^s_{ff'} &= \frac{-8s'^2c'^2}{F^2}\left(Y^f_2\frac{s'}{c'} - Y^f_1\frac{c'}{s'}\right)\left(Y^{f'}_2\frac{s'}{c'} - Y^{f'}_1\frac{c'}{s'}\right).
\end{aligned}
\quad (1)
$$

- To obtain bounds on physical mass:

$$
M_{W'} = \frac{gF}{2sc}; \quad M_{t'} \geq \sqrt{2}\lambda_t F, \text{ take } M_{t'} = \sqrt{2}F
$$

Bounds

- To suppress the corrections? $Y_1^f = Y_2^f$, $s' = c'$.

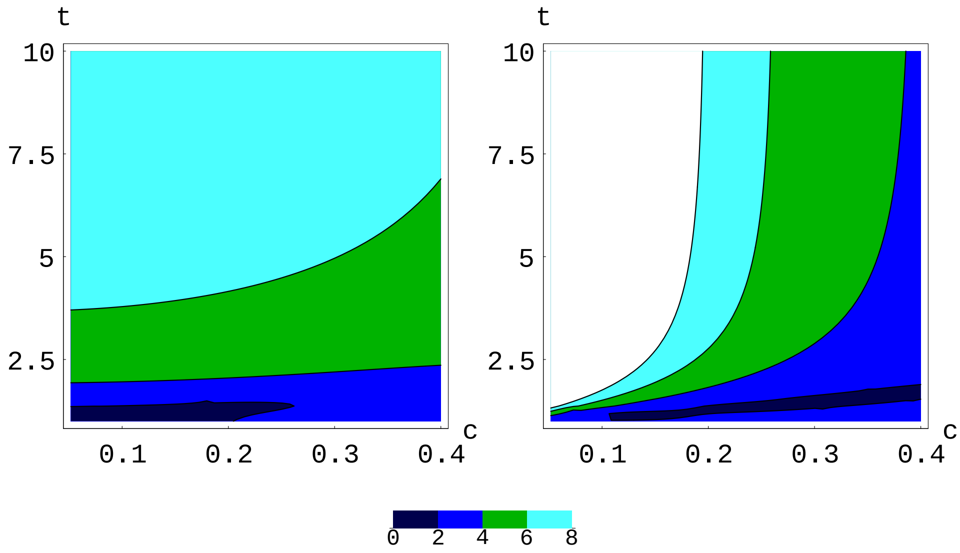


Figure 2: 95% CL lower bounds in TeV on $M_{t'}$ (left) and $M_{W'}$ (right) as functions of c and $t \equiv \tan\beta$ for $Y_1^f = Y_2^f$ and $s' = c'$.

$SU(2) \times SU(2) \times U(1)$ model *D. Morrissey, T. Tait and C. Wagner*

- $SU(2)_1 \times SU(2)_2 \times U(1)_Y \to SU(2)_L \times U(1)_Y$
  by $\langle \Sigma \rangle = \text{diag}\{u, u\}$.
- $Q : (2,1)_{1/6}, \quad L : (2,1)_{-1/2}, \quad q : (1,2)_{1/6}, \quad l : (1,2)_{-1/2}$.
- The "heavy" case: $h = (2,1)_{1/2}$.
  The "light" case: $h = (1,2)_{1/2}$.
- $g = g_1 g_2 / \sqrt{g_1^2 + g_2^2}$.

  $c = g_1/\sqrt{g_1^2 + g_2^2}, \quad s = g_2/\sqrt{g_1^2 + g_2^2}$.
- $M_{Z'}^2 = M_{W'^\pm}^2 = (g_1^2 + g_2^2)u^2$.

The operators

Light case:

$$a^t_{ll} = a^t_{lq} = a^t_{hl} = a^t_{hq} = -\frac{1}{4u^2} s^4,$$
$$a^t_{lL} = a^t_{lQ} = a^t_{hL} = a^t_{hQ} = \frac{1}{4u^2} s^2 c^2.$$

Heavy case:

$$a^t_{ll} = a^t_{lq} = -\frac{1}{4u^2} s^4,$$
$$a^t_{lL} = a^t_{lQ} = a^t_{hl} = a^t_{hq} = \frac{1}{4u^2} s^2 c^2,$$
$$a^t_{hL} = a^t_{hQ} = -\frac{1}{4u^2} c^4.$$

Compare. . .

Heavy case:

$$M_W = (M_W)_{SM}\left[1 - 0.219(1-c_\varphi^4)\delta\right]$$

$$\begin{aligned}
\Gamma_Z &= (\Gamma_Z)_{SM}\left[1 + \left(-1.348 + 0.790c_\varphi^4 + 1.684s_\varphi^2c_\varphi^2\right)\ \delta\right] \\
\Gamma_{had} &= (\Gamma_{had})_{SM}\left[1 + \left(-1.478 + 0.974c_\varphi^4 + 1.828s_\varphi^2c_\varphi^2\right)\ \delta\right] \\
\Gamma_{e,\mu} &= (\Gamma_{e,\mu})_{SM}\left[1 + \left(-1.175 + 1.175c_\varphi^4 + 2.122s_\varphi^2c_\varphi^2\right)\ \delta\right] \\
\Gamma_{inv} &= (\Gamma_{inv})_{SM}\left[1 + \left(-1.000 + 0.333c_\varphi^4 + 1.333s_\varphi^2c_\varphi^2\right)\ \delta\right]
\end{aligned}$$

$$\begin{aligned}
R_b &= (R_b)_{SM}\left[1 + \left(\ 0.059 - 1.846c_\varphi^4 - 1.828s_\varphi^2c_\varphi^2\right)\ \delta\right] \\
R_c &= (R_c)_{SM}\left[1 + \left(-0.114 + 0.618c_\varphi^4 + 0.583s_\varphi^2c_\varphi^2\right)\ \delta\right] \\
R_\tau &= (R_\tau)_{SM}\left[1 + \left(-0.302 + 1.921c_\varphi^4 + 1.828s_\varphi^2c_\varphi^2\right)\ \delta\right] \\
R_{e,\mu} &= (R_{e,\mu})_{SM}\left[1 + \left(-0.302 - 0.201c_\varphi^4 - 0.293s_\varphi^2c_\varphi^2\right)\ \delta\right]
\end{aligned}$$

$$\begin{aligned}
A_b &= (A_b)_{SM}\left[1 + \left(-0.232 + 0.071c_\varphi^4\right)\ \delta\right] \\
A_c &= (A_c)_{SM}\left[1 + \left(-1.786 + 1.786c_\varphi^4 + 1.242s_\varphi^2c_\varphi^2\right)\ \delta\right]
\end{aligned}$$

$$
\begin{aligned}
A_s &= (A_s)_{SM}\left[1+\left(-0.232+0.232c_\varphi^4+0.161s_\varphi^2c_\varphi^2\right)\,\delta\right]\\
A_\tau &= (A_\tau)_{SM}\left[1+\left(-20.391+6.215c_\varphi^4\right)\,\delta\right]\\
A_{e,\mu} &= (A_{e,\mu})_{SM}\left[1+\left(-20.391+20.391c_\varphi^4+14.17s_\varphi^2c_\varphi^2\right)\,\delta\right]
\end{aligned}
$$

$$
\begin{aligned}
A_{FB}^b &= (A_{FB}^b)_{SM}\left[1+\left(-20.621+20.462c_\varphi^4+14.17s_\varphi^2c_\varphi^2\right)\,\delta\right]\\
A_{FB}^c &= (A_{FB}^c)_{SM}\left[1+\left(-22.171+22.171c_\varphi^4+15.41s_\varphi^2c_\varphi^2\right)\,\delta\right]\\
A_{FB}^s &= (A_{FB}^s)_{SM}\left[1+\left(-20.621+20.621c_\varphi^4+14.333s_\varphi^2c_\varphi^2\right)\,\delta\right]\\
A_{FB}^\tau &= (A_{FB}^\tau)_{SM}\left[1+\left(-40.771+26.602c_\varphi^4+14.17s_\varphi^2c_\varphi^2\right)\,\delta\right]\\
A_{FB}^{e,\mu} &= (A_{FB}^{e,\mu})_{SM}\left[1+\left(-40.771+40.771c_\varphi^4+28.34s_\varphi^2c_\varphi^2\right)\,\delta\right]
\end{aligned}
$$

Light case:

$$
M_W = (M_W)_{SM}\left[1+0.219s_\varphi^4\delta\right]
$$

$$
\begin{aligned}
\Gamma_Z &= (\Gamma_Z)_{SM}\left[1+\left(-1.348+1.684s_\varphi^2c_\varphi^2-0.383s_\varphi^4\right)\,\delta\right]\\
\Gamma_{had} &= (\Gamma_{had})_{SM}\left[1+\left(0.504s_\varphi^2c_\varphi^2-0.351s_\varphi^4\right)\,\delta\right]\\
\Gamma_{e,\mu} &= (\Gamma_{e,\mu})_{SM}\left[1+\left(-0.947s_\varphi^4\right)\,\delta\right]\\
\Gamma_{inv} &= (\Gamma_{inv})_{SM}\left[1+\left(0.667s_\varphi^2c_\varphi^2-0.333s_\varphi^4\right)\,\delta\right]\\
R_b &= (R_b)_{SM}\left[1+\left(1.787s_\varphi^2c_\varphi^2+1.770s_\varphi^4\right)\,\delta\right]
\end{aligned}
$$

$$
\begin{aligned}
R_c &= (R_c)_{SM}\left[1+\left(-0.504 s_\varphi^2 c_\varphi^2 - 0.469 s_\varphi^4\right)\,\delta\right] \\
R_\tau &= (R_\tau)_{SM}\left[1+\left(-1.618 s_\varphi^2 c_\varphi^2 - 1.526 s_\varphi^4\right)\,\delta\right] \\
R_{e,\mu} &= (R_{e,\mu})_{SM}\left[1+\left(0.504 s_\varphi^2 c_\varphi^2 + 0.596 s_\varphi^4\right)\,\delta\right]
\end{aligned}
$$

$$
\begin{aligned}
A_b &= (A_b)_{SM}\left[1+\left(0.161 s_\varphi^2 c_\varphi^2 + 0.232 s_\varphi^4\right)\,\delta\right] \\
A_c &= (A_c)_{SM}\left[1+\left(0.545 s_\varphi^4\right)\,\delta\right] \\
A_s &= (A_s)_{SM}\left[1+\left(0.171 s_\varphi^4\right)\,\delta\right] \\
A_\tau &= (A_\tau)_{SM}\left[1+\left(14.171 s_\varphi^2 c_\varphi^2 + 20.386 s_\varphi^4\right)\,\delta\right] \\
A_{e,\mu} &= (A_{e,\mu})_{SM}\left[1+\left(6.215 s_\varphi^4\right)\,\delta\right]
\end{aligned}
$$

$$
\begin{aligned}
A_{FB}^b &= (A_{FB}^b)_{SM}\left[1+\left(0.161 s_\varphi^2 c_\varphi^2 + 6.450 s_\varphi^4\right)\,\delta\right] \\
A_{FB}^c &= (A_{FB}^c)_{SM}\left[1+\left(6.760 s_\varphi^4\right)\,\delta\right] \\
A_{FB}^s &= (A_{FB}^s)_{SM}\left[1+\left(6.286 s_\varphi^4\right)\,\delta\right] \\
A_{FB}^\tau &= (A_{FB}^\tau)_{SM}\left[1+\left(14.171 s_\varphi^2 c_\varphi^2 + 26.602 s_\varphi^4\right)\,\delta\right] \\
A_{FB}^{e,\mu} &= (A_{FB}^{e,\mu})_{SM}\left[1+\left(12.431 s_\varphi^4\right)\,\delta\right]
\end{aligned}
$$

Bounds

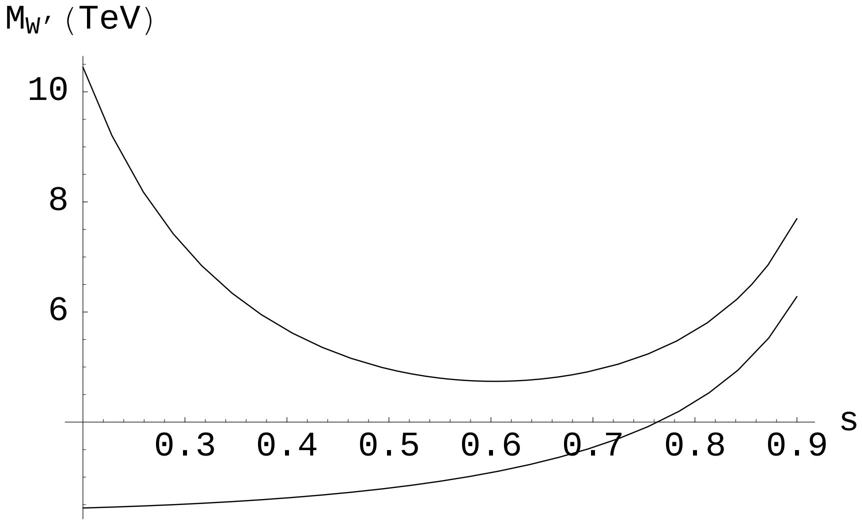


Figure 3: Lower bounds at 95% CL on $M_{W'}$ as a function of s in the $SU(2) \times SU(2) \times U(1)$ model. The upper curve corresponds to the heavy case and the lower curve corresponds to the light case.

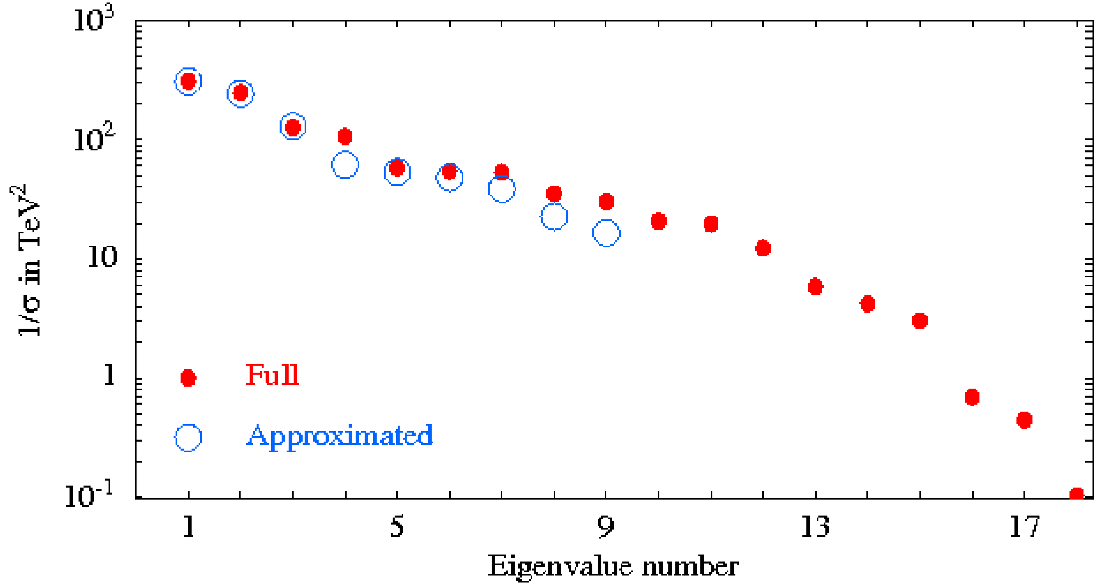
Less constrained directions *C. Grojean, W. Skiba, J. Terning*

$$-\frac{gO_{WB}}{2} + g'O_h + g'\sum_f Y^f O^s_{hf} = 2iB_{\mu\nu}D^\mu h^\dagger D^\nu h,$$

$$-g'O_{WB} + g(O^t_{hl} + O^t_{hq}) = 4iW_{\mu\nu}D^\mu h^\dagger \sigma^a D^\nu h,$$

- Triple-gauge couplings measured only from differential cross-section for W-pair production $\longrightarrow$ less constrained.

The "minimal" set *G. Gacciapaglia, C. Csaki, G. Marandella, A. Strumia*



- Some directions are more constrained than the others.
- Change the basis:

$$\hat{S}, \hat{T}, \hat{U}, V, X, W, Y, C_q, \delta\varepsilon_q, \delta\varepsilon_b$$

Conclusion

- **Electroweak precision tests can put constraints on TeV scale extensions of the SM.**
- **We have done a model-independent analysis on electroweak constraints, using the effective theory approach.**
- **Constraints on general TeV scale models can be easily obtained using our results.**