Old and new in the lattice definition of chiral gauge theories

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work in progress with Joel Giedt (Minneapolis)

on ideas from previous work with

Tanmoy Bhattacharya and Matthew Martin (Los Alamos)

hep-lat/0605003, PRD

why lattice?

QFT tools of the trade

A.) controlled expansions:

- perturbation theory
- semiclassical expansions
- 1/N

very useful divergent expansions of something:

the thing that is the **nonperturbative definition** of the theory

why lattice?

why do we need a nonperturbative definition
if various expansions work so well?

because they do not always work
e.g., QCD: ground state is highly nonperturbative

strong interactions sparked another class of QFT tools of the trade

B.) "voodoo" QCD:

models and uncontrolled "approximations:" e.g., "AdS/QCD," NJL-QCD, chiral quarks, bags, Skyrme, instanton liquid, ...

the skeptic:

*sometimes work, sometimes not;
what do we learn?*

the enthusiast:

*it's physics: experimental data is well
described!*

what nonperturbative definitions do we know of?

constructive field theory

- generally quite abstract, addresses existential questions
- one of its most useful results is the
  Osterwalder-Schrader "reconstruction theorem:" (mid 1970's)
  Euclidean Wightman functions with right properties - notably "OS positivity" - allow reconstruction of positive norm Hilbert space

string theory

- needs its own nonperturbative definition
- can be useful if enough symmetries around
- fairly helpless in non-supersymmetric situations

lattice field theory

proven very powerful in vectorlike gauge theories:

- all rigorous nonperturbative results in QFT use lattice at some point phase structure: analyticity near boundary between Higgs/confinement; confinement at strong coupling... 1970s

- actual predictions for B physics very recent!

the only one well-suited for generic QFTs

"minimal models" of 2d CFT - certainly not generic QFTs

why lattice?

by a "nonperturbative definition," we mean something like "LHS=RHS", or

arbitrary*
Green's functions

=

an object that

a.) exists

b.) can, in principle, be calculated

$$\left| \begin{array}{l} \text{couplings} \\ \text{volume} \\ \text{UV cutoff} \end{array} \right|$$

*not only a class, such as, e.g. chiral...

what nonperturbative definitions do we know of?

lattice field theory sounds great, then... **but:**

lattice breaks global symmetries!

- Poincare

- chiral (*if naive*)

- supersymmetry

furthermore:

- is at its best when Euclidean

- does not include dynamical gravity

why formulate non-QCD like theories on the lattice?

apart as a purely theoretical excersise

- standard model is a chiral gauge theory

  weakly coupled, so no really strong incentive to bother

- extensions of the standard model?

if weakly coupled, also no strong reason

if strong dynamics is shown to be relevant, the issue of non-QCD like theories on the lattice will become more prominent strong dynamics can be relevant in many ways:

supersymmetric extensions: dynamical supersymmetry breaking

some progress in lattice supersymmetry in latter years, limited to extended SUSY theories [vectorlike by nature]

see, e.g., recent review by **Joel Giedt** hep-lat/0602007

strong electroweak breaking: renaissance as AdS/EWSB (a.k.a. RS)

- weak coupling duals of large-N vectorlike theories [no useful notion of large-N in chiral gauge theories] fundamental dual 4d description strong

- other not-yet-imagined not-large-N not-QCD-like dynamics???

strong chiral gauge dynamics remains largely mysterious

- in non-SUSY case only tools are ‘t Hooft anomaly matching and MAC

- analytic methods, like large-N expansions, incl. recent “AdS/QCD dualities” do not (usefully) apply to chiral gauge theories

e.g., SU(5) with [10] and [5*]; SU(6) with [15] and 2x[6*] ... SU(n) with [n(n-1)/2] and (n-4)x[n*]

- further progress in understanding interesting supersymmetric theories on the lattice is tied to the chiral gauge theories problem

however:

numerical or analytic methods using the lattice face the difficulty of preserving chiral symmetries on the lattice

“Nielsen-Ninomiya theorem”

quickest argument:
if exact, gauge it, but where would anomalies come from?

but, in past 10 years striking progress in understanding global chiral symmetry: use it!

plan:

1 domain wall and waveguide models and their failure to obtain chiral spectrum — *"old": 1992-4*

2 "anatomy of a failure:"

i.) unitary higgs fields and symmetric phase — *"old": 1979*

ii.) strong Yukawa symmetric phases on the lattice — *"old": 1989*

3 a ***proposal*** using Ginsparg-Wilson mechanism to impose a modified exact lattice chiral symmetry — *"old": 1982 & "new": 1997*

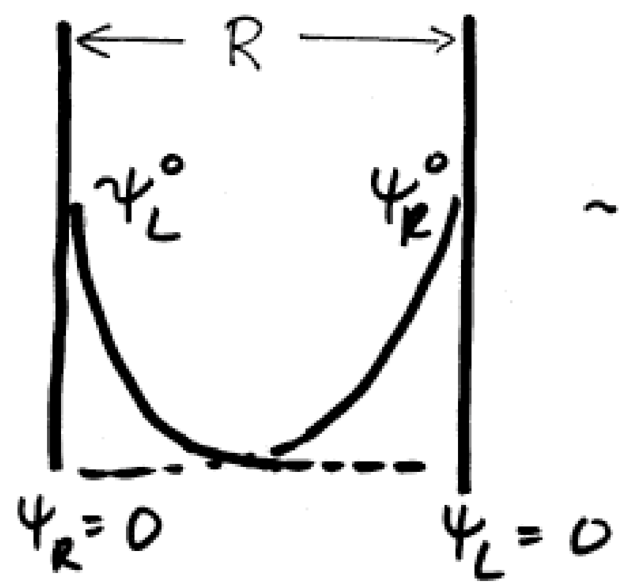
Bhattacharya, Martin, EP
hep-lat/0605003

4 recent analytical and numerical results supporting it — current work with Joel Giedt

5 outlook and remaining issues

domain wall and "waveguide" models & their failure to obtain chiral spectrum

- 5d "bulk"
- massive 5d fermion (ψ_L, ψ_R)
- Dirichlet b.c.



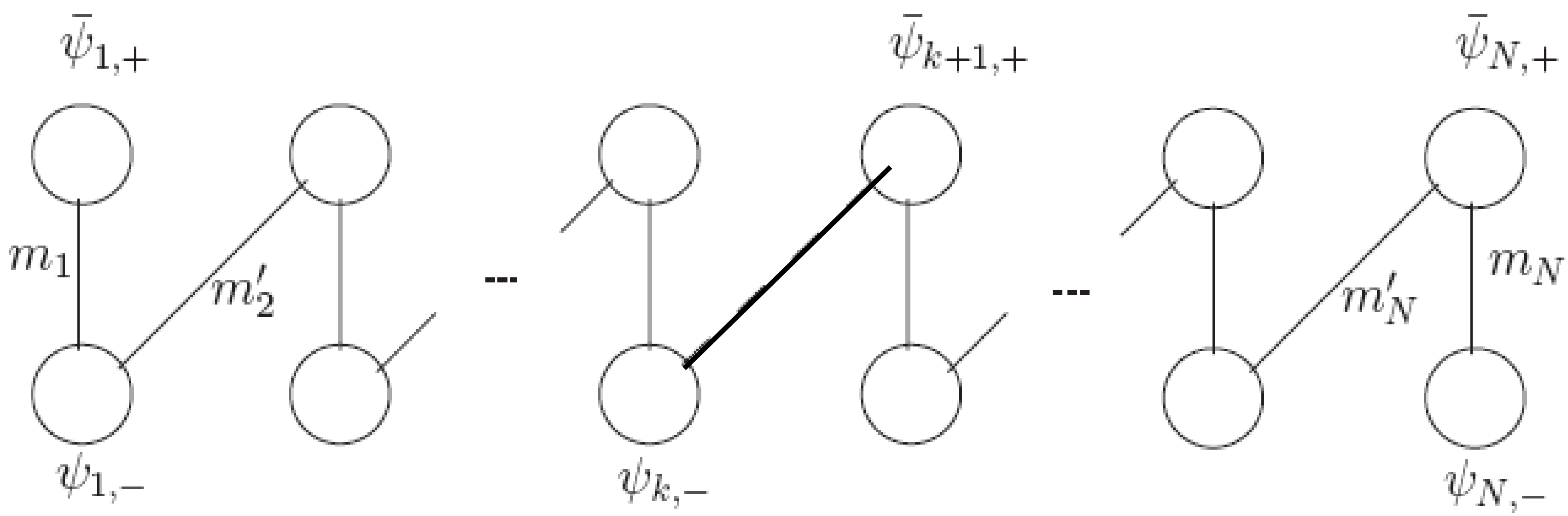
lightest states

$\sim m e^{-mR}\, \bar{\psi}_L^0 \psi_R^0$

- mass $\to 0$ as $R \to \infty$
- chiral symmetry

lattice domain wall fermions D.B. Kaplan '92

Shamir's implementation '93:

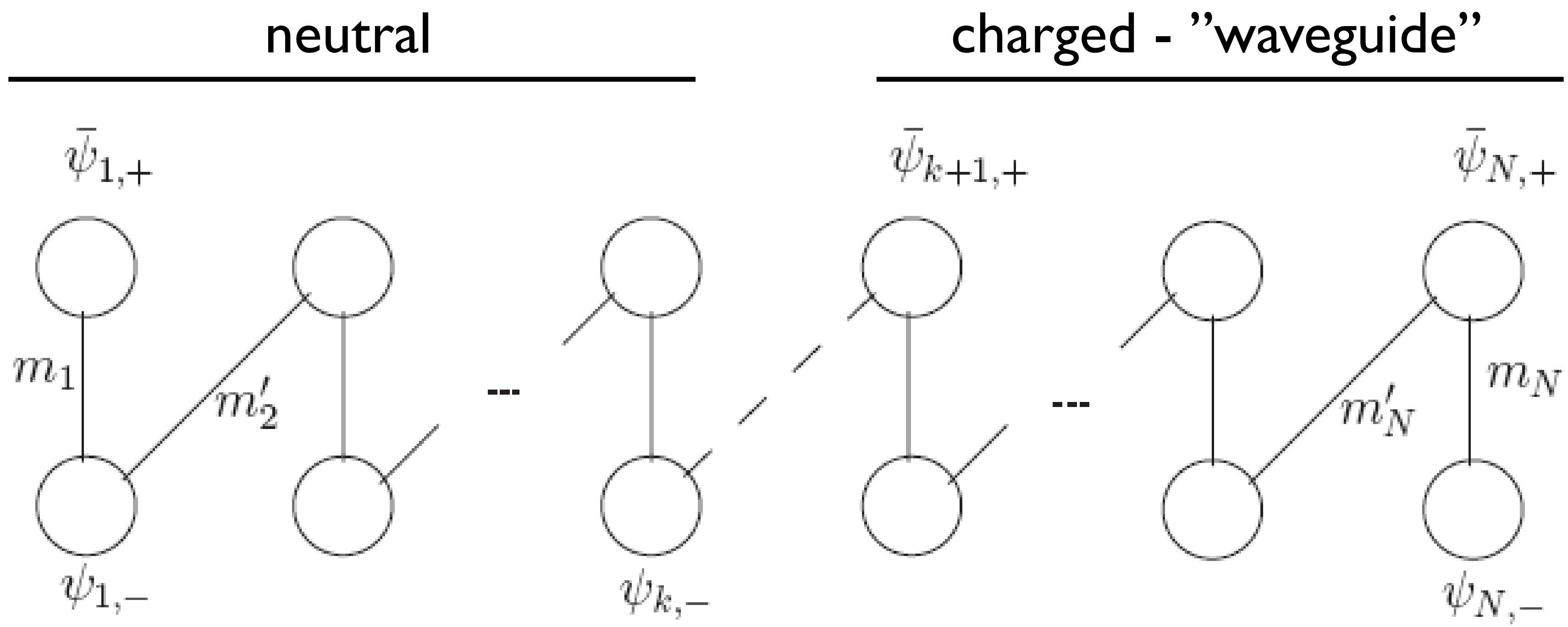


vectorlike gauge theory with exponentially light Dirac fermion;
becomes massless at infinite N, where chiral symmetry restored

waveguide domain wall fermions

D.B. Kaplan '92

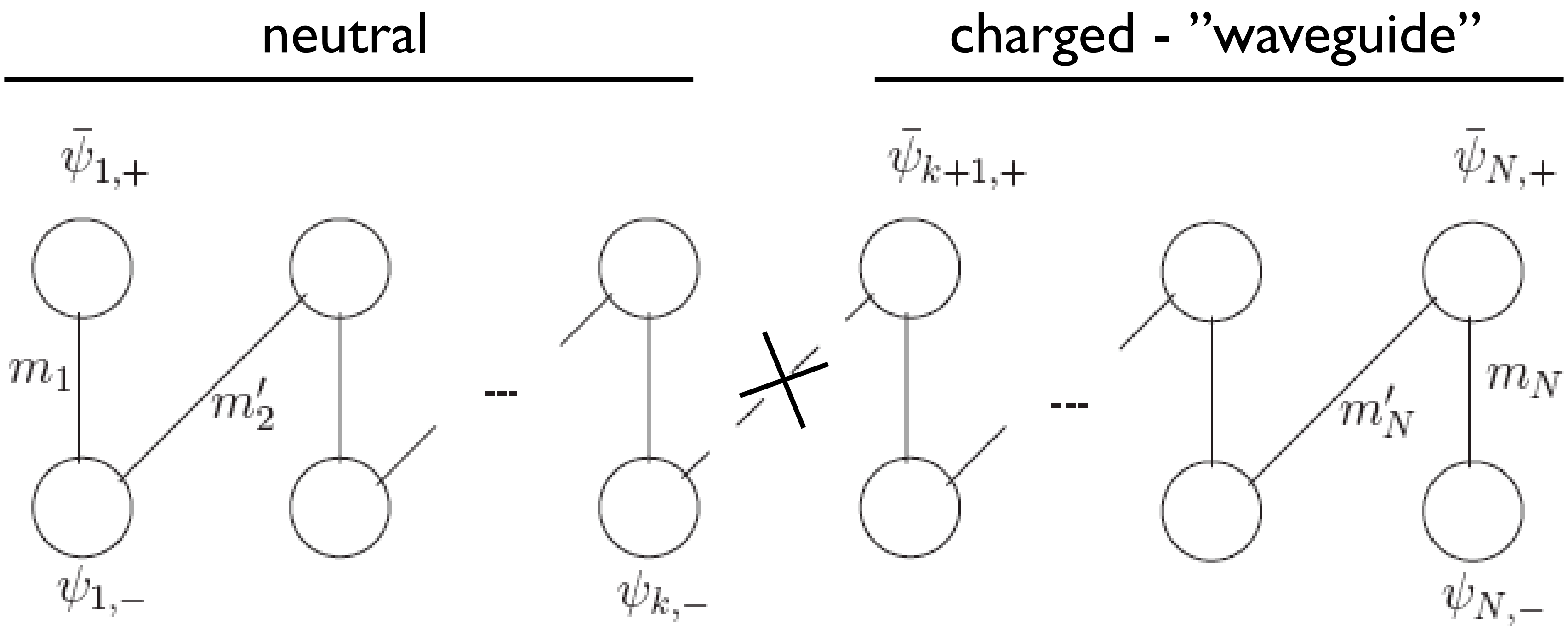
want: **A.) unbroken gauge theory**
B.) chiral light spectrum



$$y\bar\psi_{k+1,+}\phi\psi_{k,-}$$

waveguide at small Yukawa coupling

Golterman, Jansen, Petcher, Vink '93



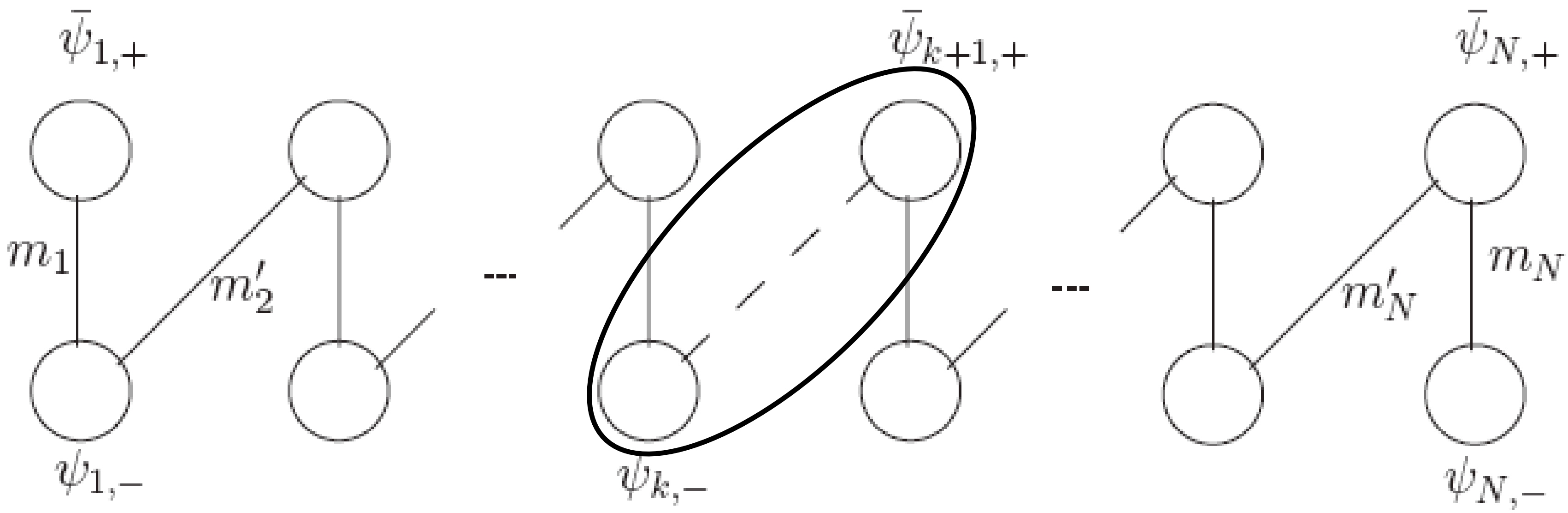
$$y\bar\psi_{k+1,+}\phi\psi_{k,-}$$

result: vectorlike fermion spectrum in the symmetric phase

waveguide at large Yukawa coupling

Golterman, Shamir '94

neutral — charged - "waveguide"



$$y\bar\psi_{k+1,+}\phi\psi_{k,-}$$ at large y $$\psi_{k+1,+} \to \frac{1}{\sqrt{y}}\psi_{k+1,+}$$

$$\bar\psi_{k+1,-}\gamma\cdot D\psi_{k+1,-} + ra\bar\psi_{k+1,+}D^2\psi_{k+1,-} \to \bar\psi_{k+1,-}\gamma\cdot D\psi_{k+1,-} + \cancel{\frac{ra}{\sqrt{y}}}\bar\psi_{k+1,+}D^2\psi_{k+1,-}$$

charged massless doublers appear due to lost Wilson term and result in: vectorlike fermion spectrum in the symmetric phase, again so far: waveguide doesn’t work at both weak and strong Yukawa coupling

“mirror” fermion and gauge boson mass both determined by Higgs vev; in the symmetric phase “mirror” becomes massless -

weak Yukawa proposal of:

Bhattacharya, Csaki, Martin, Shirman, Terning ‘05
... 4d, strong coupling issues

the use of warped domain walls

Bhattacharya, Martin, EP, ‘06
...2d study, better chance, perhaps...
(no full lattice study yet: deconstruction only
...too difficult to handle, it seems...?)

this talk

strong Yukawa-GW proposal

Bhattacharya, Martin, EP, ‘06
Giedt, EP, ‘06

first, need to understand ***what was the hope*** of the strong-Yukawa waveguide idea?

2 "anatomy of a failure:" *i.)* unitary Higgs fields and symmetric phase

Fradkin, Shenker '79 *Phase diagrams of lattice field theories with Higgs fields*

(refer to old work, '71, of F. Wegner on Z_2 gauge theory)

Foerster, Nielsen, Ninomiya '80 *Dynamical stability of local gauge symmetry:* ***creation of light from chaos***

thus, often referred to as "FNN mechanism"...

unitary Higgs field on the lattice

$$\frac{\kappa}{2}\sum_x \sum_{\hat{\mu}} [2 - (\phi(x)^* U(x,\hat{\mu})\phi(x+\hat{\mu}) + \mathrm{h.c.})]$$

symmetric phase at $\kappa \leq \kappa_c$ - use strong coupling (high-T) expansion:

find: disorder, small correlation length, integrate out Higgs, irrelevant at large scales

leading effect at small κ : $\frac{1}{g^2} \rightarrow \frac{1}{g^2} + \kappa^4$

moral: **on the lattice can have a unitary Higgs and still be in symmetric phase**

same for any compact gauge group: from Z_2 to U(N)...

2 "anatomy of a failure:" *ii.)* strong Yukawa symmetric phases on the lattice

1989
A. Hasenfratz, Neuhaus,
Stephanov, Tsypin, Aoki, Shigemitsu, Schrock ...

fermion-Higgs system on the lattice at strong Yukawa coupling has a phase with massive fermions and unbroken chiral symmetry -

pre-cursor:
1986 Eichten, Preskill
strong lattice four-Fermi interactions also exhibit a symmetric phase with massive fermions

not of interest for electroweak physics,
since it is a "lattice artifact"
- fermions are heavier than 1/a, but:

can one use it to decouple mirrors?

NO!
Golterman, Petcher, Shamir,Smit, Bock, De...
1991-94

reasons in each case similar to strong-Yukawa limit of waveguide shown before:
L-R mixing via strong Yukawa/Wilson

2 "anatomy of a failure:" *ii.)* strong Yukawa symmetric phases on the lattice

$$\sum_{x,\hat\mu} \varkappa\, \phi_x^+ U_{x,x+\hat\mu}\, \phi_{x+\hat\mu} + h.c.$$

naive fermions,
any dimension,
any compact
gauge group

$$+ \bar\psi_{L\,x}\, \gamma^\mu \left(U_{x,x+\hat\mu}\, \psi_{L\,x+\hat\mu} - U_{x,x-\hat\mu}\, \psi_{L\,x-\hat\mu} \right)$$

$$+ \bar\psi_{R\,x}\, \gamma^\mu \left(\psi_{R\,x+\hat\mu} - \psi_{R\,x-\hat\mu} \right)$$

$$+ y \left(\bar\psi_{L\,x}\, \phi_x\, \psi_{R\,x} + h.c. \right)$$

specialize (for simplicity of presentation) to

Z_2 case:

$$\left.\begin{array}{l} \psi_L \to -\psi_L \\ \psi_R \to \psi_R \\ \phi \to -\phi \end{array}\right\} \text{gauged chiral symmetry}$$

important for later:
at strong Yukawa, fermion loops
don't renormalize scalar action

symmetric phase $\quad \varkappa \longrightarrow 0$

strong Yukawa $\quad y \longrightarrow \infty$

small gauge coupling

$$\langle \phi_x\, \phi_{x'} \rangle = \delta_{x\,x'}$$

$$\langle \psi^\alpha_{L\,x}\, \bar\psi^\beta_{R\,y} \rangle_\psi = \frac{\delta_{xy}\,\delta^{\alpha\beta}}{y}\, \phi_x$$

2 "anatomy of a failure:" *ii.)* strong Yukawa symmetric phases on the lattice

to all orders in the [convergent] strong-coupling expansion $\kappa \to 0$, $y \to \infty$ and for small gauge coupling:

I) all symmetry-breaking Green functions vanish
(can, similarly, look at susceptibilities)

e.g., $\langle \Psi_{L\,x}\, \bar{\Psi}_{R\,y} \rangle = 0$

II) all matter fields are massive (m >1/a), with spectrum determined from large-t:

$\langle (\phi_0 \Psi_{L\,0})\; \bar{\Psi}_{R\,t} \rangle \neq 0$ massive neutral fermion $m_{\Psi n} \sim \frac{y}{\sqrt{\kappa}}$

$\langle \Psi_{L\,0}\; (\phi_t \bar{\Psi}_{R\,t}) \rangle \neq 0$ massive charged fermion $m_{\Psi c} \sim \frac{y}{\sqrt{\kappa}}$

degeneracy lifted at O(g^2)

$\langle (\Psi_{L\,0} \bar{\Psi}_{R\,0})\; (\bar{\Psi}_{L\,t} \Psi_{R\,t}) \rangle \neq 0$ massive charged scalar (quantum numbers of Higgs) $m_{\varphi c} \sim \frac{1}{\sqrt{\kappa}} \sim m_\phi$

$\langle (\Psi_{L\,0} \phi_0 \bar{\Psi}_{R\,0})\; (\bar{\Psi}_{L\,t} \phi_t \Psi_{R\,t}) \rangle \neq 0$ neutral massive scalar $m_{\varphi n} \sim \frac{y}{\sqrt{\kappa}}$

etc.....

III) infrared theory is that of gauge fields only

more details in, e.g. Golterman, Petcher '92, also Appendix of Eichten,Preskill '86

2 ”anatomy of a failure:” *ii.)* strong Yukawa symmetric phases on the lattice

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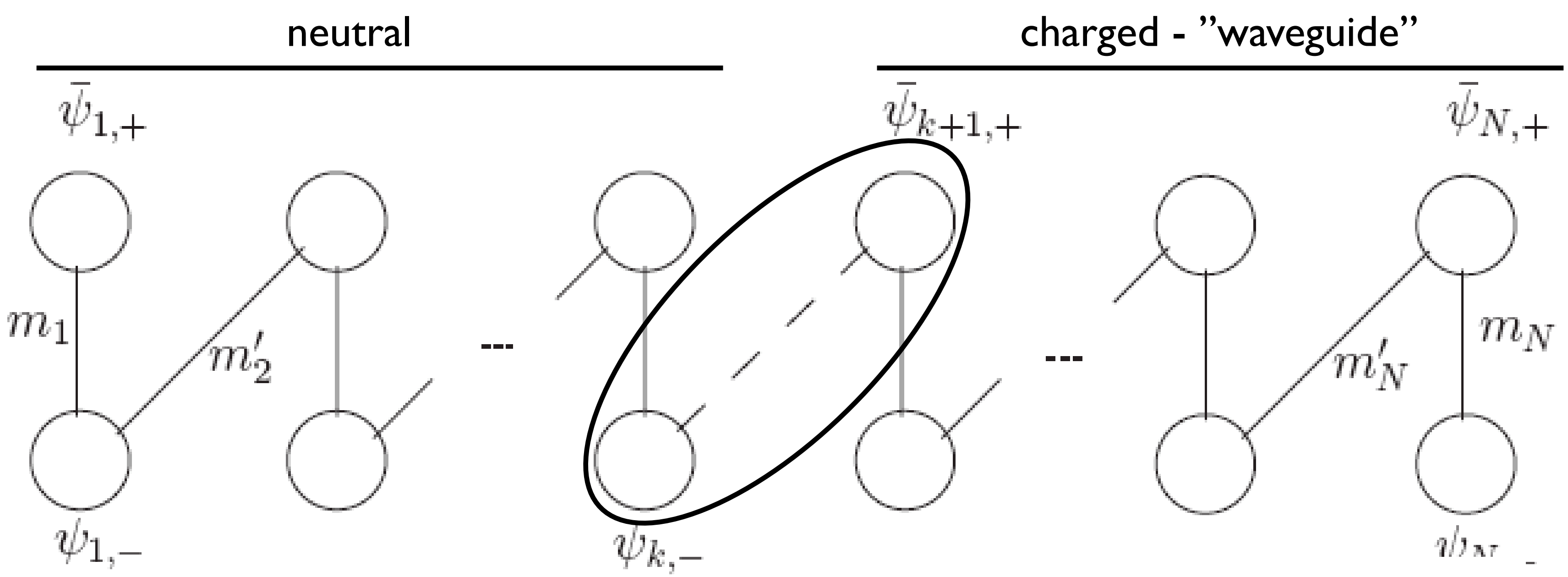
“strong coupling waveguide” ***hope:***

- ***charged mirrors and doublers would bind with scalars, pair with neutral fermions, and become massive***
- ***light charged femions stay massless***
- ***all while theory is in the symmetric phase***



$y\bar{\psi}_{k+1,+}\phi\psi_{k,-}$ at large y $\psi_{k+1,+} \to \frac{1}{\sqrt{y}}\psi_{k+1,+}$

$$\bar{\psi}_{k+1,-}\gamma\cdot D\psi_{k+1,-} + ra\bar{\psi}_{k+1,+}D^2\psi_{k+1,-} \to \bar{\psi}_{k+1,-}\gamma\cdot D\psi_{k+1,-} + \cancel{\frac{ra}{\sqrt{y}}}\bar{\psi}_{k+1,+}D^2\psi_{k+1,-}$$

Golterman, Shamir ‘94:

L-R mixing in Wilson term was ultimate cause for the appearance of massless mirrors

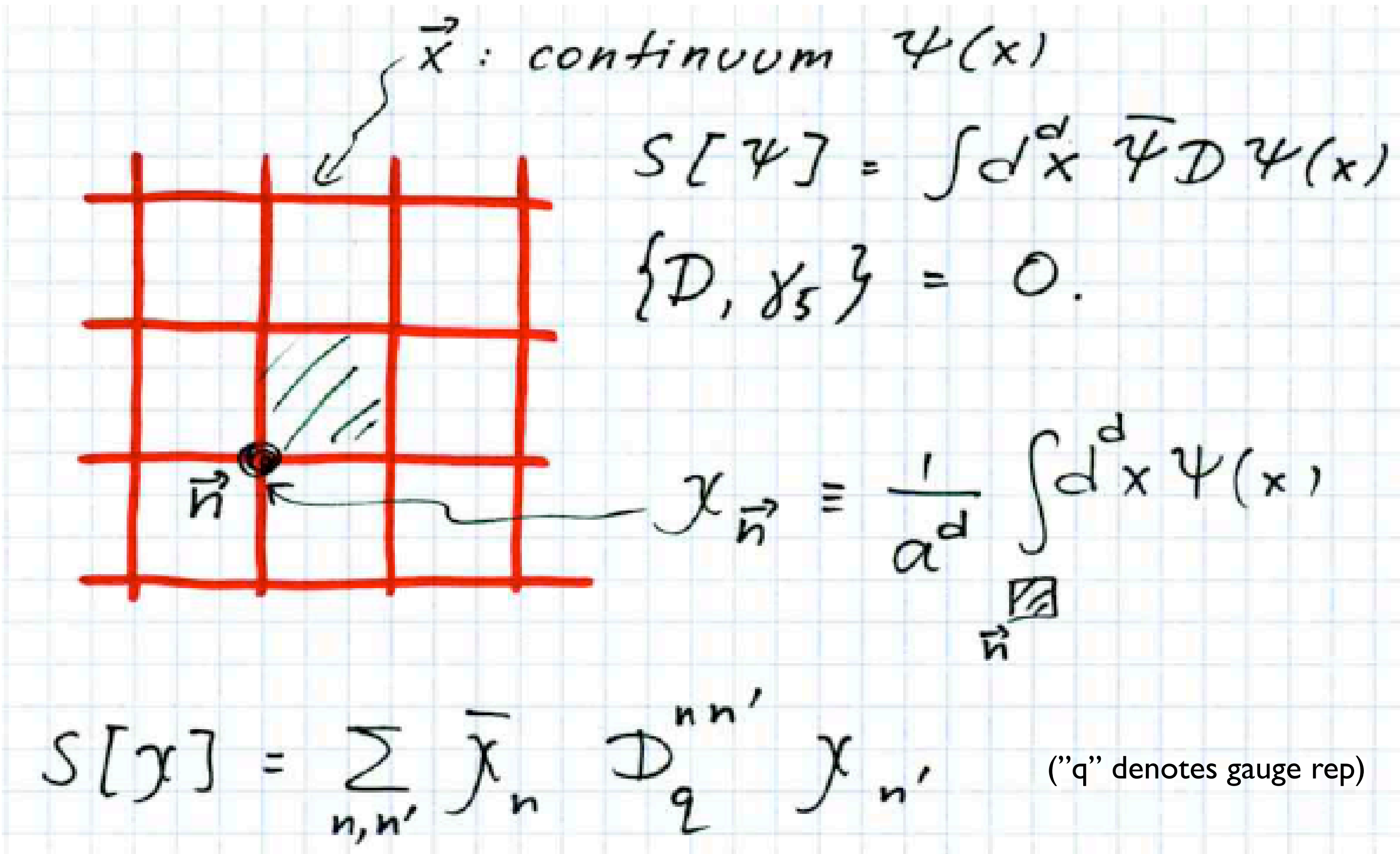
chiral spectrum fails already at g=0 for all similar attempts

Can light-mirror mixing be avoided?

3 a proposal using Ginsparg-Wilson mechanism to impose a modified exact lattice chiral symmetry

what if we use fermions where +/- mixing does not happen?

Ginsparg-Wilson fermions obey $\bar{\psi} D^{GW} \psi = \bar{\psi}_+ D^{GW} \psi_+ + \bar{\psi}_- D^{GW} \psi_-$ while having no doublers



("q" denotes gauge rep)

Ginsparg and Wilson showed (in 1982!) that $\{D_q, \gamma_5\} = D_q \gamma_5 D_q$

...OK, but what is D? - answer: Neuberger, '97;

3 a proposal using Ginsparg-Wilson mechanism to impose a modified exact lattice chiral symmetry

I = II = III:

I $\{D_q, \gamma_5\} = D_q \gamma_5 D_q$

II $\hat{\gamma}_5^2 = 1 \qquad \hat{\gamma}_5 \equiv (1 - D)\gamma_5$

III $\hat{\gamma}_5 D_q = -D_q \gamma_5$

then, there is an exact chiral symmetry (GW, 1982; formulation of Luscher, 1999)

$$\Psi \rightarrow e^{i\alpha\gamma_5}\Psi \qquad \bar{\Psi} \rightarrow \bar{\Psi} e^{i\alpha\hat{\gamma}_5}$$

of lattice action $\qquad S_{kin} = \sum_{x,y} \bar{\Psi}_x D_{qxy} \Psi_y$

note that, really, we have $\qquad \bar{\Psi}_x \rightarrow \sum_{x'} \bar{\Psi}_{x'} \left(e^{i\alpha\hat{\gamma}_5}\right)_{x'x}$

3 a proposal using Ginsparg-Wilson mechanism to impose a modified exact lattice chiral symmetry

$$\hat{P}_\pm = (1 \pm \hat{\gamma}_5)/2$$ is a projector, because of II: $$\hat{\gamma}_5^2 = 1 \qquad \hat{\gamma}_5 \equiv (1-D)\gamma_5$$

lattice action $$S_{kin} = \sum_{x,y} \bar{\Psi}_x D_{qxy} \Psi_y$$

has exact global L and R symmetries $$U(1)_{q,-} \times U(1)_{q,+} \qquad \begin{array}{rcl} \Psi_q & \rightarrow & e^{i\alpha_{q,\pm} P_\pm} \Psi_q \\ \bar{\Psi}_q & \rightarrow & \bar{\Psi}_q e^{-i\alpha_{q,\pm} \hat{P}_\mp} \end{array}$$

field dependence of transformation leads to Jacobian:
(vanishes for vector U(1)!):

$$\left[1 \pm i\alpha_{q,\pm} \mathrm{Tr}\left(\gamma_5 - \frac{1}{2} D_q \gamma_5\right)\right]$$

then properties of D are useful to to show that:

$$\mathrm{Tr}(\gamma_5 - \tfrac{1}{2} D_q \gamma_5) = n_+^0 - n_-^0$$

moral:

in lattice vectorlike theories
-exact lattice chiral symmetry (not the usual one for all modes!),
-exact lattice (anomalous) chiral Ward identities,
-axial charge violation and 't Hooft vertices

big theoretical success!!!

- tested extensively in Schwinger model (2d), works beautifully
- still more expensive to run in 4d QCD because of non-sparseness

but, our desire is not to study QCD; we want to:

start from vectorlike theory
decouple mirrors
get unbroken chiral theory

- can we do that?

finally, can explain our proposal (any dimension, but so far tests in 2d)

2-dim chiral theory: U(1) "345" theory $3_-, 4_-, 5_+$ chiral matter

133 global U(1) anomaly free
111 global U(1) anomalous, 't Hooft vertex $(3_-)^3\, \partial_+ (4_-)^4\, (\bar{5}_+)^5$

"345" theory fields: 3- 4- 5+ 0+ and mirrors: 3+ 4+ 5- 0-

spectator 0+/0- only needed in 2d for Lorentz inv

$$S_{kin} = \sum_{q=0,3,4,5} \sum_{x,y} \bar{\Psi}_q(x) D_q(x,y) \Psi_q(y)$$

8 global chiral U(1)s are symmetries of S_kin: $\prod_{q=0,3,4,5} U(1)_{q,-} \times U(1)_{q,+}$

while target 3-4-5 theory has only
4 exact classical ones $U(1)_{3,-} \times U(1)_{4,-} \times U(1)_{5,+} \times U(1)_{0,+}$

3 a proposal using Ginsparg-Wilson mechanism to impose a modified exact lattice chiral symmetry

introduce chiral components: $\Psi_\pm = P_\pm \Psi$ $\bar\Psi_\pm = \bar\Psi \hat{P}_\mp$

include Yukawa couplings involving mirrors that violate all unwanted U(1)s

e.g.: $\bar\Psi_{0,-}(\phi^*)^3\Psi_{3,+}$ (Dirac) and $\Psi^T_{3,+}\gamma_2(\phi^*)^8\Psi_{5,-}$ (Majorana)

ideologically not dissimilar from Eichten-Preskill '86

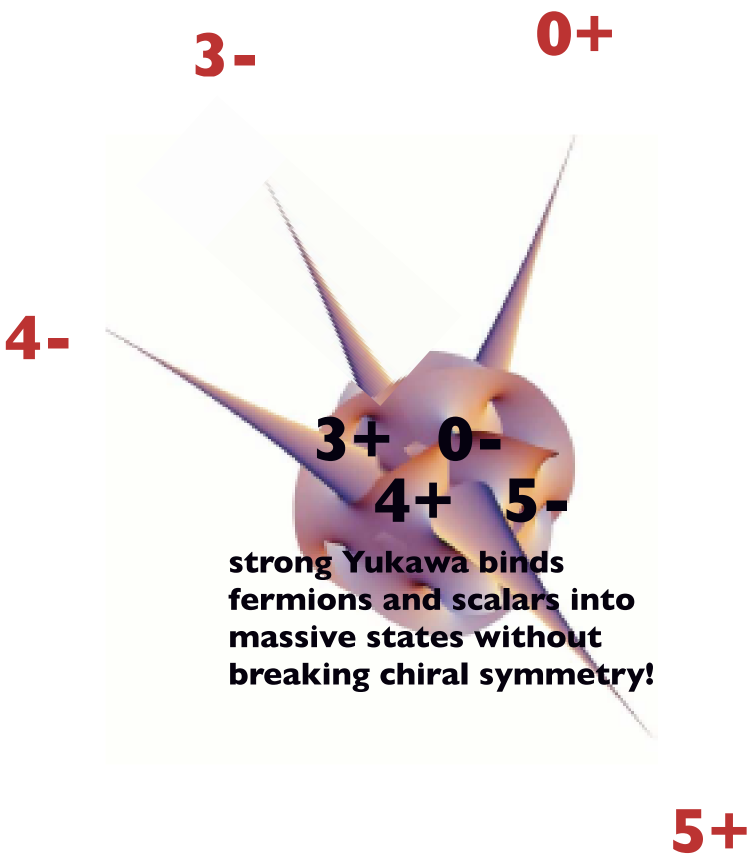
345, 133: anomaly free exact lattice chiral symmetries

lattice anomalous Ward identities for fermion number 111 symmetry

$$\langle \delta_{\alpha_{111}} \mathcal{O} \rangle = i\,\frac{\alpha}{2}\,\langle \mathcal{O}\, \mathrm{Tr}\left[\gamma_5(D_3 + D_4 - D_5)\right]\rangle$$, as in target theory

$$\mathrm{Tr}\gamma_5 D \sim \int d^2x\, \epsilon^{\mu\nu} F_{\mu\nu}$$

this completes the definition of the proposal



what have we got, so far?

- a full *lattice* proposal of action ***and*** measure
- formulated in both 2d and 4d
  [2d simulations on the "fringe" possible]
- global symmetries, *incl. anomalous ones*, are realized exactly as in continuum theory

but does it behave as we want it to?

symmetries vs. dynamics - most important issue:

- unbroken chiral (gauge) symmetry!

heavy mirrors

massless chiral matter

we don't have a proof
... but evidence ...

3 a proposal using Ginsparg-Wilson mechanism to impose a modified exact lattice chiral symmetry

$$S = S_{Wilson} + S_{kin} + S_{mass} + \frac{\kappa}{2} \sum_x \sum_{\hat{\mu}} [2 - (\phi(x)^* U(x, \hat{\mu}) \phi(x + \hat{\mu}) + \text{h.c.})]$$

since for GW fermions $\bar{\Psi}_q D_q \Psi_q = \bar{\Psi}_{q,+} D_q \Psi_{q,+} + \bar{\Psi}_{q,-} D_q \Psi_{q,-}$

no coupling of mirror and light states via the strong Yukawas

→ cause of trouble (i.e., vectorlike spectrum of light states in the symmetric phase!) for all previous Higgs/Yukawa attempts, e.g. waveguide...

except: Hernandez/Sundrum two-cutoff proposal, '96, which has its own complications

as a result, since also

$d\Psi = d\Psi_+ d\Psi_-$, when g=0 partition function factorizes:

$$Z = Z_{light} \times Z_{mirror}$$

3 a proposal using Ginsparg-Wilson mechanism to impose a modified exact lattice chiral symmetry

$$Z_{light} = \int \prod_x d\Psi_{3,-} d\Psi_{4,-} d\Psi_{5,+} d\Psi_{0,+} e^{-S_{kin}(\Psi^{light})}$$

$$Z_{mirror} = \int \prod_x d\Psi_{3,+} d\Psi_{4,+} d\Psi_{5,-} d\Psi_{0,-} d\phi$$
$$\times \quad e^{-S_{kin}^{mirror}(\Psi^{mirror}) - S_{\kappa}(\phi) - S_{mass}(\Psi^{mirror})}$$

problem at hand:

study Z_{mirror} dynamics (enough, for g = 0)
is there a phase, where as

$$\kappa \to 0 \qquad \lambda \to \infty$$

scalar has small, O(a), correlation length [=symmetric phase, no gauge boson mass]
and mirrors are heavy?

3 a proposal using Ginsparg-Wilson mechanism to impose a modified exact lattice chiral symmetry

quick argument: $$S_{mass} = \lambda \sum_x X_+(x) M Y_-(x)$$

$$Y_- = \begin{pmatrix} \Psi_{5,-} \\ \bar{\Psi}^T_{5,-} \\ \Psi_{0,-} \\ \bar{\Psi}^T_{0,-} \end{pmatrix}$$

$$X_+ = (\Psi^T_{3,+}\ \bar{\Psi}_{3,+}\ \Psi^T_{4,+}\ \bar{\Psi}_{4,+})$$

M(x) contains powers of unitary Higgs field to make Yukawa gauge invariant

- at infinite Yukawa, drop kinetic term
- mirror determinant = product of dets at each x (as in toy model)
- it is "gauge" symmetric and local (x), hence Higgs independent (since no local gauge invariant out of unitary Higgs)

→ hence, fermions are:

a.) heavy and

b.) do not effect unitary Higgs dynamics - do not drive theory into ordered ("low-T") phase:

if they did, this would mean that they generated a large kinetic term for Higgs [= gauge boson mass term, once g is turned on] requiring fine-tuning of, possibly infinitely many, operators to obtain massless gauge bosons

=> drop the proposal!

too quick!

important quick claim was:
- mirror determinant = product of dets at each x

$$S_{mass} = \lambda \sum_x X_+(x) M Y_-(x)$$ elegant, but misleading... recall:

$$\bar{\Psi}_\pm = \bar{\Psi}\hat{P}_\mp$$ is actually $$\bar{\Psi}_{\pm,x} = \sum_{x'} \bar{\Psi}_{x'} \left(\hat{P}_\mp\right)_{x'x}$$

- somewhat smeared as $$D_{x,x'} \sim e^{-\left|\frac{x-x'}{a}\right|}$$

details:
Neuberger;
Hernandez, Jansen, Luscher '98/9

and mirror determinant, even at infinite Yukawa, depends on Higgs

does it order Higgs fluctuations?

[induce large "gauge breaking" terms]

no workable *analytic only* expansion; combine strong coupling expansion with numerical "experiment:"

current work with Joel Giedt on g=0 Higgs-GW-fermion-Yukawa model

(I) *analytic:*

proper definition of measure (nontrivial because of smearing!)

$$\text{“}d\Psi = d\Psi_+ d\Psi_-\text{”}$$

and corresponding split of light and mirror action

(II) *numerical:*

use the result of (I) in simulation with backreaction of mirror fermions and study the scalar correlation length

4 recent analytic and numerical work supporting the proposal

"toy model" used in numerical study: [upon gauging = chiral Schwinger model; Jackiw/Rajaraman 1984]

$$
\begin{aligned}
S &= S_{light} + S_{mirror} \\
S_{light} &= (\bar{\psi}_+, D_1 \psi_+) + (\bar{\chi}_-, D_0 \chi_-) \\
S_{mirror} &= (\bar{\psi}_-, D_1 \psi_-) + (\bar{\chi}_+, D_0 \chi_+) \\
&+ y \left\{ (\bar{\psi}_-, \phi^* \chi_+) + (\bar{\chi}_+, \phi \psi_-) + h \left[(\psi_-^T, \phi \gamma_2 \chi_+) - (\bar{\chi}_+, \gamma_2 \phi^* \bar{\psi}_-^T) \right] \right\}
\end{aligned}
$$

for, on 16x16 lattice mirror fermion matrix in toy model is 512x512 at infinite Yukawa and 1024x1024 at finite y - already "toy model" presents a challenge for the "fringe"...

the light-mirror split of the measure is made explicit using a basis of eigenvalues of the GW operator --- skip details... mirror measure is:

$$
\prod_x d\bar{\psi}_{+\ x} d\psi_{+\ x} d\bar{\chi}_{-\ x} d\chi_{-\ x} \equiv \prod_{k_1, k_2 = 1}^{N} 16(1 - \lambda^*_{\mathbf{k}}) d\alpha_{\mathbf{k}+} d\bar{\alpha}_{\mathbf{k}+} d\beta_{\mathbf{k}-} d\bar{\beta}_{\mathbf{k}-}
$$

$$S^{kin}_{mirror} = \sum_{\mathbf{k}} \lambda_{\mathbf{k}} \left(\bar{\alpha}_{\mathbf{k}-} \alpha_{\mathbf{k}-} + \bar{\beta}_{\mathbf{k}+} \beta_{\mathbf{k}+} \right)$$

$$\frac{1}{y} S^{Dirac}_{mirror} = \frac{1}{2} \sum_{\mathbf{k},\mathbf{p}} (2 - \lambda_{\mathbf{k}}) \left(\bar{\alpha}_{\mathbf{k}-} \beta_{\mathbf{p}+} \Phi^*_{\mathbf{k}-\mathbf{p}} + \bar{\beta}_{\mathbf{k}+} \alpha_{\mathbf{p}-} \Phi_{\mathbf{k}-\mathbf{p}} e^{i(\varphi_{\mathbf{p}} - \varphi_{\mathbf{k}})} \right)$$

$$\frac{1}{yh} S^{Maj}_{mirror} =$$

$$i \sum_{\mathbf{k},\mathbf{p}} \left[\alpha_{\mathbf{k}-} \beta_{\mathbf{p}+} \Phi_{-\mathbf{k}-\mathbf{p}} e^{i\varphi_{\mathbf{k}}} - \bar{\beta}_{\mathbf{k}+} \bar{\alpha}_{\mathbf{p}-} \Phi^*_{\mathbf{k}+\mathbf{p}} \frac{(2-\lambda_{\mathbf{p}})(2-\lambda_{\mathbf{k}}) e^{-i\varphi_{\mathbf{k}}} - \lambda_{\mathbf{p}} \lambda_{\mathbf{k}} e^{-i\varphi_{\mathbf{p}}}}{4} \right]$$

$$\lambda_{\mathbf{k}} = a_{\mathbf{k}} \pm i \sqrt{b_{\mathbf{k}}^2 + c_{\mathbf{k}}^2} .$$

$$a_{\mathbf{k}} \equiv 1 - \frac{1 - 2s_1^2 - 2s_2^2}{\sqrt{1 + 8 s_1^2 s_2^2}} ,$$

$$b_{\mathbf{k}} \equiv \frac{2 s_2 c_2}{\sqrt{1 + 8 s_1^2 s_2^2}} ,$$

$$c_{\mathbf{k}} \equiv \frac{2 s_1 c_1}{\sqrt{1 + 8 s_1^2 s_2^2}} ,$$

where $$e^{i\varphi_{\mathbf{k}}} \equiv \begin{cases} \frac{i b_{\mathbf{k}} + c_{\mathbf{k}}}{\sqrt{b_{\mathbf{k}}^2 + c_{\mathbf{k}}^2}} & \text{if } \mathbf{k} \neq (N,N), (\frac{N}{2}, N), (N, \frac{N}{2}), (\frac{N}{2}, \frac{N}{2}) \\ 1 & \text{if } \mathbf{k} = (N,N), (\frac{N}{2}, N), (N, \frac{N}{2}), (\frac{N}{2}, \frac{N}{2}) \end{cases}$$

- fermion det is positive for Majorana > Dirac, for arbitrary scalar bckgd

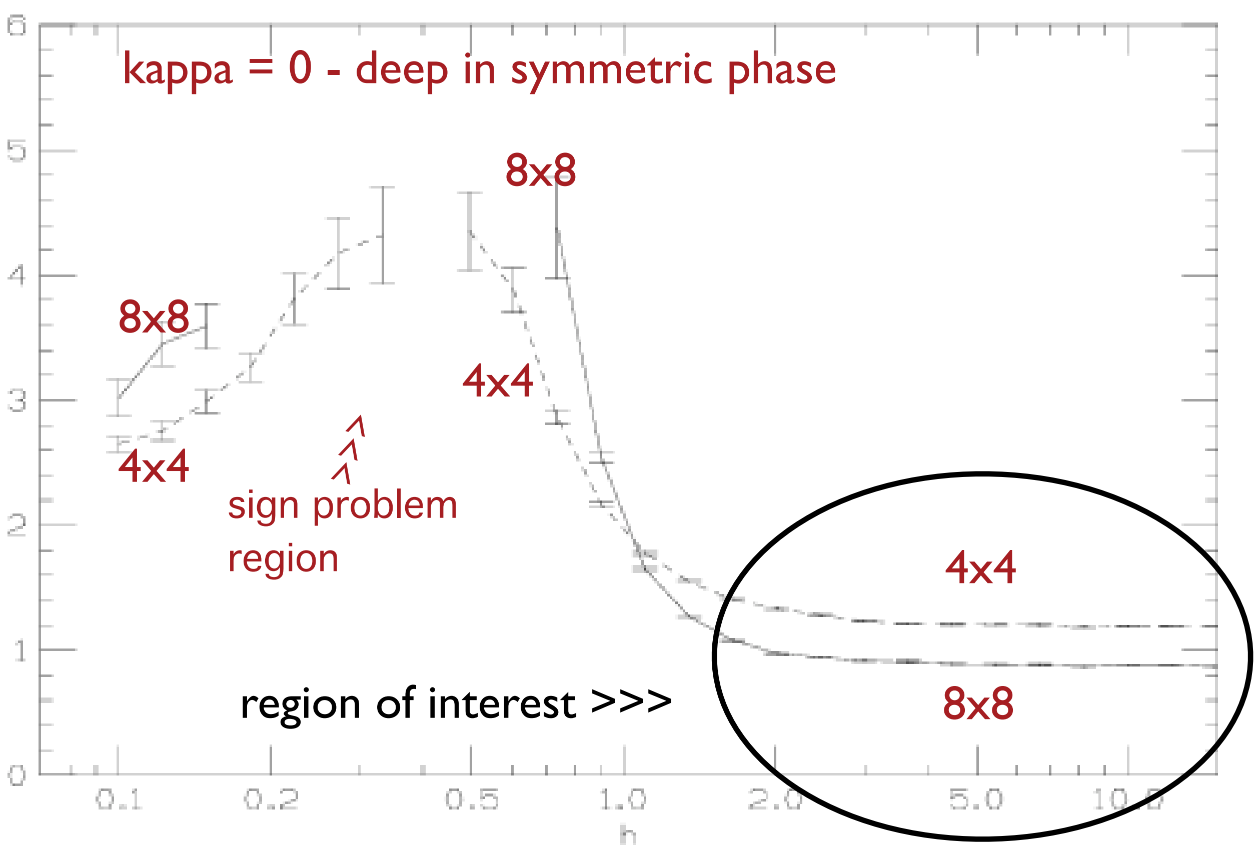
  [preliminary: appears to hold in 4d as well!]

- det. vanishes at Maj. = Dirac, sign problem for Maj. < Dirac *probe of symmetry breaking A* [KT transition, really, since 2d]

Higgs susceptibility
~ square of Higgs correlation lentgh

dynamical fermions; infinite Yukawa limit (y>10 is OK)

susceptibility as function of ratio of Majorana to Dirac mass:



Metropolis with e^{-S} = det M
...painfully slow...

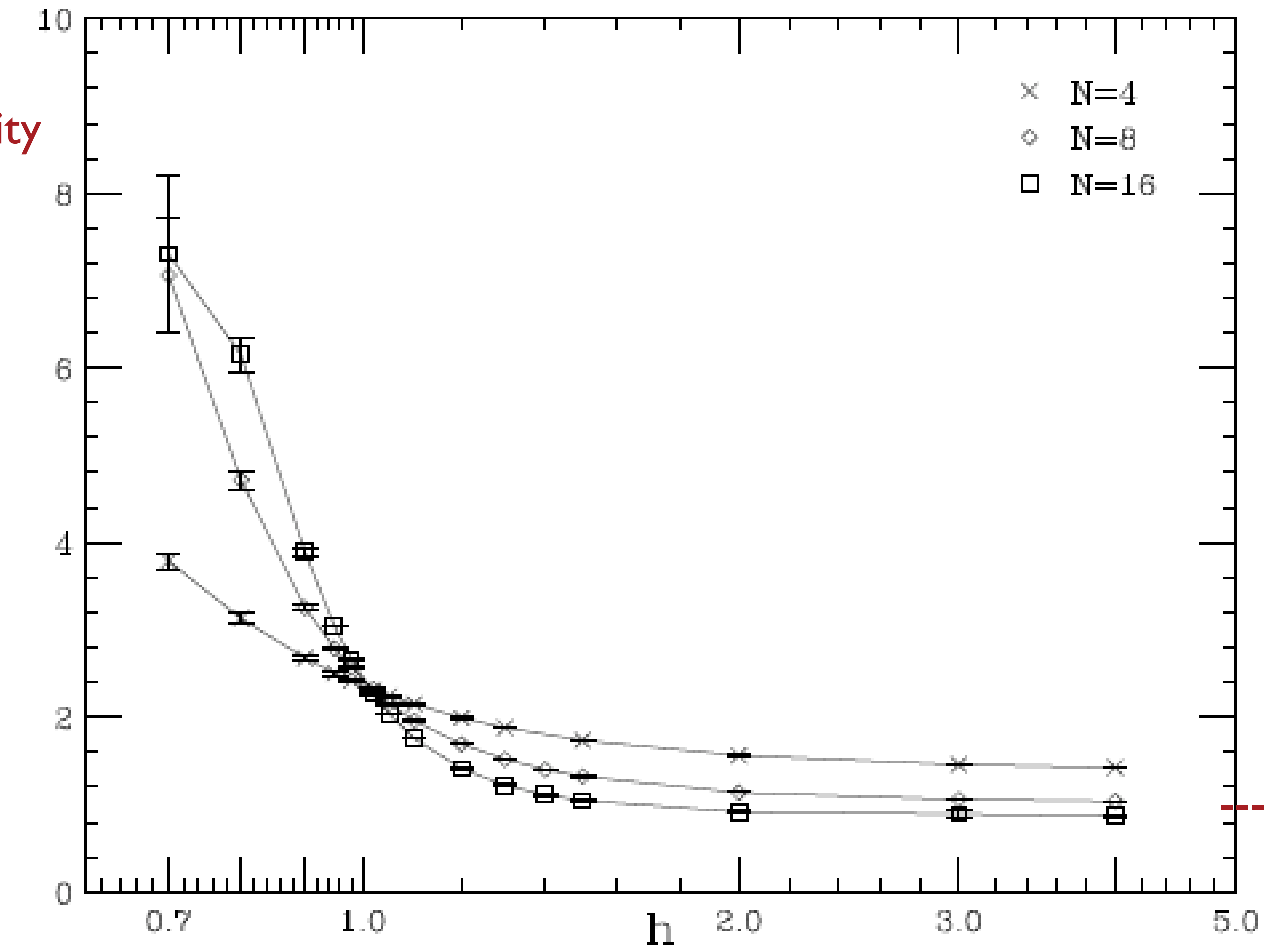
so, turn on kappa < kappa_KT and use cluster + determinant reweighting

probe of symmetry breaking A

kappa = 0.1 < kappa $_{KT}$(~ 1.1 on square lattice)

Wolff cluster (XY-model) + determinant reweigthing (much faster!)

Higgs susceptibility



--N=4 1.23
N=8 1.20
N=16 1.21

4 recent analytic and numerical work supporting the proposal

probe of symmetry breaking A

kappa = 0.5 < kappa $_{KT}$ (~ 1.1 on square lattice)

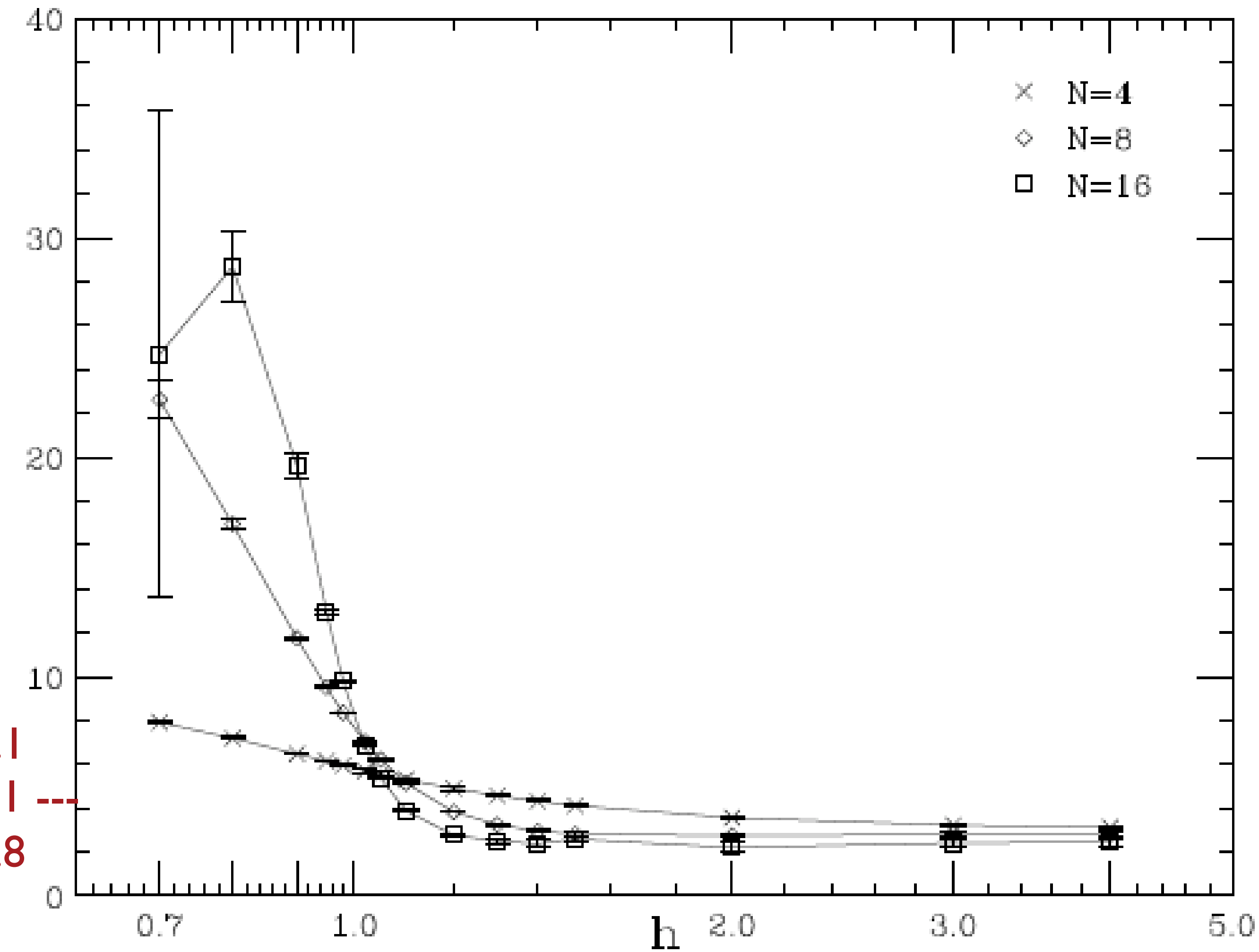
Wolff cluster (XY-model) + determinant reweigthing

larger kappa -
larger susceptibility

Higgs susceptibility

(growth with volume
- critical region)

N=8 4.1
N=16 4.1 ---
N=4 3.8



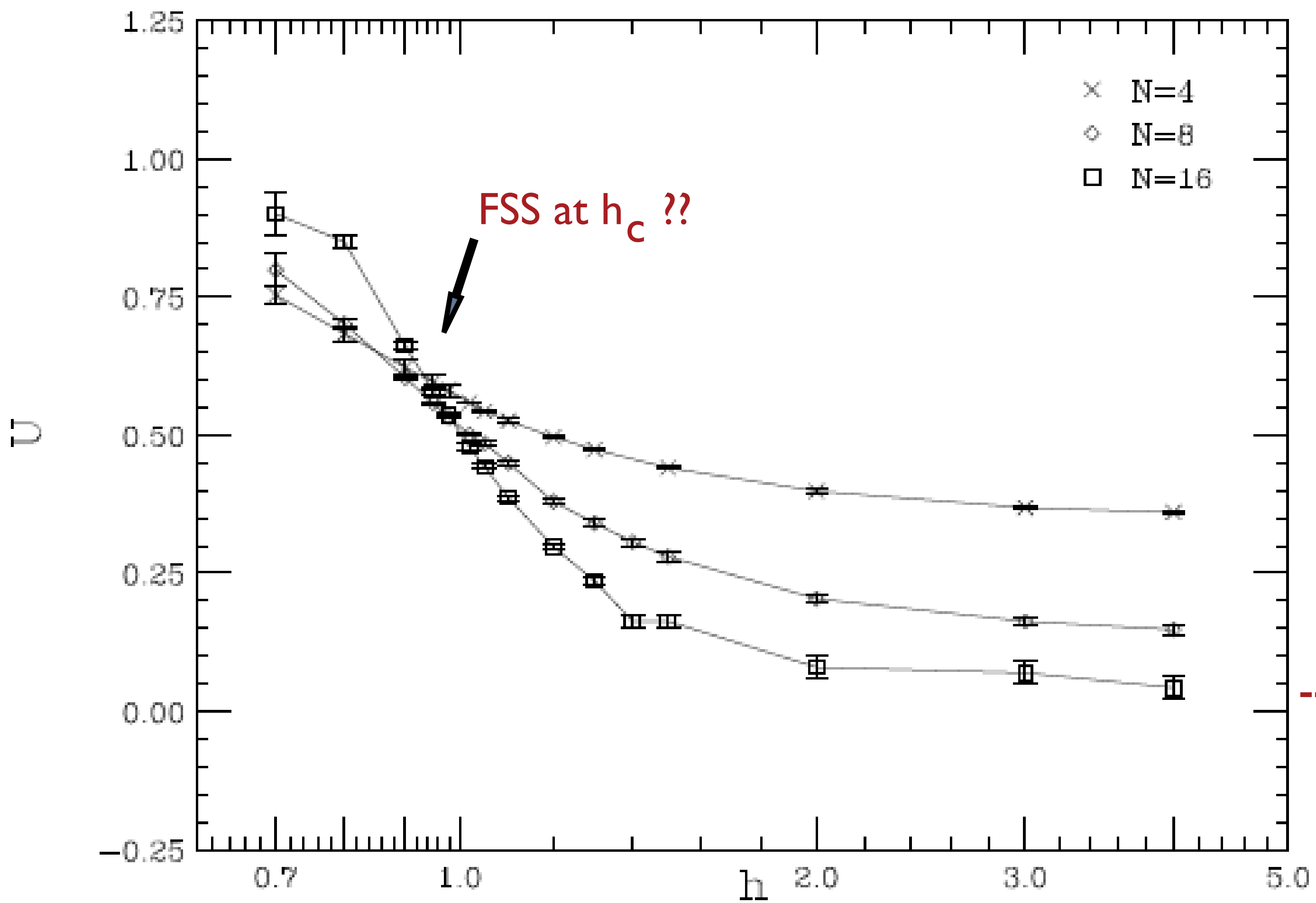
probe of symmetry breaking B

$$\lim_{\kappa \to 0} U = 0, \qquad \lim_{\kappa \to \infty} U = 1$$

$$U = 2 - \frac{\langle |M|^4 \rangle}{\langle |M|^2 \rangle^2} \qquad M = \sum_x \phi_x$$

Binder cumulant

kappa = 0.1



---N=4 .1
N=8 .05
N=16 .03

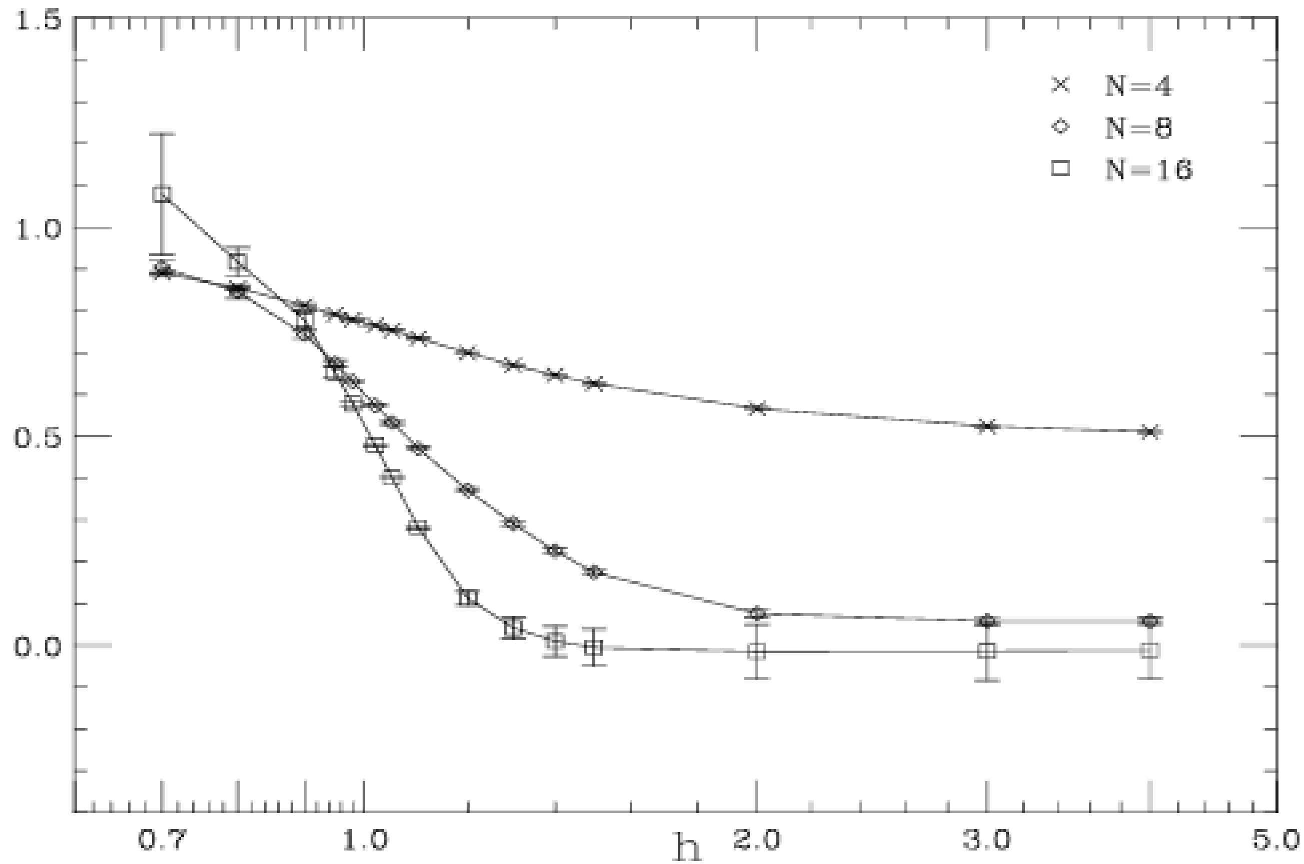
probe of symmetry breaking B

$$\lim_{\kappa \to 0} U = 0, \qquad \lim_{\kappa \to \infty} U = 1$$

$$U = 2 - \frac{\langle |M|^4 \rangle}{\langle |M|^2 \rangle^2} \qquad M = \sum_x \phi_x$$

Binder cumulant

kappa = 0.5



---N=4 .47

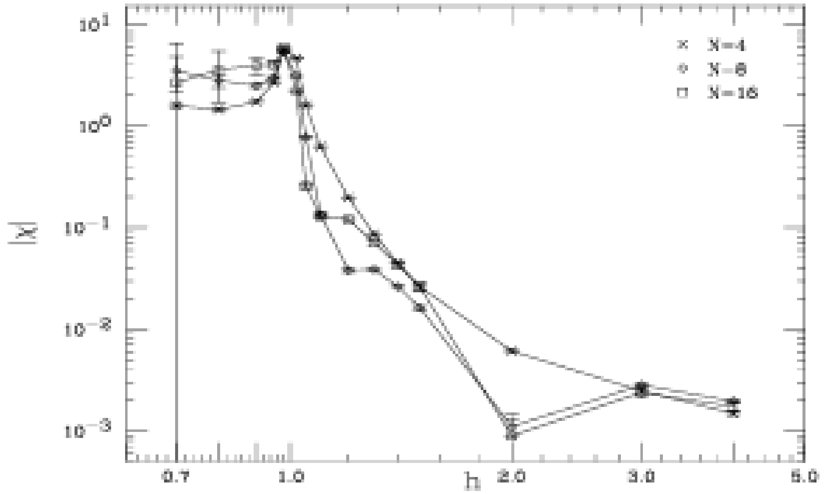
---N=8 .15

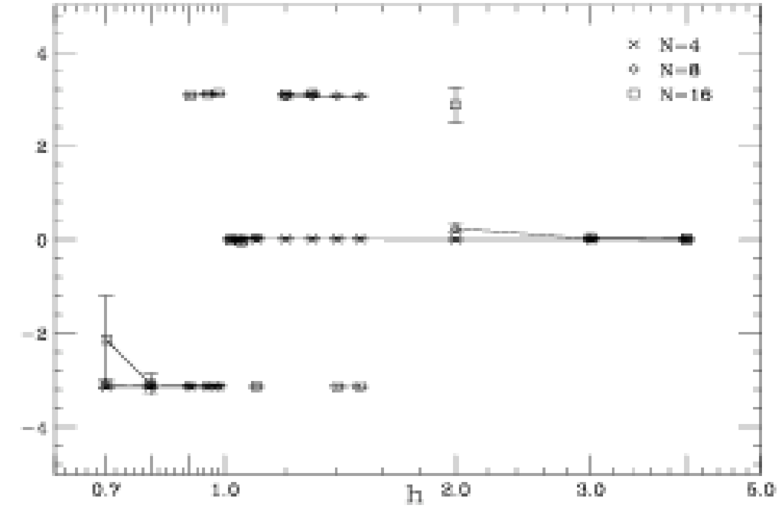
---N=16 .05

probe of symmetry breaking C

fermion-fermion bound state (1) with higgs quantum numbers
is the corresponding susceptibility large?

$$\chi_F \equiv \sum_x \langle \bar{\psi}_{-\,x} \chi_{+\,x}\, \bar{\chi}_{+\,y} \psi_{-\,y} \rangle |_{connected}$$





(based on toy model, with similar correlator

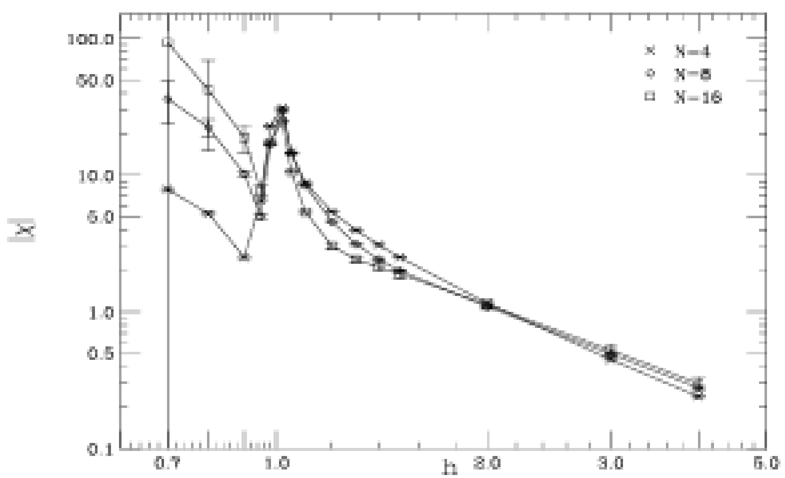
$$\langle (\Psi_{L\,0} \bar{\Psi}_{R\,0})\, (\bar{\Psi}_{L\,t} \Psi_{R\,t}) \rangle \neq 0$$

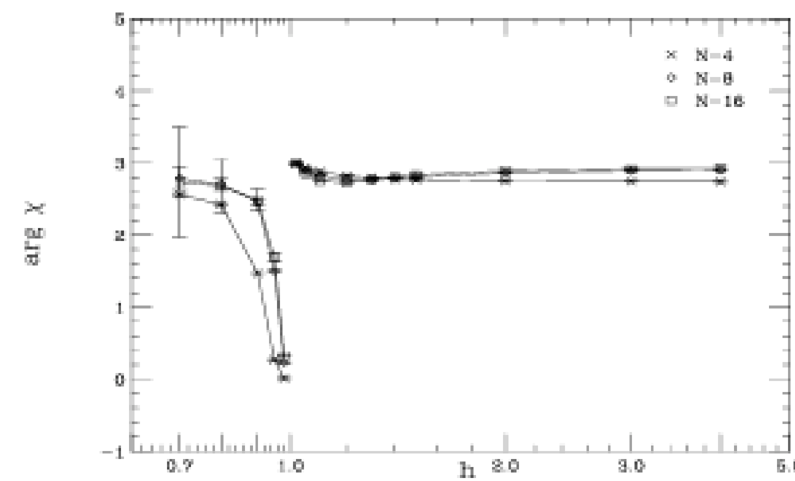
don't expect surprises - strong coupling, mixing...)

4 recent analytic and numerical work supporting the proposal

probe of symmetry breaking C

fermion-fermion bound state (2) with higgs-like quantum numbers





$$\chi'_F \equiv \sum_x \langle \psi^T_{-\,x}\gamma_2\chi_{+\,x}\,\bar{\chi}_{+\,y}\gamma_2\bar{\psi}^T_{-\,y}\rangle|_{connected}$$

fermion-susceptibility follows behavior of scalar; no growth with N at h>1 seen

probe of fermion spectrum...
(raw data)

neutral

$$S^n_{x-y} = \begin{pmatrix} \langle \chi_{+\ x}\ \phi_y \psi^T_{-\ y} \rangle & \langle \chi_{+\ x}\ \phi^*_y \bar{\psi}_{-\ y} \rangle \\ \langle \bar{\chi}^T_{+\ x}\ \phi_y \psi^T_{-\ y} \rangle & \langle \bar{\chi}^T_{+\ x}\ \phi^*_y \bar{\psi}_{-\ y} \rangle \end{pmatrix}$$

charged

$$S^c_{x-y} = \begin{pmatrix} \langle \chi_{+\ x} \phi_x\ \psi^T_{-\ y} \rangle & \langle \chi_{+\ x} \phi^*_x\ \bar{\psi}_{-\ y} \rangle \\ \langle \bar{\chi}^T_{+\ x} \phi_x\ \psi^T_{-\ y} \rangle & \langle \bar{\chi}^T_{+\ x} \phi^*_x\ \bar{\psi}_{-\ y} \rangle \end{pmatrix}$$

min(k) $\Omega^{1(2)}_{\mathbf{k}} = \dfrac{1}{\sqrt{\mathrm{Tr}\ s^{n(c)\ \dagger}_{\mathbf{k}} s^{n(c)}_{\mathbf{k}}}}$ *gives lower bound on S* ^{n,c} *eigenvalue*

first check: *fermion spectrum in broken phase (large kappa): perturbation theory, incl. spin-wave loop corrections (their scaling with h,kappa) agrees spectacularly (!) with Monte-Carlo*

probe of fermion spectrum...
raw data... lower bound on min EV of $S^{n,c}$ in units of Yukawa coupling
[y]=mass, large-(ya) >>1

fermion spectrum in symmetric phase:

$\kappa = 0.1$	$N = 4$	$N = 4$	$N = 8$	$N = 8$	$N = 16$
h	$neutr$	$chrgd$	$neutr$	$chrgd$	$neutr$
0.7	0.295	0.381	0.25	0.37	
0.8	0.198	0.272	0.198	0.269	
0.9	0.1	0.14	0.01	0.14	0.01
0.95	0.05	0.07	0.05	0.07	0.049
1.05	0.05	0.07	0.05	0.07	0.049
1.1	0.1	0.14	0.099	0.14	0.099
1.2	0.2	0.27	0.199	0.27	0.184
1.5	0.49	0.62	0.49	0.56	
2	0.94	1.1	0.73	0.8	0.44
3	1.76	1.79	1.21	1.26	0.75
4	2.39	2.41	1.68	1.72	1.1

more data near h=0 available

$\kappa = 0.5$	$N = 4$	$N = 4$	$N = 8$	$N = 8$
h	$neutr$	$chrgd$	$neutr$	$chrgd$
0.7	0.29	0.39	0.295	0.39
0.8	0.198	0.27	0.198	0.274
0.9	0.1	0.14	0.01	0.14
0.95	0.05	0.07	0.05	0.07
1.05	0.05	0.07	0.05	0.07
1.1	0.1	0.14	0.099	0.14
1.2	0.2	0.27	0.199	0.27
1.5	0.5	0.62	0.49	0.59
2	0.94	1.1	0.87	0.95
3	1.6	1.3	1.58	1.64
4	2.17	2.2	2.2	2.24

CONCLUSIONS from simulation with backreaction of mirror fermions

- good news, *positivity was somewhat unexpected, analytically not understood*

- scalar correlation length remains small, O(a),
  in infinite Yukawa limit... as per quick argument!

most important conclusion: symmetric phase

- fermions at strong Yukawa do not drive theory into broken phase

- mirror fermions massive

thus, we can continue study and address the crucial question:

will the "entire thing" work?

g=0 limit appears to work better than any Higgs/Yukawa model so far! ... so I am optimistic ...

before declaring victory, many issues to be understood, in both 2d and 4d:

- stability of next order of strong coupling, g=0, expansion
  [so far, looks good!]

- order g corrections

- behavior in nontrivial topology backgrounds and definition of Luscher construction's fermion measure!?

- if it all holds up, is there a sign problem with gauge fields?

... "old and new" in the lattice definition of chiral gauge theories ...

combining some older ideas with newer developments in understanding chiral symmetry on the lattice resulted in a

strong Yukawa proposal using Ginsparg-Wilson mechanism to impose a modified exact lattice chiral symmetry

preliminary indications and checks in 2d are promising