Little Hierarchy Problem and Little Higgs Theories

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HC & I. Low, JHEP 0309:051,2003 (hep-ph/0308199) JHEP 0408:061,2004 (hep-ph/0405243) Tevatron ($E_{CM} = 2 \text{TeV}$) is currently running. LHC ($E_{CM} = 14 \text{TeV}$) will start in 2007. It's going to be an exciting time for particle physics.

We expect that the mystery of electroweak symmetry breaking (EWSB) will be unraveled and TeV scale physics will be fully explored.

How do W, Z, quarks and leptons get their masses?



The vacuum contains a "condensate" which breaks electroweak symmetry and gives masses to all these particles.

In Standard Model (SM), electroweak symmetry breaking (EWSB) is caused by a vacuum expectation value (VEV) of a scalar Higgs field. Higgs potential: $V(H) = m_H^2 H^{\dagger} H + \frac{\lambda_H}{2} (H^{\dagger} H)^2$ $m_H^2 < 0, \quad \langle H^T \rangle = (v/\sqrt{2}, 0)$



 $v = \sqrt{2|m_H^2|/\lambda_H}$ $m_h = \sqrt{\lambda_H} v = \sqrt{2|m_H^2|} \text{ (physical Higgs boson mass)}$ Experimentally,

> v = 246 GeV m_h > 114 GeV (λ_H > 0.2)

The global fit of EW data favors a light Higgs

 $m_h = 113^{+56}_{-40} \text{ GeV},$ $m_h < 246 \text{ GeV} (95\% \text{ CL}), (RPP 2004)$ Unitary bound: $m_h \lesssim 700 \text{ GeV}$



(LEP EW Working Group '05)

The scalar Higgs field suffers from the hierarchy problem: the Higgs mass and VEV are very sensitive to Ultraviolet (UV) scale physics through quantum corrections.



$$\delta m_H^2 \sim \frac{1}{16\pi^2} \left\{ \lambda_t^2, \ g^2, \ \lambda_H \right\} \Lambda_{\text{UV}}^2$$

Naturalness requires these quadratically divergent contributions to be cut off by new physics at \sim 1 TeV.

Another hint of the TeV scale: dark matter

A weakly interacting stable neutral particle with a weak scale mass gives the right thermal relic abundance for the dark matter.

$$\Omega_{\text{wimp}} \sim \left(\frac{1}{10^2 \alpha}\right)^2 \left(\frac{m_{\text{wimp}}}{1 \text{ TeV}}\right)^2$$

Therefore, we expect to discover not only the Higgs boson, but also exciting new physics at these TeV colliders.

Although direct searches haven't reached the TeV scale (LEP, Tevatron),

new leptons	\gtrsim	100	GeV
new quarks	\gtrsim	200	GeV
leptoquarks	\gtrsim	300	GeV
W',~Z'	\gtrsim	700	GeV

we already have some experimental information about TeV physics from indirect searches. At low energies, new physics can be integrated out and encoded in higher dimensional operators made of SM fields:

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \underbrace{\sum_{i,p} \frac{c_i}{\Lambda^p} \mathcal{O}_i^{(4+p)}}_{\text{new physics}}$$

- Operators that violate the (approximate) symmetries of the SM, e.g., baryon number, flavor, CP, are strongly constrained.
 ⇒ New physics at ~ 1 TeV should also (approximately) respect these symmetries.
- Operators that do not violate the SM symmetries are also constrained by the precision electroweak measurements.

Dimension six operator	$c_i = -1$	$c_i = +1$
$\mathcal{O}_{WB} = (H^+ \sigma^a H) W^a_{\mu u} B_{\mu u}$	9.0	13
$\mathcal{O}_H = H^+ D_\mu H) ^2$	4.2	7.0
$\mathcal{O}_{LL} = rac{1}{2} (ar{L} \gamma_\mu \sigma^a L)^2$	8.2	8.8
$\mathcal{O}_{HL} = i(H^+ D_\mu H)(\bar{L}\gamma_\mu L)$	14	8.0

(Barbieri and Strumia '00)

No evidence for new physics has been found up to ~ 10 TeV (assuming $c_i \sim O(1)$).



(LEP EW Working Group '05)

The tension between the naturalness requirement of new physics at TeV and no new physics appearing in precision electroweak experiments is known as the "little hierarchy problem."

Supersymmetry (SUSY)

With SUSY, the quadratically divergent contributions to m_H^2 are canceled between the SM particles and their superpartners.



SUSY must be broken since no superpartner has been seen. The masses of the superpartners should be below ~ 1 TeV if SUSY is the solution for stabilizing the EW scale.

With R-parity conservation, the lightest SUSY particle (LSP) is stable and provides a good dark matter candidate.

Supersymmetry

EW observables receive corrections from superpartners only at the loop level. They are generally small. However, there is a related fine-tuning problem in Minimal Supersymmetric Standard Model (MSSM).

• Higgs is just one of many scalar particles. One expects $m_H^2\sim m_{\tilde{q}}^2\sim m_{\tilde{\ell}}^2$

• m_{H}^{2} receives large radiative corrections from $m_{\tilde{t}}^{2}$,

$$\delta m_H^2 \sim -\frac{6G_F m_t^2}{\sqrt{2}\pi^2} m_{\tilde{t}}^2 \ln \frac{\Lambda}{m_{\tilde{t}}}$$
$$\sim -4.5 m_{\tilde{t}}^2 \ln \frac{\Lambda}{10^{13} m_{\tilde{t}}}$$

Naturalness prefers $m_{\tilde{t}} \sim 100$ GeV.

Experimental data are in favor of large $m_{\tilde{t}}$.

• Small $\Delta \rho (\Delta T)$

• There is a tree-level bound on the light Higgs mass, $m_{h,\text{tree}} \leq M_Z = 91.2$ GeV in MSSM. To raise m_h above the direct search limit, 114 GeV, large loop contribution to the quartic coupling is needed.



but at the same time t



 $m_h^2 \approx M_Z^2 + \frac{3}{2\pi^2} m_t^2 \ln \frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}$

For $m_h \gtrsim 115$ GeV, $m_{\tilde{t}} \gtrsim 500$ GeV.

• No superpartner has been found.

MSSM is fine-tuned at a few % level.

How should we take on this little hierarchy problem?

Possibilities:

- 1. New physics \gtrsim 10 TeV. Electroweak scale is fine-tuned (but the natural explanation of dark matter with TeV scale WIMPs is also lost).
- The little hierarchy problem is giving us some hint about new physics at the TeV scale.

A historic lesson from strong interactions

Among the large number of hadrons, pions are significantly lighter than all the other hadrons.

 $m_\pi \sim 140 \; {
m MeV} \; \ll \; m_
ho (\sim 770 \; {
m MeV}), \ m_{p,n} (\sim 940 \; {
m MeV}),$

An accidental cancelation between the constituent mass and the binding energy?

. . .

By taking this hint seriously, it was realized that the lightness of the pions is due to spontaneous chiral symmetry breaking:

Pions are the (pseudo-)Nambu-Goldstone bosons (PNGBs) of $SU(2)_L \times SU(2)_R \rightarrow SU(2)_V$ chiral symmetry breaking.

It laid the foundation for our understanding of the strong interaction.

It is natural to consider the possibility that Higgs mass is small because Higgs is a PNGB of some spontaneously broken symmetry. (Georgi, Pais, '75, Kaplan, Georgi '84, ...)

However, Higgs as a PNGB requires a "smarter" structure than the pion chiral Lagrangian.

• Pion masses are quadratically sensitive to the cutoff at one-loop, e.g., the one-loop EM corrections contribute to the $m_{\pi^+}-m_{\pi^0}$ difference.

• For $m_h \sim v$, a sizable self quartic coupling is needed, but at the same time no large Higgs mass term should be induced.

The two problems are related and they are solved by the little Higgs mechanism (collective symmetry breaking).

In fact, the idea came from extra dimensions! (Arkani-Hamed, Cohen, Georgi, '01)

Consider a (non-Abelian) gauge in a compact extra dimension of size R. The extra component of the gauge field A_5 becomes a scalar in 4D below the scale R^{-1} . However, it can not have a local mass term because it's part of the gauge field at high energies. A non-local mass for A_5 is generated due to compactification.

A pure 4D theory can be obtained upon deconstructing (latticizing) the extra dimension. A_5 becomes the PNGB in 4D.

Non-locality \Rightarrow Collective symmetry breaking

A quartic coupling can come from deconstructing 2 extra dimensions:

 $F_{56}^2 = (\partial_5 A_6 - \partial_6 A_5 + i g [A_5, A_6])^2 \supset g^2 [A_5, A_6]^2$

Little Higgs theories

Higgs arises as a pseudo-Nambu-Goldstone boson (PNGB) of a spontaneously broken global symmetry, $G \rightarrow H$, with a special property that its mass is protected from one-loop quadratic divergences induced by the explicit symmetry breaking couplings.

The global symmetry is explicitly broken by 2 sets of interactions, with each set preserving a subset of the symmetry.

$$\mathcal{L} = \mathcal{L}_0 + \lambda_1 \mathcal{L}_1 + \lambda_2 \mathcal{L}_2$$

The Higgs is an exact NGB when either set of couplings is absent.

$$\delta m_H^2 \sim \left(\frac{\lambda_1^2}{16\pi^2}\right) \left(\frac{\lambda_2^2}{16\pi^2}\right) \Lambda^2$$

The cutoff Λ can be raised above 10 TeV, beyond the scale probed by the current electroweak data.

Collective symmetry breaking:



Collective symmetry breaking:

Little Higgs flat direction



Bottoms of the mexican hat potentials



Ex: Minimal moose model (Arkani-Hamed, Cohen, Katz, Nelson, Gregoire, Wacker, '02) A generalization of QCD chiral Lagrangian, $SU(3)_L \times SU(3)_R \rightarrow SU(3)_V$



Global symmetry: $[SU(3)]^8 \rightarrow [SU(3)]^4$ Gauge symmetry: $[SU(2) \times U(1)]^2 \rightarrow [SU(2) \times U(1)]$

$$\Sigma_i \to L_i \Sigma_i R_i^{\dagger}$$

$$\Sigma_{i} = \exp\left(\frac{2i\pi_{i}}{f}\right), \quad \pi_{i} = \left(\begin{array}{cc}\phi_{i} + S_{i} & H_{i}\\H_{i}^{\dagger} & -2S_{i}\end{array}\right)$$
$$\mathcal{L} \supset \kappa f^{4} \left[\operatorname{Tr}\Sigma_{1}\Sigma_{2}^{\dagger}\Sigma_{3}\Sigma_{4}^{\dagger} + \operatorname{Tr}\Sigma_{1}\Sigma_{4}^{\dagger}\Sigma_{3}\Sigma_{2}^{\dagger} + h.c.\right]$$

Top Yukawa coupling:

$$\lambda_1 f\left(b, t, T\right) \Sigma_1 \Sigma_2^{\dagger} \left(\begin{array}{c} 0\\ 0\\ u_3^c \end{array}\right) + \lambda_2 f T t^{\prime c}$$

$$\supset \lambda_t(b, t) H t^c + M_T T T^c$$

$$\lambda_t = \frac{\lambda_1 \lambda_2}{\sqrt{\lambda_1^2 + \lambda_2^2}}, \quad t^c = \frac{\lambda_2 u_3^c - \lambda_1 t'^c}{\sqrt{\lambda_1^2 + \lambda_2^2}},$$
$$M_T = \sqrt{\lambda_1^2 + \lambda_2^2} f, \quad T^c = \frac{\lambda_1 u_3^c + \lambda_2 t'^c}{\sqrt{\lambda_1^2 + \lambda_2^2}}$$

Each coupling preserves some global SU(3) which protects the Higgs mass.

The low energy (\sim 100GeV) theory contains 2 Higgs doublets with a quartic potential,

$$\kappa \mathrm{Tr} (H_{\mathrm{I}} H_{\mathrm{I}}^{\dagger} - H_{\mathrm{II}} H_{\mathrm{II}}^{\dagger})^{2} + \kappa (H_{\mathrm{I}}^{\dagger} H_{\mathrm{I}} - H_{\mathrm{II}}^{\dagger} H_{\mathrm{II}})^{2}$$

Ex: Littlest Higgs (most popular) (Arkani-Hamed, Cohen, Katz, Nelson '02) Global symmetry: SU(5)/SO(5)Gauge symmetry: $[SU(2) \times U(1)]^2 / [SU(2) \times U(1)]$ Σ : 5 × 5 symmetric matrix, transforming under the global SU(5) as $\Sigma \to V\Sigma V^T$, with a VEV

$$\Sigma_0 = \begin{pmatrix} & \mathbf{1} \\ & \mathbf{1} \\ \mathbf{1} & & \end{pmatrix},$$

which breaks $SU(5) \rightarrow SO(5)$. The Goldstone fields Π can be parametrized as

$$\Sigma(x) = e^{i\Pi/f} \Sigma_0 e^{i\Pi^T/f} = e^{i2\Pi/f} \Sigma_0$$
$$\Pi = \begin{pmatrix} \frac{H}{\sqrt{2}} & \phi \\ \frac{H^{\dagger}}{\sqrt{2}} & \frac{H^T}{\sqrt{2}} \\ \phi^{\dagger} & \frac{H^*}{\sqrt{2}} \end{pmatrix},$$

Two different $SU(2) \times U(1)$ subgroups of SU(5) are gauged with equal strength:

$$Q_{1}^{a} = \begin{pmatrix} \sigma^{a}/2 \\ \end{pmatrix},$$

$$Q_{2}^{a} = \begin{pmatrix} & & \\ & -\sigma^{a*}/2 \end{pmatrix},$$

$$Y_{1} = \text{diag}(3, 3, -2, -2, -2)/10,$$

$$Y_{2} = \text{diag}(2, 2, 2, -3, -3)/10.$$

Each gauge generator preserves an SU(3) global symmetry which protects the Higgs mass.

Top Yukawa coupling arises from

 $\lambda_1 f \epsilon_{ijk} \epsilon_{xy} Q_{3i} \Sigma_{jx} \Sigma_{ky} u_3^c + \lambda_2 f T t^{\prime c},$

 $Q_3 = (b, t, T), \quad i, j, k = 1, 2, 3, x, y = 4, 5$

The low energy theory contains only one Higgs doublet.

One-loop quadratic divergences are canceled by new particles at the TeV scale with the same spins as the corresponding SM particles.



 $m_{W_H} \sim gf, \; m_T \sim \lambda_t f, \ldots, \; f \sim 1 \; {\rm TeV}, \; \Lambda \sim 4\pi f$

Relations among couplings are ensured by nonlinearly realized (approximate) global symmetry.

Generic spectrum for little Higgs theories:

UV completion

↑

 $\Lambda \sim 4\pi f \sim$ 10 TeV — UV cutoff

100 GeV — SM with 1 or 2 Higgs Doublets The new particles at \sim 1 TeV can also contribute to EW observables.



Generically f needs to be \gtrsim a few TeV to avoid the constraints from EW data, potentially reintroducing the fine-tuning problem. (Csaki, Hubisz, Kribs, Meade, Terning, '02, Hewett, Petriello, Rizzo,'02,...)



(Marandella, Schappacher, Strumia, '05)

T-parity (HC & Low, '03, '04)

The couplings which contribute to EW observables at tree level are not necessary for canceling the 1-loop quadratic divergence. They can be eliminated by a symmetry, T-parity:

$SM \rightarrow +SM$

 $W_H, Z_H, A_H, \phi \rightarrow -(W_H, Z_H, A_H, \phi)$

It is analogous to the R-parity in supersymmetric theories.

It can be imposed in many little Higgs models, in a similar way that Parity is conserved in QCD.

Minimal moose:



For $g_a = g_b$, there is a reflection symmetry, $a \leftrightarrow b, \ \Sigma_i \leftrightarrow \Sigma_i^{\dagger}$

$$W, Z, \gamma \rightarrow +(W, Z, \gamma)$$

$$W_H, Z_H, A_H \rightarrow -(W_H, Z_H, A_H)$$

$$\pi_i (\supset H_i, \phi_i, S_i) \rightarrow -\pi_i$$

Including a discrete hypercharge rotation,

$$\Omega = \begin{pmatrix} -1 & & \\ & -1 & \\ & & 1 \end{pmatrix}, \quad \Sigma_i \leftrightarrow \Omega \Sigma_i^{\dagger} \Omega$$

the parity of H_i is flipped to +,

$$\begin{array}{rcl} \phi_i, S_i & \to & -(\phi_i, S_i) \\ H_i & \to & +H_i \end{array}$$

so that it's preserved even after EWSB.

Incorporating fermions is a little more subtle. One can introduce one more site,



Mirror fermions $\overline{\psi}_c$ marry with $\psi_a + \psi_b$ becoming heavy ~ TeV (T-odd partners of SM fermions). $\psi_a - \psi_b$ remain light and are identified as SM fermions.

Gauge couplings on site-c are taken to be large. The corresponding gauge bosons are heavy and decouple from the low-energy theory. Site-c can be integrated out and we recover the 2site model.

In the low energy theory, fermions do not need to transform linearly under $SU(3)_a$, $SU(3)_b$.

It's analogous to baryons in QCD. Parity is preserved by QCD and baryons do not transform linearly under $SU(3)_L$ or $SU(3)_R$.

Callan, Coleman, Wess and Zumino (1969) (CCWZ) showed us how to construct the most general low-energy effective Lagrangian with fields transforming nonlinearly under the full (broken) symmetry.

CCWZ:

For a broken symmetry $G \rightarrow H$, starting with fields transforming only under H, and \mathcal{L}_H only invariant under H, we can construct the most general low-energy effective Lagrangian invariant under G, with the help of the Goldstone bosons which parametrize the coset space G/H.

For
$$SU(3)_a \times SU(3)_b \to SU(3)_V$$
,
 $\Sigma_i = \xi_i^2$. (More generally, $g \cdot \xi = \xi \cdot h$).
 $\xi^{\dagger} D_{\mu} \xi = \xi^{\dagger} (\partial_{\mu} + i A^a_{\mu} Q^a_V + i A'^a_{\mu} Q^a_A) \xi$
 $\equiv v^a_{\mu} T^a + p^a_{\mu} X^a$,
 $\xi D_{\mu} \xi^{\dagger} = v^a_{\mu} T^a - p^a_{\mu} X^a$.

 v_{μ} transforms like a gauge field

$$v^a_\mu T^a \to U v^a_\mu T^a U^\dagger + U(\partial_\mu U^\dagger),$$

and p_{μ} transforms linearly

$$p^a_\mu X^a \to U p^a_\mu X^a U^{\dagger}.$$

For ψ transforming under $H = SU(3)_{\text{diag}}$, Lagrangian can be promoted to be invariant under $G = SU(3)_a \times SU(3)_b$,



Parity is preserved for c = 0.

T-parity can be imposed on any little Higgs model based on a symmetric space: T: unbroken generators (generators for H) X: broken generators (generators for G/H) The (schematic) commutation relations

 $[T, T] \sim T, \quad [T, X] \sim X, \quad [X, X] \sim T$

admits an automorphism, $T \rightarrow T$, $X \rightarrow -X$, which is the basis of the T-parity (up to a hypercharge rotation).

This includes SU(5)/SO(5), SU(6)/Sp(6),... Exceptions: Little Higgs from a simple group, $[SU(3)/SU(2)]^2$,... (Kaplan & Schmaltz, '03)

Phenomenology of little Higgs theories with T-parity:

- Contributions to EW observables are loopsuppressed. *f* can be ≤ 1TeV without violating EW precision data ⇒ no fine tuning and new particles more accessible at colliders.
- Lightest T-odd particle (LTP) is stable. It can be a good dark matter candidate if it's neutral. (A likely candidate is the A_H gauge boson.)
- T-odd particles are pair-produced (traditional Z', W' searches don't apply), then they cascade decay down to LTP. Typical collider signals are jets/leptons+ \$\vec{E}\$, which mimic supersymmetry with R-parity.

Lightest T-odd particle as the dark matter:

 A_H is often the lightest T-odd particle.

Annihilation channels: $A_{H} \xrightarrow{Z(W)} A_{H} \xrightarrow{h} A_{H$ It's somewhat similar to the Kaluza-Klein dark matter, (Servant, Tait, '02, HC, Feng, Matchev, '02) except that there is no reason for a degenerate spectrum.

Indirect detections of e^+ , γ , and ν through dark matter annihilations may be distinctively promising.

(Positions from KK dark matter annihilations:)



(HC, Feng, Matchev, '02)

Collider phenomenology for generic little Higgs models:

• Top partner T:



(Azuelos et al, '04) Reach \sim 2 TeV at LHC (300 fb $^{-1}).$



(Azuelos et al, '04) Reach \sim 5 TeV at LHC (300 fb $^{-1}).$

Collider phenomenology with T-parity:

T-odd particles, including W_H , Z_H , A_H , ϕ , and ψ_{odd} , have to be pair-produced. Then they cascade decay down to the LTP, which escapes the detector, giving rise to missing energies.

The collider signatures are similar to SUSY with R-parity, and also to universal extra dimensions (UEDs) (Appelquist, HC, Dobrescu, '00, HC, Matchev, Schmaltz, '02)

Distinguishing different models poses additional challenges after the discovery. For example,

$$3\ell + \not\!\!\!E_T \Rightarrow \begin{cases} \tilde{\chi}^{\pm} \tilde{\chi}^0_2 & \text{SUSY?} \\ W^{(1)}_{\mathsf{K}\mathsf{K}} Z^{(1)}_{\mathsf{K}\mathsf{K}} & \text{UED?} \\ W_H Z_H & \text{Little Higgs with T-parity?} \end{cases}$$

More information on spins and couplings of the new particles are needed. A lepton collider with enough E_{CM} will be helpful.

Conclusions

- 1. Little hierarchy problem may be an important clue to new physics at the TeV scale.
- Little Higgs theories provide a new mechanism to address the naturalness of the electroweak scale. All tree-level contributions to the EW observables from TeV physics can be removed with a T-parity, which provides a natural solution to the little hierarchy problem.
- Dark matter in the universe provides another hint for new physics at the TeV scale. The lightest T-odd particle serves as a natural dark matter candidate.

4. We expect to discover the new particles responsible for cutting off the quadratically divergent contributions to the Higgs mass-squared at LHC. In little Higgs theories they are new quarks, gauge bosons, and scalars. The collider signatures are different for models with and without T-parity. With T-parity, collider signals contain missing energies, similar to SUSY. Distinguish it from SUSY could be a challenge at hadron colliders. More hard work is needed.