Detecting Light Dark Matter With Phonons and Magnons

Zhengkang "Kevin" Zhang (Caltech)

*Based on: 1905.13744, 1910.08092, **1910.10716**, 2005.10256, **2009.13534**.*

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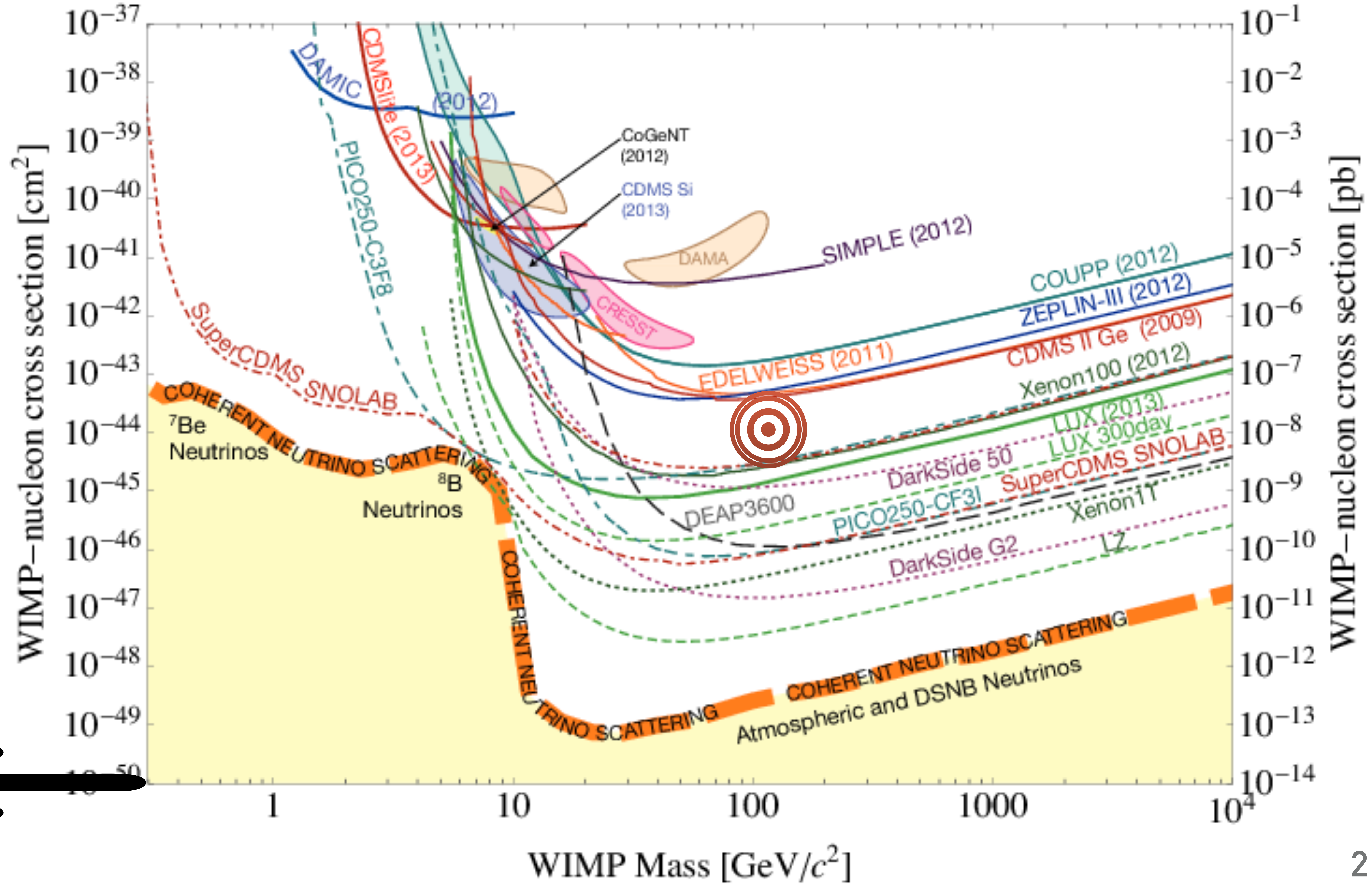
Caltech

Toward lighter DM in direct detection

- Conventional WIMP searches.
  - Nuclear recoils.
  - Lose sensitivity below DM mass ~GeV.



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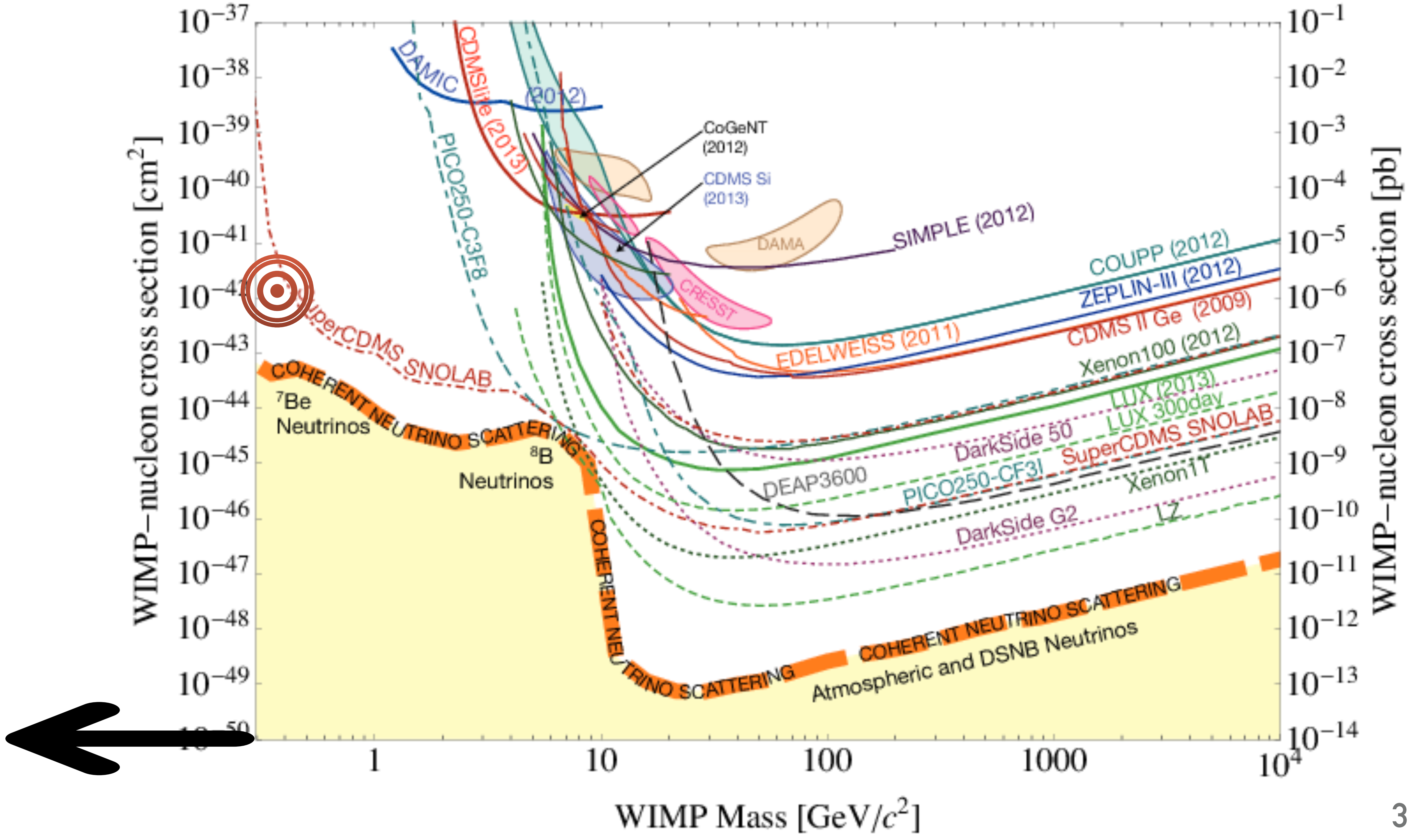
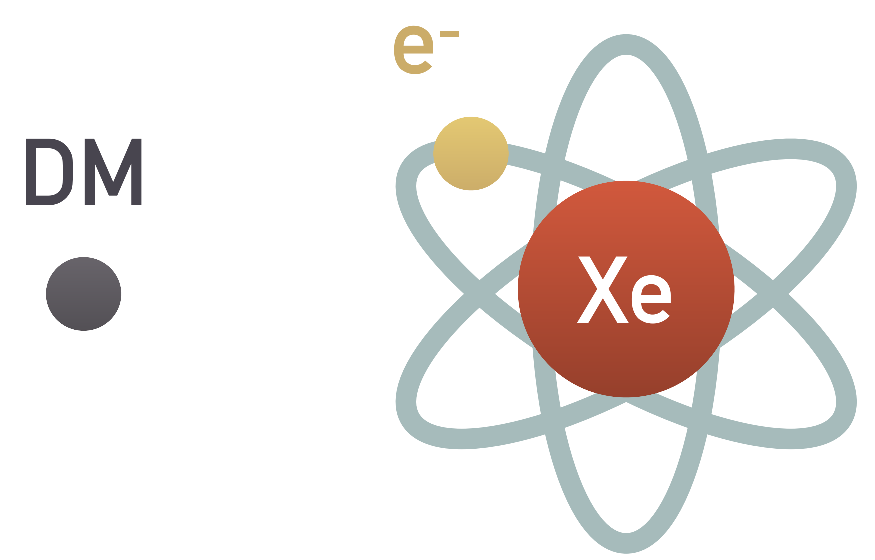
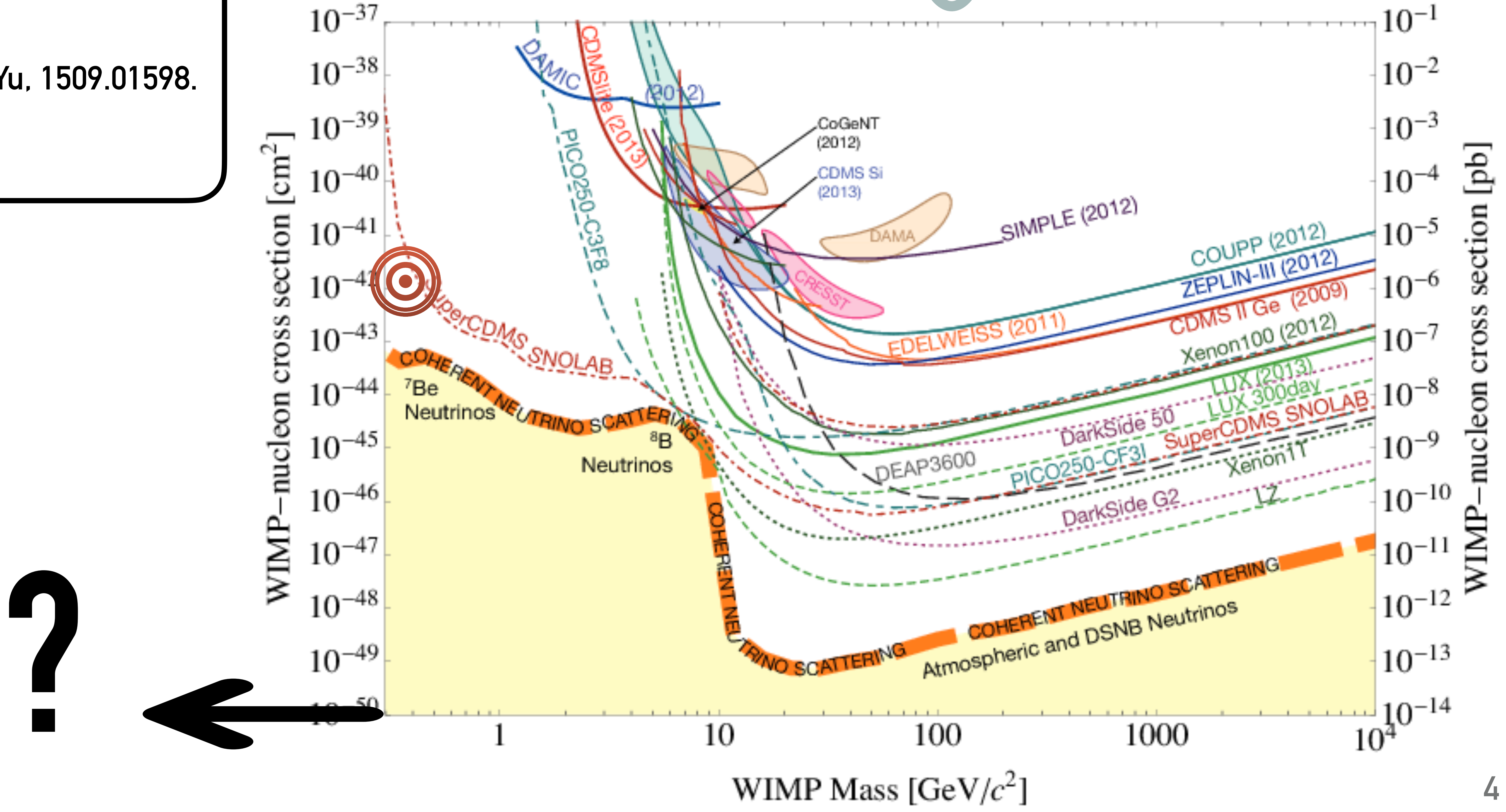


Toward lighter DM in direct detection

- Electron excitations.
  - Atoms (binding energy $\sim$ 10eV).

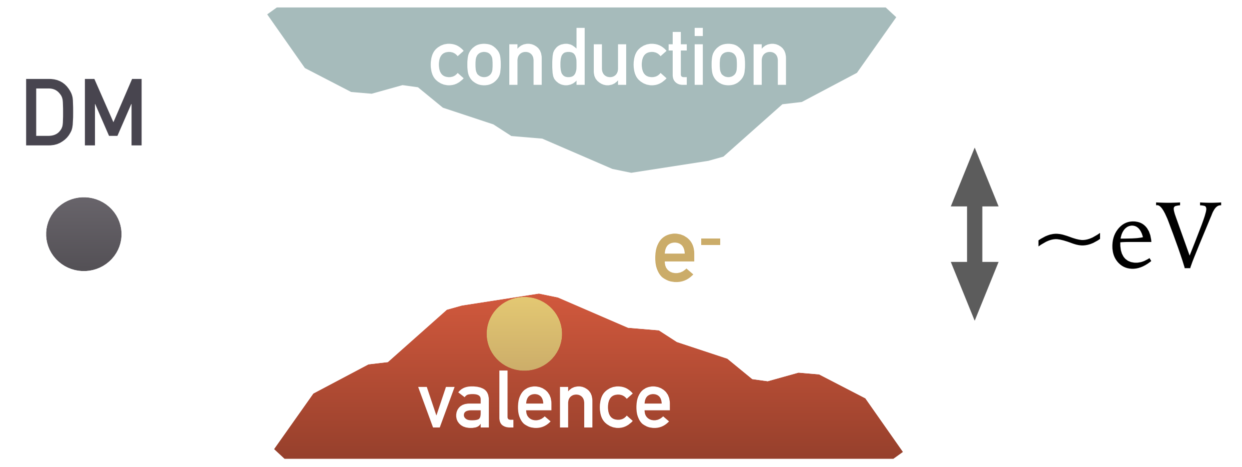
Essig, Mardon, Volansky, 1108.5383.
Graham, Kaplan, Rajendran, Walters, 1203.2531.
Lee, Lisanti, Mishra-Sharma, Safdi, 1508.07361.
Essig, Fernandez-Serra, Mardon, Soto, Volansky, Yu, 1509.01598.
Essig, Volansky, Yu, 1703.00910.
Catena, Emken, Spaldin, Tarantino, 1912.08204.





Toward lighter DM in direct detection

- ➤ Electron excitations.
  - ➤ Atoms (binding energy ~ 10eV).
  - ➤ Semiconductors (gap ~ eV).
  - ➤ Reach down to DM mass ~MeV.



Essig, Mardon, Volansky, 1108.5383.
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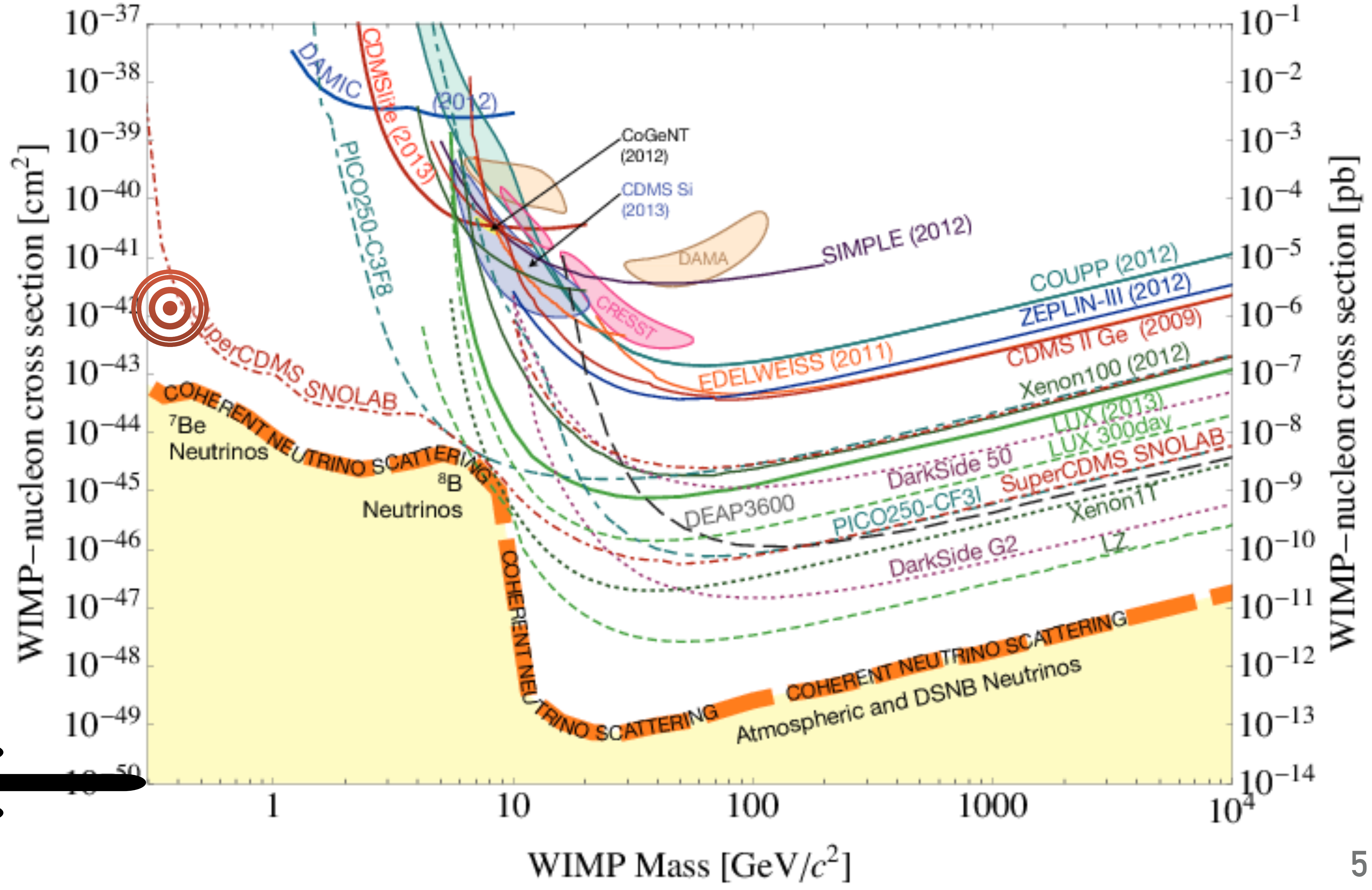
Lee, Lisanti, Mishra-Sharma, Safdi, 1508.07361.
Essig, Fernandez-Serra, Mardon, Soto, Volansky, Yu, 1509.01598.
Derenzo, Essig, Massari, Soto, Yu, 1607.01009.
Kurinsky, Yu, Hochberg, Cabrera, 1901.07569.
Griffin, Inzani, Trickle, ZZ, Zurek, 1910.08092 + 1910.10716.
Andersson, Bökmark, Catena, Emken, Moberg, Åstrand, 2001.08910.

SuperCDMS, 1804.10697.
SENSEI, 1901.10478 + 2004.11378,
DAMIC, 1907.12628.

Other similar proposals
[Graphene] Hochberg, Kahn, Lisanti, Tully, Zurek, 1606.08849.
[Aromatic organic targets] Blanco, Collar, Kahn, Lillard, 1912.02822.
…

~MeV



Toward lighter DM in direct detection

- Electron excitations.
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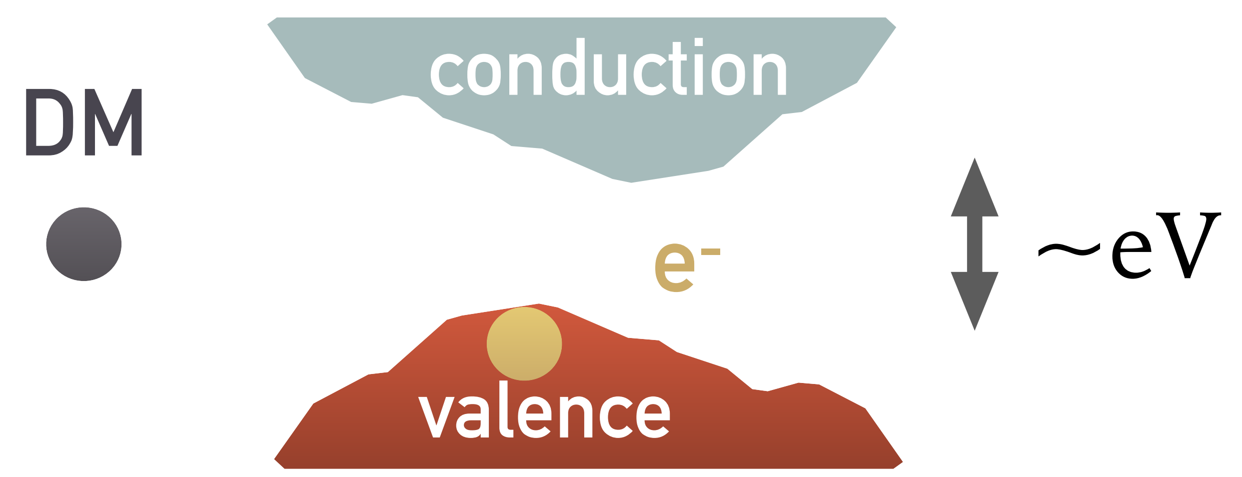
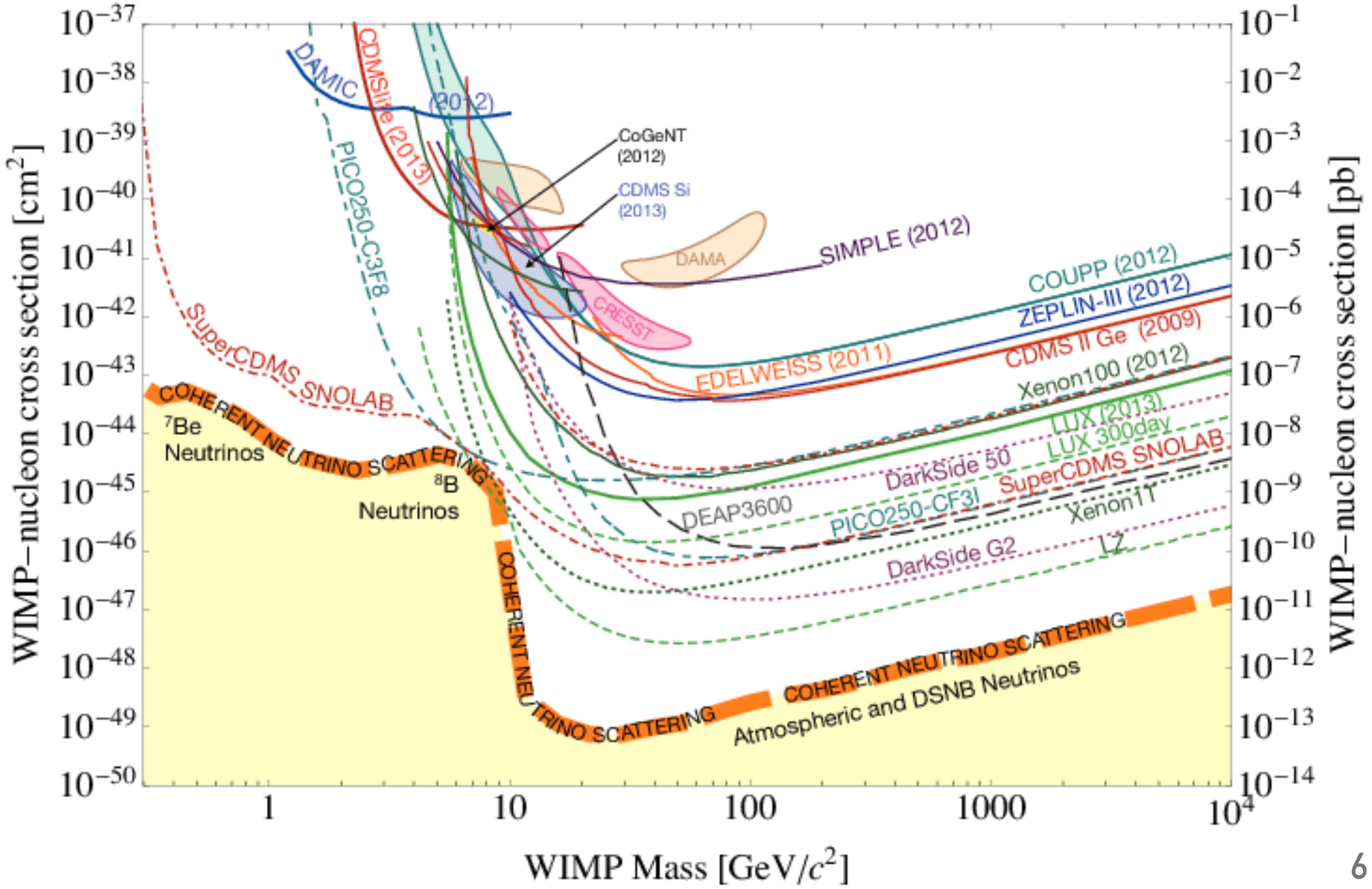
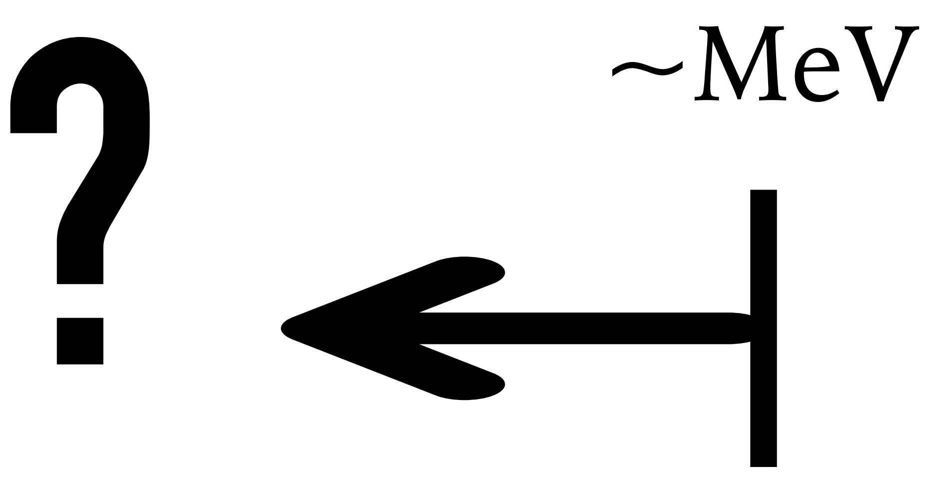
  - Semiconductors (gap $\sim$ eV).
  - Reach down to DM mass$\sim$MeV.



Proposed meV-gap targets (somewhat futuristic)
[Superconductors]
Hochberg, Zhao, Zurek, 1504.07237.
Hochberg, Pyle, Zhao, Zurek, 1512.04533.
[Dirac materials]
Hochberg et al, 1708.08929.
Geilhufe, Kahlhoefer, Winkler, 1910.02091.
Coskuner, Mitridate, Olivares, Zurek, 1909.09170.

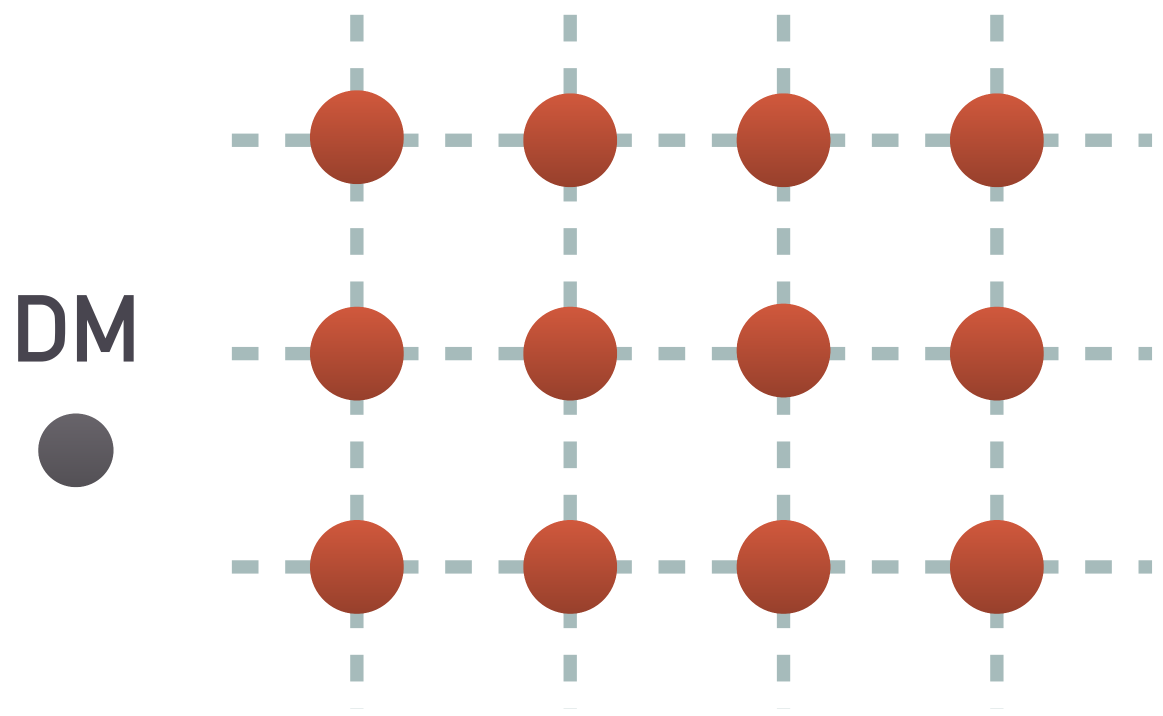




Toward lighter DM in direct detection

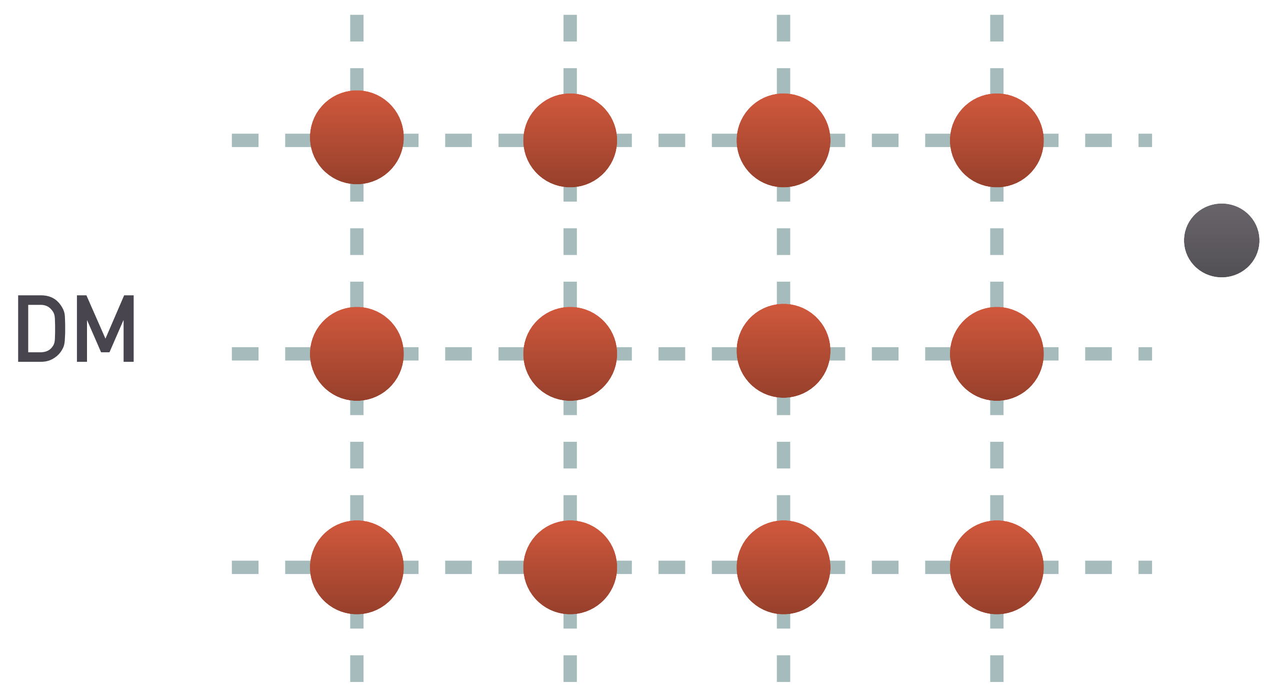
- Collective excitations (sub-eV energies).
  - Phonons in crystals.
    - Collective oscillations of ions.

Knapen, Lin, Pyle, Zurek, 1712.06598.
Griffin, Knapen, Lin, Zurek, 1807.10291.
Griffin, Inzani, Trickle, ZZ, Zurek, 1910.08092 + 1910.10716.
Campbell-Deem, Cox, Knapen, Lin, Melia, 1911.03482.
Griffin, Hochberg, Inzani, Kurinsky, Lin, Yu, 2008.08560.



Toward lighter DM in direct detection

- Collective excitations (sub-eV energies).
  - Phonons in crystals.
    - Collective oscillations of ions.
    - Experiment in active R&D.



Snowmass2021 - Letter of Interest

The TESSERACT Dark Matter Project

Thematic Areas:

- IF1 Quantum Sensors
- IF8 Noble Elements
- CF1 Dark Matter: Particle-like
- CF2 Dark Matter: Wavelike

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TESSERACT Collaboration

Authors:
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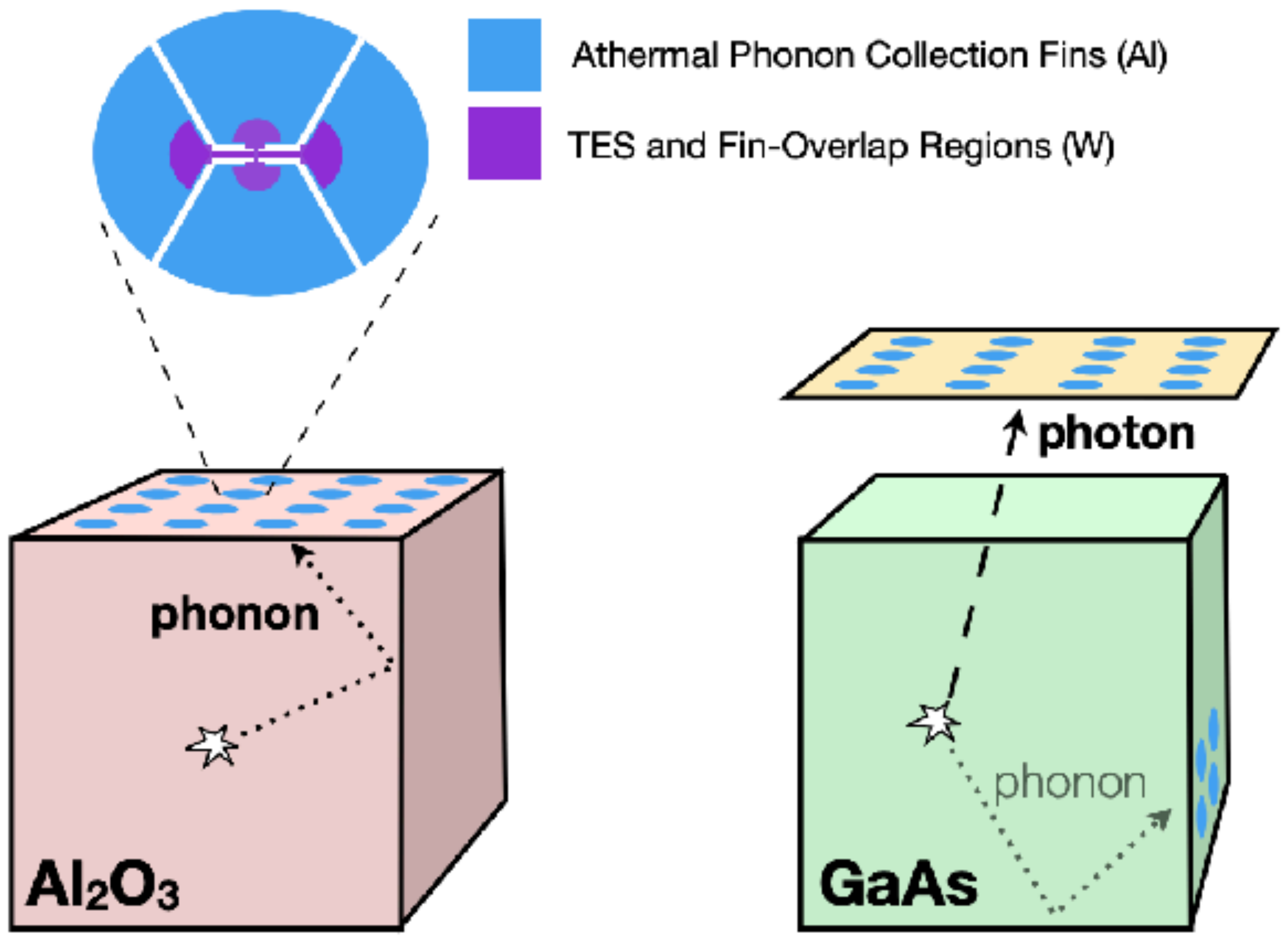


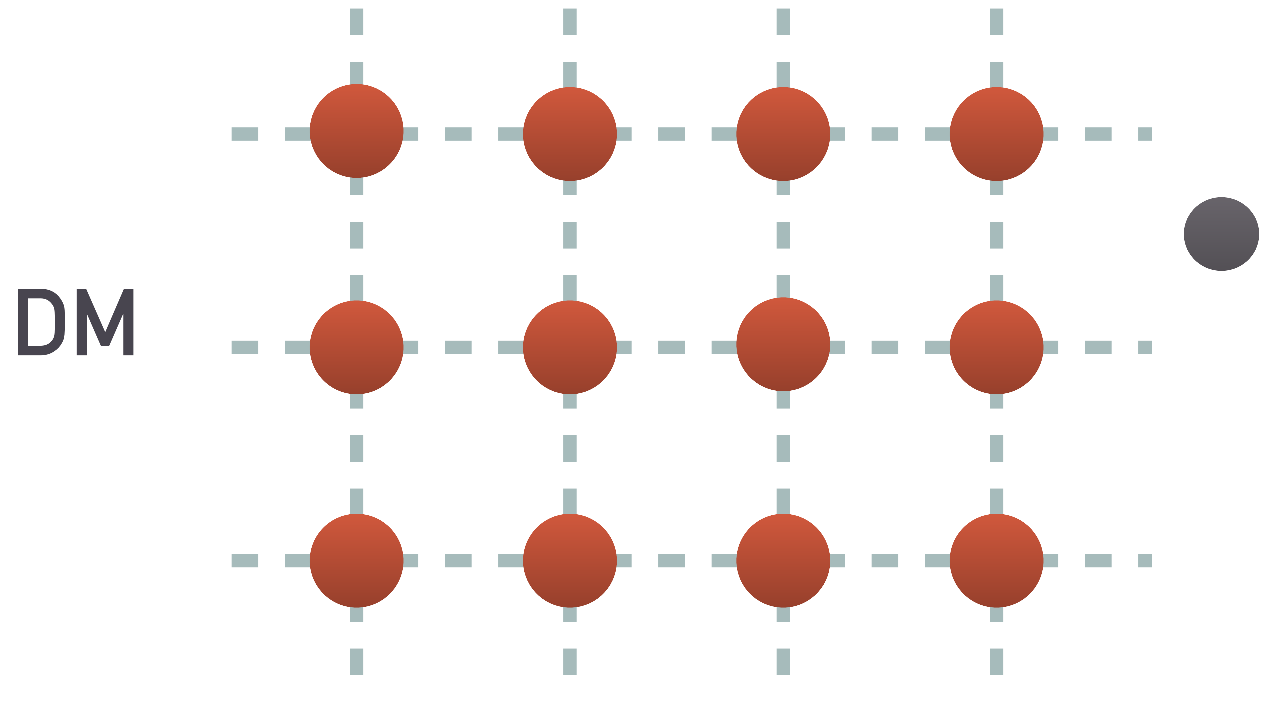
Toward lighter DM in direct detection

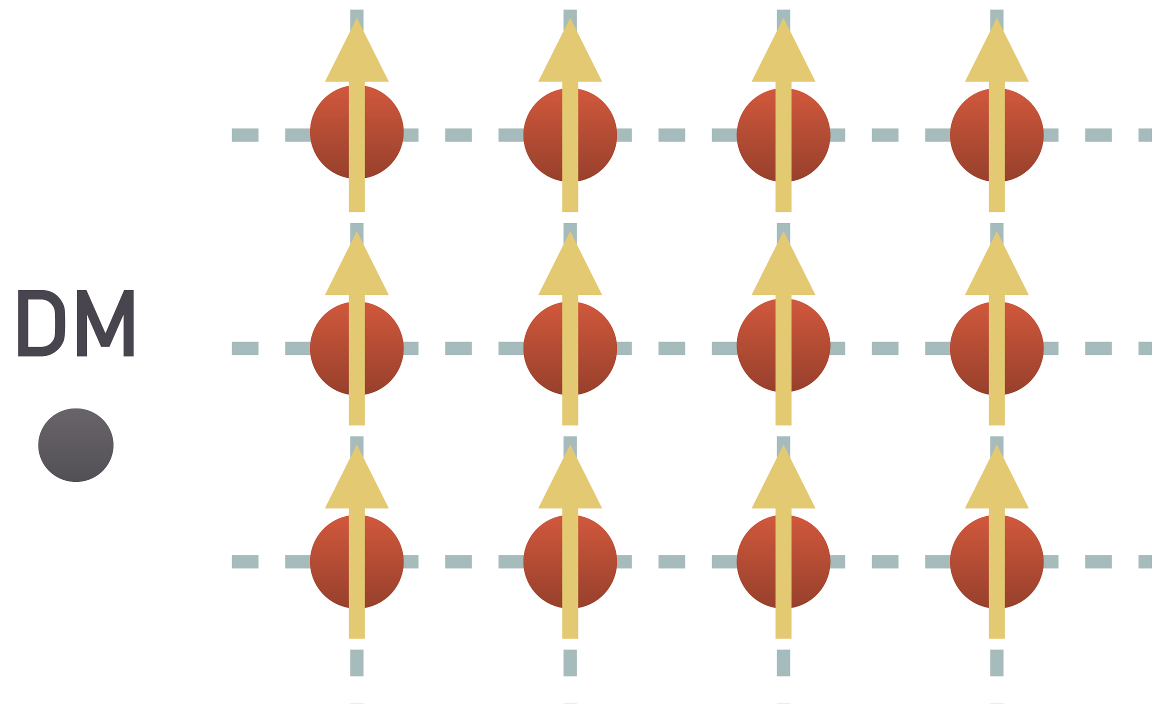
- Collective excitations (sub-eV energies).
  - Phonons in crystals.
    - Collective oscillations of ions.
    - Experiment in active R&D.
  - Magnons.
    - Collective spin excitations in magnetically ordered materials.

Trickle, ZZ, Zurek, 1905.13744.

Also discussed for axion detection.
Chigusa, Moroi, Nakayama, 2001.10666.
Mitridate, Trickle, ZZ, Zurek, 2005.10256.

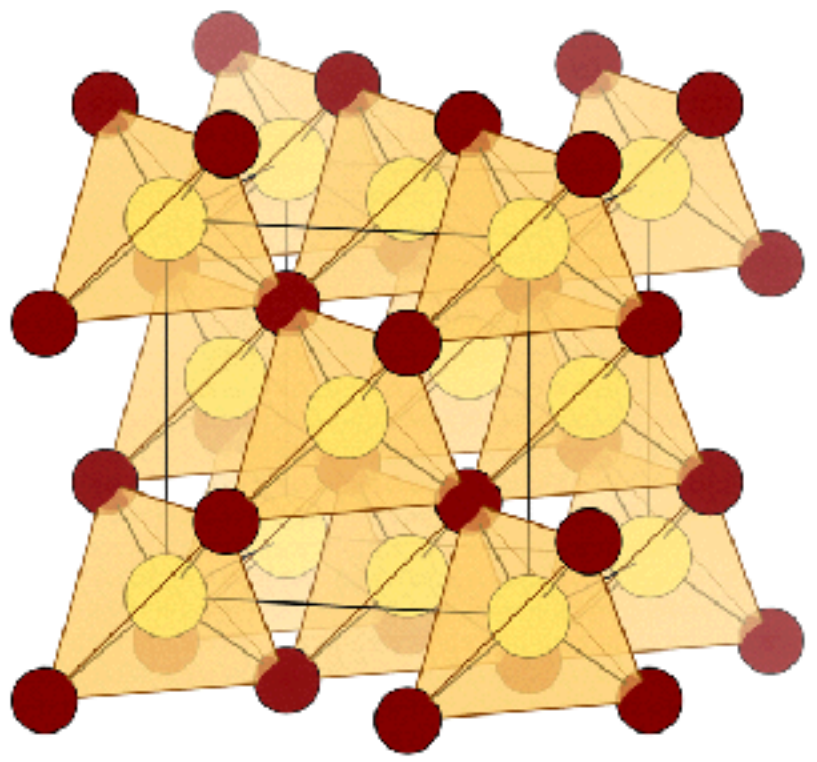
Ongoing experiment: QUAX (1511.09461, 1606.02201, 1806.00310, 1903.06547, 2001.08940) — cannot yet achieve single magnon sensitivity.

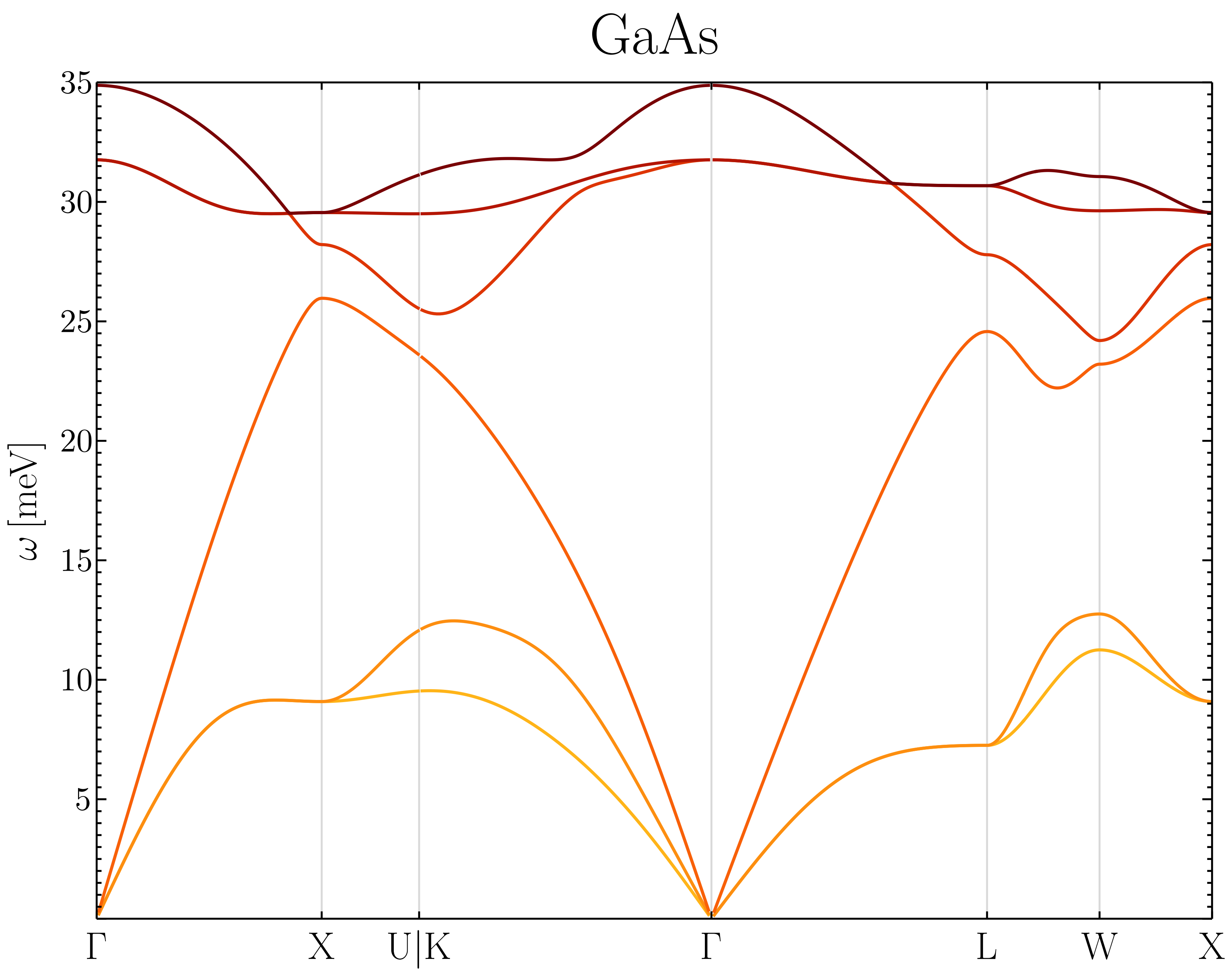




Phonons in crystals

- Examples:
  - GaAs: 1 Ga + 1 As per primitive cell => 3 acoustic + 3 optical.





Griffin, Inzani, Trickle, ZZ, Zurek, 1910.10716.

Phonons in crystals

- Examples:
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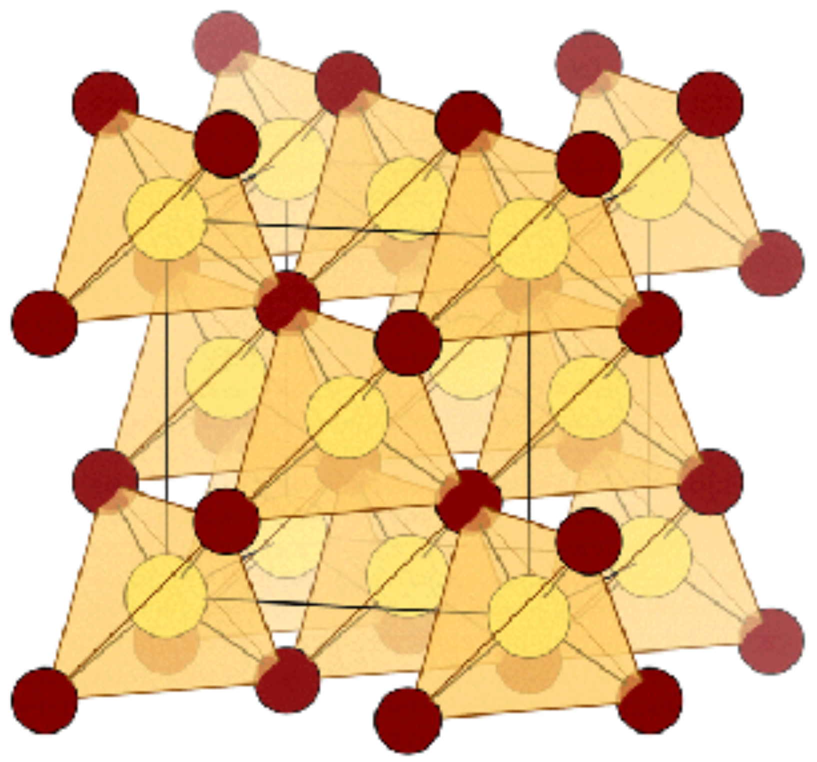
    - Acoustic phonons: in-phase oscillations (gapless).



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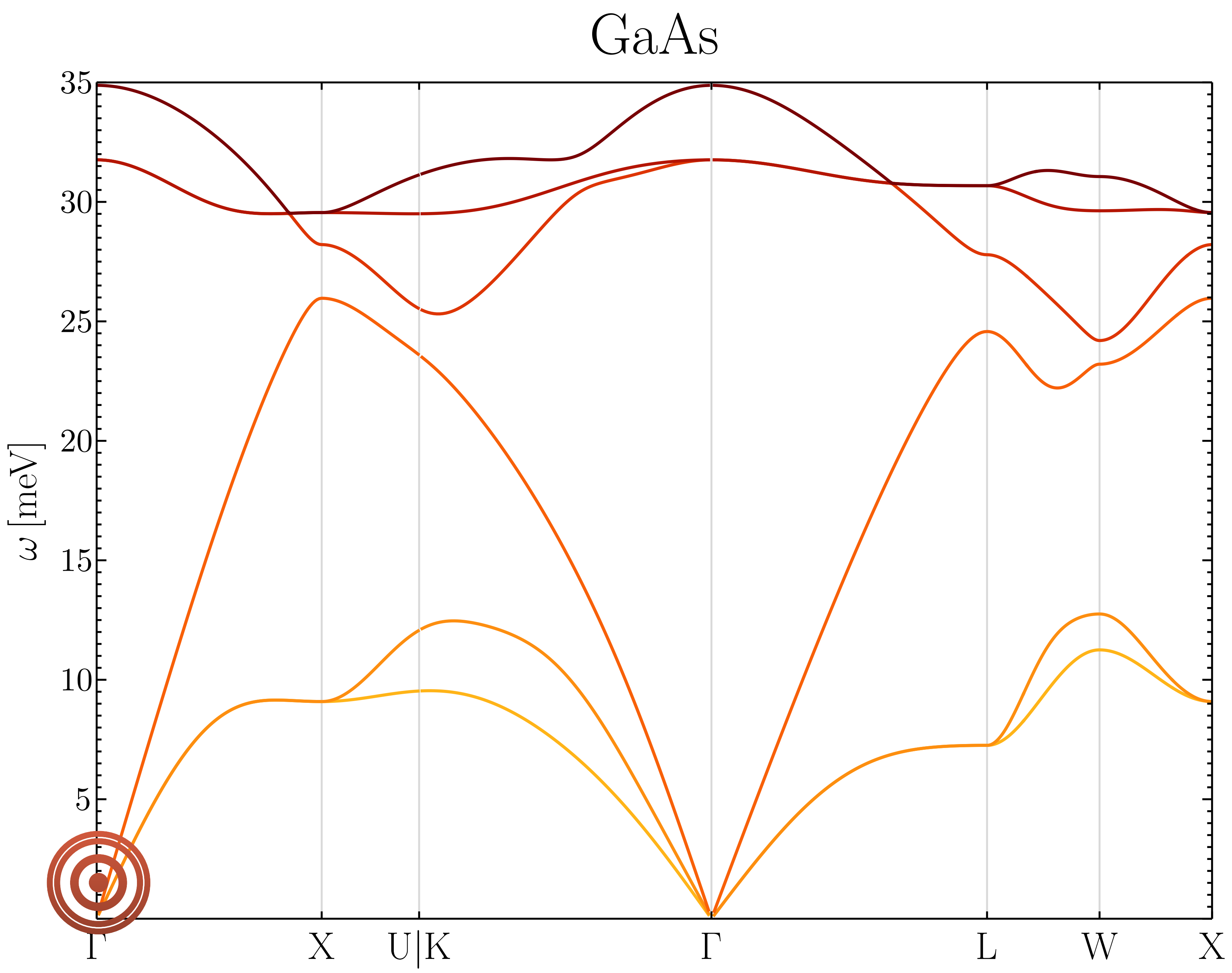
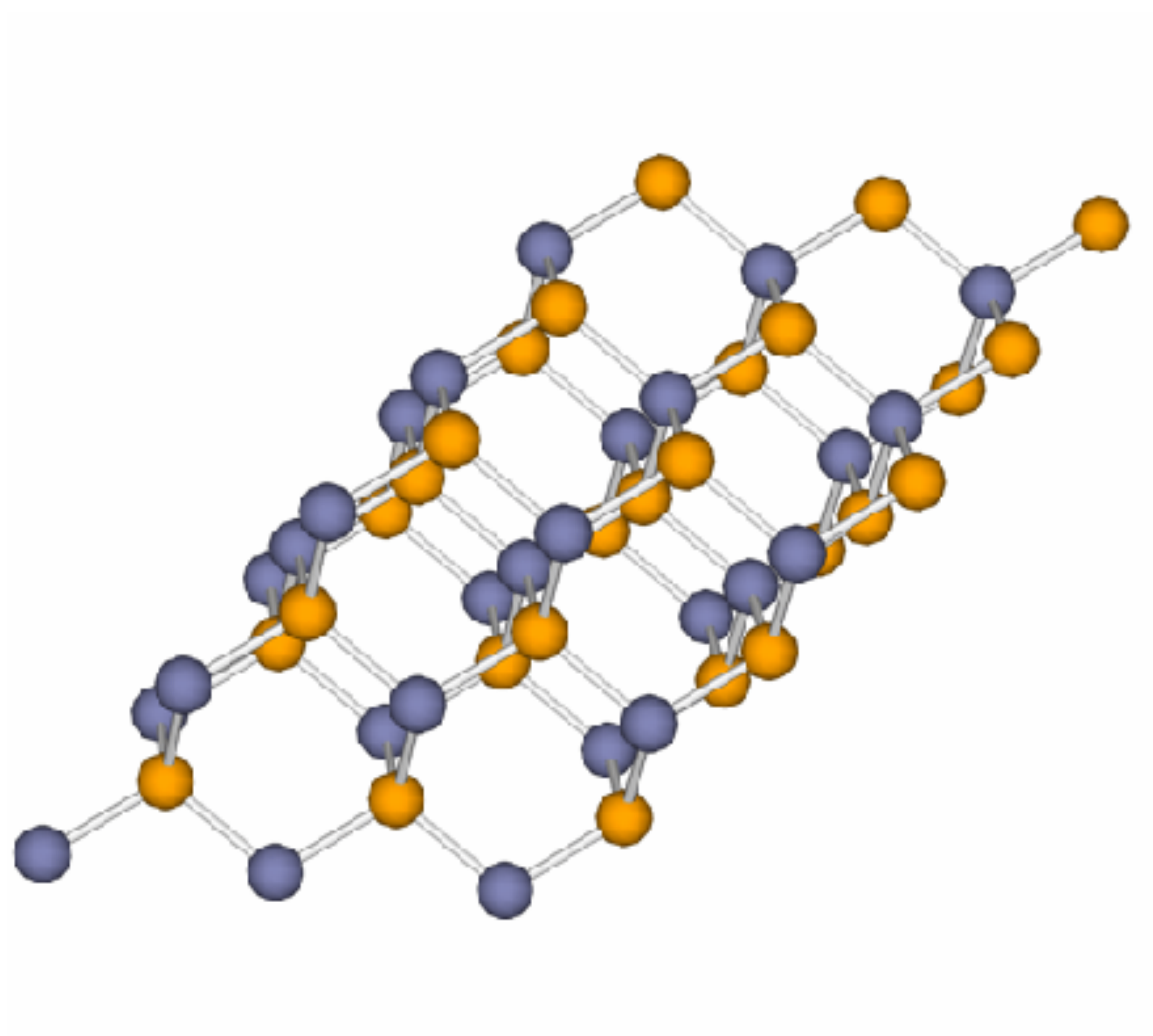
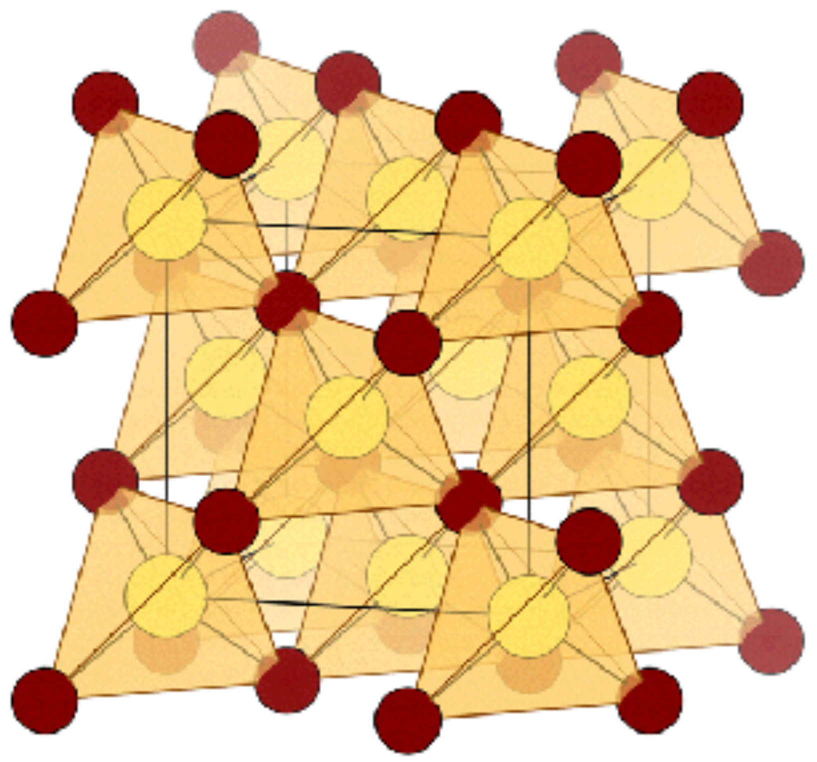




Animation from: henriquemiranda.github.io.

Phonons in crystals

- Examples:
  - GaAs: 1 Ga + 1 As per primitive cell => 3 acoustic + 3 optical.
    - Acoustic phonons: in-phase oscillations (gapless).
    - Optical phonons: out-of-phase oscillations (gapped).



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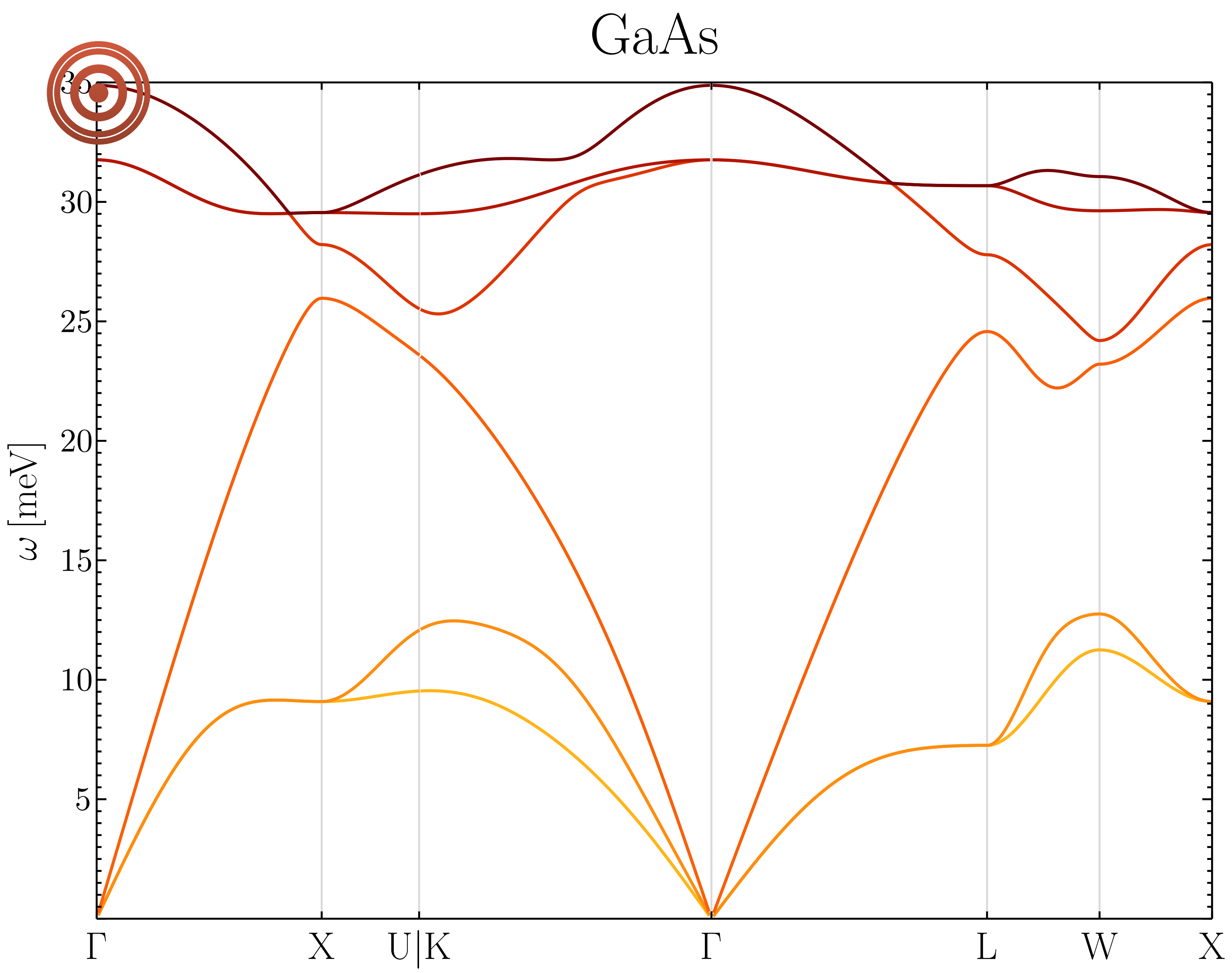
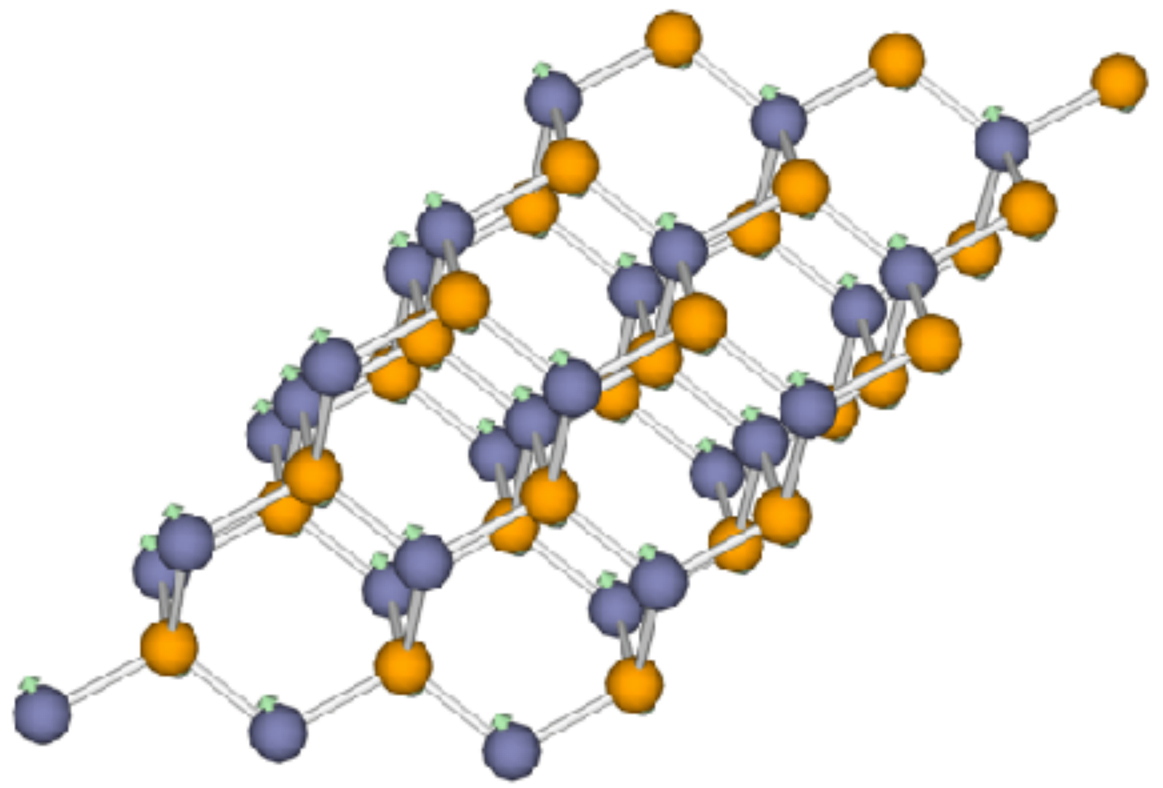
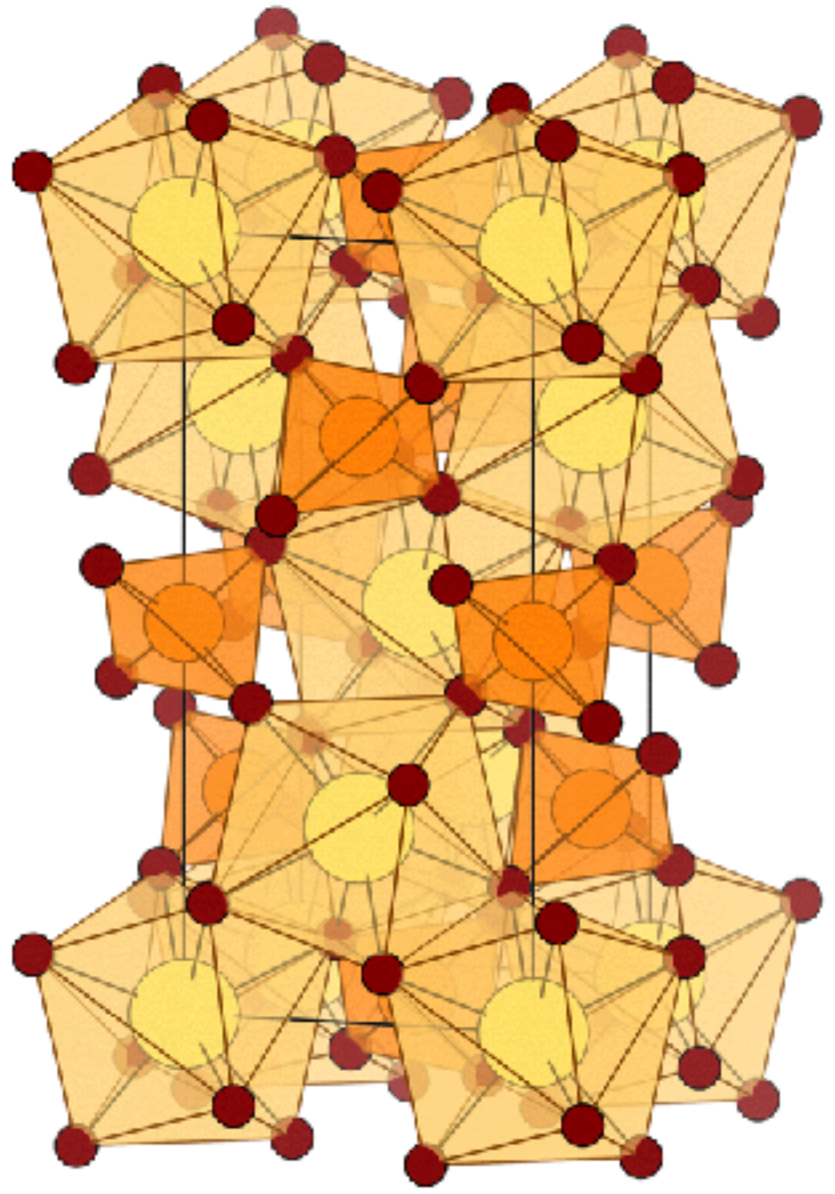


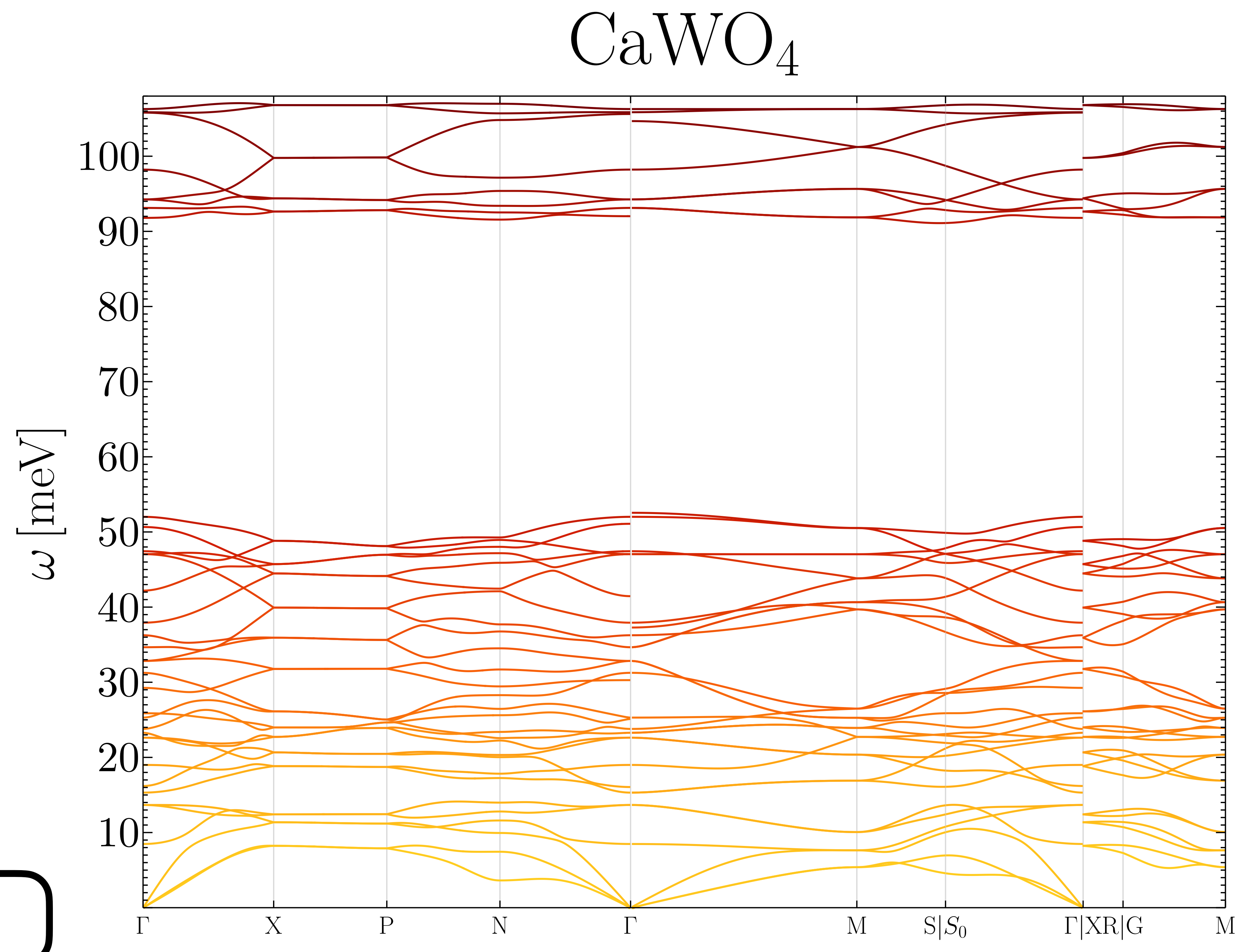


Animation from: henriquemiranda.github.io.

Phonons in crystals

- Examples:
  - GaAs: 1 Ga + 1 As per primitive cell => 3 acoustic + 3 optical.
  - $CaWO_4$: 2 Ca + 2 W + 8 O per primitive cell => 3 acoustic + 33 optical.





Griffin, Inzani, Trickle, ZZ, Zurek, 1910.10716.

Phonons in crystals

- Concretely, **phonons** arise from **lattice displacements**.

$$\boldsymbol{u}_{lj} = \boldsymbol{x}_{lj} - \boldsymbol{x}^0_{lj} = \sum_{\nu=1}^{3n} \sum_{\boldsymbol{k}\in 1\mathrm{BZ}} \frac{1}{\sqrt{2N m_j \omega_{\nu,\boldsymbol{k}}}} \left(\hat{a}_{\nu,\boldsymbol{k}} \boldsymbol{\epsilon}_{\nu,\boldsymbol{k},j} e^{i\boldsymbol{k}\cdot\boldsymbol{x}^0_{lj}} + \hat{a}^\dagger_{\nu,\boldsymbol{k}} \boldsymbol{\epsilon}^*_{\nu,\boldsymbol{k},j} e^{-i\boldsymbol{k}\cdot\boldsymbol{x}^0_{lj}} \right)$$

  - We compute crystal Hamiltonians using density functional theory (DFT).
  - Then solve the eigensystem using the phonopy program.

- DM detection rate:

$$\Gamma(\boldsymbol{v}) = \frac{1}{\Omega} \int \frac{d^3q}{(2\pi)^3} \sum_{\nu=1}^{3n} 2\pi\, \delta\left(\omega_{\nu,\boldsymbol{k}} - \omega_{\boldsymbol{q}}\right) \frac{1}{2\omega_{\nu,\boldsymbol{k}}} \left| \sum_j e^{-W_j(\boldsymbol{q})} e^{i\boldsymbol{G}\cdot\boldsymbol{x}^0_j} \frac{\boldsymbol{q}\cdot\boldsymbol{\epsilon}^*_{\nu,\boldsymbol{k},j}}{\sqrt{m_j}} \widetilde{\mathcal{V}}_j(-\boldsymbol{q},\boldsymbol{v}) \right|^2$$

DM-ion couplings

sum over phonon branches

phonon eigenenergies & eigenvectors

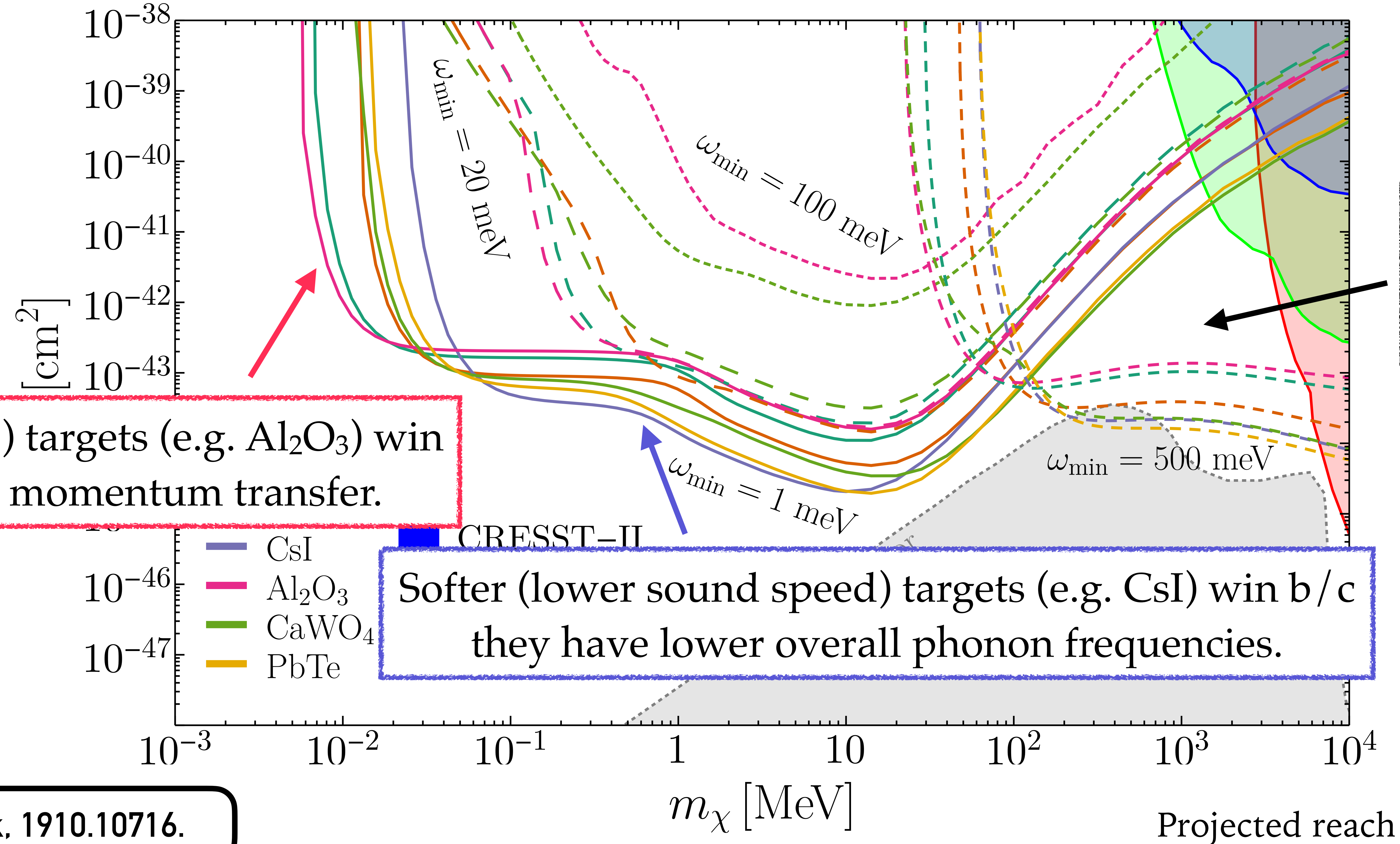
sum over individual ion amplitudes

reminiscent of a harmonic oscillator

- **Magnon** calculation is somewhat similar, but involves expanding **ionic spin operators** around ordered ground state (Holstein-Primakoff transformation).

Target comparison: acoustic vs. optical phonons

- If DM couples to all ions with the **same sign**, it would dominantly excite **acoustic** phonons (in-phase oscillations).
  - Example: coupling to nucleon number via a heavy scalar mediator.



Harder (higher sound speed) targets (e.g. Al_2O_3) win b/c they can reach lower momentum transfer.

Softer (lower sound speed) targets (e.g. CsI) win b/c they have lower overall phonon frequencies.

Phonon reach weakens. Transition to nuclear recoil description (dashed curves).

Griffin, Inzani, Trickle, ZZ, Zurek, 1910.10716.

Projected reach assumes 3 events/kg-yr.

Target comparison: acoustic vs. optical phonons

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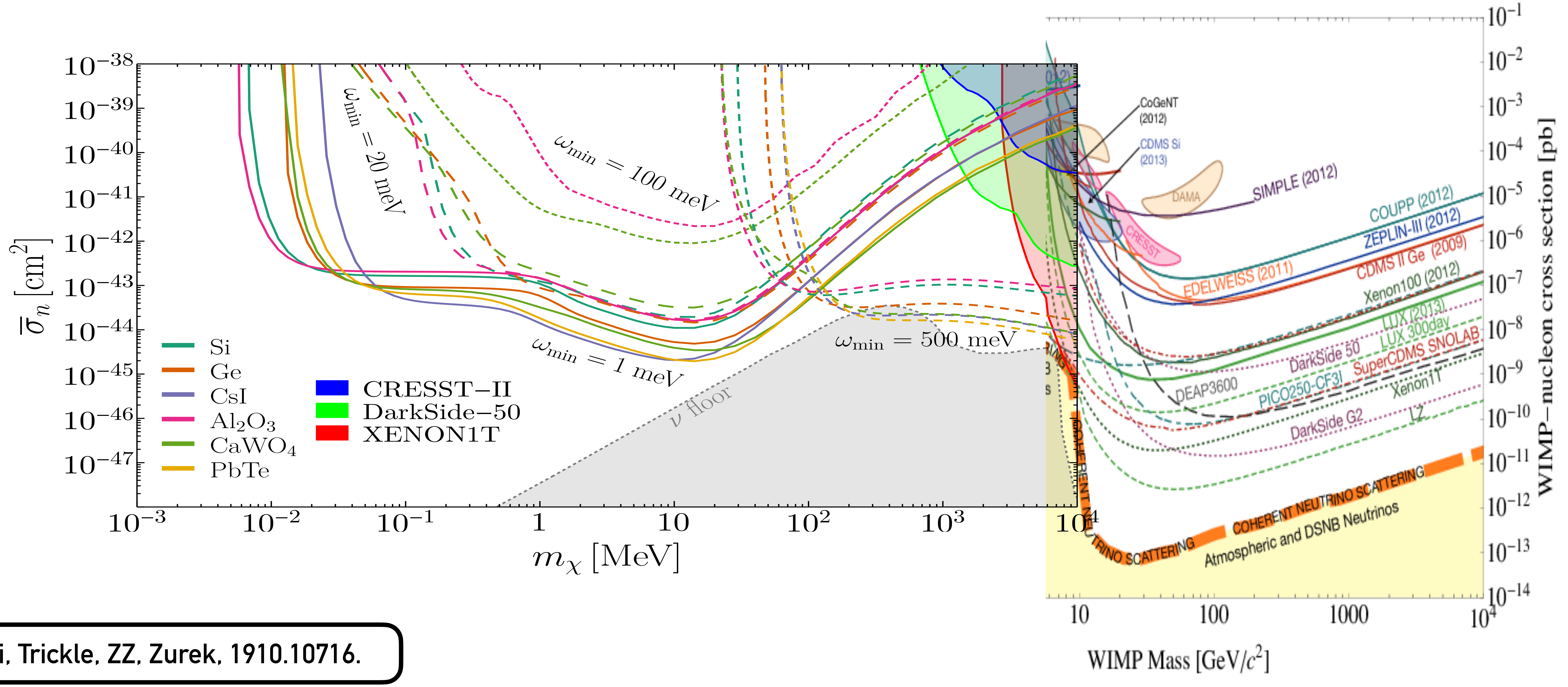
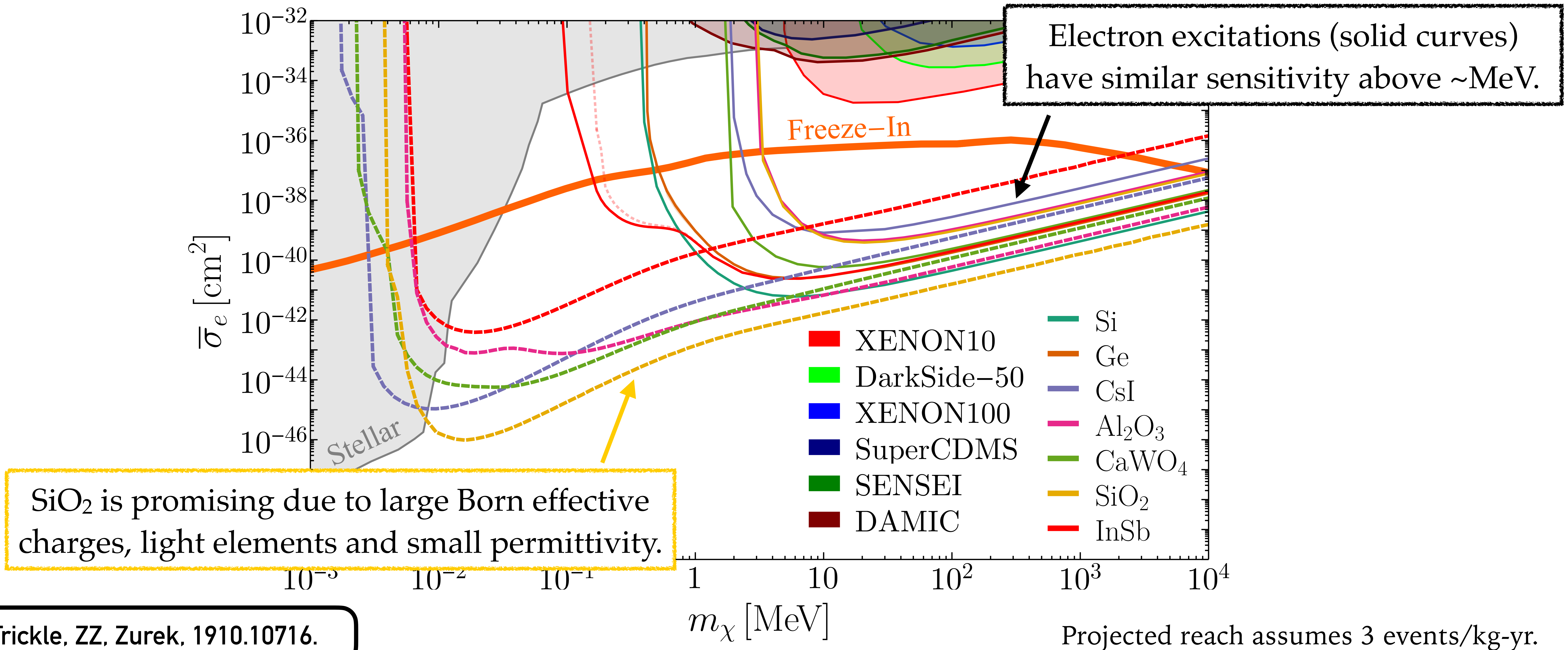


Griffin, Inzani, Trickle, ZZ, Zurek, 1910.10716.

Target comparison: acoustic vs. optical phonons

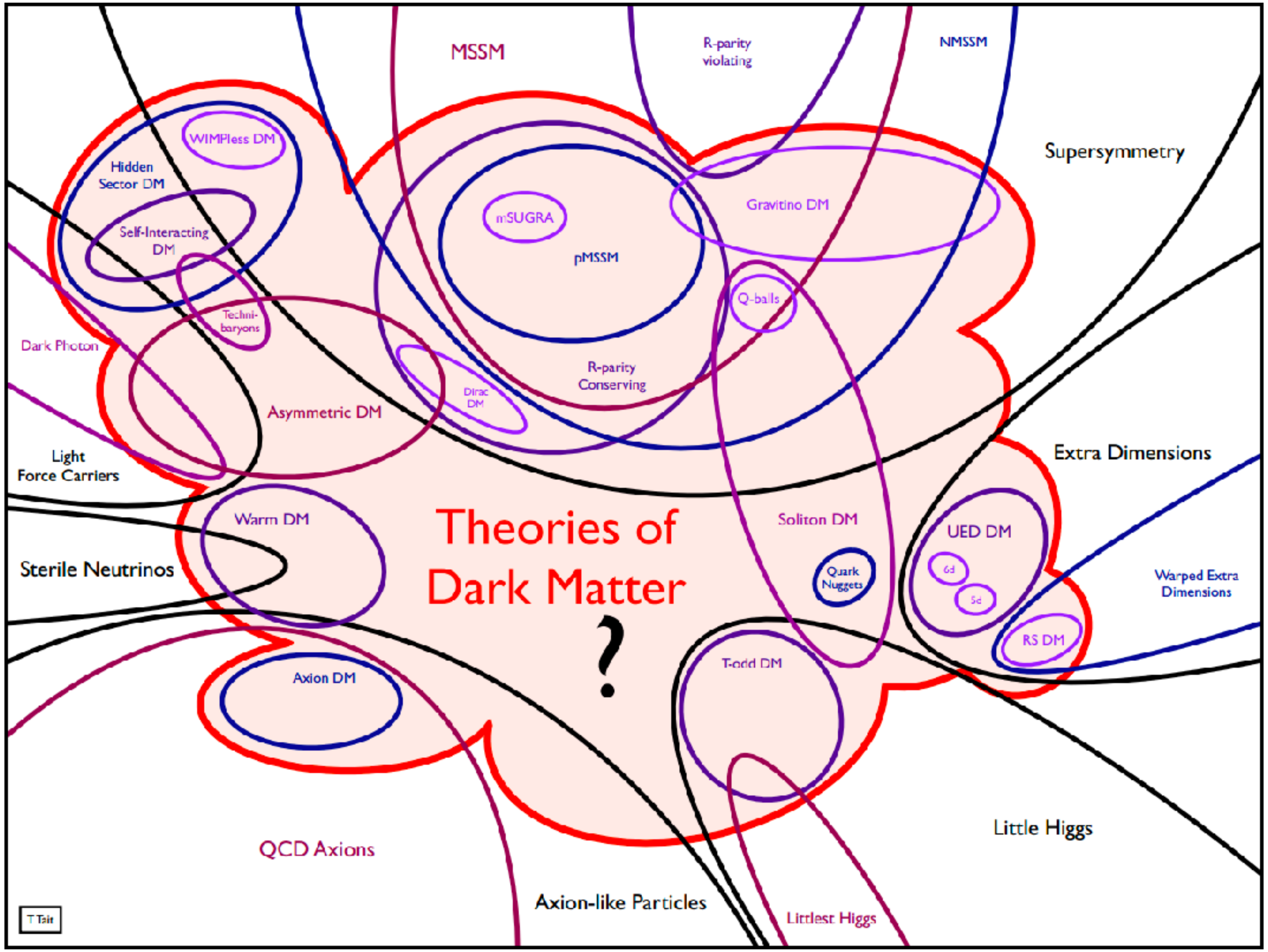
- If DM couples to all ions with **opposite signs**, it would dominantly excite **optical** phonons (out-of-phase oscillations).
  - Example: coupling to electric charge via a light dark photon mediator.



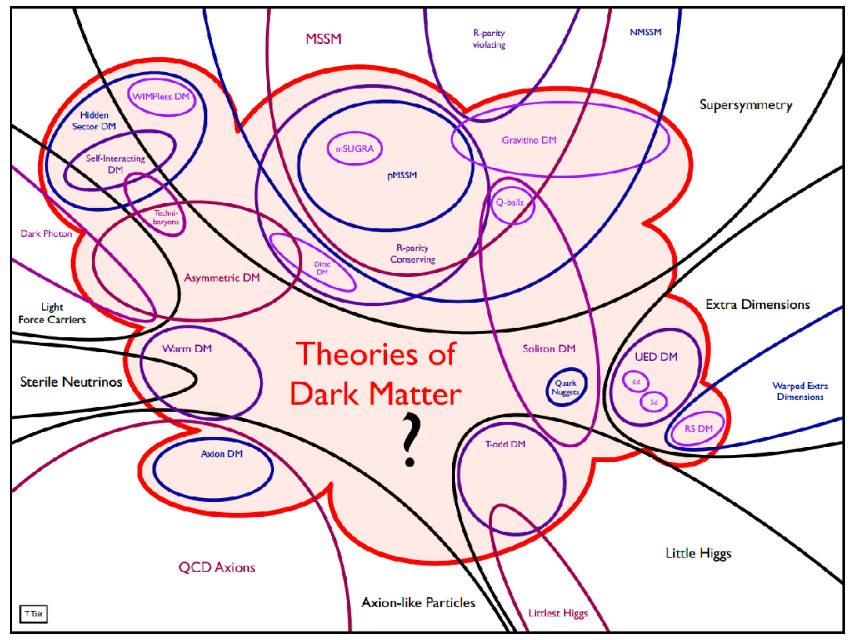
Griffin, Inzani, Trickle, ZZ, Zurek, 1910.10716.

Projected reach assumes 3 events/kg-yr.

Beyond these simplest benchmarks…



A common description at low energy



Nonrelativistic (NR) EFT of DM-SM interactions

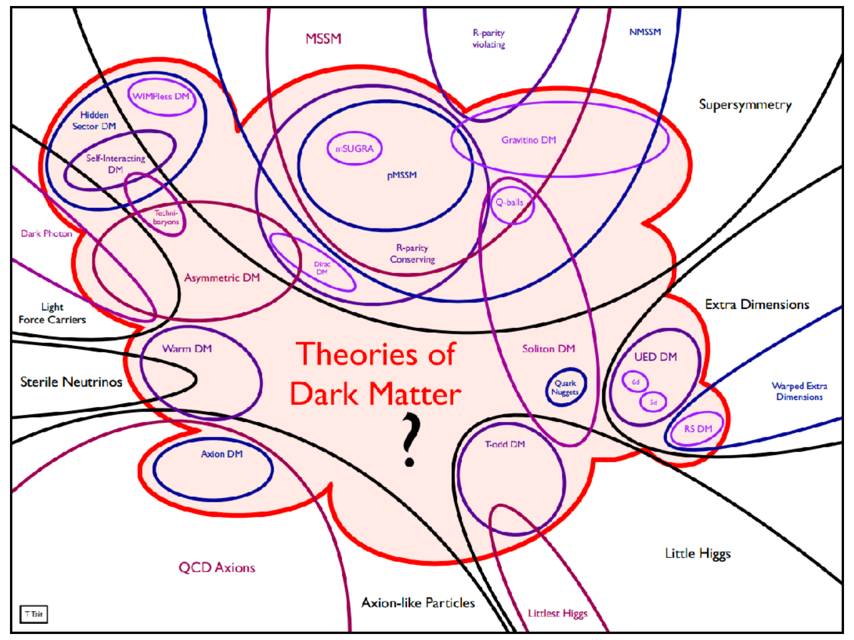
$$\mathcal{O}_1^{(\psi)} = \mathbb{1}$$
$$\mathcal{O}_{11}^{(\psi)} = \boldsymbol{S}_\chi \cdot \frac{i\boldsymbol{q}}{m_\psi}$$
$$\mathcal{O}_5^{(\psi)} = \boldsymbol{S}_\chi \cdot \left(\frac{i\boldsymbol{q}}{m_\psi} \times \boldsymbol{v}^\perp\right)$$
$$\mathcal{O}_8^{(\psi)} = \boldsymbol{S}_\chi \cdot \boldsymbol{v}^\perp$$

$$\mathcal{O}_3^{(\psi)} = \boldsymbol{S}_\psi \cdot \left(\frac{i\boldsymbol{q}}{m_\psi} \times \boldsymbol{v}^\perp\right)$$
$$\mathcal{O}_7^{(\psi)} = \boldsymbol{S}_\psi \cdot \boldsymbol{v}^\perp$$
$$\mathcal{O}_{12}^{(\psi)} = \boldsymbol{S}_\chi \cdot \left(\boldsymbol{S}_\psi \times \boldsymbol{v}^\perp\right)$$
$$\mathcal{O}_{13}^{(\psi)} = \left(\boldsymbol{S}_\chi \cdot \boldsymbol{v}^\perp\right)\left(\boldsymbol{S}_\psi \cdot \frac{i\boldsymbol{q}}{m_\psi}\right)$$
$$\mathcal{O}_{14}^{(\psi)} = \left(\boldsymbol{S}_\psi \cdot \boldsymbol{v}^\perp\right)\left(\boldsymbol{S}_\chi \cdot \frac{i\boldsymbol{q}}{m_\psi}\right)$$
$$\mathcal{O}_{15}^{(\psi)} = \left(\boldsymbol{S}_\chi \cdot \left(\frac{i\boldsymbol{q}}{m_\psi} \times \boldsymbol{v}^\perp\right)\right)\left(\boldsymbol{S}_\psi \cdot \frac{i\boldsymbol{q}}{m_\psi}\right)$$

$$\mathcal{O}_4^{(\psi)} = \boldsymbol{S}_\chi \cdot \boldsymbol{S}_\psi$$
$$\mathcal{O}_6^{(\psi)} = \left(\boldsymbol{S}_\chi \cdot \frac{\boldsymbol{q}}{m_\psi}\right)\left(\boldsymbol{S}_\psi \cdot \frac{\boldsymbol{q}}{m_\psi}\right)$$
$$\mathcal{O}_9^{(\psi)} = \boldsymbol{S}_\chi \cdot \left(\boldsymbol{S}_\psi \times \frac{i\boldsymbol{q}}{m_\psi}\right)$$
$$\mathcal{O}_{10}^{(\psi)} = \boldsymbol{S}_\psi \cdot \frac{i\boldsymbol{q}}{m_\psi}$$

EFT of DM direct detection w/ phonons & magnons

Trickle, ZZ, Zurek, 2009.13534.



Nonrelativistic (NR) EFT of DM-SM interactions

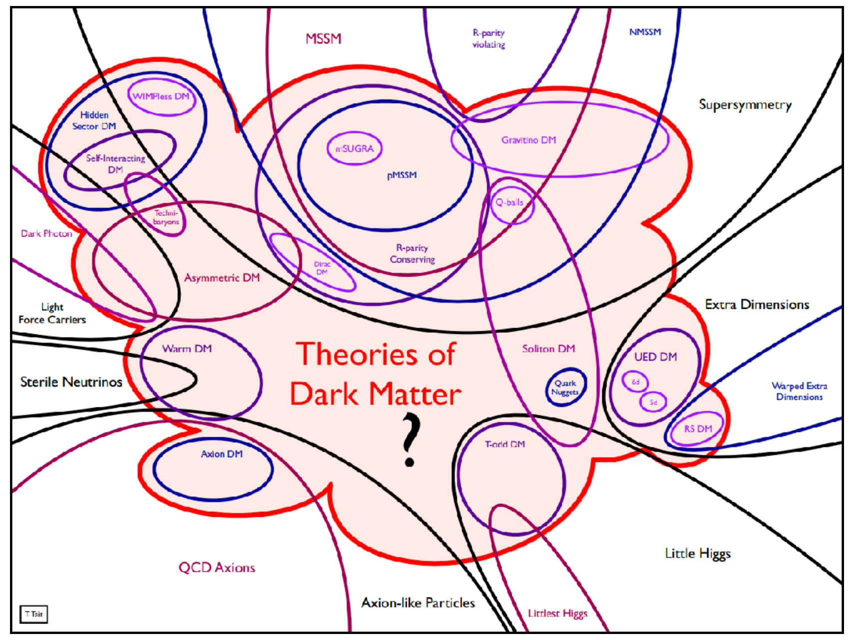
Crystal responses

DM couplings to lattice d.o.f.

$\boldsymbol{N}$ (particle number)	$\boldsymbol{S}$ (spin)
$\boldsymbol{L}$ (orbital angular momentum)	$\boldsymbol{L} \otimes \boldsymbol{S}$ (spin-orbit coupling)

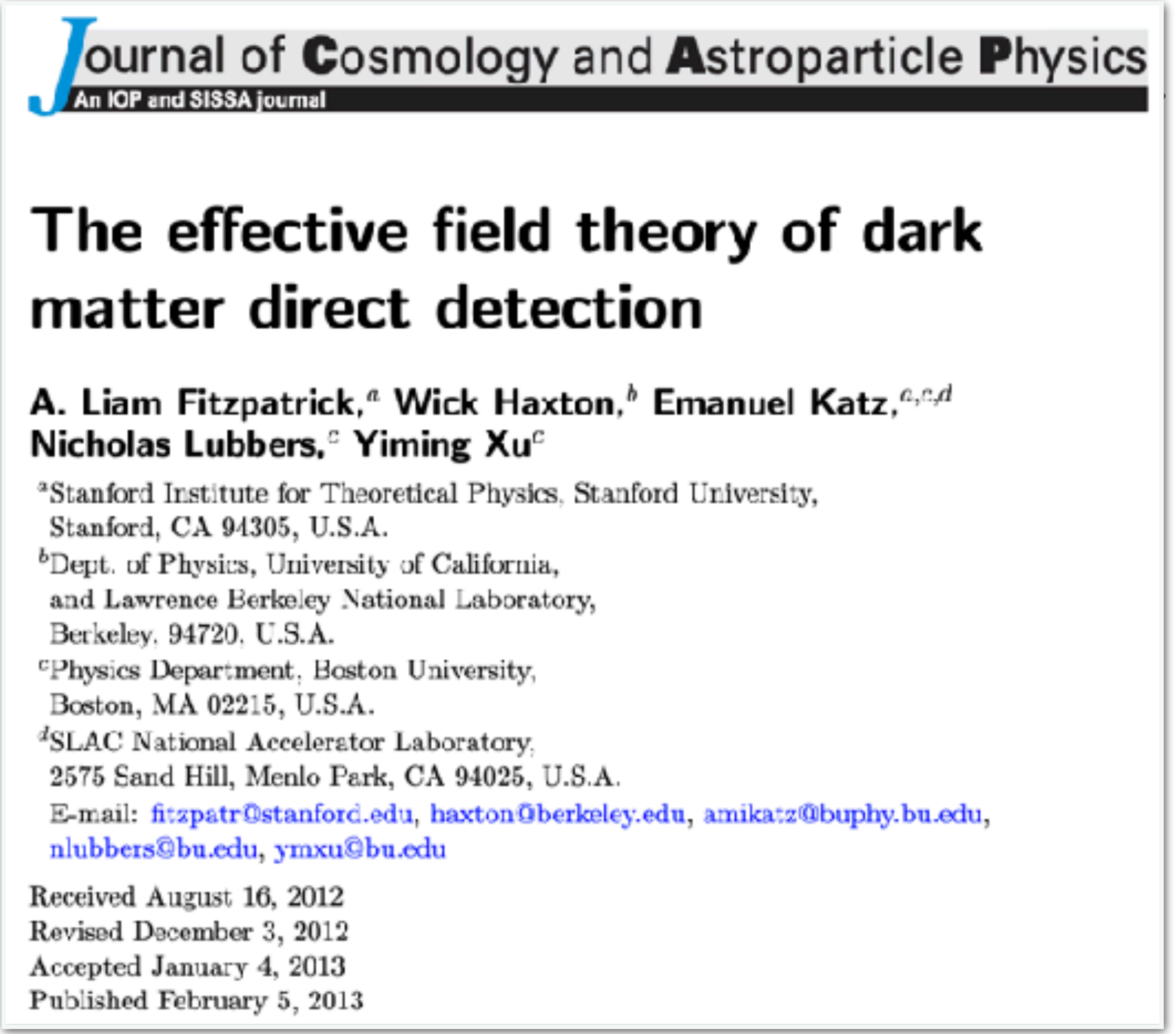
EFT of DM direct detection w/ phonons & magnons

Trickle, ZZ, Zurek, 2009.13534.



Similar situation in nuclear recoil calculations.

- At first, just spin-independent (SI) and spin-dependent (SD) benchmarks.
- Later on, extended to EFT.
- UV model ⇒ EFT ⇒ nuclear responses ⇒ rates.


Journal of Cosmology and Astroparticle Physics
An IOP and SISSA journal

The effective field theory of dark matter direct detection

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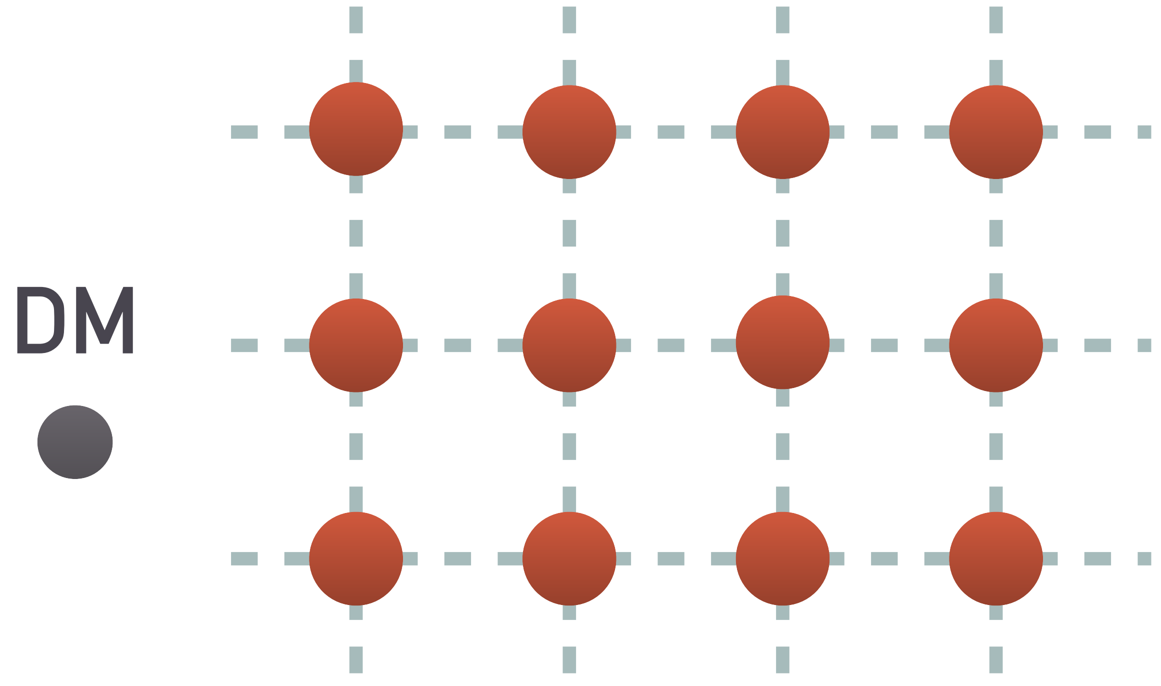
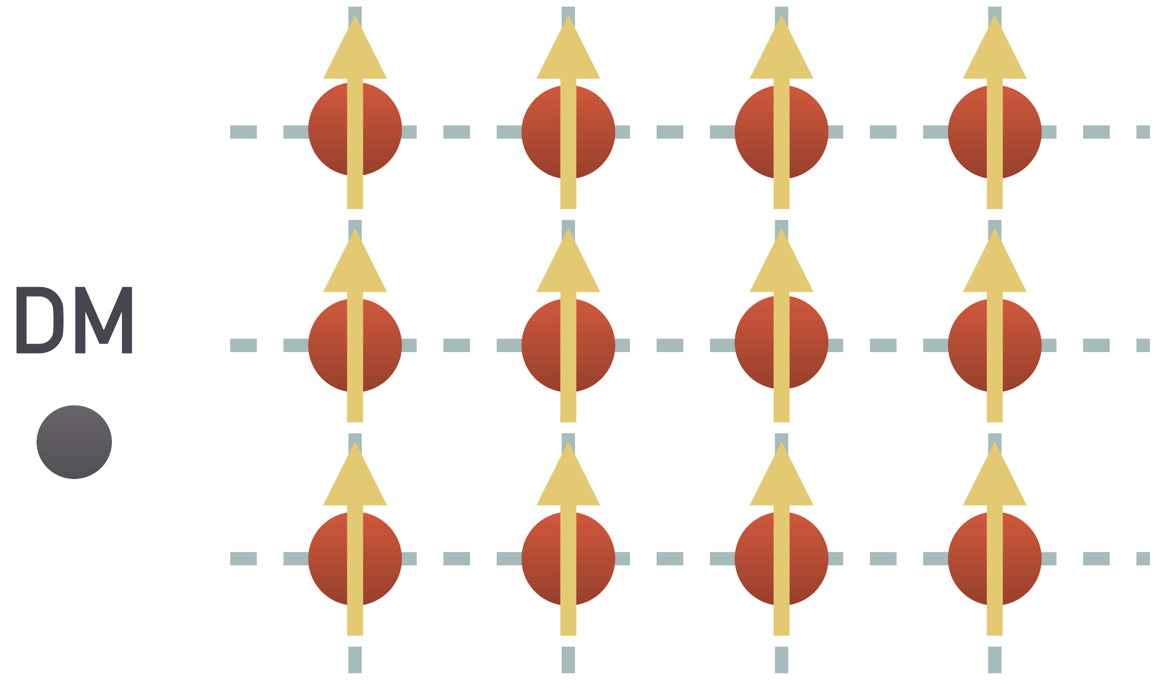
Crystal responses

Nonrelativistic (NR) EFT of DM-SM interactions

Phonon & magnon excitation rates



(via N, S, L, $L\otimes S$)



(via S_e, L_e)

See also:
Cirelli, Del Nobile, Panci, 1307.5955.
Anand, Fitzpatrick, Haxton, 1308.6288 + 1405.6690.
Gresham, Zurek, 1401.3739.
Del Nobile, 1806.01291.

Similar calculation for electron excitations in atoms:
Catena, Emken, Spaldin, Tarantino, 1912.08204.

Example

Trickle, ZZ, Zurek, 2009.13534.

- Dark photon mediator models.

$$\mathcal{L} \supset -g_e V_\mu J^\mu_{\rm EM} + \dots$$

  - Several possibilities on how the DM couples.

Millicharged DM (SI)

$$g_\chi V_\mu \bar{\chi}\gamma^\mu \chi$$

$$\Rightarrow \quad c_1^{(\psi)} = -\frac{g_\chi g_\psi^{\rm eff}}{\boldsymbol{q}^2 + m_V^2} \qquad \text{(standard SI)}$$

where $g_e^{\rm eff} = \frac{\boldsymbol{q}^2}{\boldsymbol{q}\cdot\boldsymbol{\varepsilon}\cdot\boldsymbol{q}}\, g_e = -g_p^{\rm eff}$ (screened couplings)

Interaction Type	NR Operators
Coupling to *charge*, $\boldsymbol{v}^\perp$-*independent* $\Rightarrow N$	$\mathcal{O}_1^{(\psi)} = \mathbb{1}$ $\mathcal{O}_{11}^{(\psi)} = \boldsymbol{S}_\chi \cdot \frac{i\boldsymbol{q}}{m_\psi}$
Coupling to *charge*, $\boldsymbol{v}^\perp$-*dependent* $\Rightarrow N, L$	$\mathcal{O}_5^{(\psi)} = \boldsymbol{S}_\chi \cdot \left(\frac{i\boldsymbol{q}}{m_\psi} \times \boldsymbol{v}^\perp\right)$ $\mathcal{O}_8^{(\psi)} = \boldsymbol{S}_\chi \cdot \boldsymbol{v}^\perp$
Coupling to *spin*, $\boldsymbol{v}^\perp$-*independent* $\Rightarrow S$	$\mathcal{O}_4^{(\psi)} = \boldsymbol{S}_\chi \cdot \boldsymbol{S}_\psi$ $\mathcal{O}_6^{(\psi)} = \left(\boldsymbol{S}_\chi \cdot \frac{\boldsymbol{q}}{m_\psi}\right)\left(\boldsymbol{S}_\psi \cdot \frac{\boldsymbol{q}}{m_\psi}\right)$ $\mathcal{O}_9^{(\psi)} = \boldsymbol{S}_\chi \cdot \left(\boldsymbol{S}_\psi \times \frac{i\boldsymbol{q}}{m_\psi}\right)$ $\mathcal{O}_{10}^{(\psi)} = \boldsymbol{S}_\psi \cdot \frac{i\boldsymbol{q}}{m_\psi}$
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Example

Trickle, ZZ, Zurek, 2009.13534.

- Dark photon mediator models.

$$\mathcal{L} \supset -g_e V_\mu J^\mu_{\rm EM} + \dots$$

  - Several possibilities on how the DM couples.

Millicharged DM (SI)

$$c_1^{(\psi)} = -\frac{g_\chi g_\psi^{\rm eff}}{\boldsymbol{q}^2 + m_V^2}$$

Electric dipole DM

$$\frac{g_\chi}{4m_\chi} V_{\mu\nu}\bar{\chi}\sigma^{\mu\nu} i\gamma^5 \chi$$

$$\Rightarrow \quad c_{11}^{(\psi)} = -\frac{m_\psi}{m_\chi}\frac{g_\chi g_\psi^{\rm eff}}{\boldsymbol{q}^2 + m_V^2}$$

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where $\tilde{\mu}_p^{\rm eff} \simeq 1 + 1.8\,(\hat{\boldsymbol{q}} \cdot \boldsymbol{\varepsilon}_\infty \cdot \hat{\boldsymbol{q}})\,, \quad \tilde{\mu}_e^{\rm eff} \simeq 1$

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Anapole DM

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Crystal responses

$$\begin{aligned}
\widetilde{\mathcal{V}}_{lj}(-\boldsymbol{q},\boldsymbol{v}) = \sum_{\psi=p,n,e} & c_1^{(\psi)}\langle N_\psi\rangle_{lj} \\
&+ c_3^{(\psi)}\left[-\frac{iq}{m_\psi}\boldsymbol{v}'\cdot\left(\hat{\boldsymbol{q}}\times\langle \boldsymbol{S}_\psi\rangle_{lj}\right) + \frac{q^2}{2m_\psi^2}\left(\delta^{ik}-\hat{q}^i\hat{q}^k\right)\left(\langle \boldsymbol{L}_\psi\otimes\boldsymbol{S}_\psi\rangle_{lj}\right)^{ik}\right] \\
&+ c_4^{(\psi)}\boldsymbol{S}_\chi\cdot\langle \boldsymbol{S}_\psi\rangle_{lj} \\
&+ c_5^{(\psi)}\left[\frac{i\boldsymbol{q}}{m_\psi}\cdot\left(\boldsymbol{v}'\times\boldsymbol{S}_\chi\right)\langle N_\psi\rangle_{lj} + \frac{q^2}{2m_\psi^2}\boldsymbol{S}_\chi\cdot\left(\mathbb{1}-\hat{\boldsymbol{q}}\hat{\boldsymbol{q}}\right)\cdot\langle \boldsymbol{L}_\psi\rangle_{lj}\right] \\
&+ c_6^{(\psi)}\frac{q^2}{m_\psi^2}\left(\hat{\boldsymbol{q}}\cdot\boldsymbol{S}_\chi\right)\left(\hat{\boldsymbol{q}}\cdot\langle \boldsymbol{S}_\psi\rangle_{lj}\right) \\
&+ c_7^{(\psi)}\left[\boldsymbol{v}'\cdot\langle \boldsymbol{S}_\psi\rangle_{lj} + \epsilon^{ikk'}\frac{iq^{k'}}{2m_\chi}\left(\langle \boldsymbol{L}_\psi\otimes\boldsymbol{S}_\psi\rangle_{lj}\right)^{ik}\right] \\
&+ c_8^{(\psi)}\left[\left(\boldsymbol{v}'\cdot\boldsymbol{S}_\chi\right)\langle N_\psi\rangle_{lj} + \frac{iq}{2m_\psi}\boldsymbol{S}_\chi\cdot\left(\hat{\boldsymbol{q}}\times\langle \boldsymbol{L}_\psi\rangle_{lj}\right)\right] \\
&+ c_9^{(\psi)}\frac{iq}{m_\psi}\boldsymbol{S}_\chi\cdot\left(\langle \boldsymbol{S}_\psi\rangle_{lj}\times\hat{\boldsymbol{q}}\right) \\
&+ c_{10}^{(\psi)}\frac{i\boldsymbol{q}}{m_\psi}\cdot\langle \boldsymbol{S}_\psi\rangle_{lj} \\
&+ c_{11}^{(\psi)}\frac{i\boldsymbol{q}}{m_\psi}\cdot\boldsymbol{S}_\chi\langle N_\psi\rangle_{lj} \\
&+ c_{12}^{(\psi)}\left[\left(\boldsymbol{v}'\times\boldsymbol{S}_\chi\right)\cdot\langle \boldsymbol{S}_\psi\rangle_{lj} + \frac{iq}{2m_\psi}\left(\left(\hat{\boldsymbol{q}}\cdot\boldsymbol{S}_\chi\right)\delta^{ik}-\hat{q}^k S_\chi^i\right)\left(\langle \boldsymbol{L}_\psi\otimes\boldsymbol{S}_\psi\rangle_{lj}\right)^{ik}\right] \\
&+ c_{13}^{(\psi)}\left[\frac{iq}{m_\psi}\left(\boldsymbol{v}'\cdot\boldsymbol{S}_\chi\right)\left(\hat{\boldsymbol{q}}\cdot\langle \boldsymbol{S}_\psi\rangle_{lj}\right) + \frac{q^2}{2m_\psi^2}\left(\hat{\boldsymbol{q}}\times\boldsymbol{S}_\chi\right)\cdot\langle \boldsymbol{L}_\psi\otimes\boldsymbol{S}_\psi\rangle_{lj}\cdot\hat{\boldsymbol{q}}\right] \\
&+ c_{14}^{(\psi)}\left[\frac{iq}{m_\psi}\left(\hat{\boldsymbol{q}}\cdot\boldsymbol{S}_\chi\right)\left(\boldsymbol{v}'\cdot\langle \boldsymbol{S}_\psi\rangle_{lj}\right) - \epsilon^{ikk'}\frac{q^2}{2m_\psi^2}\hat{q}^{k'}\left(\hat{\boldsymbol{q}}\cdot\boldsymbol{S}_\chi\right)\left(\langle \boldsymbol{L}_\psi\otimes\boldsymbol{S}_\psi\rangle_{lj}\right)^{ik}\right] \\
&+ c_{15}^{(\psi)}\left[-\frac{q^2}{m_\psi^2}\left(\hat{\boldsymbol{q}}\cdot\left(\boldsymbol{v}'\times\boldsymbol{S}_\chi\right)\right)\left(\hat{\boldsymbol{q}}\cdot\langle \boldsymbol{S}_\psi\rangle_{lj}\right)\right. \\
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Anapole DM

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&+ c_6^{(\psi)}\frac{q^2}{m_\psi^2}\left(\hat{\boldsymbol{q}}\cdot\boldsymbol{S}_\chi\right)\left(\hat{\boldsymbol{q}}\cdot\langle\boldsymbol{S}_\psi\rangle_{lj}\right) \\
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$\boldsymbol{S}$, $\boldsymbol{L}$ responses $\Rightarrow$ magnons.

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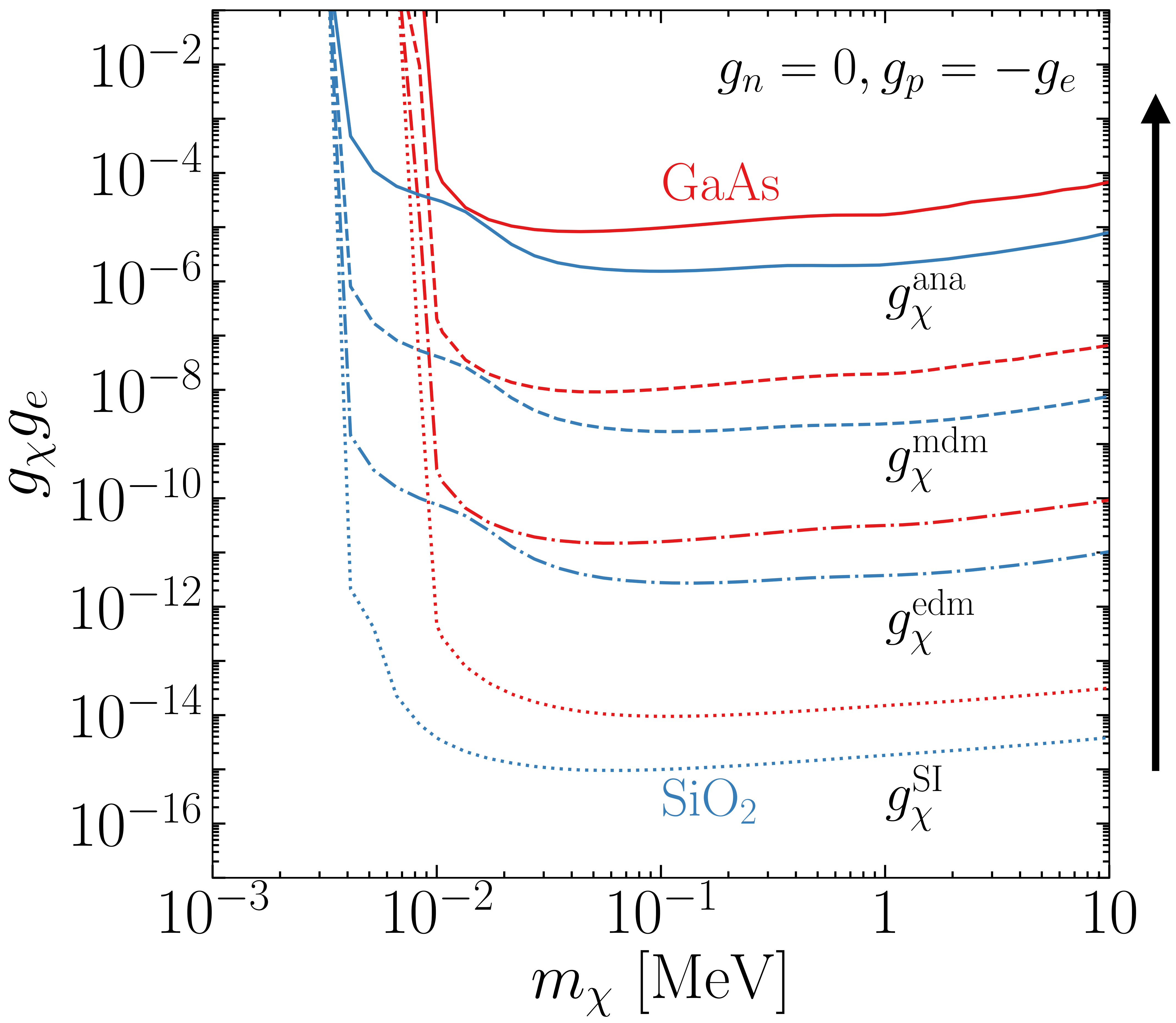
& + c_6^{(\psi)}\frac{q^2}{m_\psi^2}\left(\hat{\boldsymbol{q}}\cdot\boldsymbol{S}_\chi\right)\left(\hat{\boldsymbol{q}}\cdot\langle\boldsymbol{S}_\psi\rangle_{lj}\right) \\
& + c_7^{(\psi)}\left[\boldsymbol{v}'\cdot\langle\boldsymbol{S}_\psi\rangle_{lj} + \epsilon^{ikk'}\frac{iq^{k'}}{2m_\chi}\left(\langle\boldsymbol{L}_\psi\otimes\boldsymbol{S}_\psi\rangle_{lj}\right)^{ik}\right] \\
& + c_8^{(\psi)}\left[\left(\boldsymbol{v}'\cdot\boldsymbol{S}_\chi\right)\langle N_\psi\rangle_{lj} + \frac{iq}{2m_\psi}\boldsymbol{S}_\chi\cdot\left(\hat{\boldsymbol{q}}\times\langle\boldsymbol{L}_\psi\rangle_{lj}\right)\right] \\
& + c_9^{(\psi)}\frac{iq}{m_\psi}\boldsymbol{S}_\chi\cdot\left(\langle\boldsymbol{S}_\psi\rangle_{lj}\times\hat{\boldsymbol{q}}\right) \\
& + c_{10}^{(\psi)}\frac{i\boldsymbol{q}}{m_\psi}\cdot\langle\boldsymbol{S}_\psi\rangle_{lj} \\
& + c_{11}^{(\psi)}\frac{i\boldsymbol{q}}{m_\psi}\cdot\boldsymbol{S}_\chi\langle N_\psi\rangle_{lj} \\
& + c_{12}^{(\psi)}\left[\left(\boldsymbol{v}'\times\boldsymbol{S}_\chi\right)\cdot\langle\boldsymbol{S}_\psi\rangle_{lj} + \frac{iq}{2m_\psi}\left(\left(\hat{\boldsymbol{q}}\cdot\boldsymbol{S}_\chi\right)\delta^{ik}-\hat{q}^kS_\chi^i\right)\left(\langle\boldsymbol{L}_\psi\otimes\boldsymbol{S}_\psi\rangle_{lj}\right)^{ik}\right] \\
& + c_{13}^{(\psi)}\left[\frac{iq}{m_\psi}\left(\boldsymbol{v}'\cdot\boldsymbol{S}_\chi\right)\left(\hat{\boldsymbol{q}}\cdot\langle\boldsymbol{S}_\psi\rangle_{lj}\right) + \frac{q^2}{2m_\psi^2}\left(\hat{\boldsymbol{q}}\times\boldsymbol{S}_\chi\right)\cdot\langle\boldsymbol{L}_\psi\otimes\boldsymbol{S}_\psi\rangle_{lj}\cdot\hat{\boldsymbol{q}}\right] \\
& + c_{14}^{(\psi)}\left[\frac{iq}{m_\psi}\left(\hat{\boldsymbol{q}}\cdot\boldsymbol{S}_\chi\right)\left(\boldsymbol{v}'\cdot\langle\boldsymbol{S}_\psi\rangle_{lj}\right) - \epsilon^{ikk'}\frac{q^2}{2m_\psi^2}\hat{q}^{k'}\left(\hat{\boldsymbol{q}}\cdot\boldsymbol{S}_\chi\right)\left(\langle\boldsymbol{L}_\psi\otimes\boldsymbol{S}_\psi\rangle_{lj}\right)^{ik}\right] \\
& + c_{15}^{(\psi)}\left[-\frac{q^2}{m_\psi^2}\left(\hat{\boldsymbol{q}}\cdot\left(\boldsymbol{v}'\times\boldsymbol{S}_\chi\right)\right)\left(\hat{\boldsymbol{q}}\cdot\langle\boldsymbol{S}_\psi\rangle_{lj}\right)\right. \\
& \qquad \left. + \frac{iq^3}{2m_\psi^3}\boldsymbol{S}_\chi\cdot\left(\mathbb{1}-\hat{\boldsymbol{q}}\hat{\boldsymbol{q}}\right)\cdot\langle\boldsymbol{L}_\psi\otimes\boldsymbol{S}_\psi\rangle_{lj}\cdot\hat{\boldsymbol{q}}\right],
\end{aligned}$$

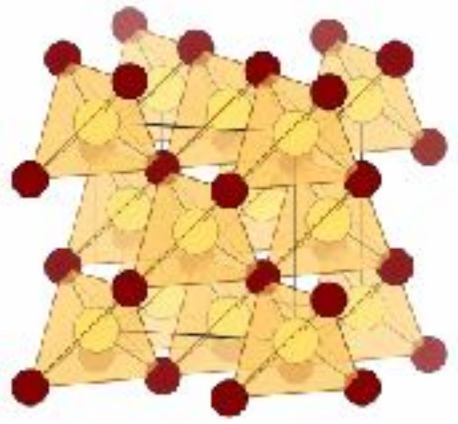
Example

Trickle, ZZ, Zurek, 2009.13534.

- Phonon reach for kg-yr exposure, assuming background-free.

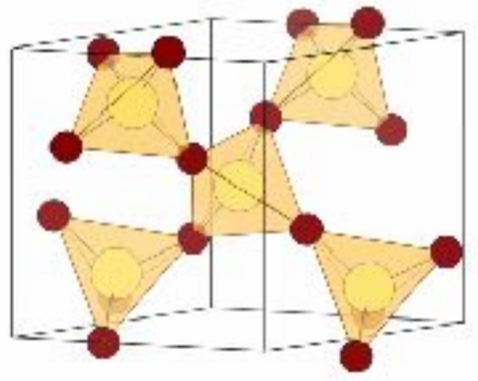


GaAs



Used in the SPICE experiment.

$\mathrm{SiO_2}$



"Optimal" phonon target.

These targets do not have ***S*** or ***L*** order.

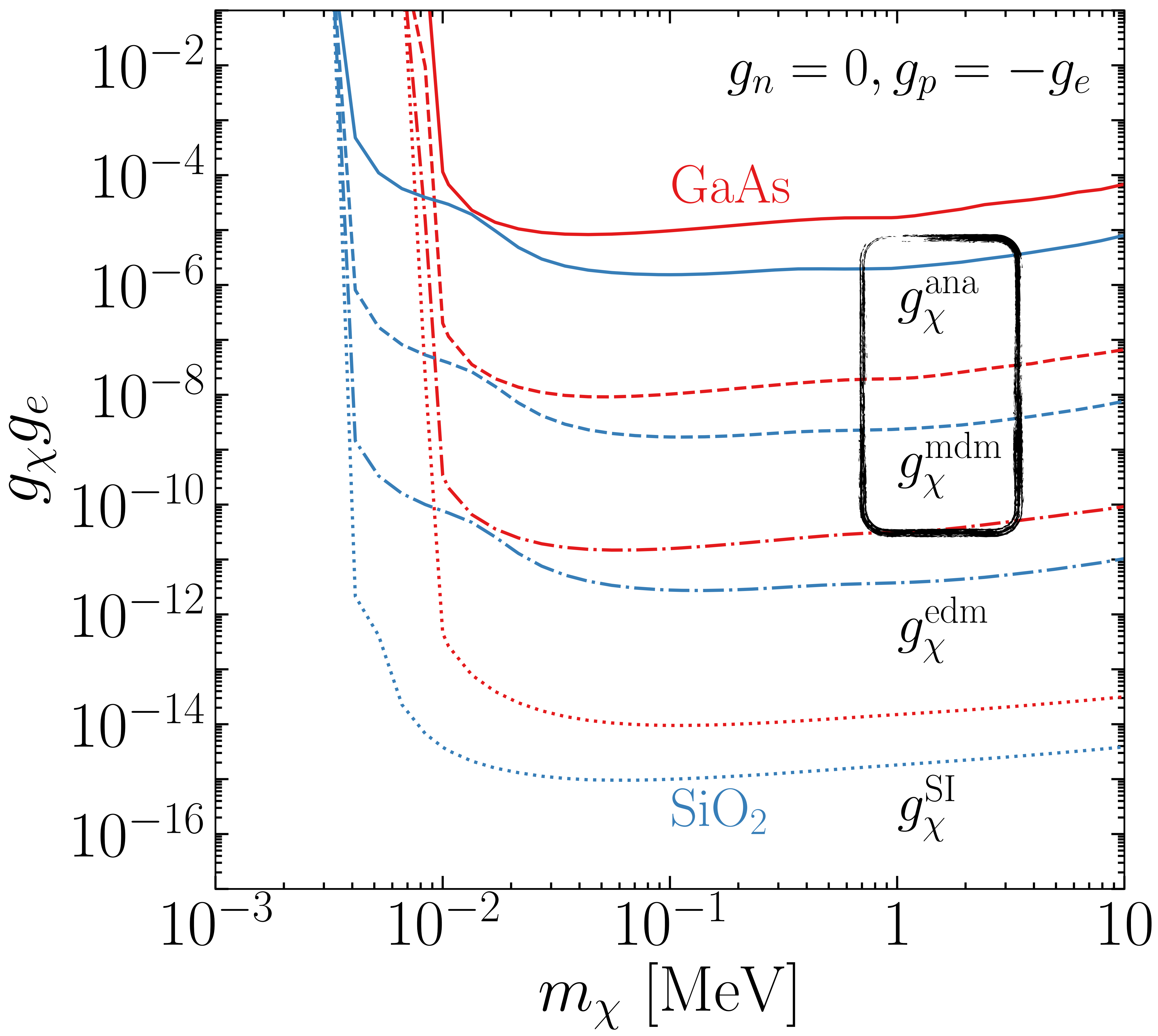
$\Rightarrow$ Phonon excitations only via the N response

(DM couplings to particle numbers).

Example

Trickle, ZZ, Zurek, 2009.13534.

➤ Phonon reach for kg-yr exposure, assuming background-free.



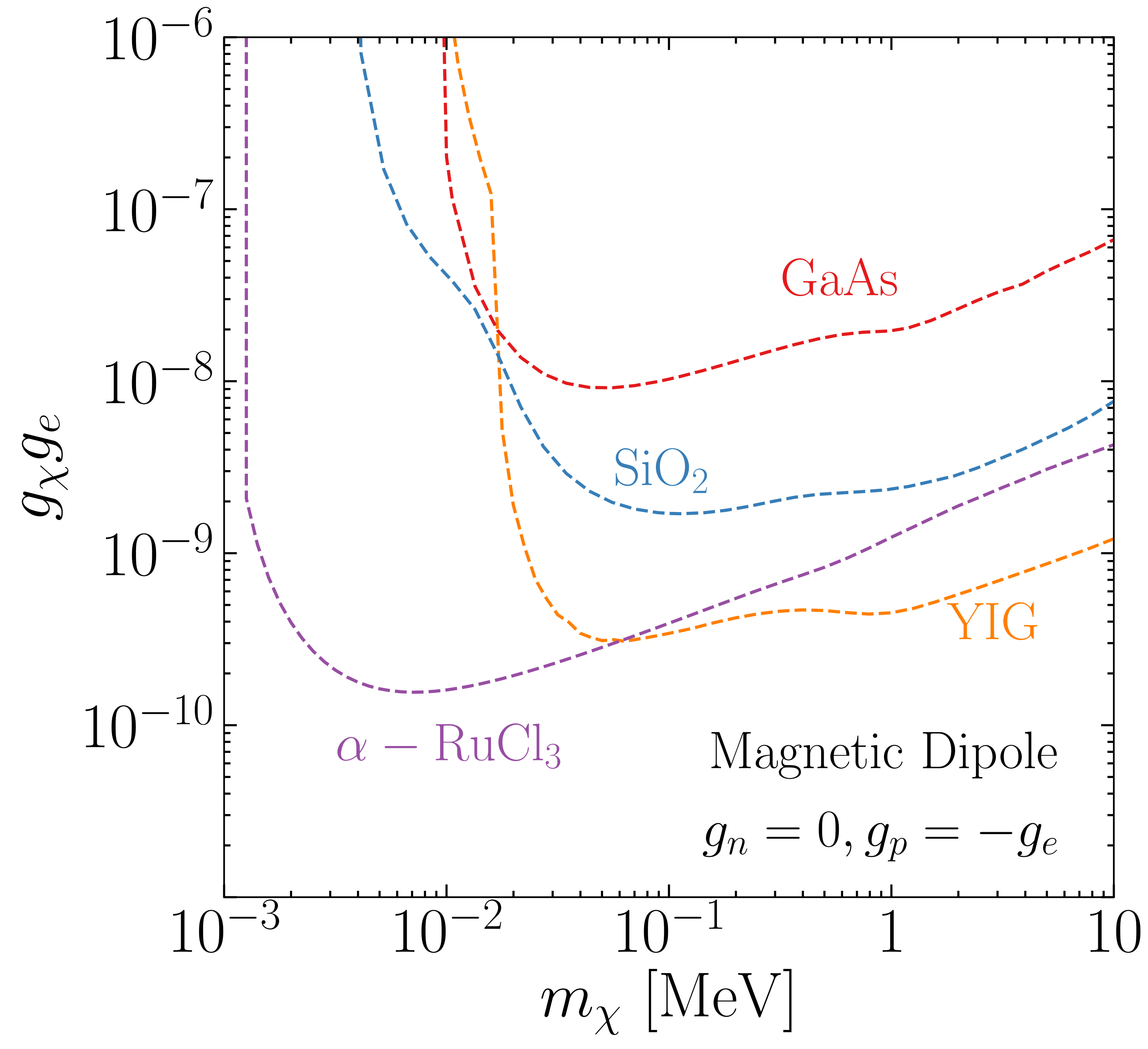
These models also generate couplings to $\boldsymbol{S}$ and $\boldsymbol{L}$.

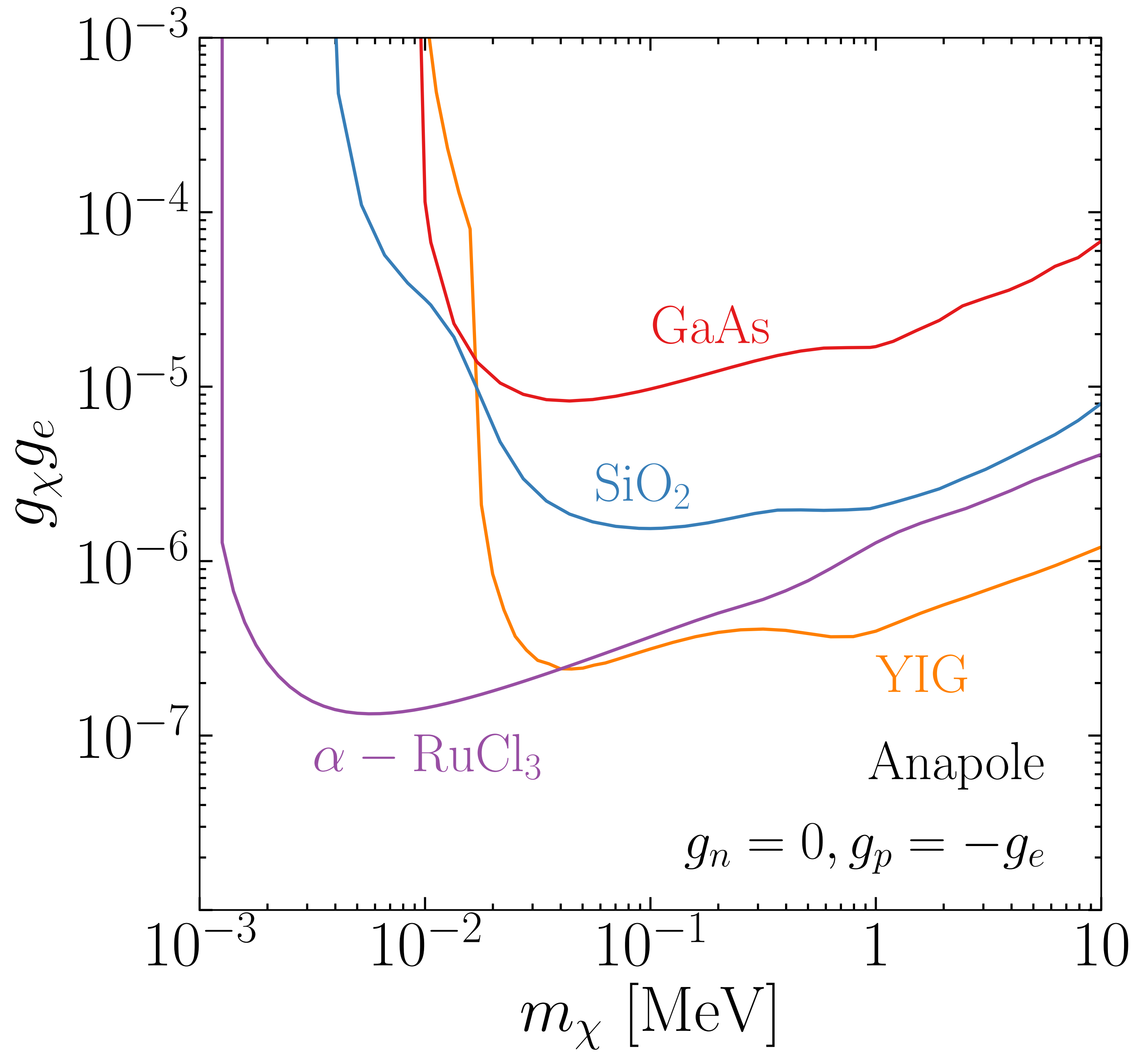
$\Rightarrow$ Best probed by magnons.

Example

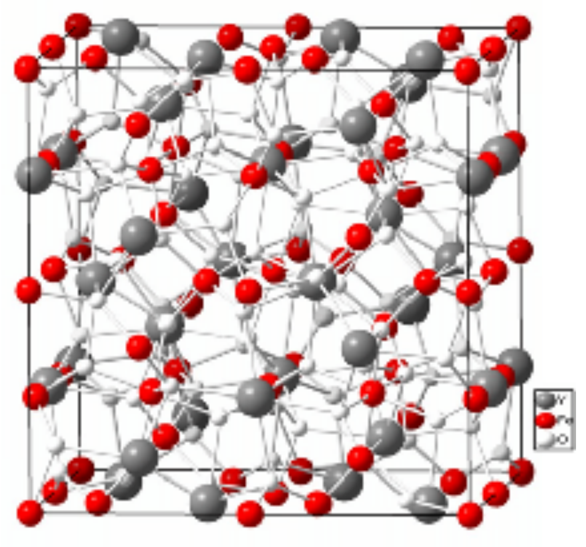
Trickle, ZZ, Zurek, 2009.13534.

- Zoom in on these two models.
- Compare phonon reach (from previous plot) vs. magnon reach.





YIG

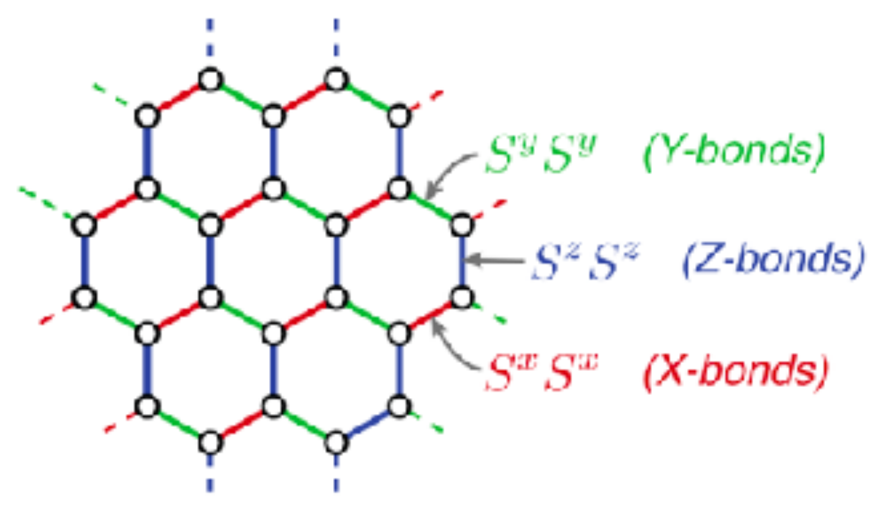


($Y_3Fe_5O_{12}$)

S order.

Used in QUAX.

α-RuCl3



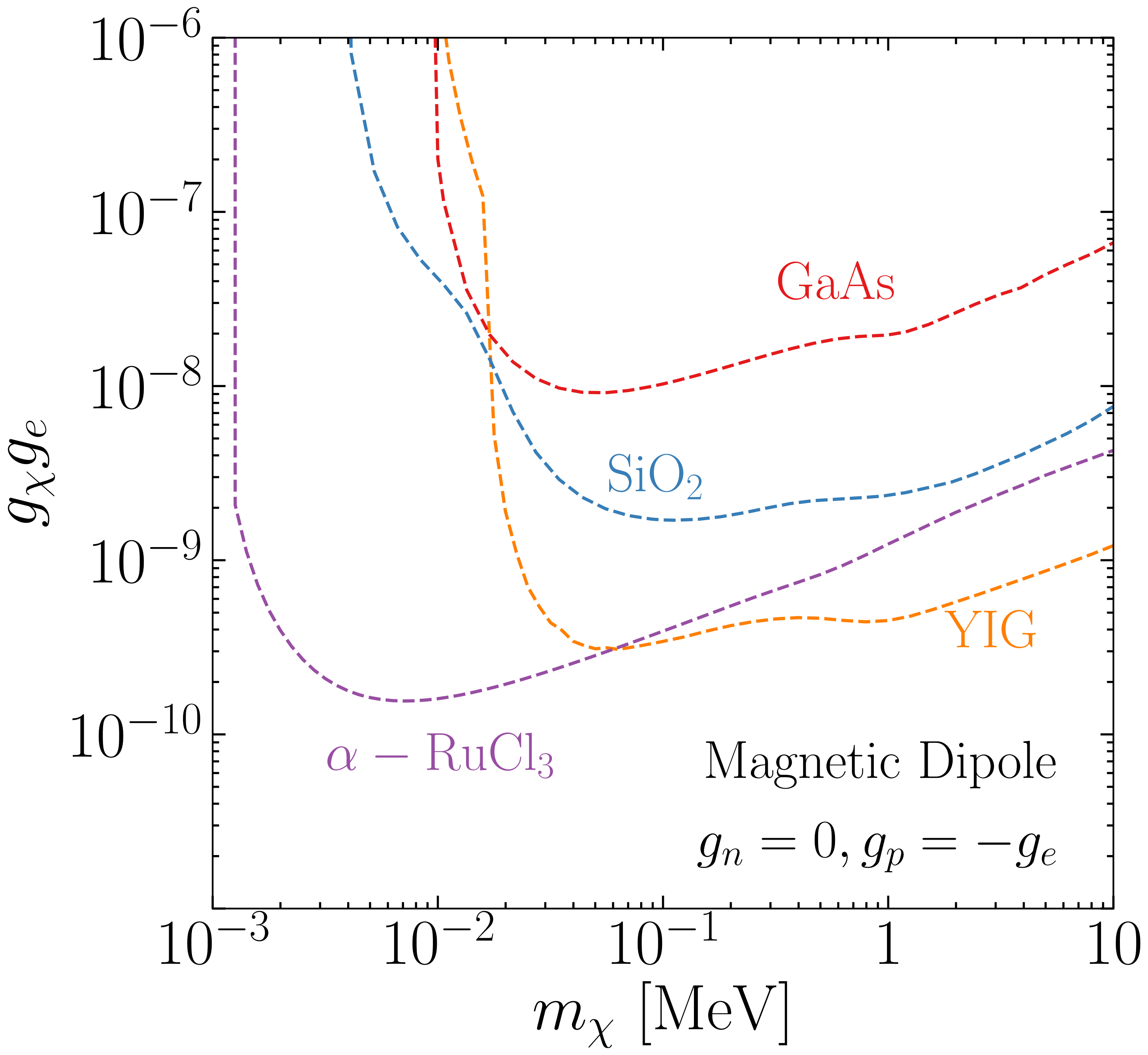
S & ***L*** order.

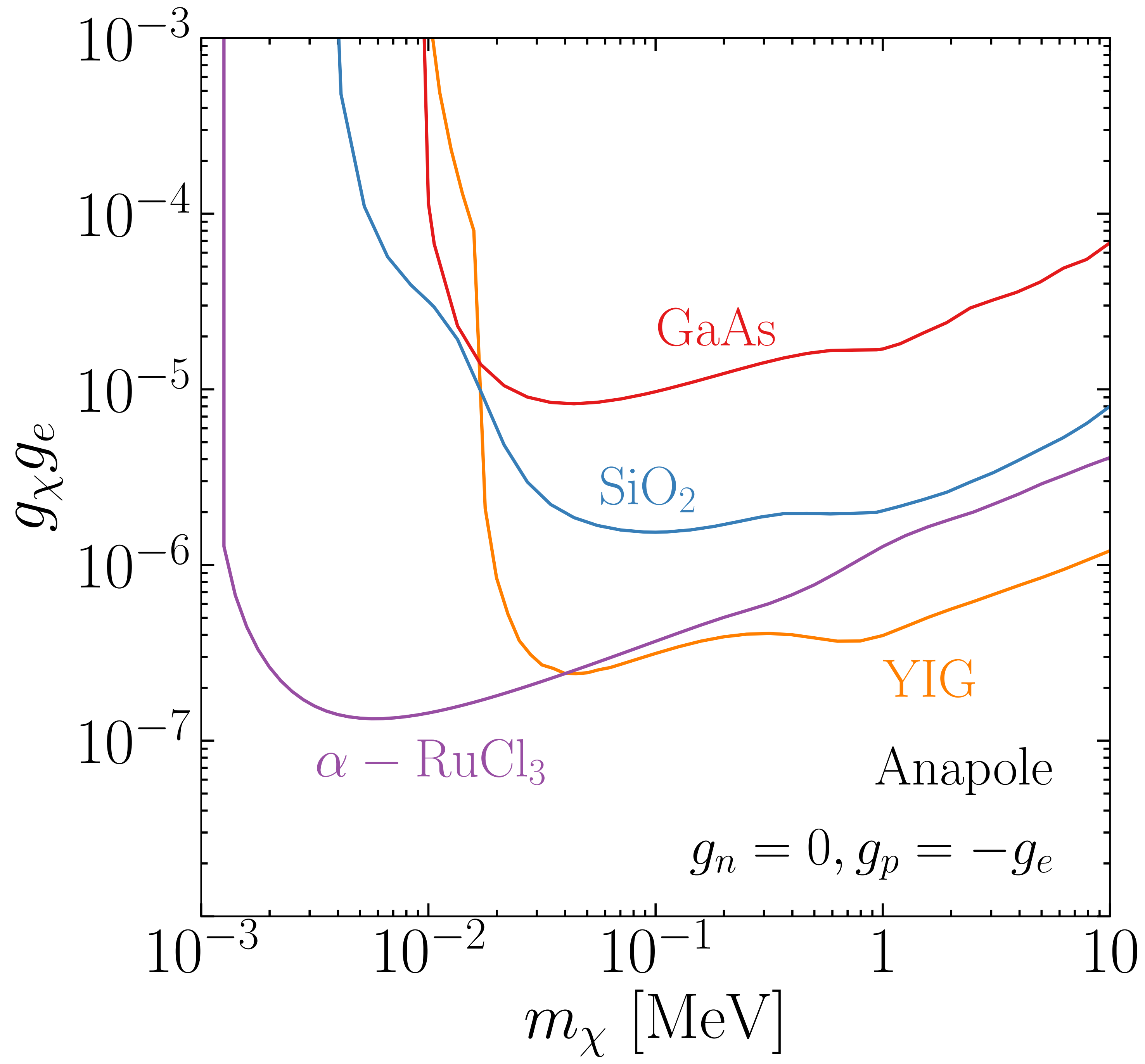
Features low energy gapped magnons.

Example

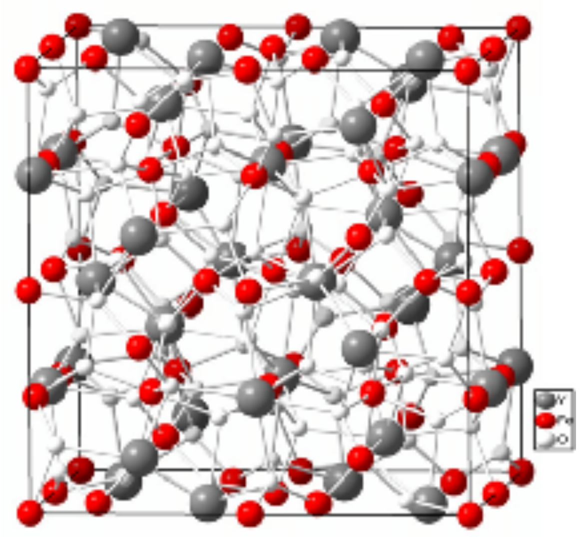
Trickle, ZZ, Zurek, 2009.13534.

- Zoom in on these two models.
- Compare phonon reach (from previous plot) vs. magnon reach.





YIG

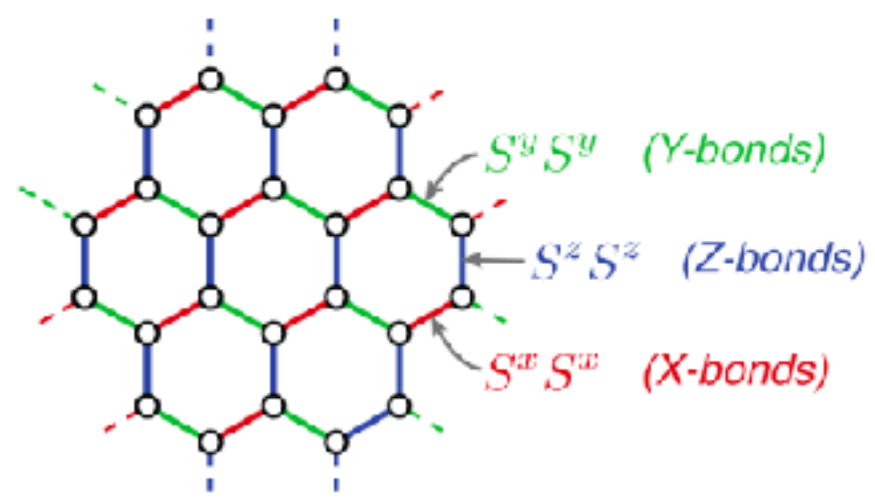


($Y_3Fe_5O_{12}$)

S order.

Used in QUAX.

α-$\mathrm{RuCl_3}$



S & ***L*** order.

Features low energy gapped magnons.

- Magnon reach is parametrically better, but $\mathrm{SiO_2}$ (optimal phonon target) is not too far behind.
- Encouraging for the technically more mature phonon experiments.

Take-home messages

Collective excitations such as phonons and magnons offer a novel path to detect light DM.

New experiments such as SPICE (TESSERACT) are moving forward.

We have developed the EFT tools for computing detection rates for general DM models.

THANK YOU