

Dynamics of the Fermion-Rotor System

arXiv:2408.04577, arXiv:2508.21059

In collaboration with: Takemichi Okui and David Tong



UNIVERSITY OF
OXFORD

QMAP Seminar

Speaker: Vazha Loladze

Plan

- “Semiton”/Unitarity Puzzle in Monopole-Fermion scattering
- The Fermion-Rotor system and resolution of the puzzle
- The connection to boundary conformal field theory

Fermion-Monopole system is interesting

- Angular momentum in EM field:

$$\Delta J = -eg\hat{r}$$

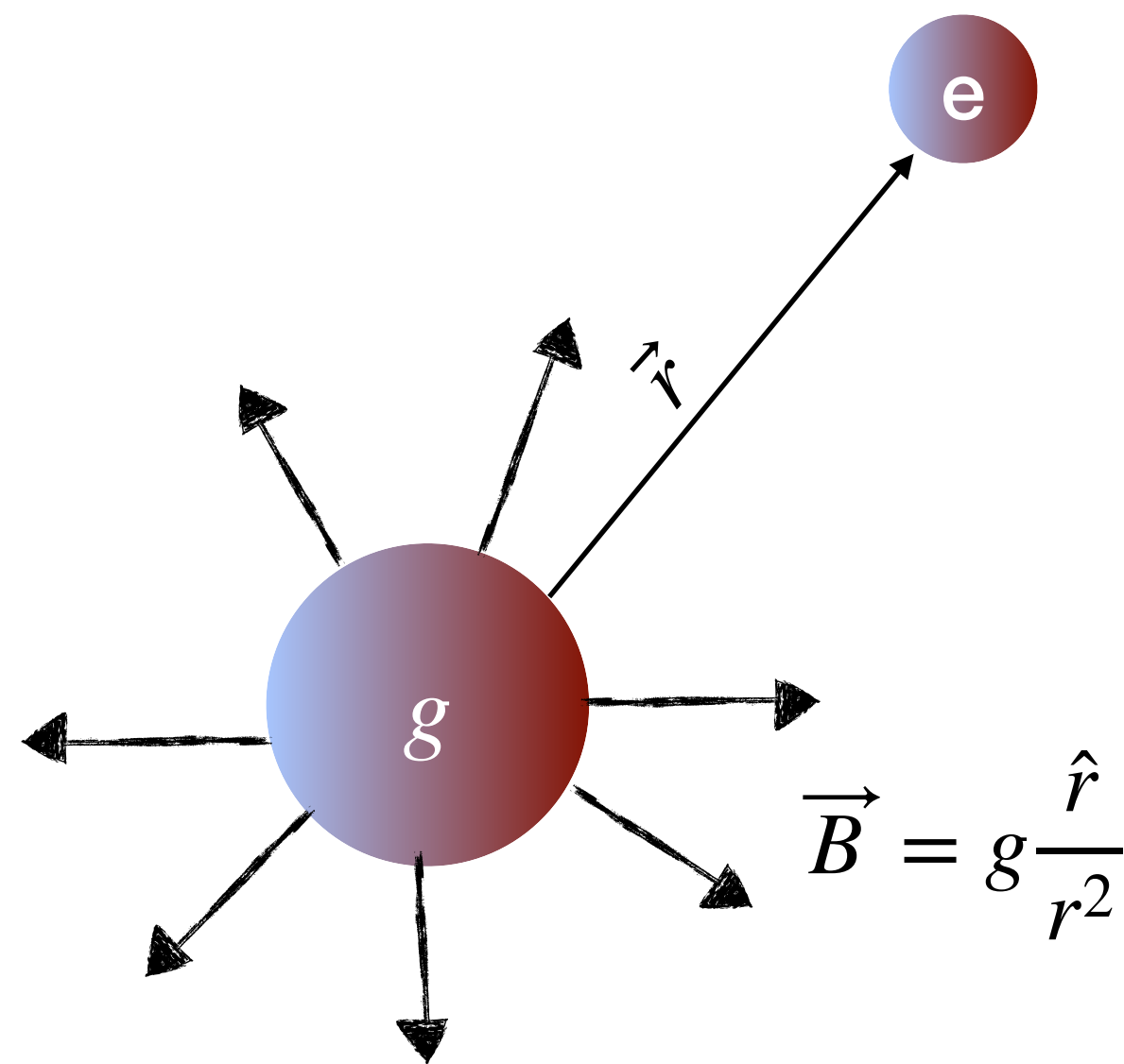
- Conserved angular momentum:

$$J = L + S - eg\hat{r}$$

- Multi-particle states:

$$|eg\rangle \neq |e\rangle \times |g\rangle$$

C. Csáki, S. Hong, Y. Shirman, O. Telem, J. Terning,
arXiv:2010.13794



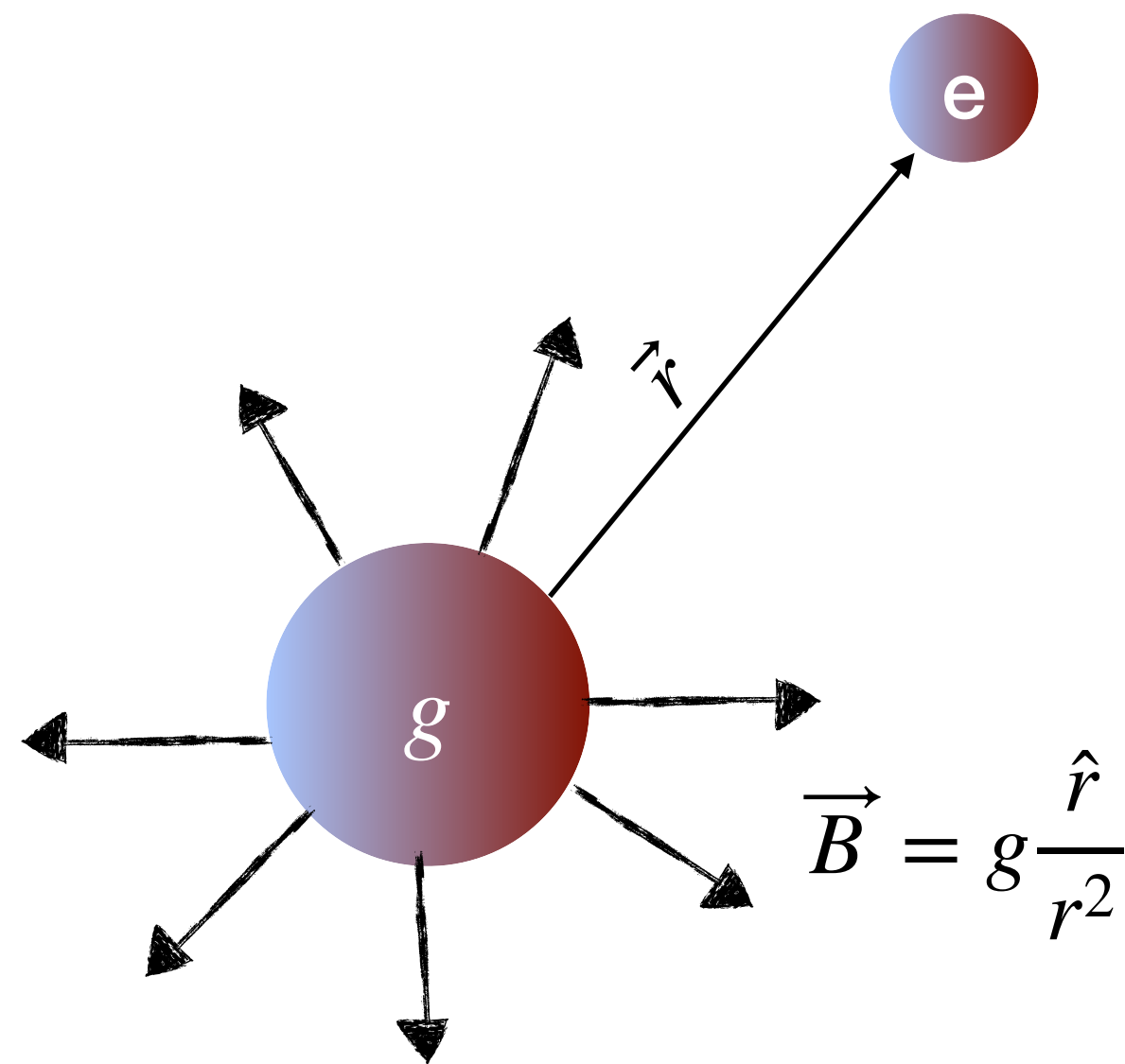
Fermion-Monopole scattering

Scattering of Fermion on Magnetic monopole:

- Fermion energy: $M_{EW} \ll E_\psi \ll M_{GUT}$
- Partial wave decomposition is useful:

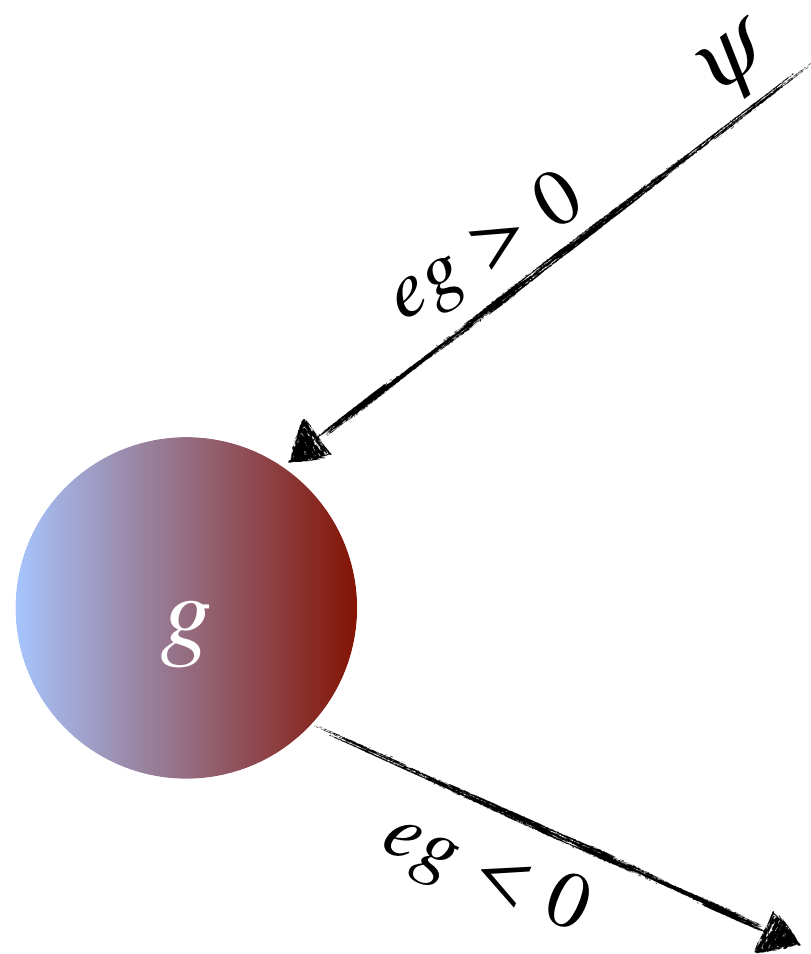
$$\psi(x, t) = \sum_{\{n\}} \frac{\chi_{\{n\}}(r)}{r} \Phi_{\{n\}}(\varphi, \theta)$$

- Modes with $J > J_{min} = |eg| - \frac{1}{2}$ decouple



Fermion-Monopole scattering

Study scattering of $J = J_{min}$ on monopole ($J_{min} = 0$ rest of the talk)



$$\mathcal{L} \supset \chi^\dagger (\partial_t - \text{sign}(eg) \partial_r) \chi$$

Fields with $eg > 0$ are ingoing

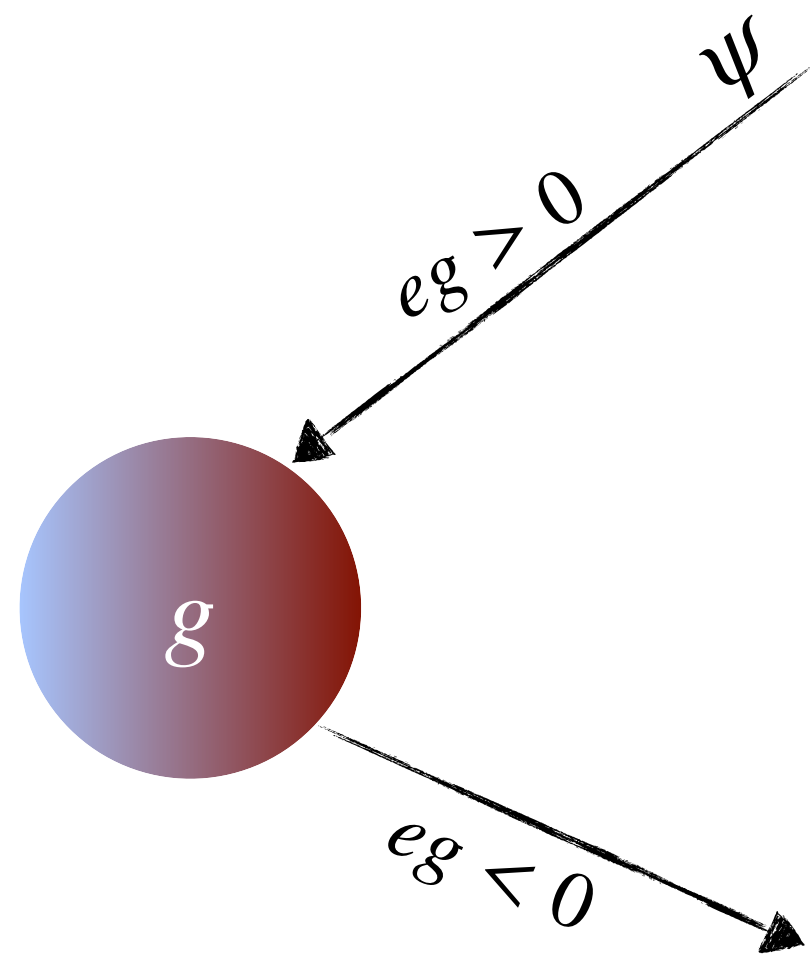
Fields with $eg < 0$ are outgoing

Fermion-Monopole scattering in GUT

$$SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$$

$$SU(2)_M \rightarrow U(1)_M$$

$$\begin{array}{l} \text{out} \\ \text{in} \end{array} \begin{pmatrix} e \\ d_3^c \end{pmatrix}_L, \begin{pmatrix} d^3 \\ e^c \end{pmatrix}_L, \begin{pmatrix} u_1^c \\ u^2 \end{pmatrix}_L, \begin{pmatrix} u_2^c \\ u^1 \end{pmatrix}_L \begin{array}{l} e, \\ +1 \\ -1 \end{array} \quad g < 0$$



Symmetry: $SU(4) \times U(1)_M$

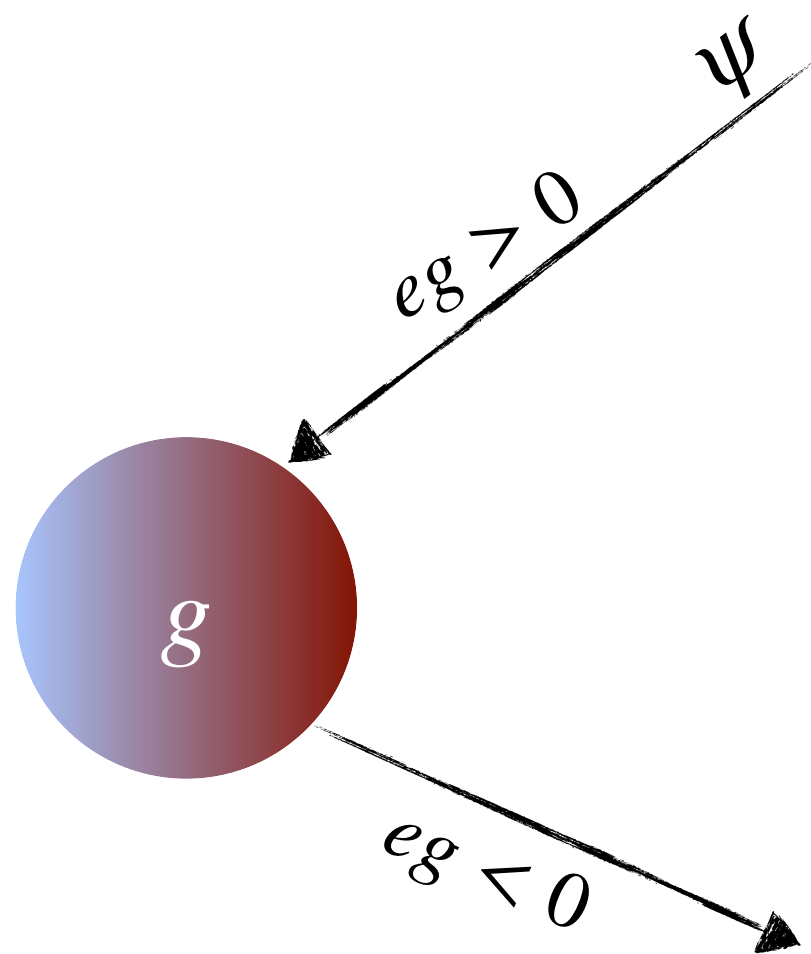
Motivation: Fermion-Monopole scattering in GUT

$$\begin{array}{c} \text{out} \\ \text{in} \end{array} \begin{pmatrix} e \\ d_3^c \end{pmatrix}_L, \begin{pmatrix} d^3 \\ e^c \end{pmatrix}_L, \begin{pmatrix} u_1^c \\ u^2 \end{pmatrix}_L, \begin{pmatrix} u_2^c \\ u^1 \end{pmatrix}_L \begin{array}{c} e, \\ +1 \\ -1 \end{array} \quad g < 0$$

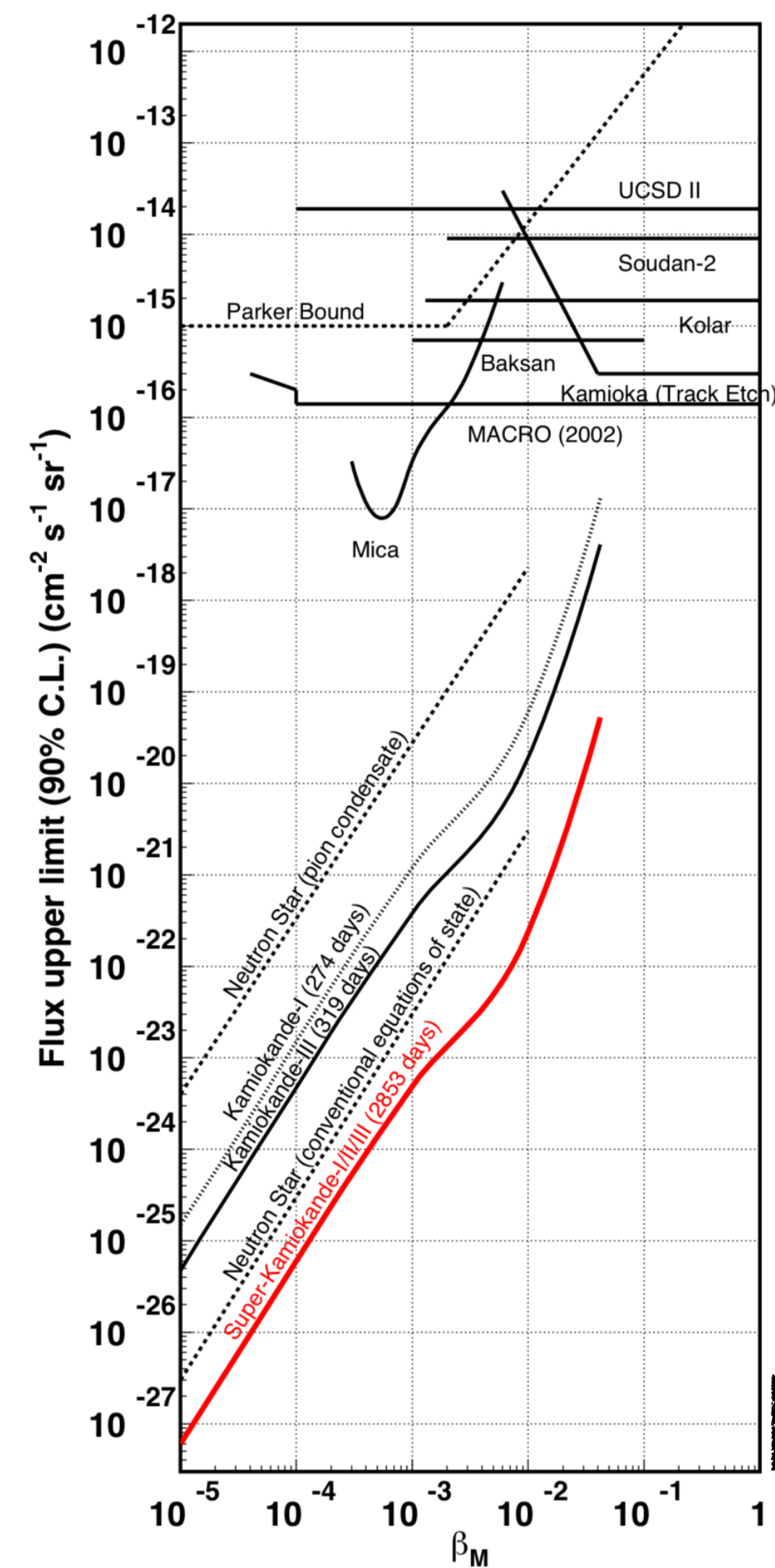
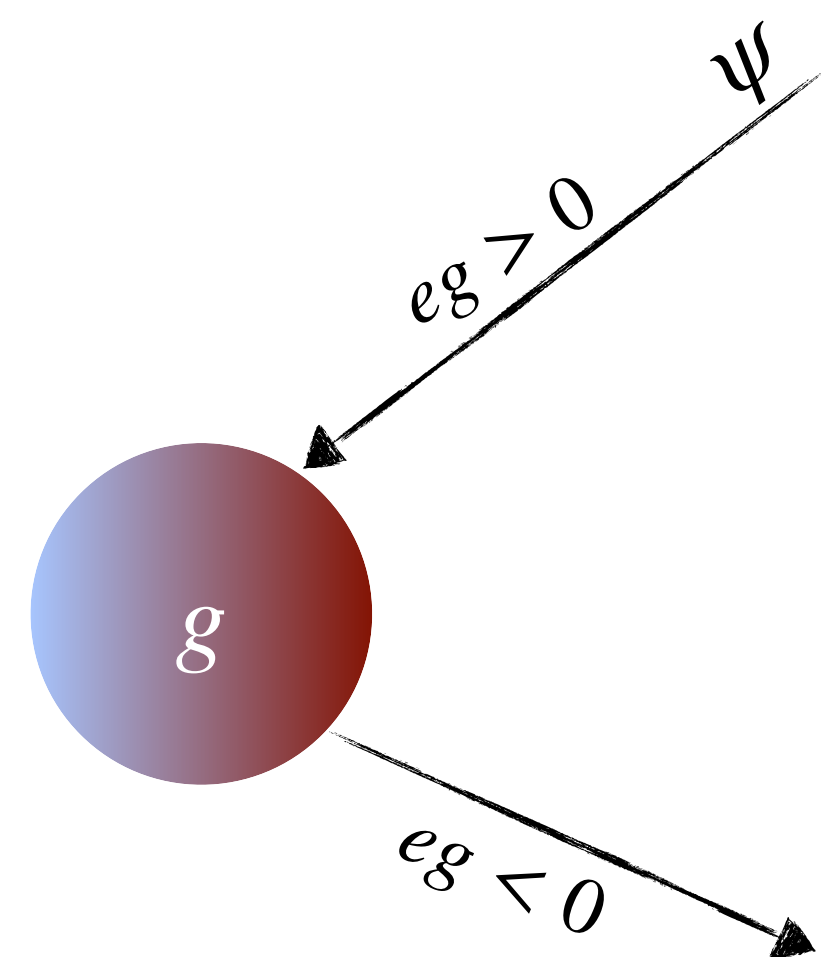
Symmetry: $SU(4) \times U(1)_M$

$u^1 + u^2 + M \rightarrow \bar{e} + \bar{d}_3 + M$ - Callan-Rubakov process

$p + M \rightarrow \bar{e} + M$ - Proton decay!



Monopole Catalysis of Nucleon Decay

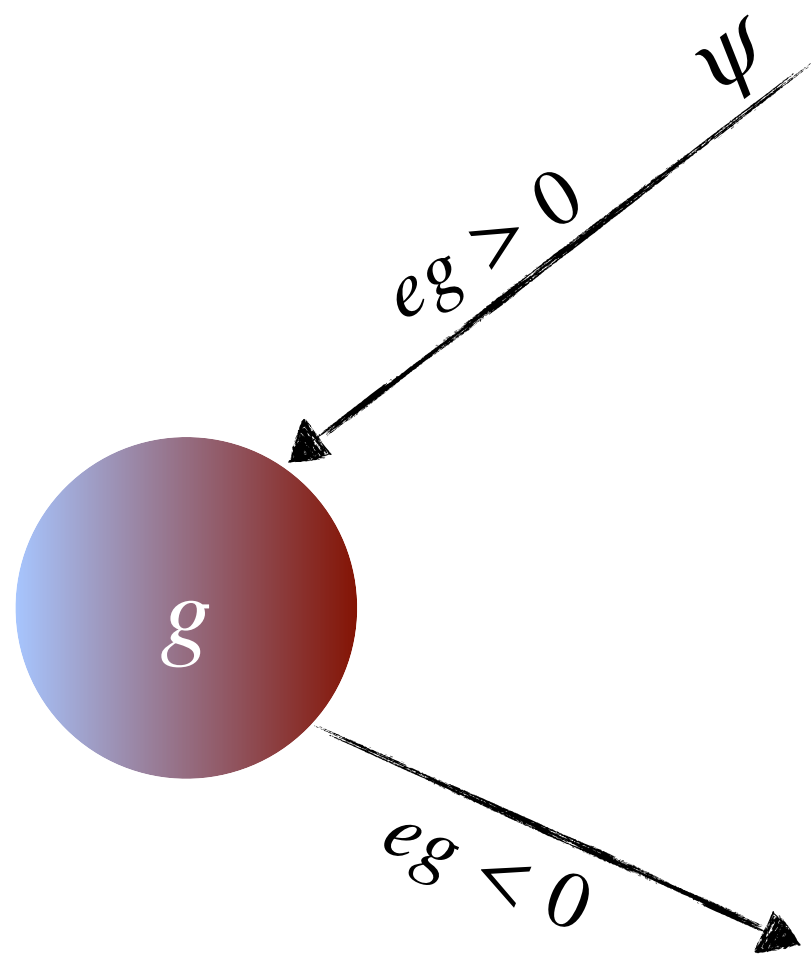


Super-Kamiokande arXiv:1203.0940

Semiton/Unitarity Puzzle

$$\begin{array}{c} \text{out} \\ \text{in} \end{array} \begin{pmatrix} e \\ d_3^c \end{pmatrix}_L, \begin{pmatrix} d^3 \\ e^c \end{pmatrix}_L, \begin{pmatrix} u_1^c \\ u^2 \end{pmatrix}_L, \begin{pmatrix} u_2^c \\ u^1 \end{pmatrix}_L \begin{array}{c} e, \\ +1 \\ -1 \end{array} \quad g < 0$$

Symmetry: $SU(4) \times U(1)_M$



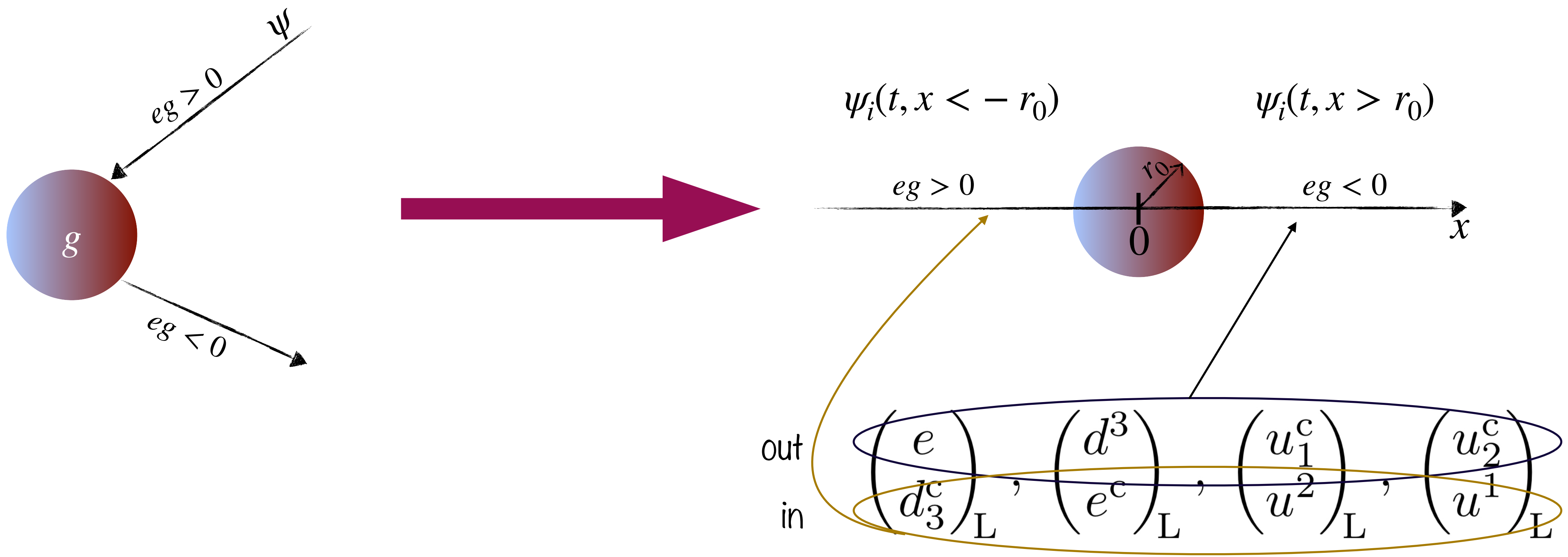
$$u_1 + u_2 + M \rightarrow \bar{e} + \bar{d}_3 + M$$

$$p + M \rightarrow \bar{e} + M \quad \text{- Proton decay!}$$

$$e^c + M \rightarrow \frac{1}{2}(d_3 + \bar{e} + \bar{u}_1^c + \bar{u}_2^c) + M$$

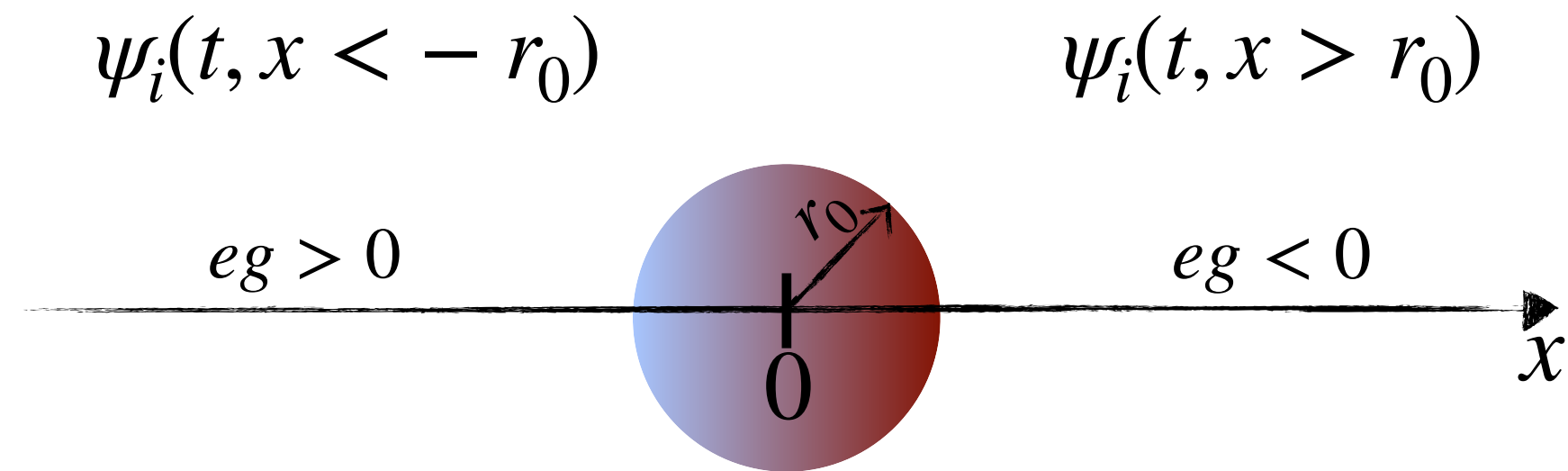
The Fermion-Rotor System

J.Polchinski Nucl. Phys. B 242 (1984)



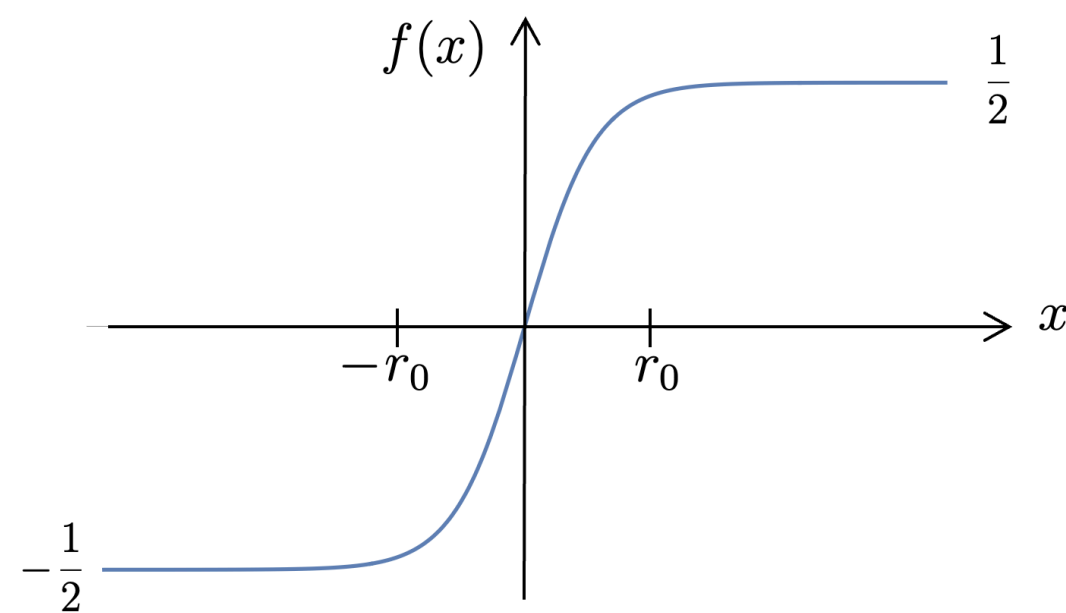
The Fermion-Rotor System

J. Polchinski Nucl. Phys. B 242 (1984)



Effective theory of N - $\psi_i(t, x)$ fields with symmetry:

$$U(1)_M : \psi_i(t, x) \rightarrow e^{if(x)\beta} \psi_i(t, x)$$

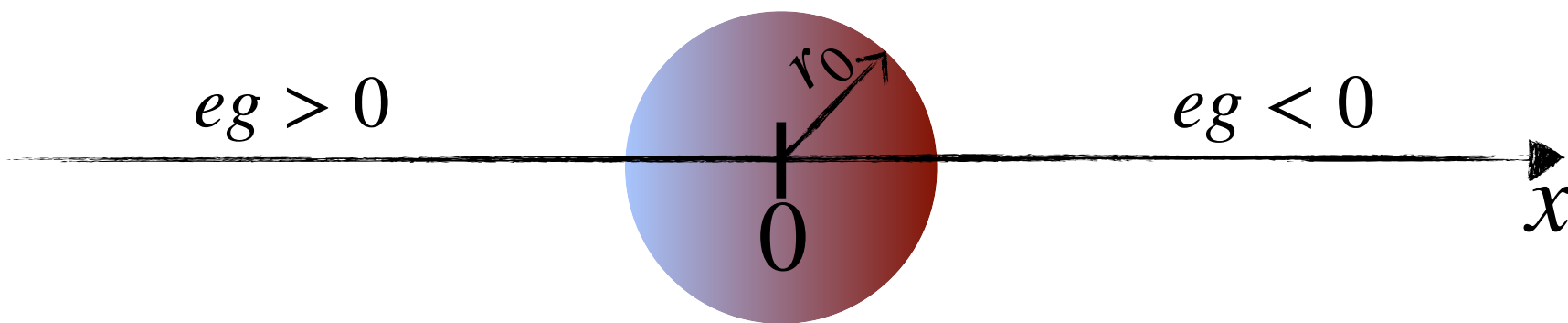


$\psi_i(t, x < -r_0)$ - lower component of doublet (ingoing)

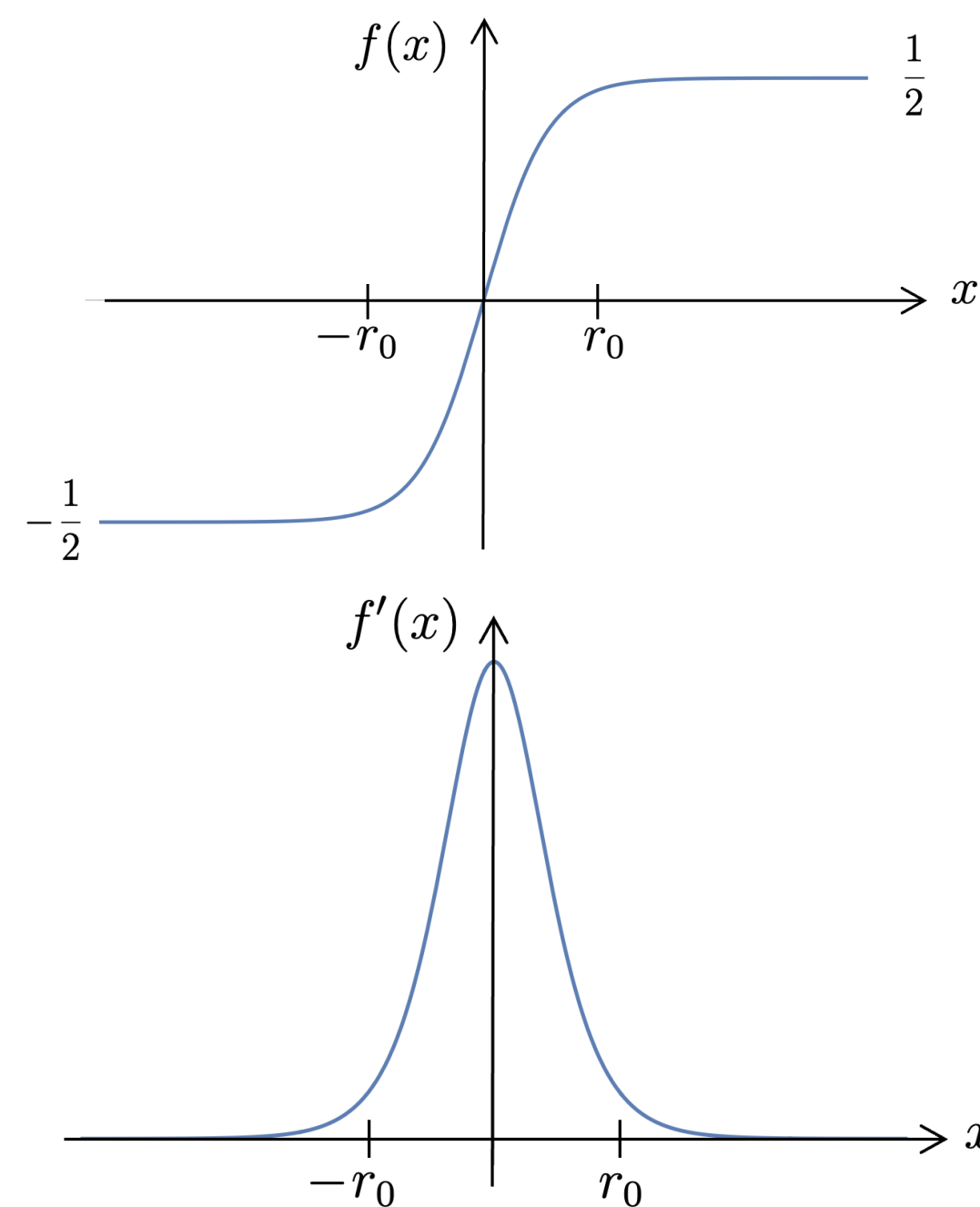
$\psi_i(t, x < r_0)$ - upper component of doublet (outgoing)

The Fermion-Rotor System

J. Polchinski Nucl. Phys. B 242 (1984)



$$S = \int d^2x \, i \sum_{i=1}^N \psi_i^\dagger (\partial_+ + i\alpha(t)f'(x))\psi_i + \int dt \, \frac{I}{2} \dot{\alpha}^2$$



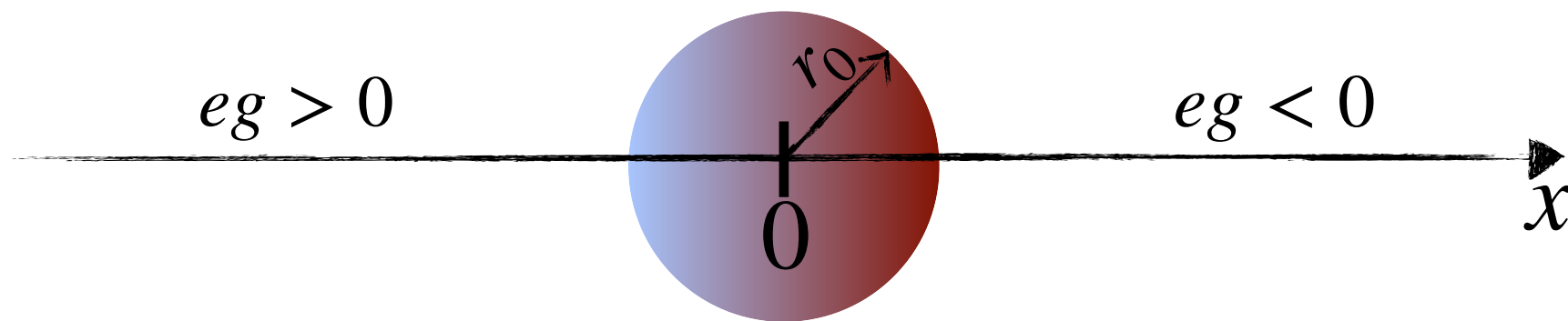
$$U(1)_M : \psi_i(t, x) \rightarrow e^{if(x)\beta} \psi_i(t, x), \quad \alpha(t) \rightarrow \alpha(t) - \beta$$

Anomaly free symmetry: $SU(N) \times U(1)_M$

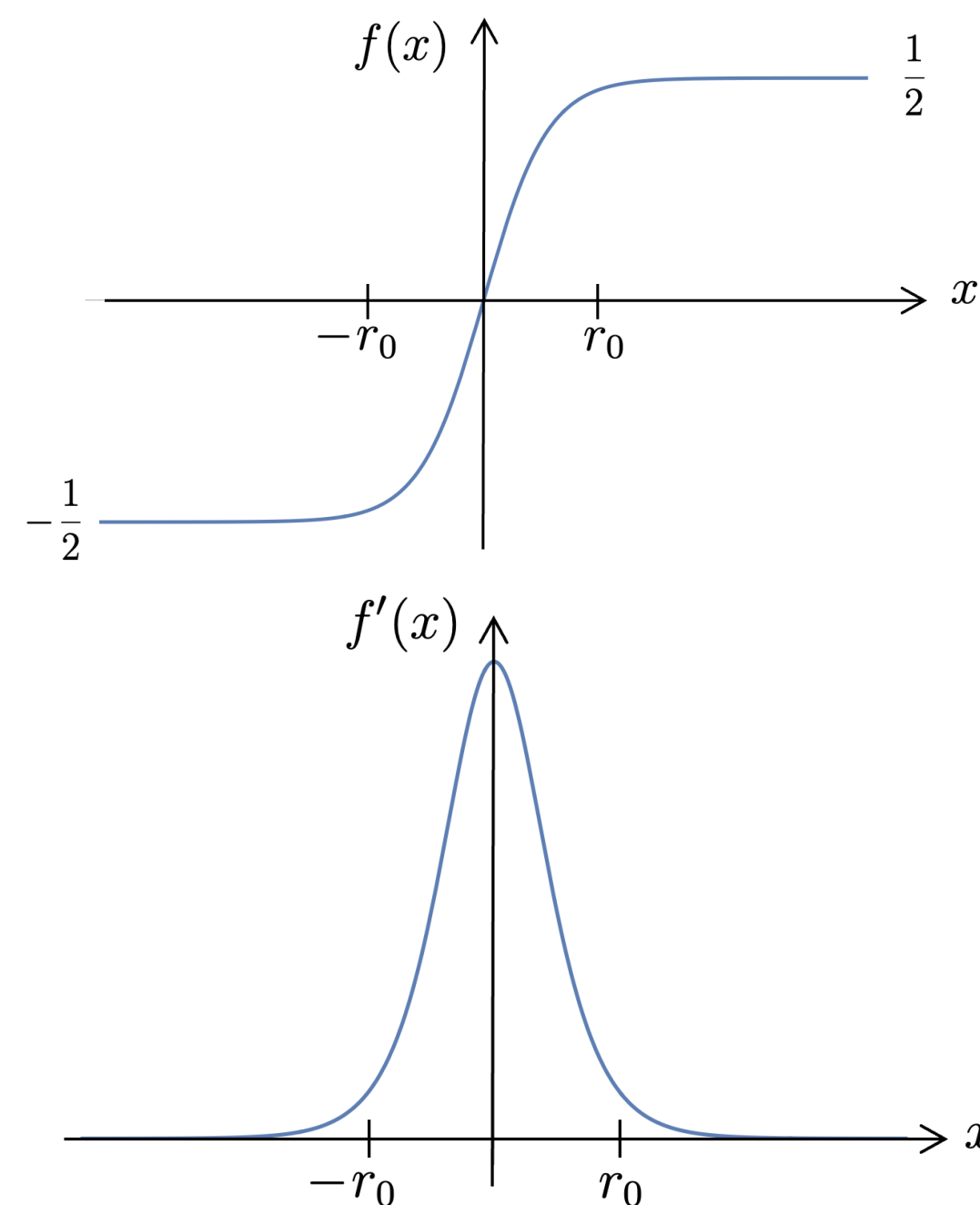
$$[U(1)_M]^2 \text{ anomaly} \sim \int f(x)f'(x) = 0$$

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Anomaly free symmetry: $SU(N) \times U(1)_M$

$U(1)_\psi : \psi_i(t, x) \rightarrow e^{i\gamma} \psi_i(t, x)$ is anomalous:

$$\partial_+ J(t, x) = \frac{N}{2\pi} \dot{\alpha}(t) f'(x) \Rightarrow \Delta Q_\psi = \frac{2\pi}{N} \Delta \alpha$$

Instantons are represented by $\Delta \alpha \neq 0$ configurations

The Fermion-Rotor System

J. Polchinski Nucl. Phys. B 242 (1984)

$$S = \int d^2x \, i \sum_{i=1}^N \psi_i^\dagger (\partial_+ + i\alpha(t)f'(x))\psi_i + \int dt \, \frac{I}{2} \dot{\alpha}^2$$

$$U(1)_M : \quad Q = I\dot{\alpha}(t) + \sum_{k=1}^N \int_{-\infty}^{\infty} dx f(x) \psi_k^\dagger \psi_k(t, x)$$

Charge on the rotor: $I\dot{\alpha}(t) = 1 \rightarrow E \sim \frac{1}{I} \gg E_\psi \Rightarrow$ the rotor is gapped!

The rotor describes dyon excitations (**but there is more**)!

The Fermion-Rotor System

J. Polchinski Nucl. Phys. B 242 (1984)

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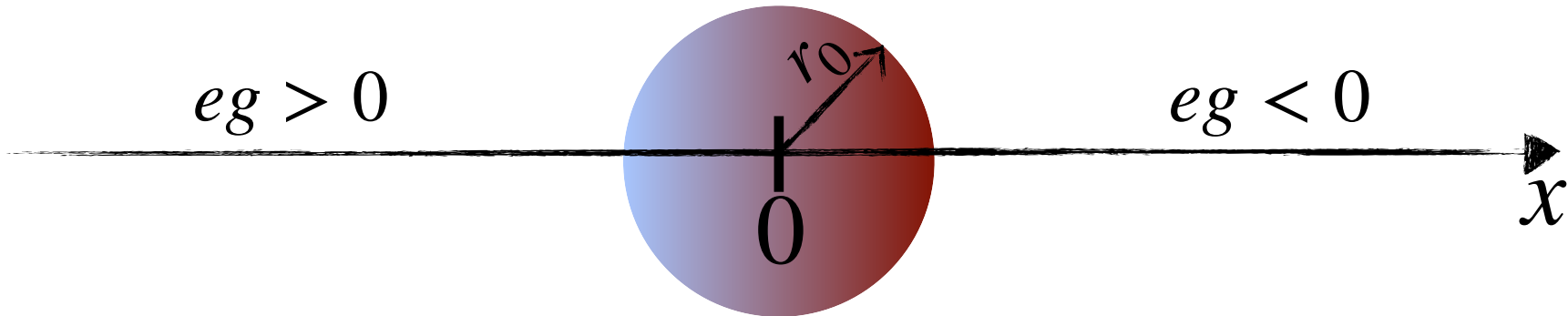
The rotor describes dyon excitations (**but there is more!**)

1) Any on-shell excitations associated to rotor decouples

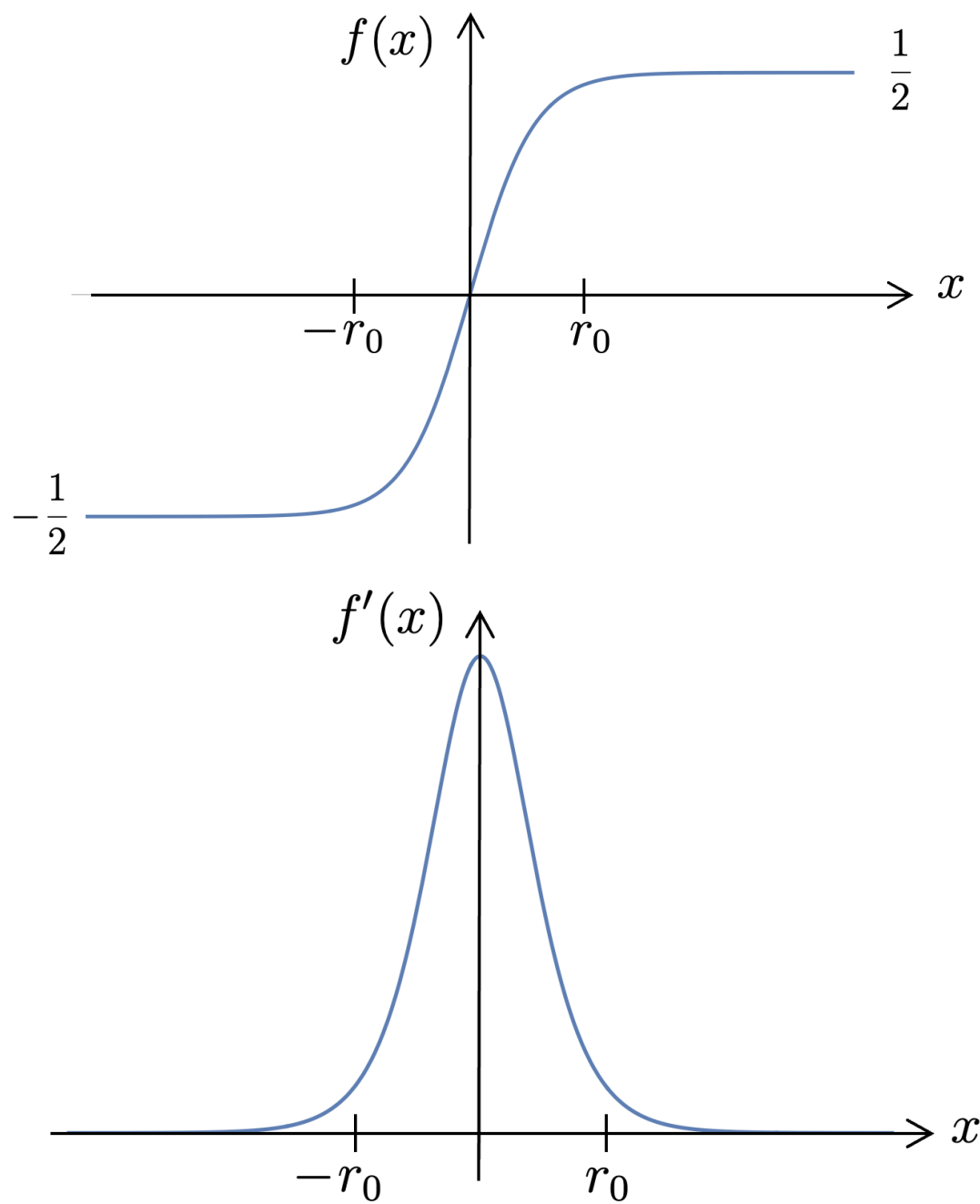
2) Rotor is very important : **by construction the theory conserves $U(1)_M$ charge!**

The Fermion-Rotor System

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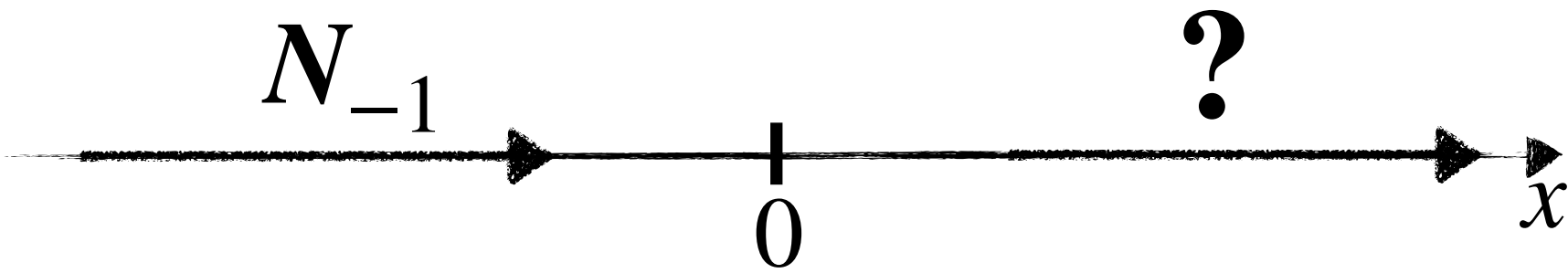
$$S = \int d^2x \, i \sum_{i=1}^N \psi_i^\dagger (\partial_+ + i\alpha(t)f'(x))\psi_i + \int dt \, \frac{I}{2} \dot{\alpha}^2$$



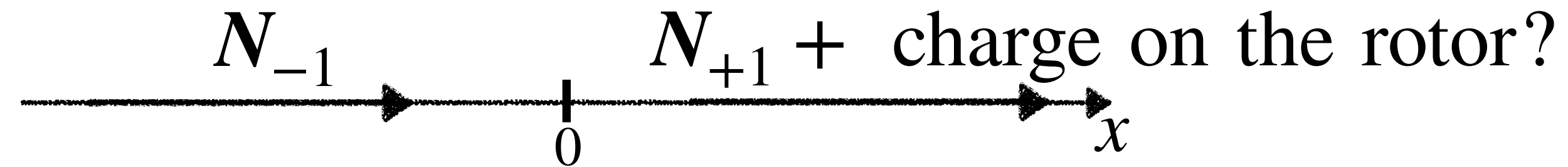
$SU(N) \times U(1)_M$ representation:

$$\psi_i(t, x < -r_0) : N_{-1}, \quad \psi_i(t, x > r_0) : N_{+1}$$

Unitarity Puzzle:



The Fermion-Rotor System



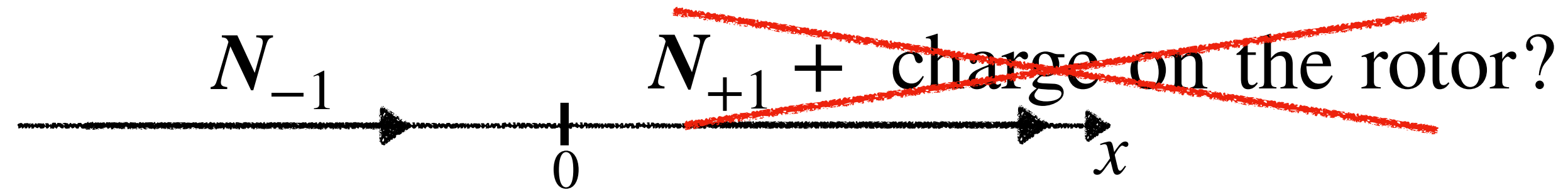
Can we deposit charge on the rotor?

$$I\ddot{\alpha}(t) = -\tilde{J}_{\text{in}}(t) - \frac{N}{4\pi}\dot{\alpha}(t)$$

$$\tilde{J}_{\text{in}}(t) = \lim_{r_0 \rightarrow 0} \sum_{k=1}^N \psi_k^\dagger \psi_k(t - r_0, -r_0)$$

Any charge on the rotor decays in $t \sim \frac{4\pi I}{N}$

The Fermion-Rotor System



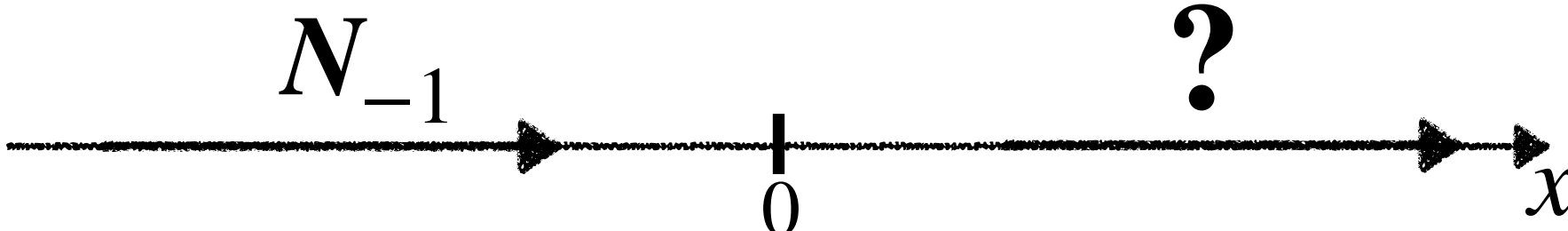
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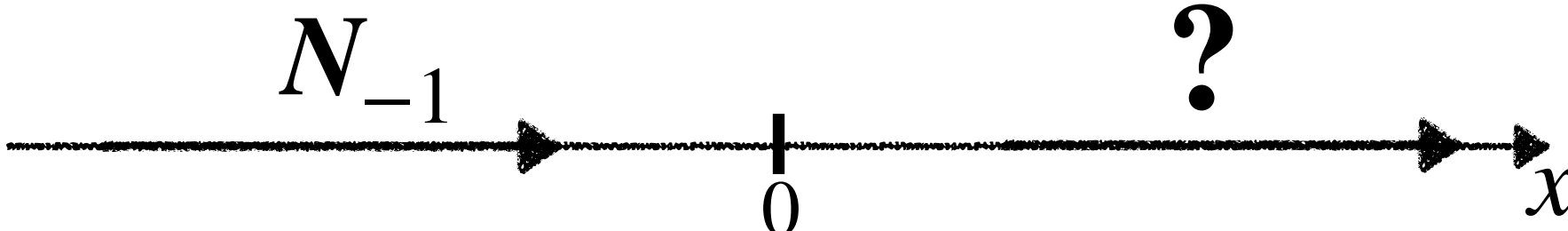
$$S = \int d^2x \, i \sum_{i=1}^N \psi_i^\dagger (\partial_+ + i\alpha(t)f'(x)) \psi_i + \int dt \, \frac{I}{2} \dot{\alpha}^2$$


Charge can not be deposited on the monopole

We have hermitian hamiltonian that conserves the charge

What is wrong? how can we have consistent scattering?

The Fermion-Rotor System

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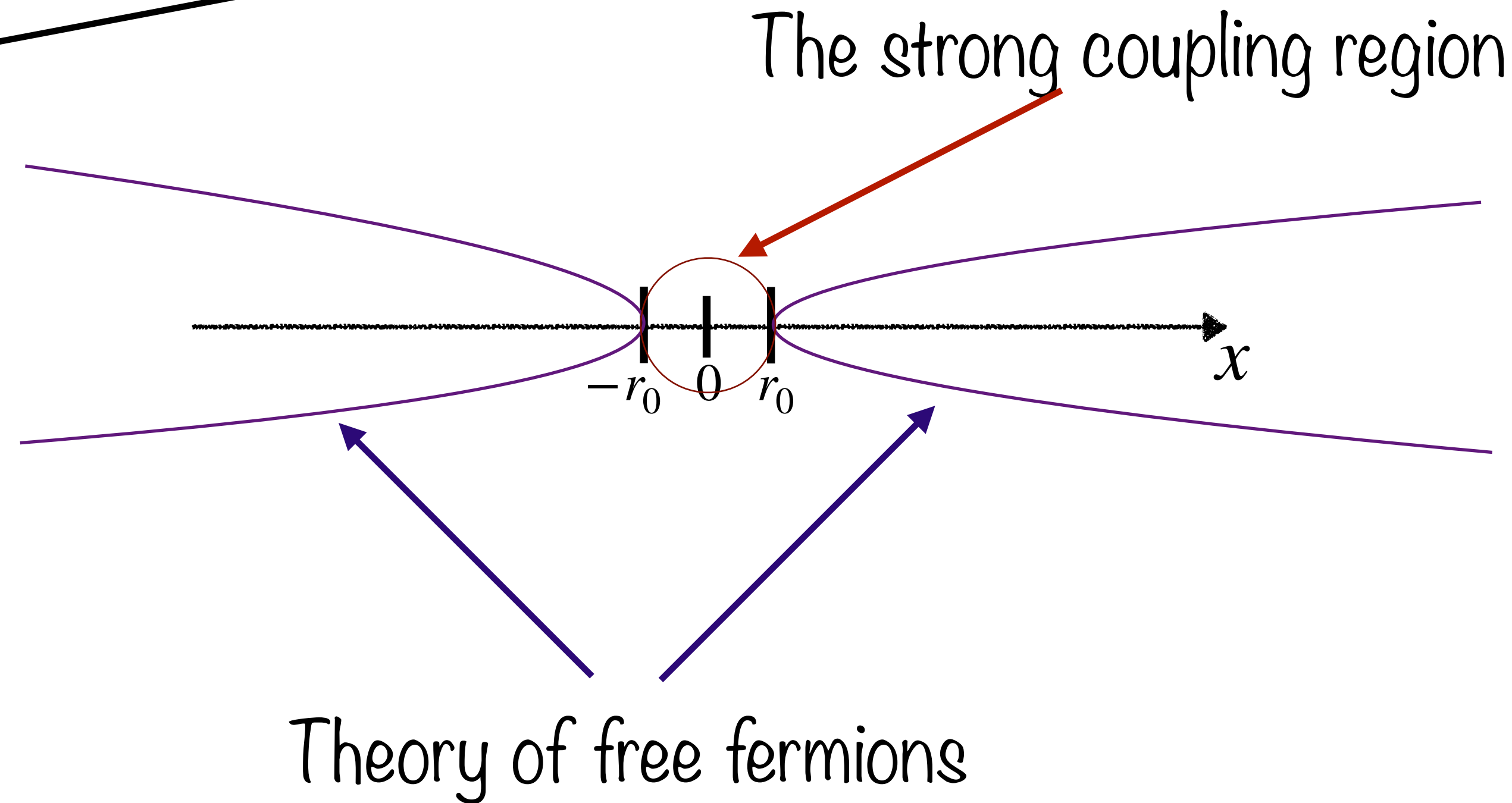
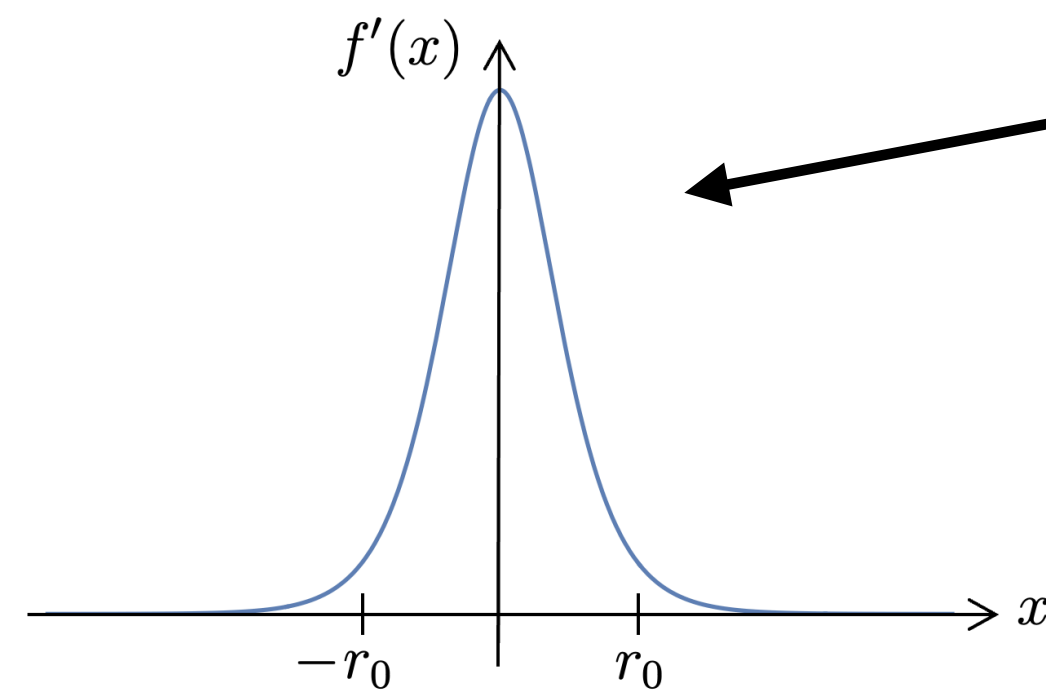
What is wrong? how can we have consistent scattering?

$$(\partial_+ + i\alpha(t)f'(x))\psi_i = 0, \quad I\ddot{\alpha}(t) = - \sum_{k=1}^N \int_{-\infty}^{+\infty} dx f'(x) \psi_k^\dagger \psi_k(t, x)$$

The theory is strongly coupled we need to carefully quantize it before asking any scattering questions!

The Fermion-Rotor System

$$S = \int d^2x \, i \sum_{i=1}^N \psi_i^\dagger (\partial_+ + i\alpha(t)f'(x))\psi_i + \int dt \, \frac{I}{2} \dot{\alpha}^2$$



The Fermion-Rotor System

$$S = \int d^2x \, i \sum_{i=1}^N \psi_i^\dagger (\partial_+ + i\alpha(t)f'(x))\psi_i + \int dt \, \frac{I}{2} \dot{\alpha}^2 \quad \xrightarrow{N_{-1}} \quad \begin{array}{c} \text{?} \\ \hline 0 \end{array} \xrightarrow{\quad} x$$

Simplification: the strong coupling region is localised $-r_0 < x < r_0$ and the theory is free for $|x| > r_0$

As a result **path integral becomes Gaussian** and one can find:

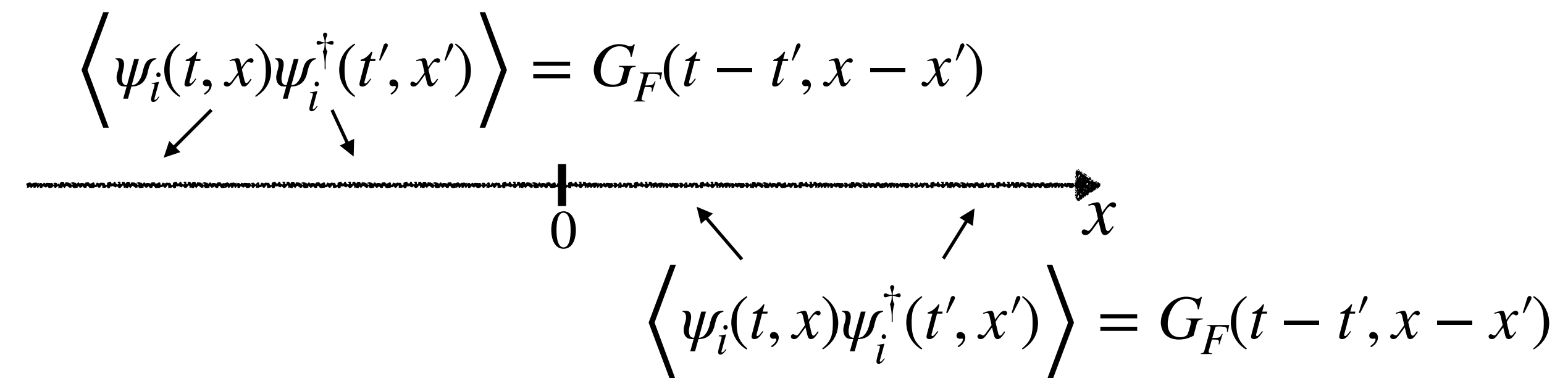
$$\left\langle \prod_{j=1}^n \psi_{i_j}(t_j, x_j) \prod_{k=1}^{n'} \psi_{i'_k}^\dagger(t'_k, x'_k) \right\rangle = (\text{free correlators}) \times \mathcal{Z}$$

$$\mathcal{Z} = \exp \left(\frac{2}{N} \int_0^\infty \frac{d\omega}{\omega} AB \right)$$

$$A = \sum_{j=1}^n \theta(x_j) e^{-i\omega(t_j - x_j)} - \sum_{k=1}^{n'} \theta(x'_k) e^{-i\omega(t'_k - x'_k)}, \quad B = \sum_{j=1}^n \theta(-x_j) e^{+i\omega(t_j - x_j)} - \sum_{k=1}^{n'} \theta(-x'_k) e^{+i\omega(t'_k - x'_k)}$$

The Fermion-Rotor System

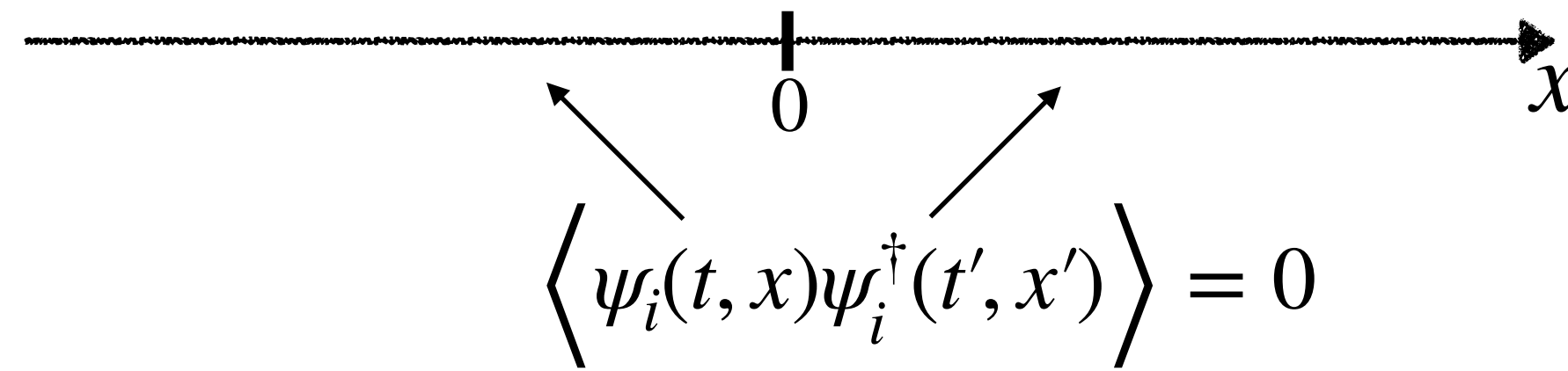
- two point function: $\langle \psi_i(t, x) \psi_j^\dagger(t', x') \rangle = \theta(xx') \delta_{ij} G_F(t - t', x - x')$

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The Fermion-Rotor System

- two point function: $\langle \psi_i(t, x) \psi_j^\dagger(t', x') \rangle = \theta(xx') \delta_{ij} G_F(t - t', x - x')$



$\langle \psi_i(t, x) \psi_i^\dagger(t', x') \rangle = 0$

The Callan-Rubakov Effect

- $N=4$ on this slide

Only non-vanishing correlation function involving insertion of $\psi_3^\dagger \psi_4^\dagger$ left to the rotor is

$$\langle \psi_1^\dagger(t, x > 0) \psi_2^\dagger(t, x > 0) \psi_3^\dagger(t, x < 0) \psi_4^\dagger(t, x < 0) \rangle$$

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$\downarrow$
 $\begin{pmatrix} e \\ d_3^c \end{pmatrix}_L$

$\downarrow$
 $\begin{pmatrix} d^3 \\ e^c \end{pmatrix}_L$

$\downarrow$
 $\begin{pmatrix} u_1^c \\ u^2 \end{pmatrix}_L$

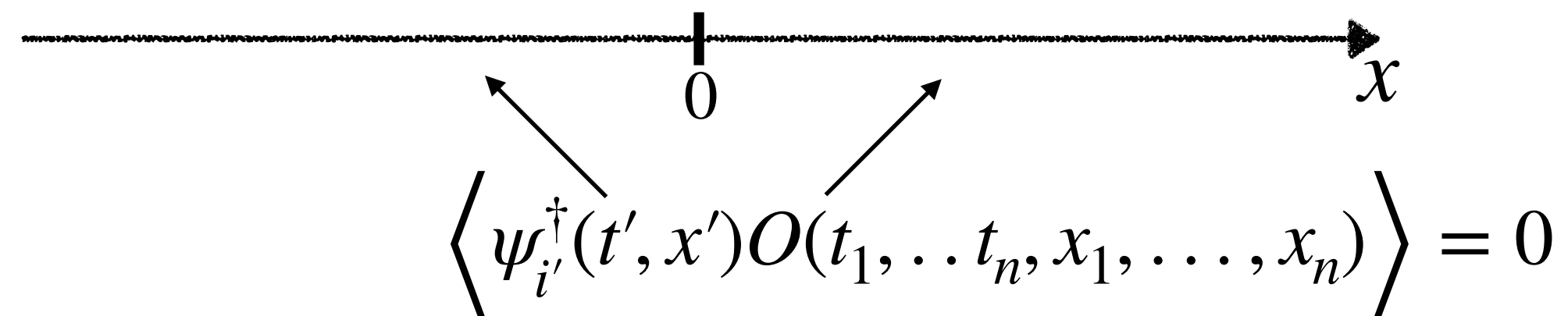
$\downarrow$
 $\begin{pmatrix} u_2^c \\ u^1 \end{pmatrix}_L$

so we have:

$$u^1 + u^2 + M \rightarrow \bar{e} + \bar{d}_3 + M$$

The Unitarity Puzzle

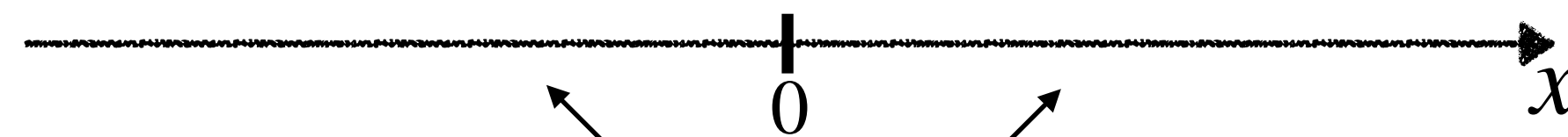
- $\left\langle \psi_{i'}^\dagger(t', x') O(t_1, \dots, t_n, x_1, \dots, x_n) \right\rangle = 0$, for $x' < 0, x_i > 0$ and $O(t_1, \dots, t_n, x_1, \dots, x_n)$ is any local operator containing just fermion fields!



The Solution to the Puzzle

- Final state is created by composite fermions rotor-operator:

$$\left\langle \psi_i^\dagger(t', x') [\psi_i(t, x) e^{i\alpha(t-x)}] \right\rangle = G_F(t - t', x - x'), \quad x > 0, \quad x' < 0.$$



$$\left\langle \psi_i^\dagger(t', x') [\psi_i(t, x) e^{i\alpha(t-x)}] \right\rangle = G_F(t - t', x - x')$$

The Solution to the Puzzle

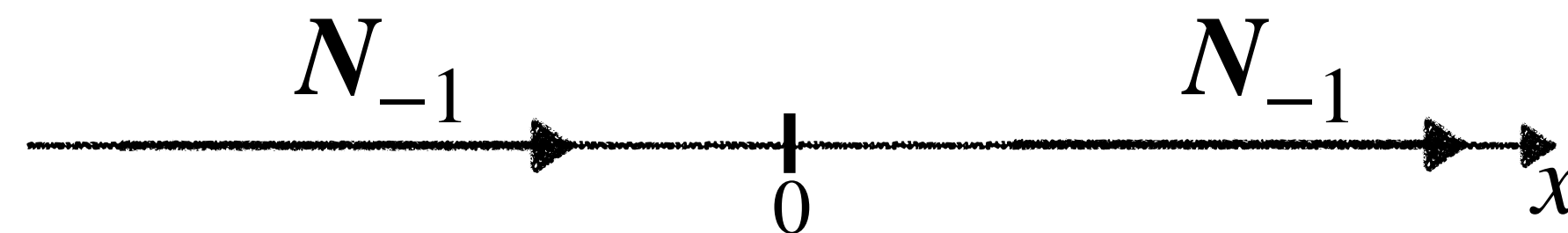
VL, T. Okui, arXiv:2408.04577

- Final State is created by $O_i(t, x) = \psi_i(t, x)e^{i\alpha(t-x)}$ and under symmetry group it transforms as:

$$SU(N) : N \times \mathbf{1} = N$$

$$U(1)_M : 1 - 2 = -1$$

- $O_i(t, x)$ creates **one particle excitation**: $\langle O_i(t, x)O_j^\dagger(t', x') \rangle = \delta_{ij}G_F(t - t', x - x'), \quad x > 0, \quad x' > 0.$



The Solution to the Puzzle

VL, T. Okui, arXiv:2408.04577

- We can define composite operators:

$\Psi_i(t, x) = \psi_i(t, x)e^{i\theta(x)\alpha(t-x)}$ - interpolating N_{-1} on the whole axis

$\tilde{\Psi}_i(t, x) = \psi_i(t, x)e^{-i\theta(-x)\alpha(t-x)}$ - interpolating N_{+1} on the whole axis

At $x < 0$:

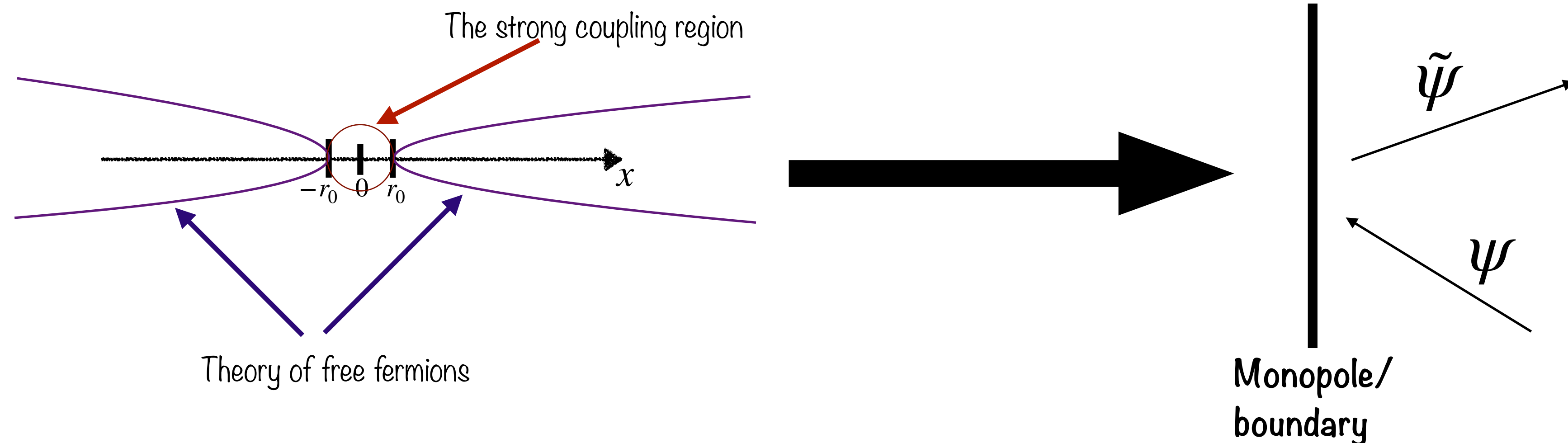
$$SU(N) : N \times \mathbf{1} = N$$

$$U(1)_M : -1 + 2 = +1$$

Boundary Conformal Field Theory

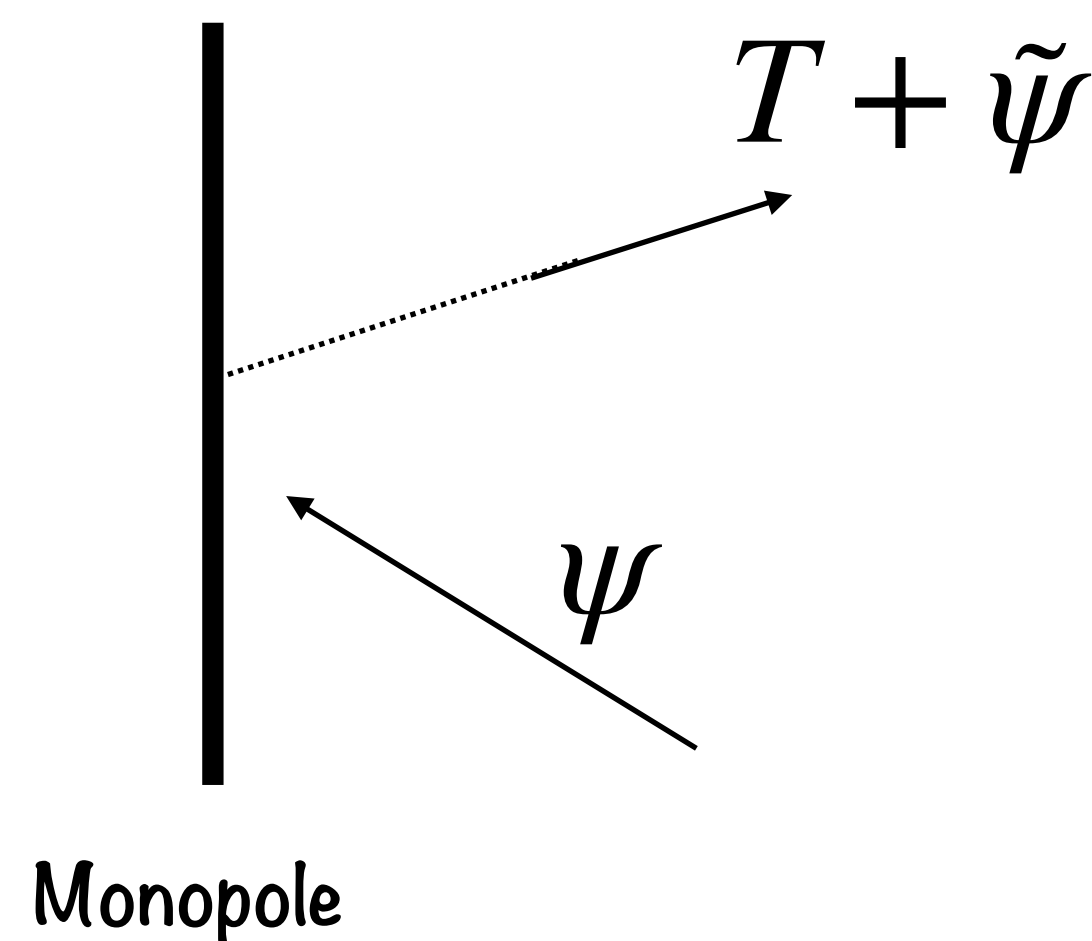
In $\mathbb{R}$ the only role of rotor is to guarantee conservation of $U(1)_M$

Equivalent to: CFT_1 (ingoing) and $\widetilde{CFT}_2$ (outgoing) coupled with the boundary that conserves $SU(N) \times U(1)_M$



Boundary Conformal Field Theory

M. van Beest, P. Boyle Smith, D. Delmastro, Z. Komargodski and D. Tong, arXiv:2306.07318



$$\Delta(T + \tilde{\psi}) = \frac{1}{2} \quad - \text{ free fermion}$$

$T + \tilde{\psi}$ transforms as N_{-1} under $SU(N) \times U(1)_M$

Mild non-locality:

$$T(t, x)\tilde{\psi}(t, y) = \tilde{\psi}(t, y)T(t, x), \quad y > x,$$

$$T(t, x)\tilde{\psi}(t, y) = e^{-4\pi i/N} \tilde{\psi}(t, y)T(t, x), \quad y < x.$$

Rotor $\leftrightarrow$ BCFT

The Fermion-Rotor system

$$O_i(t, x) = e^{i\alpha(t-x)} \psi_i(t, x)$$

$$N_{-1}$$

$$\left\langle O_i(t, x) O_j^\dagger(t', x') \right\rangle = \delta_{ij} G_F(t - t', x - x')$$

BCFT

$$T(t, x) + \tilde{\psi}(t, x)$$

$$N_{-1}$$

$$\Delta(T + \tilde{\psi}) = \frac{1}{2}$$

Rotor $\leftrightarrow$ BCFT

The Fermion-Rotor system

BCFT

$$O_i(t, x) = e^{i\alpha(t-x)} \psi_i(t, x)$$

$$T(t, x) + \tilde{\psi}(t, x)$$

$$N_{-1}$$

$$N_{-1}$$

$$\langle O_i(t, x) O_j^\dagger(t', x') \rangle = \delta_{ij} G_F(t - t', x - x')$$

$$\Delta(T + \tilde{\psi}) = \frac{1}{2}$$

?

$$\begin{aligned} T(t, x) \tilde{\psi}(t, y) &= \tilde{\psi}(t, y) T(t, x), \quad y > x, \\ T(t, x) \tilde{\psi}(t, y) &= e^{-4\pi i/N} \tilde{\psi}(t, y) T(t, x), \quad y < x. \end{aligned}$$

Is $e^{i\alpha(t-x)}$ the twist operator?

VL, T. Okui, D. Tong, arXiv:2508.21059

Exchange relations in Fermion-Rotor system:

$$\dot{\alpha}(t) = \frac{4\pi}{N} \lim_{r_0 \rightarrow 0} J(t + r_0, r_0) \quad = > \quad \alpha(t - x) = \alpha(t) - \frac{4\pi}{N} \int_{t-x}^t dt' \lim_{r_0 \rightarrow 0} J(t' + r_0, r_0)$$

$\parallel$
 $\vee$

$$[\alpha(t - x), \psi_i(t, y)] = -\frac{4\pi}{N} \theta(y) \theta(x - y) \psi_i(t, y)$$

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VL, T. Okui, D. Tong, arXiv:2508.21059

Exchange relations in Fermion-Rotor system:

$$[\alpha(t-x), \psi_i(t, y)] = -\frac{4\pi}{N} \theta(y) \theta(x-y) \psi_i(t, y)$$

$\parallel$
 $\vee$

$$e^{i\alpha(t-x)} \psi_i(t, y) = \psi_i(t, y) e^{i\alpha(t-x)}, \quad y > x,$$

$$e^{i\alpha(t-x)} \psi_i(t, y) = e^{-4\pi i/N} \psi_i(t, y) e^{i\alpha(t-x)}, \quad y < x.$$

Is $e^{i\alpha(t-x)}$ the twist operator?

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We can bosonize fermions:

$$\psi_i(t, x < -r_0) = \frac{1}{\sqrt{2\pi}} e^{i\chi_i(t-x)} \quad \text{and} \quad \psi_i(t, x > r_0) = \frac{1}{\sqrt{2\pi}} e^{i\tilde{\chi}_i(t-x)}$$

$\parallel$
 $\vee$

$$e^{i\alpha(t-x)} = e^{\left(-\frac{2i}{N} \sum_i \tilde{\chi}_i(t-x)\right)}$$

$e^{i\alpha(t-x)}$ is the twist operator!

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We can bosonize fermions:

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$\parallel$
 $\vee$

$$e^{i\alpha(t-x)} = e^{\left(-\frac{2i}{N} \sum_i \tilde{\chi}_i(t-x)\right)}$$

$$T(t, x) \equiv e^{i\alpha(t-x)}$$

Multiple Rotors:

$$S = \int d^2x \, i \sum_{i=1}^N \psi_i^\dagger \left(\partial_+ + i \sum_{a=1}^r q_{ai} \alpha_a(t) f'_a(x) \right) \psi_i + \sum_{a=1}^r \int dt \, \frac{I_a}{2} \dot{\alpha}_a^2$$

$$U(1)^r : \psi_i(t, x) \rightarrow e^{iq_{ai}\beta_a f_a(x)} \psi_i(t, x) \quad \text{and} \quad \alpha_a(t) \rightarrow \alpha_a(t) - \beta_a \quad \text{with } a = 1, \dots, r$$

$$U(1)^{N-r} : \psi_i(t, x) \rightarrow e^{ip_{a'i}\beta_{a'}} \psi_i(t, x) \quad \text{with } a' = r + 1, \dots, N$$

$$\text{with } \sum_{i=1}^N q_{ai} q_{bi} = 0 \quad \text{for } a \neq b \quad \text{and} \quad \sum_{i=1}^N q_{ai} p_{a'i} = 0$$

Rotor $\rightarrow$ BCFT

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In IR the system flows to BCFT conserving $U(1)^r \times U(1)^{N-r}$ on the boundary

We have set of twist operators:

$$T_a(t, x) = e^{i\alpha_a(t-x)}$$

With exchange relations:

$$T_a(t, x)\psi_i(t, y) = \psi_i(t, y)T_a(t, x) \quad \text{for } y > x,$$

$$T_a(t, x)\psi_i(t, y) = \exp\left(-\frac{4\pi i q_{ai}}{\sum_k q_{ak}^2}\right) \psi_i(t, y)T_a(t, x) \quad \text{for } y < x$$

The Fermion-Rotor System Remembers about Witten Anomaly

VL, T. Okui, D. Tong, arXiv:2508.21059

For Fermion-Rotor systems which descends from the theories with the **Witten anomaly** (system with N odd ...) Grassmann-odd operators get the VeV.

For example, for $N=1$:

$$\langle \psi(t, x) e^{i \text{sign}(x) \alpha(\tau)} \rangle \sim \frac{\sqrt{I}}{t - x - \tau}$$

The system compactified on the circle has **odd number of Majorana zero modes** as also previously found in BCFT context.

Summary

- The Fermion-Rotor system provides effective description of the scattering of lowest partial waves of fermions on the monopole
- The well known “Unitarity” puzzle is resolved by recovering “missing” states in the problem interpolated by composite operators.
- In far IR any physical excitation related to rotor (dyon) decouples but the rotor plays crucial role in guarantying conservation of charges!
- In far IR the Fermion-Rotor system flows to BCFT

Open Questions

- What does this mean in 3+1D?
- What happens for small non-zero fermion mass?
- How is the story connected to the abelian instantons?
- What happens with monopole catalysis of nucleon decay?

Thank you for your attention!