Phy 110B Lec 1

Ohm's "Law"

for most substances $\vec{J} = \sigma \vec{f}$

- σ: conductivity
- $\vec{f}$: force per unit charge

electrons randomly scatter

$v_{drift} \sim 10^{-5}$ m/s

for electromagnetic forces

$$\vec{J} = \sigma(\vec{E} + \vec{v} \times \vec{B})$$

($\vec{v} \times \vec{B}$: usually neglect)

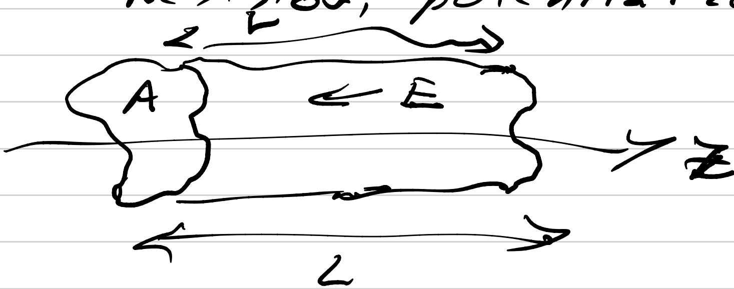
$$\vec{J} = \sigma \vec{E}$$

microscopic Ohm's Law

wires ≈ perfect conductors $\sigma \approx \infty$

resistors ≈ good conductors

Ex 7.1 Resistor, potential constant on ends



current $I = JA = \sigma E A = \frac{\sigma A}{L} V$

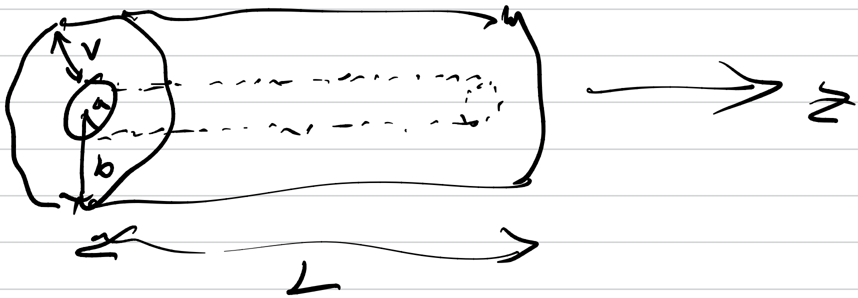
$$\frac{\sigma A}{L} = \frac{1}{R}$$

resistance $= R = \frac{L}{\sigma A} = \rho \frac{L}{A}$ $[\sigma] = \frac{1}{\Omega \cdot m}$

$[\rho] = \Omega \cdot m$

$V = IR$ Ohm's Law (macroscopic)

Ex 7.2 coaxial metal cylinders, separated by conductivity σ, find $I(V)$



$$\vec{E} = \frac{\lambda}{2\pi\epsilon_0 s}\hat{s}$$, λ is charge per unit length at a

$$I = \int \vec{J} \cdot d\vec{a} = \sigma \int \vec{E} \cdot d\vec{a} = \frac{\sigma\lambda}{2\pi\epsilon_0 s} 2\pi s L$$

$$= \frac{\sigma}{\epsilon_0} \lambda L$$

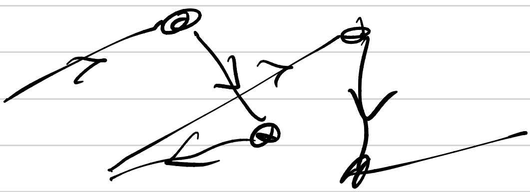
$$V = -\int_b^a \vec{E} \cdot d\vec{l} = \frac{-\lambda}{2\pi\epsilon_0} \int_b^a \frac{ds}{s} = \frac{\lambda}{2\pi\epsilon_0} \ln\left(\frac{b}{a}\right)$$

$$\lambda = \frac{2\pi\epsilon_0 V}{\ln(b/a)}$$

$$I = \frac{\sigma}{\epsilon_0} \lambda L = \frac{2\pi\sigma}{\ln(b/a)} V$$

$$V = IR \quad , \quad R = \frac{\ln(b/a)}{2\pi\sigma L}$$

Ohm's "law" is a good rule of thumb that applies to lots of materials



$V_{thermal} \sim 100$ m/s

$V_{drift} \sim 10^{-5}$ m/s

collisions transfer energy to lattice

work done per unit charge = V
charge per unit time = I

Power = rate of work = $VI = I^2 R$

Joule heating "law"

Problem 7.1 two metal spheres
weak conductor between
at potential difference V
what is the current

if Q is the charge on the inner shell

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{r}$$

$$V_a - V_b = -\int_b^a \vec{E}\cdot d\vec{r} = \frac{-1}{4\pi\epsilon_0} Q \int_b^a \frac{1}{r^2}\,dr$$

$$= \frac{-Q}{4\pi\epsilon_0}\left[-\frac{1}{r}\right]_b^a = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$

$$I = \int \vec{J}\cdot d\vec{a} = \sigma \int \vec{E}\cdot d\vec{a} = \sigma \frac{Q}{\epsilon_0}$$

$$= \frac{\sigma}{\epsilon_0} \frac{4\pi\epsilon_0}{\left(\frac{1}{a} - \frac{1}{b}\right)}(V_a - V_b)$$

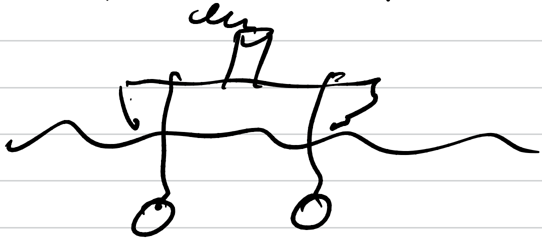
b) What is the resistance?

$$R = \frac{V_a - V_b}{I} = \frac{1}{4\pi\sigma}\left(\frac{1}{a} - \frac{1}{b}\right)$$

c) What happens for $b \gg a$

$$R \approx \frac{1}{4\pi\sigma a}$$

all the resistance is near the sphere

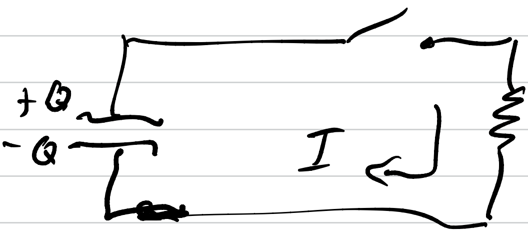


$$R = \frac{2}{4\pi\sigma a}$$

$$I = \frac{V}{R} = 2\pi\sigma a V$$

$$\sigma = \frac{I}{2\pi a V}$$

Prob 7.2 A capicitor, C, is charged up to V_0 at $t=0$ it is connected to a resistor



a) determine $Q(t)$ and $I(t)$

$$V = Q/C = IR$$

$$\frac{dQ}{dt} = -I = -\frac{V}{R} = -\frac{Q}{RC}$$

$$Q = Q_0 e^{-t/RC} = CV_0 e^{-t/RC}$$

$$I = -\frac{dQ}{dt} = \frac{V_0}{R} e^{-t/RC}$$

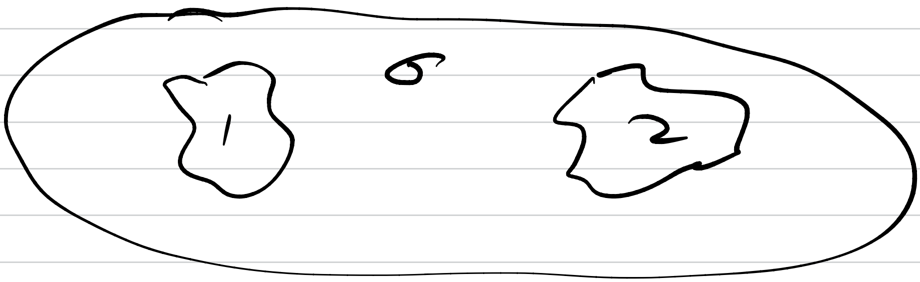
b) confirm that the heat delivered to the resistor is equal to the initial energy of the capacitor

$$U = \frac{1}{2} C V_0^2$$

$$\int_0^\infty P\,dt = \int_0^\infty I^2 R\,dt = \frac{V_0^2}{R}\int_0^\infty e^{-2t/RC}\,dt$$

$$= \frac{V_0^2}{R}\left[-\frac{RC}{2} e^{-2t/RC}\right]_0^\infty = \frac{1}{2} C V_0^2$$

Problem 7.3 two metal objects in a weakly conducting material find the resistance between them



start with ~~charg~~ Q on 1 and -Q on 2

$I = \int \vec{J} \cdot d\vec{a}$ for surface enclosing 1

$= \sigma \int \vec{E} \cdot d\vec{a} = \sigma \int \vec{E} \cdot d\vec{a} = \sigma \frac{Q}{\epsilon_0}$

$Q = CV = CIR$

$R = \frac{Q}{CI} = \frac{\epsilon_0}{\sigma C}$

b) charge up to potential diffence V_0 disconnect battery, find $V(t)$

$\frac{dQ}{dt} = -I = \frac{-Q}{CR} \Rightarrow Q = Q_0 e^{-t/RC}$

$V = \frac{Q}{C} = V_0 e^{-t/RC}$ $RC = \frac{\epsilon_0}{\sigma}$