Chiral lagrangian from quarks with dynamical mass

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We develop a simple model for coupling the pseudo-Goldstone bosons of QCD (π's, K's, η) to quark fields. This nonlocal model incorporates a momentum dependent dynamical quark mass $\Sigma(p)$ in a manner consistent with global chiral symmetry. Explicit current quark masses and gauge fields are also introduced. All low energy parameters of the chiral lagrangian are expressed in terms of $\Sigma(p)$. In particular we obtain the ten coefficients of order p^4 terms in the energy expansion. The physical values for m_{π^0}, m_{K^+}, m_{K^0}, f_π, and f_K are used to determine the three current quark masses, the constituent quark mass, and the quark condensate. The results are in surprisingly good agreement with the experimental values.

In this paper we will present a simple nonlocal model for pseudo-Goldstone bosons (PGBs) coupled to massive quarks. Upon integrating out the quarks and performing a derivative expansion the model yields an effective low energy chiral lagrangian. We will make no attempt to "derive" the model as a controlled approximation to QCD, and there is no reason a priori why the chiral lagrangian we obtain should resemble the low energy effective theory of QCD. But the model does incorporate a nontrivial momentum dependence for the quark mass and thereby represents the dynamical quark mass in QCD. This is accomplished in a manner consistent with the dynamical breakdown of chiral symmetry and it improves previous derivations of chiral lagrangians in chiral quark models. We find, rather surprisingly, that the model does well at reproducing what is currently known about low energy QCD.

An effective chiral lagrangian may be used to encode the physics of the PGBs of QCD in a systematic low-energy expansion. In the notation of Gasser and Leutwyler [1], for QCD with three light quarks the leading (order p^2) and next-to-leading (order p^4) terms of the chiral lagrangian are the following:

$$\begin{aligned} L = {} & \tfrac{1}{4}F_0^2\{\langle\nabla^\mu U^\dagger\nabla_\mu U\rangle + \langle\chi^\dagger U + \chi U^\dagger\rangle\} + L_1\langle\nabla^\mu U^\dagger\nabla_\mu U\rangle^2 + L_2\langle\nabla_\mu U^\dagger\nabla_\nu U\rangle\langle\nabla^\mu U^\dagger\nabla^\nu U\rangle + L_3\langle\nabla^\mu U^\dagger\nabla_\mu U\nabla^\nu U^\dagger\nabla_\nu U\rangle \\ & + L_4\langle\nabla^\mu U^\dagger\nabla_\mu U\rangle\langle\chi^\dagger U + \chi U^\dagger\rangle + L_5\langle\nabla^\mu U^\dagger\nabla_\mu U(\chi^\dagger U + \chi U^\dagger)\rangle + L_6\langle\chi^\dagger U + \chi U^\dagger\rangle^2 + L_7\langle\chi^\dagger U - \chi U^\dagger\rangle^2 \\ & + L_8\langle\chi^\dagger U\chi^\dagger U + \chi U^\dagger\chi U^\dagger\rangle - \mathrm{i}L_9\langle F^{\mathrm{R}}_{\mu\nu}\nabla^\mu U\nabla^\nu U^\dagger + F^{\mathrm{L}}_{\mu\nu}\nabla^\mu U^\dagger\nabla^\nu U\rangle + L_{10}\langle U^\dagger F^{\mathrm{R}}_{\mu\nu} U F^{\mathrm{L}\mu\nu}\rangle + L_{\mathrm{WZ}}. \end{aligned} \quad (1)$$

We may express U in terms of the PGBs (here π_i represents the pions, the kaons, and the eta) as follows:

$$U \equiv \exp\left(\sum_i \frac{2\mathrm{i}\pi_i(x)\lambda_i}{F_0}\right), \quad (2)$$

where F_0 is the decay constant in the chiral limit and $\mathrm{Tr}\,\lambda_i\lambda_k = \frac{1}{2}\delta_{ik}$. χ is defined by

$$\chi \equiv 2B_0 M, \qquad M = \begin{pmatrix} m_{\mathrm{u}} & 0 & 0 \\ 0 & m_{\mathrm{d}} & 0 \\ 0 & 0 & m_{\mathrm{s}} \end{pmatrix}. \quad (3)$$

The Wess–Zumino term L_{WZ} introduces no further parameters and thus to this order in the energy expansion

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the chiral lagrangian contains 14 independent parameters:

$$F_0, \quad B_0 m_u, \quad B_0 m_d, \quad B_0 m_s, \quad L_1\text{-}L_{10}. \tag{4}$$

All low energy physical quantities are functions of these parameters, and their values have been obtained in the phenomenological analysis of Gasser and Leutwyler. L_1-L_{10} are scale dependent and these authors choose to renormalize these quantities at the η mass. Their values are included in table 1.

More recently various groups [2,3] have argued that some of these parameters, in particular L_1, L_2, L_3, L_9, L_{10}, are in fact saturated by the lowest lying spin-1 resonances of QCD. The authors of ref. [2] develop a chiral lagrangian model including vector and axial-vector mesons; integrating out the spin-1 fields yields results which are reproduced in table 1 (renormalized at the η mass).

In this paper we argue that all the quantities in (4) are quite consistent with a different picture in which the essential dynamics occurs at the quark level. The main point of our model will be to incorporate a momentum dependent dynamical quark mass $\Sigma(p)$ characteristic of the dynamical chiral symmetry breaking of QCD [4]. $\Sigma(p)$ represents an order parameter and it is natural for PGBs to appear as fluctuations of this order parameter. $\Sigma(p)$ will also serve as a natural regulator in our calculations, thereby avoiding the introduction of an independent ultraviolet cutoff. (This is also a feature of the work in ref. [5].)

Our model is closely related to the Pagels–Stokar (PS) approximation [6] for QCD in which all corrections beyond those contained in $\Sigma(p)$ are dropped. In particular we will reproduce the PS formula for F_0 (see (16) below). Recently [7], F_0 has been calculated from the Bethe–Salpeter equation in the ladder approximation. This goes beyond the PS formula and includes corrections beyond those contained in $\Sigma(p)$. These corrections are found to be surprisingly small, thus lending some support to the PS approximation.

The gauging of our model will be nontrivial since it is nonlocal, but this will be necessary to obtain L_9 and L_{10}. To obtain L_4, L_5, L_6, L_7 and L_8, current quark masses are introduced in an obvious way. The model then clearly distinguishes between the dynamical, momentum-dependent quark mass and the current quark mass.

This distinction is lost if $\Sigma(p)$ is replaced by a constant. Then the model reproduces previous calculations in local chiral quark models [8]. They yield values for the quantities L_1, L_2, L_3, L_9, L_{10} as shown in table 1. In fact these quantities may be derived in models making no mention of quark mass. In that context there is no natural scale and it would be difficult to associate these quantities with the scale dependent couplings of a chiral lagrangian. Even so, there is surprisingly fair agreement between the values listed and the physical L_i's renormalized at m_η.

Other quantities are a more sensitive reflection of the details of chiral symmetry breaking. For example, when $\Sigma(p)$ is a constant, f_π and other L_i's acquire dependence on an ultraviolet cutoff. But we shall see that this cutoff dependence is just a consequence of the unrealistic, constant $\Sigma(p)$.

Upon integrating out the quark fields the dynamical quark mass (or more precisely twice this mass) will provide a natural scale at which to match the effective theory to the underlying theory. Thus our model will express parameters of the effective chiral lagrangian, when renormalized at this scale, purely in terms of integrals of $\Sigma(p)$ and its derivatives.

Expanding L_{WZ} produces terms with, respectively, five PGBs, three PGBs and one gauge field, and one PGB and two gauge fields. The coefficients of these three terms in our model turn out to be independent of $\Sigma(p)$ as they should; the associated integrals of $\Sigma(p)$ involve total derivatives and are identically equal to the correct constants [4,5].

We use physical values for masses and decay constants (m_{π^0}, m_{K^+}, m_{K^-}, f_π, and f_K) to determine the five parameters of our model: the constituent quark mass, the quark condensate (or equivalently the parameter B_0), and the three current quark masses. In our model the current quark masses and the parameter B_0 are independent parameters, unlike the chiral lagrangian (1), and this will allow us to determine the overall scale of the quark masses as well as their ratios.

As for the 10 L_i's we find that one combination of L_5 and L_8, and all the other L_i's are independent of the above mentioned parameters. (The running of the L_i's does of course depend on the parameters.) We present results for two different $\Sigma(p)$, both of which are consistent with the asymptotic behavior implied by QCD (additional logs are safely ignored).

$$\Sigma(p)_1 = \frac{2m^3}{m^2+p^2}, \qquad \Sigma(p)_2 = \frac{4m^3}{3m^2+p^2}. \tag{5}$$

$\Sigma(p)$ is normalized by setting $\Sigma(m)=m$ and we have identified m with the constituent quark mass.

Table 1 gives the L_i's and table 2 gives the five parameters. As we have said, the model yields values for the running couplings effectively renormalized at $\mu \approx 2m$ (we actually use $\mu = 2m + m_u + m_d$). To compare to the Gasser and Leutwyler values we run our values down to their choice, $\mu = m_\eta$. It may be noted that many of the quantities display little sensitivity to the choice for $\Sigma(p)$. The most sensitive is L_{10}, and its value shows a marked improvement from the constant $\Sigma(p)$ case.

We now describe our model. Incorporating a momentum dependent self-energy $\Sigma(p)$ requires a bilocal term bilinear in the fermion field. All $\pi_i(x)$ dependence occurs in this term; that is, PGBs appear as fluctuations of the order parameter. Our model lagrangian then takes the form, in euclidean space,

$$L(x,y) = \bar\psi(x)\delta(x-y)\not\partial\psi(y) + \bar\psi(x)\Sigma_\pi(x,y)\psi(y). \tag{6}$$

$\Sigma_\pi(x,y)$ is a function of $\pi(x)$ and $\pi(y)$ such that $\Sigma_\pi(x,y) \Rightarrow \Sigma(x-y)$ for $\pi=0$, where $\Sigma(x-y)$ is Fourier transform of $\Sigma(p)$. The quarks carry both color and flavor but $\Sigma_\pi(x,y)$ is nontrivial only in flavor space. "color" is only a global symmetry.

The model must retain a global chiral invariance and this will constrain $\Sigma_\pi(x,y)$. Even so, our choice for $\Sigma_\pi(x,y)$ will not be unique, and we can only argue that we are making a minimal choice. Our choice follows from the standard derivative coupling construction for coupling PGBs to matter fields [9]. And as we shall see, this choice will reproduce the Pagels-Stokar formula for F_0.

Under a global chiral transformation the quantity $\xi(x) = \exp[-i\pi(x)/F_0]$, where $\pi(x) \equiv \sum_i \pi_i(x)\lambda_i\gamma_5$, transforms as

$$\xi(x) \Rightarrow g(x,\alpha,\pi(x))\xi(x)\exp(-i\alpha\gamma_5) = \exp(-i\alpha\gamma_5)\xi(x)g^\dagger(x,\alpha,\pi(x)). \tag{7}$$

g is a nonlinear function of $\alpha = \sum_i \alpha_i\lambda_i$ and $\pi(x)$. The vector field

$$V_\mu(x) = \tfrac{1}{2}i(\partial_\mu\xi\xi^\dagger + \partial_\mu\xi^\dagger\xi) \tag{8}$$

Table 1
Values, multiplied by 10^3, for the coefficients of the order p^4 chiral lagrangian. GL: experimental values, Gasser and Leutwyler [1]; VMD: vector and axial vector meson model [2]; LCQ: local chiral quark models [8]; last two columns are values from our model with quark self energies $\Sigma(p)_1$ and $\Sigma(p)_2$ given in eq. (5).

	GL	VMD	LCQ	$\Sigma(p)_1$	$\Sigma(p)_2$
L_1	0.9 ± 0.3	1.1	0.79	0.97	0.90
L_2	1.7 ± 0.7	2.2	1.58	1.95	1.80
L_3	-4.4 ± 2.5	−5.5	−3.17	−4.20	−3.90
L_4	0 ± 0.5	-	-	0.16	0.13
L_5	2.2 ± 0.3	-	-	2.04	2.07
L_6	0 ± 0.3	-	-	0.10	0.08
L_7	-0.4 ± 0.15	-	-	−0.249	−0.252
L_8	1.1 ± 0.7	-	-	1.04	1.06
L_9	7.4 ± 0.7	7.8	6.33	6.27	6.11
L_{10}	-6.0 ± 0.7	−6.0	−3.17	−7.09	−5.23

Table 2
Values, in MeV, for the current quark masses, the constituent quark mass m, and the quark condensate $\langle\bar\psi\psi\rangle_\mu$.

	m_u	m_d	m_s	m	$\lvert\langle\bar\psi\psi\rangle_\mu^{1/3}\rvert$
$\Sigma(p)_1$	4.7	8.1	160	331	211
$\Sigma(p)_2$	5.0	8.7	170	292	208

transforms as $V_\mu(x)\Rightarrow gV_\mu(x)g^\dagger+\mathrm{i}(\partial_\mu g)g^\dagger$. In terms of these quantities it is straightforward to construct a $\Sigma_\pi(x,y)$ which transforms under a global chiral transformation as

$$\Sigma_\pi(x,y)\Rightarrow\exp(-\mathrm{i}\alpha\gamma_5)\Sigma_\pi(x,y)\exp(-\mathrm{i}\alpha\gamma_5). \tag{9}$$

We may take

$$\Sigma_\pi(x,y)=\Sigma(x-y)\xi(x)X(x,y)\xi(y),\qquad \text{where } X(x,y)=\mathrm{P}\exp\left(\mathrm{i}\int_x^y V_\mu\,\mathrm{d}x^\mu\right). \tag{10}$$

$X(x,y)$ is the standard path ordered exponential. This is a minimal choice for $\Sigma_\pi(x,y)$ in the sense that it reduces to a very simple and suggestive form, $\xi(x)\Sigma(x-y)\xi(y)$, when V_μ happens to vanish. V_μ vanishes when $[\partial_\mu\xi,\xi^\dagger]=0$ or when $[\pi(x),\pi(y)]=0$ for all x, y.

But as it stands (10) is not terribly useful for obtaining an explicit power series expansion in $\pi(x)$ and $\pi(y)$ for $\Sigma_\pi(x,y)$. We will instead write $\Sigma_\pi(x,y)$ as the following explicit function of $\pi(x)$ and $\pi(y)$, and then impose the constraint that $\Sigma_\pi(x,y)\Rightarrow\xi(x)\Sigma(x-y)\xi(y)$ when $[\pi(x),\pi(y)]=0$.

$$\Sigma_\pi(x,y)=\Sigma(x-y)G(\pi(x),\pi(y)),$$
$$G(\pi(x),\pi(y))=\tfrac{1}{2}\{[a_1V(x)+a_2V(x)V(y)^\dagger V(x)+a_3V(x)V(y)^\dagger V(x)V(y)^\dagger V(x)+\cdots]+(x\Leftrightarrow y)\}. \tag{11}$$

$V(x)\equiv\exp[-2\mathrm{i}\pi(x)\gamma_5/F_0]$; this explicitly transforms in the correct manner since $V(x)\Rightarrow\exp(-\mathrm{i}\alpha\gamma_5)V(x)\exp(-\mathrm{i}\alpha\gamma_5)$ under a chiral transformation. The constraint on $G(\pi(x),\pi(y))$ implies that the a_k must satisfy

$$\sum_{k=1}^{\infty}a_k(2k-1)^{2n}=1\qquad\text{for } n=0,$$
$$=0\qquad\text{for } n=1,2,3,\ldots\,. \tag{12}$$

These equations do in fact have solutions of the form

$$a_k=\frac{4(-1)^{(k-1)}}{\pi(2k-1)}\cos((2k-1)t),\qquad t<\pi/2. \tag{13}$$

But we are more interested in the explicit expansion in $\pi(x)$ and $\pi(y)$, and for our purposes we require this expansion to order π^4. To this order we need only satisfy the first three equations in (12), ($n=0,1,2$). We find that any a_k which satisfies these equations gives a unique expansion for $\Sigma_\pi(x,y)$ to $\mathrm{O}(\pi^4)$. We find (leaving out the factors of F_0^{-1})

$$\begin{aligned}G(\pi(x),\pi(y))=&\,1-\mathrm{i}\pi(x)-\mathrm{i}\pi(y)-\pi(x)^2/2-\pi(y)^2/2-\pi(x)\pi(y)/2-\pi(y)\pi(x)/2+\mathrm{i}\pi(x)^3/6+\mathrm{i}\pi(y)^3/6\\&+\mathrm{i}\pi(x)\pi(y)\pi(x)/2+\mathrm{i}\pi(y)\pi(x)\pi(y)/2+\pi(x)^4/24+\pi(y)^4/24-\pi(x)^2\pi(y)^2/8-\pi(y)^2\pi(x)^2/8\\&-\pi(x)\pi(y)^3/24-\pi(x)^3\pi(y)/24-\pi(y)\pi(x)^3/24-\pi(y)^3\pi(x)/24\\&+\pi(x)\pi(y)\pi(x)^2/8-\pi(x)\pi(y)^2\pi(x)/8+\pi(x)^2\pi(y)\pi(x)/8\\&+\pi(y)\pi(x)\pi(y)^2/8-\pi(y)\pi(x)^2\pi(y)/8+\pi(y)^2\pi(x)\pi(y)/8\\&+3\pi(x)\pi(y)\pi(x)\pi(y)/8+3\pi(y)\pi(x)\pi(y)\pi(x)/8+(\text{higher order in }\pi).\end{aligned} \tag{14}$$

With this we may integrate out the quark fields and obtain the chiral lagrangian $L_{\mathrm{eff}}(\pi(x))$:

$$\exp\left(-\int\mathrm{d}^4x\,L_{\mathrm{eff}}(\pi(x))\right)\equiv\int\mathrm{D}\bar\psi\,\mathrm{D}\psi\,\exp\left(-\int\mathrm{d}^4x\,\mathrm{d}^4y\,L(x,y)\right),$$
$$\int\mathrm{d}^4x\,L_{\mathrm{eff}}(\pi(x))=-\mathrm{tr}\log S_\pi^{-1},\qquad S_\pi^{-1}\equiv\not\partial_x\delta(x-y)+\Sigma_\pi(x,y). \tag{15}$$

This may be expanded simultaneously in powers of π and in powers of derivatives. This involves quark loop diagrams with π's attached in all possible ways according to (14). Fortunately the result is constrained by chiral symmetry to take the form of a standard chiral lagrangian, a function of the $U(x)$ in (2).

We will insert current quark masses below but for now there are no zero derivative terms, i.e. no π mass. For the two derivative term in the chiral lagrangian it is sufficient to look at the two PGB diagrams. Standard normalization of the kinetic term in the chiral lagrangian implies a formula for F_0,

$$F_0^2 = \frac{N_c}{(2\pi)^2} \int dq^2\, q^2 \Sigma(q) \left(\frac{\Sigma(q) - \frac{1}{2} q^2 \Sigma'(q)}{[q^2 + \Sigma(q)^2]^2} \right), \qquad \text{where } \Sigma'(q) \equiv \frac{d\Sigma(q)}{dq^2}. \tag{16}$$

This happens to be the result derived by Pagels and Stokar by quite different methods. It is related to our particular choice for $\Sigma_\pi(x, y)$, other choices can give different formulas for F_0 [4]. (The physical f_π and f_K are related to F_0 but incorporate quark mass and PGB loop corrections.)

To find the four derivative terms in the chiral lagrangian we must look at both the two PGB and the four PGB diagrams in the derivative expansion. The two PGB-four derivative piece will fix the coefficient of the $\mathrm{Tr}(\partial^2 U^\dagger \partial^2 U)$ term in the chiral lagrangian. But as Gasser and Leutwyler described [1], the equations of motion may be used to remove this term and in the process adjust the parameter L_3 (along with L_7 and L_8 when there are quark masses). The four PGB-four derivative piece will then determine L_1, L_2, L_3. We eventually arive at lengthy integral expressions involving $\Sigma(p)$ and its first four derivatives. Table 1 gives the result of evaluating these expressions for the two different choices of $\Sigma(p)$. Note that for a given form for $\Sigma(p)$, L_1, L_2, L_3 are independent of m.

Now let us incorporate a current quark mass matrix M (as given in (3)). We simply add a PGB independent quark mass term to the model; that is, the S_π^{-1} appearing in (15) takes the modified form

$$S_\pi^{-1}(x, y) \equiv \not{\partial}_x \delta(x-y) + \Sigma_\pi(x, y) + M\delta(x-y). \tag{17}$$

We first find the term in the chiral lagrangian (1) which depends linearly on M; this determines the quantity B_0.

$$B_0 = -\frac{\langle \bar\psi\psi \rangle}{F_0^2}, \qquad \text{where } \langle \bar\psi\psi \rangle = -\frac{N_c}{4\pi^2} \int d^4q \frac{\Sigma(q)}{q^2 + \Sigma(q)^2}. \tag{18}$$

The standard relation $F_0^2 m_\pi^2 = -(m_u + m_d)\langle \bar\psi\psi \rangle$ follows. But the expression for $\langle \bar\psi\psi \rangle$ in (18) is divergent and must of course be renormalized. The prescription in QCD leads to a result which is related to the high energy behavior of $\Sigma(p)$. For our purposes we will simply take the renormalized B_0 to be a parameter, to be determined, and define $\langle \bar\psi\psi \rangle_\mu = -F_0^2 B_0$.

Thus for a given form for $\Sigma(p)$ with $\Sigma(m) = m$ the model has two parameters B_0 and m, besides the three quark masses m_u, m_d, m_s (F_0 is related to M through (16)). We find that all but one combination of the L_i's are independent of these parameters. L_4, L_5, L_6, L_7, L_8 are defined in the lagrangian (1) with appropriate factors of B_0 inserted to make up the correct dimensions. In our derivative expansion factors of m appear in place of B_0, and thus it seems that we need the ratio B_0/m to determine these L_i's. But it turns out that these L_i's are independent of B_0/m except for one combination of L_5 and L_8.

L_4 and L_6 are simply zero (up to PGB loop corrections) and this is consistent with Zweig's rule. ($2L_1 - L_2 = 0$ is also a reflection of Zweig's rule.) L_7 is nonzero only due to the shift introduced by the $\mathrm{Tr}(\partial^2 U^\dagger \partial^2 U)$ term mentioned above. But the coefficient of $\mathrm{Tr}(\partial^2 U^\dagger \partial^2 U)$, like the other four derivative terms, is independent of B_0 and m and thus so is L_7. Note that in QCD L_7 is sensitive to physics in the flavor singlet channel including η' exchange.

The model yields the following relation between L_5 and L_8 which is also independent of B_0 and m.

$$L_5^2 + c_1 L_8 = c_2, \tag{19}$$

$(c_1, c_2) = (-1.55 \times 10^{-2}, -1.16 \times 10^{-5})$, $(-1.43 \times 10^{-2}, -1.08 \times 10^{-5})$ for $\Sigma(p)_1$, $\Sigma(p)_2$ respectively. In either case, this relation is well satisfied by the experimental values renormalized at $\mu \approx 2m$.

The parameters m_u, m_d, m_s, m, and $\langle\bar{\psi}\psi\rangle_\mu$ in table 2 are found by calculating m_π, $m_{K^\pm}$, m_{K^0}, f_π, f_K and then fitting the resulting expressions to the experimental values. Second order quark mass effects and PGB loop corrections as described in ref. [1] are included. The quark mass ratios are very stable against changes in $\Sigma(p)$: $m_u/m_d=(0.58, 0.58)$ and $m_s/m_d=(19.7, 19.6)$. These values agree with the analysis of ref. [1] which includes input from baryon octet mass splittings. In addition the model yields the overall scale of the quark masses. We obtain $B_0=(1420, 1340)$ and $F_0=(81.5, 82.0)$ and this yields the values for $\langle\bar{\psi}\psi\rangle_\mu$ in table 2.

To find L_9 and L_{10} it is sufficient to gauge the model with a non-abelian vector gauge field A_μ. This is done by introducing appropriate factors of

$$Y(x, y)=\mathrm{P}\exp\left(-\mathrm{i}\int_x^y gA_\mu\,\mathrm{d}x^\mu\right). \tag{20}$$

The expansion in (11) now reads

$$\tfrac{1}{2}\{[a_1 V(x)+a_2 V(x)Y(x, y)V(y)^\dagger Y(y, x)V(x)+\cdots]+(x\Leftrightarrow y)\}. \tag{21}$$

But since the PGBs themselves transform linearly under vector transformations, factors of $Y(x, y)$ may be inserted directly into the expansion for $G(\pi(x), \pi(y))$ in (14). This is then translated into the nonlocal multi-A_μ-multi-π-$\bar{\mathrm{q}}\mathrm{q}$ vertices appearing in the two sets of quark loop diagrams used to determine L_9 and L_{10}. The former set involves two π's and one A_μ attached in all possible ways and the latter involves two π's and two A_μ's.

We will elaborate on the derivation of the gauge field vertices elsewhere. Schematically, we transform the action to momentum space and expand $\Sigma(p)$ in powers of p. We replace powers of p by derivatives which, when acting on $Y(x, y)$, yield $A_\mu(y)$ and its derivatives [10]. The resulting derivative expansion for each gauge vertex is then summed to all orders. These vertices satisfy the appropriate Ward-Takahashi identities. There are additional checks since coefficients of various terms in the chiral lagrangian may be determined in more than one way, by considering appropriate quark loop diagrams with and without gauge fields attached.

We now note the following fact in QCD; the dynamical mass $\Sigma(x-y)$ is expected to have some dependence on the current quark mass. This would result in M dependence in $\Sigma_\pi(x, y)$ and produce M dependent PGB-quark couplings. Does this make our results for L_5 and L_8 necessarily inconsistent with QCD? The answer is no since the model can incorporate an M dependent dynamical mass without changing the results. This occurs if $\Sigma_\pi(x, y)$ depends only on $MM^\dagger$ or $M^\dagger M$. ($\Sigma_\pi(x, y)$ cannot have a linear dependence on M without disrupting the standard relation following (18).) The correction term would take the form

$$\tilde{\Sigma}(x-y)[MM^\dagger G(\pi(x), \pi(y))+G(\pi(x), \pi(y))M^\dagger M]. \tag{22}$$

It is not difficult to see that this will not contribute to any of the L_i.

In conclusion we have found that a simple model which incorporates a momentum dependent constituent quark mass reproduces low energy QCD quite well. This is surprising given that our model has completely ignored all QCD physics associated with confinement. This seems to suggest that QCD corrections beyond those already contained in $\Sigma(p)$ are relatively unimportant for low energy chiral dynamics.

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Note added in proof. A quark-based model of the QCD chiral lagrangian was also recently discussed in ref. [11].

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