ISOSPIN BREAKING AND THE TOP-QUARK MASS IN MODELS OF DYNAMICAL ELECTROWEAK SYMMETRY BREAKING[*]

R.S. Chivukula, B.A. Dobrescu, and J. Terning
Physics Department, Boston University
590 Commonwealth Ave.
Boston, MA 02215 USA
E-mail: sekhar@bu.edu, dobrescu@budoe.bu.edu, terning@calvin.bu.edu

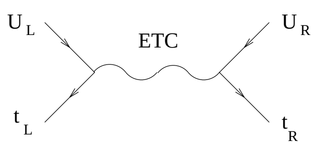
BUHEP-95-22 & hep-ph/9506450

ABSTRACT

In this talk we review the physics of top-quark mass generation in models of dynamical electroweak symmetry breaking and the constraints on this physics arising from limits on the deviation of the weak interaction ρ-parameter from one. We then discuss top-color assisted technicolor in this context.

1. m_t in Models of Dynamical Electroweak Symmetry Breaking

In technicolor models, the masses of the ordinary fermions are due to their coupling, through additional, broken, extended-technicolor (ETC) gauge-interactions[1], to the technifermions:



(1)

which leads to a mass for the top-quark

$$m_t \approx \frac{g^2}{M_{ETC}^2}\langle \bar{U}U\rangle_{M_{ETC}} \ , \tag{2}$$

where we have been careful to note that it is the value of the technifermion condensate renormalized at the scale M_{ETC} which is relevant.

For a QCD-like technicolor, where there is no substantial difference between $\langle \bar{U}U\rangle_{M_{ETC}}$ and $\langle \bar{U}U\rangle_{\Lambda_{TC}}$, we can use naive dimensional analysis[2] to estimate the technifermion condensate, arriving at a top-quark mass

$$m_t \approx \frac{g^2}{M_{ETC}^2} 4\pi F^3 \ , \tag{3}$$

[*]Talk presented by R. S. Chivukula at the *Workshop on Top Quark Physics*, Iowa State University, Ames, IA, May 25-26, 1995 and the *Yukawa International Seminar '95*, Yukawa Institute, Kyoto, Aug. 21-25, 1995

where F is the analog of the pion decay-constant (93 MeV, in my normalization) in QCD. We can invert this relation to find the characteristic mass-scale of top-quark mass-generation

$$\frac{M_{ETC}}{g} \approx 1 \text{ TeV} \left(\frac{F}{246 \text{ GeV}}\right)^{\frac{3}{2}} \left(\frac{175 \text{ GeV}}{m_t}\right)^{\frac{1}{2}} . \tag{4}$$

We immediately see that the scale of top-quark mass generation is likely to be *quite* low, unless the value of the technifermion condensate ($\langle \bar{U}U \rangle_{M_{ETC}}$) can be raised significantly above the value predicted by naive dimensional analysis. The prospect of such a low ETC-scale is both tantalizing and problematic. As we will see in the next section, constraints from the deviation of the weak interaction ρ parameter from one suggest that the scale may have to be larger than one TeV.

The most promising approach[3] to enhance the technifermion condensate and accommodate the top-quark mass is "strong-ETC". As the size of the ETC-coupling approaches the critical value for chiral symmetry breaking, it is possible to enhance the running technifermion self-energy $\Sigma(k)$ at large momenta (see Figure 1).

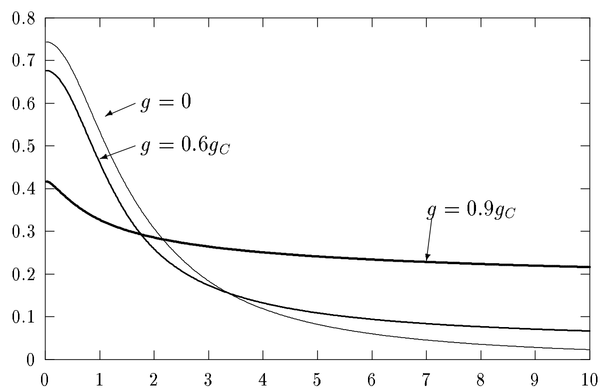


Figure 1: Plot[4] of technifermion self energy $\Sigma(k)$ vs. momentum (both in TeV), as predicted by the gap-equation in the rainbow approximation, for various strengths of the ETC coupling relative to their critical value g_C.

Since the technifermion condensate is related to the trace of the fermion propagator.

$$\langle \bar{U}U \rangle_{M_{ETC}} \propto \int^{M^2_{ETC}} dk^2 \Sigma(k) \ , \tag{5}$$

a slowly-falling running-mass translates to an enhanced condensate.[a]

Unfortunately, there is no such thing as a free lunch. As we see from Fig. 2, the enhancement of the technifermion self-energy in strong-ETC theories comes at the

[a]More physically, in terms of the relevant low-energy theory, it can be shown that the enhancement of the top-quark mass is due to the dynamical generation of a light scalar state[5]

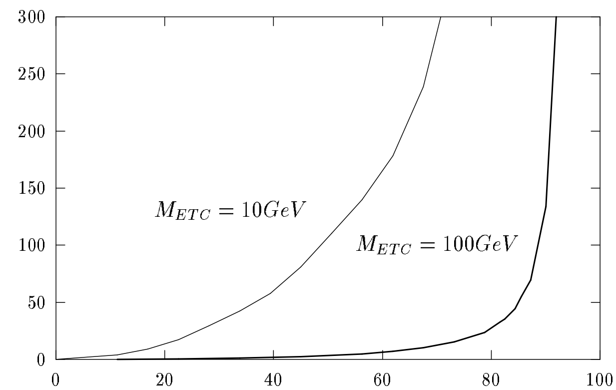


Figure 2: Plot[4] of top mass (in GeV) vs. ETC coupling (g/g_c in %), as predicted by gap-equation in the rainbow approximation, for ETC scales of 10 and 100 TeV.

cost of a "fine-tuning" of the strength of the ETC coupling relative to the critical value where the ETC interactions would, in and of themselves, generate chiral symmetry breaking. In the context of the NJL approximation[6], we find that enhancement of the top quark mass is directly related to the severity of this adjustment,

$$\frac{\langle \bar{U}U \rangle_{\Lambda_{TC}}}{\langle \bar{U}U \rangle_{M_{ETC}}} \approx \frac{\Delta g^2}{g_c^2} \ , \tag{6}$$

where $\Delta g^2 \equiv g^2 - g_C^2$.

2. $\Delta\rho_*$

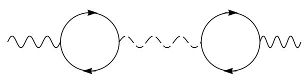
The physics which is responsible for top-quark mass generation must violate custodial $SU(2)$ since, after all, this physics must give rise to the disparate top- and bottom-quark masses.

2.1. Direct Contributions

ETC operators which violate custodial isospin by two units ($\Delta I = 2$) are particularly dangerous[7]. Denoting the right-handed technifermion doublet by ψ_R, consider the operator

$$\frac{g^2}{M^2}\left(\bar{\Psi}_R \gamma_\mu \sigma_3 \Psi_R\right)^2 \quad , \tag{7}$$

which can be interpreted as the (mass-)mixing of the Z with an isosinglet ETC gauge-boson



(8)

and give rise to a contribution to $\Delta\rho_*$.[b]

If there are N_D doublets of the technifermions and they give rise to a contribution to M_W^2 proportional to $N_D F^2$, the contribution of the operator in eqn. (7) to the ρ parameter can be calculated to be

$$\Delta\rho_* \approx \frac{2g^2}{M^2}\frac{N_D^2 F^4}{v^2} \tag{9}$$

$$\approx 12\%\ g^2 \left(\frac{N_D F^2}{(246\ \mathrm{GeV})^2}\right)^2 \left(\frac{1\ \mathrm{TeV}}{M}\right)^2 . \tag{10}$$

Current limits[11] on the parameter T ($\Delta\rho_* = \alpha T$) imply that $\Delta\rho_* \lesssim 0.4\%$.

There are two ways in which one may try to satisfy this constraint. The equation above implies

$$\frac{M}{g} \gtrsim 5.5\ \mathrm{TeV} \left(\frac{N_D F^2}{(246\ \mathrm{GeV})^2}\right) . \tag{11}$$

If $N_D F^2 \approx (246\ \mathrm{GeV})^2$, that is if the sector giving rise to the top-quark mass is responsible for the bulk of EWSB, then the scale M must be much larger than the naive 1 TeV expectation in QCD-like technicolor. Comparing this with eqns. (4) and (6) above, we see that the enhancement of the condensate needed requires a fine-tuning of order 3% ($\approx (1/5.5)^2$) in order to produce a top-quark mass of order 175 GeV. Alternatively, we may re-write the bound as

$$F \lesssim \frac{105\ \mathrm{GeV}}{\sqrt{N_D}} \left(\frac{M/g}{1\ \mathrm{TeV}}\right)^{\frac{1}{2}} \tag{12}$$

If M/g is of order 1 TeV, it is necessary that the sector responsible for top quark mass generation *not* give rise to the bulk of EWSB. This is essentially what happens in multiscale models [12,13] and in top-color assisted technicolor [9].

2.2. Indirect Contributions

A second class of potentially dangerous contributions come from isospin violation in the technifermion mass spectrum. In a manner analogous to the contribution[14] of the $t-b$ mass splitting to $\Delta\rho$, any difference in the dynamical masses of two

[b] Contributions of this sort occur naturally in ETC-models which give rise to the top-quark mass[8].

technifermions in the *same* doublet will give rise to deviations in the ρ parameter from one. The size of this effect can be estimated à la Pagels-Stokar[15],

$$\Delta\rho_* \propto \frac{N_D\, d}{16\pi^2}\left(\frac{\Sigma_U(0)-\Sigma_D(0)}{v}\right)^2 \quad , \tag{13}$$

where N_D and d are the the number of doublets and dimension of the technicolor representation respectively. Since we require $\Delta\rho_* \lesssim 0.4\%$, the equation above implies

$$N_D\, d\left(\frac{\Delta\Sigma(0)}{m_t}\right)^2 \lesssim 1.3 \ . \tag{14}$$

From this we see that, $\Delta\Sigma(0)$ must be less than of order m_t [c].

However, if the t and b get their mass from the same technidoublet, then at the ETC-scale we expect that there is no difference between the t, b and the corresponding technifermions[3]

$$\begin{aligned} \Delta\Sigma(M_{ETC}) &\equiv \Sigma_U(M_{ETC}) - \Sigma_D(M_{ETC}) \approx \\ \Delta m(M_{ETC}) &\equiv m_t(M_{ETC}) - m_b(M_{ETC}) \ . \end{aligned} \tag{15}$$

Furthermore, if QCD is the only interaction which contributes to the scaling of the t and b masses, we expect $\Delta m(M_{ETC}) \approx m_t^{pole}$, and from scaling properties of the technifermion self-energies, we expect $\Delta\Sigma(0) \gtrsim \Delta\Sigma(M_{ETC})$.

There are two ways to avoid these constraints. One is that perhaps there are *additional* interactions which contribute to the scaling of the top- and bottom-masses below the ETC scale, and hence that $\Delta m(M_{ETC}) \ll m_t^{pole}$. This would be the case if the t and/or b get only a *portion* of their mass from the technicolor interactions, and would imply that the third generation must have (strong) interactions different from the technifermions (and possibly from the first and second generations). Another possibility is that the t and b get mass from *different* technidoublets, each of which have isospin-symmetric masses. The first alternative is the solution chosen in top-color assisted technicolor models (see below), while the latter has only recently begun to be explored[16].

3. Case Study: Top-Color Assisted Technicolor

Recently[9], Hill has introduced a model in which a top-condensate is driven by the combination of a strong, but spontaneously broken and non-confining, isospin-symmetric top-color interaction and an additional (either weak or strong) isospin-breaking $U(1)$ interaction which couple only to the third generation quarks. At low-

[c] Perhaps, given the crude approximations involved, one may be able to live with $d = 2$ in the fundamental of and $SU(2)$ technicolor group with one doublet.

energies, the top-color and hypercharge interactions of the third generation quarks may be approximated by four-fermion operators

$$\mathcal{L}_{4f} = -\frac{4\pi\kappa_{tc}}{M^2}\left[\overline{\psi}\gamma_\mu\frac{\lambda^a}{2}\psi\right]^2 - \frac{4\pi\kappa_1}{M^2}\left[\frac{1}{3}\overline{\psi_L}\gamma_\mu\psi_L + \frac{4}{3}\overline{t_R}\gamma_\mu t_R - \frac{2}{3}\overline{b_R}\gamma_\mu b_R\right]^2 \ , \tag{16}$$

where ψ represents the top-bottom doublet, κ_{tc} and κ_1 are related respectively to the top-color and $U(1)$ gauge-couplings squared, and where (for convenience) we have assumed that the top-color and $U(1)$ gauge-boson masses are comparable and of order M. In order to produce a large top quark mass without giving rise to a correspondingly large bottom quark mass, the combination of the top-color and extra hypercharge interactions are assumed to be critical in the case of the top quark but not the bottom quark:

$$\kappa^t_{eff} = \kappa_{tc} + \frac{1}{3}\kappa_1 > \kappa_c = \frac{3\pi}{8} > \kappa^b_{eff} = \kappa_{tc} - \frac{1}{6}\kappa_1 \ . \tag{17}$$

The contribution of the top-color sector to electroweak symmetry breaking can be quantified by the F-constant of this sector. In the NJL approximation[6], for M of order 1 TeV, and $m_t \approx 175$ GeV, we find

$$f_t^2 \equiv \frac{N_c}{8\pi^2}\, m_t^2 \log\left(\frac{M^2}{m_t^2}\right) \approx (64\ \text{GeV})^2 \ . \tag{18}$$

3.1. Direct Isospin Violation

As f_t is small compared to 246 GeV, technicolor is necessary to produce the bulk of EWSB and to give mass to the light fermions. However, the heavy and light fermions must mix — hence, we would naturally expect that at least some of the *technifermions* carry the extra $U(1)$ interaction. If the additional $U(1)$ interactions violate custodial symmetry,[d] the $U(1)$ coupling will have to be quite small to keep this contribution to $\Delta\rho_*$ small[11]. We will illustrate this in the one-family technicolor[10] model, assuming that techniquarks and technileptons carry $U(1)$-charges proportional to the hypercharge of the corresponding ordinary fermion.[e] We can rewrite the effective $U(1)$ interaction of the technifermions as

$$\mathcal{L}_{4T1} = -\frac{4\pi\kappa_1}{M^2}\left[\frac{1}{3}\overline{\Psi}\gamma_\mu\Psi + \overline{\Psi}_R\gamma_\mu\sigma^3\Psi_R - \overline{L}\gamma_\mu L + \overline{L}_R\gamma_\mu\sigma^3 L_R\right]^2 \ , \tag{19}$$

where Ψ and L are the techniquark and technilepton doublets respectively.

[d]It has been noted[16] that if the top- and bottom-quarks receive their masses from *different* technidoublets, it is possible to assign the extra $U(1)$ quantum numbers in a custodially invariant fashion.
[e]Note that this choice is anomaly-free.

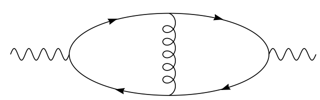
From the analysis given above (eqn. (10)), we see that the contribution to $\Delta\rho_*$ is[11]:

$$\Delta\rho_*^{\mathrm{T}} \approx 152\% \ \kappa_1 \left(\frac{1\ \mathrm{TeV}}{M}\right)^2 \ . \tag{20}$$

Therefore if M is of order 1 TeV, and the extra $U(1)$ has isospin-violating couplings to technifermions, κ_1 must be extremely small.

3.2. Indirect Isospin Violation

In principle, since the isospin-splitting of the top and bottom are driven by the combination of top-color and the extra $U(1)$, the technifermions can be degenerate. In this case, the only indirect contribution to the ρ parameter at one-loop is the usual contribution coming from loops of top- and bottom-quarks[14]. However, since there are additional interactions felt by the third-generation of quarks, there are "two-loop" contributions of the form



(21)

This contribution yields[11]

$$\Delta\rho_*^{\mathrm{tc}} \approx 0.53\% \left(\frac{\kappa_{tc}}{\kappa_c}\right) \left(\frac{1\ \mathrm{TeV}}{M}\right)^2 \left(\frac{f_t}{64\ \mathrm{GeV}}\right)^4 \ . \tag{22}$$

Combining this with eqn. (18), we find that

$$M \gtrsim 1.4\ \mathrm{TeV} \tag{23}$$

for $\kappa_{tc} \approx \kappa_c$. This immediately puts a constraint on the mass of the top-color gluon which is comparable to the direct limits currently obtained by CDF[17].

3.3. Fine-Tuning

Finally, we must require that the sum of the effects of eqns. (20) and (22) do not give rise to an experimentally disallowed contribution to the ρ parameter. Equation (20) implies that κ_1 must either be very small, or M very large. However, we must also simultaneously satisfy the constraint of eqn. (17), which implies that

$$\frac{\Delta\kappa_{tc}}{\kappa_c} = \left|\frac{\kappa_{tc} - \kappa_c}{\kappa_c}\right| \le \frac{1}{3}\frac{\kappa_1}{\kappa_c} \ , \tag{24}$$

Therefore, if M is low and κ_1 is small, the top-color coupling must be tuned close to the critical value for chiral symmetry breaking. On the other hand, if κ_1 is not small and M is relatively large the *total* coupling of the top-quark must be tuned close to the critical NJL value[6] for chiral symmetry breaking in order to keep the top-quark mass low,

$$\frac{\Delta\kappa_{eff}}{\kappa_c} = \frac{\kappa^t_{eff} - \kappa_c}{\kappa_c} = \frac{\frac{m_t^2}{M^2}\log\frac{M^2}{m_t^2}}{1 - \frac{m_t^2}{M^2}\log\frac{M^2}{m_t^2}} \ . \tag{25}$$

These two constraints are shown in Fig. 3. For $M > 1.4$ TeV, we find that either $\Delta\kappa_{tc}/\kappa_c$ or $\Delta\kappa_{eff}/\kappa_c$ must be tuned to less than 1%. This trade-off in fine tunings is displayed in figure 4. For the "best" case where both tunings are of order 1%, $M = 4.5$ TeV.

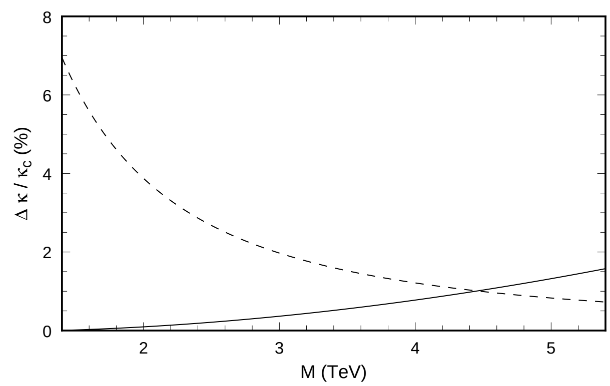


Figure 3: The amount of fine-tuning required[11] in the TC^2 model. The dashed line is the amount of fine-tuning in $\Delta\kappa_{eff}$ required to keep m_t much lighter than M, see equation (25). The solid curve shows the amount of fine-tuning (see equation (24)) in $\Delta\kappa_{tc}$ required to satisfy the bound $\Delta\rho_* < 0.4\%$. The region excluded by the experimental constraint on $\Delta\rho_*$ is above the solid curve.

4. Conclusions

We have seen that a large top quark mass has a number of important implications for dynamical electroweak symmetry breaking:

- A large top-quark mass naturally implies, in models of dynamical electroweak symmetry breaking, the possibility of a correspondingly low scale for the scale of top flavor-physics.
- The physics responsible for the large isospin breaking in the $t-b$ mass splitting can lead to potentially dangerous "direct" and "indirect" effects in the W and Z masses.

- The direct and indirect effects can be mitigated if the sector which is responsible for the top- and bottom-masses does *not* provide the bulk of electroweak symmetry breaking and, conversely, if the sector responsible for the W and Z masses gives rise to only a *small portion* of the top- and bottom-masses. This can happen only if the top and bottom feel *strong* interactions which are not shared by the technifermions and, possibly, the first two generations.

- In top-color assisted technicolor, the extra top-color interactions give rise to additional indirect contributions to $\Delta\rho$, and we must require that $M_g \gtrsim 1.4$ TeV. Furthermore, If the extra $U(1)$ has isospin-violating couplings to technifermions, we require fine-tuning of order 1%.

Acknowledgments

We thank Tom Appelquist, Nick Evans, and Ken Lane for helpful conversations, and Mike Dugan for help in preparing the manuscript. R.S.C. acknowledges the support of an NSF Presidential Young Investigator Award, and a DOE Outstanding Junior Investigator Award. *This work was supported in part by the National Science Foundation under grant PHY-9057173, and by the Department of Energy under grant DE-FG02-91ER40676.*

1. S. Dimopoulos and L. Susskind, *Nucl. Phys.* **B155** (1979) 237; E. Eichten and K. Lane, *Phys. Lett.* **B90** (1980) 125.
2. A. Manohar and H. Georgi, *Nucl. Phys.* **B234** (1984) 189.
3. T. Appelquist, M. Einhorn, T. Takeuchi, and L.C.R. Wijewardhana, *Phys. Lett.* **220B**, 223 (1989); V.A. Miransky and K. Yamawaki, *Mod. Phys. Lett.* **A4**, 129 (1989); K. Matumoto *Prog. Theor. Phys. Lett.* **81**, 277 (1989) .
4. N. Evans, *Phys. Lett.* **B331** (1994) 378.
5. R.S. Chivukula, K. Lane, and A.G. Cohen, *Nucl. Phys.* **B 343**, 554 (1990); T. Appelquist, J. Terning, and L. Wijewardhana, *Phys. Rev.* **44**, 871 (1991).
6. Y. Nambu and G. Jona-Lasinio, *Phys. Rev.* **122** (1961) 345.
7. T. Appelquist et al., *Phys. Rev.* **D31** (1985) 1676.
8. See, for example, G.-H. Wu, these proceedings and hep-ph/9412206.
9. C.T. Hill, these proceedings and *Phys. Lett.* **B345** (1995) 483.
10. E. Farhi and L. Susskind, *Phys. Rev.* **D20** (1979) 3404.
11. R.S. Chivukula, B.A. Dobrescu, and J. Terning, hep-ph/9503203.
12. K. Lane and E. Eichten, *Phys. Lett.* **B222** (1989) 274.
13. J. Terning, hep-ph/9410233, *Phys. Lett.* **B344** (1995) 279.
14. M. Einhorn, D. Jones, and M. Veltman, *Nucl. Phys.* **B191** (1981) 146.
15. H. Pagels and S. Stokar, *Phys. Rev.* **D20** (1979) 2947; B. Holdom, *Phys. Lett.* **B226** (1989) 137.
16. K. Lane and E. Eichten, hep-ph/9503433.
17. A. Beretvas, these proceedings, and F. Abe *et. al.*, CDF collaboration, FERMILAB-PUB-94/405-E.