Final Exam Due Thu. June 9, at 5 pm. Return to Rachel Houtz.

Part I: Consider the SUSY breaking model

	$Sp(8)$	$SU(5)_L$	$SU(5)_R$	$SU(2)$
Q	$\square$	$\square$	$\mathbf{1}$	$\mathbf{1}$
$\overline{Q}$	$\square$	$\mathbf{1}$	$\overline{\square}$	$\mathbf{1}$
Σ	$\mathbf{1}$	$\overline{\square}$	$\square$	$\mathbf{1}$
S	$\mathbf{1}$	$\overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}}$	$\mathbf{1}$	$\mathbf{1}$
$\overline{S}$	$\mathbf{1}$	$\mathbf{1}$	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$\mathbf{1}$
ϕ^a	$\mathbf{1}$	$\overline{\square}$	$\mathbf{1}$	$\square$
$\overline{\phi}^a$	$\mathbf{1}$	$\mathbf{1}$	$\square$	$\square$

(where $SU(2)$ is a global symmetry, $Sp(8)$ is more strongly coupled than the $SU(5)$'s, and the standard model gauge groups will be embedded in the diagonal subgroup of $SU(5)_L \times SU(5)_R$) with a superpotential

$$W = \frac{1}{2} h_L S Q Q + \frac{1}{2} h_R \overline{S Q Q} + \lambda \Sigma \overline{Q} Q \qquad (1)$$

1) Explain how the $Sp(8)$ dynamics coupled with the superpotential breaks supersymmetry.

Assuming that λ and the $SU(5)$ gauge couplings have values such that the minimum occurs at $\Sigma = \text{diag}(v, v, v, v, v)$ with $v \gg \Lambda$ (where Λ is the intrinsic scale of the $Sp(8)$ interactions.)

2) Using a one-loop analysis, determine (parametrically) the vacuum energy density (and hence the F component of Σ) in terms of λ and Λ.

3) Assuming that S and ϕ have non-renormalizable couplings to Σ suppressed by M_{Pl}, use gauge invariance and dimensional analysis to show that

$$m_S \sim v \left(\frac{v}{M_{\text{Pl}}} \right)^{n_S}$$
$$m_\phi \sim v \left(\frac{v}{M_{\text{Pl}}} \right)^{n_\phi}$$

and determine the integers n_S and n_ϕ.

Part II: Consider the SUSY model:

	$SU(6)$	$SU(F)$	$SU(F)$	$U(1)_A$	$U(1)_B$	$U(1)_R$
Q	$\square$	$\square$	$\mathbf{1}$	-3	1	$\frac{F-6}{F}$
$\overline{Q}$	$\overline{\square}$	$\mathbf{1}$	$\square$	-3	-1	$\frac{F-6}{F}$
A	$\begin{smallmatrix}\square\\\square\\\square\end{smallmatrix}$	$\mathbf{1}$	$\mathbf{1}$	F	0	1

1) Check that the mixed $U(1)$-gauge anomalies vanish.
2) At what value of F is asymptotic freedom lost?
3) For $F = 4$ show that this theory satisfies the index constraint for S-confinment.
4) Consider the the general R-charge

$$R_g = R + b\, q_A + c\, q_B \ , \tag{2}$$

where b and c are, at the moment, arbitrary coefficients. Since the theory is vector-like we know that to find the superconformal R-charge we can take $c = 0$ since we need $R(Q) = R(\overline{Q})$. Perform a-maximation for $F = 8$ and $F = 9$ to determine the dimension of the composites $Q\overline{Q}$, AA, and $AQQQ$. Are any of these fields free for $F = 8$?