Exercise 3

1. Check that the SUSY transformation of the gauge field A_μ^a in

$$\mathcal{L}_{\psi\ gauge\ int.} = -\psi^\dagger \overline{\sigma}^\mu g A_\mu^a T^a \psi \ , \tag{1}$$

cancels against the SUSY transformation of ϕ and ϕ^* in

$$\mathcal{L}_{Yukawa} = -\sqrt{2} g \left[(\phi^* T^a \psi)\lambda^a + \lambda^{\dagger a}(\psi^\dagger T^a \phi) \right] . \tag{2}$$

You will need to use the generalized Pauli identity (A.27). Note that the cancellation relates two terms where two fields are replaced by superpartners.

2. For the superpotential

$$W = m\, \phi_2 \phi_3 + \frac{y}{2} \phi_1 \phi_3^2 \ ; \tag{3}$$

a) calculate the scalar potential;
b) show that the SUSY vacua are given by $\langle\phi_1\rangle = v$; $\langle\phi_2\rangle = 0$, $\langle\phi_3\rangle = 0$, for arbitray v;
c) expanding around the vacua $\phi_1 = v + \tilde{\phi}_1$, and writing the scalar potential out to quadratic order, find the mass squared matrix, M_ϕ^2, for the scalars;
d) find the fermion mass matrix M_ψ and verify that $M_\psi M_\psi^\dagger = M_\phi^2$.

3. Using

$$V^a = \theta\sigma^\mu\bar{\theta} A_\mu^a + \theta^2 \bar{\theta}\lambda^{\dagger a} + \bar{\theta}^2 \theta \lambda^a + \frac{1}{2}\theta^2\bar{\theta}^2 D^a \ , \tag{4}$$

perform the superspace integration for the Lagrangian

$$\mathcal{L} = \int d^4\theta\ \Phi^\dagger e^{2gT^aV^a} \Phi \tag{5}$$

keeping only terms of order g or higher. (We did the g independent terms in class.)