Exercise 2

1. Consider a Wess-Zumino-like model with the superpotential

$$W = \frac{y}{3}\phi^3 + \frac{\lambda}{4M}\phi^4 \ . \tag{1}$$

   What are the off-shell SUSY transformations of the the scalar ϕ and its superpartner fermion ψ expressed only in terms of ϕ and ψ?

2. For the superpotential given in (1) what is the corresponding Lagrangian in terms of ϕ and ψ?

3. Schematically (include the parametric dependence on the couplings) what are the Feynman rules for the cubic and quartic interactions?

4. Check that

$$S = \int d^4x \, (-\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} + i\lambda^{\dagger a}\overline{\sigma}^\mu D_\mu \lambda^a + \frac{1}{2}D^a D^a) \tag{2}$$

   is a SUSY invariant using eqns (2.91)-(2.94). After doing the SUSY transformations you can go to a gauge where at the point of interest, x_0^μ, the gauge field vanishes ($A^a_\nu(x_0) = 0$). You will need to use the Bianchi identity $\epsilon^{\mu\nu\alpha\beta}(D_\nu F_{\alpha\beta})^a = 0$.