115B Midterm 1 Formulae

$E = h\,f = \hbar\omega$, $p = \frac{h}{\lambda} = \hbar k$

$-\frac{\hbar^2}{2m}\nabla^2\Psi(\vec{r},t) + V(\vec{r})\Psi(\vec{r},t) = i\hbar\frac{\partial\Psi(\vec{r},t)}{\partial t}$

$-\frac{\hbar^2}{2m}\nabla^2\psi(\vec{r}) + V(\vec{r})\psi(\vec{r}) = E\psi(\vec{r})$ if $\Psi(\vec{r},t) = e^{-iEt/\hbar}\,\psi(\vec{r})$

$[x, p_x] = i\hbar$, $[L_x, L_y] = i\hbar L_z$, $[L_y, L_z] = i\hbar L_x$, $[L_z, L_x] = i\hbar L_y$,

$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$V(x) = \begin{cases} 0 & 0 < x < L \\ \infty & x < 0, x > L \end{cases}$, $\quad \psi_n(x) = \begin{cases} \sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L} & 0 < x < L \\ 0 & x < 0, x > L \end{cases}$, $\quad E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}$

$V(x) = \frac{1}{2}m\omega^2x^2$, $E = (n+\frac{1}{2})\hbar\omega$, $\quad \psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4}\frac{1}{\sqrt{2^n n!}}\,H_n(\xi)\,e^{-\xi^2/2}$, $\xi = x\sqrt{\frac{m\omega}{\hbar}}$

$H_0 = 1,\ H_1 = 2\xi,\ H_2 = 4\xi^2 - 2,\ H_3 = 8\xi^3 - 12\xi,\ H_4 = 16\xi^4 - 48\xi^2 + 12$

$V(r) = -\frac{\alpha\hbar c}{r}$, $\quad -\frac{\hbar^2}{2m}\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d}{dr}\right)R_{n\ell}(r) + \frac{\hbar^2\ell(\ell+1)}{2mr^2}R_{n\ell}(r) + V(r)R(r) = E\,R_{n\ell}(r)$

$\int_0^\infty r^2 R_{n\ell}(r)^2 dr = 1$, $\quad \int_0^{2\pi}\int_0^{\pi}|Y_\ell^m(\theta,\phi)|^2\sin\theta\,d\theta d\phi = 1$, $\quad R_{n,n-1}(r) = A\,r^{n-1}e^{-\frac{r}{na}}$

$a = \frac{\hbar}{\alpha mc} = 0.0529$ nm, $\quad \alpha = 1/137.04$, $E_n = -\frac{mc^2\alpha^2}{2n^2} = -\frac{13.6\text{eV}}{n^2}$,

$\psi_{n\ell m}(\vec{r}) = R_{n\ell}(r)Y_\ell^m(\theta,\phi)$

$R_{10} = \frac{2}{a^{3/2}}\,e^{-r/a}$, $R_{20} = \frac{1}{\sqrt{2a^3}}\left(1 - \frac{1}{2}\frac{r}{a}\right)\,e^{-r/2a}$, $R_{21} = \frac{1}{\sqrt{24a^3}}\,\frac{r}{a}\,e^{-r/2a}$

$R_{30} = \frac{2}{\sqrt{27a^3}}\left(1 - \frac{2}{3}\frac{r}{a} + \frac{2}{27}\frac{r^2}{a^2}\right)\,e^{-r/3a}$, $R_{31} = \frac{8}{27\sqrt{6a^3}}\left(1 - \frac{1}{6}\frac{r}{a}\right)\frac{r}{a}\,e^{-r/3a}$, $R_{32} = \frac{4}{81\sqrt{30a^3}}\,\frac{r^2}{a^2}\,e^{-r/3a}$

$Y_0^0 = \frac{1}{\sqrt{4\pi}}$, $Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$, $Y_1^{\pm1} = \mp\sqrt{\frac{3}{8\pi}}\sin\theta\,e^{\pm i\phi}$, $\quad Y_2^0 = \sqrt{\frac{5}{16\pi}}\,(3\cos^2\theta - 1)$

$Y_2^{\pm1} = \mp\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\,e^{\pm i\phi}$, $\quad Y_2^{\pm2} = \mp\sqrt{\frac{15}{32\pi}}\sin^2\theta\,e^{\pm 2i\phi}$

$L_\pm|\ell\ m\rangle = \hbar\sqrt{\ell(\ell+1) - m(m\pm1)}|\ell\ (m\pm1)\rangle$

$S_\pm|s\ m\rangle = \hbar\sqrt{s(s+1) - m(m\pm1)}|s\ (m\pm1)\rangle$

$\int_0^\infty x^m\,e^{-bx}\,dx = \frac{m!}{b^{m+1}}$, $\quad \sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$

$\int\cos(a\theta)\,d\theta = \frac{1}{a}\sin(a\theta)$, $\quad \int\sin(a\theta)\cos(a\theta)\,d\theta = -\frac{\cos^2(a\theta)}{2a}$, $\ e^{i\theta} = \cos\theta + i\sin\theta$

$\hbar = 1.05\times10^{-34}\text{J}\cdot\text{s} = 6.58\times10^{-16}\text{eV}\cdot\text{s}$, $\quad c = 3\times10^8\,\text{m/s}$

$m_e = 9.11\times10^{-31}\,\text{kg} = 511\,\text{keV/c}^2$, $\quad 1\,\text{eV} = 1.602\times10^{-19}$ J

Name: ________________________________ ID# ________________________________

1. Fill in the blank.

The Bohr radius gives the typical ________________________________ of the electron

in the hydrogen ________________________________ state.

A non-zero eigenvalue of L^2 is never equal to the square of the eigenvalue of L_z because

the components of $\vec{L}$ do not ________________________________.

The eigenstates of orbital angular are ________________________________ of position

while the eigenstates of spin 1/2 are ________________________________.

A beam of spin s particles is split into ________________________________ beams by a Stern-Gerlach experiment.

For bosons the overall wavefunction of two particles is ________________________________

under interchange.

A particle of spin s_1 and a particle of spin s_2 can combine to form states with maximum

total spin ________________________________ and minimum total spin

________________________________.

The radial (Schrödinger) equation is independent of the two ________________________________ variables.

2. A particle of spin 1/2 is in a magnetic field pointing in the z direction:

$$\vec{B} = B_0 \hat{z} \ .$$

The Hamiltonian is $H = -\gamma \, \vec{B} \cdot \vec{S}$. At $t = 0$ the spinor wavefunction is given by

$$\chi = \begin{pmatrix} a \\ b \end{pmatrix} \ .$$

(a) What is the spinor wavefunction at an arbitary time t?
(b) What are the expectation values of S_x, S_y, and S_z at time t? (Express the time dependence in terms of cos and sin functions.)

3. A an electron is in a stationary state of a hydrogen atom with no orbital angular momentum ($\ell = 0$) and with 2 nodes in its radial wavefunction. One of these nodes is at $r = \infty$.

(a) What is the energy of this state?

(b) Is the wave function an eigenstate of momentum squared (p^2)? Why?

(c) What is the expectation value of the kinetic energy operator?

(d) Express the previous answer in eV.

4. Given an electron in a hydrogen atom with $n = 3$ and $\ell = 2$,
(a) What is the most probable value for distance (in units of the Bohr radius) between the proton and the electron?
(b) What is the expectation value for the distance (in units of the Bohr radius) between the proton and the electron?

5. Consider two particles, one with spin s, and one with spin $\frac{1}{2}$, with total angular momentum $\vec{J} = \vec{S}^{(1)} + \vec{S}^{(2)}$. Use the total angular momentum lowering operator to find the Clebsch-Gordan coefficents (up to an overall sign) for the state with total $j = s - \frac{1}{2}$ and $m = s - \frac{3}{2}$. (You can assume that $s \geq 1$).

extra space