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QM 115B   Lec 7

Spin $\vec{S}$, intrinsic angular momentum

$[S_x, S_y] = i\hbar S_z \qquad [S_y, S_z] = i\hbar S_x \qquad [S_z, S_x] = i\hbar S_y$

$$S^2 |sm\rangle = \hbar^2 s(s+1) |sm\rangle$$
$$S_z |sm\rangle = \hbar m |sm\rangle$$

Prob 4.18 $\quad S_\pm |sm\rangle = \hbar \sqrt{s(s+1) - m(m\pm 1)}\, |s, m\pm 1\rangle$

$$S_\pm = S_x \pm i S_y$$

$s = 0, \frac{1}{2}, 1, 3/2, 2, \ldots \qquad m = -s, s+1, \ldots, s$

Higgs (0); electrons, quarks, neutrinos ($\frac{1}{2}$); photon (1); graviton (2)

integer bosons $\qquad \frac{1}{2}$integer fermions

spin 1/2
$$|\tfrac{1}{2}\,\tfrac{1}{2}\rangle = |\uparrow\rangle = |+\rangle$$
$$|\tfrac{1}{2}\,-\tfrac{1}{2}\rangle = |\downarrow\rangle = |-\rangle$$

spinor $\quad \chi = \begin{pmatrix} a \\ b \end{pmatrix} = a\chi_\uparrow + b\chi_\downarrow = a|+\rangle + b|-\rangle$

$$\chi_\uparrow = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \chi_\downarrow = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$S^2 \chi_\uparrow = \hbar^2 (\tfrac{1}{2})(\tfrac{1}{2}+1)\chi_\uparrow = \tfrac{3}{4}\hbar^2 \chi_\uparrow \qquad S^2 \chi_\downarrow = \tfrac{3}{4}\hbar^2 \chi_\downarrow$

$$S^2 = \frac{3}{4}\hbar^2 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$S_z \chi_\uparrow = \frac{\hbar}{2}\chi_\uparrow \qquad S_z \chi_\downarrow = -\frac{\hbar}{2}\chi_\downarrow$$

$$S_z = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$S_+ \chi_\downarrow = \hbar \chi_\uparrow \qquad S_- \chi_\uparrow = \hbar \chi_\downarrow$$

$$S_+ \chi_\uparrow = 0 \qquad S_- \chi_\downarrow = 0$$

$$S_+ = \hbar\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \qquad S_- = \hbar\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$S_x = \frac{1}{2}(S_+ + S_-) \qquad S_y = \frac{1}{2i}(S_+ - S_-)$$

$$S_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad S_y = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\vec{S} = \frac{\hbar}{2}\vec{\sigma}$$

Pauli:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\chi = a\chi_\uparrow + b\chi_\downarrow$$

$$S_z \chi \rightarrow \begin{cases} \text{Prob } |a|^2 & +\hbar/2 \\ \text{Prob } |b|^2 & -\hbar/2 \end{cases}$$

$$|a|^2 + |b|^2 = 1$$

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measure S_x

$S_x \chi = \lambda \chi \rightarrow (S_x - \lambda)\chi = 0 \quad \det(S_x - \lambda) = 0$

$$\begin{vmatrix} -\lambda & \hbar/2 \\ \hbar/2 & -\lambda \end{vmatrix} = 0 \qquad \lambda^2 - \frac{\hbar^2}{4} = 0$$

$\lambda = \pm \hbar/2$

$$\frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \pm \frac{\hbar}{2}\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

$$\begin{pmatrix} \beta \\ \alpha \end{pmatrix} = \pm \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

$\beta = \pm \alpha$

$$|+\rangle_x = \chi_\uparrow^{(x)} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \qquad \chi_\downarrow^{(x)} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\chi = \begin{pmatrix} a \\ b \end{pmatrix} = \left(\frac{a+b}{\sqrt{2}}\right)\chi_\uparrow^{(x)} + \left(\frac{a-b}{\sqrt{2}}\right)\chi_\downarrow^{(x)}$$

S_x Prob $\frac{1}{2}|a+b|^2 \rightarrow +\hbar/2$

Prob $\frac{1}{2}|a-b|^2 \rightarrow -\hbar/2$

$$1 \stackrel{?}{=} \frac{1}{2}\left(Re(a+b)^2 + Im(a+b)^2 + Re(a-b)^2 + Im(a-b)\right.$$

$b = c e^{i\theta} a$

$a+b = a(1+ce^{i\theta}) \qquad a-b = a(1-ce^{i\theta})$

$|a+b|^2 = |a|^2(1+ce^{-i\theta} + ce^{i\theta} + c^2)$

$|a-b|^2 = |a|^2(1-ce^{-i\theta} - ce^{i\theta} + c^2)$

$|a+b|^2 + |a-b|^2 = 2|a|^2(1+c^2) = 2(|a|^2 + |b|^2) = 2$