

QM115B Lec 27

$$\nabla^2 \phi(r) = - \frac{\rho(r)}{\epsilon}$$

$$\rho(r) = 4\pi e \delta(r) - 4\pi e (n(r) - n_0)$$

are equilibrium value

guess $\phi(r) = - \frac{e}{\epsilon r} g(r)$

$$\nabla^2 \phi(r) = - \frac{e}{\epsilon} \left(4\pi \delta(r) g(r) + \frac{4\pi}{r} \frac{\partial^2 g}{\partial r^2} \right)$$

$$- \frac{1}{r} \frac{\partial^2 g}{\partial r^2} = (n(r) - n_0)$$

$$n(r) = n_0 e^{+e\phi/k_B T}$$

$$= n_0 \left(1 + \frac{e\phi}{k_B T} + \dots \right)$$

$$- \frac{1}{r} \frac{\partial^2 g}{\partial r^2} \approx n_0 \frac{e}{k_B T} \left(- \frac{e}{\epsilon r} g(r) \right)$$

$$g(r) = e^{-q_D r}$$

Debye screening length $\frac{1}{q_D}$

$$q_D^2 = \frac{n_0 e^2}{\epsilon k_B T}$$

(123)

take the approx that electrons are in a fermi gas

$$E_F = \frac{\hbar^2 k_F^2}{2m^*} \approx \frac{3}{2} k_B T$$

$$n_0 = \frac{k_F^3}{3\pi^2}$$

$$q_D^2 = \frac{n_0 e^2}{\epsilon k_B T} = \frac{k_F^3 e^2}{3\pi^2 \epsilon \left(\frac{2}{3} \frac{\hbar^2 k_F^2}{2m^*} \right)}$$
$$= \frac{k_F m^* e^2}{\epsilon \pi^2 \hbar^2} = q_{TF}^2$$

$$f(\theta) = \frac{2m\alpha c}{\epsilon_r \hbar (q_{TF}^2 + K^2)}$$

$$K = 2k \sin(\theta/2)$$

CF Coulomb $f(\theta) = \frac{2m\alpha c}{\hbar K^2}$

$$\epsilon_{eff} = \epsilon_r \epsilon_0 \left(1 + \frac{q_{TF}^2}{K^2} \right) \quad \text{dielectric function}$$

(24)

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2 = \frac{4m^2 \alpha^2 c^2}{\epsilon_r^2 k^2 (q_{TF}^2 + k^2)^2}$$

$$\sigma = \int \frac{d\sigma}{d\Omega} d\Omega = \frac{16\pi m^2 \alpha^2 c^2}{k^2 \epsilon_r^2 q_{TF}^2 (q_{TF}^2 + 4k^2)}$$

$$= \frac{4\pi^2}{k_F a^* (q_{TF}^2 + 4k^2)}$$

$$n = \frac{10^{18}}{\text{cm}^3}$$

$$k = k_F = \frac{1}{3 \text{ nm}}$$

$$q_{TF} = \frac{1}{5 \text{ nm}}$$

$$\sigma = 24 \text{ nm}^2$$

$$\begin{aligned} L &= \frac{1}{n \cdot \sigma} = \frac{1}{\frac{10^{18}}{\text{cm}^3} \left(\frac{100 \text{ cm}}{1 \text{ m}} \frac{1 \text{ m}}{10^9 \text{ nm}} \right)^3 24 \text{ nm}^2} \\ &= 42 \text{ nm} \end{aligned}$$

Hyperfine triplet \rightarrow singlet lifetime

$$\Delta E = \frac{4g_p \hbar^4}{3m_p m_e^2 \alpha^4 c^2} \quad \omega_0 = \frac{\Delta E}{\hbar}$$

EM energy density $u = \frac{\epsilon_0 E^2}{2} + \frac{B^2}{2\mu_0} \quad \epsilon_0 \mu_0 = \frac{1}{c^2}$

electric dipole $H' = \vec{p} \cdot \vec{E}$ magnetic $H' = -\frac{\gamma \hbar}{2} \vec{\sigma} \cdot \vec{B}$

$$A_{\text{dipole}} = \frac{\omega_0^3 |\langle \vec{p} \rangle|^2}{3\pi \epsilon_0 \hbar c^3} \quad A_{\text{magnetic}} = \frac{\omega_0^3 \mu_0 |\langle \uparrow | -\frac{\gamma \hbar}{2} \vec{\sigma} | \downarrow \rangle|^2}{3\pi \hbar c^3}$$

$$\langle \uparrow | \vec{\sigma} | \downarrow \rangle = (10) \begin{pmatrix} \hat{z} & \hat{x} - i\hat{y} \\ \hat{x} + i\hat{y} & -\hat{z} \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \hat{x} - i\hat{y}$$

$$|\langle \uparrow | -\frac{\gamma \hbar}{2} \vec{\sigma} | \downarrow \rangle|^2 = \frac{\gamma^2 \hbar^2}{4} (1^2 + |i|^2) = \frac{\gamma^2 \hbar^2}{2} = \frac{e^2 \hbar^2}{2m_e^2} = \frac{4\pi \epsilon_0 \hbar^3 c \alpha}{2m_e^2}$$

$$A_{\text{magnetic}} = \frac{\omega_0^3 4\pi \epsilon_0 \hbar^3 c \alpha \mu_0}{2m_e^2 3\pi \hbar c^3} = \left(\frac{\Delta E}{\hbar}\right)^3 \frac{4\hbar^2 \alpha}{6m_e^2 c^2} \frac{1}{c^2}$$

$$= (5.9 \times 10^{-6} \text{ eV})^3 \frac{2}{3} \frac{1}{137} \frac{1}{(511 \times 10^3 \text{ eV})^2} \frac{1}{\hbar}$$

$$= \frac{3.8 \times 10^{-30} \text{ eV}}{6.6 \times 10^{-16} \text{ eV}\cdot\text{s}} = 5.8 \times 10^{-15} \text{ 1/s}$$

$$\tau = \frac{1}{A} = 1.7 \times 10^4 \text{ s} \left(\frac{1 \text{ yr}}{\pi \times 10^7 \text{ s}} \right) = 5.5 \times 10^6 \text{ yr}$$