QM 115B Lec. 23

$H = -\gamma \vec{B}\cdot\vec{S}$ spin 1/2

$\vec{B} = B_{rf}\cos(\omega t)\hat{x} - B_{rf}\sin(\omega t)\hat{y} + B_0\hat{z}$

$$\chi(t) = \begin{pmatrix} a(t) \\ b(t) \end{pmatrix}$$

$$a(t) = e^{\frac{i\omega t}{2}}\left(a_0\cos\left(\frac{\omega' t}{2}\right) + \frac{i}{\omega'}\left(a_0(\omega_0-\omega) + b_0\Omega\right)\sin\left(\frac{\omega' t}{2}\right)\right)$$

$$b(t) = e^{-\frac{i\omega t}{2}}\left(b_0\cos\left(\frac{\omega' t}{2}\right) + \frac{i}{\omega'}\left(-b_0(\omega_0-\omega) + a_0\Omega\right)\sin\left(\frac{\omega' t}{2}\right)\right)$$

$\omega_0 = \gamma B_0 \qquad \Omega = \gamma B_{rf}$

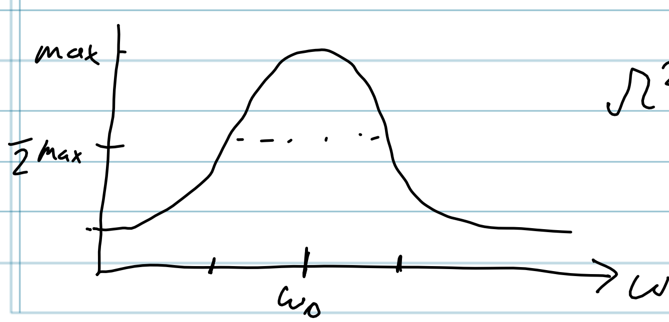
$\omega' = \sqrt{(\omega-\omega_0)^2 + \Omega^2}$

start with particle spin up at $t=0$ $\quad a_0 = 1$, $b_0 = 0$

$$P_{\uparrow\to\downarrow}(t) = |b(t)|^2 = \left|\frac{i}{\omega'}\Omega\sin\left(\frac{\omega' t}{2}\right)e^{-\frac{i\omega t}{2}}\right|^2$$

$$= \frac{\Omega^2}{(\omega-\omega_0)^2+\Omega^2}\sin^2\left(\frac{\omega' t}{2}\right)$$

resonance curve $\quad P_{\uparrow\to\downarrow}(\omega) = \dfrac{\Omega^2}{(\omega-\omega_0)^2+\Omega^2}$



$\Omega^2 = \frac{1}{2}\left((\omega-\omega_0)^2 + \Omega^2\right)$

$\frac{1}{2}\Omega^2 = \frac{1}{2}(\omega-\omega_0)^2$

$\omega = \omega_0 \pm \Omega$

$\Delta\omega = 2\Omega$

(105)

for a proton $\gamma = \frac{g_p e}{2m_p}$ $g_p = 5.59$

$B_0 = 1T$ $B_{rf} = 10^{-6}\,T$

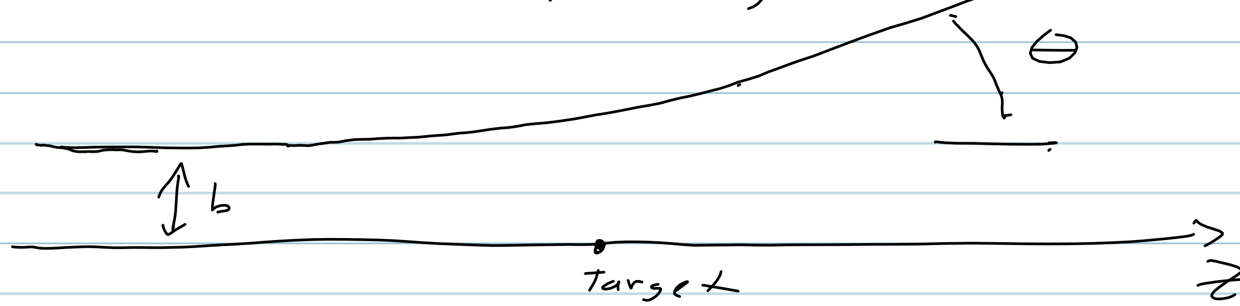
$$\nu_{res} = \frac{\omega_0}{2\pi} = \frac{g_p e B_0}{4\pi m_p} = 42\,MHz$$

$$\Delta\nu = \frac{\Delta\omega}{2\pi} = \frac{\Omega}{\pi} = \frac{\gamma B_{rf}}{\pi} = \frac{\gamma B_0}{\pi}\frac{B_{rf}}{B_0}$$

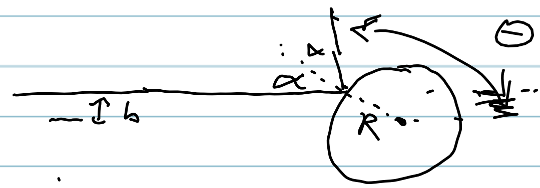
$$= 2\nu_{res}\frac{B_{rf}}{B_0} = \nu_{res}\,2\times10^{-6}$$

$$= 85\,Hz$$

Classical Scattering



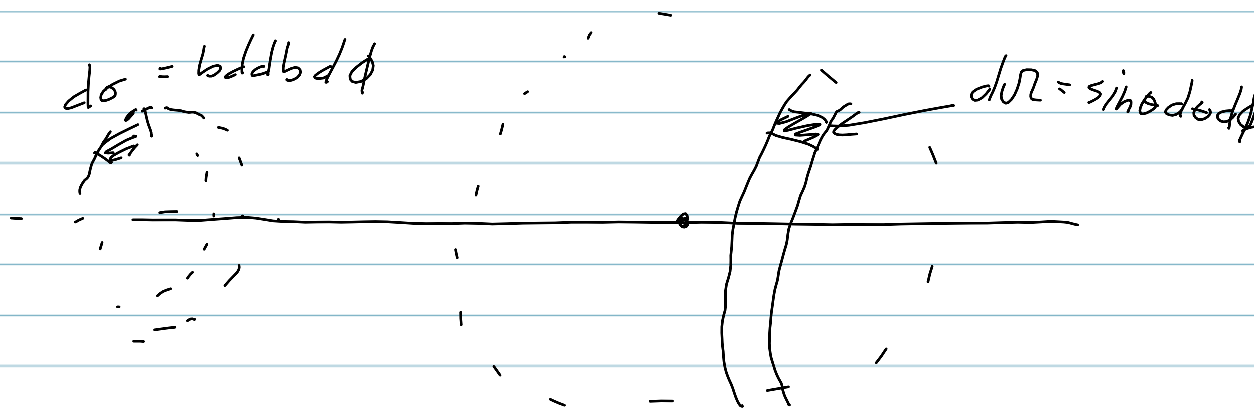
e.g. hard sphere



$\theta = \pi - 2\alpha$

$$b = R \sin\alpha$$
$$= R \sin\left(\frac{\pi - \theta}{2}\right) = R\cos\left(\frac{\theta}{2}\right)$$

$$\theta = \begin{cases} 2\cos^{-1}(b/R) & b \le R \\ 0 & b > R \end{cases}$$



differential-scattering cross-section

$$d\sigma = D(\theta)\,d\Omega$$

$$D(\theta) = \frac{d\sigma}{d\Omega} = \frac{b\,db\,d\phi}{\sin\theta\,d\theta\,d\phi} = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right|$$

(107)

hard sphere $\quad b = R \cos\frac{\theta}{2}$

$$\frac{db}{d\theta} = -\frac{1}{2} R \sin\theta/2$$

$$D(\theta) = \frac{R\cos\theta/2}{\sin\theta}\left(\frac{1}{2}R\sin\frac{\theta}{2}\right) = \frac{R^2}{4}$$

total cross-section

$$\sigma \equiv \int D(\theta)\, d\Omega = \frac{R^2}{4}\int d\Omega = \frac{R^2 4\pi}{4}$$

$$= \pi R^2$$

cross-sectional area of a sphere

beam of particles with uniform "luminosity"

$\mathcal{L} \equiv$ # of incident particles per unit area per unit time

number enter $d\sigma$ scatter to $d\Omega$

$$dN = \mathcal{L}\, d\sigma = \mathcal{L}\, D(\theta)\, d\Omega$$

$$D(\theta) = \frac{1}{\mathcal{L}}\frac{dN}{d\Omega}$$

count number per unit time
divide by $d\Omega$
divide by Luminosity

nuclear physics $1 \text{barn} = 10^{-28} m^2$

LHC one year of running $10 \frac{1}{fb}$

$$\mathcal{L} = 10 \frac{1}{fb \text{ year}}$$

to get 1000 events

$$\frac{d\sigma}{d\Omega} = \frac{1}{\mathcal{L}} \cdot \frac{1000}{d\Omega}$$

$$\sigma = \frac{1000 \, 4\pi}{\mathcal{L}} = \frac{1000 \cdot 10}{10} fb$$

$$= 1000 \, fb = 10^{-12} \, b$$

$$= 10^{-40} m^2$$

direct detection of dark matter

$$\sigma \sim 10^{-46} m^2 - 10^{-50} m^2$$