QM 115B Lec 21

Light $\lambda \sim 5000\,\text{Å}$ atoms $1\,\text{Å}$

$\lambda \gg$ atomic size

$\vec{k}\cdot\vec{x} \ll 1 \quad \cos(\omega t - \vec{k}\cdot\vec{x}) \approx \cos(\omega t)$

$$H' = -qE_0 z \cos(\omega t)$$

$$H'_{ba} = -q \underbrace{\langle \psi_b | z | \psi_a \rangle}_{p} E_0 \cos(\omega t)$$

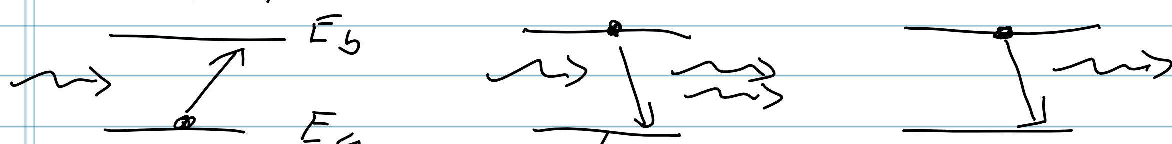
Parity: ψ even or odd $\rightarrow z|\psi|^2$ odd

$$\int z |\psi|^2 d^3r = 0$$

diagonal $H'_{aa} = 0$

$$V_{ba} = -pE_0$$

Absorption, Emission



absorption stim. emission spont. emission

$$P_{a\to b}(t) = \left(\frac{|p|E_0}{\hbar}\right)^2 \frac{\sin^2[(\omega_0-\omega)t/2]}{(\omega_0-\omega)^2}$$

$P_{b\to a}(t) = P_{a\to b}(t)$ Einstein

population inversion $\Rightarrow$ amplification

Laser

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non-monochromatic, unpolarized, incoherent radition eg, thermal radiation

energy density in EMwave

$$u = \frac{\epsilon_0 E_0^2}{2} \qquad p = q \langle \psi_b | z | \psi_a \rangle$$

monochromatic: $$P_{b\to a}(t) = \frac{2u}{\epsilon_0 \hbar} |p|^2 \frac{\sin^2[(\omega_0-\omega)t/2]}{(\omega_0-\omega)^2}$$

$$u \Rightarrow \rho(\omega)\, d\omega$$

$$P_{b\to a}(t) = \frac{2}{\epsilon_0 \hbar^2} |p|^2 \int_0^\infty \underset{\text{broad}}{\rho(\omega)} \underset{\text{peaked}}{\frac{\sin^2[(\omega_0-\omega)t/2]}{(\omega_0-\omega)^2}}\, d\omega$$

$$= \frac{2|p|^2}{\epsilon_0 \hbar^2} \rho(\omega_0) \int_0^\infty \frac{\sin^2[(\omega_0-\omega)t/2]}{(\omega_0-\omega)^2}\, d\omega$$

$$x = (\omega_0-\omega)t/2 \qquad dx = -d\omega\, t/2$$

$$\int_{\omega_0 t/2}^{-\infty} \approx \int_{\infty}^{-\infty}$$

$$\left(\frac{2}{t}\right) \frac{1}{\left(\frac{2}{t}\right)^2} \int_{-\infty}^{\infty} \frac{\sin^2 x}{x^2}\, dx = \frac{t}{2}\pi$$

$$P_{b\to a}(t) \approx \frac{\pi |p|^2}{\epsilon_0 \hbar} \rho(\omega_0)\, t$$

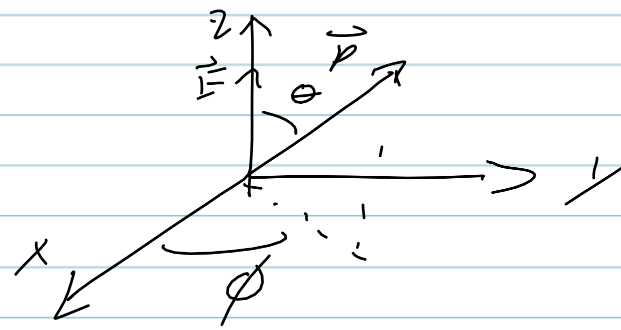
$$R_{b\to a} = \frac{d}{dt} P_{b\to a} = \frac{\pi}{\epsilon_0 \hbar} |p|^2 \rho(\omega_0)$$

unpolarized radiation from all directions

$$|p|^2 \to |\vec{p}\cdot\hat{n}|^2 \quad \text{average over } \hat{n}$$

$$\vec{p} = q \langle \psi_b | \vec{r} | \psi_a \rangle$$

propagation is along $\hat{x}$
polarization $\hat{n}$ is along $\hat{z}$



$$\vec{p}\cdot\hat{n} = |\vec{p}|\cos\theta$$

$$|\vec{p}\cdot\hat{n}|^2_{ave} = \frac{1}{4\pi}\int |\vec{p}|^2 \cos^2\theta \sin\theta \, d\theta \, d\phi$$

$$= \frac{2\pi}{4\pi}|p|^2 \left[-\frac{\cos^3\theta}{3} \right]_0^{\pi} = \frac{|p|^2}{3}$$

$$R_{b\to a} = \frac{\pi}{3\epsilon_0 \hbar^2} |p|^2 \rho(\omega_0)$$

dipole moment matrix element

energy density of EM field per unit frequency $\omega_0 = \frac{E_b - E_a}{\hbar}$

Special case Fermi's Golden Rule

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Spontaneous Emission

N_a atoms in ψ_a
N_b atoms in ψ_b

spontaneous emission rate A

$$N_{b\to a} = N_b \cdot A$$

stimulated emission rate $B_{ba}\,\rho(\omega_0)$
absorption rate $B_{ab}\,\rho(\omega_0)$

$$\frac{dN_b}{dt} = -N_b A - N_b B_{ba}\,\rho(\omega_0) + N_a B_{ab}\,\rho(\omega_0)$$

therm equil. $\dfrac{dN_b}{dt} = 0$

$$\rho(\omega_0) = \frac{N_b A}{N_a B_{ab} - N_b B_{ba}} = \frac{A}{\frac{N_a}{N_b} B_{ab} - B_{ba}}$$

Boltzmann $\dfrac{N_a}{N_b} = \dfrac{e^{-E_a/k_B T}}{e^{-E_b/k_B T}} = e^{\hbar\omega_0/k_B T}$

$$\rho(\omega_0) = \frac{A}{e^{\hbar\omega_0/k_B T} B_{ab} - B_{ba}}$$

Planck: $\rho(\omega) = \dfrac{\hbar}{\pi^2 c^3} \dfrac{\omega^3}{e^{\hbar\omega/k_B T} - 1}$

$B_{ab} = B_{ba}$ $\qquad A = \dfrac{\omega_0^3 \hbar}{\pi^2 c^3} B_{ba}$

Lifetime excited state $dN_b = -A N_b\, dt$

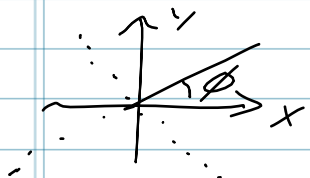
$N_b(t) = N_b(0)\, e^{-At}$ $\qquad \tau = \dfrac{1}{A}$ lifetime

$$\psi_b \to \psi_{a_1},\ \psi_{a_2},\ \psi_{a_3}$$

$$\tau = \frac{1}{A_1 + A_2 + A_2}$$

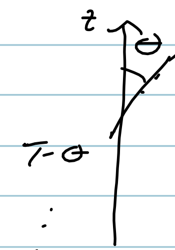
9.11 Calculate the lifetimes of the hydrogen $n=2$ states

$$\langle \psi_{100} | x | \psi_{2\ell m} \rangle \quad \langle \psi_{100} | y | \psi_{2\ell m} \rangle \quad \langle \psi_{100} | z | \psi_{2\ell m} \rangle$$



$x \to -x$ $\quad$ $y \to -y$ $\quad$ $z \to -z$

$\phi \to \phi + \frac{\pi}{2}$ $\quad$ $\phi \to \phi - \frac{\pi}{2}$ $\quad$ $\theta \to \pi - \theta$



$|\psi|^2$ even $\begin{cases} x \to -x \\ y \to -y \\ z \to -z \end{cases}$ $\quad H'_{ii} = 0$

$$\psi_{100} = \frac{1}{\sqrt{\pi a^3}} e^{-r/a} \qquad \psi_{200} = \frac{1}{\sqrt{8\pi a^3}} \left(1 - \frac{r}{2a}\right)^{-r/2a}$$

both even under $\begin{cases} x \to -x \\ y \to -y \\ z \to -z \end{cases}$

$$\psi_{210} = \frac{1}{\sqrt{32\pi a^3}} \frac{r}{a} e^{-r/2a} \cos\theta$$

even under $\begin{cases} x \to -x \\ y \to -y \end{cases}$

odd under $z \to -z$

$$\psi_{21\pm1} = \mp \frac{1}{\sqrt{64\pi a^3}} \frac{r}{a} e^{-r/2a} \sin\theta\, e^{\pm i\phi}$$

odd under $\begin{cases} x \to -x \\ y \to -y \end{cases}$

even under $z \to -z$

$$r \sin\theta\, e^{\pm i\phi} = x \pm iy$$