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QM 115B Lec 18

Hyperfine Splitting

$$\vec{\mu}_e = - \frac{g_e e \hbar}{2m_e} \vec{S}_e$$

$$g_e \approx 2$$

$$\vec{\mu}_p = \frac{g_p e \hbar}{2m_p} \vec{S}_p$$

$$g_p = 5.58$$

$$\vec{B}_p = \frac{\mu_0}{4\pi r^3} (3(\vec{\mu} \cdot \hat{r})\hat{r} - \vec{\mu}) + \frac{2\mu_0}{3} \vec{\mu} \delta^3(\vec{r})$$

$$H'_{hf} = -\vec{\mu}_e \cdot \vec{B}_p$$

$$H'_{hf} = \frac{\mu_0 g_p e^2 \hbar^2}{8\pi m_p m_e} \left(\frac{3(\vec{S}_p \cdot \hat{r})(\vec{S}_e \cdot \hat{r}) - \vec{S}_p \cdot \vec{S}_e}{r^3} + \frac{\mu_0 g_p e^2}{3m} \vec{S}_p \cdot \vec{S}_e \delta^3(\vec{r}) \right)$$

$$E'_{hf} = \frac{\mu_0 g_p e^2 \hbar^2}{8\pi m_p m_e} \left\langle \frac{3(\vec{S}_p \cdot \hat{r})(\vec{S}_e \cdot \hat{r}) - \vec{S}_p \cdot \vec{S}_e}{r^3} \right\rangle + \frac{\mu_0 g_p e^2 \hbar^2}{3 m_p m_e} \langle \vec{S}_p \cdot \vec{S}_e \rangle |4_{100}(0)|^2 = 0 \text{ for } l=0$$

Prob 6.27

$$\vec{S} = \vec{S}_p + \vec{S}_e \quad S^2 = S_p^2 + S_e^2 + 2\vec{S}_p \cdot \vec{S}_e$$

$$\vec{S}_p \cdot \vec{S}_e = \frac{1}{2} (S^2 - S_p^2 - S_e^2) = \frac{1}{2} (S^2 - \frac{3}{4}\hbar^2 - \frac{3}{4}\hbar^2)$$

$$E'_{hf} = \frac{4}{3} g_p \alpha^4 \frac{m_e^2 c^2}{m_p \hbar^2} \left\{ \begin{array}{l} \hbar^2 \frac{1}{2} (1(1+1) - \frac{3}{2}) \text{ triplet} \\ \hbar^2 \frac{1}{2} (0 - \frac{3}{2}) \text{ singlet} \end{array} \right.$$

$$= \alpha^4 m_e c^2 \left(\frac{4}{3} g_p \frac{m_e}{m_p} \right) \left\{ \begin{array}{l} \frac{1}{4} \text{ triplet} \\ -\frac{3}{4} \text{ singlet} \end{array} \right.$$

$$\Delta E = 5.88 \times 10^{-6} \text{ eV}$$

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→ Stark Effect

$$\mathcal{E} = \mathcal{E}_z$$

$$H' = e \mathcal{E} z$$

find first order shifts in $n=1$ $n=2$ levels

$n=1$ groundstate $l=0$ $m=0$
non-degenerate

$$E'_{100} = \langle 100 | H' | 100 \rangle$$

$$= e \mathcal{E} \langle 100 | z | 100 \rangle$$

$$= e \mathcal{E} \int \psi_{100}^* z \psi_{100} d^3\vec{r} = 0$$

Parity $z \rightarrow -z$

$$|\psi_{100}|^2 \rightarrow |\psi_{100}|^2$$

Proof; $Y_0^0 = \frac{1}{\sqrt{4\pi}}$ $z = r \cos \theta$

$$\int_0^{2\pi} \int_0^\pi |\psi_0^0|^2 \cos \theta \sin \theta d\theta d\phi = \frac{2\pi}{4\pi} \int_0^\pi \cos \theta \sin \theta d\theta$$

$$x = \cos \theta$$

$$dx = -\sin \theta d\theta$$

$$= \frac{1}{2} \int_{-1}^1 f(x) dx$$

$$= \frac{1}{2} \left[\frac{x^2}{2} \right]_{-1}^1$$

$$= 0$$

$h=2$ ψ_{200} ψ_{211} ψ_{210} ψ_{21-1}

4-fold degeneracy

$\langle 2 \ell' m' | z | 2 \ell m \rangle$

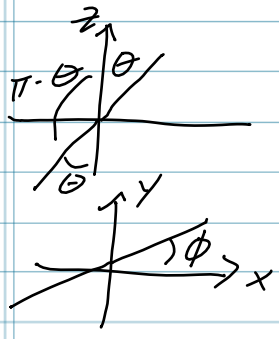
$z = r \cos \theta$
 z independent of ϕ

16 3D integrals

$\int Y_{\ell' m'}^* Y_{\ell m} f(\theta) \sin \theta d\theta d\phi$
 $\propto \delta_{m m'}$

$P \quad \vec{r} \rightarrow -\vec{r}$
 $z \rightarrow -z$

$P \psi_{n \ell' m'} \psi_{n \ell m} = -\psi_{n \ell' m'} \psi_{n \ell m}$



$P \quad r \rightarrow r$
 $\theta \rightarrow \pi - \theta$
 $\phi \rightarrow \phi + \pi$

Y_0^0 even

$Y_1^0 = c \cos \theta \rightarrow c \cos(\pi - \theta) = -c \cos \theta$
 $Y_1^{\pm 1} = c \sin \theta e^{\pm i\phi} \rightarrow c \sin(\pi - \theta) e^{\pm i(\pi + \phi)}$
 $= c \sin \theta e^{\pm i\pi} e^{\pm i\phi} = -Y_1^{\pm 1}$

$Y_{\ell m} \rightarrow (-1)^m Y_{\ell m}$

only $\langle 200 | z | 210 \rangle = \langle 210 | z | 200 \rangle^* \neq 0$

$\langle 200 | z | 210 \rangle = \int_0^\infty r^2 R_{20}^*(r) R_{21}(r) r dr \int \cos \theta Y_0^0 Y_1^0 d\Omega$
 $= \int_0^\infty r^3 \frac{1}{\sqrt{2} a^3} \left(1 - \frac{r}{2a}\right) e^{-r/2a} \frac{1}{\sqrt{24} a^3} \frac{r}{a} e^{-r/2a} dr \int_0^{2\pi} \int_0^\pi \cos \theta \frac{\sqrt{3} \cos \theta}{\sqrt{4\pi}} \frac{\sqrt{3} \cos \theta \sin \theta}{\sqrt{4\pi}} d\theta d\phi$
 $= \frac{1}{\sqrt{48}} \int_0^\infty \left(\frac{r^4}{a^4} - \frac{r^5}{2a^5}\right) e^{-r/a} dr \frac{2\pi \sqrt{3}}{4\pi} \int_0^\pi \cos^2 \theta \sin \theta d\theta$

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$$\int_0^{\infty} dy y^m e^{-by} = \frac{m!}{b^{m+1}}$$

$$y = r/a$$

$$x = \cos \theta$$

$$\langle 200 | z | 210 \rangle = \frac{a}{\sqrt{48}} \int_0^{\infty} (y^4 - \frac{y^5}{2}) e^{-y} dy \frac{\sqrt{3}}{2} \int_{-1}^1 dx x^2$$

$$= \frac{a}{\sqrt{48}} \left(4! - \frac{5!}{2} \right) \frac{\sqrt{3}}{2} \left[\frac{x^3}{3} \right]_{-1}^1$$

$$= \frac{a}{\sqrt{48}} (24 - 60) \frac{1}{\sqrt{3}} = -\frac{36a}{4 \cdot 3} = -3a$$

$$W = -3a \in \mathcal{E} \begin{pmatrix} 200 & 211 & 210 & 21-1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{vmatrix} -\lambda & 1 \\ 1 & -\lambda \end{vmatrix} = \lambda^2 - 1 \quad \lambda = 0$$

eigenvec. $\lambda = 0$

$$\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\lambda = 1 \quad \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ \sqrt{2} \\ 0 \end{pmatrix} \quad \lambda = -1 \quad \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ -\sqrt{2} \\ 0 \end{pmatrix}$$

$$|\psi_2\rangle_A = \frac{1}{\sqrt{2}} (|200\rangle + |210\rangle) \quad E_{2A} = -\frac{mc^2\alpha^2}{8} - 3eE$$

$$|\psi_2\rangle_B = |211\rangle$$

$$E_{2C} = E_{2B} = -\frac{mc^2\alpha^2}{8}$$

$$|\psi_2\rangle_C = |21-1\rangle$$

$$|\psi_2\rangle_D = \frac{1}{\sqrt{2}} (|200\rangle - |210\rangle) \quad E_{2D} = -\frac{mc^2\alpha^2}{8} + 3eE$$

