QM 115B Lec. 14

linear perturbation of Harmonic Oscillator

$$\cancel{H} V = \frac{1}{2}\mu\omega^2 x^2 \underbrace{- qEx}_{H'}$$

$$x' = x - \frac{qE}{\mu\omega^2} \qquad x = x' + \frac{qE}{\mu\omega^2}$$

$$V = \frac{1}{2}\mu\omega^2\left(x'^2 + \frac{2qEx'}{\mu\omega^2} + \frac{q^2E^2}{\mu^2\omega^4}\right)$$

$$-qE\left(x' + \frac{qE}{\mu\omega^2}\right)$$

$$= \frac{1}{2}\mu\omega^2 x'^2 + qEx' - qEx' \frac{1}{2}\frac{q^2E^2}{\mu\omega^2} - \frac{q^2E^2}{\mu\omega^2}$$

$$= \frac{1}{2}\mu\omega^2 x'^2 - \frac{\frac{1}{2}q^2E^2}{\mu\omega^2}$$

$$E_n = (n+\tfrac{1}{2})\hbar\omega - \frac{1}{2}\frac{q^2E^2}{\mu\omega^2}$$

perturbation theory $\quad E_n' = \langle \psi_n^0 | H' | \psi_n^0 \rangle$

$$= -qE\langle n|x|n\rangle$$

$$a_\pm = \frac{1}{\sqrt{2\hbar\mu\omega}}(\mp ip + \mu\omega x)$$

$$x = \sqrt{\frac{\hbar}{2\mu\omega}}(a_+ + a_-)$$

$$a_+|n\rangle = \sqrt{n+1}\,|n+1\rangle$$

$$a_-|n\rangle = \sqrt{n}\,|n-1\rangle$$

$$\langle n|x|n'\rangle = \sqrt{\frac{\hbar}{2\mu\omega}}\langle n|a_+ + a_-|n'\rangle$$

$$= \sqrt{\frac{\hbar}{2\mu\omega}}\left(\langle a_- n|n'\rangle + \langle n|a_- n'\rangle\right)$$

$$= \sqrt{\frac{\hbar}{2\mu\omega}}\left(\sqrt{n}\langle n-1|n'\rangle + \sqrt{n'}\langle n|n'-1\rangle\right)$$

$$= \sqrt{\frac{\hbar}{2\mu\omega}}\left(\sqrt{n}\,\delta_{n',n-1} + \sqrt{n'}\,\delta_{n,n'-1}\right)$$

$$E_n^1 = -qE\langle n|x|n\rangle = 0$$

$$E_n^2 = \sum_{m\neq n}\frac{|\langle m|-qEx|n\rangle|^2}{(n+\frac{1}{2})\hbar\omega-(m+\frac{1}{2})\hbar\omega}$$

$$= q^2E^2\sum_{m\neq n}\frac{|\langle m|x|n\rangle|^2}{(n-m)\hbar\omega}$$

$$= \frac{q^2E^2}{\hbar\omega}\sum_{m\neq n}\frac{\hbar}{2\mu\omega}\left(\frac{\sqrt{m}\,\delta_{n,m-1} + \sqrt{n}\,\delta_{m,n-1}}{n-m}\right)^2$$

$$= \frac{q^2E^2}{2\mu\omega^2}\left(\frac{n+1}{n-(n+1)} + \frac{n}{n-(n-1)}\right)$$

$$= \frac{-q^2E^2}{2\mu\omega^2}$$

$$E = (n+\tfrac{1}{2})\hbar\omega - \frac{q^2E^2}{2\mu\omega^2} + \dots$$

53

cubic perturbation to S.H.O.

$$H' = 2^{3/2} b \sqrt{\frac{\mu^3 \omega^5}{\hbar}} x^3$$

dimensionless

units $\frac{kg^{3/2} (\frac{1}{s})^{5/2} m^3}{(kg\, m^2/s)^{1/2}} = kg\, m^2/s^2$

$$x^3 = \left(\sqrt{\frac{\hbar}{2m\omega}}\right)^3 (a_+ + a_-)^3$$

$$H' = b\hbar\omega\, (a_+ + a_-)^3$$

$$(a_+ + a_-)^3 = (a_+^2 + a_+a_- + a_-a_+ + a_-^2)(a_+ + a_-)$$

$$= a_+^3 + a_+^2 a_- + a_+a_-a_+ + a_-a_+^2 + a_-^2 a_+ + a_-a_+a_- + a_+a_-^2 + a_-^3$$

$a_+a_- |n\rangle$ $\quad$ $a_-a_+|n\rangle$

$a_+ \sqrt{n}\, |n-1\rangle$ $\quad$ $a_- \sqrt{n+1}\, |n+1\rangle$

$\sqrt{n}\sqrt{n}\, |n\rangle$ $\quad$ $\sqrt{n+1}\sqrt{n+1}\, |n\rangle$

$\hat{n} = a_+a_-$ $\quad$ $\hat{n} + 1 = a_-a_+$ $\quad$ $[a_-, a_+] = 1$

$$(a_+ + a_-)^3 = a_+^3 + a_+\hat{n} + \hat{n}a_+ + \hat{n}a_- + (\hat{n}+1)a_+ + (\hat{n}+1)a_- + a_-(\hat{n}+1) + a_-^3$$

$$[\hat{n}, a_-] = [a_+a_-, a_-] = a_+[a_-, a_-] + [a_+, a_-]a_- = -a_-$$

$$[\hat{n}, a_+] = [a_+a_-, a_+] = a_+[a_-, a_+] + [a_+, a_+]a_- = a_+$$

$$(a_+ + a_-)^3 = a_+^3 + (\hat{n}a_+ - a_+) + \hat{n}a_+ + \hat{n}a_- + (\hat{n}+1)a_+ + (\hat{n}+1)a_- + \hat{n}a_- + a_- + a_- + a_-^3$$

$$= a_+^3 + 3\hat{n}a_+ + 3(\hat{n}+1)a_- + a_-^3$$

$$E_n^1 = \langle n | H' | n \rangle = 0$$

$$E_n^2 = \sum_{m \neq n} \frac{|\langle m | H' | n \rangle|^2}{E_n^0 - E_m^0}$$

$$\langle m = n-3 | H' | n \rangle = b\hbar\omega \langle n-3 | \sqrt{n(n-1)(n-2)} | n-3 \rangle$$
$$= b\hbar\omega \sqrt{n(n-1)(n-2}$$

$$\langle m = n-1 | H' | n \rangle = b\hbar\omega \langle n-1 | 3(n-1+1)\sqrt{n} | n-1 \rangle$$
$$= b\hbar\omega \; 3n\sqrt{n}$$

$$\langle m = n+1 | H' | n \rangle = b\hbar\omega \langle n+1 | 3\hat{n}\, a_+ | n \rangle$$
$$= b\hbar\omega \; 3(n+1)\sqrt{n+1}$$

$$\langle m = n+3 | H' | n \rangle = b\hbar\omega \sqrt{(n+1)(n+2)(n+3)}$$

$$E_n^2 = \frac{b^2\hbar^2\omega^2}{\hbar\omega} \left( \frac{n(n-1)(n-2)}{(n+1/2) - (n-3+1/2)} + \frac{9n^2 n}{n-(n-1)} \right.$$
$$\left. \frac{9(n+1)^2(n+1)}{n-(n+1)} + \frac{(n+1)(n+2)(n+3)}{n-(n+3)} \right)$$

$$= b^2\hbar\omega \left( \frac{n(n^2-3n+2)}{3} + 9n^3 \right.$$
$$\left. + \frac{9(n+1)^3}{-1} + \frac{(n+1)(n^2+5n+6)}{-3} \right)$$

$$= b^2\hbar\omega \left( \frac{n^3 - 3n^2 + 2n - (n^3 + 5n^2 + 6n + n^2 + 5n + 6)}{3} \right.$$
$$\left. + 9(n^3 - (n^3 + 3n^2 + 3n + 1) \right)$$

55

$$E_n^2 = -b^2\hbar\omega\left(\frac{9n^2+9n+6}{3} + 27n^2+27n+9\right)$$

$$= -b^2\hbar\omega\left(30n^2+30n+11\right)$$

$$= -b^2\hbar\omega\left(30\left(n+\tfrac{1}{2}\right)^2 + \tfrac{7}{2}\right)$$

$$E_n = \left(n+\tfrac{1}{2}\right)\hbar\omega - b^2\hbar\omega\left(30\left(n+\tfrac{1}{2}\right)^2 + \frac{7}{2}\right)$$