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QM 115B Lec 10

Free Electron Gas   ignore interactions

box $V: \begin{cases} 0 & 0<x<l_x,\ 0<y<l_y,\ 0<z<l_z \\ \infty & \text{otherwise} \end{cases}$

inside $-\frac{\hbar^2\nabla^2}{2m}\psi = E\psi$

$$\psi(x,y,z) = X(x)Y(y)Z(z)$$

$$-\frac{\hbar^2}{2m}\frac{d^2X}{dx^2} = E X$$

$$\frac{d^2X}{dx^2} = k_x^2 X$$

$$X(x) = A_x\sin(k_x x) + B_x\cos(k_x x)$$

$X(0)=0 \Rightarrow B_x = 0$

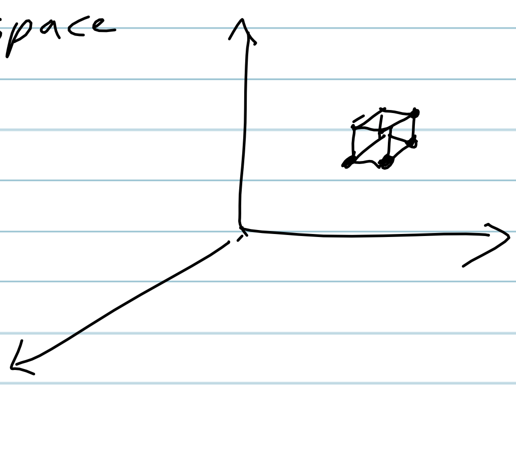
$X(l_x)=0 \Rightarrow k_x l_x = n_x\pi$

$$\psi_{n_x n_y n_z} = \sqrt{\frac{8}{l_x l_y l_z}}\sin\left(\frac{n_x\pi x}{l_x}\right)\sin\left(\frac{n_y\pi y}{l_y}\right)\sin\left(\frac{n_z\pi z}{l_z}\right)$$

$$E_{n_x n_y n_z} = \frac{\hbar^2\pi^2}{2m}\left(\frac{n_x^2}{l_x^2}+\frac{n_y^2}{l_y^2}+\frac{n_z^2}{l_z^2}\right) = \frac{\hbar^2 \vec{k}^2}{2m}$$

$$\vec{k} = (k_x, k_y, k_z) = \left(\frac{n_x\pi}{l_x}, \frac{n_y\pi}{l_y}, \frac{n_z\pi}{l_z}\right)$$

k-space



8 corners / box

each corner has 8 neighboring boxes

associate 1 box per state

volume in k-space $\frac{\pi^3}{l_x l_y l_z} = \frac{\pi^3}{V}$

N atoms, each atom contribute q free electrons
$N \sim 6\times10^{23}$ $\quad q \sim 1, 2$

bigger q harder metal $\quad$ Na $q=1$
higher melting temp $\quad$ Mg $q=2$

only two electrons per state $\uparrow\downarrow$
$Nq/2$ states filled

ground state fill up 1/8 sphere in k-space

radius k_F

$$\frac{1}{8}\left(\frac{4}{3}\pi k_F^3\right) = \frac{Nq}{2}\left(\frac{\pi^3}{V}\right)$$

$$k_F = \left(\frac{3Nq}{\cancel{\pi}}\frac{\pi^{\cancel{3}\,2}}{V}\right)^{1/3}$$

$$= \left(\frac{Nq}{V}3\pi^2\right)^{1/3} \qquad \rho = \frac{Nq}{V}$$

density of free electrons

boundary of sphere is called Fermi surface

$$E_F = \frac{\hbar^2 k_F^2}{2m} = \frac{\hbar^2}{2m}\left(3\rho\pi^2\right)^{2/3}$$

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Total energy of electron gas
shell of thickness dk

shell volume $\frac{1}{8}(4\pi k^2)\,dk$

electron states in shell = $\frac{\text{shell vol.}}{\text{vol./state}}\ \frac{2}{\text{state}}$

$$= \frac{2}{8}\frac{(4\pi k^2)\,dk}{\pi^3/V} = \frac{Vk^2dk}{\pi^2}$$

each state has energy $= \frac{\hbar^2 k^2}{2m}$

$$dE = \frac{\hbar k^2}{2m}\ \frac{V}{\pi^2}k^2dk$$

$$E_{tot} = \frac{\hbar^2 V}{2\pi^2 m}\int_0^{k_F} k^4\,dk = \frac{\hbar^2 V}{2\pi^2 m}\frac{k_F^5}{5}$$

$$= \frac{\hbar^2 V}{10\pi^2 m}\left(\frac{3N_q\pi^2}{V}\right)^{5/3} \qquad k_F = \left(\frac{3N_q\pi^2}{V}\right)^{1/3}$$

$$= \frac{\hbar^2}{10\pi^2 m}(3\pi^2 N_q)^{5/3}\ V^{-2/3}$$

$\frac{dE}{dV} < 0$

positive quantum degeneracy pressure

5.16 Cu: $E_F = 7eV = k_B T_F$

$$T_F = 8 \times 10^4 K$$

melting point of Cu is $1.3 \times 10^3 K$