115B Final Formulae

Please tear off this sheet.

\[ E = h f = h \frac{c}{\lambda} = h \omega, \quad p = \frac{h}{\lambda} = \hbar k, \quad E^2 = p^2 c^2 + m^2 c^4 \]

\[ -\frac{\hbar^2}{2m} \nabla^2 \Psi(r, t) + V(r) \Psi(r, t) = i\hbar \frac{\partial \Psi(r, t)}{\partial t} \]

\[ -\frac{\hbar^2}{2m} \nabla^2 \psi(\vec{r}) + V(\vec{r}) \psi(\vec{r}) = E \psi(\vec{r}) \text{ if } \Psi(\vec{r}, t) = e^{-iEt/h} \psi(\vec{r}) \]

\[ x, p_x = i\hbar, \quad [L_x, L_y] = i\hbar L_z, \quad [L_y, L_z] = i\hbar L_x, \quad [L_z, L_x] = i\hbar L_y, \]

\[ \text{spin 1/2: } \vec{S} = (\hbar/2)\vec{\sigma}, \quad \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \]

\[ J_{\pm j} m) = h\sqrt{j(j + 1) - m(m + 1)|j (m + 1)} \]

\[ V(x) = \begin{cases} 0 & 0 < x < L \\ \infty & x < 0, x > L \end{cases}, \quad \psi_n(x) = \begin{cases} \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L} & 0 < x < L \\ 0 & x < 0, x > L \end{cases}, \quad E_n = \frac{\pi^2 \hbar^2}{2mL^2} \]

\[ H = \frac{\hbar^2}{2m} \nabla^2 + \frac{1}{2} m \omega^2 x^2 = (a_+ a_- + \frac{1}{2}) h\omega, \quad E_n = (n + \frac{1}{2}) h\omega, \quad [a_-, a_+] = 1, \]

\[ a_+ \psi_n = \sqrt{n + 1} \psi_{n+1}, \quad a_- \psi_n = \sqrt{n} \psi_{n-1}, \quad \psi_n(x) = (\frac{m\omega}{\pi \hbar})^{1/4} \frac{1}{\sqrt{2n!}} H_n(\xi) e^{-\xi^2/2}, \quad \xi = x \sqrt{\frac{m\omega}{\hbar}} \]

\[ H_0 = 1, \quad H_1 = 2\xi, \quad H_2 = 4\xi^2 - 2, \quad H_3 = 8\xi^3 - 12\xi, \quad H_4 = 16\xi^4 - 48\xi^2 + 12 \]

\[ V(r) = -\frac{\hbar c}{r}, \quad -\frac{\hbar^2}{2m} \frac{d^2}{dr^2} R_n(r) + h^2 \xi^2(r) + V(r) R_n(r) = E_n R_n(r) \]

\[ a = \frac{n}{amc} = 0.0529 \text{ nm}, \quad \alpha = 1/137.04, \quad E_n = -\frac{\alpha^2 m^2 c^2}{2 m} \left[ 1 + \frac{2n}{n} \left( \frac{n}{j+1/2} - \frac{3}{4} \right) \right] \approx 13.6 \text{ eV} / n^2 \]

\[ \alpha = e^2/(4\pi \epsilon_0 h c), \quad \int_0^{2\pi} \int_0^\infty |\psi_{nlm}(\vec{r})|^2 r^2 dr \sin \theta d\theta d\phi = 1, \quad \psi_{nlm}(\vec{r}) = R_n(r) Y_l^m(\theta, \phi), \]

\[ R_{n,n-1}(r) = A r^{n-1} e^{-r/a}, \quad R_{10} = \frac{2}{a} e^{-r/a}, \quad R_{20} = \frac{2}{\sqrt{2} a} e^{-r/a}, \quad R_{21} = \frac{2}{\sqrt{2} a} e^{-r/a}, \quad R_{31} = \frac{8}{27a^3} e^{-r/a}, \quad R_{32} = \frac{4}{81a^3} e^{-r/a} \]

\[ Y_0^0 = \frac{1}{\sqrt{\pi}}, \quad Y_1^0 = \frac{1}{\sqrt{3}} \cos \theta, \quad Y_1^1 = \frac{1}{\sqrt{3}} \sin \theta e^{i\phi}, \quad Y_0^0 = \frac{1}{\sqrt{10}} (3 \cos^2 \theta - 1) \]

\[ Y_2^1 = \frac{1}{\sqrt{15}} \sin \theta \cos \theta e^{i\phi}, \quad Y_2^2 = \frac{1}{\sqrt{15}} \sin \theta e^{2i\phi}, \quad dE = \frac{h^2 k^2}{2m} \frac{V}{\pi} k^2 dk, \]

\[ E_F = \frac{h^2}{2m} \left( \frac{3N\pi^2}{V} \right)^{2/3} = \frac{h^2 k_F}{2m}, \quad \rho(\omega) = \frac{h^3}{\pi^2 \hbar^2} \int_{\omega = \Omega}^{\infty} \frac{1}{\bar{n}} \frac{d\bar{n}}{d\omega} \]

\[ n(\epsilon) = \begin{cases} \frac{1}{e^{(\epsilon - \mu)/k_B T} - 1} & \text{Fermi – Dirac} \\ \frac{1}{e^{(\epsilon - \mu)/k_B T} + 1} & \text{Bose – Einstein} \end{cases}, \quad \int_0^\infty d\nu \frac{b^{\nu - 1}}{e^{b\nu + 1}} = a^{-\nu} \Gamma(\nu) \zeta(\nu) \left\{ \begin{array}{l} 1 \quad 2^{\nu - 1} \\ 0 \end{array} \right\} \]

\[ c_a^1 = 1, \quad c_b^1 = -\frac{i}{\hbar} \int_0^\infty H_{ba}(t') e^{i\omega_a t'} dt', \quad \omega_0 = (E_b - E_a)/\hbar, \quad H_{ba} = V_{ab} \cos \omega t \]

\[ P_{a \rightarrow b} \approx \frac{|V_{ab}|^2 \sin^2(\omega_{\gamma} - \omega_{\theta}/2)}{\omega_{\gamma}^2}, \quad R_{a \rightarrow b} = B_{ba} \rho(\omega_0), \quad B_{ab} = \pi \rho(\omega_0) \int_0^\infty \frac{b^{\nu - 1}}{e^{b\nu + 1}} A \left( \frac{\omega_0^2}{3\pi \epsilon_0 \hbar^2} \right)^3 \]

\[ \Delta \ell = \pm 1, \quad \Delta m = \pm 1, 0; \quad \sigma = |f(\theta, \phi)|^2 d\Omega = dN/L F(\theta, \phi) = -\frac{m}{2\pi \hbar^2} \int e^{i\vec{r} \cdot \vec{r} V(\vec{r})} d^3 r \]

\[ \vec{k} = k' - \vec{k}, \quad f(\theta) = -\frac{2m}{\hbar^2} \int_0^\infty R(r) \sin(\kappa r) dr, \quad \kappa = 2k \sin \theta/2; \]

\[ e^{i\theta} = \cos \theta + i \sin \theta \int_0^\infty \sin(\kappa r) r^{n-1} dr = \frac{p^{n-1}}{2} \cos(n\pi/2) \Gamma(1-n), \quad 0 < n < 2 \]

\[ J = 1.05 \times 10^{-34} \text{ J} \cdot \text{s}, \quad c = 3 \times 10^8 \text{ m/s}, \quad \omega_0 = 9.11 \times 10^{-31} \text{ kg} = 511 \text{ keV}/c^2, \quad 1 \text{ eV} = 1.602 \times 10^{-19} \text{ J} \]
1. Fill in the blank.

When applying perturbation theory to non-degenerate energy levels the correction to the energy is the _______________ ________________ of the perturbation Hamiltonian in the unperturbed state.

The variational method gives a/an _______________ bound on the ground state energy.

The three ways atoms interact with photons are absorption, _______________ emission, and _______________ emission.

The _______________ ________________ tells us the probability amplitude to for a incoming wavefunction to scatter of a target in a particular direction.

If the spatial wavefunction of two electrons is symmetric under interchange, the spin wavefunction must be _______________.

We can’t simultaneously have eigenstate of $L_x$ and $L_z$ because these operators do not _______________.

For degenerate perturbation theory we find the “good” states by _______________ the perturbation Hamiltonian in the degenerate subspace.
2. A hydrogen atom is prepared with in the second excited state with the maximal value of the z-component of orbital angular momentum $L_z$.

a) What are the corresponding quantum numbers $n, \ell, m$?
b) What is the expectation value of rotational kinetic energy?
3. Consider a spin $1/2$ particle in one dimensional harmonic potential

$$H = -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + \frac{1}{2} m \omega_0^2 x^2 = \hbar \omega_0 \left( a_+ a_- + \frac{1}{2} \right).$$

Now we perturb this system by a magnetic field $\vec{B} = B \hat{y}$ so that there is an additional term in the Hamiltonian:

$$H' = -\gamma \vec{B}.\vec{S}.$$

a) To first order in $B$ (assume $|\gamma B| \ll \omega_0$) how many levels does the first excited state of the oscillator split into and what are the energies of each of them? 

b) Now suppose that $B = 0$ and we turn on a new time dependent perturbation at $t = 0$

$$H = V(a_+ e^{-i\omega t} + a_- e^{i\omega t}).$$

If the particle starts in the ground state what is the probability amplitude to find the particle in the first excited state after a very short time $t$?
4. Consider a particle, with orbital angular momentum $\ell = 4$ and spin $s = 2$ and total angular momentum $\vec{J} = \vec{L} + \vec{S}$. Use the total angular momentum lowering operator to find the eigenstate of total angular momentum with $j = 5$ and $z$-component of total angular momentum $m = 4$. 
5. A hydrogen atom is prepared in the $n = 3, \ell = 2, m = 0$ state and decays by emitting a photon through a dipole transition to an $m = 0$ state. What is the transition rate for this process? (Hint: since $\Delta m = 0$, only the $z$ component of the dipole transition matrix element is non-zero.)
6. Consider scattering off of a spherically symmetric potential

\[ V(r) = \frac{\beta}{r^\delta}, \]

with \(1 < \delta < 3\).

a) What is the scattering amplitude as a function of the scattering angle \(\theta\)?

b) If the scattered wave and the incident wave are observed to interfere destructively behind the target for \(\delta = 1.15\) should \(\beta\) be a positive or negative number? (Hint: \(\Gamma(x) > 0\) for \(x > 0\).)
extra space