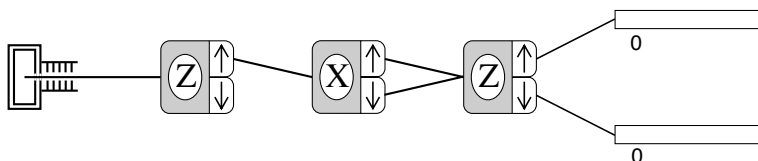


### Phys 115B Problem Set 3, due Oct. 8 in class

From the textbook do problems 4.13, 4.16,  
as well as the attached exercise using the SPINS software.

3. Make an interferometer as shown:



Use Random as the initial state. Measure the relative probabilities of spin up and spin down after the final SG device. Do this for the three cases where (1) the spin up beam from the middle SG device is used, (2) the spin down beam from the middle SG device is used, and (3) both beams from the middle SG device are used (as the figure shows). Put your experimental results in the table on the worksheet. Repeat for the case where the spin down state is used from the first SG device.

Now calculate the theoretically expected probabilities and fill in the theory part of the worksheet. When both beams from the middle SG device are used, the input state to the last SG device (call it  $|\psi\rangle$ ) is a combination (superposition) of the spin up and spin down states with respect to the axis of the middle SG device. To calculate  $|\psi\rangle$  properly, use the projection postulate (5th postulate), which says that the ket after a measurement is the normalized projection onto the subspace corresponding to the measured results. For the case where both spin up and spin down results are obtained (because both beams are sent to the next device), the projector is  $P = |+\rangle_{xx}\langle+| + |-\rangle_{xx}\langle-|$ . For the case where the input to the middle SG device is  $|+\rangle$ , the projection is

$$P|+\rangle = |+\rangle_{xx}\langle+|+\rangle + |-\rangle_{xx}\langle-|+\rangle = {}_x\langle+|+\rangle|+\rangle_x + {}_x\langle-|+\rangle|-\rangle_x.$$

To find  $|\psi\rangle$ , just normalize this new ket.

Spin 1/2 Interferometer

| Beams used  |                            | Experiment     |                  | Theory         |                  |
|-------------|----------------------------|----------------|------------------|----------------|------------------|
| SG1         | SG2                        | $P_{\uparrow}$ | $P_{\downarrow}$ | $P_{\uparrow}$ | $P_{\downarrow}$ |
| $ +\rangle$ | $ +\rangle_x$              |                |                  |                |                  |
|             | $ -\rangle_x$              |                |                  |                |                  |
|             | $ +\rangle_x,  -\rangle_x$ |                |                  |                |                  |
| $ -\rangle$ | $ +\rangle_x$              |                |                  |                |                  |
|             | $ -\rangle_x$              |                |                  |                |                  |
|             | $ +\rangle_x,  -\rangle_x$ |                |                  |                |                  |

1. Choose the Spin-1 case under the Design menu. Set up an experiment for successive measurement of spin projections. The first analyzer will prepare the atom in a state  $|\psi\rangle$ . The second analyzer will then project this state vector onto one of the eigenstates corresponding to the spin projection along the axis of the second analyzer. If we call this eigenstate  $|\phi\rangle$ , then the probability that the atom is detected in that channel is  $|\langle\phi|\psi\rangle|^2$ . Your experiment will allow you to measure this probability. Remember that this is the probability that an atom leaving the first analyzer also makes it through the second analyzer to the appropriate detector, and not the total probability for getting from the oven to the detector. Fill in the experimental table on the worksheet with your measured values corresponding to all three choices of axes for each analyzer. Use the expressions for the spin eigenkets on the spin reference sheet to calculate all these same probabilities and fill in the theory table.

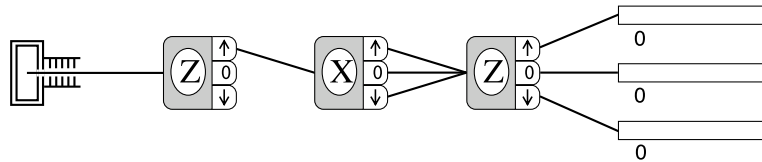
Experimental Table

| $\langle\phi \psi\rangle^2$ | $ 1\rangle$ | $ 0\rangle$ | $ -1\rangle$ | $ 1\rangle_x$ | $ 0\rangle_x$ | $ -1\rangle_x$ | $ 1\rangle_y$ | $ 0\rangle_y$ | $ -1\rangle_y$ |
|-----------------------------|-------------|-------------|--------------|---------------|---------------|----------------|---------------|---------------|----------------|
| $\langle 1 $                |             |             |              |               |               |                |               |               |                |
| $\langle 0 $                |             |             |              |               |               |                |               |               |                |
| $\langle -1 $               |             |             |              |               |               |                |               |               |                |
| ${}_x\langle 1 $            |             |             |              |               |               |                |               |               |                |
| ${}_x\langle 0 $            |             |             |              |               |               |                |               |               |                |
| ${}_x\langle -1 $           |             |             |              |               |               |                |               |               |                |
| ${}_y\langle 1 $            |             |             |              |               |               |                |               |               |                |
| ${}_y\langle 0 $            |             |             |              |               |               |                |               |               |                |
| ${}_y\langle -1 $           |             |             |              |               |               |                |               |               |                |

Theory Table

| $\langle\phi \psi\rangle^2$ | $ 1\rangle$ | $ 0\rangle$ | $ -1\rangle$ | $ 1\rangle_x$ | $ 0\rangle_x$ | $ -1\rangle_x$ | $ 1\rangle_y$ | $ 0\rangle_y$ | $ -1\rangle_y$ |
|-----------------------------|-------------|-------------|--------------|---------------|---------------|----------------|---------------|---------------|----------------|
| $\langle 1 $                |             |             |              |               |               |                |               |               |                |
| $\langle 0 $                |             |             |              |               |               |                |               |               |                |
| $\langle -1 $               |             |             |              |               |               |                |               |               |                |
| ${}_x\langle 1 $            |             |             |              |               |               |                |               |               |                |
| ${}_x\langle 0 $            |             |             |              |               |               |                |               |               |                |
| ${}_x\langle -1 $           |             |             |              |               |               |                |               |               |                |
| ${}_y\langle 1 $            |             |             |              |               |               |                |               |               |                |
| ${}_y\langle 0 $            |             |             |              |               |               |                |               |               |                |
| ${}_y\langle -1 $           |             |             |              |               |               |                |               |               |                |

3. Make an interferometer as shown:



Use **Random** as the initial state. Measure the relative probabilities after the final SG device. Do this for the seven possible cases where one beam, a pair of beams, or all three beams from the middle SG device are used. Record your results in the experiment part of the worksheet table. Use the projection postulate to calculate and fill in the theory part of the table and compare to the experiment.

Spin 1 Interferometer

| Beams                                    | Experiment       |                  |                   | Theory           |                  |                   |
|--|------------------|------------------|-------------------|------------------|------------------|-------------------|
|  | $\mathcal{P}(1)$ | $\mathcal{P}(0)$ | $\mathcal{P}(-1)$ | $\mathcal{P}(1)$ | $\mathcal{P}(0)$ | $\mathcal{P}(-1)$ |
| $ 1\rangle_x$                            |                  |                  |                   |                  |                  |                   |
| $ 0\rangle_x$                            |                  |                  |                   |                  |                  |                   |
| $ -1\rangle_x$                           |                  |                  |                   |                  |                  |                   |
| $ 1\rangle_x,  0\rangle_x$               |                  |                  |                   |                  |                  |                   |
| $ 1\rangle_x,  -1\rangle_x$              |                  |                  |                   |                  |                  |                   |
| $ 0\rangle_x,  -1\rangle_x$              |                  |                  |                   |                  |                  |                   |
| $ 1\rangle_x,  0\rangle_x,  -1\rangle_x$ |                  |                  |                   |                  |                  |                   |