Phys 115B Problem Set 12, due Nov. 12 in class.

Do the following four questions which should be good practice for the midterm.

1. Suppose there was an attractive force between electrons and protons with a potential given by

$$V(r) = \frac{\beta M^2 c^3 r}{\hbar} \ ,$$

where M is a mass.

a) What would be the first order correction in β to the hydrogen energy levels (work to order α^2)? You can use

$$\langle \psi_{n\ell m} | \, r \, | \psi_{n\ell m} \rangle = \frac{a}{2} \left[3n^2 - \ell(\ell+1) \right] \ .$$

b) How many levels does the third excited state split into?

c) What is the energy of the photon emitted when a transition is made from the lowest $n = 3$ level to the highest $n = 2$ level.

2. Consider a one dimensional harmonic oscillator with a potential

$$H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{1}{2} m\omega^2 x^2 = \hbar\omega \left(a_+^{(x)} a_-^{(x)} + \frac{1}{2} \right) \ ,$$

with a small perturbation

$$H' = b \, \hbar\omega \left(a_+^{(x)} a_+^{(x)} a_+^{(x)} + a_-^{(x)} a_-^{(x)} a_-^{(x)} \right) \ .$$

To second order in b, what is the energy of the n-th level?

3. Consider a three dimensional harmonic oscillator with a potential

$$H = -\frac{\hbar^2 \nabla^2}{2m} + \frac{1}{2} m\omega^2 (x^2 + y^2 + z^2) = \hbar\omega \left(a_+^{(x)} a_-^{(x)} + a_+^{(y)} a_-^{(y)} + a_+^{(z)} a_-^{(z)} + \frac{3}{2} \right) \ .$$

a) What are the degeneracies of the ground state and the first excited state? Now consider the perturbation

$$H' = b \, \hbar\omega \left(a_+^{(x)} a_+^{(x)} a_-^{(y)} a_-^{(y)} + a_+^{(y)} a_+^{(y)} a_-^{(x)} a_-^{(x)} \right) \ .$$

b) To first order in b, what is the energy of the ground state?
c) To first order in b, how many levels does the second excited state split into and what are their energies?

4. A system of electrons is in thermal equilibrium.
a) At a temperature where $k_B T$ is equal to the chemical potential μ, what fraction of states with an energy equal to 2μ should be filled?
b) At $T = 0$ what is the average momentum squared of the fermions expressed in terms of the Fermi momentum?