Hadron Physics and 3d dualities

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Based on collaboration with Ryo Yokokura, Naoto Kan (KEK)
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θ term in QCD

$$S_{\rm QCD} = \int d^4x \left(\frac{1}{4g^2} F^2 + \frac{i\theta}{32\pi^2} F\tilde{F} + \bar{\psi}(D+m)\psi \right)$$

$$Z_{\rm QCD}(\theta) = \int [dA][d\psi][d\bar{\psi}] e^{-S_{\rm QCD}}$$

instanton number: $Q = \int d^4x \frac{1}{32\pi^2} F\tilde{F}$ (integer!)

There are many mysteries

Is θ physical? (Is the up quark massive?)

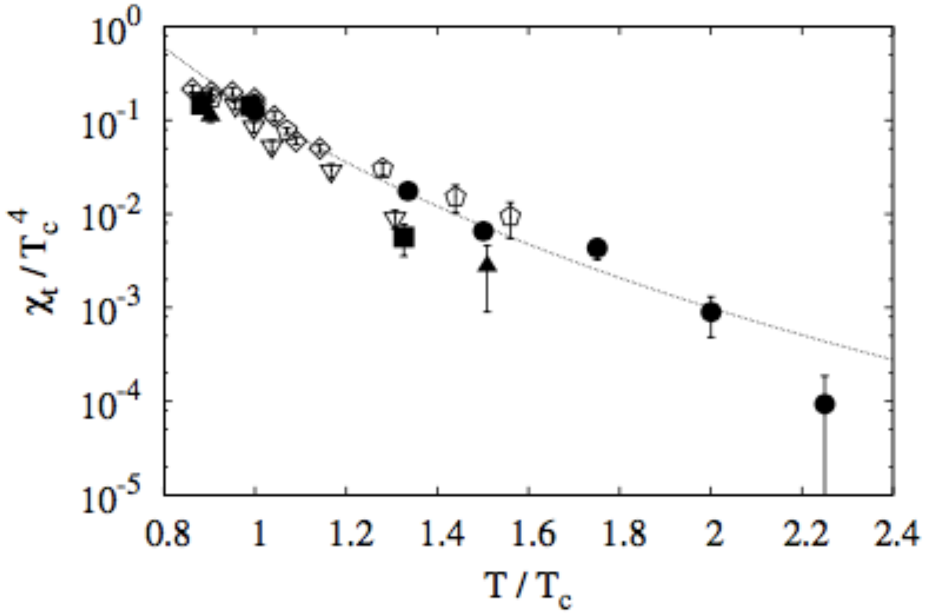
If so, why small?

What happens at θ= π ?

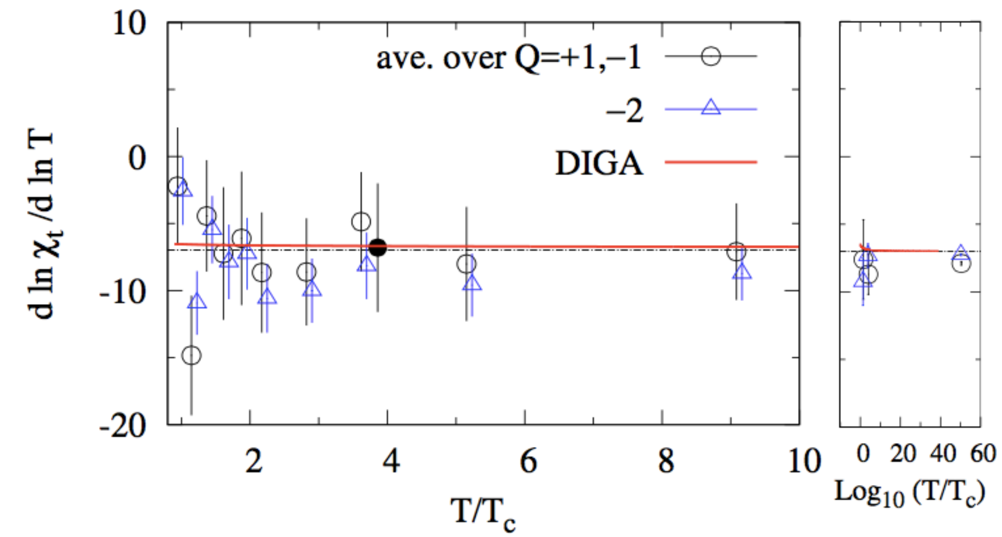
Does θ dependence disappear when chiral symmetry is restored at high temperatures?

θ dependence in SU(2) theories?

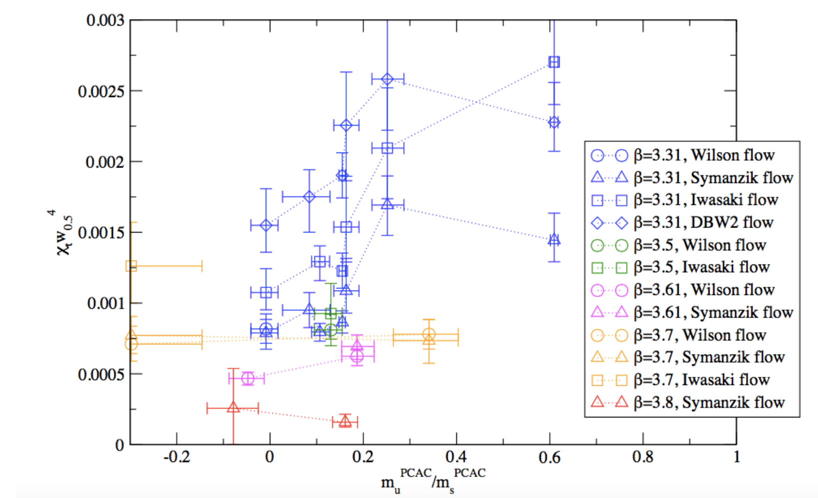
[RK, Yamada '15]



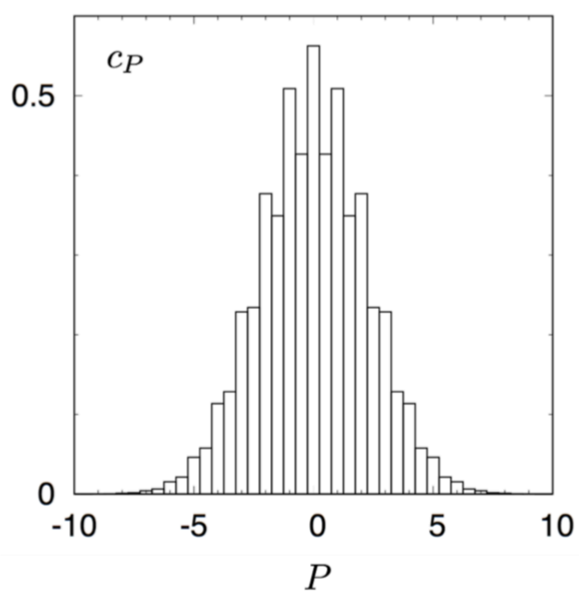
[Frison, RK, Matsufuru, Mori, Yamada '16]



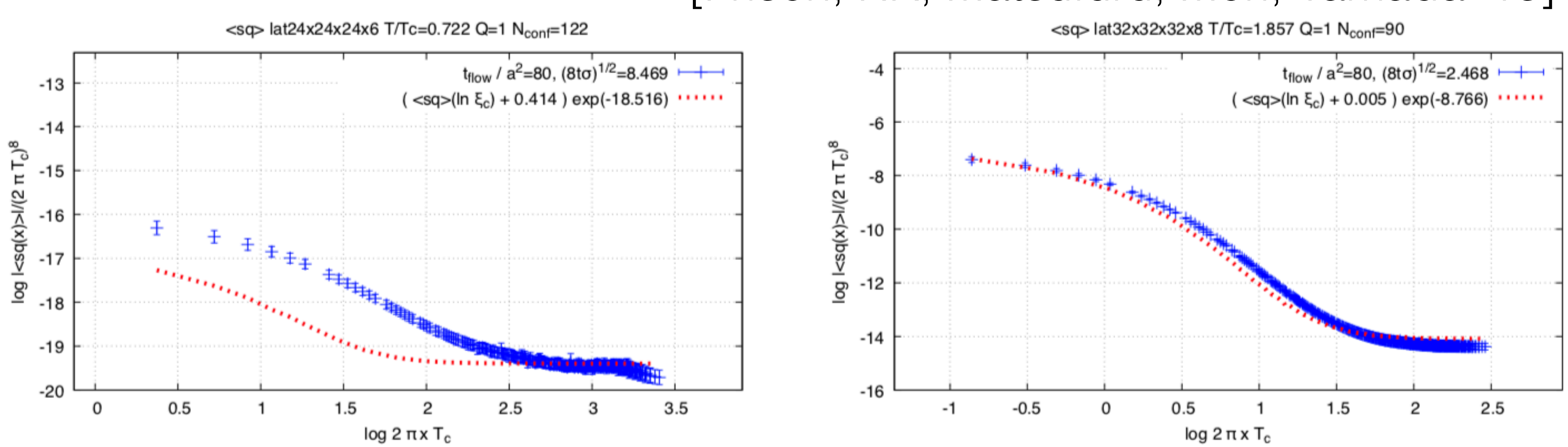
[RK, Yamada, Frison '16]



[RK, Suyama, Yamada '17]



[Frison, RK, Matsufuru, Mori, Yamada '18]



(a) CPV 2pt-function in $24^3 \times 6$ with $T/T_c \simeq 0.72$

(b) CPV 2pt-function in $32^3 \times 8$ with $T/T_c \simeq 1.9$

Today, I discuss some **exotic** possibilities of hadron properties in QCD based on the consideration of the θ term.

It is motivated by the recent discussions on 3-dim QCD, where various dualities are proposed.

Let me start with a review of the 3d world.

(There is a nice review on-line by David Tong. I stole many items from the review.)

[Peskin '78]

Particle-Vortex Duality

$$S_{XY} = \int d^3x \left(|\partial_\mu \tilde{\phi}|^2 - \tilde{m}^2 |\tilde{\phi}|^2 - \frac{\tilde{\lambda}}{2} |\tilde{\phi}|^4 \right)$$

Particle ⟷ Vortex

Vortex (Confined) ⟷ Particle (Confined)

Wow.

$$S_{AH} = \int d^3x \left(-\frac{1}{4e^2} F_{\mu\nu}^2 + |\mathcal{D}_\mu \phi|^2 - m^2 |\phi|^2 - \frac{\lambda}{2} |\phi|^4 \right)$$

Global U(1): $J^\mu_{\rm top} = \frac{1}{4\pi} \epsilon^{\mu\nu\rho} F_{\nu\rho}$

duality: U(1) + a scalar <—-> a scalar
(near the WF fixed point)

Chern-Simons theory

$$S_{CS} = \frac{k}{4\pi} \int d^3x \; \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho$$

k: CS level. need to be **integers** for gauge invariance.

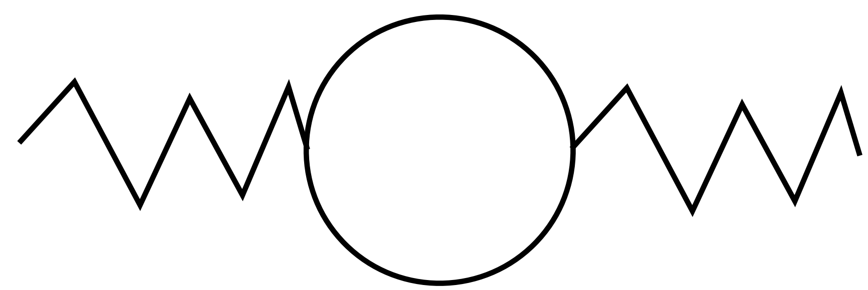
we call it $U(1)_k$ → Maxwell+CS —> CS at low energy.

photon gets mass: $m_{CS} = \frac{ke^2}{2\pi}$

topological. Observables are correlators of Wilson loops which depends only on topology.

CS + fermions

Integrating out fermions shifts the CS level.



$U(1)_k$ + N_f fermions —> $U(1)_{k \pm N_f/2}$

± depends on sign of fermion mass

Fermions don't just decouple in 3d.

Boson? Fermion?

$$S = \int d^3x \,(-\frac{1}{4e^2} F_{\mu\nu}F^{\mu\nu} + \frac{k}{4\pi}\epsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho + \sum_{i=1}^{N_f} i\bar{\psi}_i \not{D}\psi_i + m_i\bar{\psi}_i\psi_i \,)$$

At low energy, one can ignore the Maxwell term.

Gauss law constraint: $\frac{k}{2\pi}B = \rho$ → $Q = kQ_M$

k x (magnetic charge density) = number density

fermion is attached to a magnetic flux.

exchanging two particles gets the Aharonov-Bohm phase.

k=1: Fermions obey Bose statistics.
Bosons obey Fermi statistics.

[Polyakov '88]

Boson-Fermion Duality

Maybe there is a theory to describe CS+scalar by fermion d.o.f.

$$S_A[\phi, a] = \int d^3x \, (-\frac{1}{4e^2} f_{\mu\nu} f^{\mu\nu} + \frac{1}{4\pi} \epsilon^{\mu\nu\rho} a_\mu \partial_\nu a_\rho + |\mathcal{D}_\mu \phi|^2 - m^2 |\phi|^2 - \frac{\lambda}{2} |\phi|^4 \,)$$

$U(1)_1$ + a scalar — global sym. $U(1)_{top}$

↕ **dual**

a free fermion — global sym. $U(1)_{fermion}$

This type of duality is promoted to 4d Electric-Magnetic duality!

Deconstructing S-Duality

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Abstract: We use the technique of deconstruction to lift dualities from 2+1 to 3+1 dimensions. In this work we demonstrate the basic idea by deriving S-duality of maximally supersymmetric electromagnetism in 3+1 dimensions from mirror symmetry in 2+1. We also study the deconstruction of a non-supersymmetric duality in 3+1 dimensions using Abelian bosonization in 2+1.

Interesting.

Non-abelian dualities

There are non-abelian versions proposed:

$$SU(N)_k \text{ with } N_f \text{ scalars} \quad \longleftrightarrow \quad U(k)_{-N+\frac{N_f}{2}} \text{ with } N_f \text{ fermions}$$

$$U(N)_k \text{ with } N_f \text{ scalars} \quad \longleftrightarrow \quad SU(k)_{-N+\frac{N_f}{2}} \text{ with } N_f \text{ fermions}$$

$$U(N)_{k,k\pm N} \text{ with } N_f \text{ scalars} \quad \longleftrightarrow \quad U(k)_{-N+\frac{N_f}{2},-N\mp k+\frac{N_f}{2}} \text{ with } N_f \text{ fermions}$$

This is based on the **level-rank** dualities of CS theories:

$$SU(N)_k \leftrightarrow U(k)_{-N}$$

* The previously discussed U(1) duality is a part of them.

2nd duality with $(N,k,N_f)=(1,1,1)$

Let's see what's going on.

$$U(N)_k \text{ with } N_f \text{ scalars} \quad \longleftrightarrow \quad SU(k)_{-N+\frac{N_f}{2}} \text{ with } N_f \text{ fermions}$$

$m^2_{\text{scalar}} \gg g^4$ low energy is $U(N)_k$ <—> $SU(k)_{-N}$

$m^2_{\text{scalar}} \ll -g^4$ low energy is $U(N-N_f)_k$ <—> $SU(k)_{-N+N_f}$

Consistent with $\tilde{k} = -N + \frac{N_f}{2} \pm \frac{N_f}{2}$

Both theories have the **same** global symmetry:

$SU(N_f) \times U(1)$ (naive)

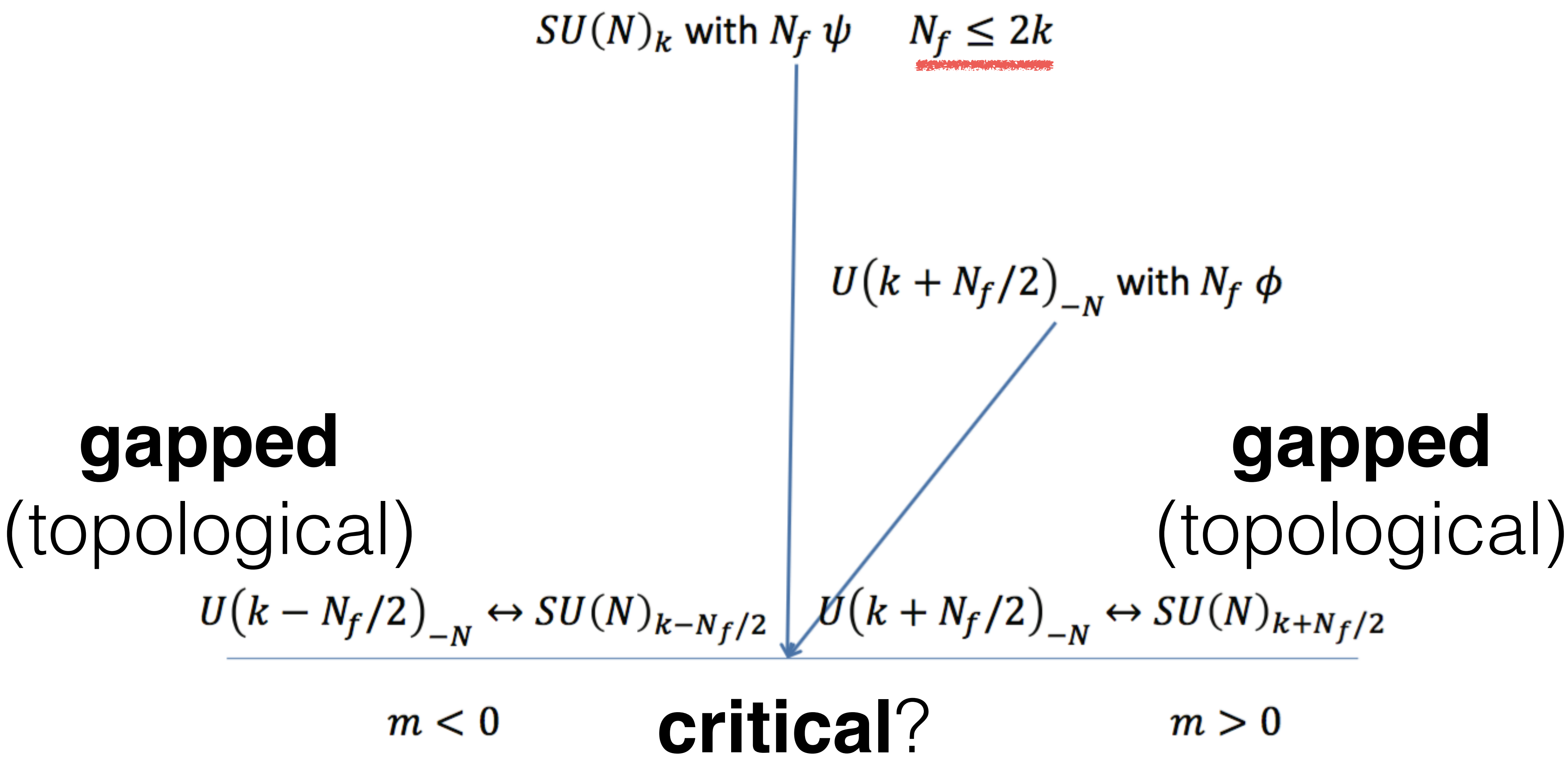
* $U(1)_{top}$ symmetry <—> $U(1)_{baryon}$ in U(N) - SU(k) dualities.

monopole <—> baryon

[Komargodski, Seiberg ’17]

phase diagram

QCD$_3$: SU(N)$_k$ + N$_f$ fermions



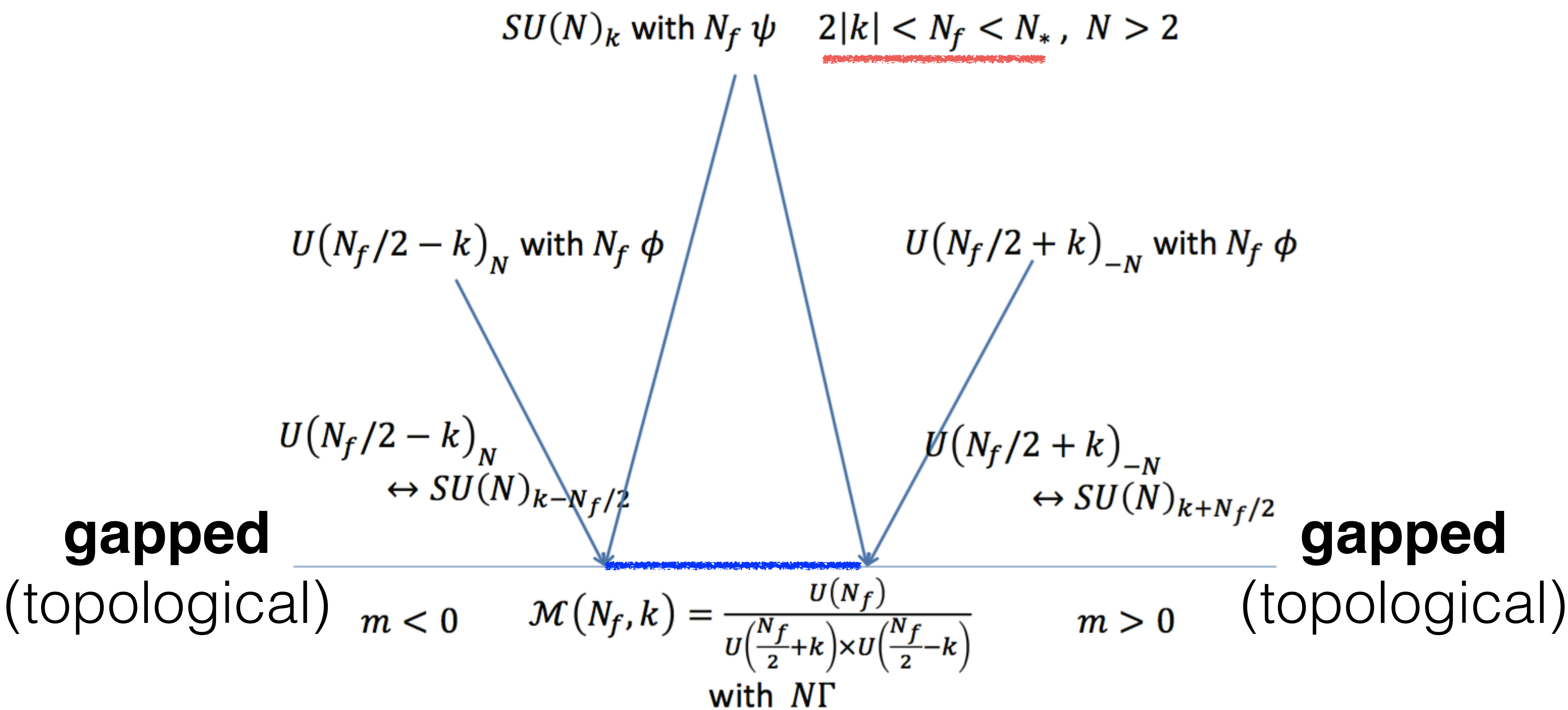
monopole=baryon

U(N$_f$) global symmetry unbroken everywhere

[Komargodski, Seiberg ’17]

What about $N_f > 2|k|$?

yet another conjecture



Non-linear sigma model

I’d like to know whether these interesting 3d dynamics have something to do with 4d world.

S^1 compactified QCD

Let's consider QCD in 4d under a background:

$$S = \int d^3x \int_{-\pi R}^{\pi R} dx_3 \Big[-\frac{1}{2g_4^2}\mathrm{Tr}\left(F_{MN}F^{MN}\right) + \frac{\theta(x_3)}{32\pi^2}\epsilon_{MNPQ}\mathrm{Tr}\left(F^{MN}F^{PQ}\right)$$

$$+ i\bar{\Psi}_i\gamma^M(\partial_M - iA_M)\Psi_i$$

$$- \partial_\mu\alpha_L^i(x_3)\bar{\Psi}_i\gamma^\mu P_L\Psi_i - \partial_\mu\alpha_R^i(x_3)\bar{\Psi}_i\gamma^\mu P_R\Psi_i \Big].$$

b.c. for fermions: $\Psi_i(x_3 + 2\pi R) = e^{i\nu}\Psi_i(x_3),$

$$\int_{S^1} d\theta = 2\pi k, \quad \int_{S^1} d\alpha_{L,R}^i = 2\pi m_{L,R}^i,$$

$$\bar{\theta}(x_3) = \theta(x_3) + \sum_i(\alpha_R^i(x_3) - \alpha_L^i(x_3)),$$

Integer parameter:

$$\bar{k} = k + \sum_i(m_R^i - m_L^i)$$

Small and Large radius

Small radius
$\Lambda_4 R \ll 1$

One can do KK decomposition of QCD.

Large radius
$\Lambda_4 R \gg 1$

One can do KK decomposition of the Chiral Lagrangian.

We can analyze the theory in the both limits.

Small radius

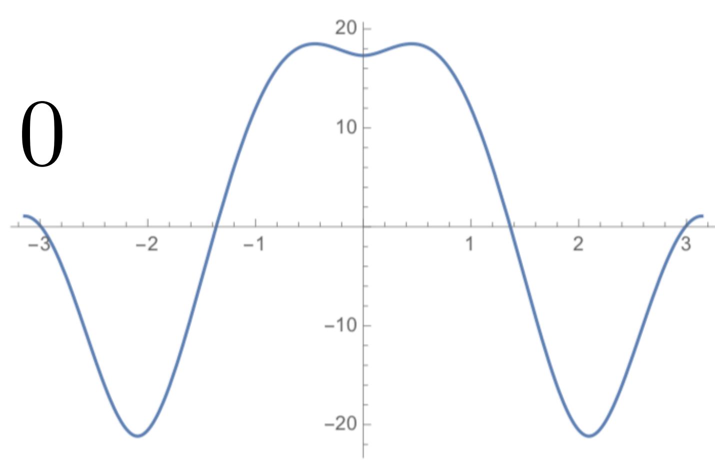
KK decomposition: (k=0 basis)

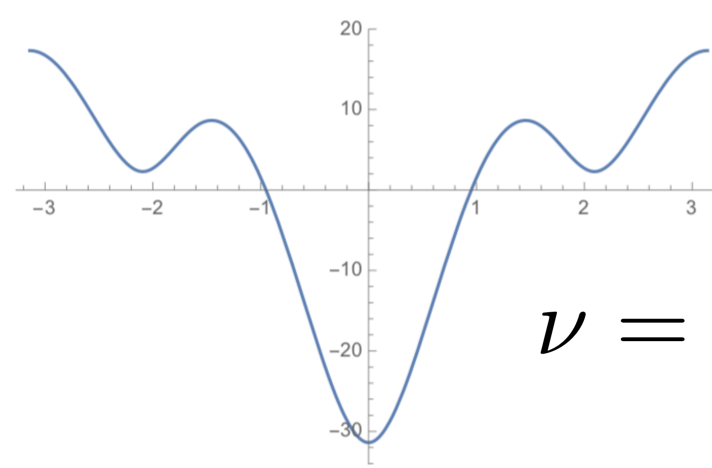
$$
\begin{aligned}
S_{\text{eff}} = \int d^3x \Big[& -\frac{1}{2g_3^2}\mathrm{Tr}\,(F_{\mu\nu}F^{\mu\nu}) \\
& + \sum_n i\bar{\psi}_n \gamma^\mu(\partial_\mu - iA_\mu)\psi_n + \sum_n i\bar{\tilde{\psi}}_n \gamma^\mu(\partial_\mu - iA_\mu)\tilde{\psi}_n \\
& - \sum_n \left(-\frac{m_L}{R} - \frac{n}{R} - \frac{\nu}{2\pi R}\right)\bar{\psi}_n\psi_n \\
& - \sum_n \left(\frac{m_R}{R} + \frac{n}{R} + \frac{\nu}{2\pi R}\right)\bar{\tilde{\psi}}_n\tilde{\psi}_n \\
& + \frac{1}{2g_3^2}\mathrm{Tr}(D_\mu A_3 D^\mu A_3) - (\bar{\psi}A_3\psi - \bar{\tilde{\psi}}A_3\tilde{\psi}) - V(A_3)\Big].
\end{aligned}
$$

An example
of the potential for A_3 ($N=3$, $N_f=2$)

$$A_3 = \frac{1}{2\pi R}\begin{pmatrix} \xi & 0 & 0 \\ 0 & \xi & 0 \\ 0 & 0 & -2\xi \end{pmatrix}$$

$\nu = 0$





$\nu = \pi$

[Hosotani '83]

SU(N) is unbroken while fermions obtain masses
for all the boundary conditions.

The m_L and m_R shifts the level of KK modes.

The low energy limit of the theory is

$SU(N)_{\bar{k}}$.

$\updownarrow$

$U(\bar{k})_{-N}$

Large radius

Chiral Lagrangian with the θ term:

$$U = e^{2i\pi^a T^a + 2i\eta}$$

$$S_{\text{eff}} = \int d^3x \int_0^{2\pi R} dx_3 \times \left[\frac{f_\pi^2}{4} \text{Tr}|\partial_\mu U|^2 - \frac{m_\eta^2 f_\pi^2}{4N_f} \left| \log(e^{-i\bar{\theta}} \det U) \right|^2 + \cdots \right].$$

Eq. of motion for η

$$\eta'' = m_\eta^2 \left(\eta - \frac{\bar{\theta}}{2N_f} \right)$$

η gets winding! $\eta(x_3 + 2\pi R) = \eta(x_3) + \frac{\bar{k}\pi}{N_f}$

(For $\bar{k}/N_f$ non-integer, pions also get winding.)

4d WZW term

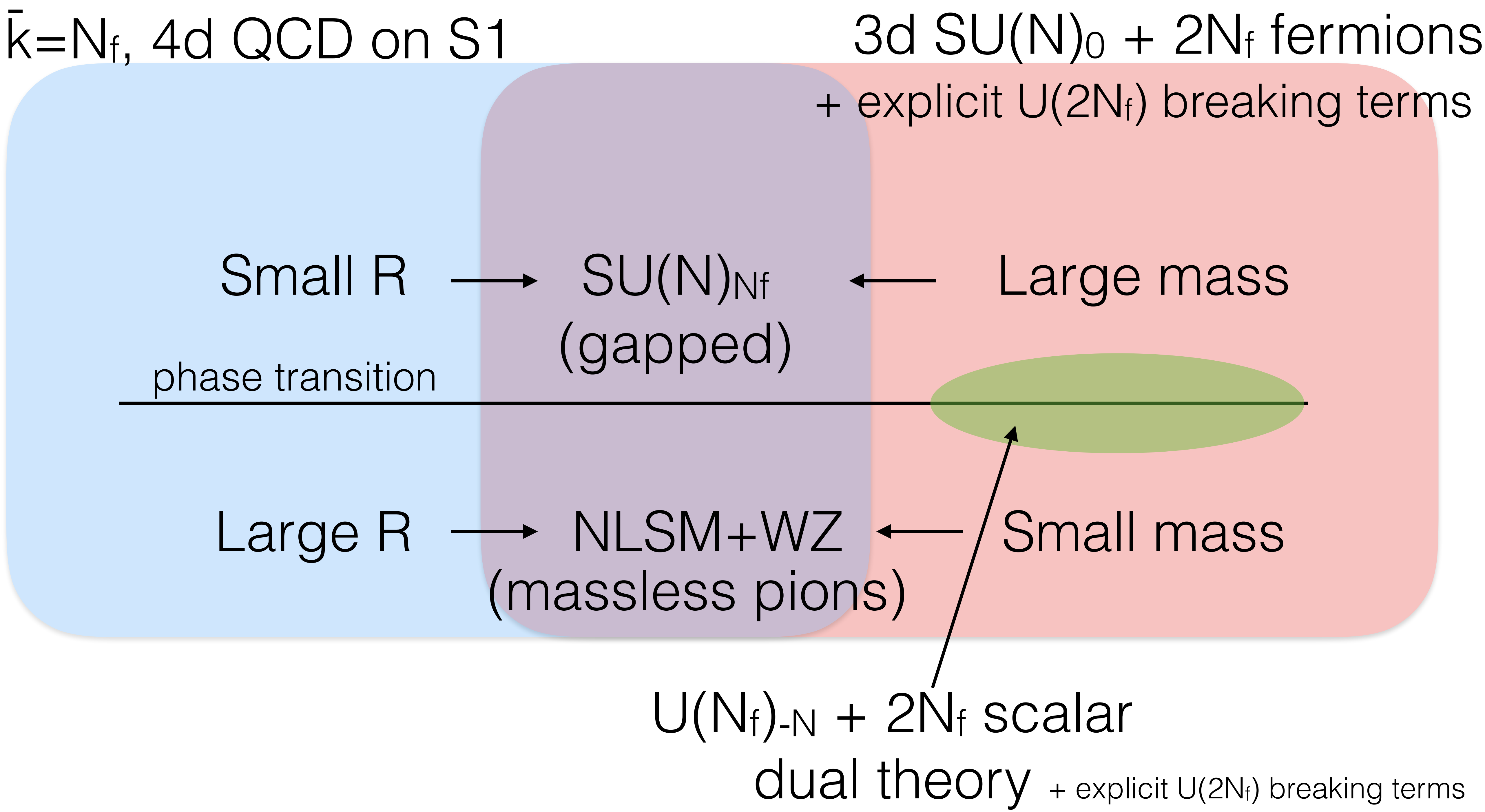
There are WZW terms in 4d. There are terms like:

$$S_{\mathrm{WZW}} = -\frac{N}{4\pi^2}\int_{M^3\times S^1} \mathrm{Tr}\left(A_{\mathrm{ext}}dA_{\mathrm{ext}} + \frac{2}{3}A_{\mathrm{ext}}^3\right) d\eta$$

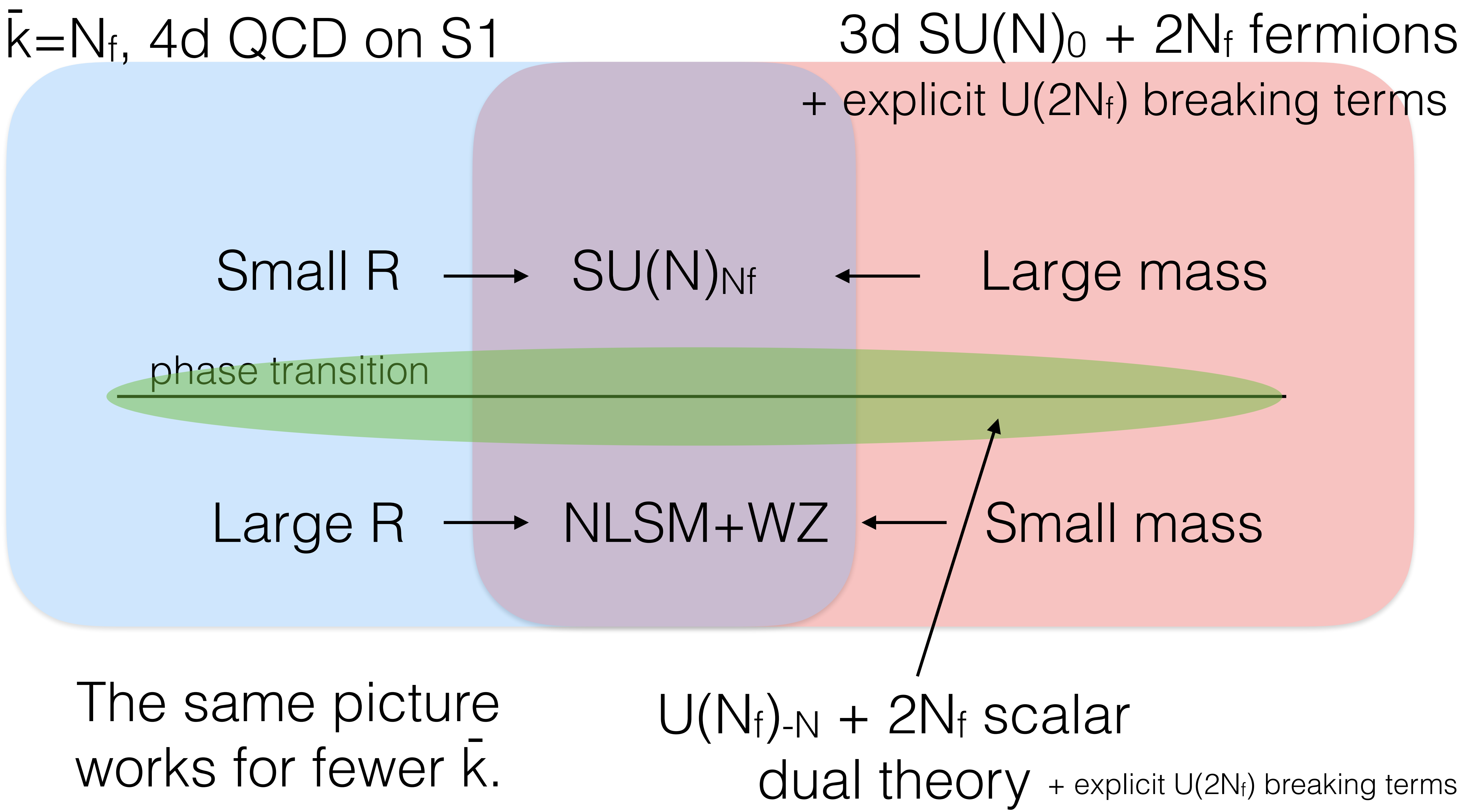
Under the winding, we obtain the 3d WZ terms.

Baryon = monopole (string along the S^1)
is also realized in 4d.

We found


$\bar{k}=N_f$, 4d QCD on S1
3d $SU(N)_0$ + $2N_f$ fermions
+ explicit $U(2N_f)$ breaking terms
Small R
$SU(N)_{Nf}$
(gapped)
Large mass
phase transition
Large R
NLSM+WZ
(massless pions)
Small mass
$U(N_f)_{-N}$ + $2N_f$ scalar
dual theory
+ explicit $U(2N_f)$ breaking terms

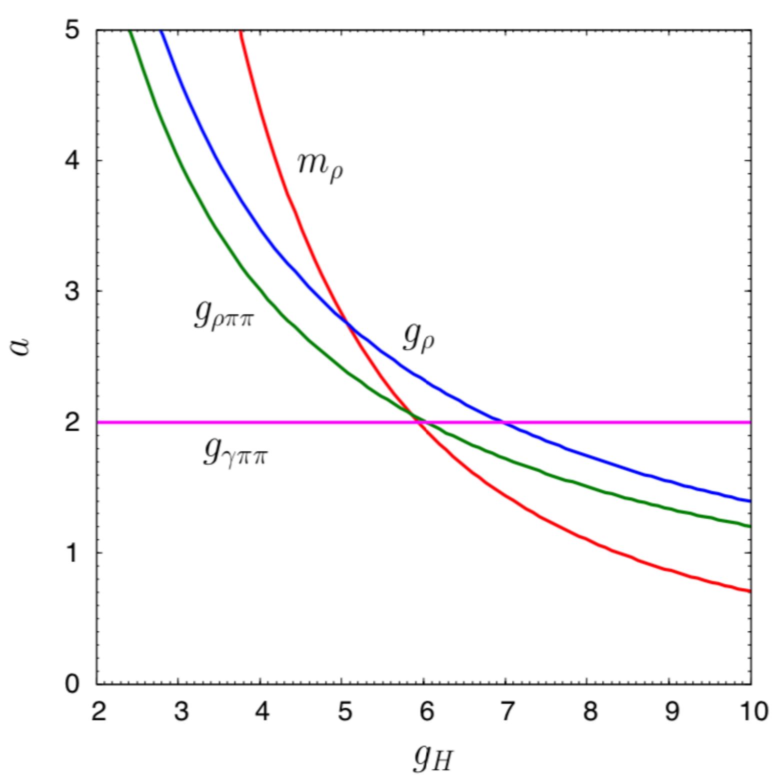
Maybe



The same picture works for fewer $\bar{k}$.

What’s interesting?

The 3d “chiral symmetry breaking” may be described by the Higgs mechanism of $U(N_f)$ gauge theory.



Actually, there is a similar picture in 4d QCD, where the gauge bosons are the ρ and ω mesons.

Figure 1: The predictions of the hidden local symmetry. We have used $m_\rho = 776$ MeV, $f_\pi = 92.4$ MeV, $g_{\rho\pi\pi} = 6.03$, $g_\rho = (345\ \text{MeV})^2$, $g_{\gamma\pi\pi} \sim 0$. Values are taken from Ref. [3].

Vector mesons as gauge bosons?

$\rho(770)$ [h] $I^G(J^{PC}) = 1^+(1^{--})$

Mass $m = 775.26 \pm 0.25$ MeV
Full width $\Gamma = 149.1 \pm 0.8$ MeV
$\Gamma_{ee} = 7.04 \pm 0.06$ keV

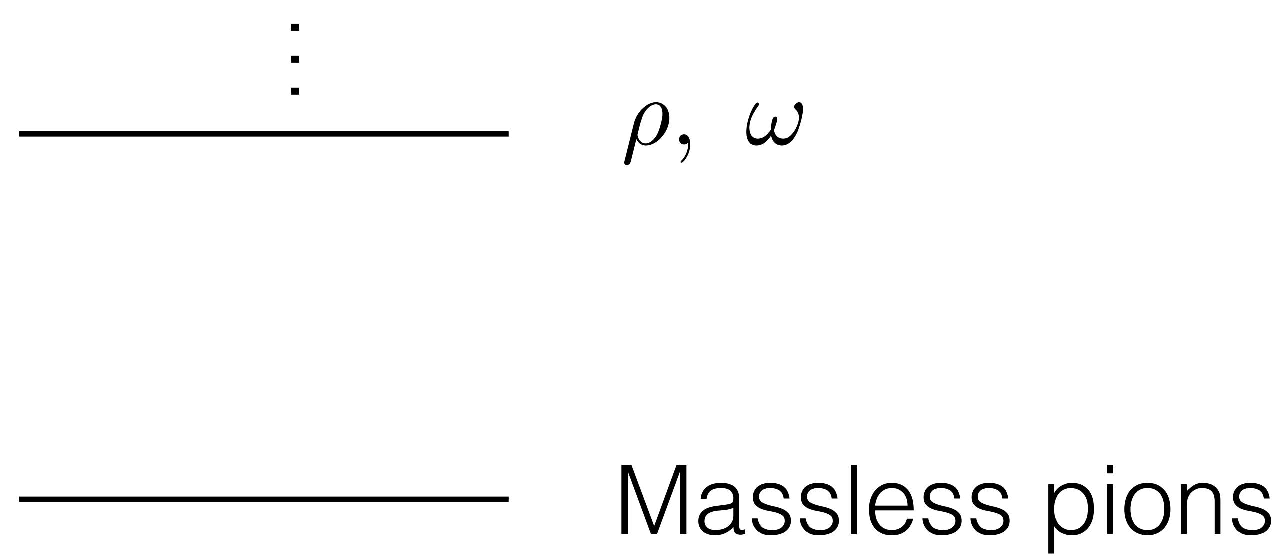
$\omega(782)$ $I^G(J^{PC}) = 0^-(1^{--})$

Mass $m = 782.65 \pm 0.12$ MeV (S = 1.9)
Full width $\Gamma = 8.49 \pm 0.08$ MeV
$\Gamma_{ee} = 0.60 \pm 0.02$ keV

Sakurai 60's
Bando, Kugo, Uehara,Yamawaki, Yanagida '85
Son, Stephanov '03
Sakai, Sugimoto '04
...

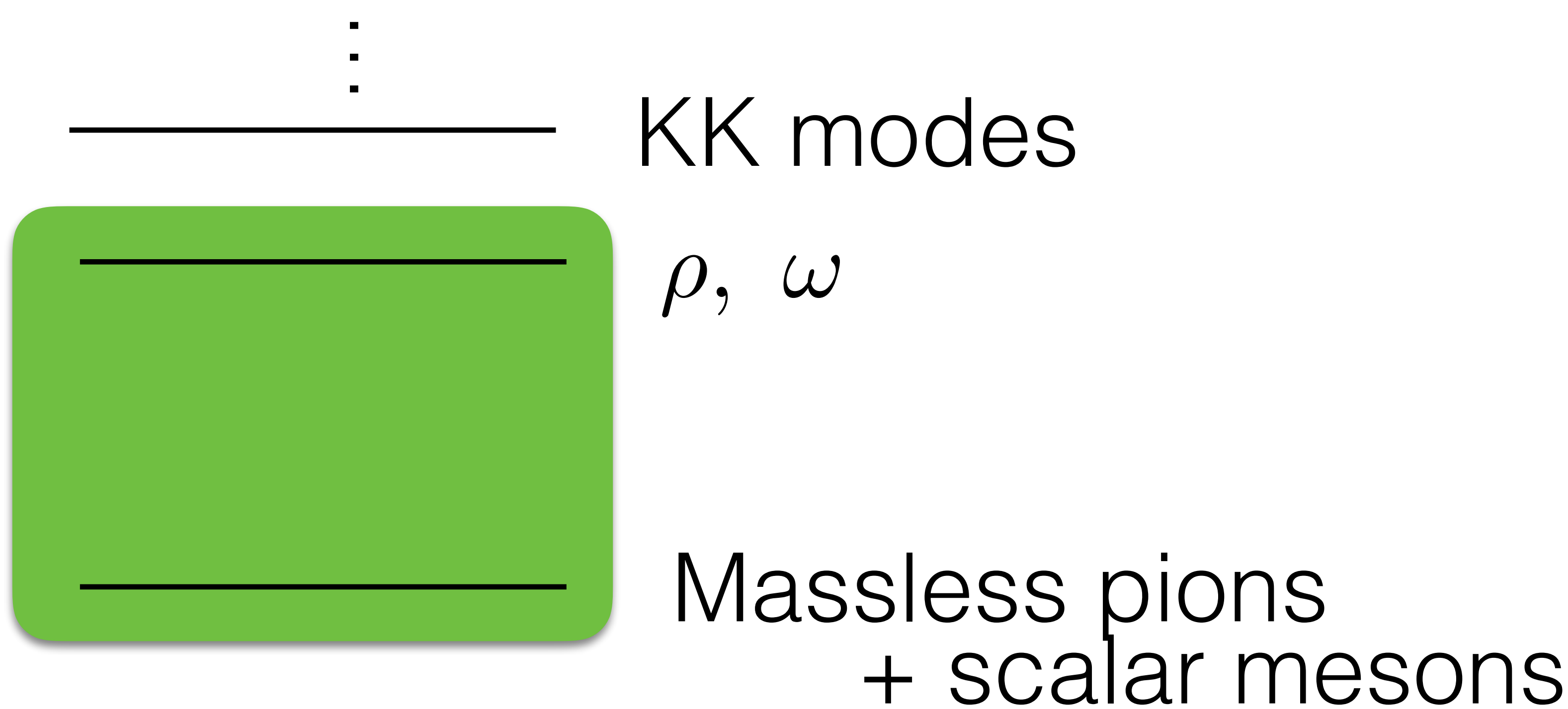
Behavior of Hadrons?

Large radius (4d)



Behavior of Hadrons?

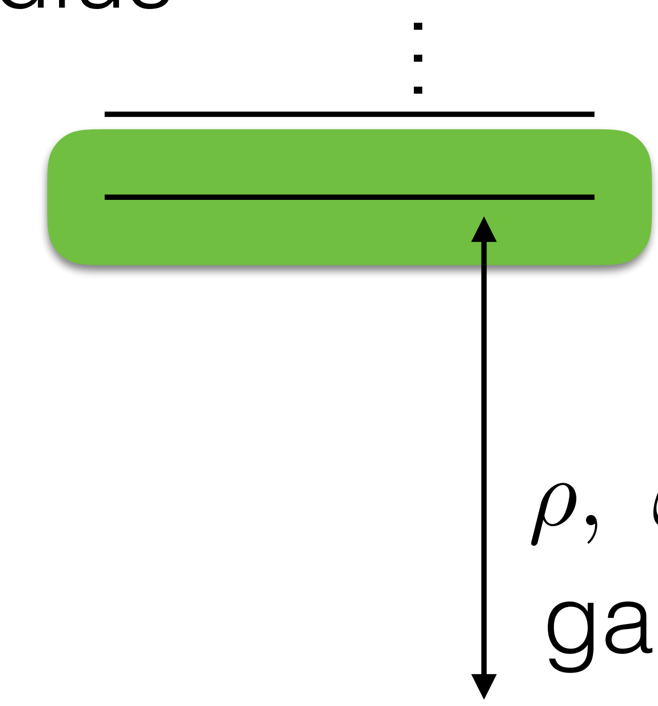
Critical radius



Form a $U(N_f)_{-N}$ gauge theory
+ $2N_f$ scalars

Behavior of Hadrons?

Small radius

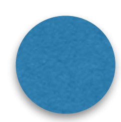


$U(N_f)_{-N}$ theory $\longleftrightarrow$ $SU(N)_{N_f}$

Gluons are replaced by the $\rho, \ \omega$ mesons!

Baryons

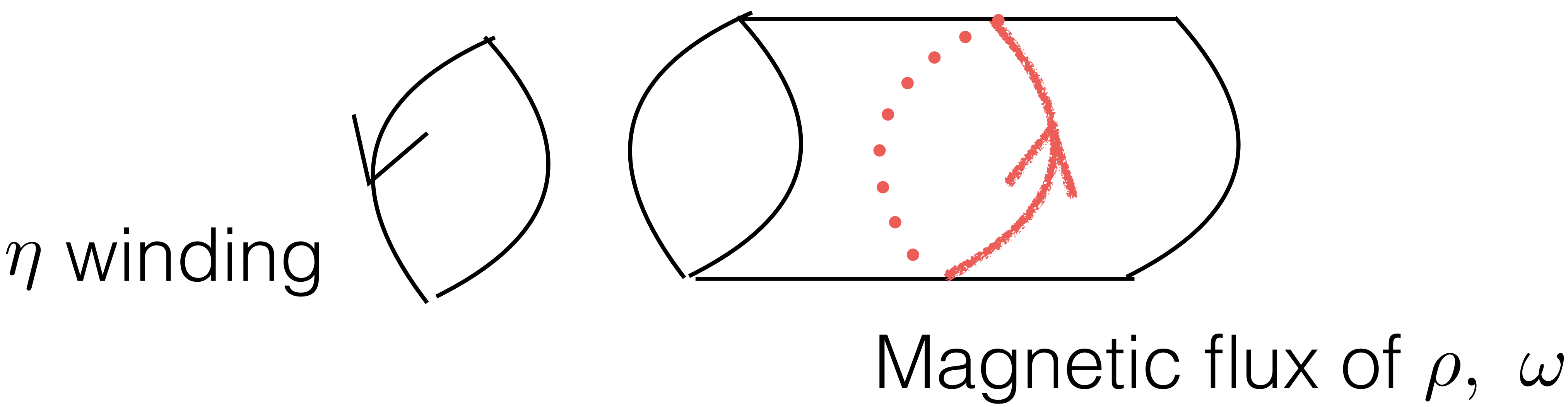
Large radius



Baryon number is carried by baryons.

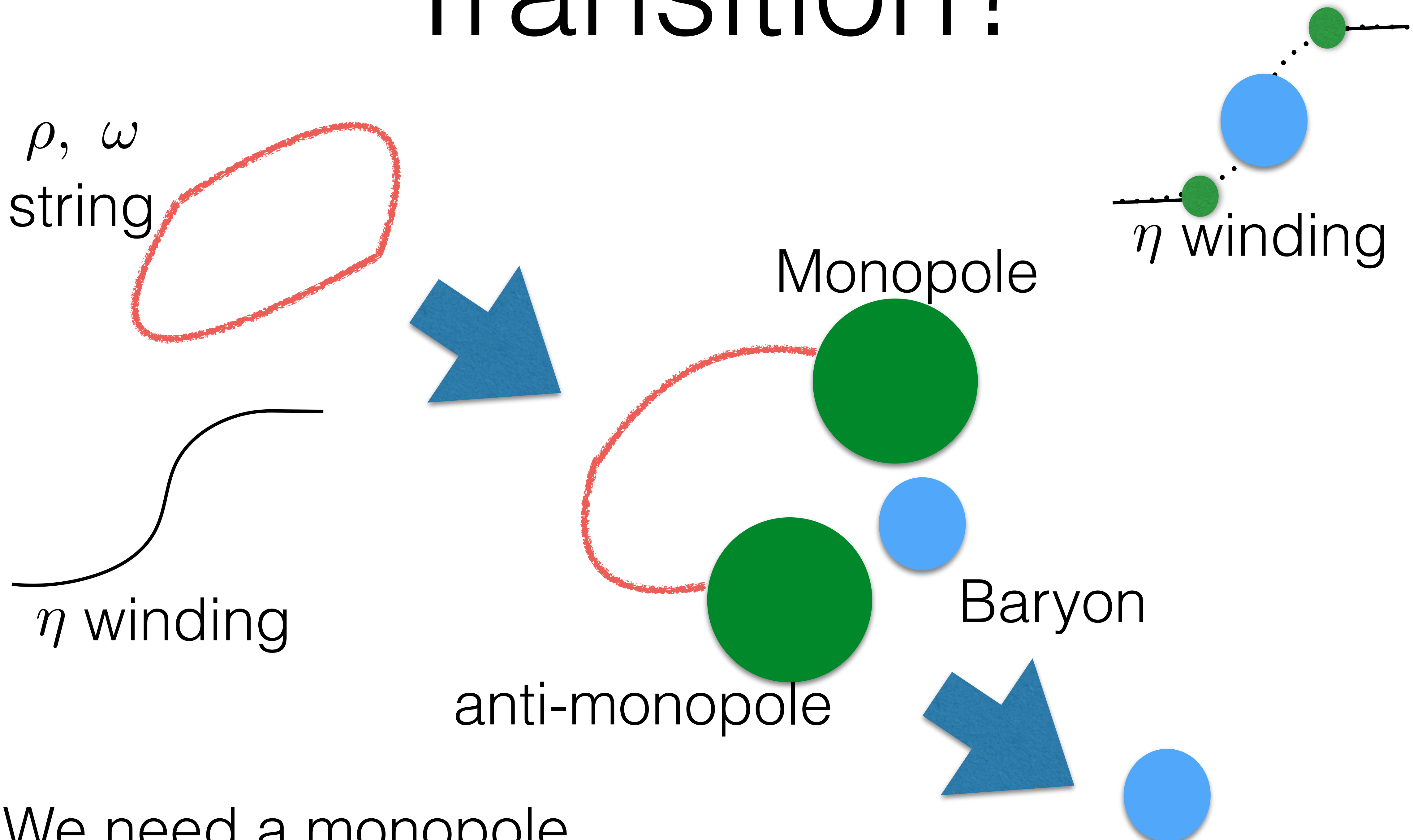
Baryons

Near the critical radius



Baryon number is also carried by vortex strings of $\rho,\ \omega$.

Transition?



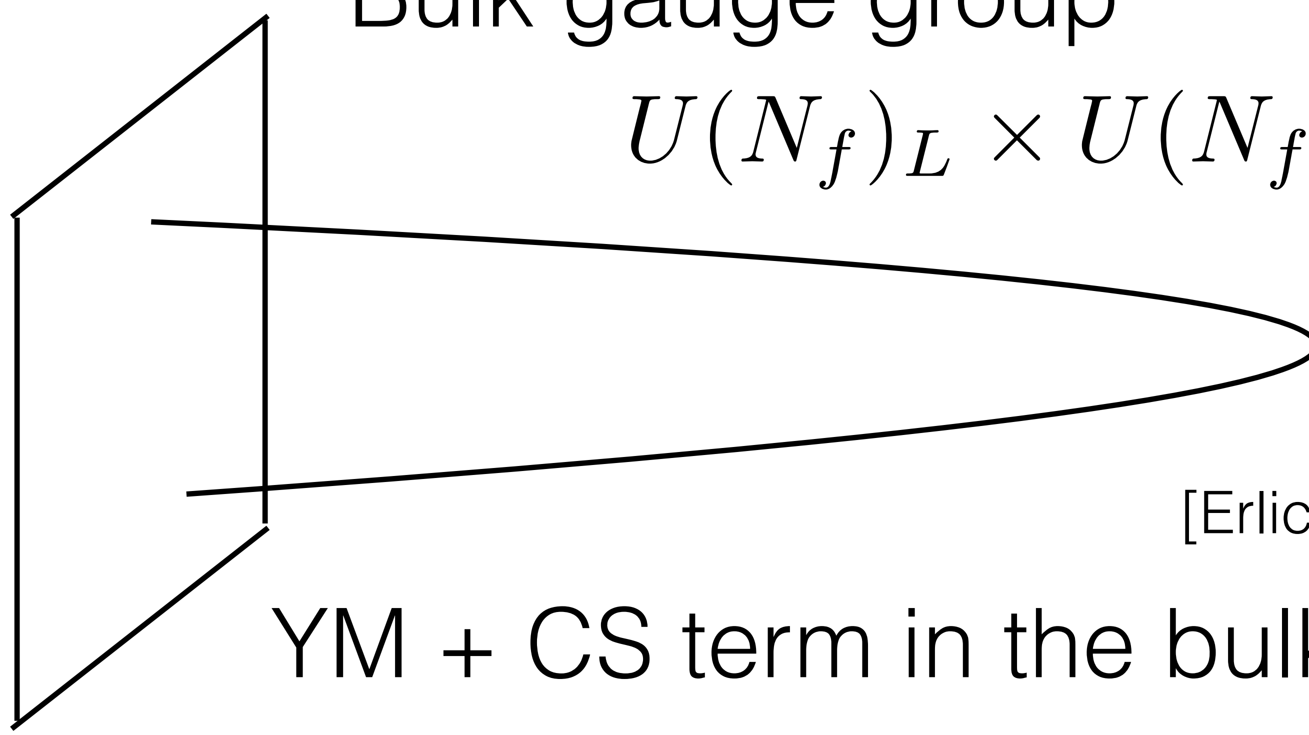
We need a monopole
to cut the string.

There is a good candidate
'Hall droplet' [Komargodski '18]

4d (or 5d) dual theory?

Bulk gauge group

$U(N_f)_L \times U(N_f)_R$



YM + CS term in the bulk

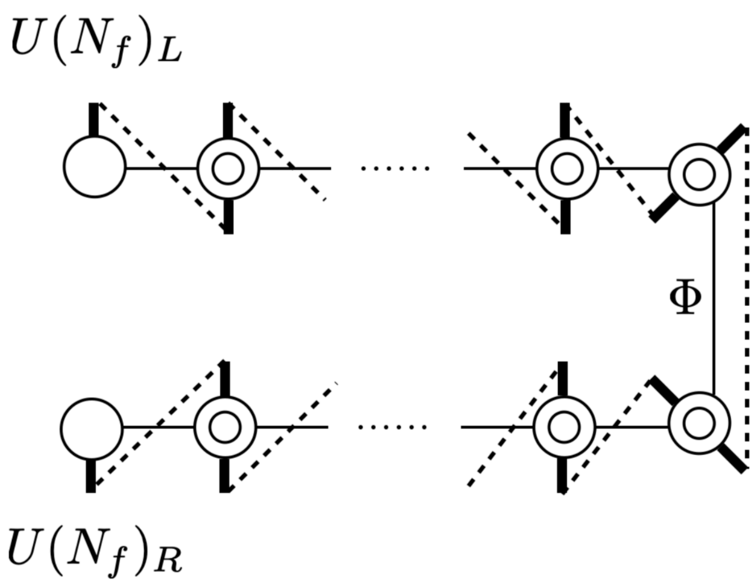
[Sakai, Sugimoto '04]
[Erlich, Katz, Son, Stephanov '05]
[Da Rold, Pomarol '05]

Natural candidate is the holographic QCD.

We know that this model gives the chiral Lagrangian at low energy with the correct WZW term.

$\rho, \ \omega$ are gauge bosons.

Deconstructed version

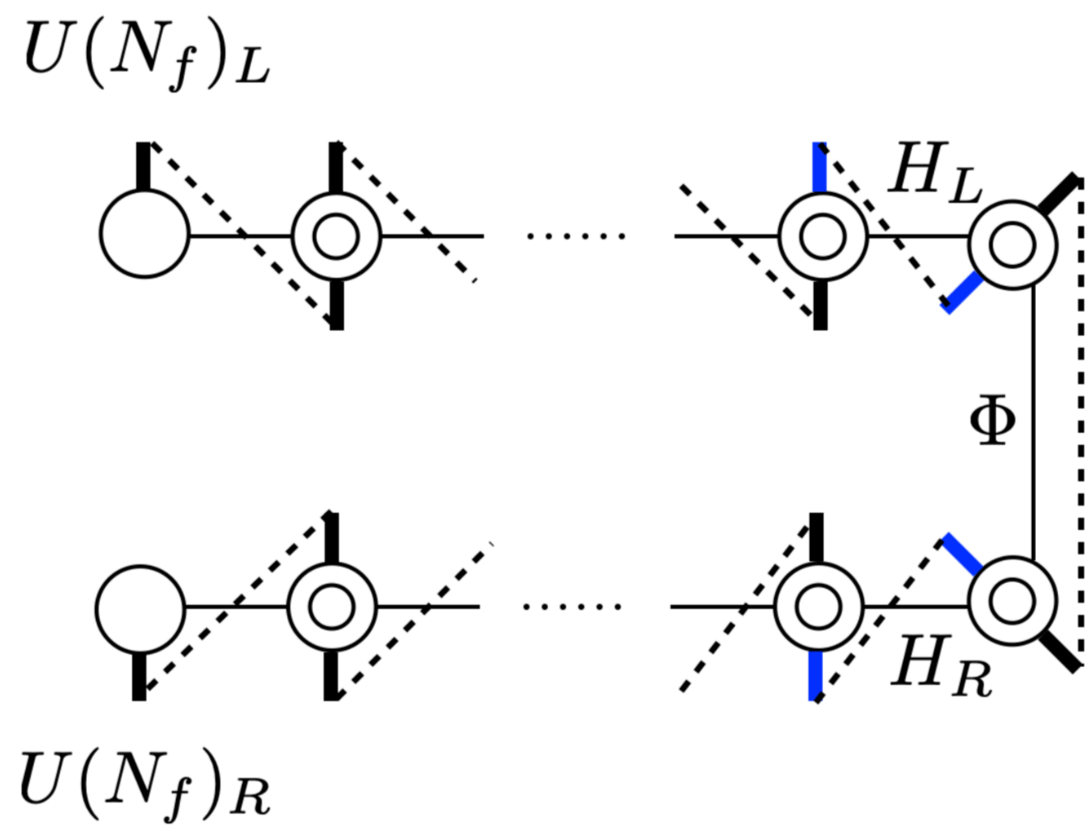


[Son, Stephanov '03]

near the
critical radius

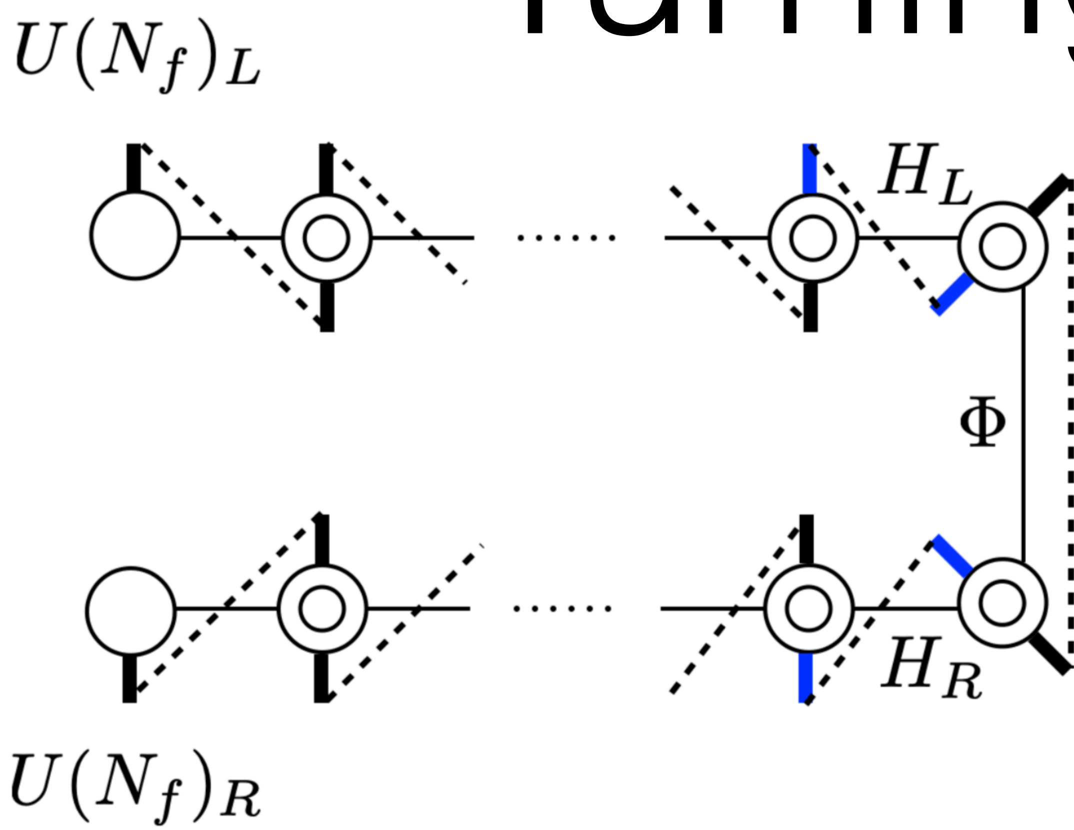
When two of the links are cut,
we obtain unbroken $U(N_f)$.

We have N fermions to
reproduce 5d CS term.



(We need to gauge Z_N
to have $U(1)_B/Z_N$ as global sym.)

Turning on $\bar{\theta}$



We have a term

$$|\log e^{-i\bar{\theta}} \det(\Phi)|^2$$

5d CS term: $$S_{\text{bulk}} = -\frac{N}{8\pi^2}\int_{M^3\times S^1} \text{Tr}\left(\mathcal{A}d\mathcal{A} + \frac{2}{3}\mathcal{A}^3\right) d(-i\log\det\Phi)/N_f$$

By the same story as the eta winding,
we obtain the 3d CS term at low energy.

This model can describe the symmetric phase with the correct low energy limit.
(fermions decouple by holonomy)

$U(N_f)_{-N}$

What are you talking about?

1. In QCD, there must be a phase transition between the large and small radius of S^1.
2. The 3d $U(N_f)_{-N}$ + $2N_f$ scalar theory consistently describes the phase transition.
3. The holographic picture of QCD can be the origin of the $U(N_f)$ group.

Actually the discussion works for $|\bar{k}|<N_f$.

$\bar{k}=0$?

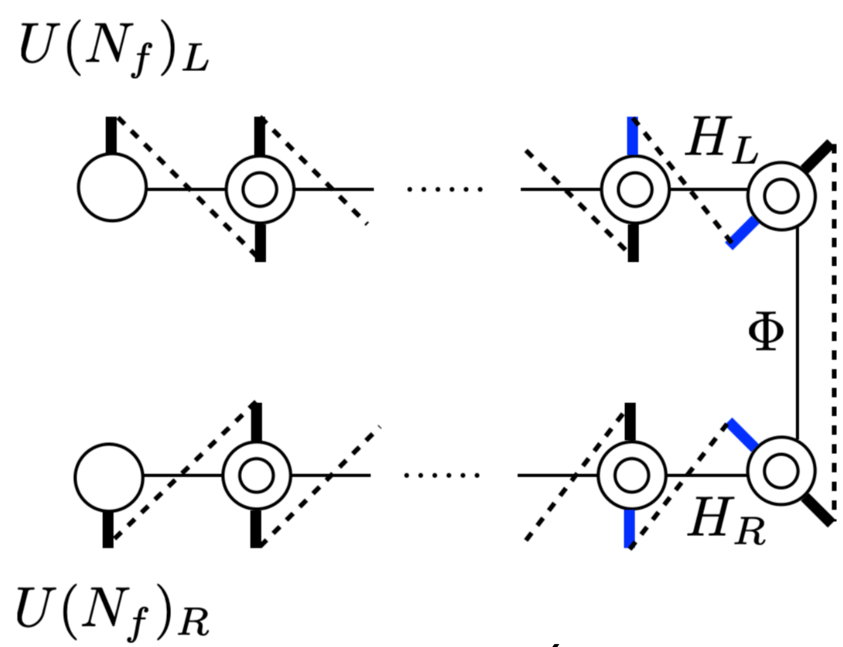
So far, I was talking about QCD on funny backgrounds.
But for $\bar{k}=0$, with the anti-periodic boundary condition,
the theory is the same as the finite temperature QCD.

If my picture is correct,

1. The ρ, ω mesons get massless(!) (in 4d) at the critical temperature of the chiral phase transition, and
2. the axial U(1) should not be restored at the critical temperature.

These items should be testable by the lattice QCD.

Summary



This model is a good candidate
of the NJL-type model
to describe the chiral phase transition.

(anomaly matches and the same low energy
limits as QCD.)

It is qualitatively different from conventional models.
Especially, there are U(N_f) gauge theory in the topological or
confining phase above the critical temperature.