Two Tales of Baryogenesis

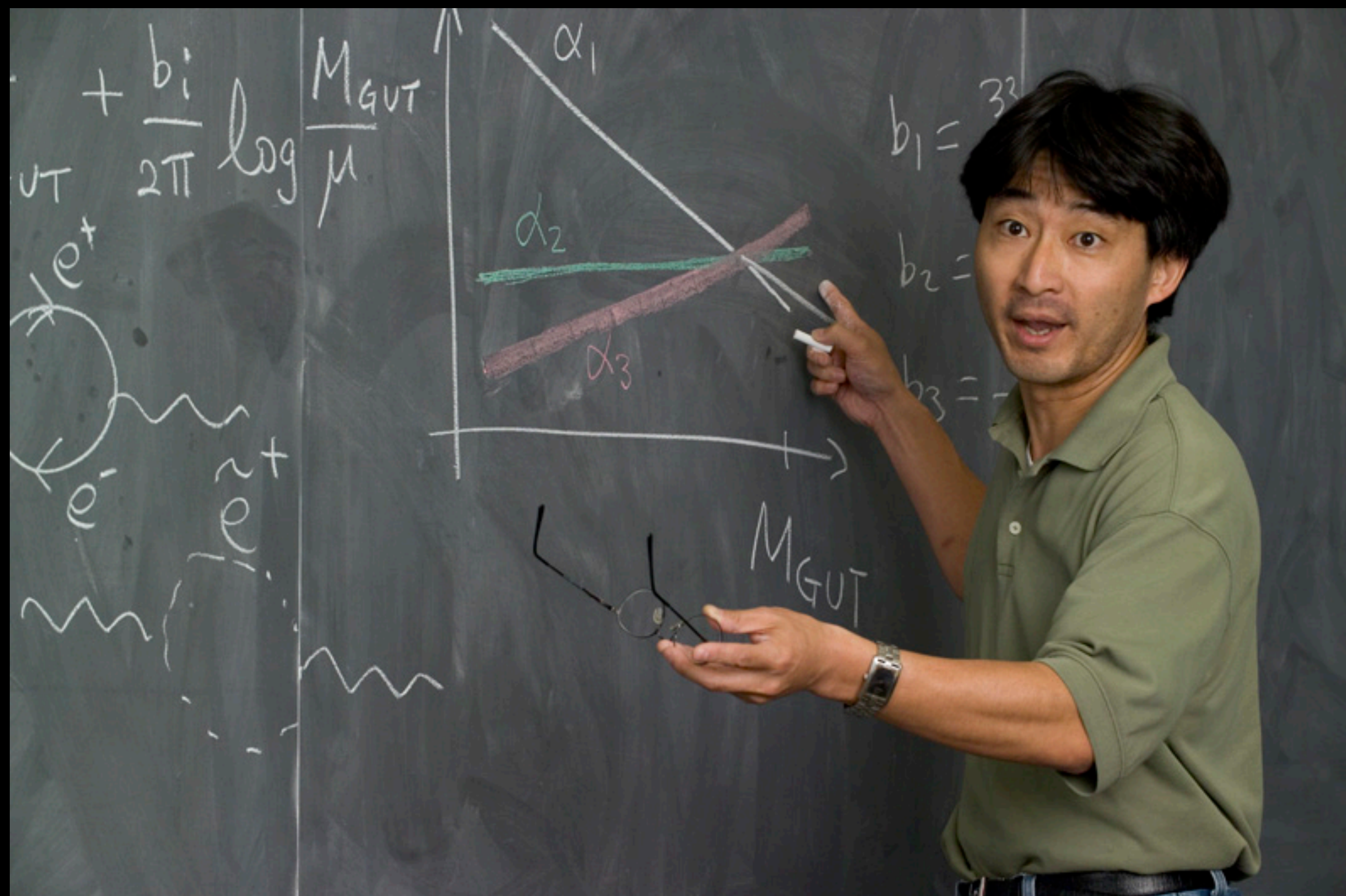
Hitoshi Murayama (Berkeley, Kavli IPMU)

UC Davis Joint Theory Seminar, Jan 27, 2020

Director of the Institute for the Physics

and Mathematics of the Universe

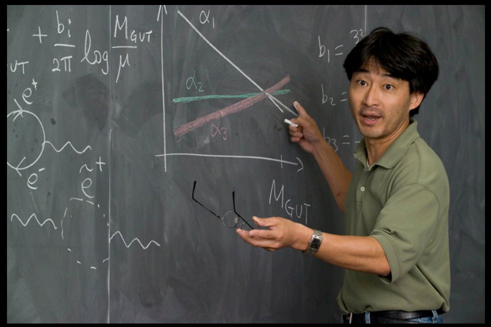
A lot of responsibility!



Director of the Universe

Not high enough authority

Voice of God

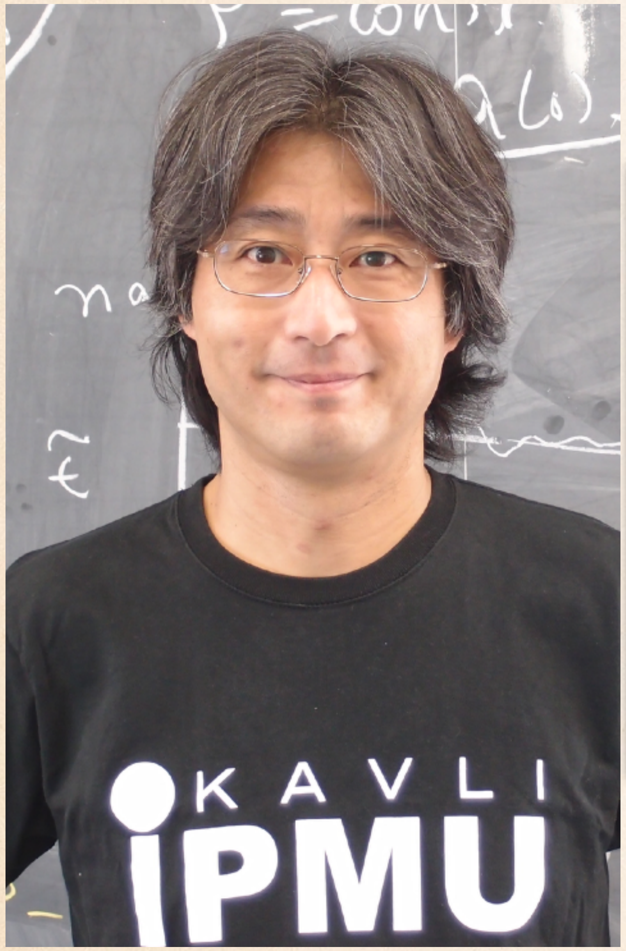


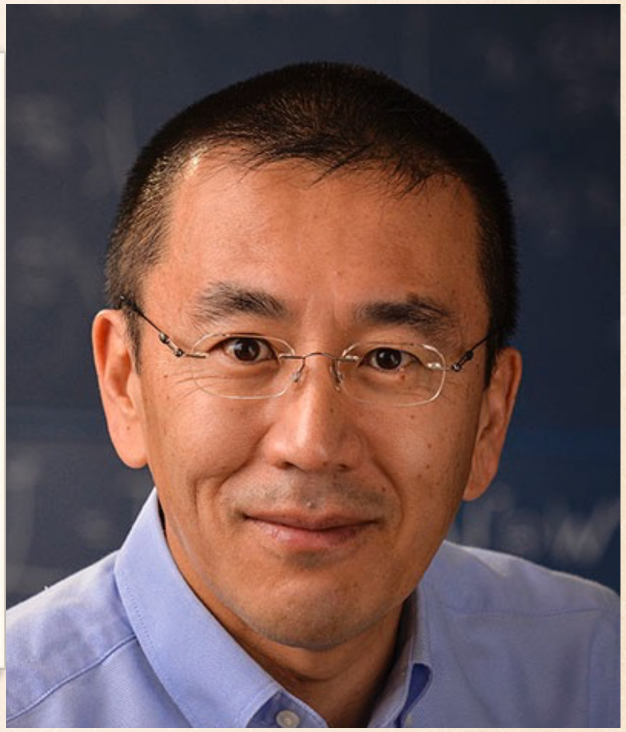
Director of the Universe

Oct 15, 2018

Kavli IPMU became officially a permanent institute on April 1, 2018

PASSING THE TORCH

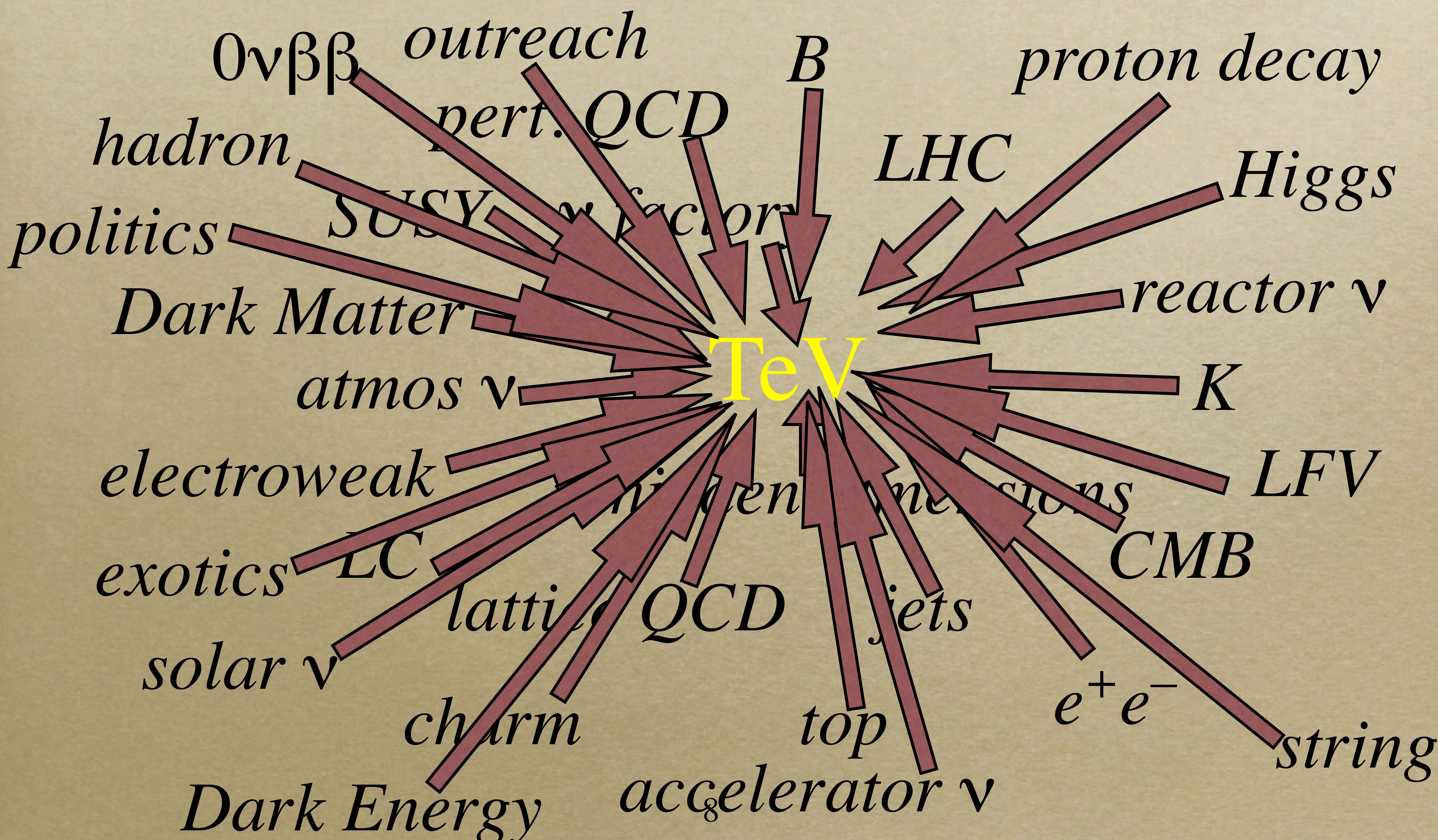


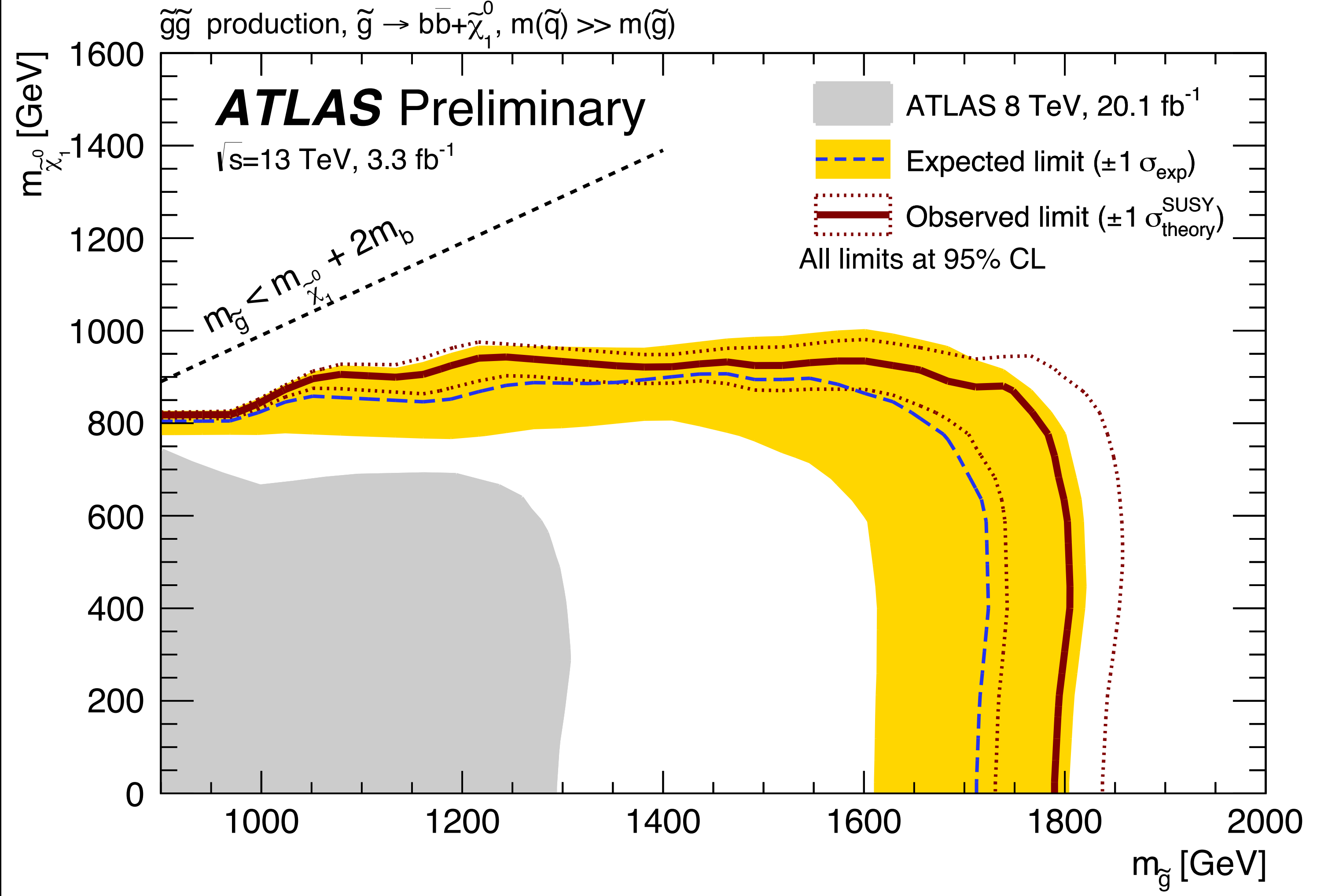


Oct 15, 2018

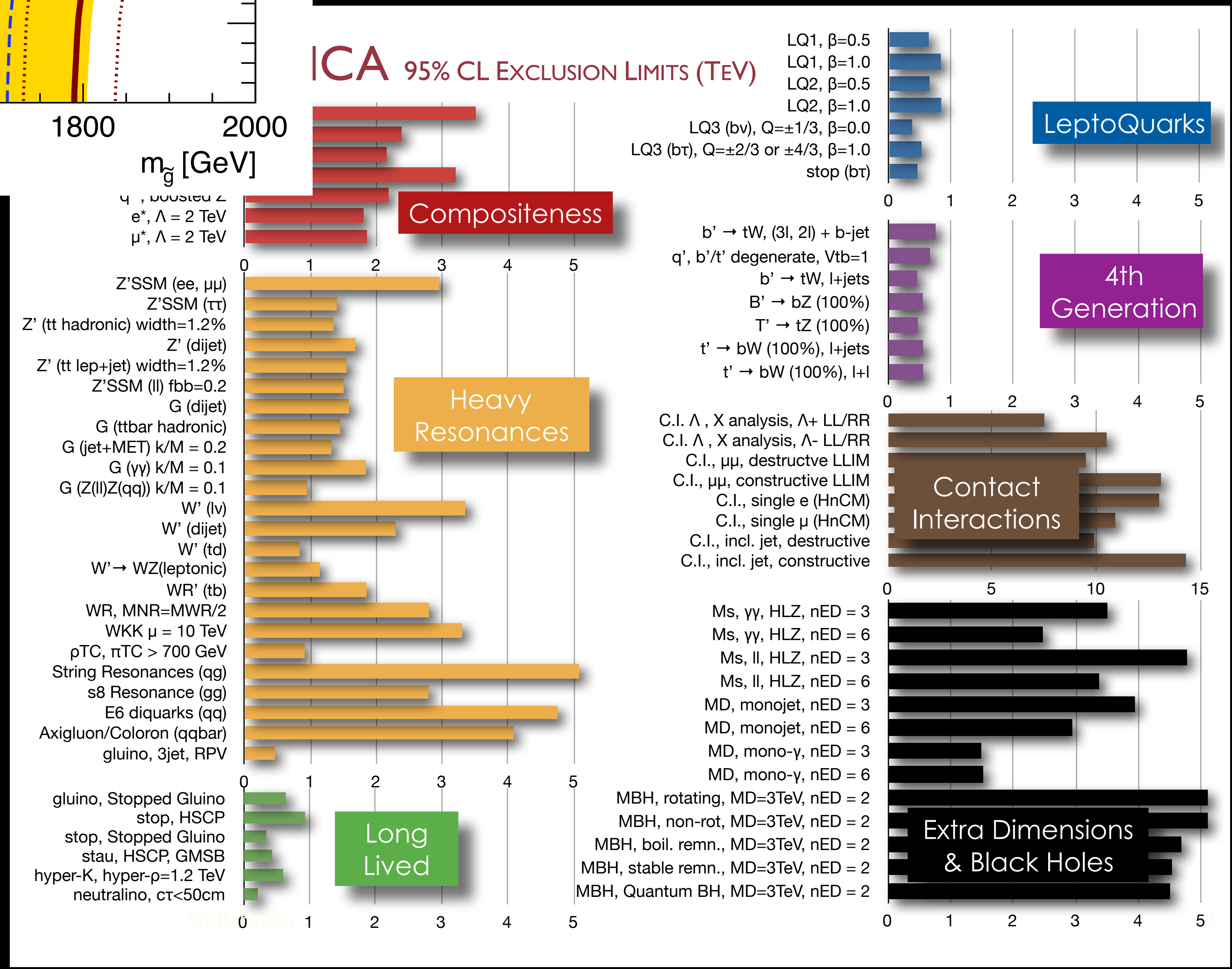

KAVLI
東京大学

Need a broad program



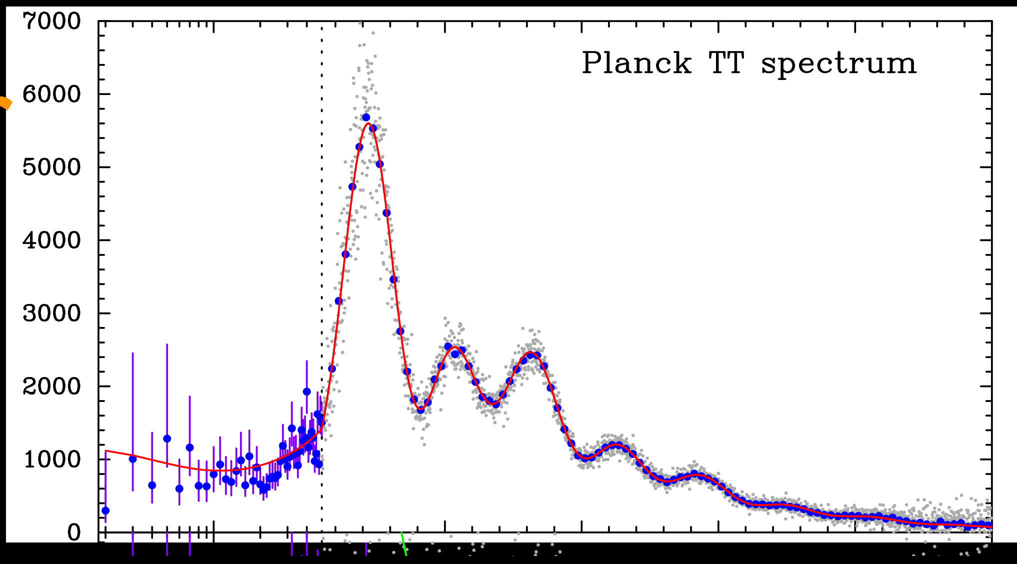


no sign of
new physics that
makes Higgs natural!



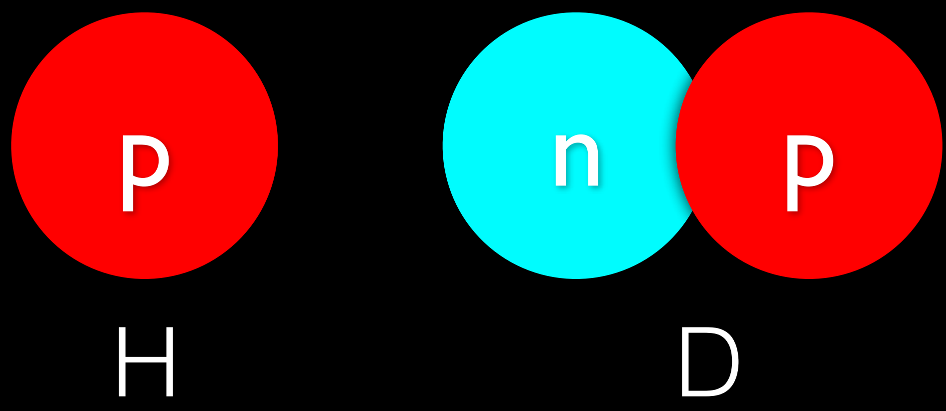
Five evidences for physics beyond SM

- Since 1998, it became clear that there are at least five missing pieces in the SM
  - non-baryonic dark matter
  - neutrino mass
  - dark energy
  - apparently acausal density fluctuations
  - baryon asymmetry

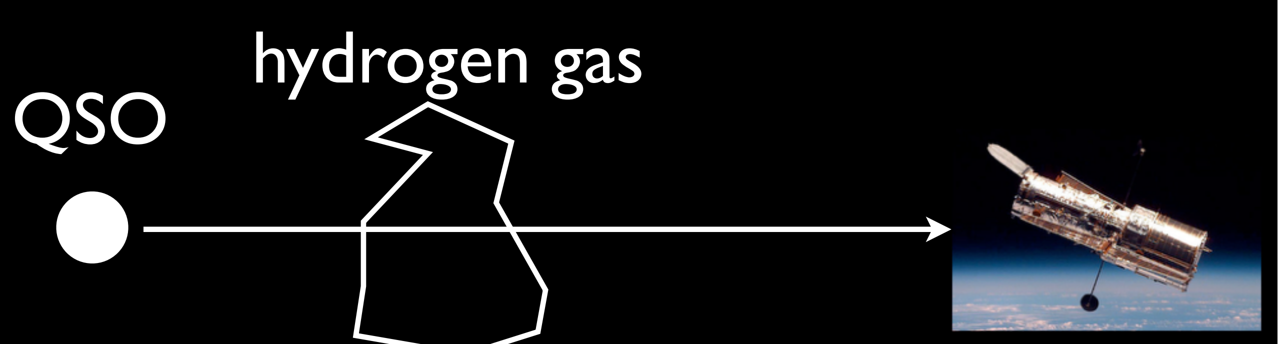


We don't really know their energy scales...

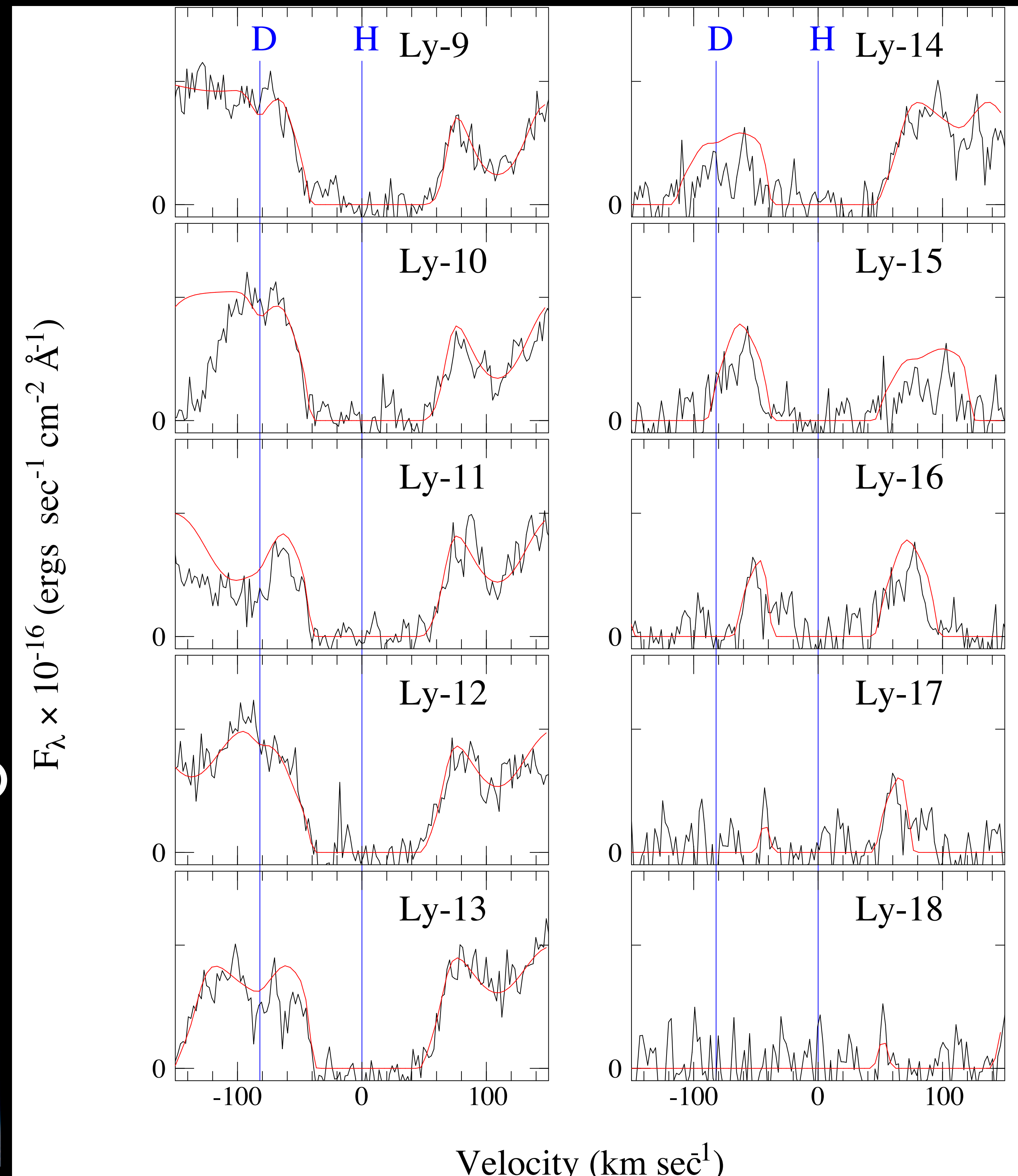
deuterium

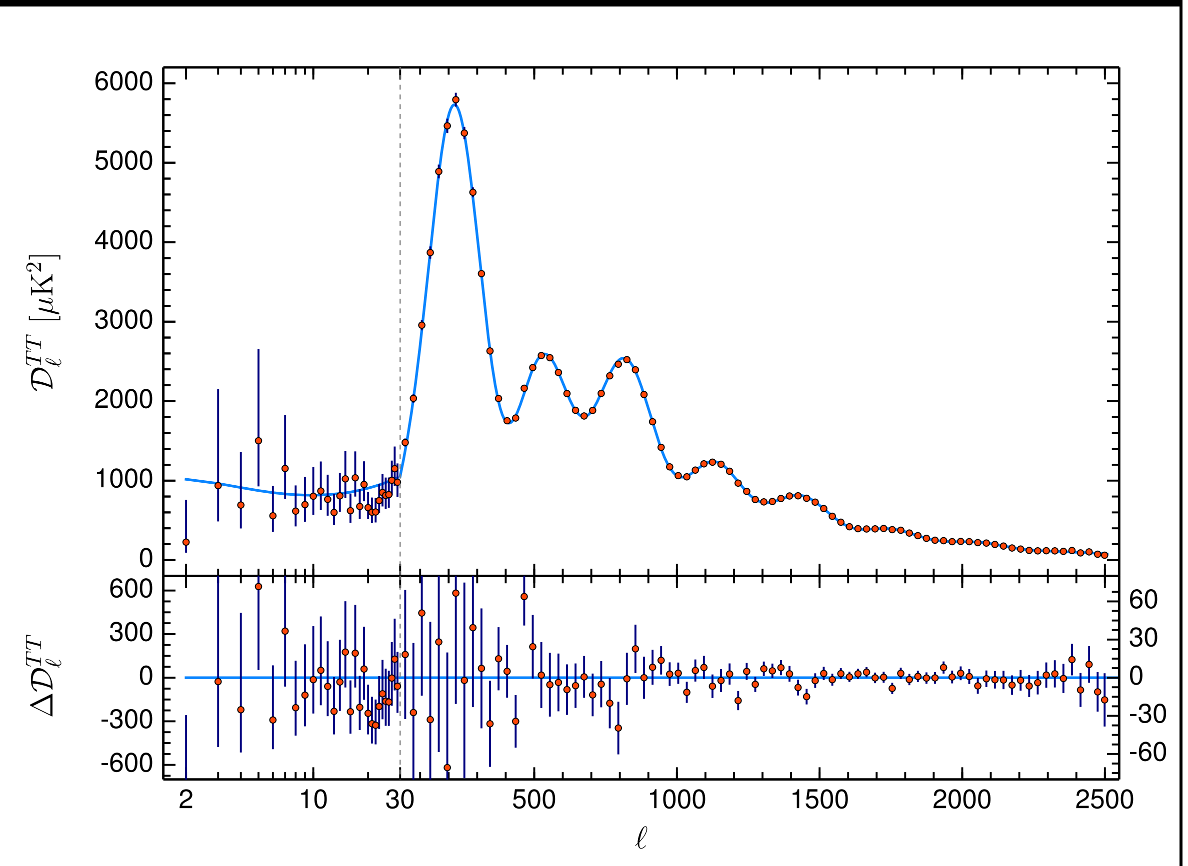


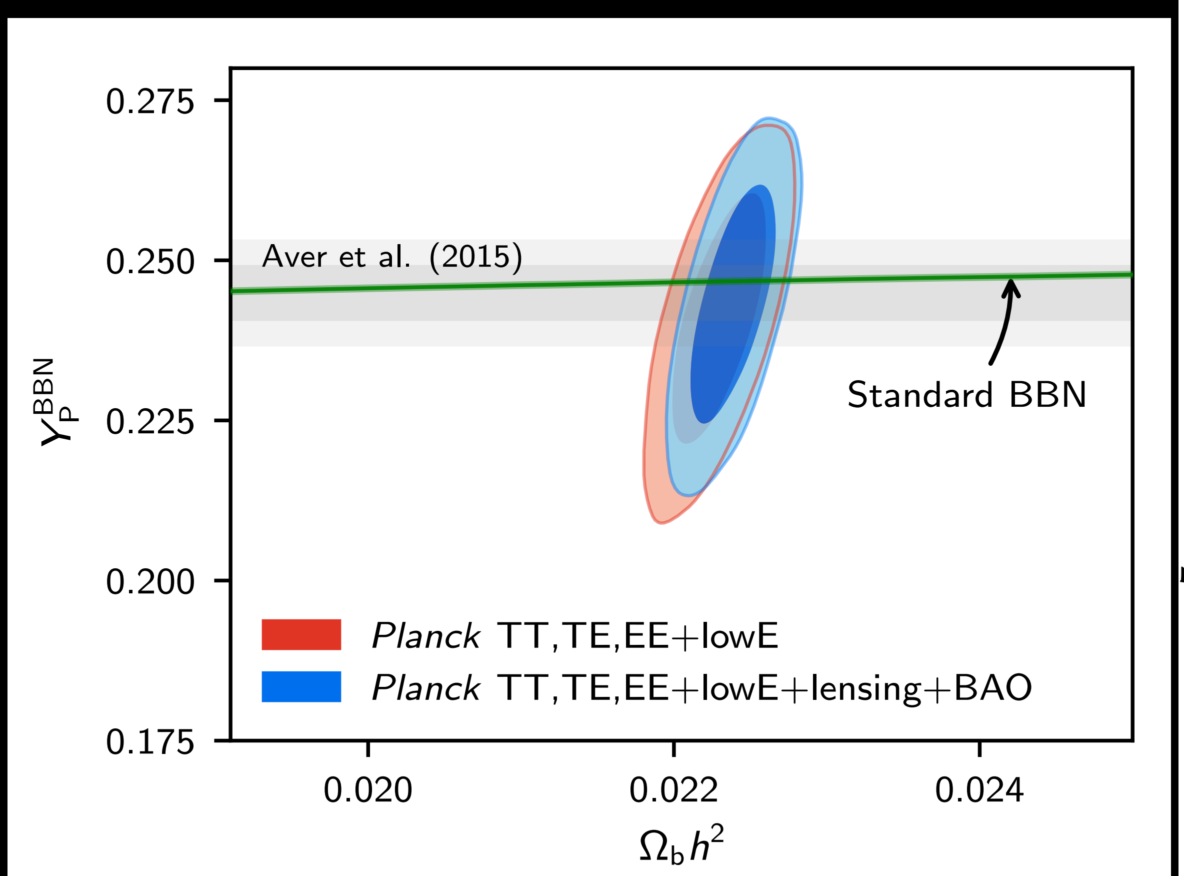
- the same chemically
- energy levels
  $E_n = -\alpha^2 \mu c^2/2$
- reduced mass differs by ~1/4000 between H & D

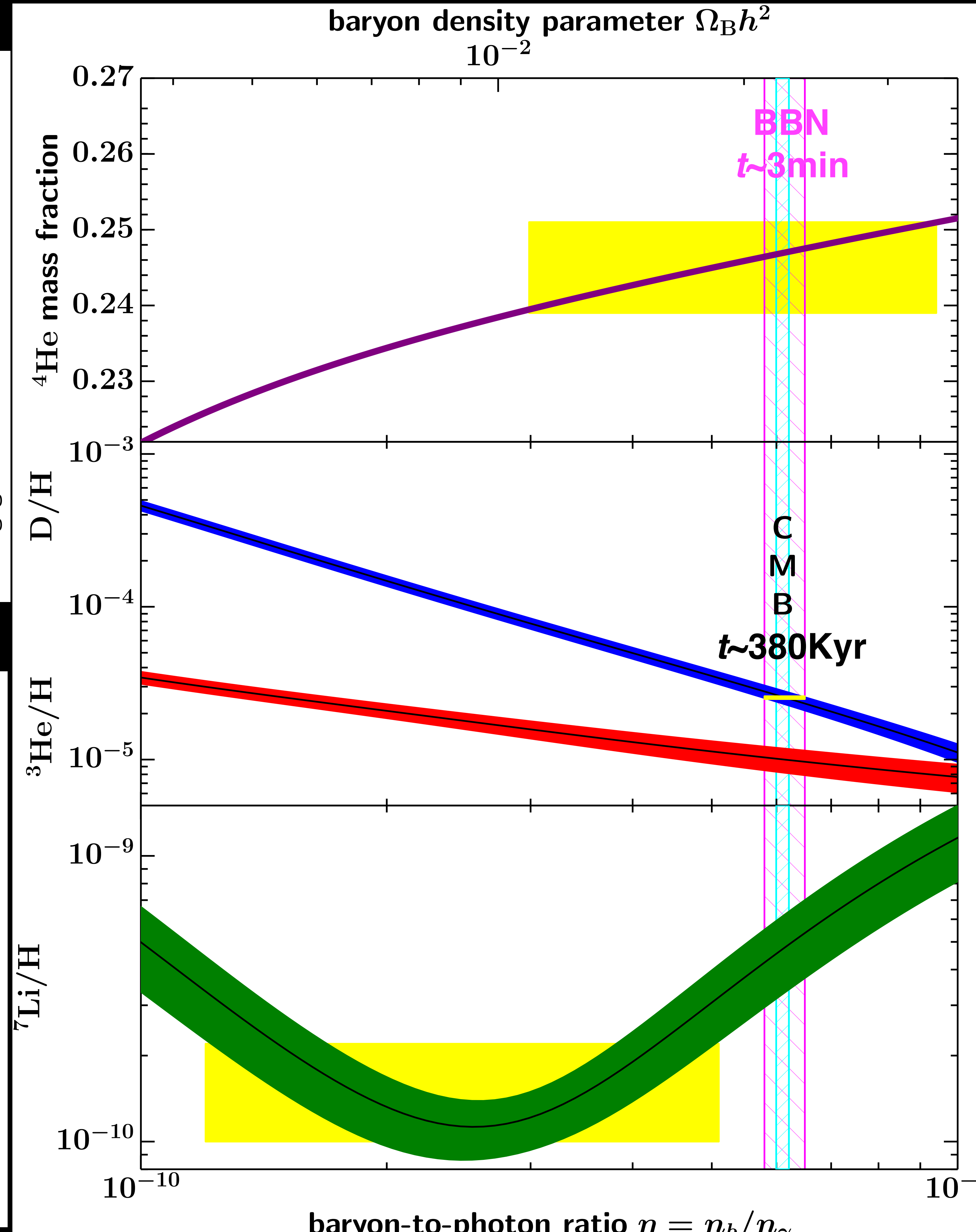


Kirkman, Tytler, Suzuki, O'Meara, Lubin




6000
5000
4000
3000
2000
1000
0
$\mathcal{D}_\ell^{TT}$ [μK^2]
600
300
0
-300
-600
$\Delta\mathcal{D}_\ell^{TT}$
60
30
0
-30
-60
2
10
30
500
1000
1500
2000
2500
ℓ


0.275
0.250
0.225
0.200
0.175
Y_P^{BBN}
Aver et al. (2015)
Standard BBN
0.020
0.022
0.024
$\Omega_b h^2$
Planck TT,TE,EE+lowE
Planck TT,TE,EE+lowE+lensing+BAO


baryon density parameter $\Omega_B h^2$
10^{-2}
0.27
0.26
0.25
0.24
0.23
^{4}He mass fraction
BBN
t~3min
10^{-3}
10^{-4}
10^{-5}
D/H
^{3}He/H
C
M
B
t~380Kyr
10^{-9}
10^{-10}
^{7}Li/H
10^{-10}
10^{-9}
baryon-to-photon ratio $\eta = n_b/n_\gamma$

Beginning of Universe

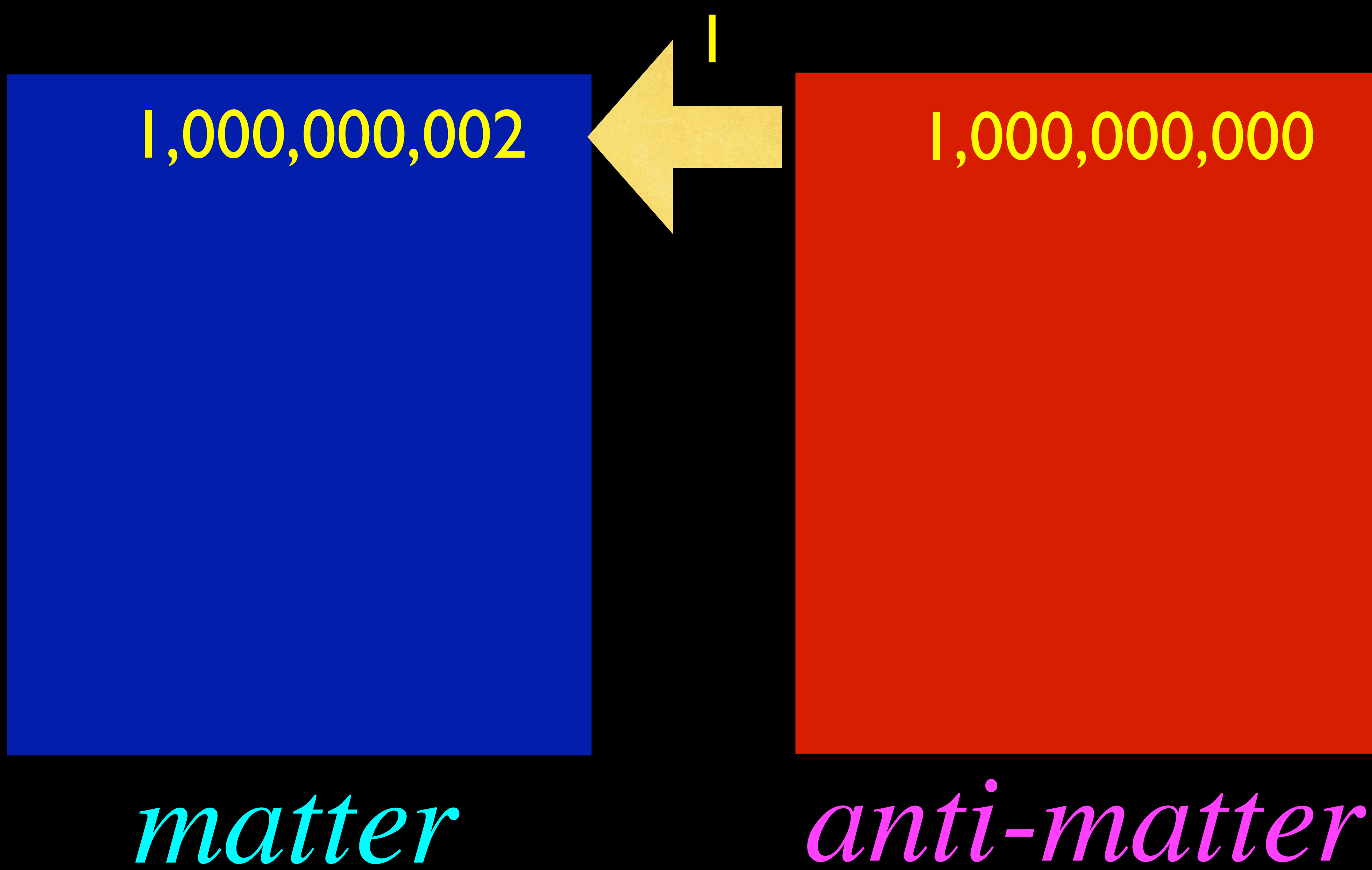
1,000,000,001

matter

1,000,000,001

anti-matter

fraction of second later



turned a billionth of anti-matter to matter

Universe Now

2

•

us

matter *anti-matter*

we were saved from the complete annihilation!


Who saved us from
a complete annihilation?

too many theories for a single number

Two tales

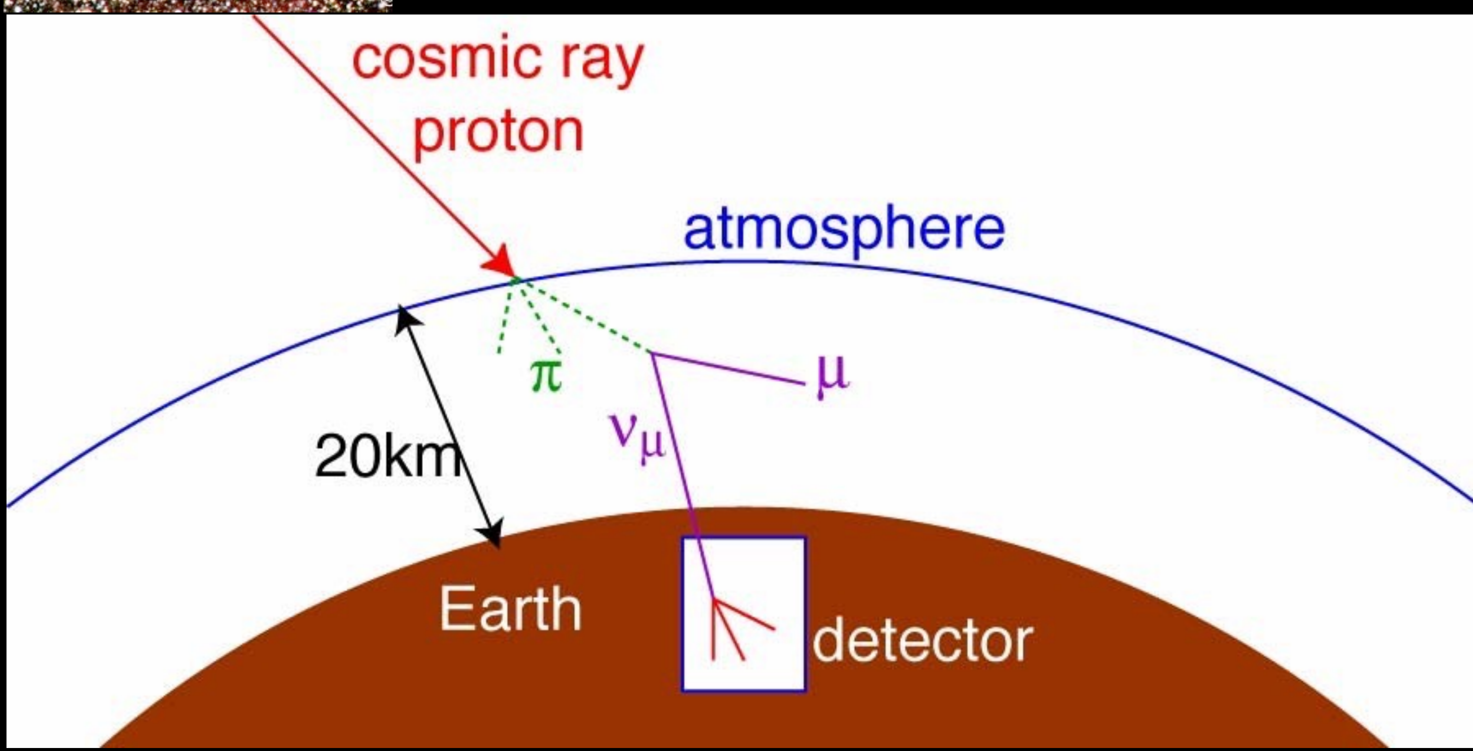
- Testing Leptogenesis with gravitational waves
  - +Jeff Dror (Berkeley), Takashi Hiramatsu (ICRR), Kazunori Kohri (KEK), Graham White (TRIUMF)
  - arXiv:1908.03227 accepted for PRL, *Editors' Suggestion*
- Asymmetric Matters from a dark first-order phase transition
  - +Eleanor Hall (Berkeley), Thomas Konstandin (DESY), Robert McGehee (Berkeley)
  - arXiv:1911.12342

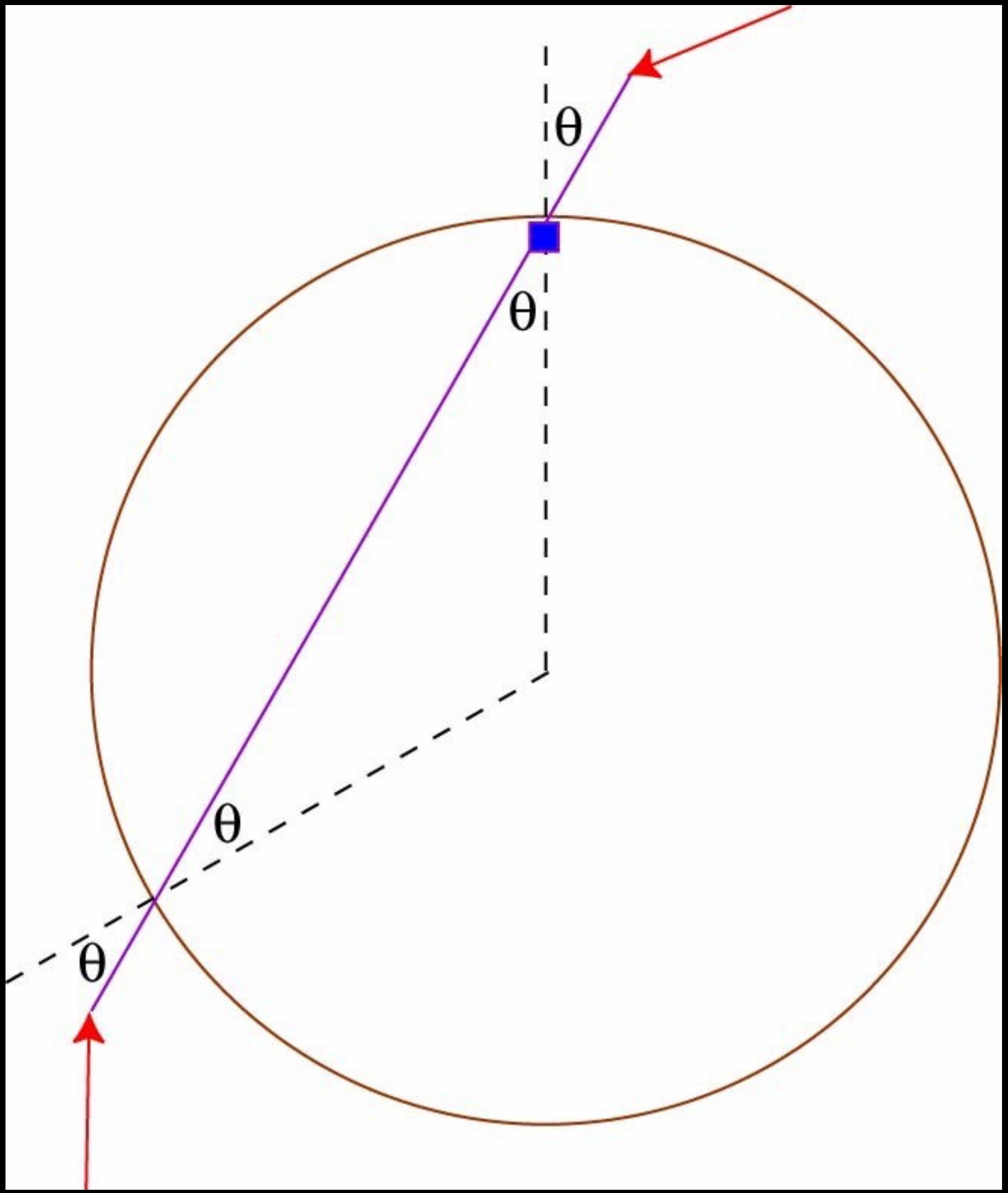
Testing seesaw and leptogenesis by gravitational wave

Hitoshi Murayama (Berkeley, Kavli IPMU)
+Jeff Dror (Berkeley), Takashi Hiramatsu (ICRR), Kazunori Kohri (KEK), Graham White (TRIUMF)

arXiv:1908.03227, accepted for PRL

Atmospheric Neutrinos



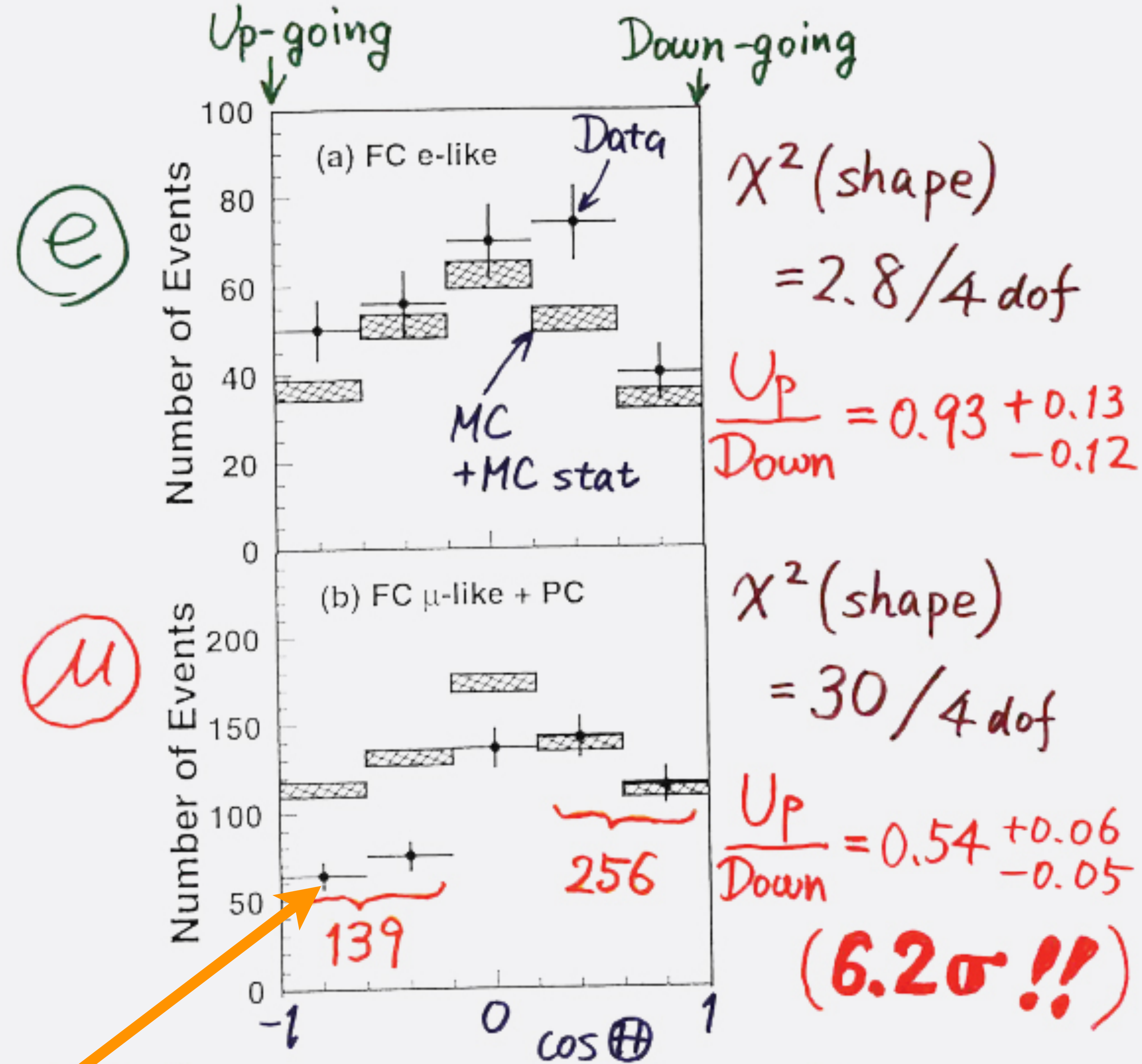


neutrinos morph

1998

a half of expected

Zenith angle dependence (Multi-GeV)

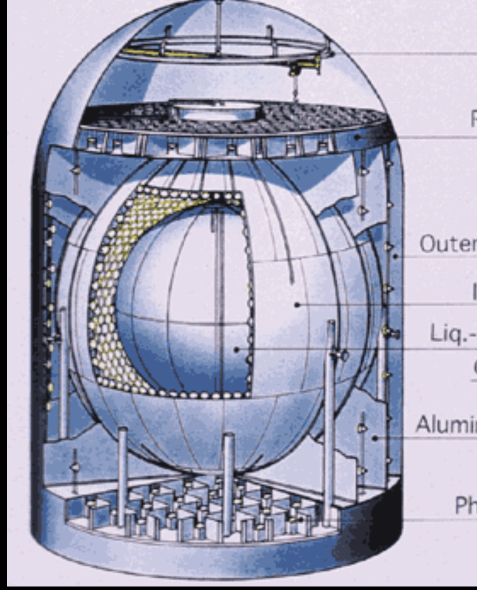


※ Up/Down syst. error for μ-like

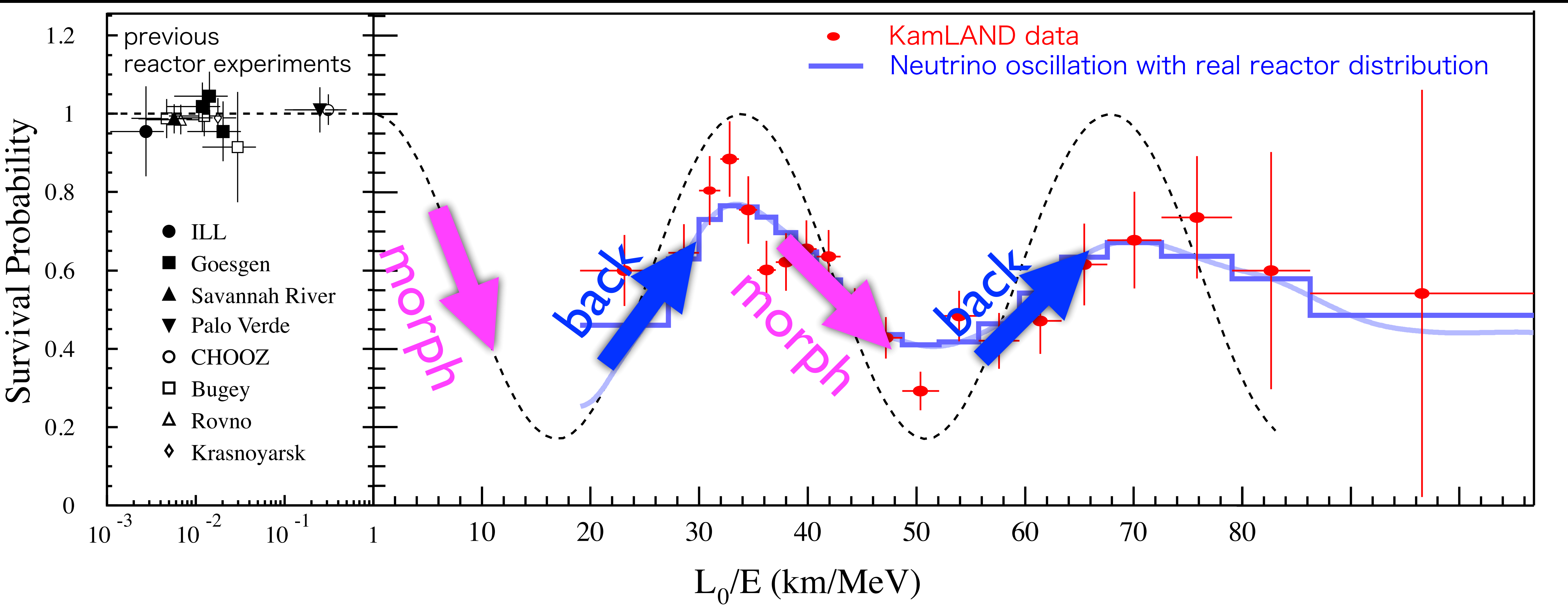
Prediction (flux calculation ...... ≲ 1% ; 1km rock above SK .... 1.5%) 1.8%

Data (Energy calib. for ↑↓ .... 0.7% ; Non ν Background ...... < 2%) 2.1%

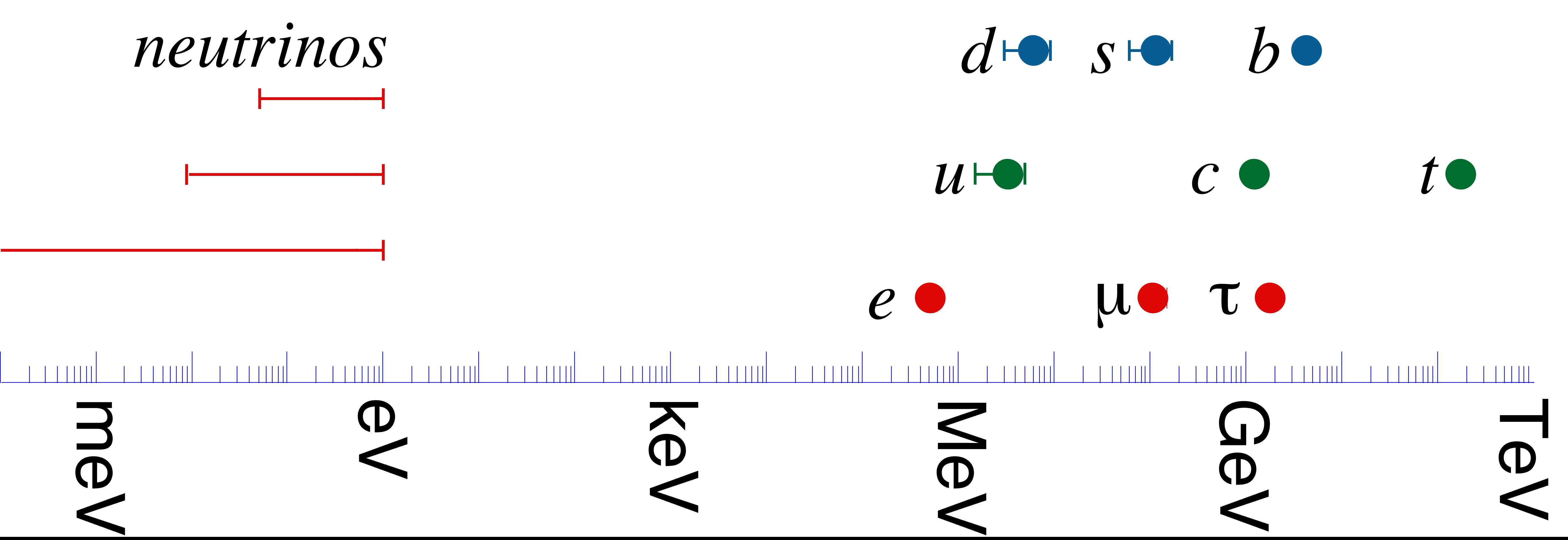
reactor neutrinos



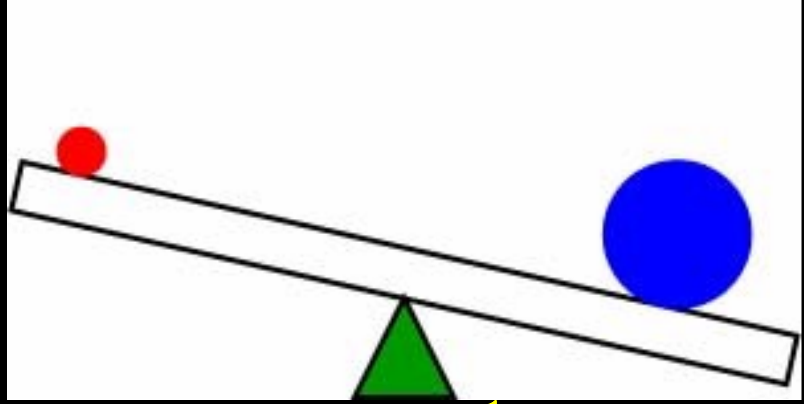
- KamLAND experiment
- a ring of reactors with average *L*~175 km



very light



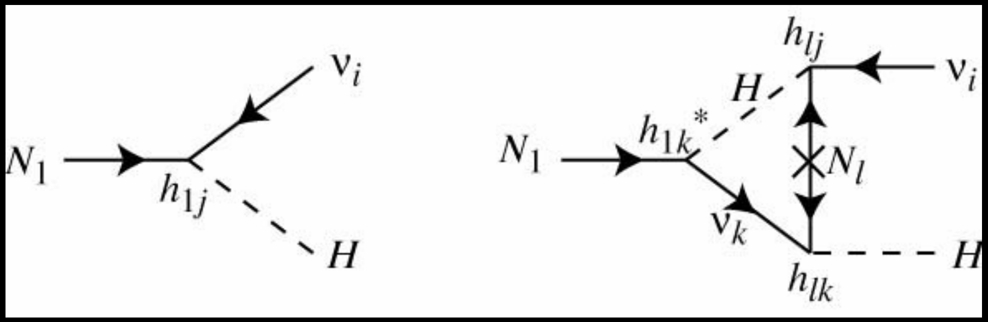
Seesaw



$$\mathcal{L} = -yLNH - \frac{1}{2}MNN$$

$$\begin{pmatrix} \nu & N \end{pmatrix} \begin{pmatrix} 0 & yv \\ yv & M \end{pmatrix} \begin{pmatrix} \nu \\ N \end{pmatrix}$$

- Seesaw mechanism explains
  - small but finite neutrino masses $m_\nu \sim v^2 / M$
  - baryon asymmetry of the Universe through leptogenesis



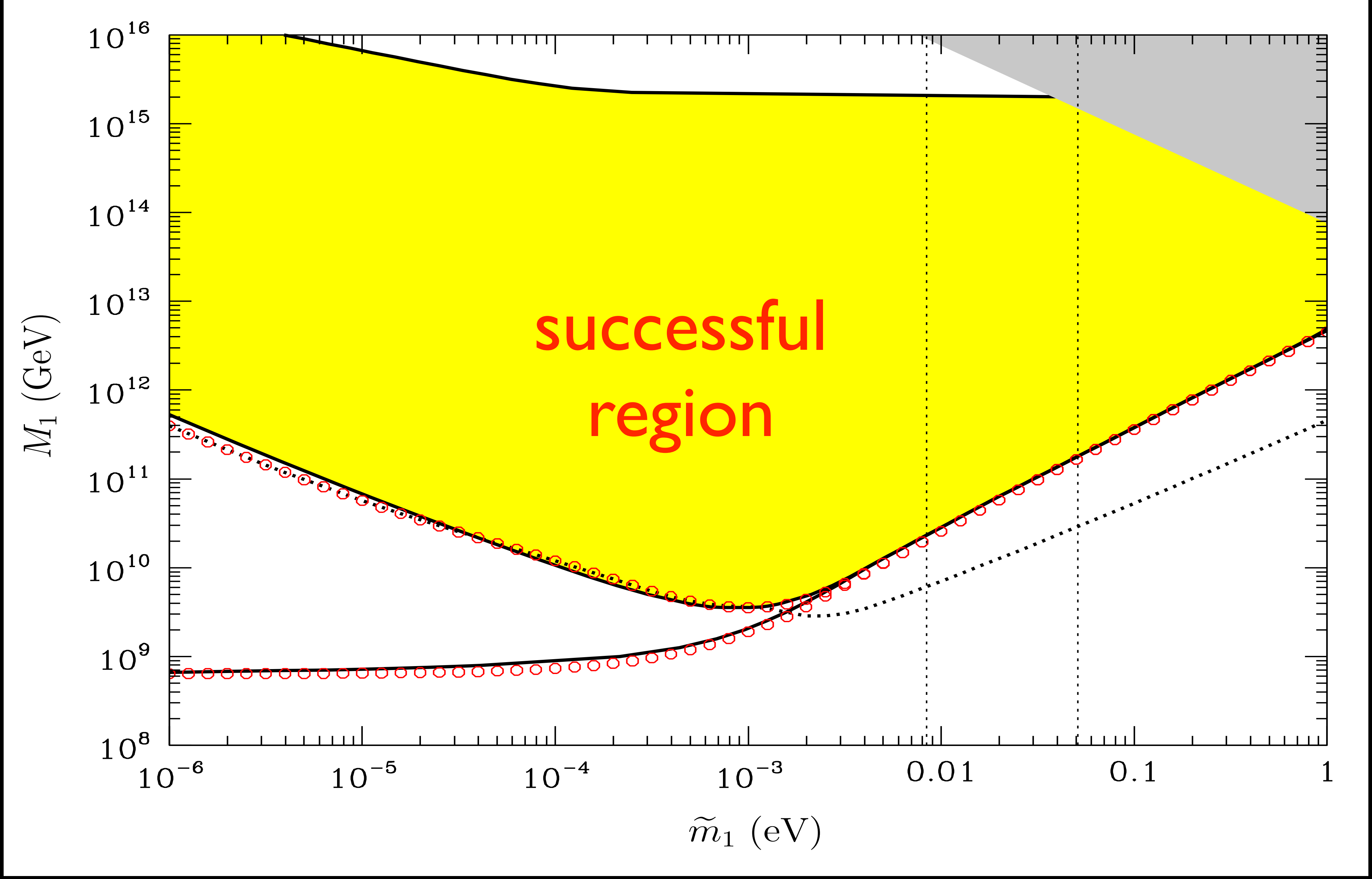
$$\Gamma(N_1 \to \nu_i H) - \Gamma(N_1 \to \bar{\nu}_i H^*) \propto \Im m(h_{1j} h_{1k} h_{lk}^* h_{lj}^*)$$

- the dominant paradigm in neutrino physics
- probe to very high-energy scale
- notoriously difficult to test

Sakharov Conditions

- all three ingredients satisfied
- Baryon number violation
  - lepton number violation + Electroweak anomaly (sphaleron effect)
- CP violation
  - Yukawa couplings $y_{i\alpha} L_i N_\alpha H + M_\alpha N_\alpha N_\alpha$
  - even two generations sufficient
- Non-equilibrium
  - out-of-equilibrium decay of N_α due to long lifetimes

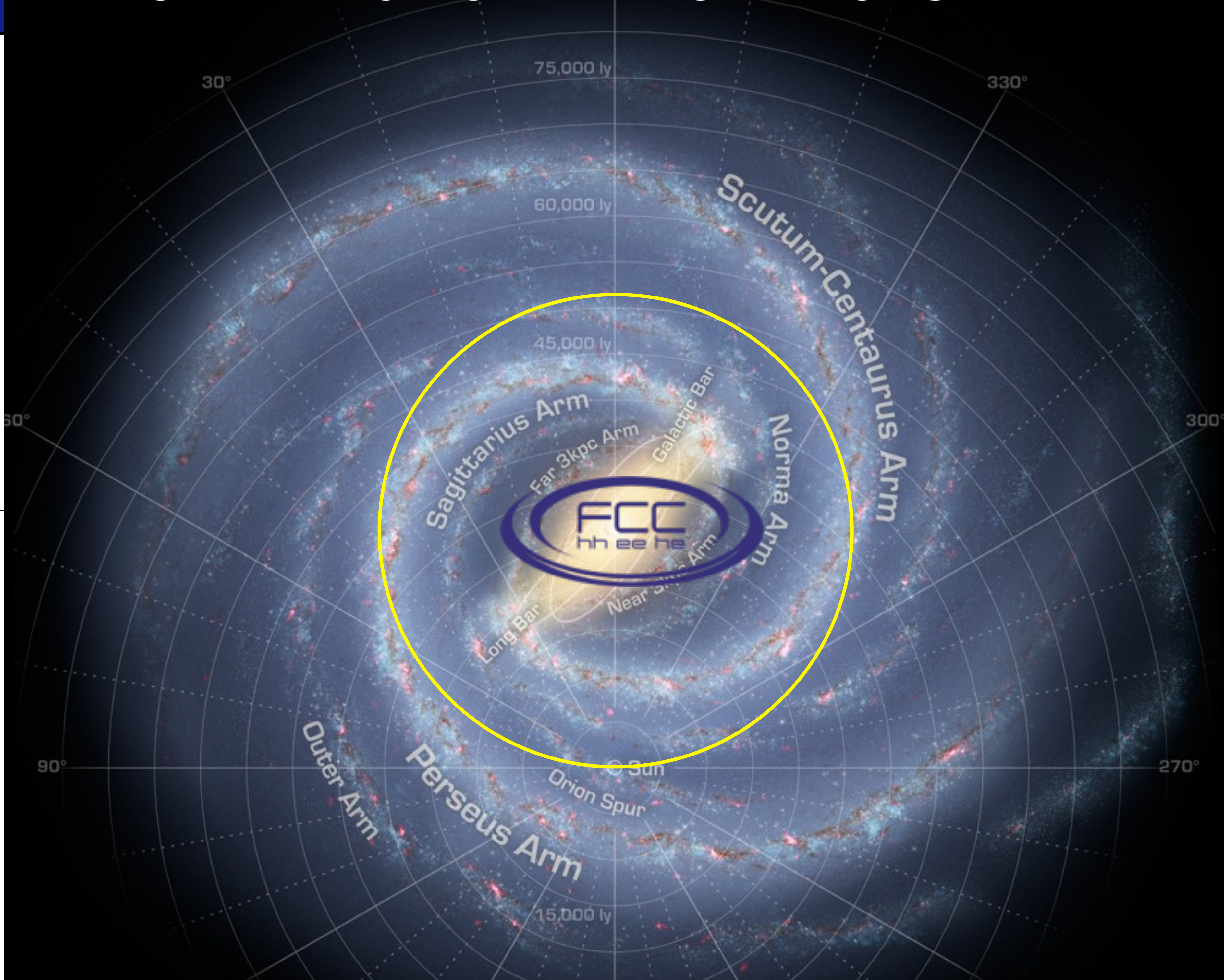
Leptogenesis



$$\tilde{m}_1 = \frac{(m_D^\dagger m_D)_{11}}{M_1}$$

di Bari, Plümacher, Buchmüller

How do we test it?



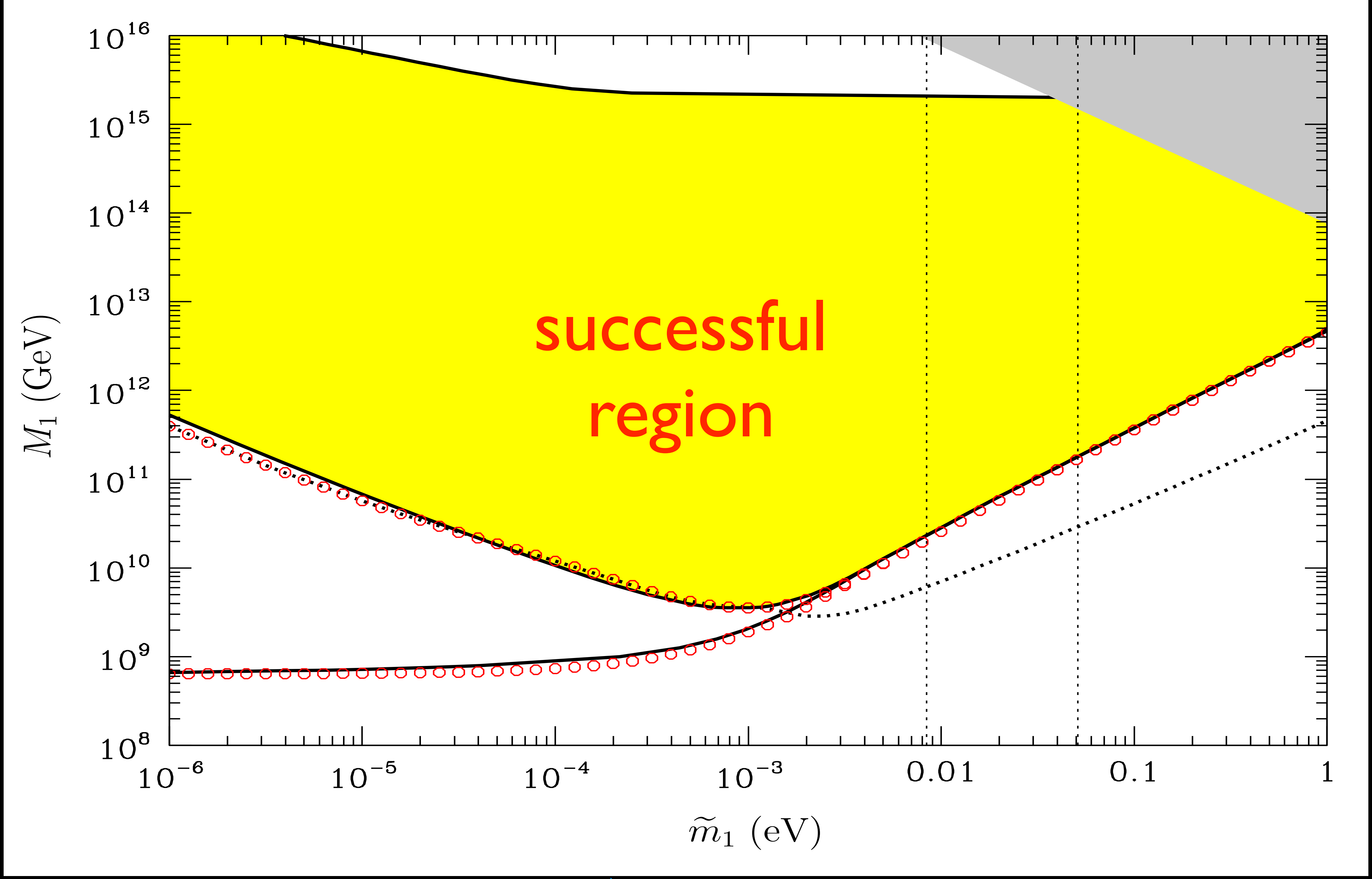
build a 10^{14} GeV collider

how do we test it?

- possible three circumstantial evidences
  - $0\nu\beta\beta$
  - CP violation in neutrino oscillation
  - other impacts e.g. LFV (requires new particles/interactions < 100 TeV)
- *archeology*
- *any more circumstantial evidences?*

M_{Pl}

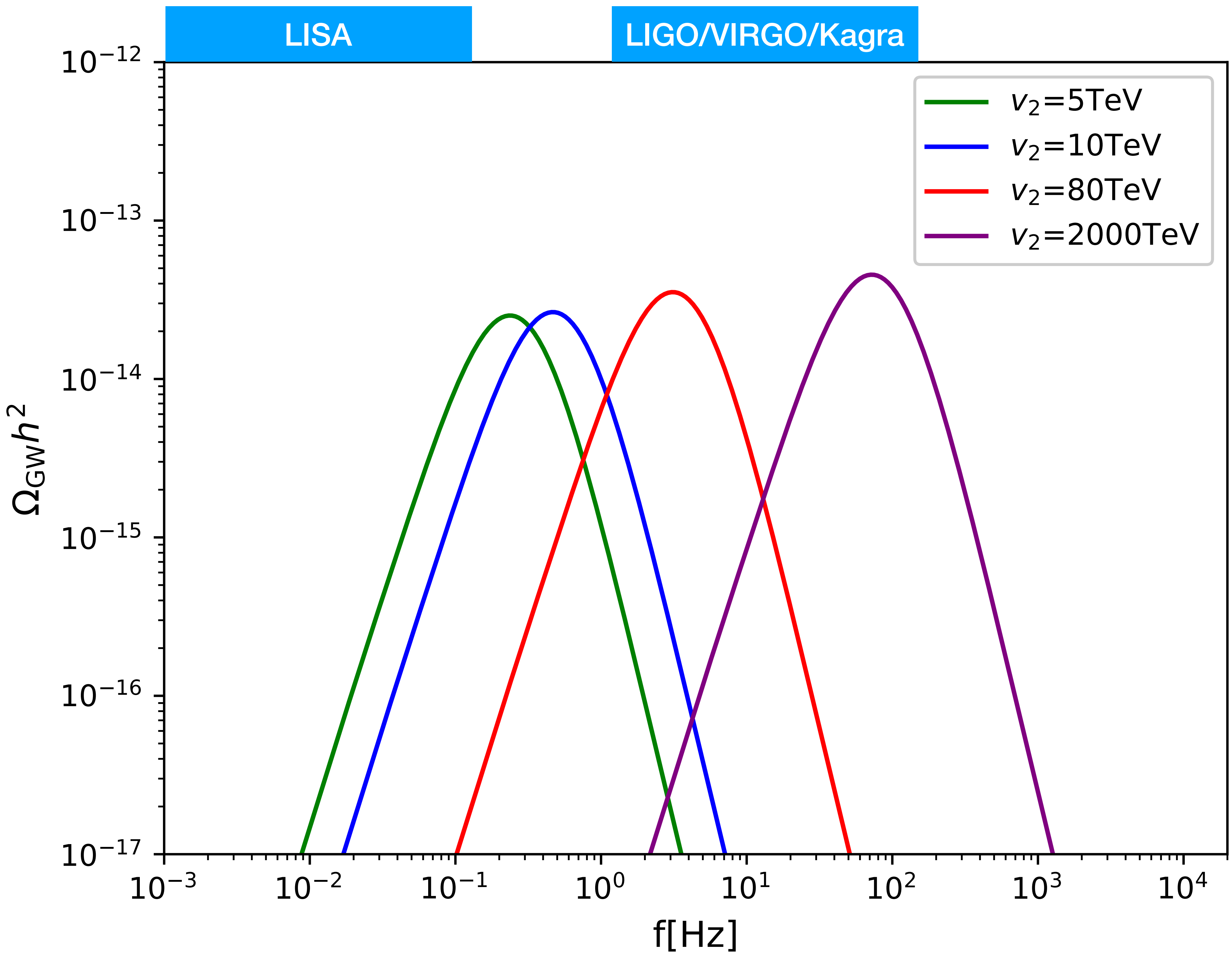
Natural to think M is induced from symmetry breaking
e.g. $\mathcal{L}=-y\langle\varphi\rangle N\,N$



Phase Transition → **Gravitational Waves?**

1st order Phase Transition

Taiki Hasegawa, Nobuchika Okada, Osamu Seto, arXiv:1904.03020

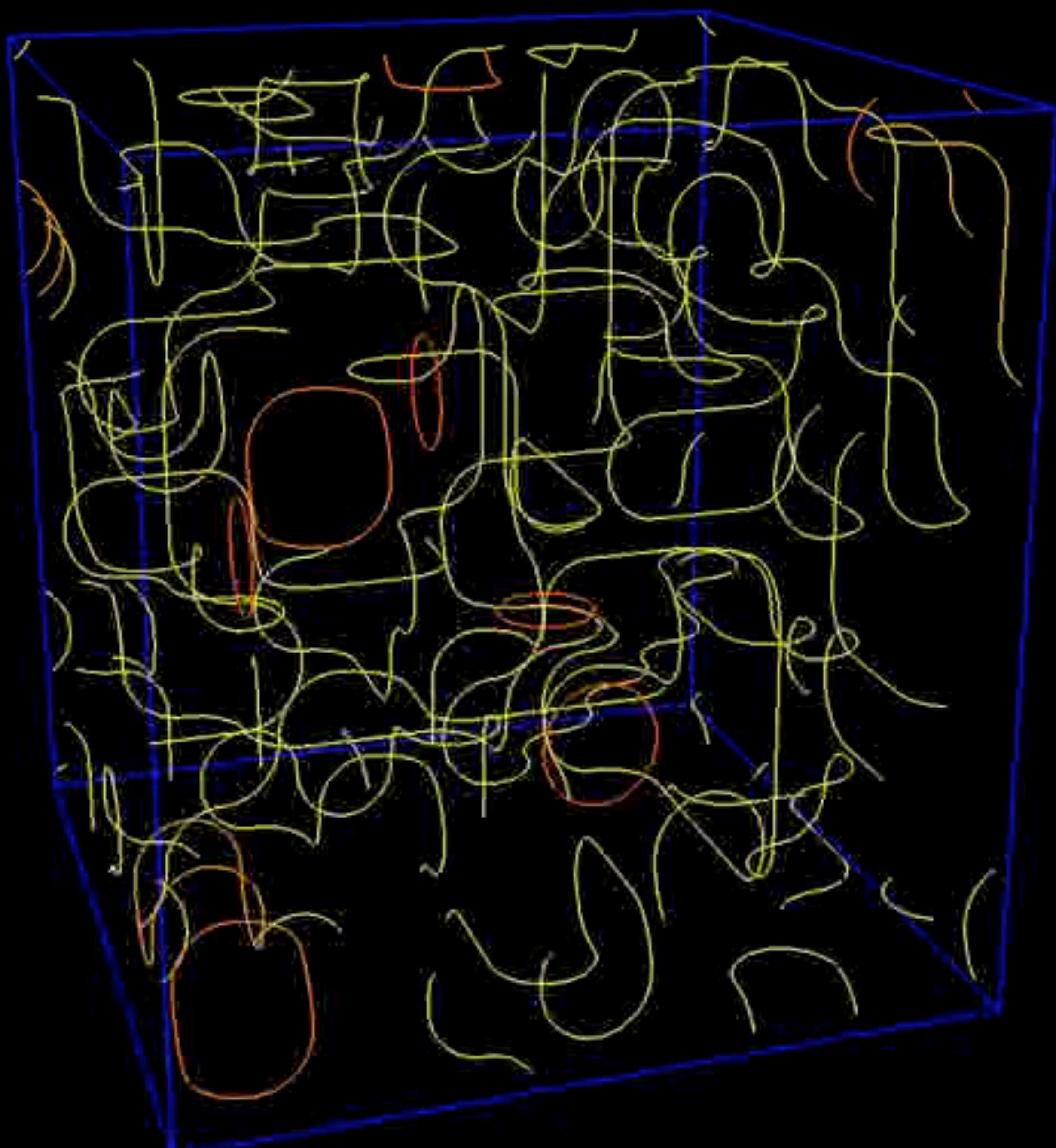


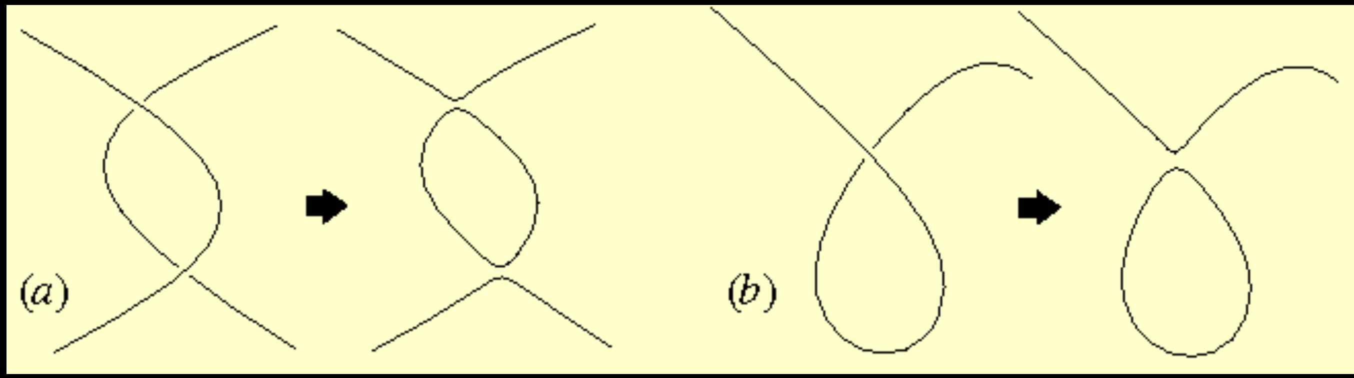
$U(1)_{B-L}$

- Consider $\langle\phi\rangle\neq 0$
  - M_R from $\langle\phi\rangle v_R v_R$ or $\langle\phi^2\rangle v_R v_R/M_{Pl}$
- U(1) breaking produces cosmic strings because $\pi_1(U(1))=Z$
- nearly scale invariant spectrum
- simplification of the network produces gravitational waves
- stochastic gravitational wave background

https://www.ligo.org/science/Publication-S5S6CosmicStrings/index.php

cosmic strings

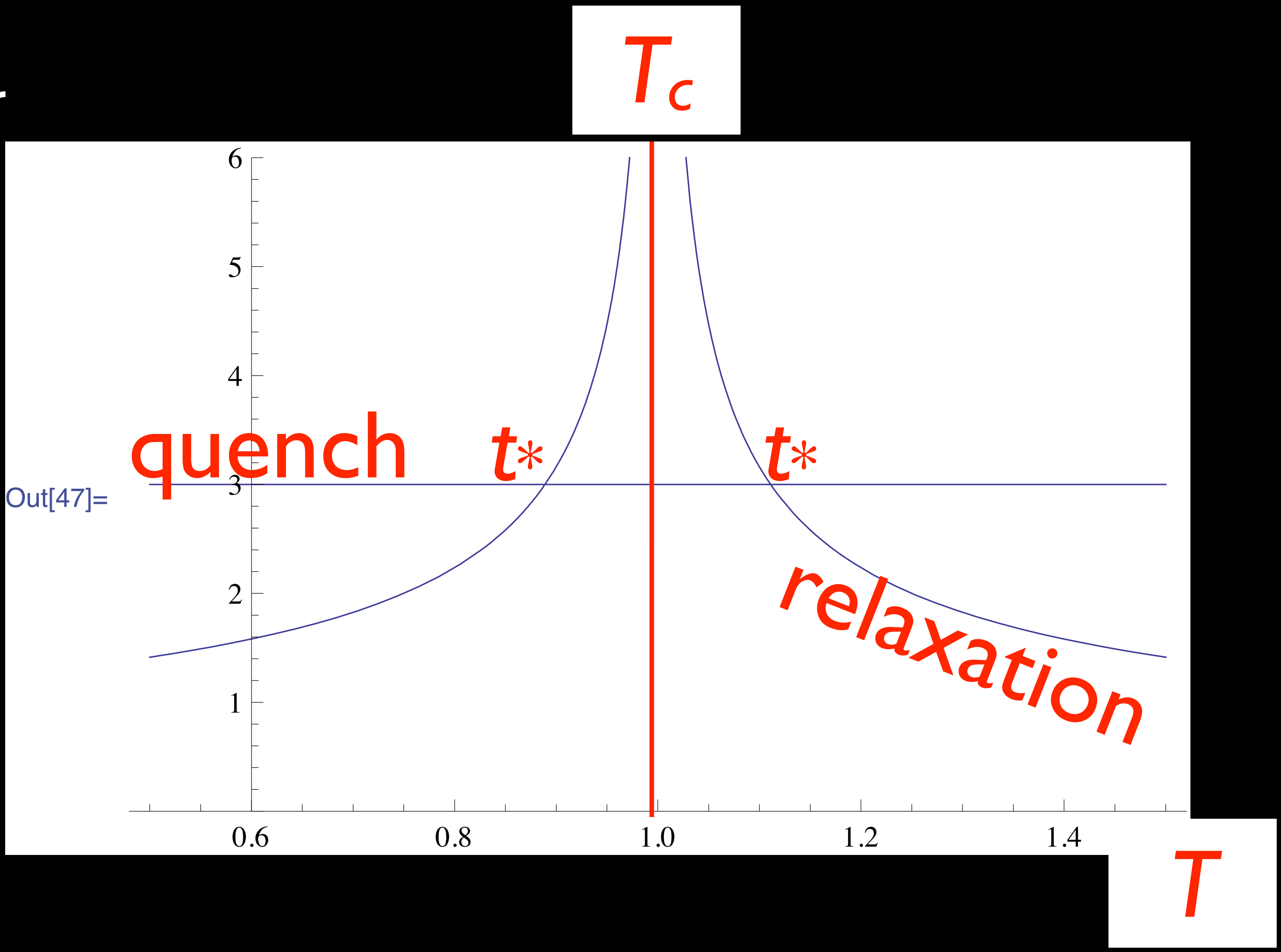




$G\mu \sim v^2/M_{Pl}^2$

Kibble-Zurek mechanism

- far more defects in 2nd order phase transition than the original Kibble mechanism
- proximity to T_c: $\varepsilon = |T_c - T|/T_c$
- relaxation time: $\tau = \tau_0\, \varepsilon^{-\mu}$
- quenching rate: $\tau_Q = (t - t_c)/\varepsilon$
- available time for relaxation: $\tau(t_*) = |t_* - t_c|$
- $\tau_0\, \varepsilon(t_*)^{-\mu} = \varepsilon(t_*)\, \tau_Q$
- $\varepsilon(t_*) = |\tau_Q/\tau_0|^{-1/(1+\mu)}$



HM, Jing Shu, arXiv:0905.1720

monopoles

- Kibble mechanism:

$$\left.\frac{n_{PD}}{s}\right|_{T=T_c} \approx \left(\frac{T_c}{M_{pl}}\right)^3 \; .$$

- Kibble-Zurek mechanism (mean field):

$$\left.\frac{n_{PD}}{s}\right|_{T=T_c} \approx 0.1\,\frac{T_c}{M_{pl}} \; .$$

overabundant if T_c>10^6 GeV

- Kibble-Zurek mechanism (quantum):

$$\left.\frac{n_{PD}}{s}\right|_{T=T_c} \approx 0.006\left(\frac{30T_c}{M_{pl}}\right)^{\frac{3\nu}{1+\nu}} \; .$$

Classification of gauge symmetries

- forbids $M\, \nu_R\, \nu_R$
- anomaly free with $QduLe+\nu_R$
- rank ≤ 5
- no magnetic monopoles

$G_{\rm disc} = G_{\rm SM} \times \mathbb{Z}_N\,,$

$G_{B-L} = G_{\rm SM} \times U(1)_{B-L}\,,$ $\quad Q(3,2,\frac{1}{6},\frac{1}{3}), d^c(3^*,1,\frac{2}{3},-\frac{1}{3}), u^c(3^*,1,-\frac{1}{3},-\frac{1}{3}), L(1,2,-\frac{1}{2},-1), e^c(1,1,+1,+1), \nu_R^c(1,1,1,+1)$

$G_{LR} = SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}\,,$ $\quad Q(3,2,1,\frac{1}{3}), (d^c+u^c)(3^*,1,2,-\frac{1}{3}), L(1,2,1,-1), (e^c+\nu_R^c)(1,1,2,+1$

$G_{421} = SU(4)_{\rm PS} \times SU(2)_L \times U(1)_Y\,,$ $\quad (Q+L)(4,2,0), (d^c+e^c)(4^*,1,\frac{1}{2}), (u^c+\nu_R^c)(4^*,1,-\frac{1}{2})$

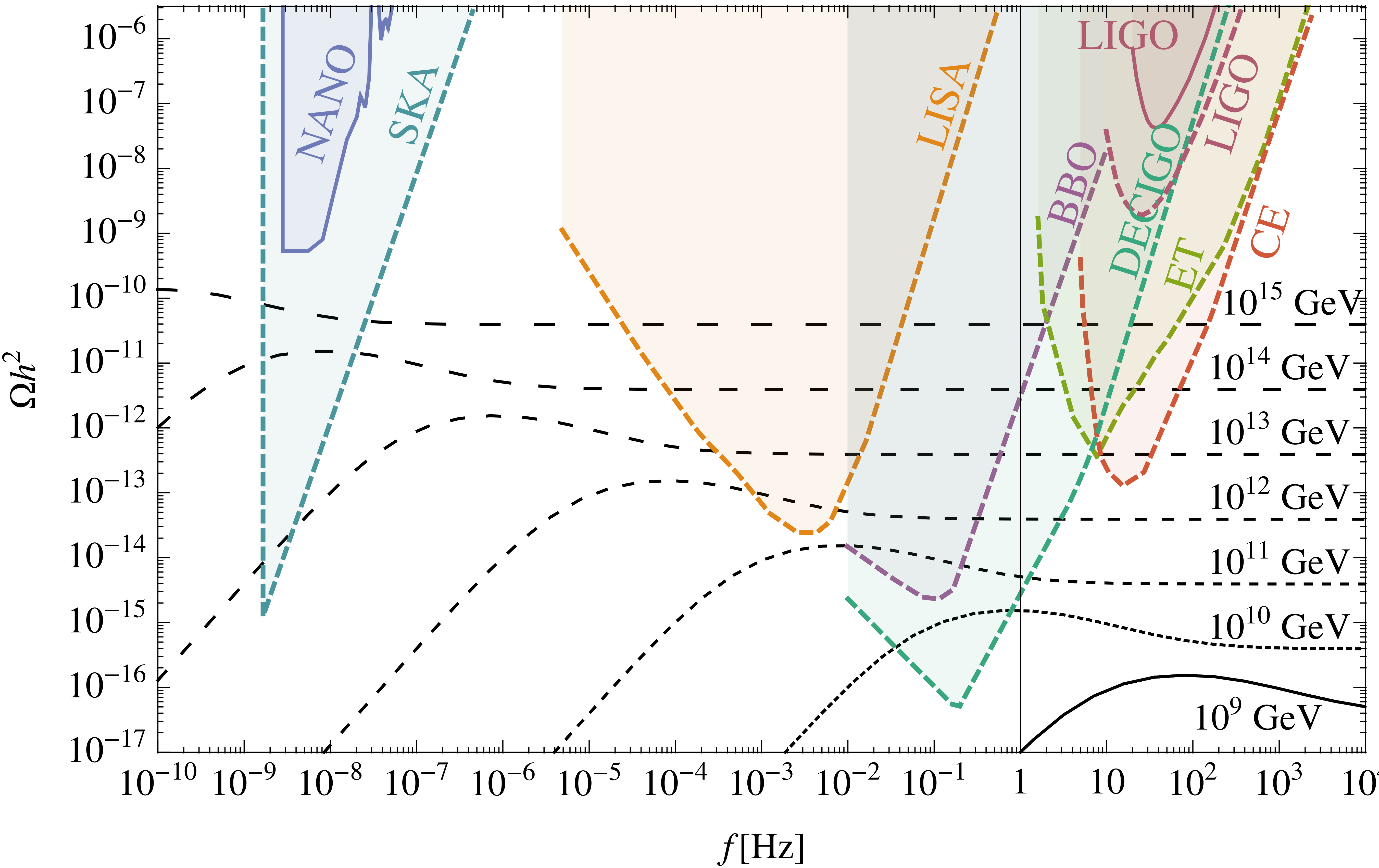
$G_{\rm flip} = SU(5) \times U(1)\,.$ $\quad (Q+d^c+\nu_R^c)(10,1), (L+u^c)(5^*,-3), e^c(1,5)$

$<\phi\phi>\nu_R\nu_R/M_{Pl}$ $\qquad$ $<\phi>\nu_R\nu_R$

	$H = G_{\rm SM}$		$H = G_{\rm SM}\times\mathbb{Z}_2$	
G	**defects**	**Higgs**	**defects**	**Higgs**
$G_{\rm disc}$	domain wall*	$B-L=1$	domain wall*	$B-L=2$
G_{B-L}	abelian string*	$B-L=1$	$\mathbb{Z}_2$ string†	$B-L=2$
G_{LR}	texture*	$(\mathbf{1},\mathbf{1},\mathbf{2},\frac{1}{2})$	$\mathbb{Z}_2$ string	$(\mathbf{1},\mathbf{1},\mathbf{3},1)$
G_{421}	none	$(\mathbf{10},\mathbf{1},2)$	$\mathbb{Z}_2$ string	$(\mathbf{15},\mathbf{1},2)$
$G_{\rm flip}$	none	$(\mathbf{10},1)$	$\mathbb{Z}_2$ string	$(\mathbf{50},2)$

Z_2 matter parity: flips signs of all fermions

$$0 = \pi_2(G) \to \pi_2(G/H) \to \pi_1(H) \to \pi_1(G) \to \pi_1(G/H) \to \pi_0(H) \to \pi_0(G) = 0$$



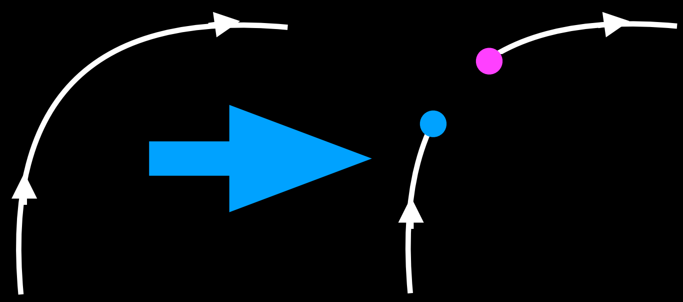
J. Dror, T. Hiramatsu, K. Kohri, HM, G. White, arXiv:1908.03227

covers pretty much the entire range for leptogenesis!

caveat: particle emission from cosmic strings

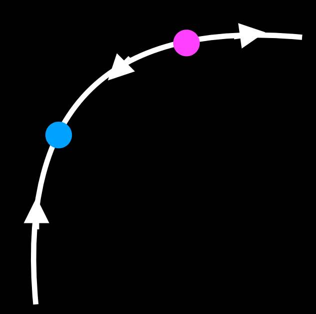
SO(10)

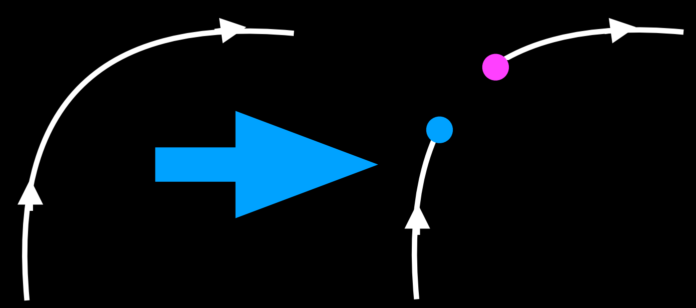
- It is natural to embed $U(1)_{B-L}$ etc into SO(10)
- However, SO(10)→SU(3)×SU(2)×U(1) doesn't lead to cosmic strings because $\pi_1(SO(10)/SU(3)\times SU(2)\times U(1))=0$
- SO(10)→SU(3)×SU(2)×U(1)×$U(1)_{B-L}$ produces monopoles
    - SO(10) scale is presumably $V\sim 10^{16}$GeV≫v
    - need inflation below this scale
- SU(3)×SU(2)×U(1)×$U(1)_{B-L}$→SU(3)×SU(2)×U(1) produces strings
    - strings can be *cut* by monopole-anti-monopole pairs through a tunneling process



monopoles

- string from $U(1)_{B-L}$ breaking is basically Abrikosov flux in a superconductor
  - For the Higgs $\phi(\pm Q)$
  - magnetic flux $h/(g\ Q) \times$ integer (Q=1, 2, …)
  - minimum monopole charge h/g
  - If Q=1, monopole can saturate the flux and cut the string
  - If Q=2, the minimum string cannot be cut by monopoles





Schwinger

- Schwinger computed the production of e^+e^- pairs in a constant electric field in 3+1 dimension
- adopt it to 1+1 dimension
- dualize it to magnetic field

$$\frac{\Gamma}{L} = \frac{eE}{4\pi^2} \sum_{n=1}^{\infty} \frac{1}{n} e^{-\pi m^2 n/eE}$$

- cross section of the string $A \sim (g\, v)^{-2}$
- $B\, A \sim 2\pi/(g\, Q)$
- length of the string $L \sim H^{-1}$
- strings get cut when $H \sim \Gamma/L \times L \sim \Gamma/L \times H^{-1}$
- string network persists until $H^2 \sim (\Gamma/L) \sim (g\, v)^2 \exp(-\pi m^2/gB)$
- monopole mass $m \sim V/g$
- survives to date if $v < 10^{15}$GeV

Hybrid inflation

- U(1)$_{B\text{-}L}$ broken after inflation

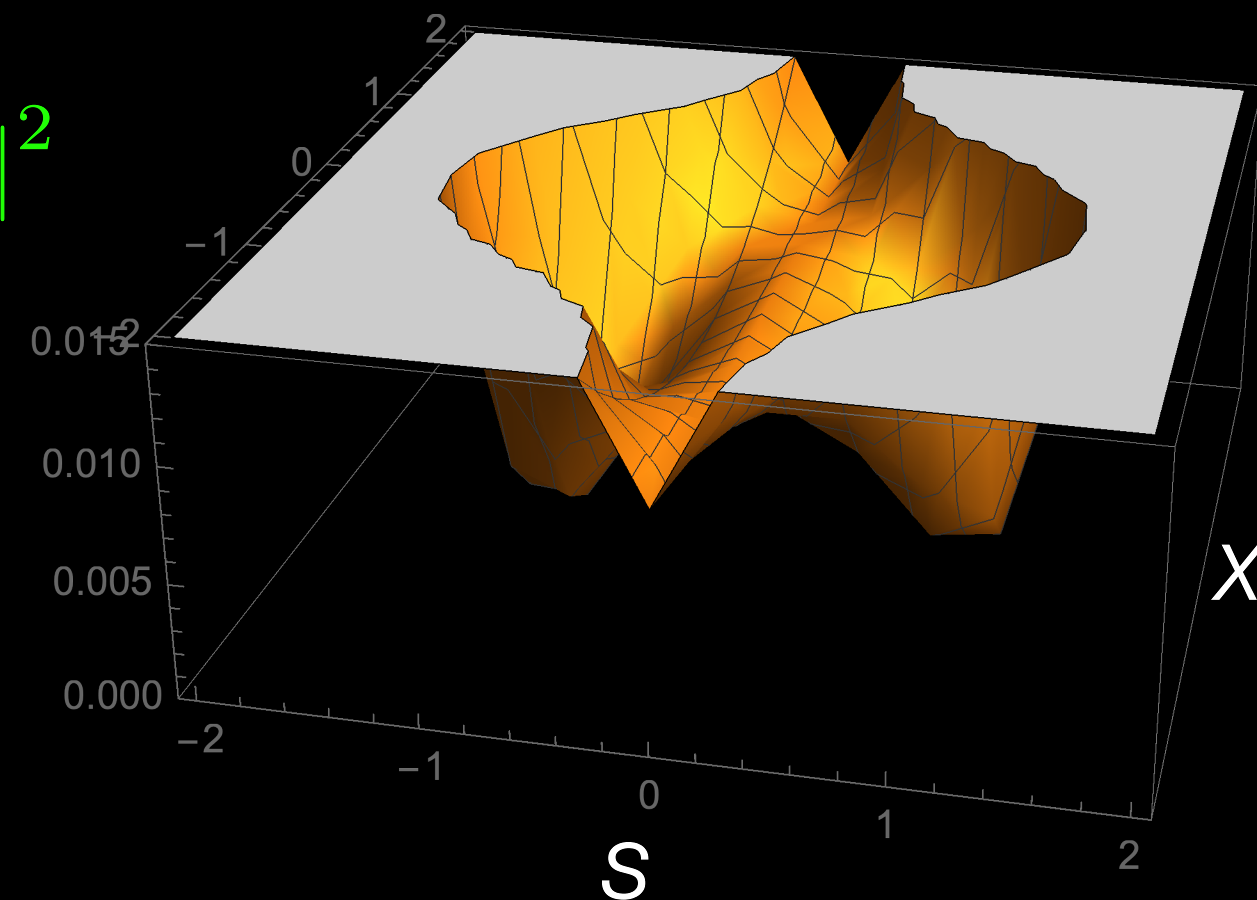
$$W = \lambda X (S^+ S^- - v^2)$$

$$V = \lambda^2 \left|S^+ S^- - v^2\right|^2 + \lambda^2 \left|X\right|^2 \left(\left|S^+\right|^2 + \left|S^-\right|^2\right) + \frac{e^2}{2}\left(\left|S^+\right|^2 - \left|S^-\right|^2\right)^2$$

- *D*-flat direction $S=S^+=S^-$

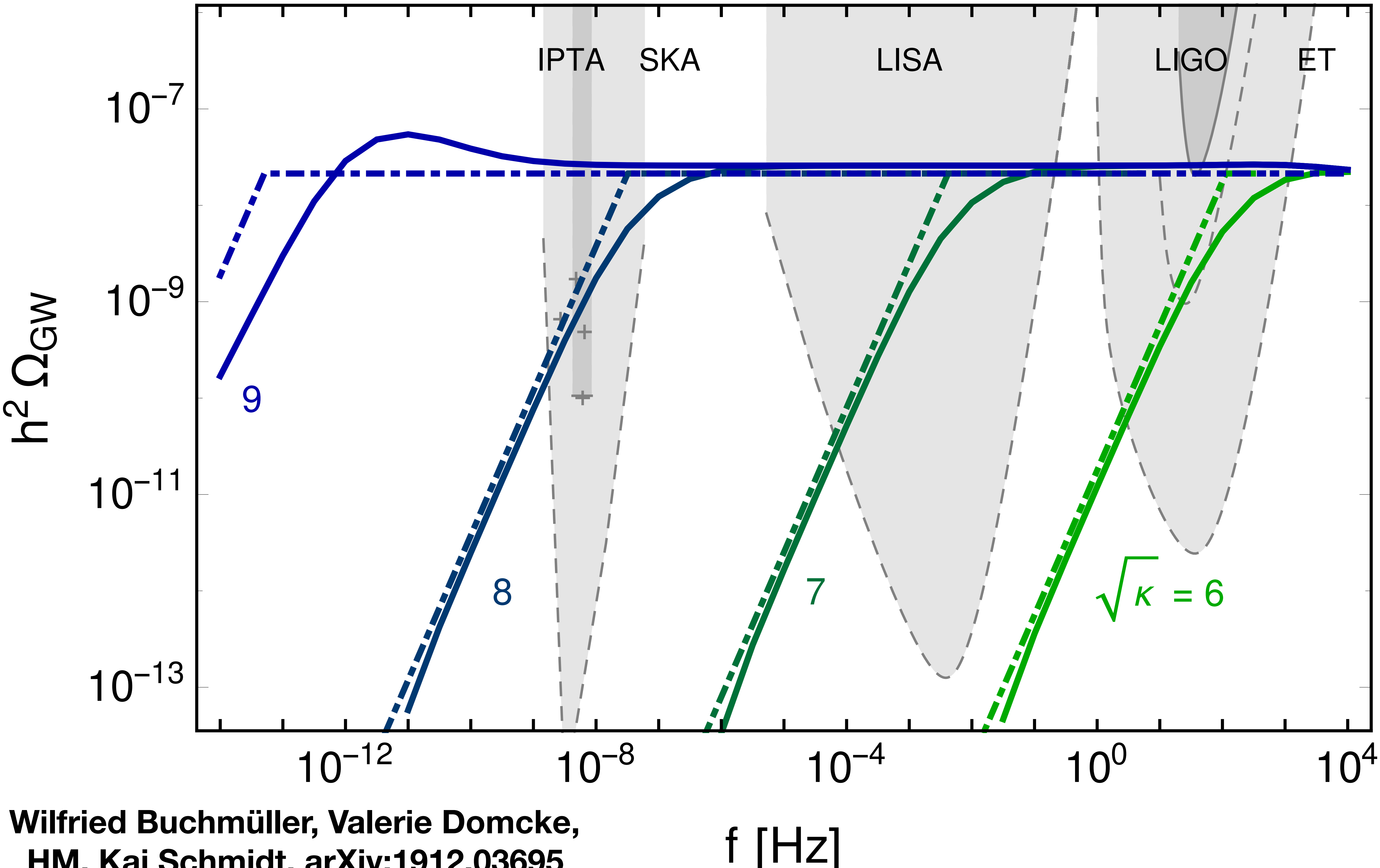
$$V = \lambda^2 \left|S^2 - v^2\right|^2 + 2\lambda^2 \left|X\right|^2 \left|S\right|^2$$

- flat: $S=0$, $V=\lambda^2 v^2$



- falls down to $S=v$ near $X\sim 0$
- forms cosmic strings
- requires high $v \geq$ a few 10^{15} GeV

hybrid inflation



Wilfried Buchmüller, Valerie Domcke, HM, Kai Schmidt, arXiv:1912.03695

Conclusions

- stochastic gravitational waves as another possible circumstantial evidence for seesaw+leptogenesis
- for rank≤5 gauge groups, more than a half of them produce cosmic strings
- future missions promising to cover most range of seesaw scales
- if we do detect scale-invariant gravitational waves, helps establish not only seesaw but also the breaking pattern
- if strings appear to break, evidence for grand unification!
- *any experimental technique to probe gravitational waves of much higher frequencies?*

Asymmetric Matters from a dark first-order phase transition

Hitoshi Murayama (Berkeley, Kavli IPMU)
+Nell Hall (Berkeley), Thomas Konstandin (DESY), Robert McGehee (Berkeley)

arXiv:1911.12342

Sakharov Conditions

- Standard Model may have all three ingredients
- Baryon number violation
  - Electroweak anomaly (sphaleron effect)
- CP violation
  - Kobayashi–Maskawa phase

$$J \propto \det[M_u^\dagger M_u, M_d^\dagger M_d]/T_{EW}^{12} \sim 10^{-20} \ll 10^{-10}$$

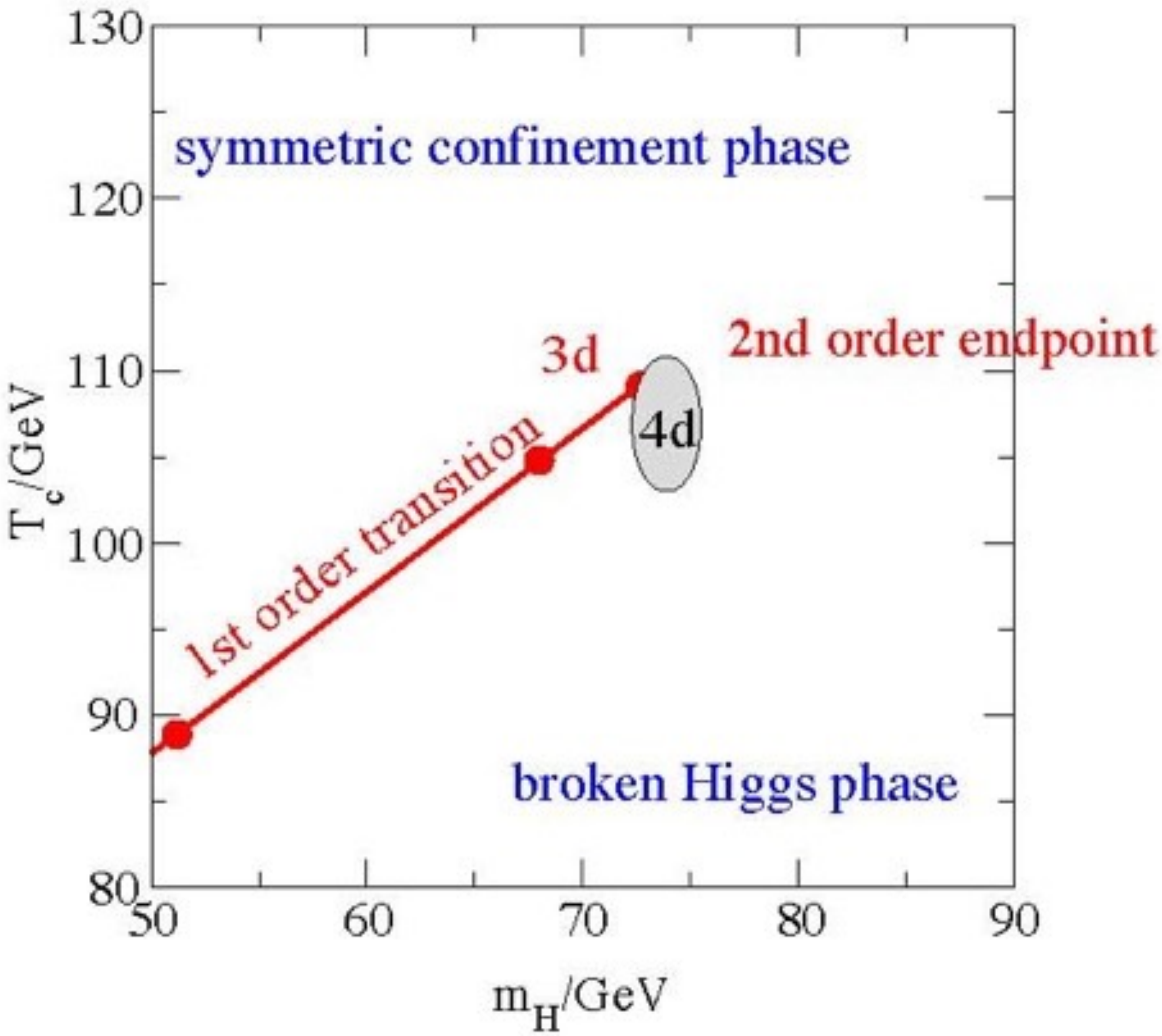
- Non-equilibrium
  - First-order phase transition of Higgs

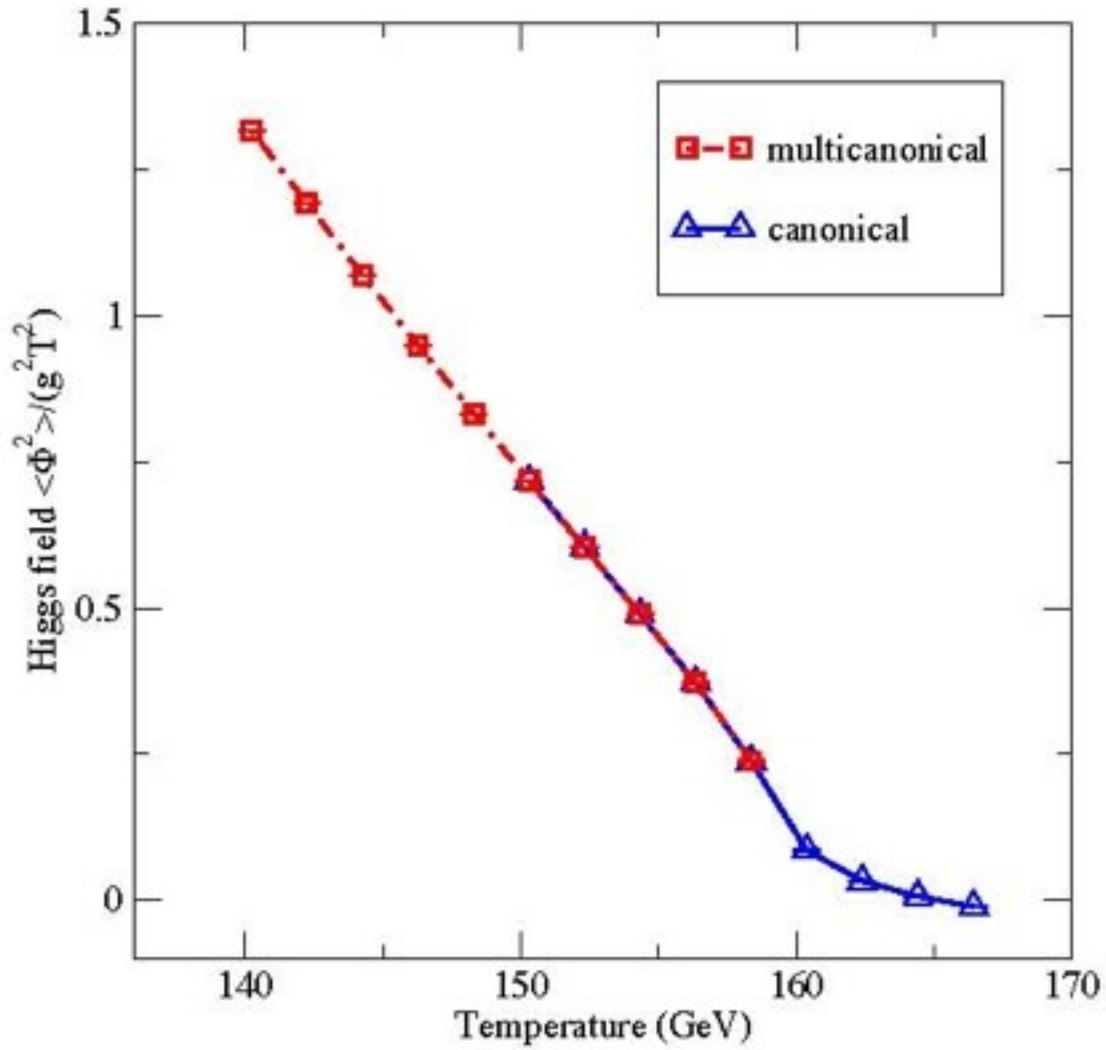
requires $m_h < 75$ GeV

- Experimentally testable?

Mikko Laine (Bern)

Phase diagram for the Standard Model:





<H>=0 from gauge invariance (Elitzur)

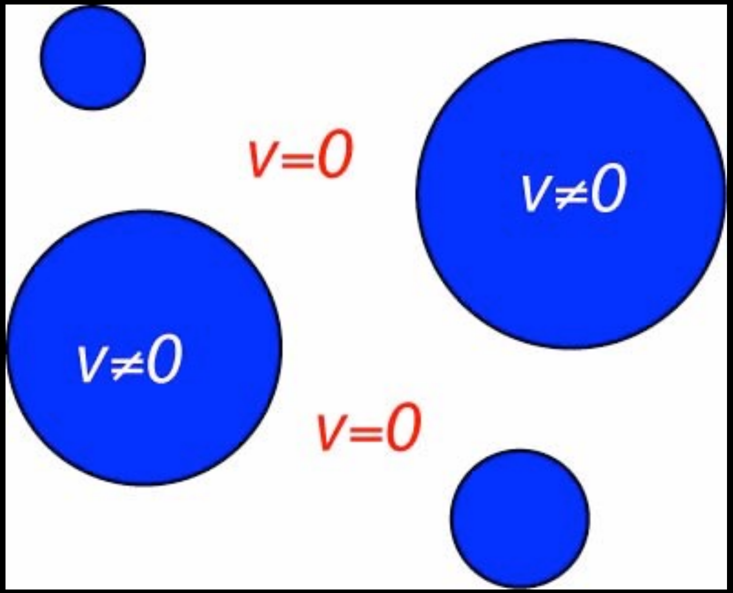
<H†H> is not an order parameter

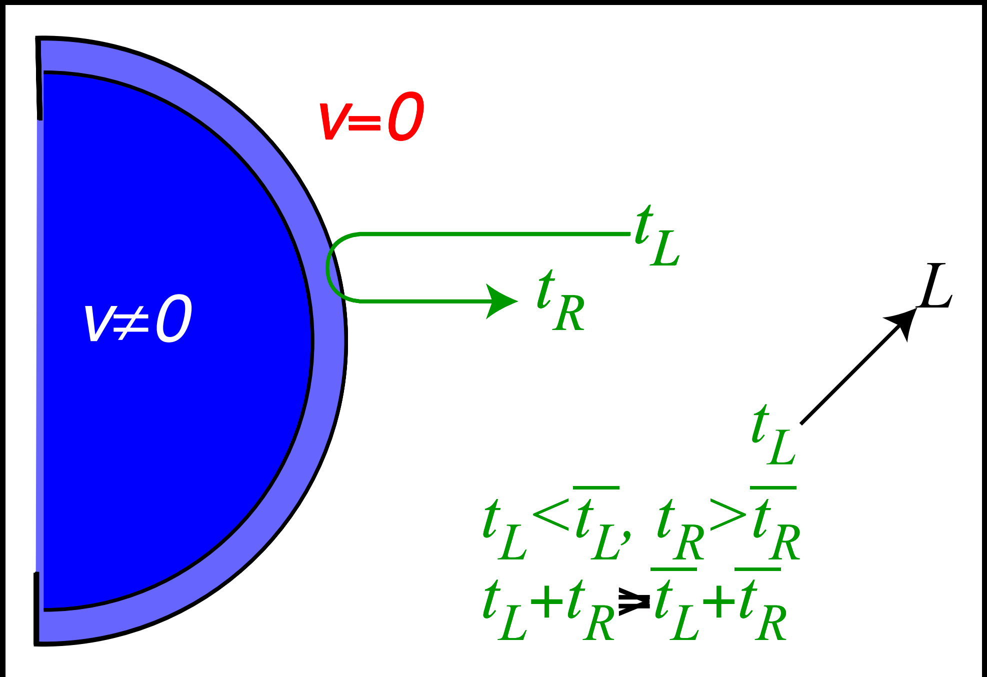
for m_h=125GeV, it is crossover

No phase transition in the Minimal Standard Model

Scenario
Cohen, Kaplan, Nelson

- First-order phase transition
- Different reflection probabilities for t_L, t_R
- asymmetry in top quark
- Left-handed top quark asymmetry partially converted to lepton asymmetry via anomaly
- Remaining top quark asymmetry becomes baryon asymmetry
- need *varying CP phase* inside the bubble wall (G. Servant)
- fixed KM phase doesn't help
- need CPV in Higgs sector





Electric Dipole Moment

- baryon asymmetry limited by the sphaleron rate
  $\Gamma \sim 20\, \alpha_W^5\, T \sim 10^{-6}\, T$
- Can't lose much more to obtain 10^{-9}
- need
  - new physics for 1st order PT at the Higgs scale v=250 GeV
  - CP violation×efficiency $\geq 10^{-3}$

Oct 2018

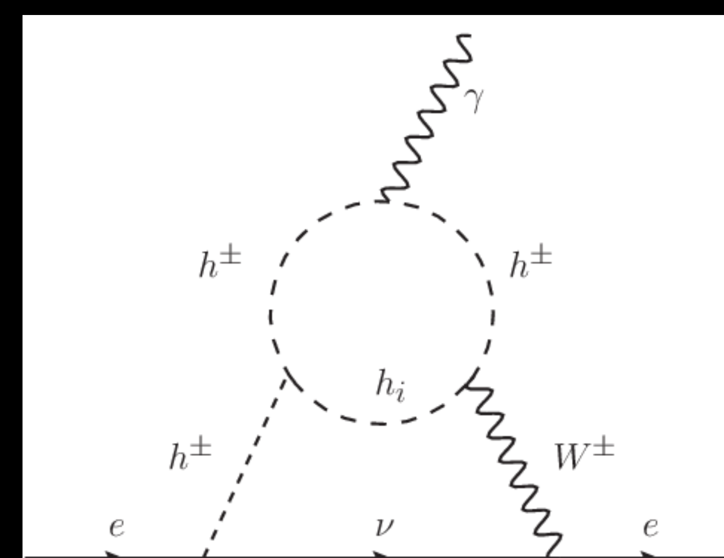
ARTICLE

https://doi.org/10.1038/s41586-018-0599-8

Improved limit on the electric dipole moment of the electron

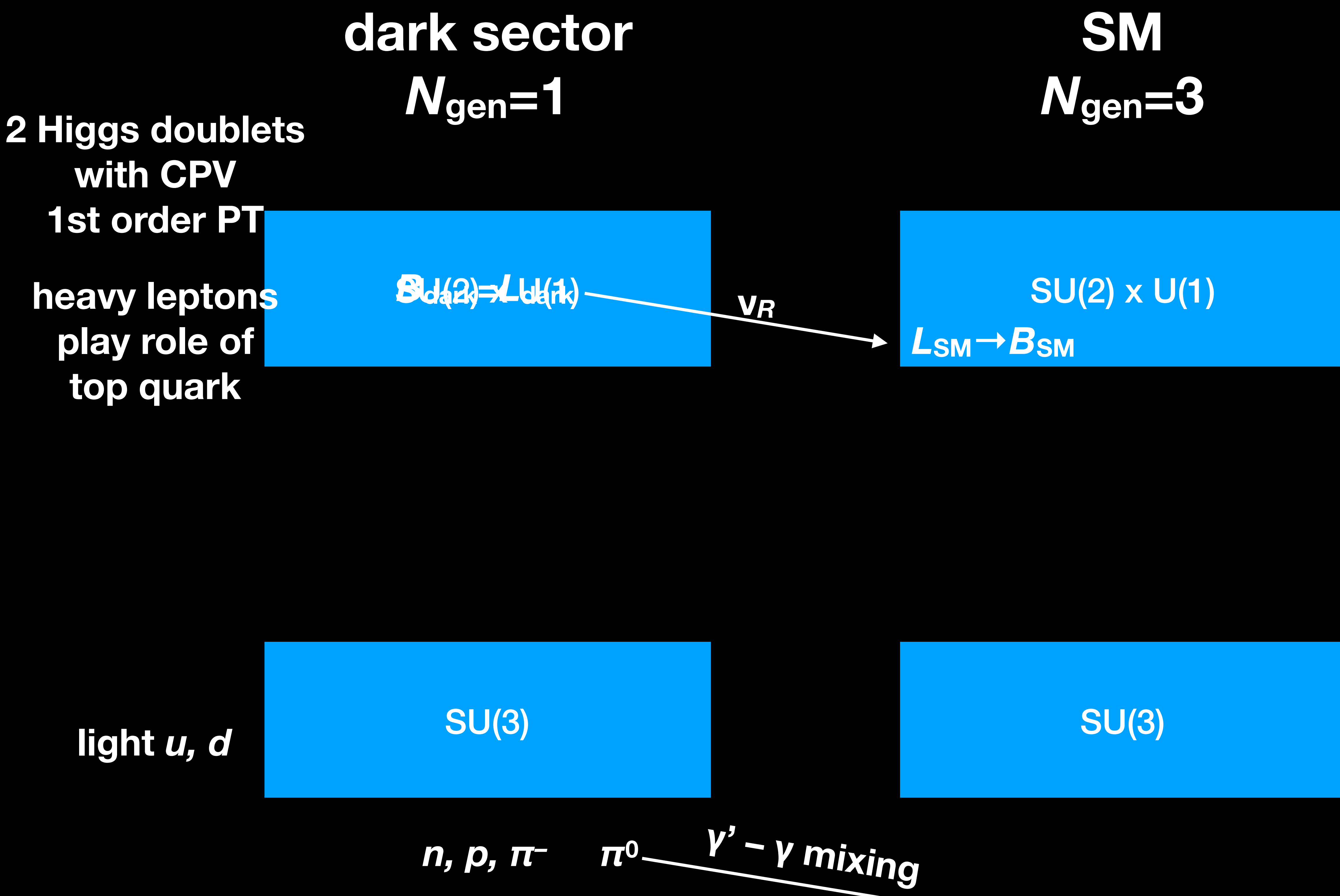
ACME Collaboration*

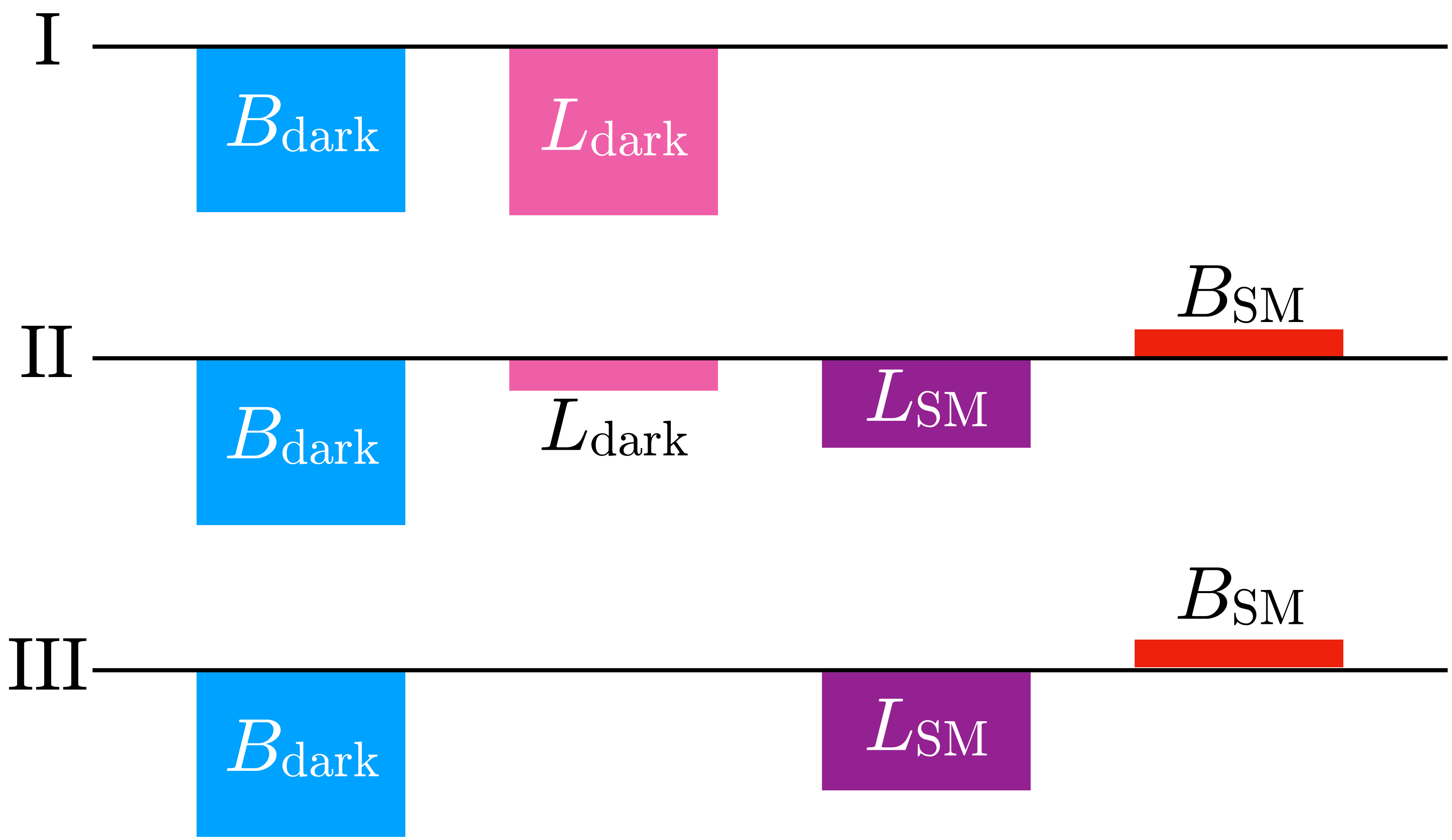
$d_e \leq 1.1\times10^{-29}\ e$ cm



Barr-Zee diagrams

$$d_e \approx \frac{e m_e}{(16\pi^2)^2}\frac{1}{v^2}\sin\delta = 1.6\times10^{-22} e\mathrm{cm}\sin\delta$$


dark sector
N_{gen}=1
SM
N_{gen}=3
2 Higgs doublets
with CPV
1st order PT
heavy leptons
play role of
top quark
SU(2) x U(1)
B_{dark} → L_{dark}
ν_R
SU(2) x U(1)
L_{SM} → B_{SM}
light u, d
SU(3)
SU(3)
n, p, π^-
π^0
$\gamma' - \gamma$ mixing
e^+e^-

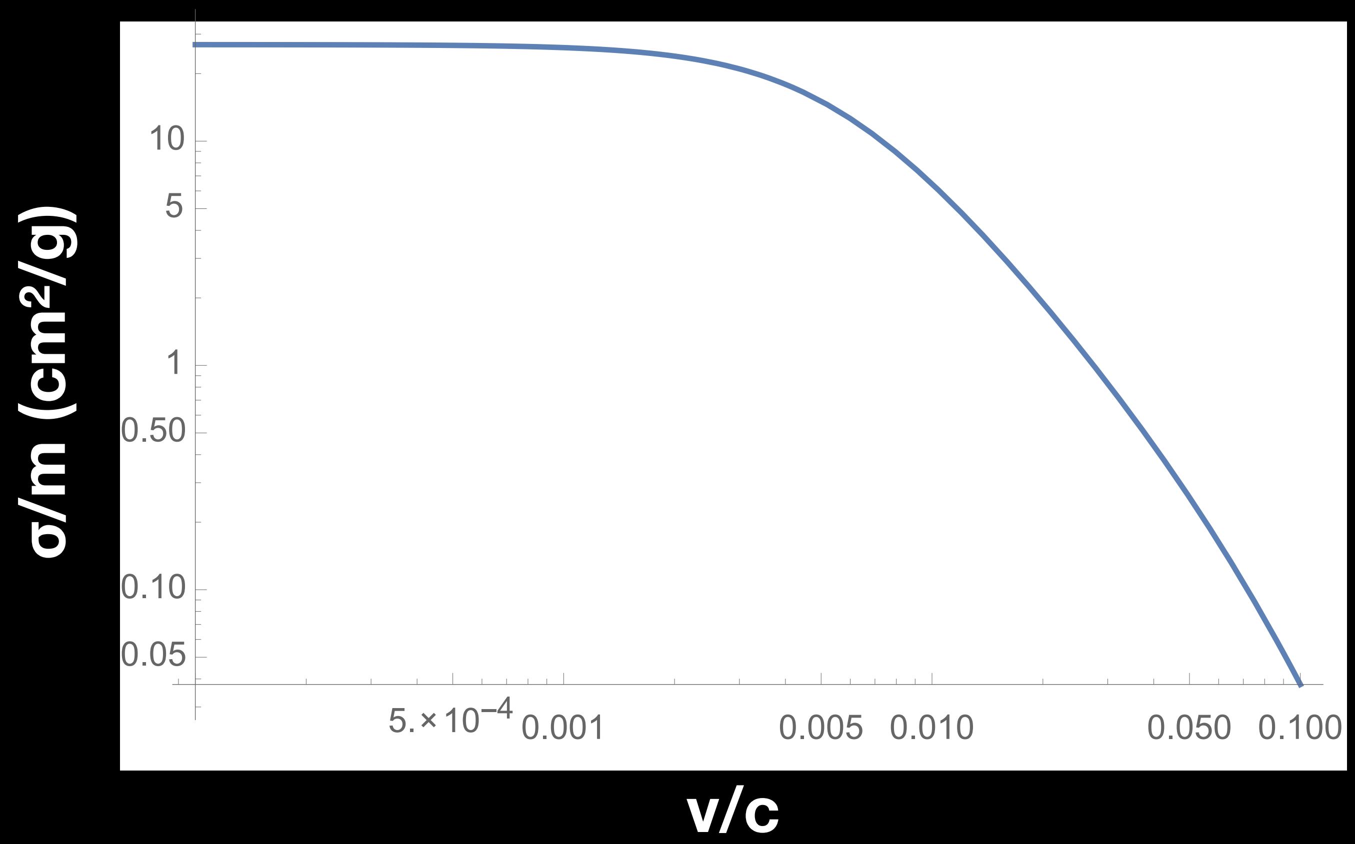


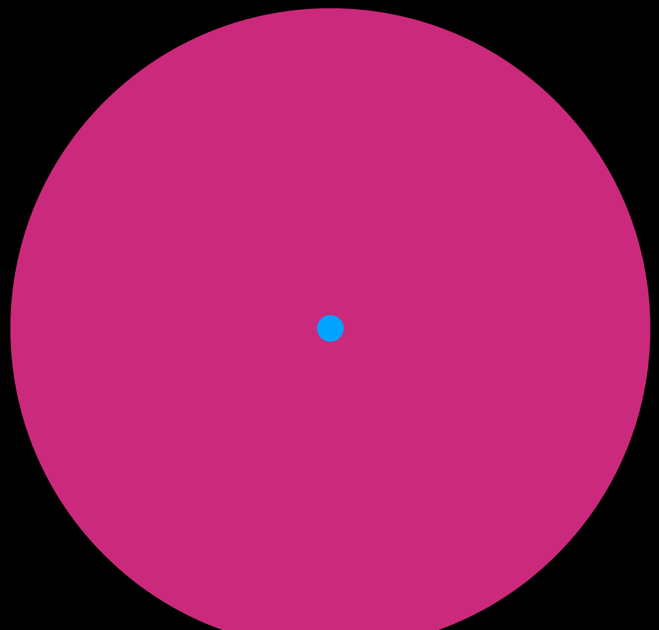
If $M_N > T_{sphaleron}$ $B_{SM} = \frac{36}{133} B_{dark}, \quad L_{S}$

If $M_N < T_{sphaleron}$ $B_{SM} = \frac{12}{37} B_{dark}, \quad L_{S}$

n-*n* scattering

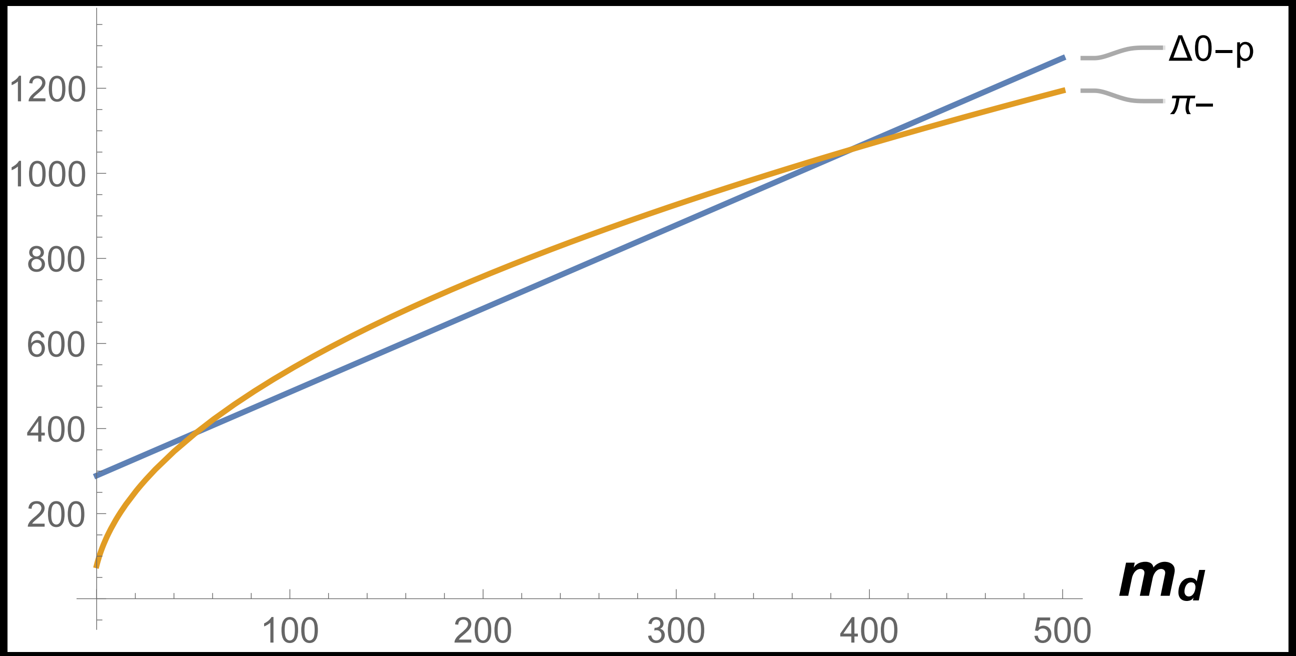
- *n*-*n* scattering has an anomalously large cross section a=18.9fm
- If so, it violates astrophysical bounds on self-interaction
- a fine cancellation between the bare and one-loop couplings in the pion-less EFT
- According to lattice simulations (HAL QCD), the cross section is more or less of the geometric size



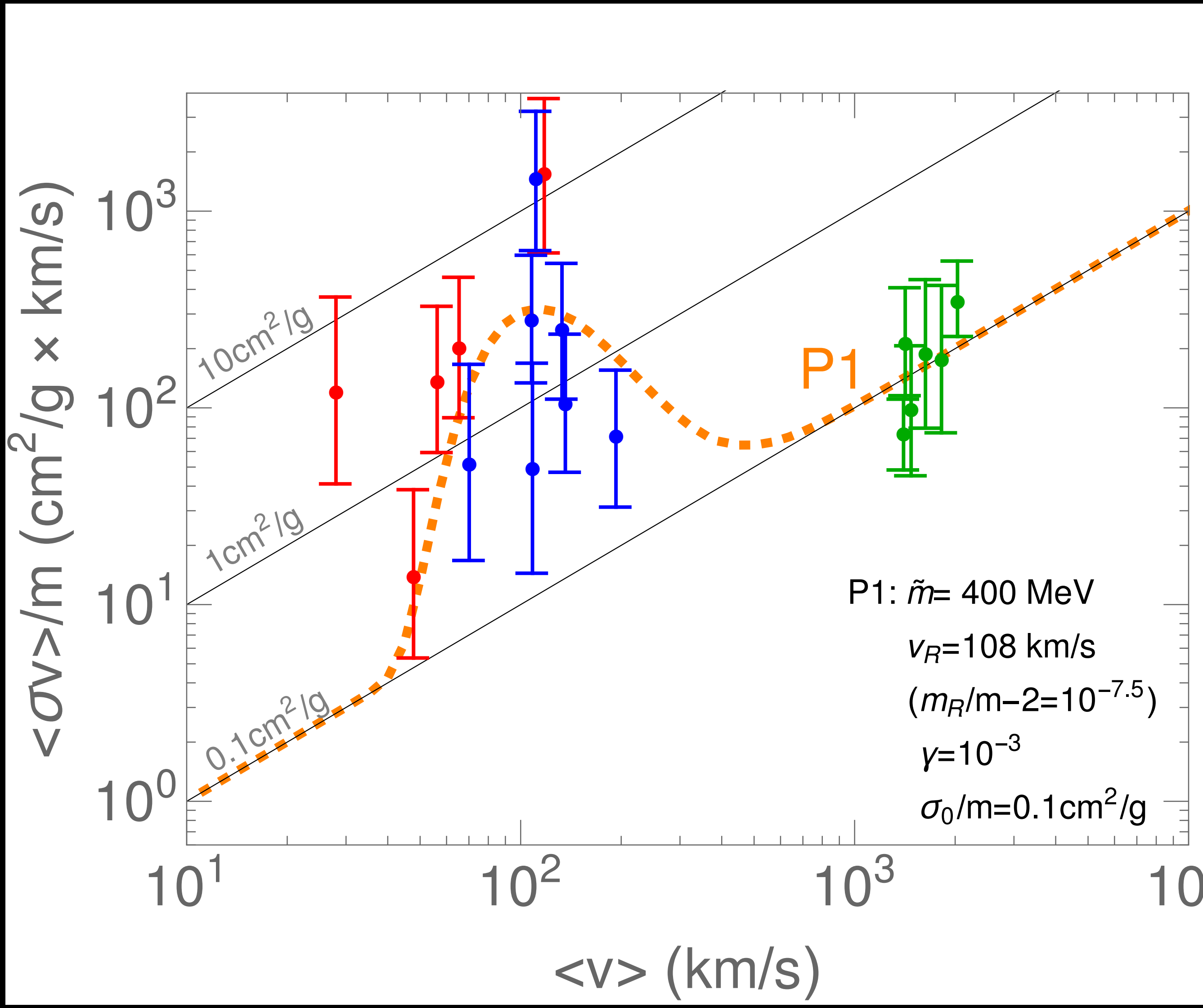


baryon spectrum

- m_u and m_d free parameters
- If $m_d \ll m_u \ll \Lambda_{QCD}$, *n'* dominates
- If $m_u \ll m_d \ll \Lambda_{QCD}$, *p'* dominates, together with π'^- for charge neutrality
  - possibly a resonant interaction $\pi'^- p' \to \Delta^0 \to \pi'^- p'$
  - may solve core/cusp problem



Robert McGehee, HM, Yu-Dai Tsai, in prep



Xiaoyong Chu, Camilo Carcia-Cely, HM, Phys.Rev.Lett. 122 (2019) no.7, 071103

some history

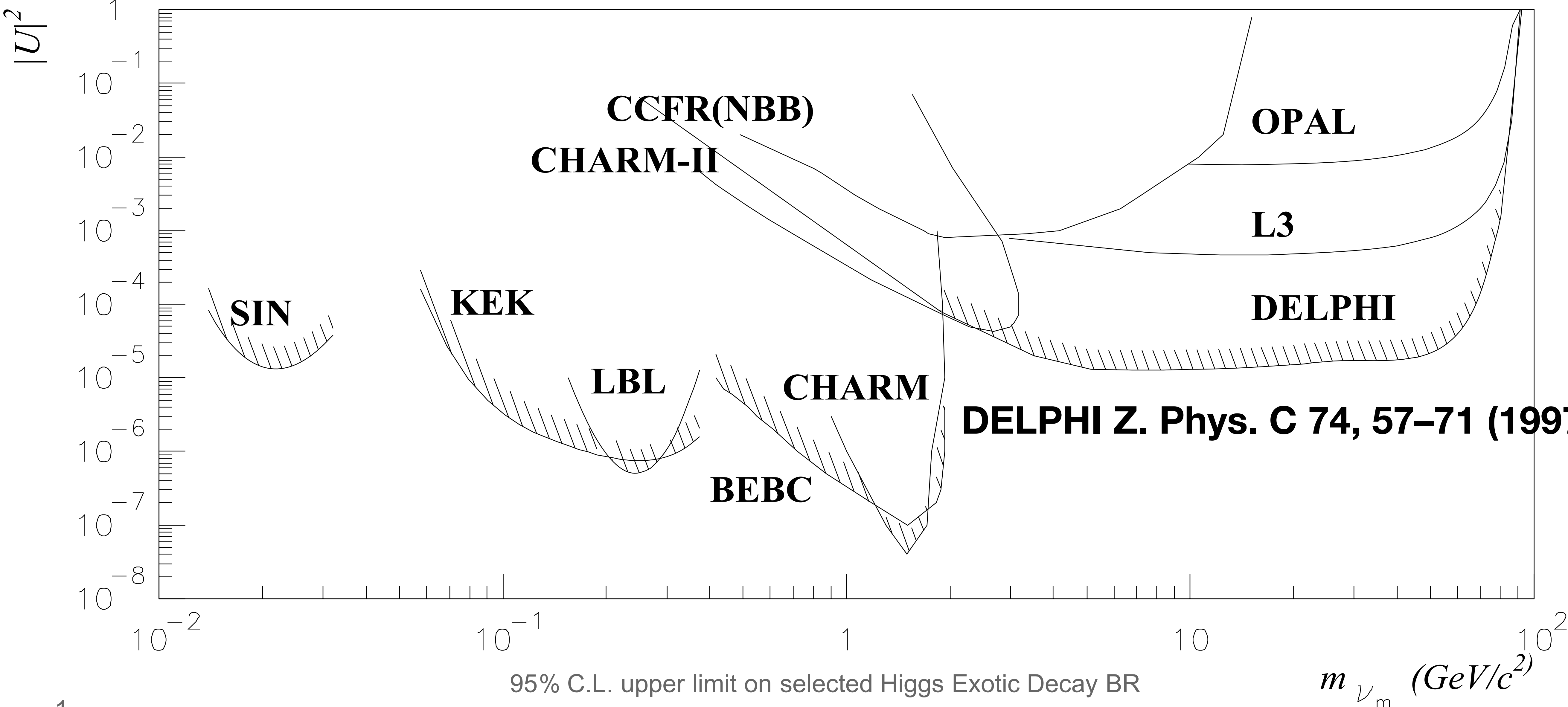
- asymmetric dark matter
  - S. Nussinov, PLB 165, 55 (1985) “technocosmology”
  - R. Kitano, HM, M. Ratz, arXiv:0807.4313, moduli decay
  - D.E. Kaplan, M. Luty, K. Zurek, arXiv:0901.4117
- darkogenesis (= “EW baryogenesis” in the dark sector)
  - J. Shelton, K. Zurek, arXiv:1008.1997

neutrino portal

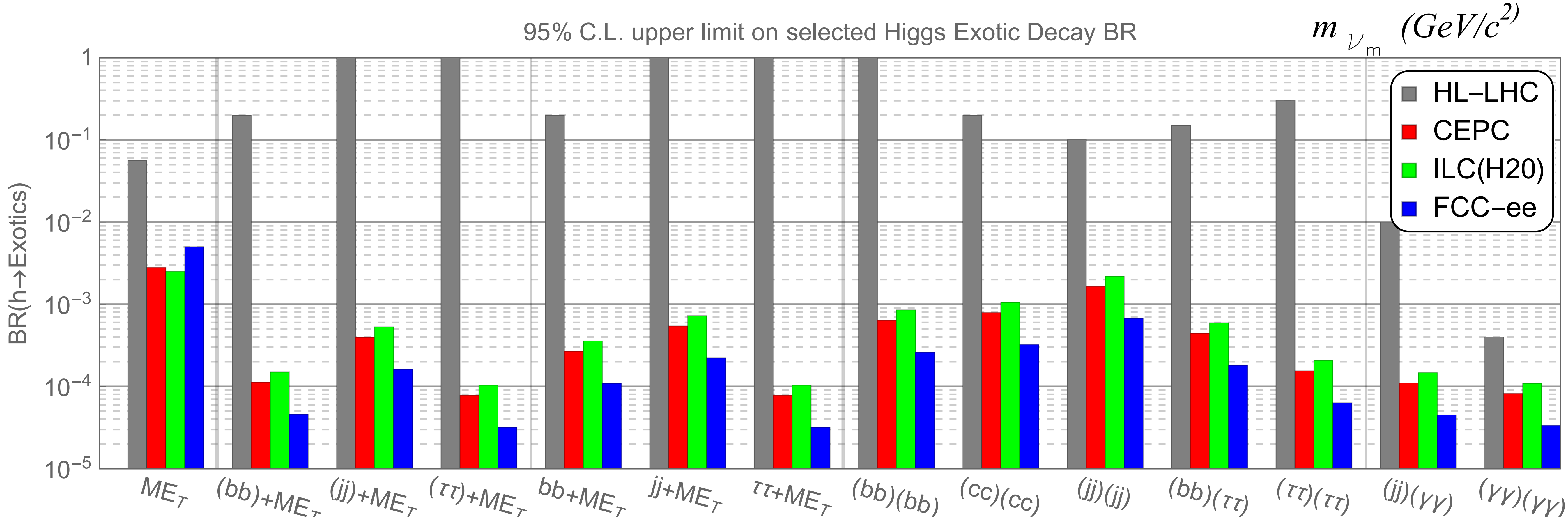
$$\mathcal{L} = y' \bar{L}' H \nu_R + y_i \bar{L}_i H \nu_R$$

$$\epsilon_i = \frac{y_i}{\sqrt{(y')^2 + (y_i)^2}} \qquad M_\nu = \sqrt{(y')^2 + (y_i)^2} v$$

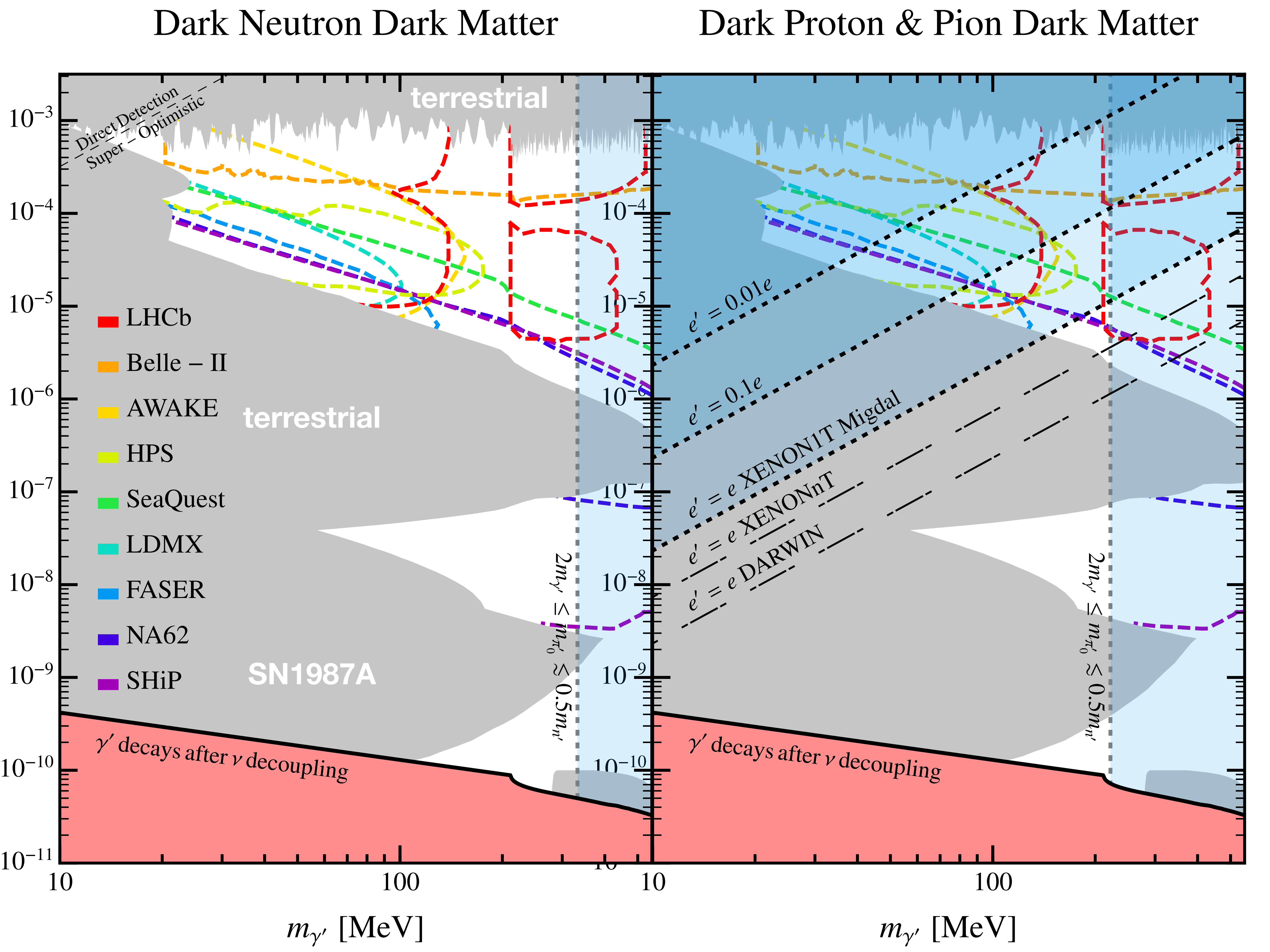
- charged current universality: $\varepsilon_i^2 < 10^{-3}$
- $\mu \to e\,\gamma$ constraint: $\varepsilon_e\,\varepsilon_\mu < 4\times 10^{-5}\,(G_F\,M_\nu)$
- $\tau \to \mu\,\gamma$ constraint: $\varepsilon_e\,\varepsilon_\mu < 0.03\,(G_F\,M_\nu)$
- If $M_\nu < 70$ GeV, $\varepsilon_i^2 < 10^{-5}$ (DELPHI: $Z \to \nu\,\nu_R$, $\nu_R \to l f f$)
- equilibration of asymmetries requires only $\varepsilon_i > 10^{-16}$ or so
- (orders of magnitude estimates so far)



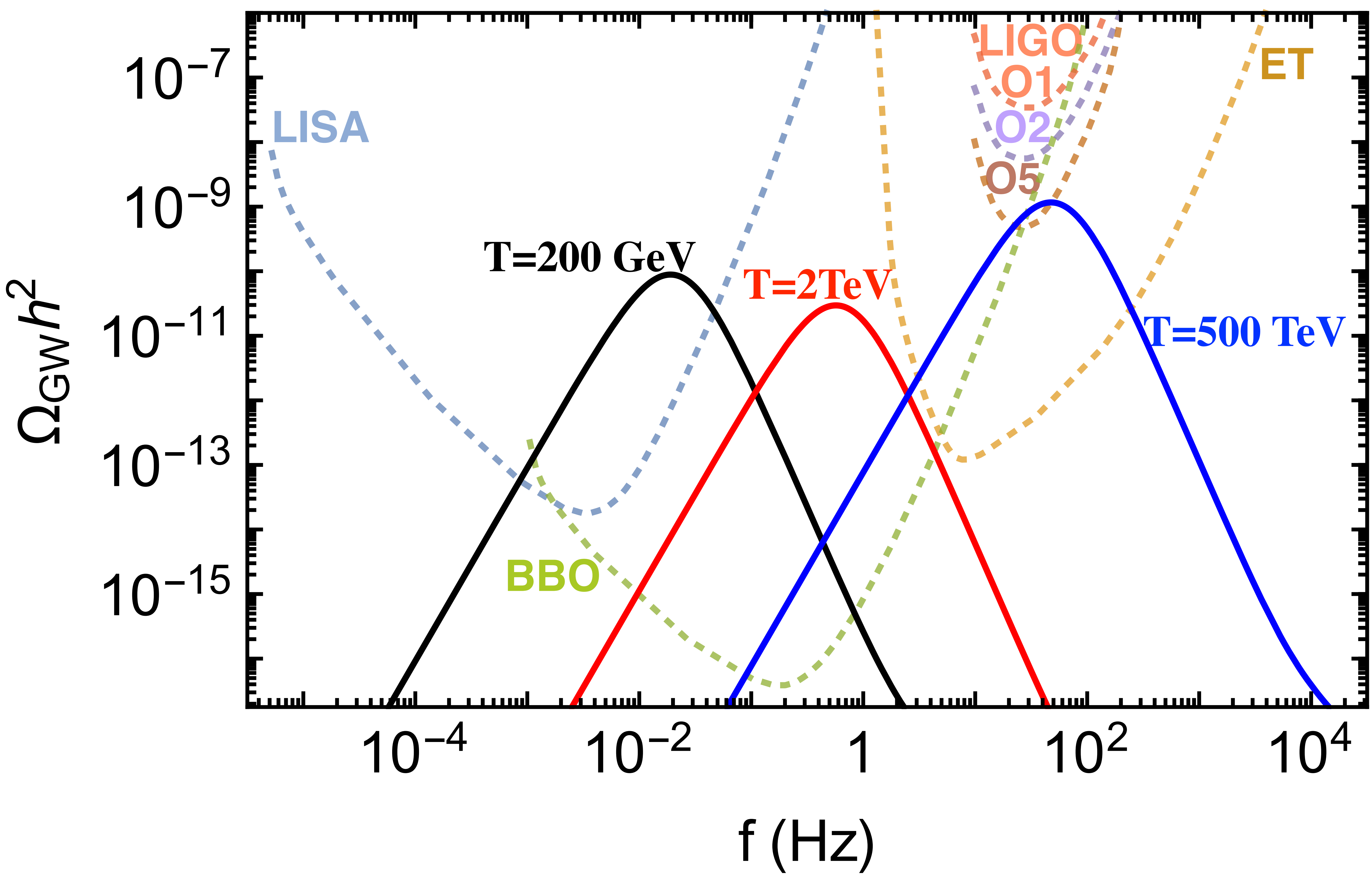
DELPHI Z. Phys. C 74, 57–71 (1997)



Zhen Liu, Lian-Tao Wang, Hao Zhang, arXiv:1612.09284



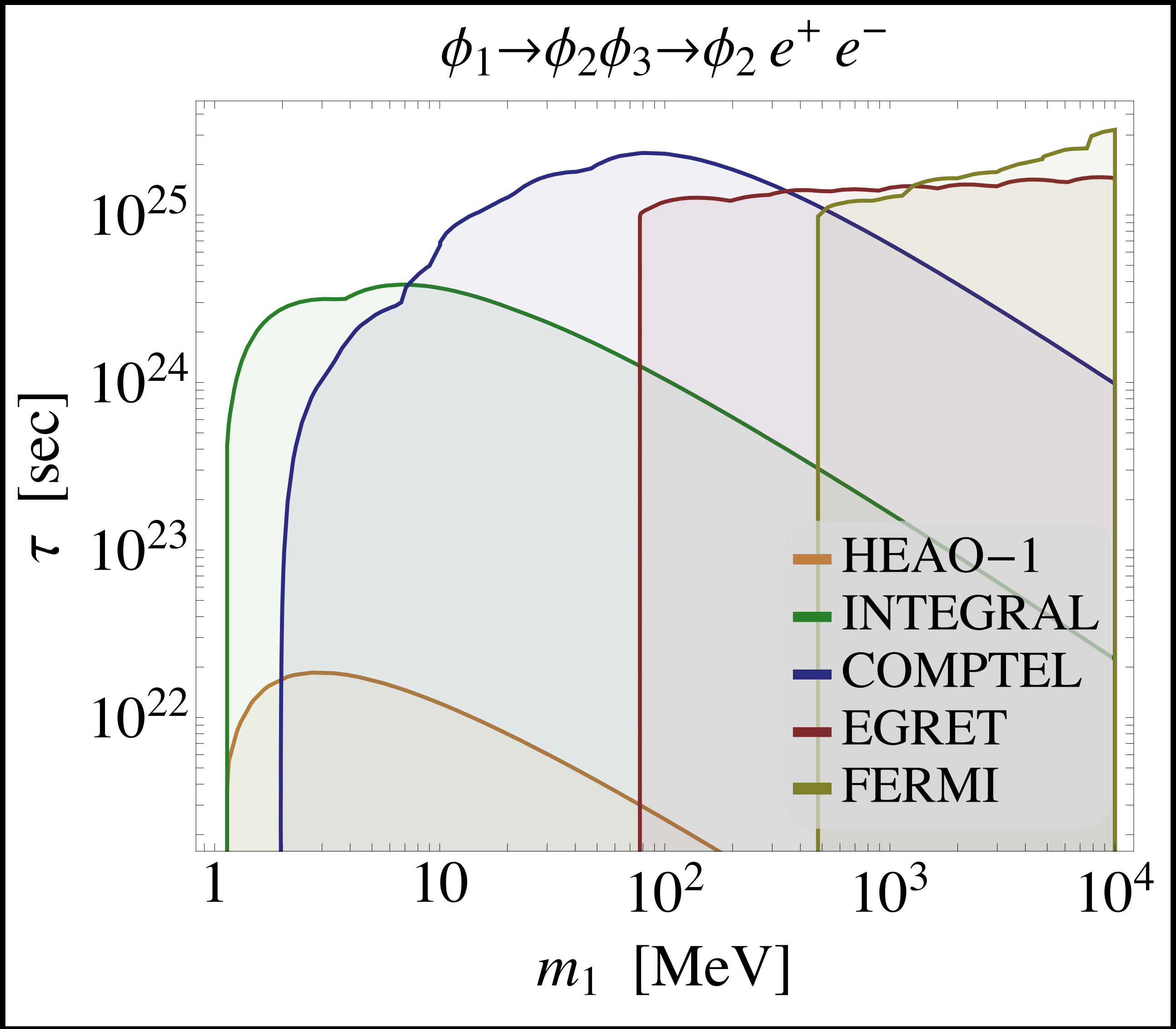
Dark Neutron Dark Matter
Dark Proton & Pion Dark Matter
terrestrial
Direct Detection
Super – Optimistic
LHCb
Belle – II
AWAKE
HPS
SeaQuest
LDMX
FASER
NA62
SHiP
terrestrial
SN1987A
$2m_{\gamma'} \leq m_{\pi_0'} \lesssim 0.5 m_{n'}$
γ' decays after ν decoupling
$e' = 0.01e$
$e' = 0.1e$
$e' = e$ XENON1T Migdal
$e' = e$ XENONnT
$e' = e$ DARWIN
$m_{\gamma'}$ [MeV]



LIGO
O1
O2
O5
ET
LISA
BBO
T=200 GeV
T=2TeV
T=500 TeV
10^{-7}
10^{-9}
10^{-11}
10^{-13}
10^{-15}
$\Omega_{GW}h^2$
10^{-4}
10^{-2}
1
10^{2}
10^{4}
f (Hz)

exotic signal

- SU(2)' instanton generates $u' \, d' \, d' \, \nu' \, e^{-8\pi^2/g^2} / v^2$
- dark neutron mixes to dark neutrino to neutrino portal to SM neutrino, decays into SM ℓ + (qqbar or $\ell\nu$)
- indirect detection of gamma's from galactic halos $\tau > 10^{25}$sec
- can happen if $\alpha'_W > 0.3$
- not possible when $N_{gen} > 1$



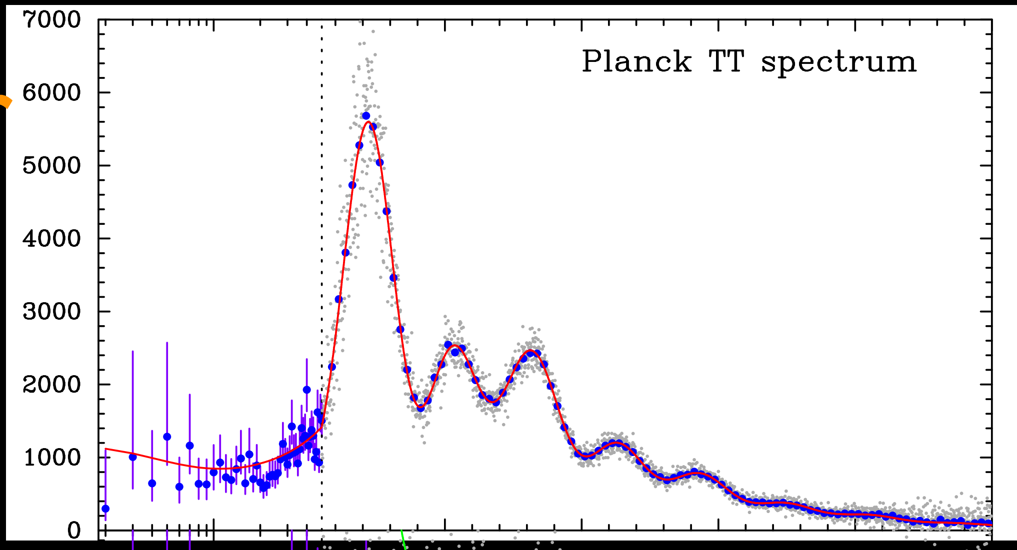
Essig, Kuflik, McDermott, Volansky, Zurek
aarXiv:1309.4091

Conclusions

- Electroweak baryogenesis *too testable*, dead?
  - do it in the dark sector
- dark SU(3)xSU(2)xU(1), one generation
  - two Higgs doublet CPV, 1st order phase transition
  - neutrino portal to transfer asymmetry to SM baryons
- dark neutron 1.33 or 1.58 GeV, or multi-component $p+\pi^-$
- amazingly wide array of experimental signatures
  - dark proton good target for direct detection
  - exotic Z-decay, h-decay (ILC, CEPC, FCC-ee)
  - dark photon search at Belle II, LHC-b, beam dump
  - gravitational wave at LIGO, LISA, Einstein Telescope, etc
  - potential instanton-induced dark neutron decay in halos
- explain coincidence Ω_{DM}~Ω_b if N_{gen}=3 and unification

Five evidences for physics beyond SM

- Since 1998, it became clear that there are at least five missing pieces in the SM
  - non-baryonic dark matter
  - neutrino mass
  - dark energy
  - apparently acausal density fluctuations
  - baryon asymmetry



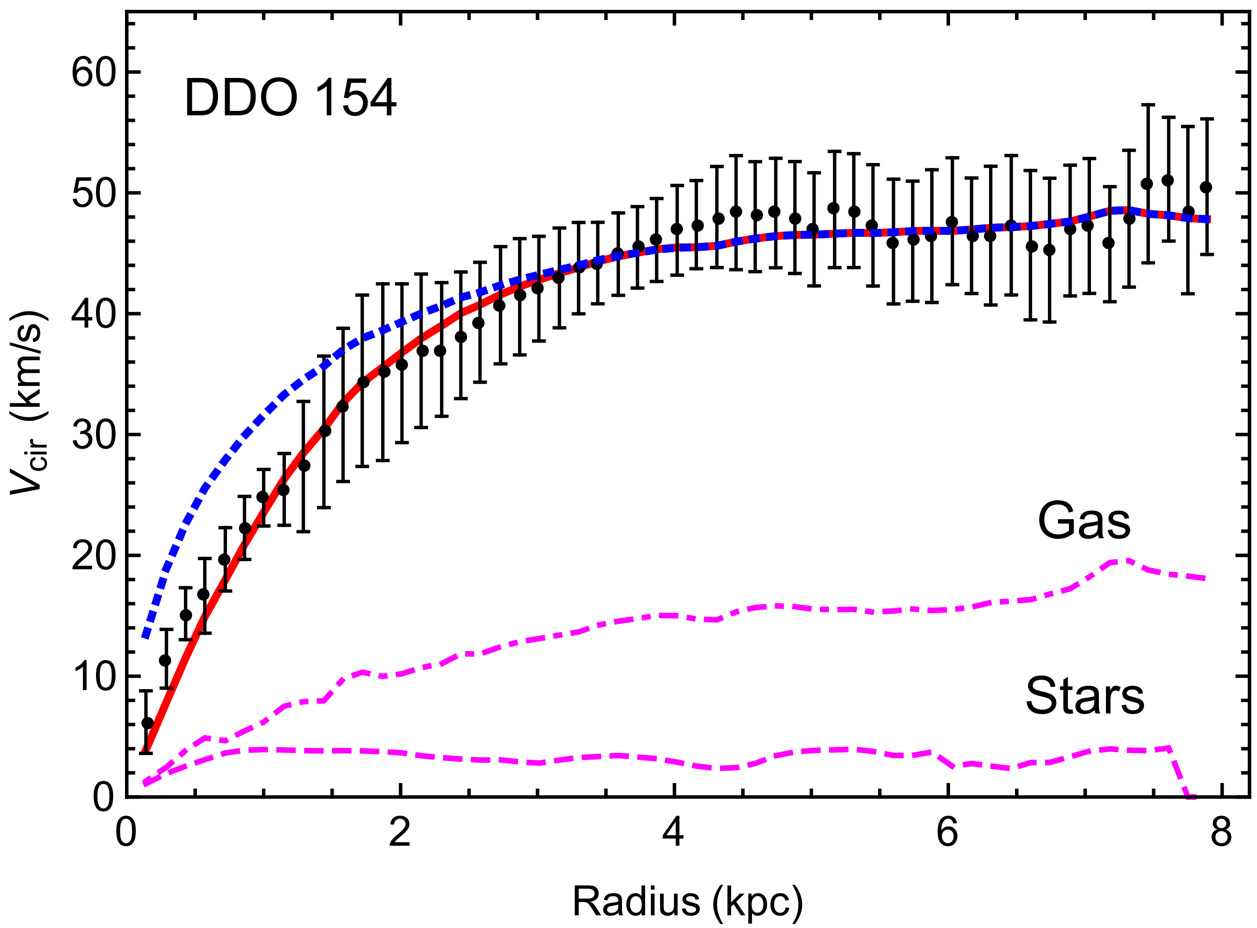
We don't really know their energy scales...

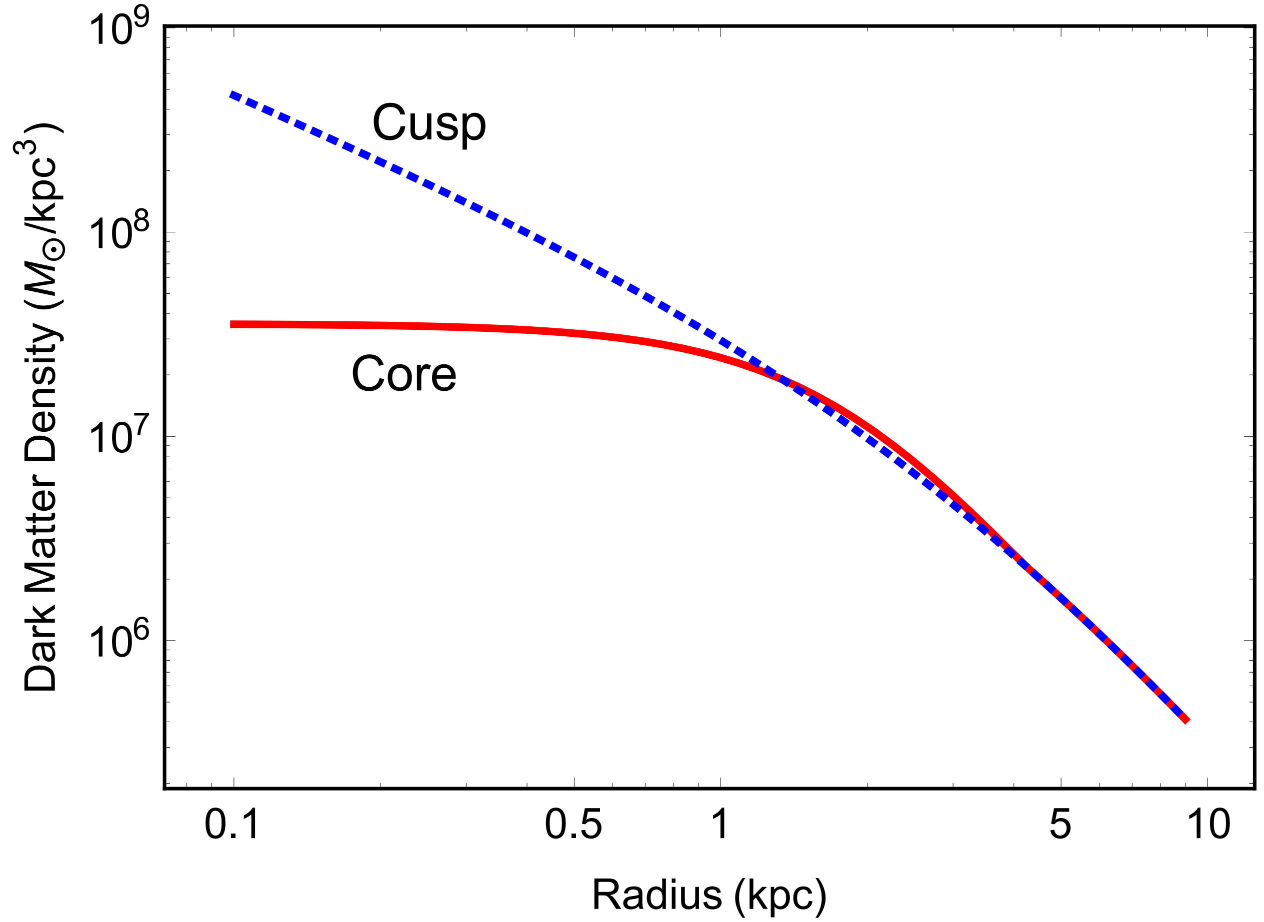
experiments

healthy field!


Still many things
to look forward to!

DDO 154 dwarf galaxy



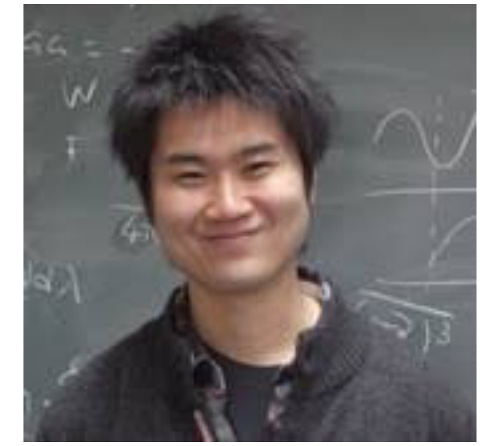


can be explained if dark matter scatters against itself

Need $\sigma/m \sim$ 1b / GeV

only astrophysical information beyond gravity

Diversity in stellar distribution

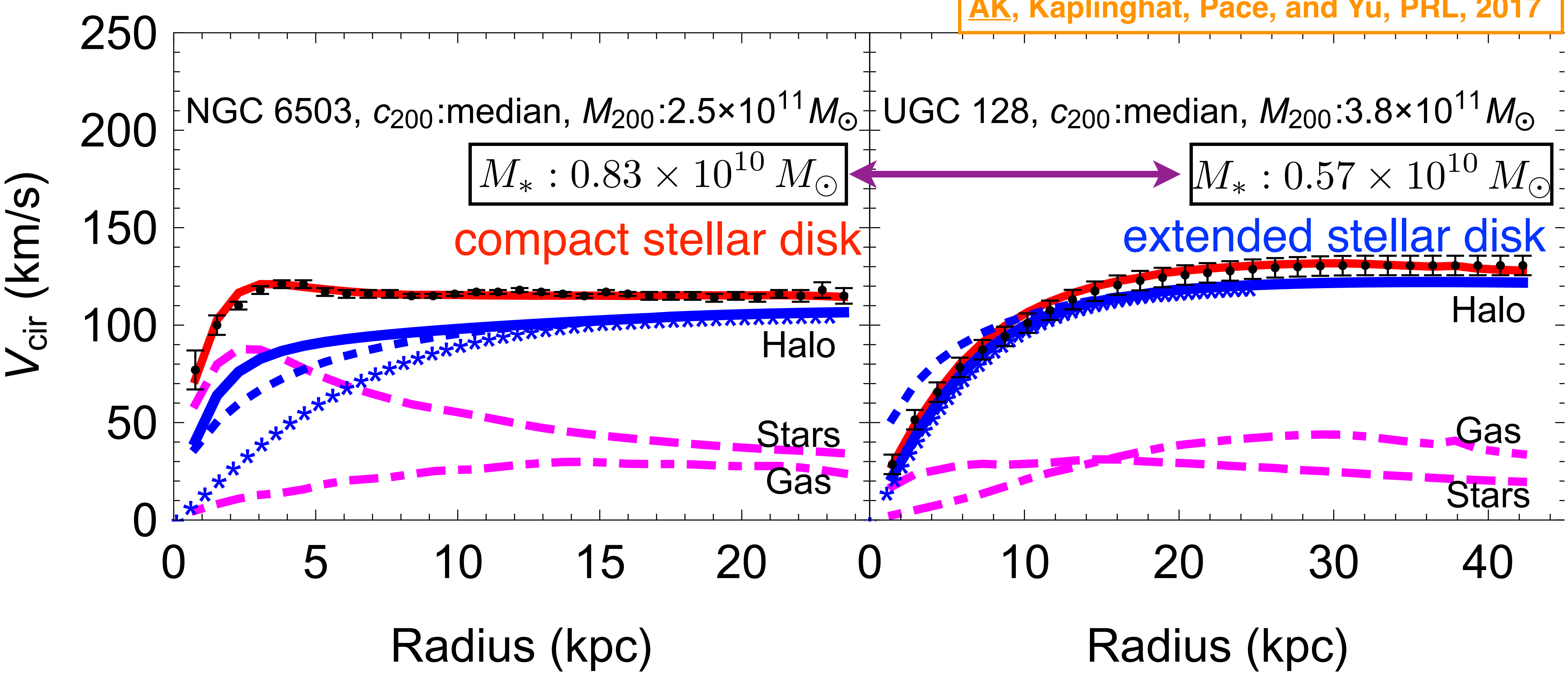


Ayuki Kamada

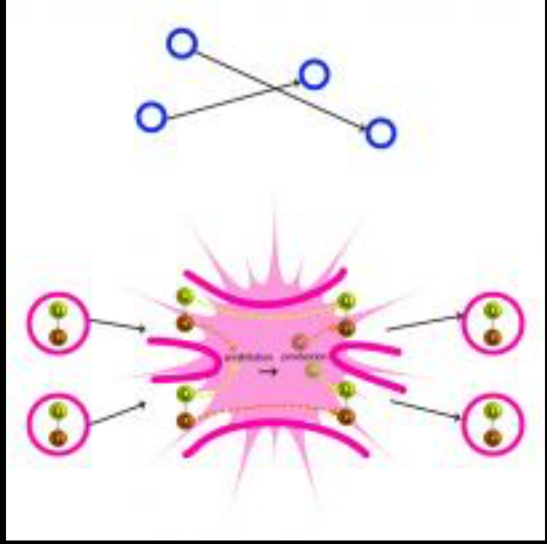
Similar outer circular velocity and stellar mass, but different stellar distribution

- compact → redistribute SIDM significantly
- extended → unchange SIDM distribution

AK, Kaplinghat, Pace, and Yu, PRL, 2017

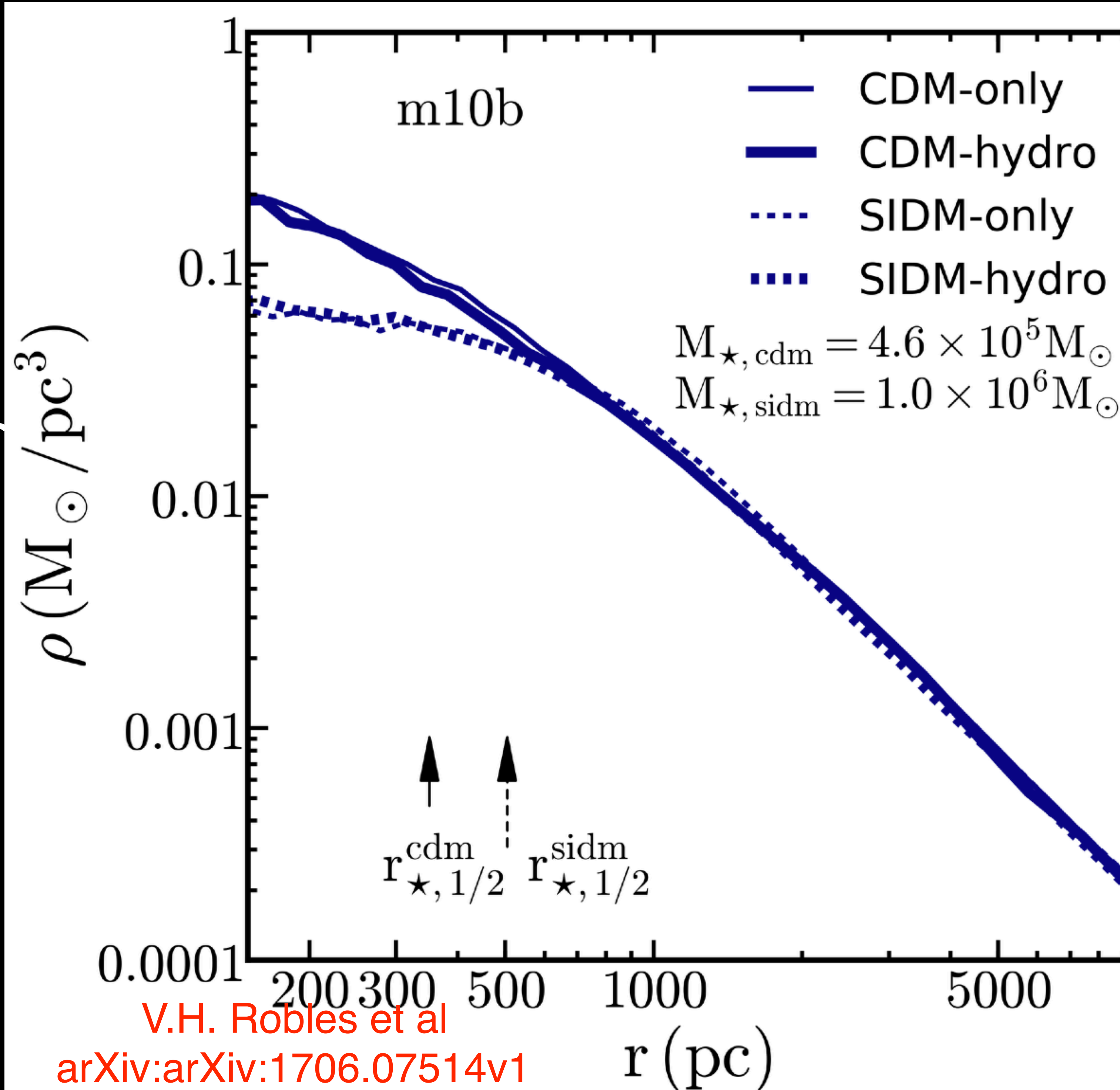


self interaction



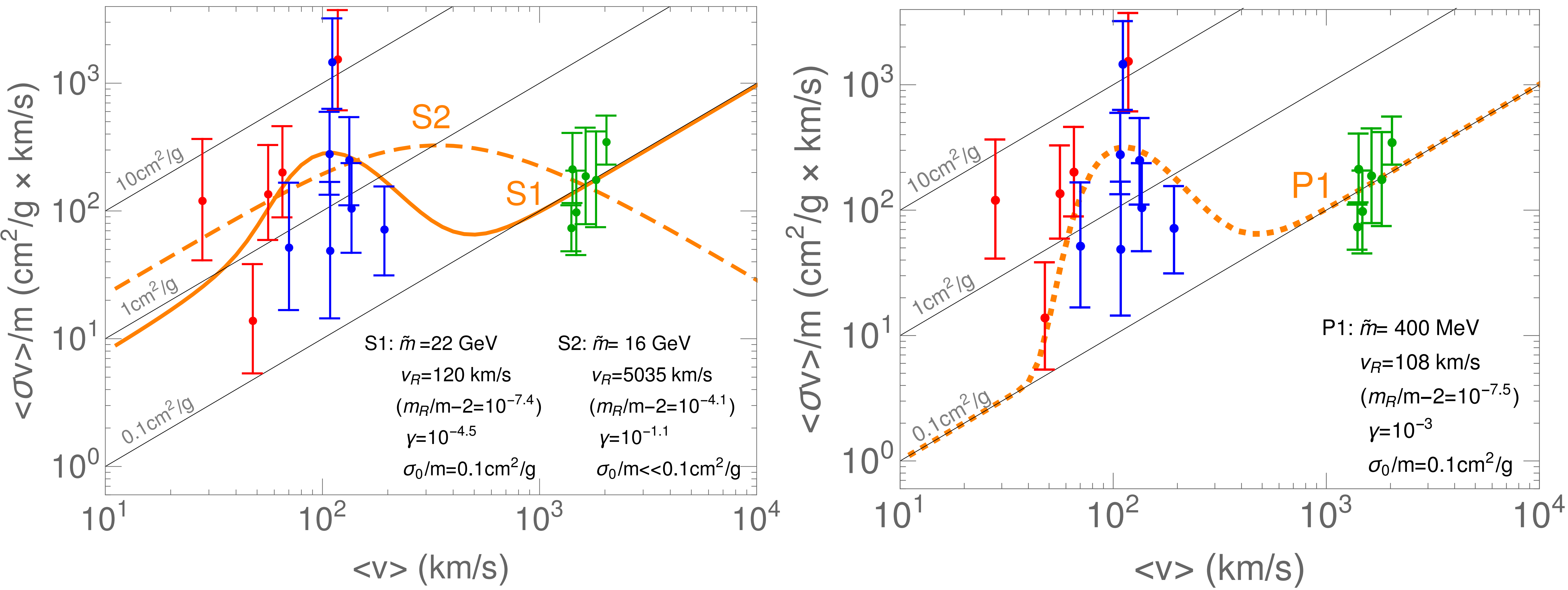
- $\sigma/m \sim cm^2/g$
  $\sim 10^{-24} cm^2$ / 300MeV
- flattens the cusps in NFW profile
- suppresses substructur
- actually desirable for dwarf galaxies?

SIDM
Spergel & Steinhardt
(2000)
now complete theory



V.H. Robles et al
arXiv:arXiv:1706.07514v1

Resonant scattering



Xiaoyong Chu, Camilo Garcia-Cely, HM,
Phys.Rev.Lett. 122 (2019) no.7, 071103

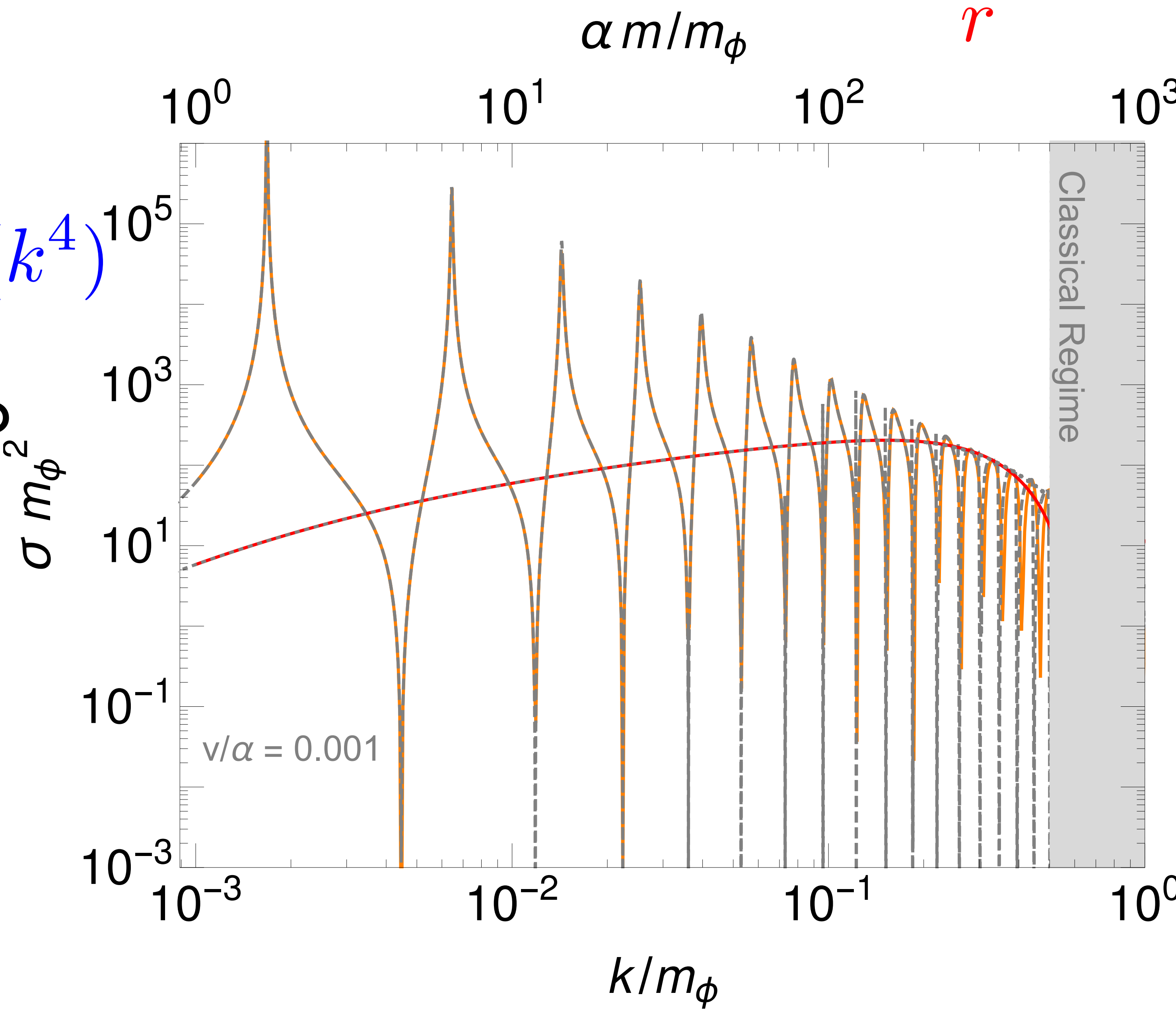
Unified description of SIDM

$$V = -\alpha \frac{e^{-m_\phi r}}{r}$$

- Hans Bethe: effective range theory

$$k \cot \delta = -\frac{1}{a} + \frac{1}{2} r_e k^2 + O(k^4)$$

- only two parameters to describe scattering at low velocities
- fully unitary and non-perturbative
- ideal for simulations!



Xiaoyong Chu, Camilo Garcia-Cely, HM, arXiv:1908.06067