

UC-Davis

QMAP Particles/Cosmology Seminar

Hunting for **Gluonic** ALPs



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When we talk about Axion-Like-Particles (ALPs)...

In the particle realm, we most of the time just talked about a pseudo scalar...

But the defining coupling is to **gluons**, to make some connections to strong CP.
Can we motivate that?

Axion needs no intro

$$L \supset \frac{\alpha_s}{8\pi} \theta \tilde{G}G + y_u \bar{Q}_L \tilde{H} u_R + y_d \bar{Q}_L H d_R$$

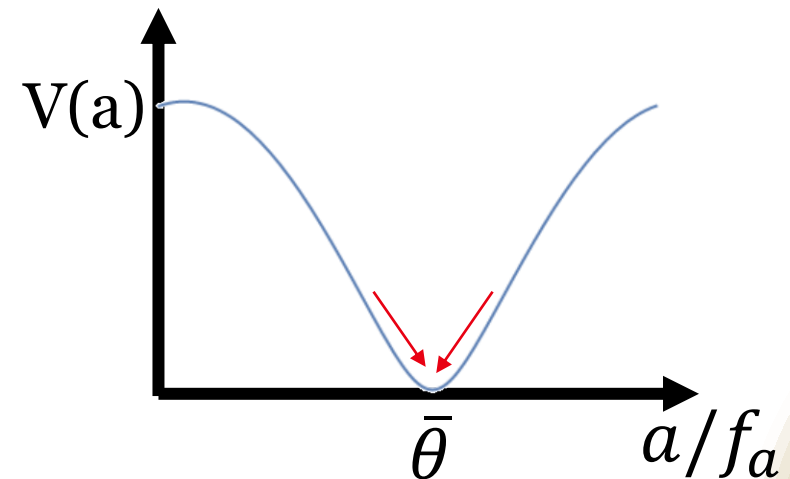
$$\bar{\theta} \equiv \theta + \text{ArgDet}[Y_u Y_d] \leq 10^{-10}$$

While $\text{ArgDet}[Y_u Y_d]$ anticipated around $\delta_{CKM} \sim O(1)$

Strong CP puzzle of QCD

Dynamical solution:
QCD Axion a as a pseudo
Nambu-Goldstone boson

$$\frac{\alpha_s}{8\pi} \left(\theta - \frac{a}{f_a} \right) \tilde{G}G$$



But a quality problem

The axion fakes a dynamical angle.
How good of an imposter is it?

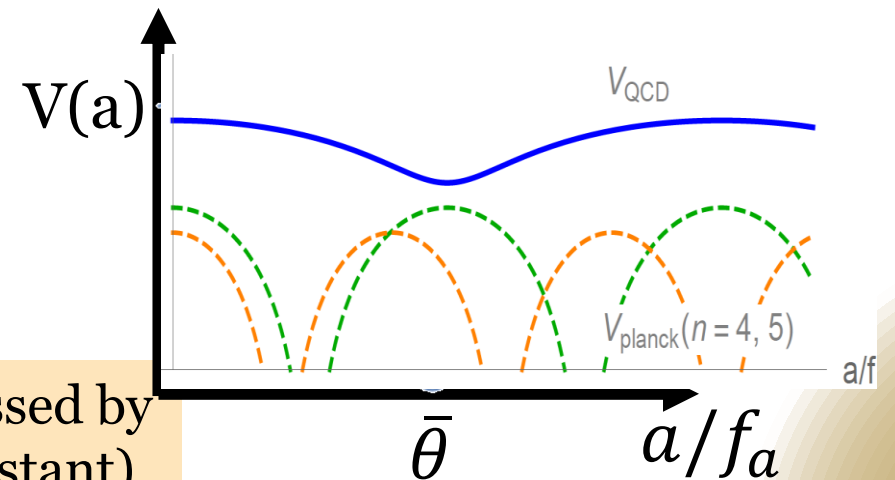
Dynamical solution:
QCD Axion a as a pseudo
Nambu-Goldstone boson

$$V \approx -(100 \text{ MeV})^4 \cos\left(\bar{\theta} - \frac{a}{f_a}\right) + \Lambda_{\text{contamination}}^4 \cos\left(\theta' - \frac{a}{f_a}\right) - \frac{\alpha_s}{8\pi} \left(\theta - \frac{a}{f_a}\right) \tilde{G}G$$

$$\Lambda_{\text{contamination}} < 0.1 \text{ MeV}$$

There are also many other scales :
GUT, Planck, Dark matter

$$V = \frac{\Phi^{14}}{M_{pl}^{10}} \quad \Phi \equiv e^{i\frac{a}{f_a}}$$

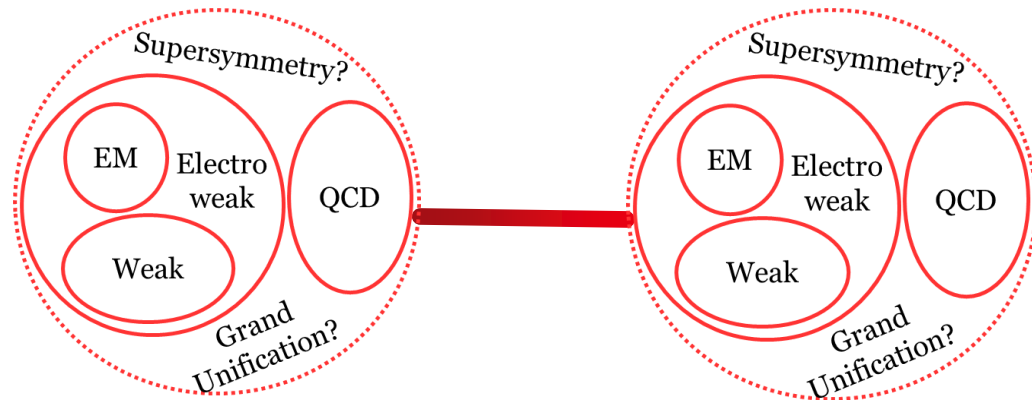


Leading order gravity suppressed operators need to be suppressed by 10 powers of the Planck scale! (for 10^{12} GeV Axion decay constant)

The Quality Problem and reinforced Axion potential

Copying Mirror Gauge QCD + Weak and Chiral Matter fields, relates the Lagrangian parameters with a Z_2 symmetry

one axion couples to both and **solve both** strong CP puzzles dynamically.



$$\begin{aligned}
 SU(2) &\leftrightarrow SU(2)' \\
 SU(3) &\leftrightarrow SU(3)' \\
 U(1) &\leftrightarrow U(1)' \text{ or } U(1) \leftrightarrow U(1) \\
 \Psi &\leftrightarrow \Psi'
 \end{aligned}$$

\leftrightarrow represents the Z_2 transformation
 X' represents the mirror sector

Softly broken by
 $\mu^2 H^\dagger H + \mu'^2 H'^\dagger H'$
 with $|\mu'^2| \gg |\mu^2|$

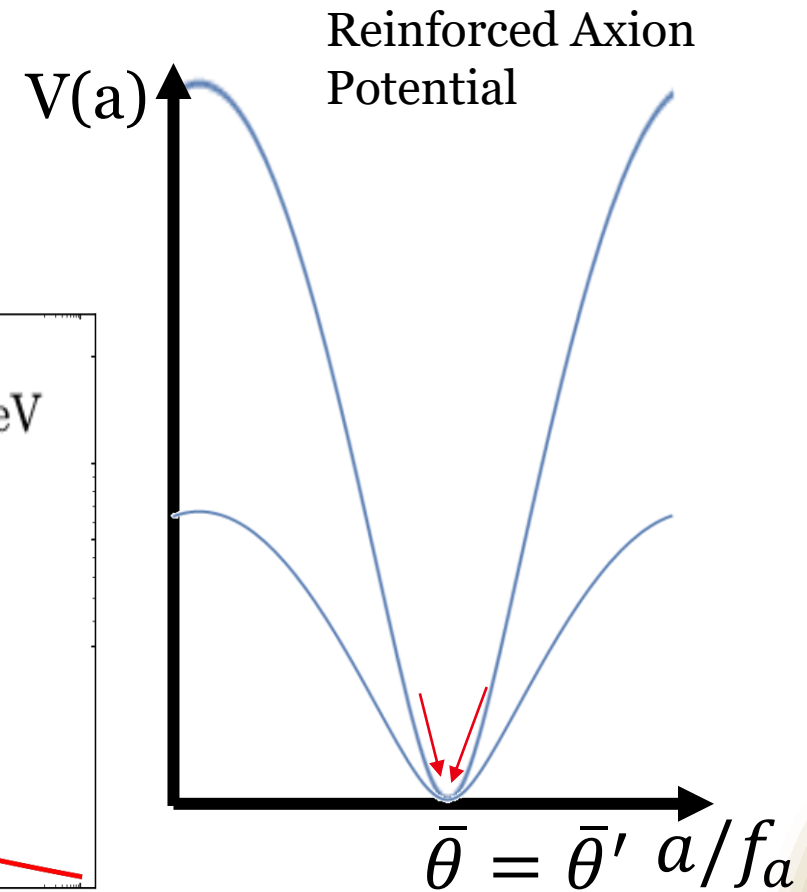
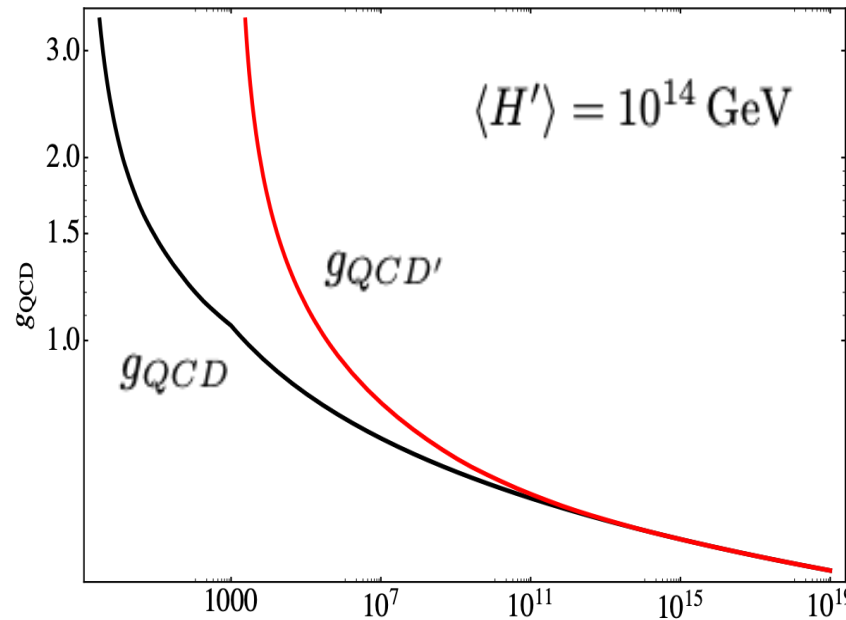
The Quality Problem and reinforced Axion potential

Soft Z2 breaking by giving Mirror Higgs large VEV
 massive fermions
 decouples earlier
 mirror QCD run fast
 and confines

$$m_a^2 \simeq \frac{m_u m_d}{(m_u + m_d)^2} \frac{m_\pi f_\pi}{f_a^2}$$



$$m_a^2 \simeq \frac{\Lambda_{QCD'}^4}{f_a^2}$$



The Quality Problem and reinforced Axion potential

Makes the Quality problem better

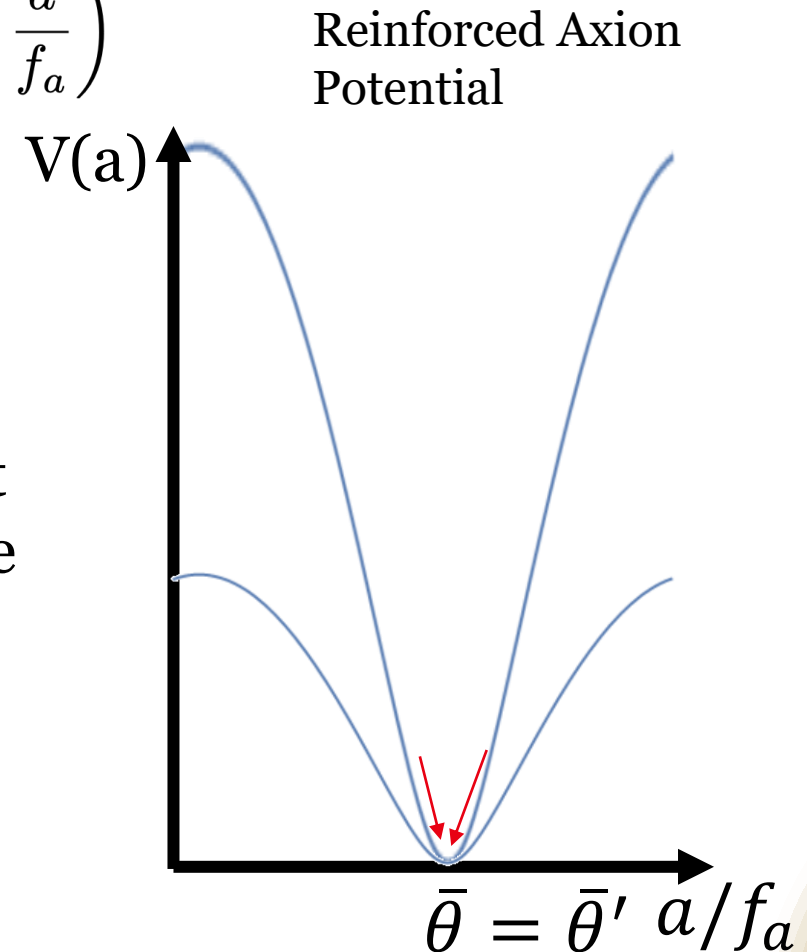
$$V \approx -(100 \text{ MeV})^4 \cos\left(\bar{\theta} - \frac{a}{f_a}\right) - (10^8 \text{ MeV})^4 \cos\left(\bar{\theta} - \frac{a}{f_a}\right) \\ + \Lambda_{\text{contamination}}^4 \cos\left(\theta' - \frac{a}{f_a}\right)$$

$$\Lambda_{\text{contamination}} < 10^5 \text{ MeV}$$

If the Higgs mass were the only thing different between the two copies, the neutron angles are still the same!

Flavor structure of the SM ensures that any change occurs in very high loop orders.

Not true for a generic theory!



The Quality Problem and reinforced Axion potential

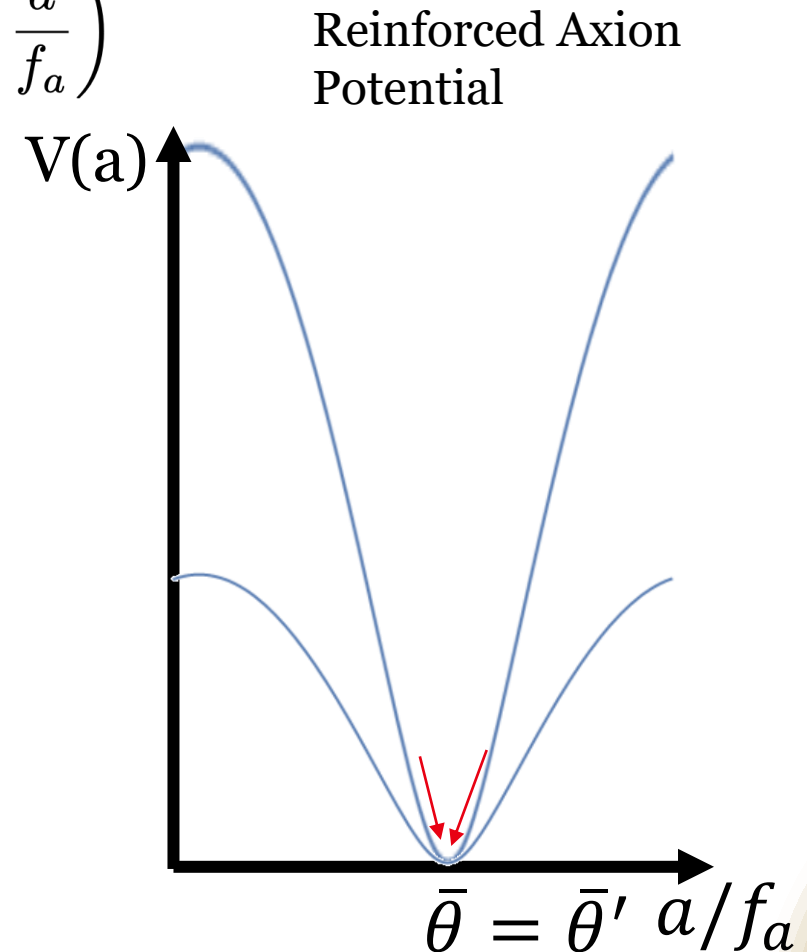
Makes the Quality problem better

$$V \approx -(100 \text{ MeV})^4 \cos\left(\bar{\theta} - \frac{a}{f_a}\right) - (10^8 \text{ MeV})^4 \cos\left(\bar{\theta} - \frac{a}{f_a}\right) \\ + \Lambda_{\text{contamination}}^4 \cos\left(\theta' - \frac{a}{f_a}\right)$$

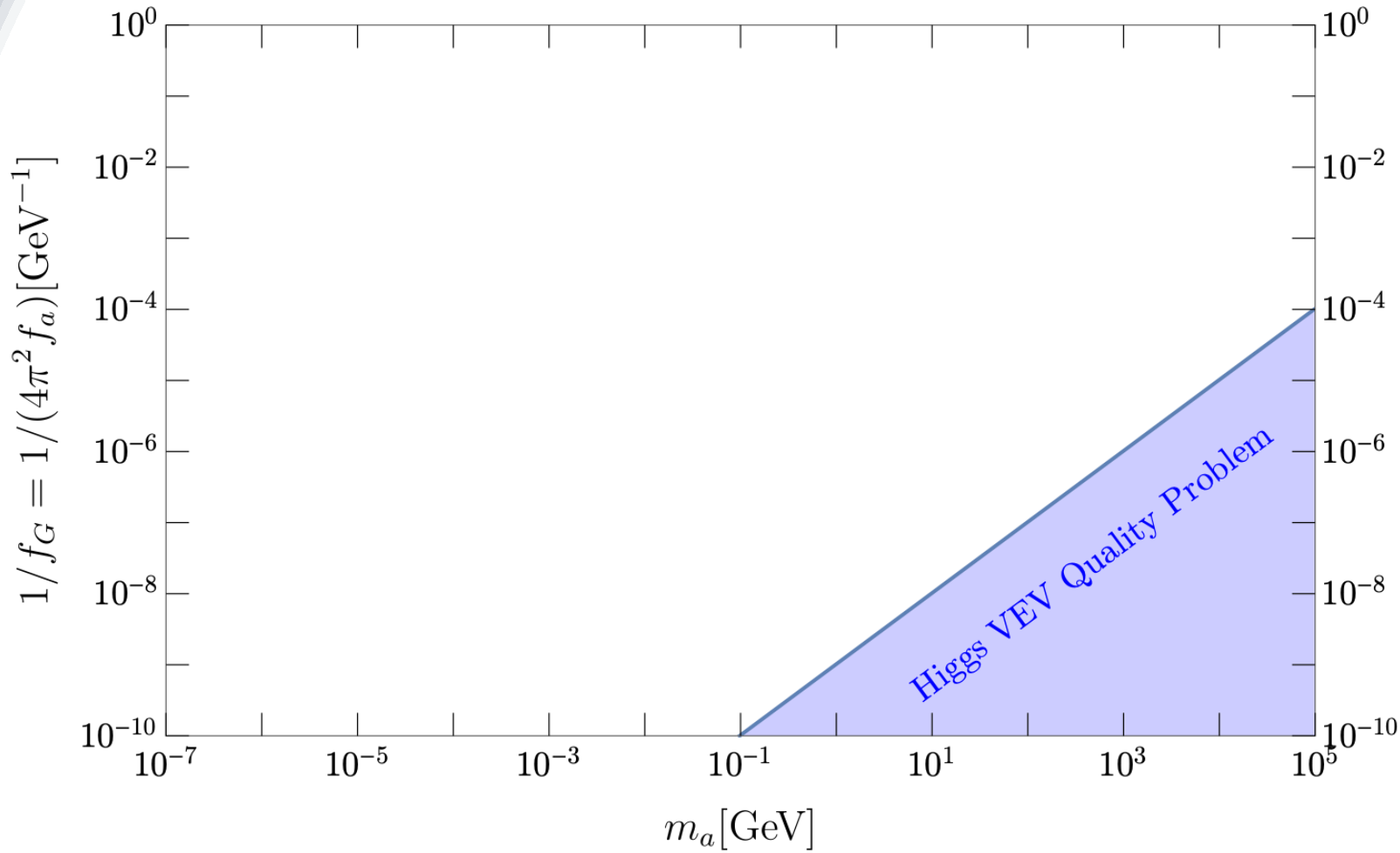
$$\Lambda_{\text{contamination}} < 10^5 \text{ MeV}$$

But generates a new quality problem:

$$\frac{g^2}{32\pi^2} \left(\frac{HH^\dagger}{M_{pl}^2} G\tilde{G} + \frac{H'H'^\dagger}{M_{pl}^2} G'\tilde{G}' \right) \\ H' \lesssim 10^{14} \text{ GeV}$$



Theory parameter space



$$m_a^2 \simeq \frac{\Lambda_{QCD'}^4}{f_a^2}$$

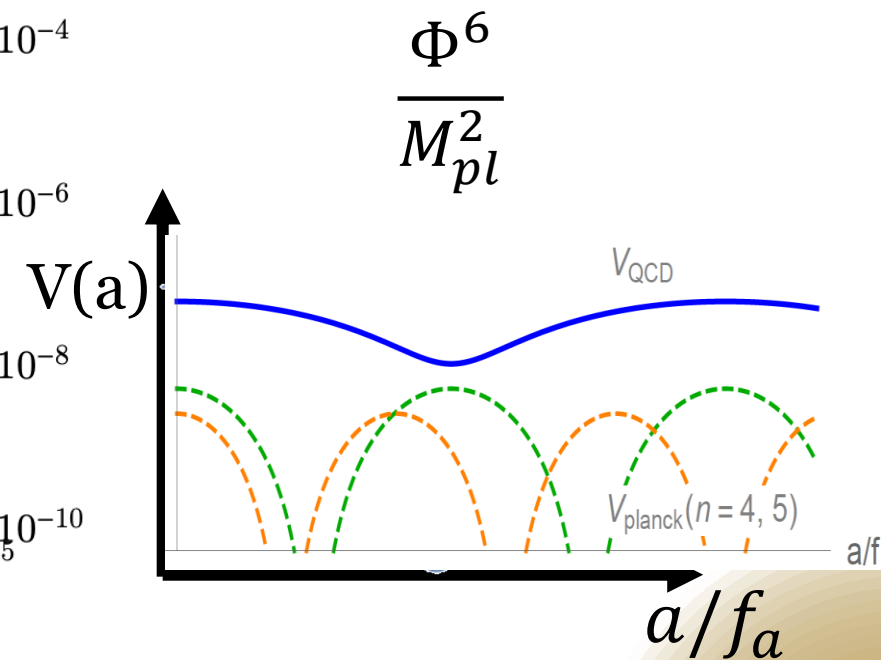
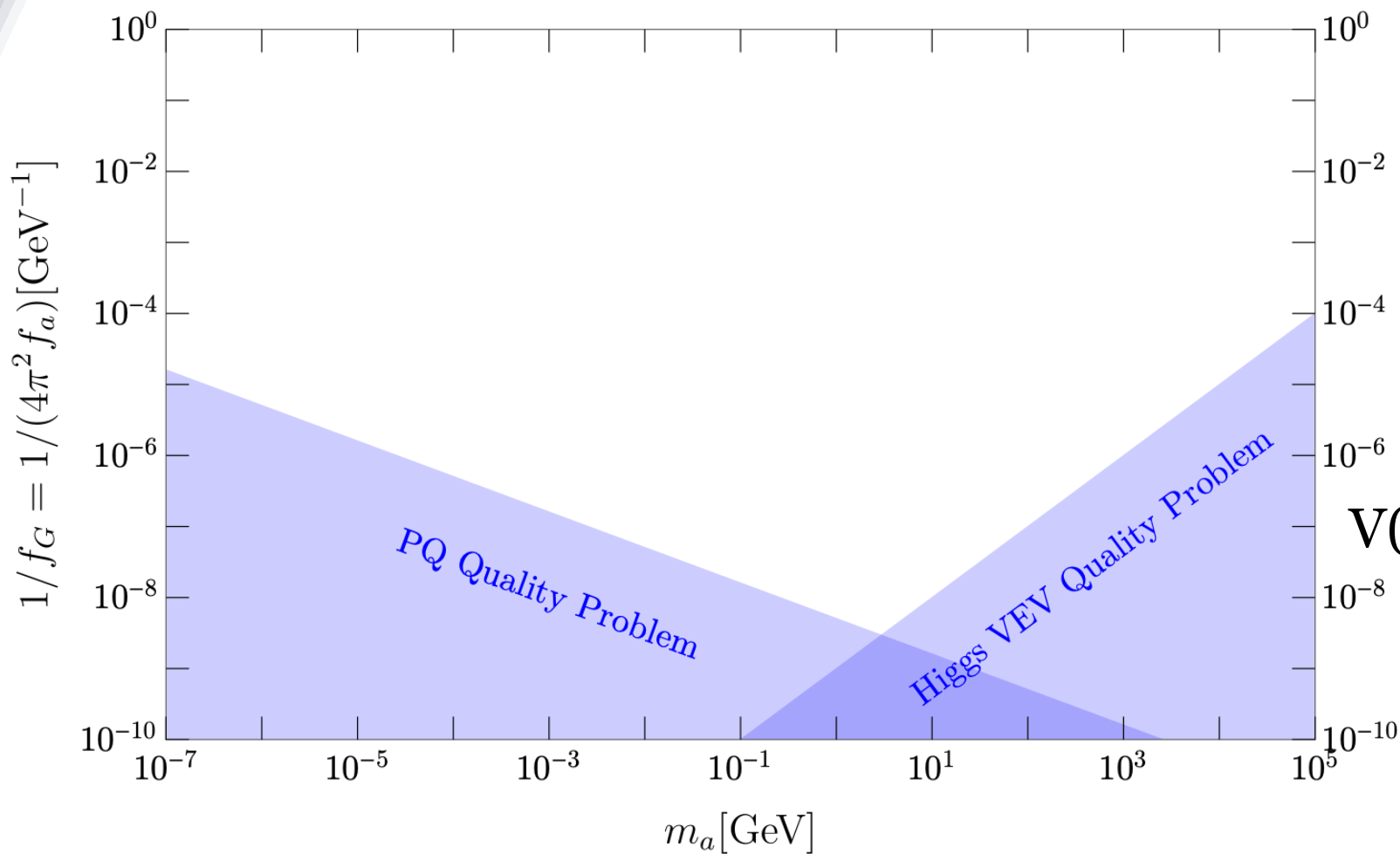
$$\frac{\alpha_3}{8\pi} \left(\frac{H^\dagger H}{M_p^2} G\tilde{G} + \frac{H'^\dagger H'}{M_p^2} G'\tilde{G}' \right)$$

$$\Rightarrow \langle H' \rangle = \mu' < 10^{14} \text{ GeV}$$

$$\frac{a}{8\pi f_a} \left(c_3 \alpha_3 G\tilde{G} + c_2 \alpha_2 W\tilde{W} + c_1 \alpha_1 B\tilde{B} \right)$$

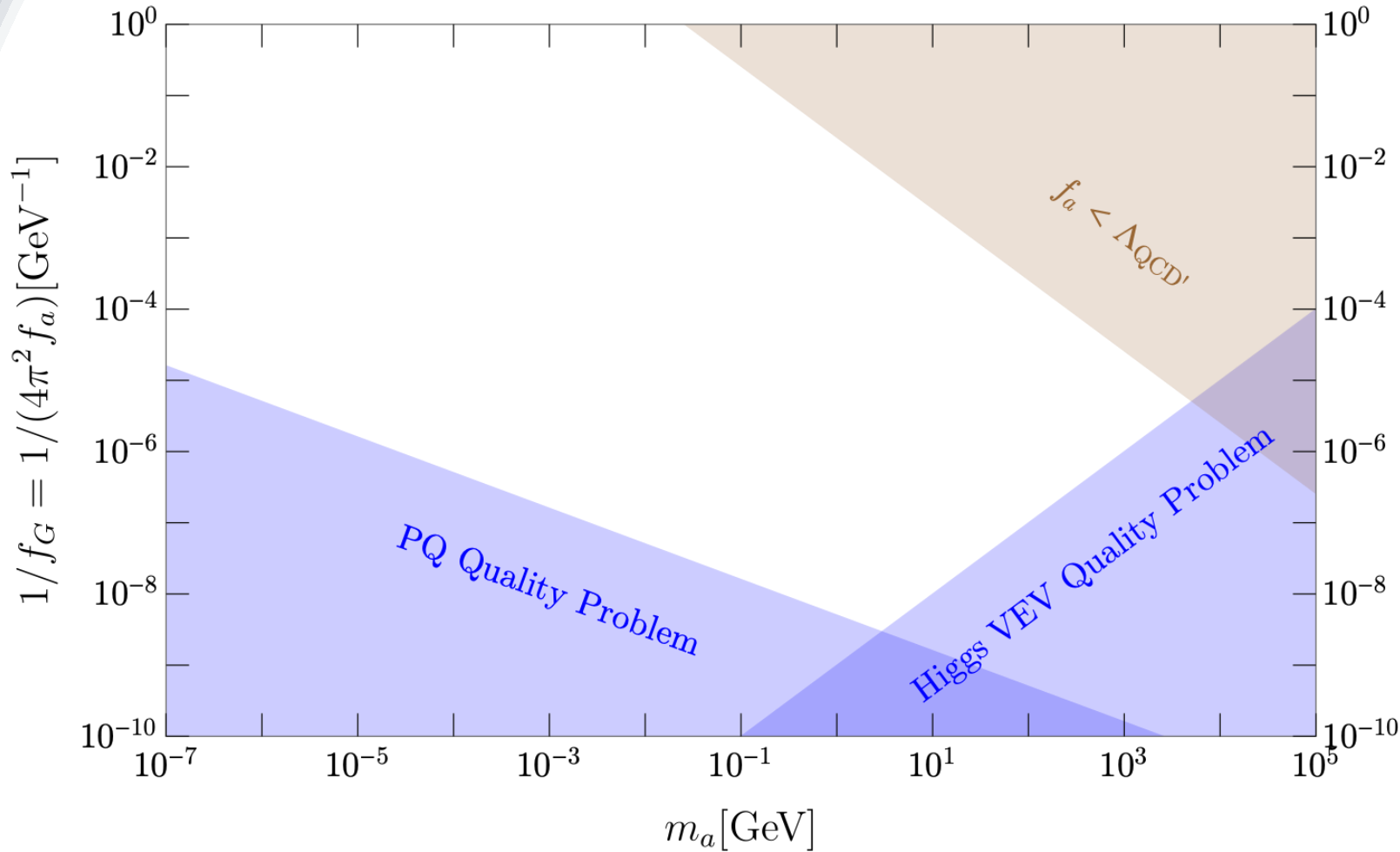
Theory parameter space

$$m_a^2 \simeq \frac{\Lambda_{QCD'}^4}{f_a^2}$$



$$\frac{a}{8\pi f_a} (c_3 \alpha_3 G\tilde{G} + c_2 \alpha_2 W\tilde{W} + c_1 \alpha_1 B\tilde{B})$$

Theory parameter space



$$m_a^2 \simeq \frac{\Lambda_{\text{QCD}'^4}}{f_a^2}$$

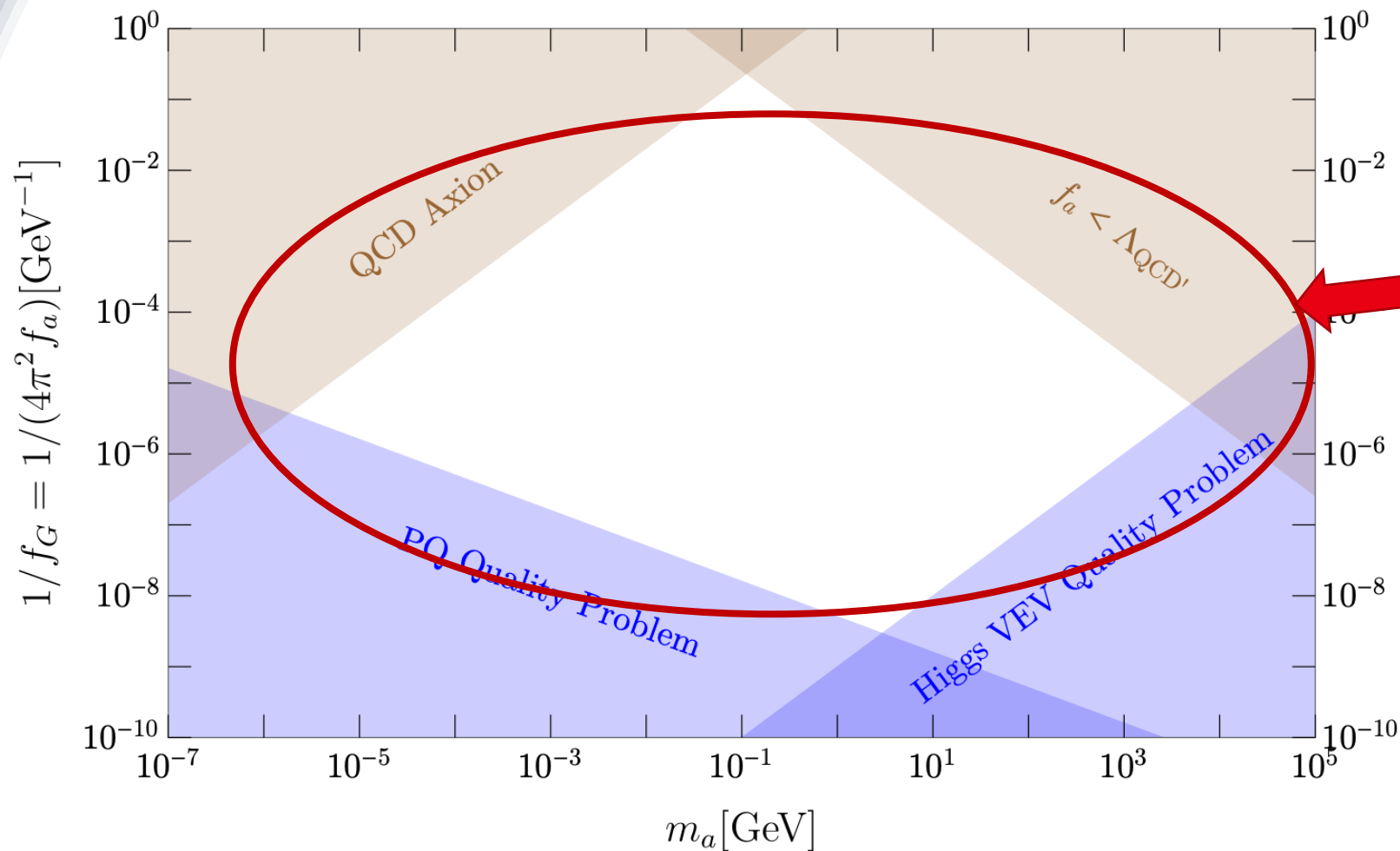
no axion EFT control

$$m_a > f_a$$

$$\frac{a}{8\pi f_a} \left(c_3 \alpha_3 G\tilde{G} + c_2 \alpha_2 W\tilde{W} + c_1 \alpha_1 B\tilde{B} \right)$$

Theory parameter space

$$m_a^2 \simeq \frac{\Lambda_{QCD'}^4}{f_a^2}$$

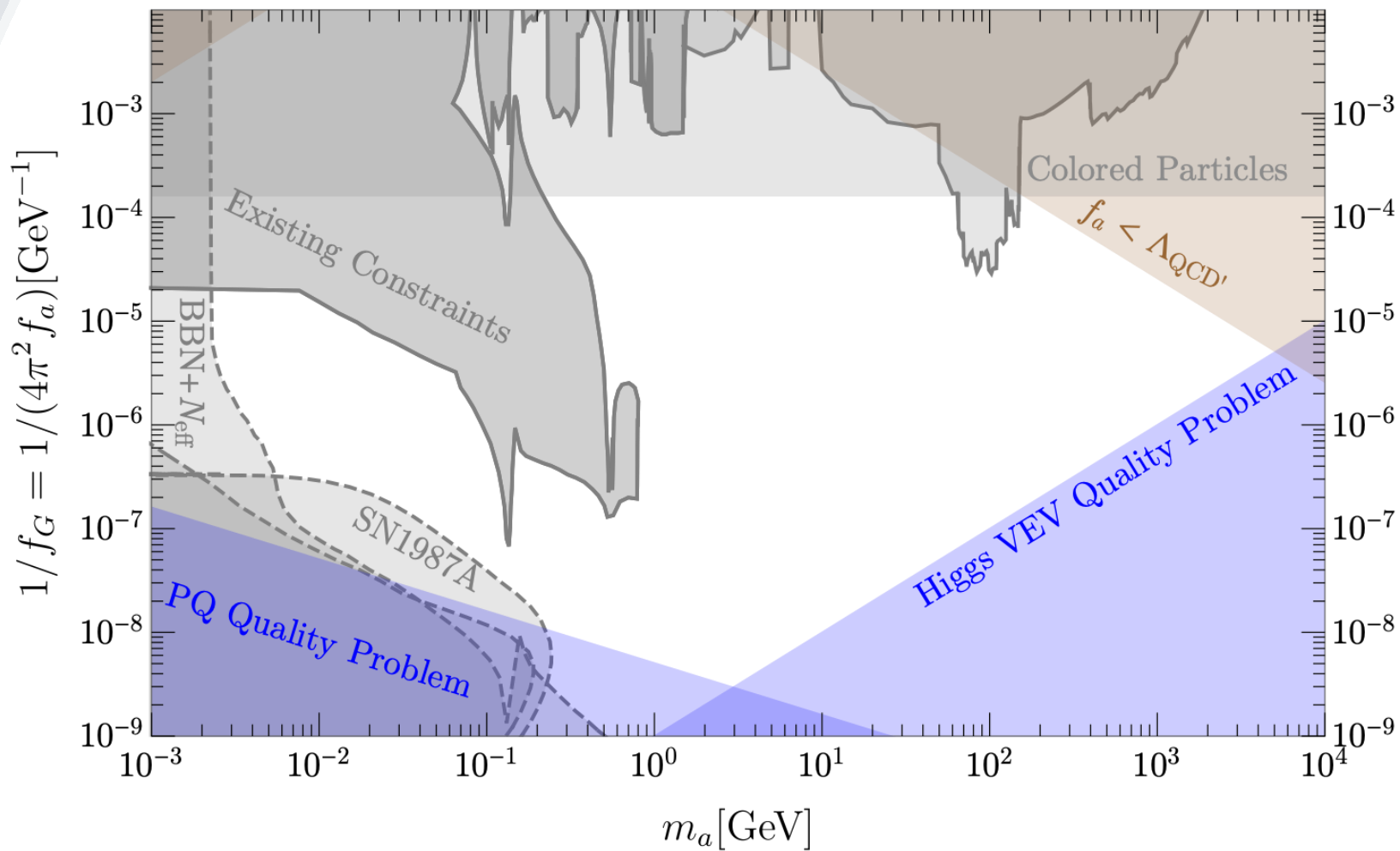


lighter than
QCD axion

Theory Land of Opportunity

$$\frac{a}{8\pi f_a} \left(c_3 \alpha_3 G\tilde{G} + c_2 \alpha_2 W\tilde{W} + c_1 \alpha_1 B\tilde{B} \right)$$

Existing constraints

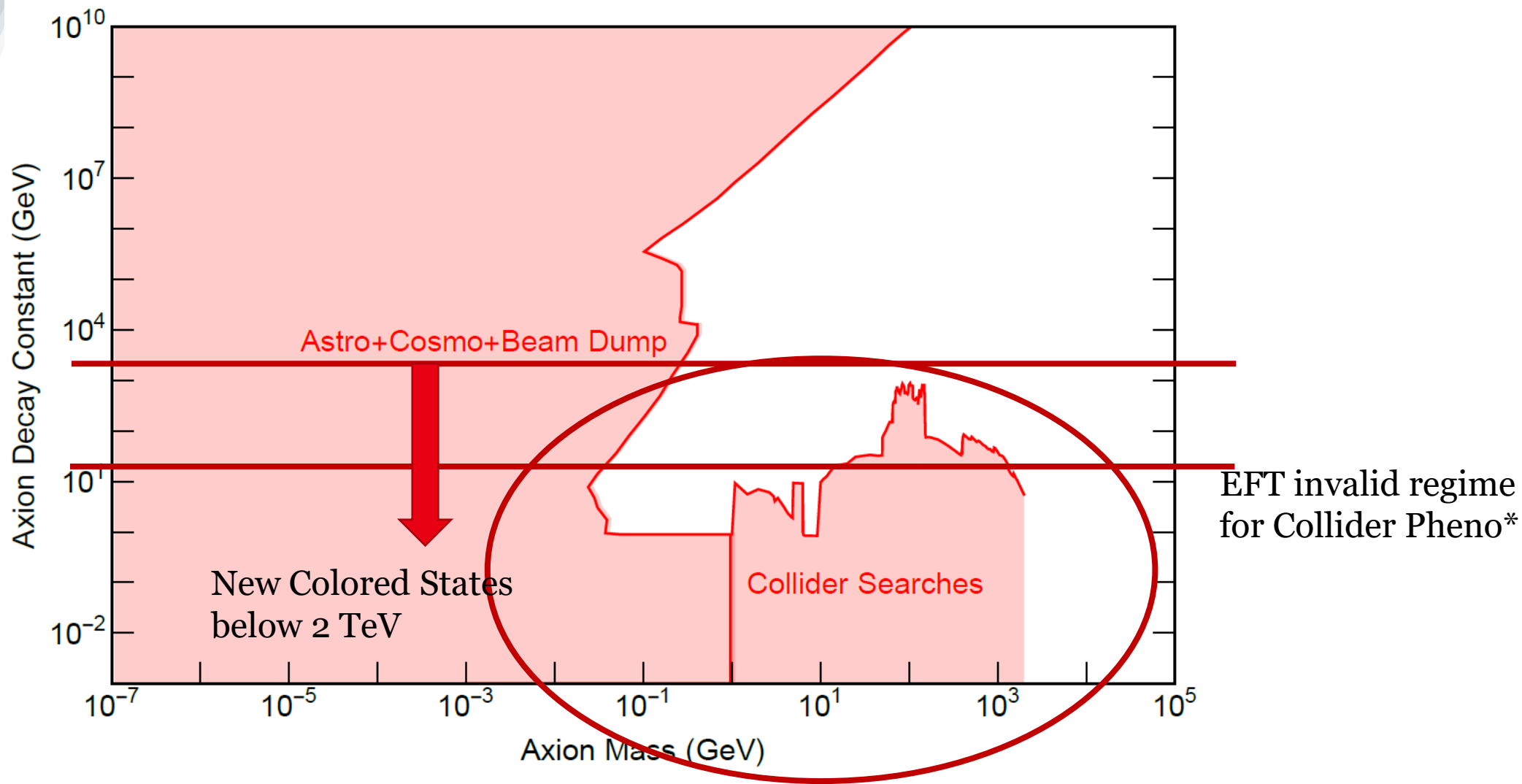


$$\frac{a}{8\pi f_a} \left(c_3 \alpha_3 G\tilde{G} + c_2 \alpha_2 W\tilde{W} + c_1 \alpha_1 B\tilde{B} \right)$$

$$c_1 = c_2 = c_3 = 1$$

$$\frac{a}{8\pi f_a} \left(c_3 \alpha_3 G\tilde{G} + c_2 \alpha_2 W\tilde{W} + c_1 \alpha_1 B\tilde{B} \right)$$

Preferred parameter space



$$\frac{a}{8\pi f_a} \left(c_3 \alpha_3 G\tilde{G} + c_2 \alpha_2 W\tilde{W} + c_1 \alpha_1 B\tilde{B} \right)$$

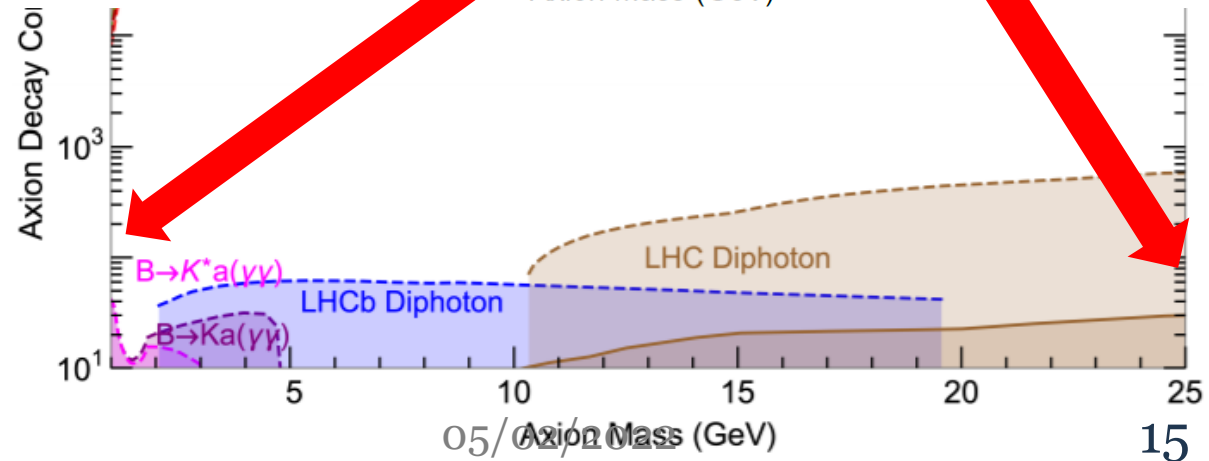
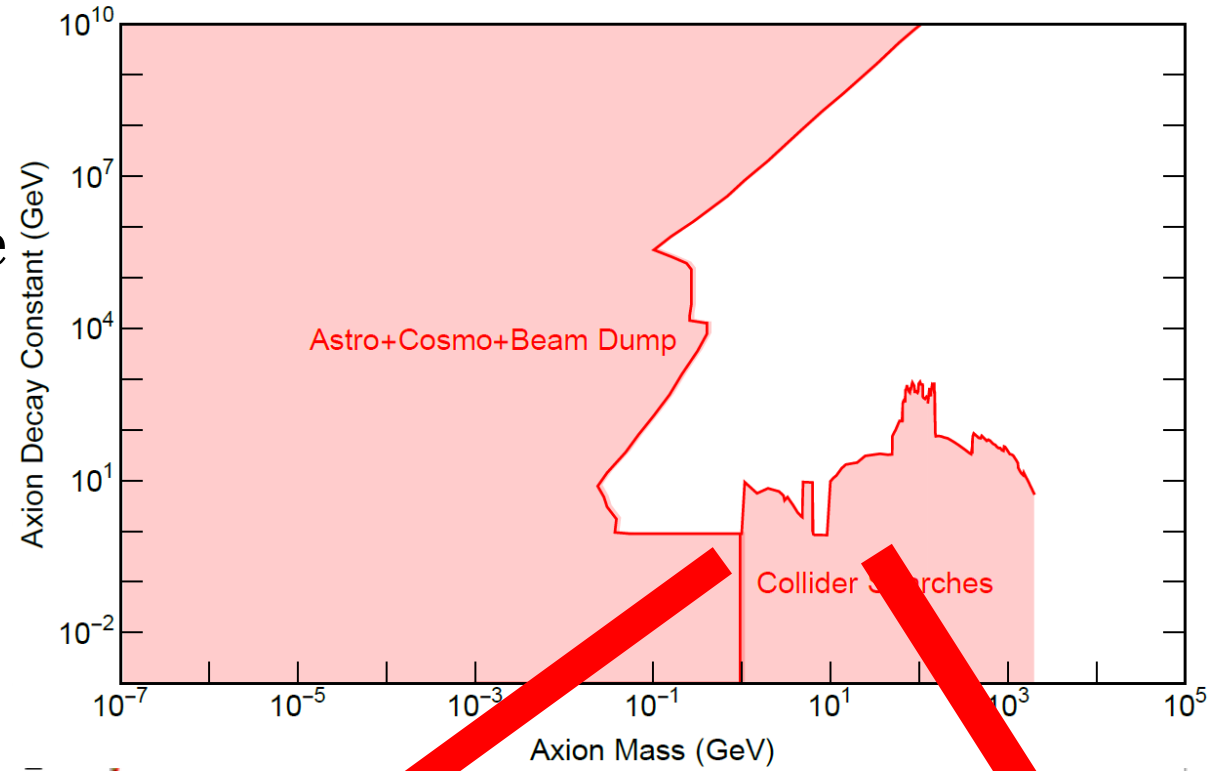
Preferred parameter space

$\tilde{G}G$ the defining coupling for strong CP
 Constraints and studies in the GeV scale
 realm **very minimal**.
 The existing studies focus on **photonic**,
Higgs decays and **leptonic** couplings.

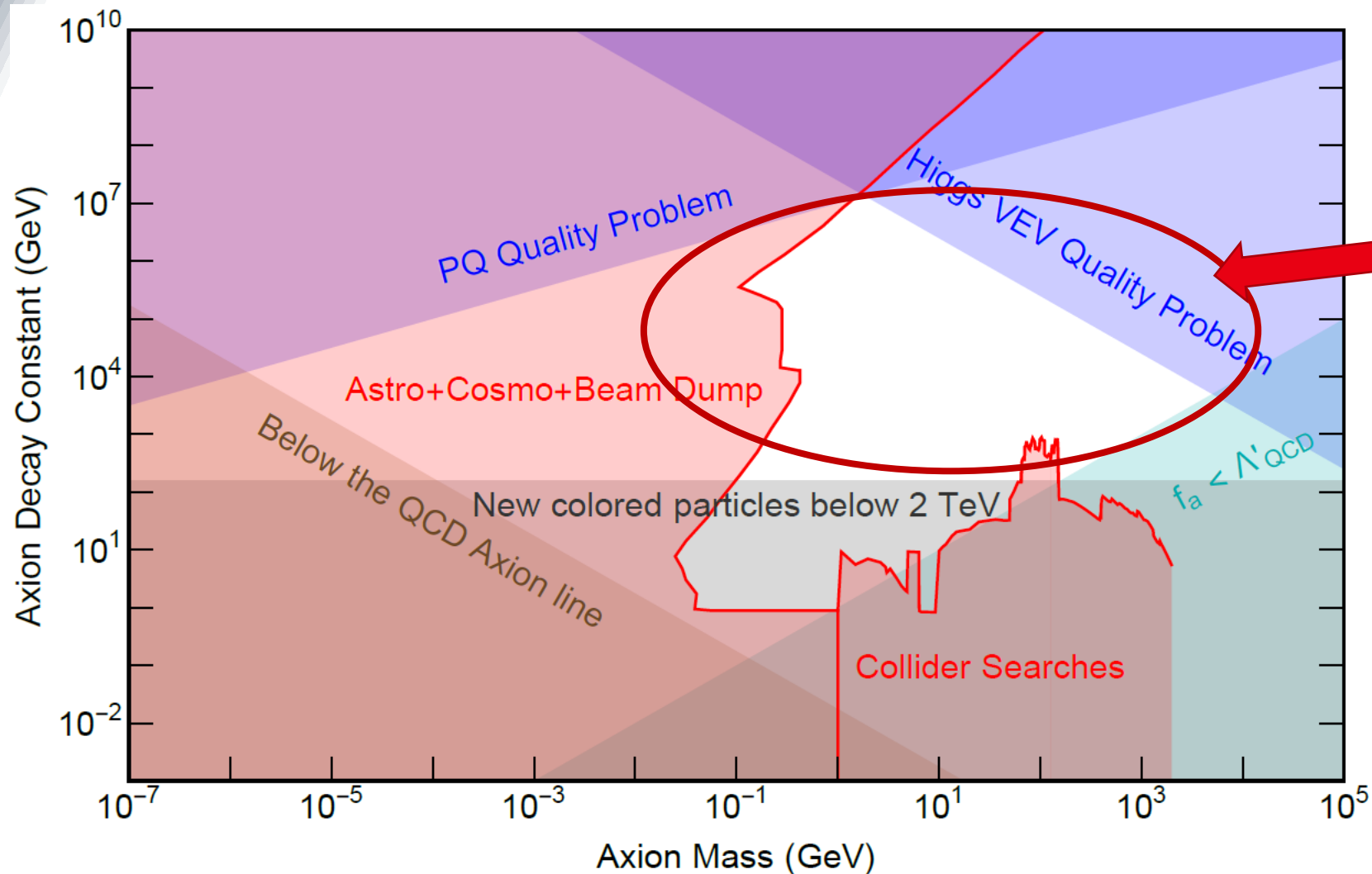


Hadronic mode buried
 under QCD background

$$\frac{\alpha_s}{8\pi} \left(\theta + \frac{a}{f_a} \right) \tilde{G}G + \frac{\alpha}{8\pi} \frac{a}{f_a} \tilde{F}F$$

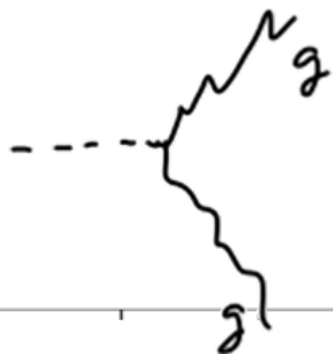


Preferred parameter space

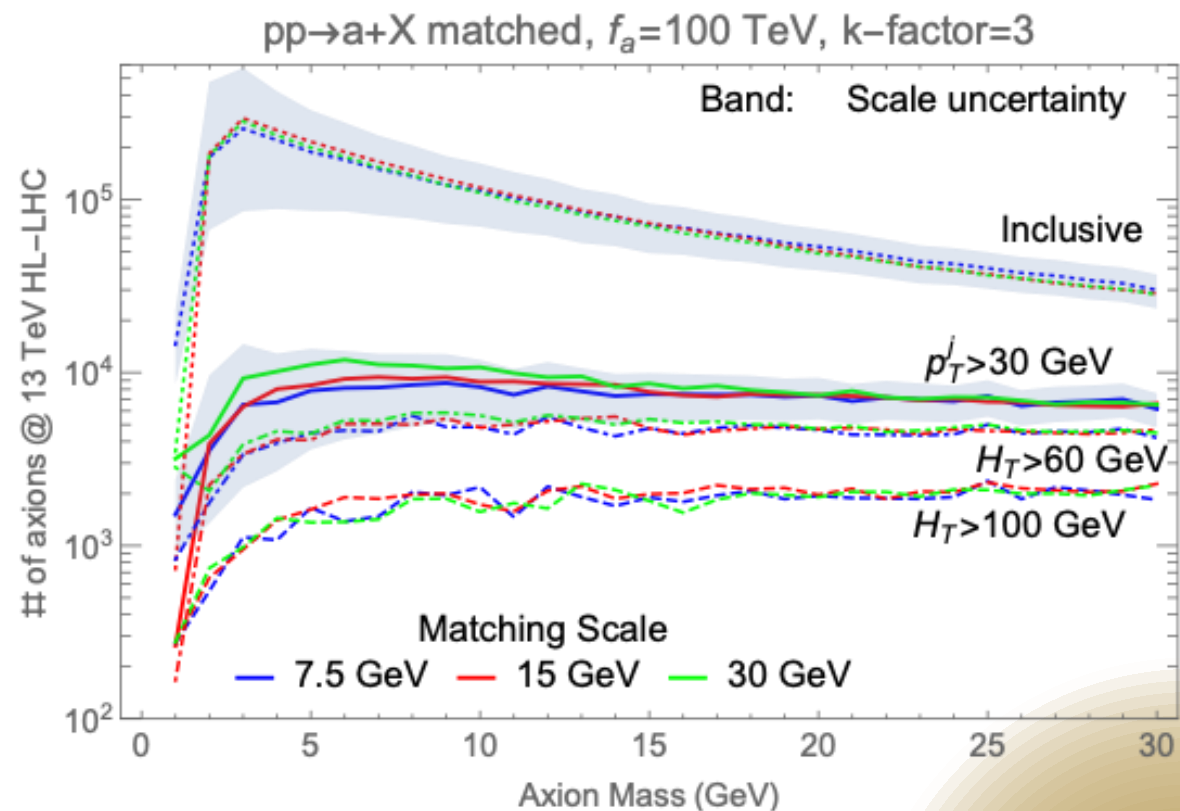
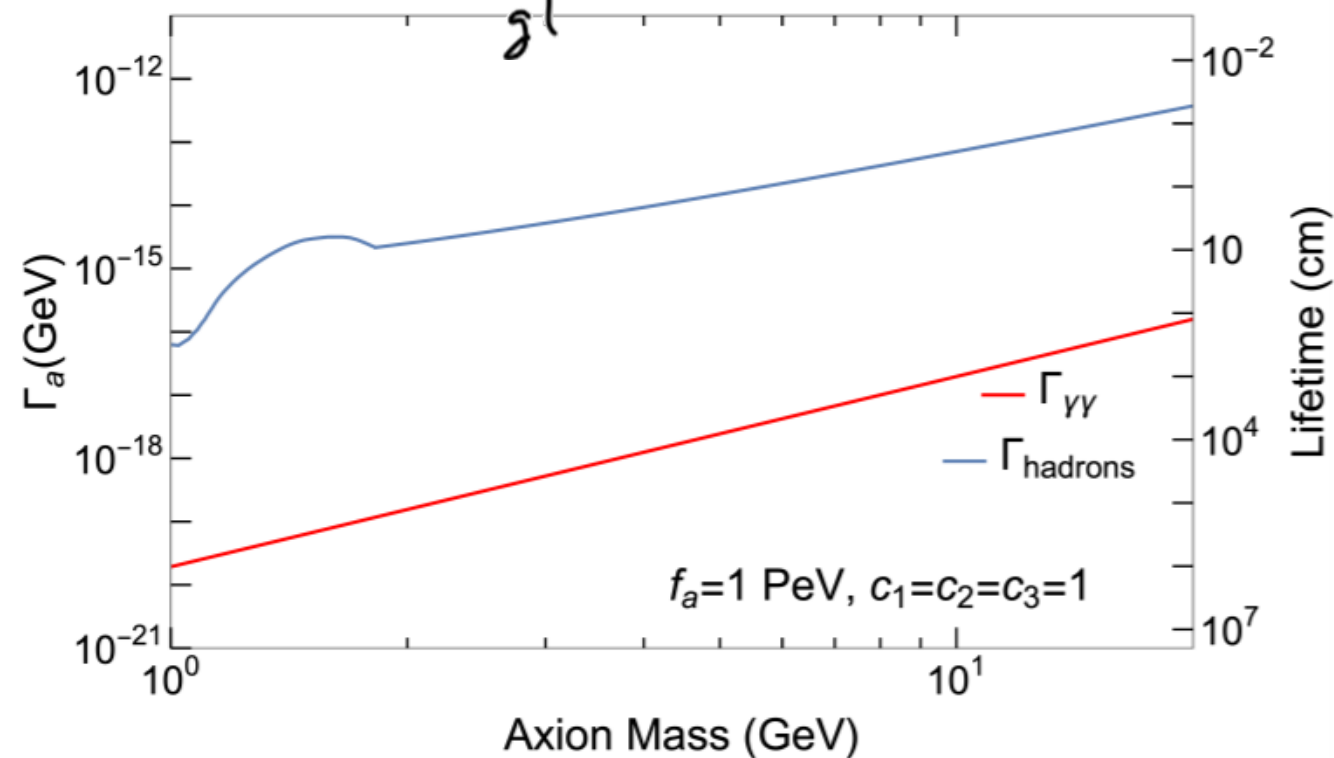


Land of Opportunity (and challenge)

Long-lived Axion

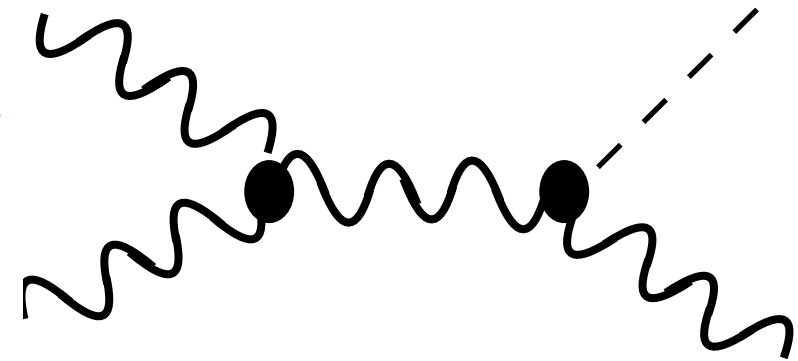


$$\frac{\alpha_s}{8\pi} \left(\theta + \frac{a}{f_a} \right) \tilde{G}G + \frac{\alpha}{8\pi} \frac{\mathbf{a}}{f_a^{\gamma\gamma}} \tilde{F}F$$



Displaced track trigger: crucial ingredient

- At least three $p_T > 2$ GeV tracks within an L1 jet;
- Amongst the above tracks, at least three of them have the transverse impact parameter $d_0 > 1$ mm;
- The pseudo-rapidity of the tracks to be $|\eta| < 2.4$;
- The event has $H_T > 100$ GeV.

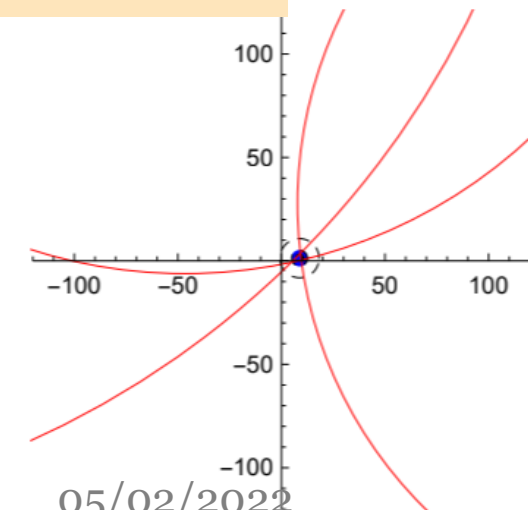
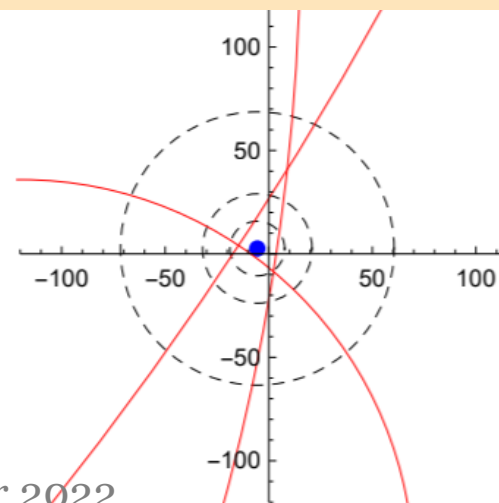


Long-lived signals are produced and decayed through a same vertex

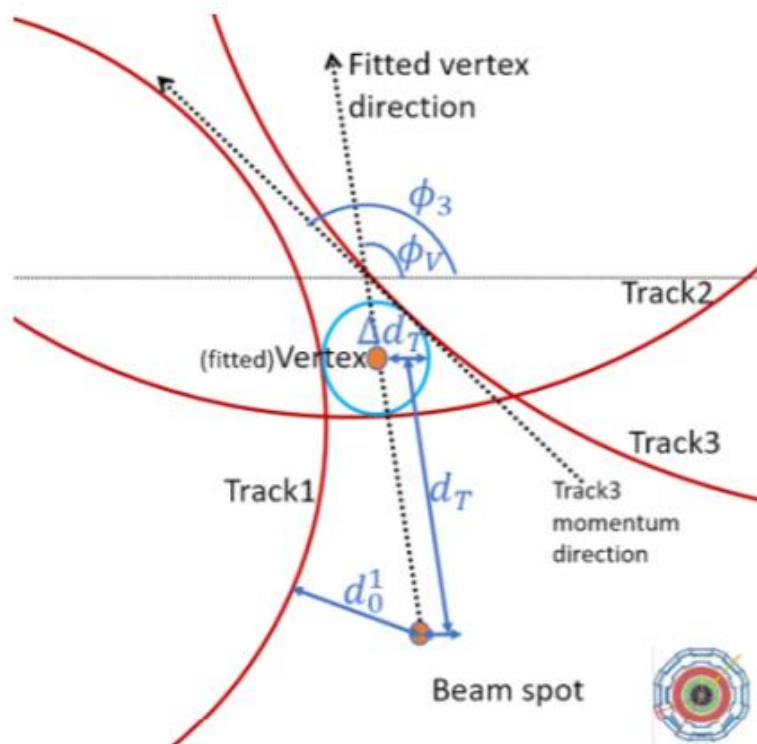
Post L1-triggering Background: 10^{12} (at HL-LHC)

Empirical modeling fake background:

- track impact parameter ($|d_0| < 15$ cm)
- track curvature ($1/R \propto q/p_T < 1/(1.8$ m))
- track eta $|\eta| < 2.4$
- track time to ($|t_0| < 6$ ns)
- track z- coordinate ($|z_0| < 15$ cm)

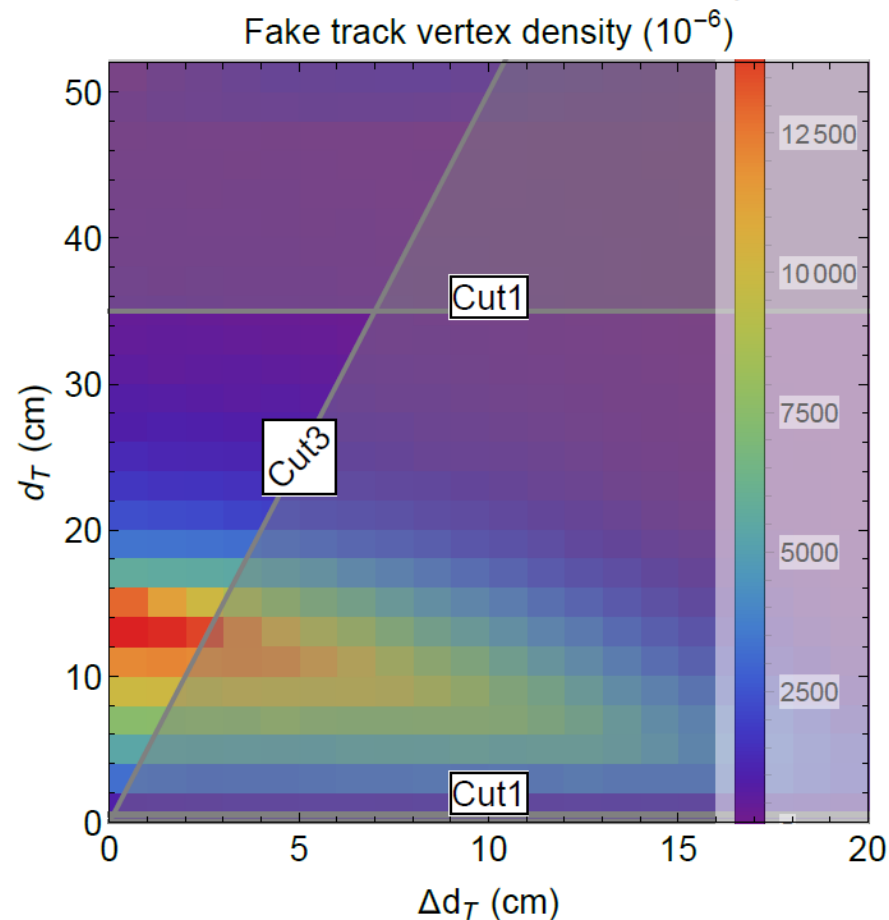


Analysis: 2D-4D vertexing selection

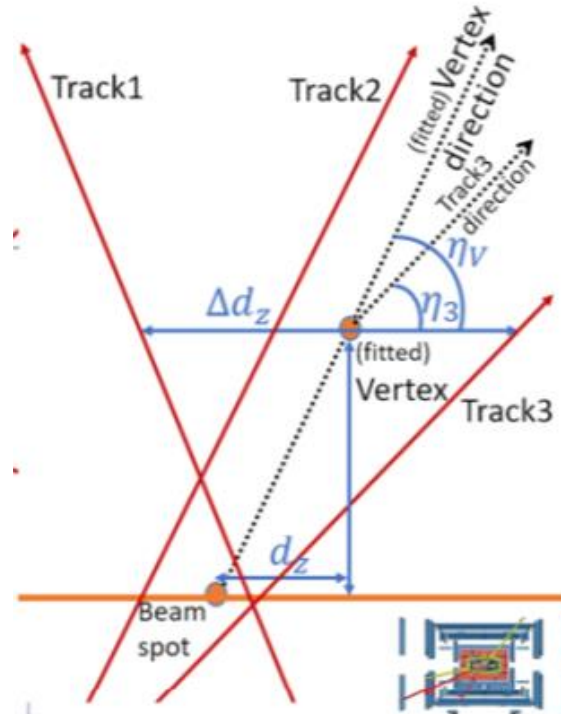


Vertex fitting in the transverse plane. Such consistency requirement provides 8.2×10^{-2} suppression.

1. The 2D common vertex has a minimal distance to the interaction point of 0.5 cm and maximal distance of 35 cm, $0.5 \text{ cm} < d_T < 35 \text{ cm}$;
2. The 2D tracks fit a common vertex with standard deviation $\Delta d_T < 1 \text{ cm}$;
3. The 2D common vertex is sufficiently displaced away from the interaction point, $d_T / \Delta d_T > 5$;



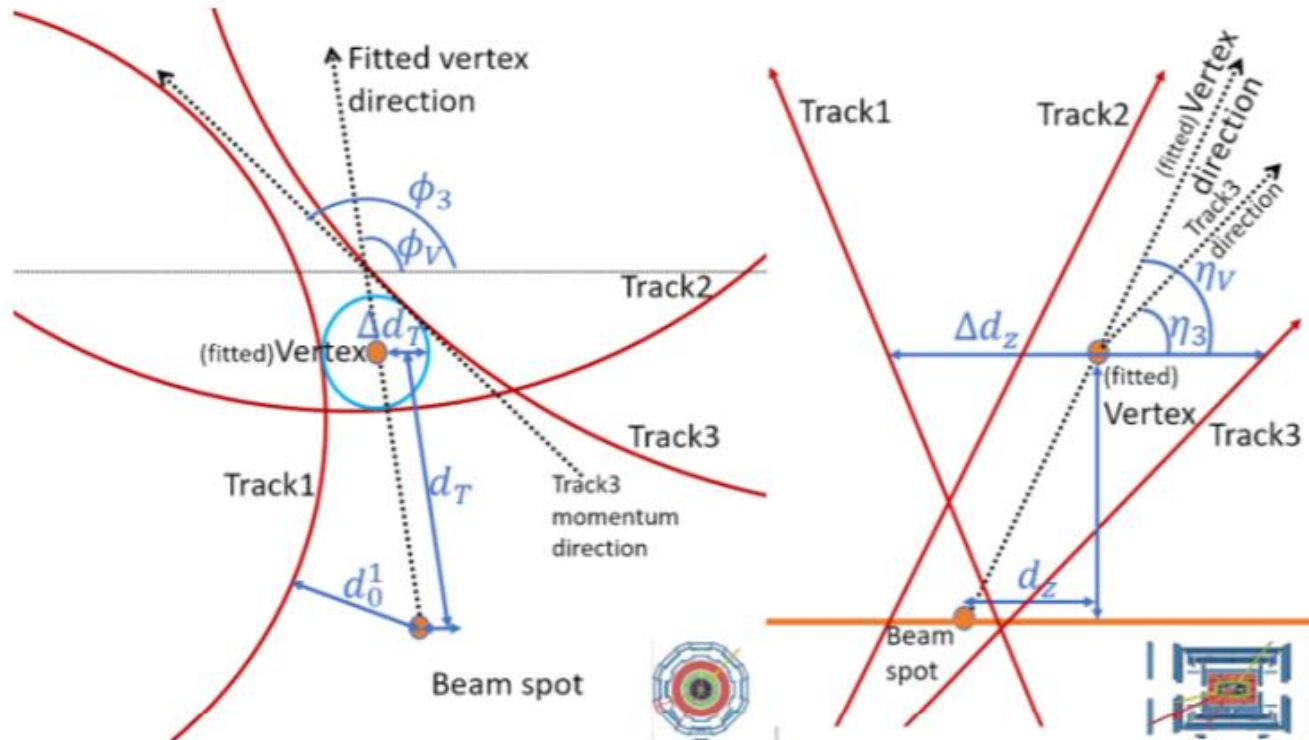
Analysis: 2D-4D vertexing selection



4. The corresponding 4D vertex has a standard deviation in z direction $\Delta d_z < 5$ cm;
5. The corresponding 4D vertex has a z -direction location $d_z < 20$ cm;
6. The corresponding 4D vertex has a standard deviation in time $\Delta d_t < 500$ ps;
7. The corresponding 4D vertex has a time $d_t < 1000$ ps;

Derived from the 2D fit results, one further imposes the consistency requirements between the three track in the z -direction and t -direction, providing 4.9×10^{-2} and 3.0×10^{-3} suppression, respectively.

Analysis: 2D-4D vertexing selection



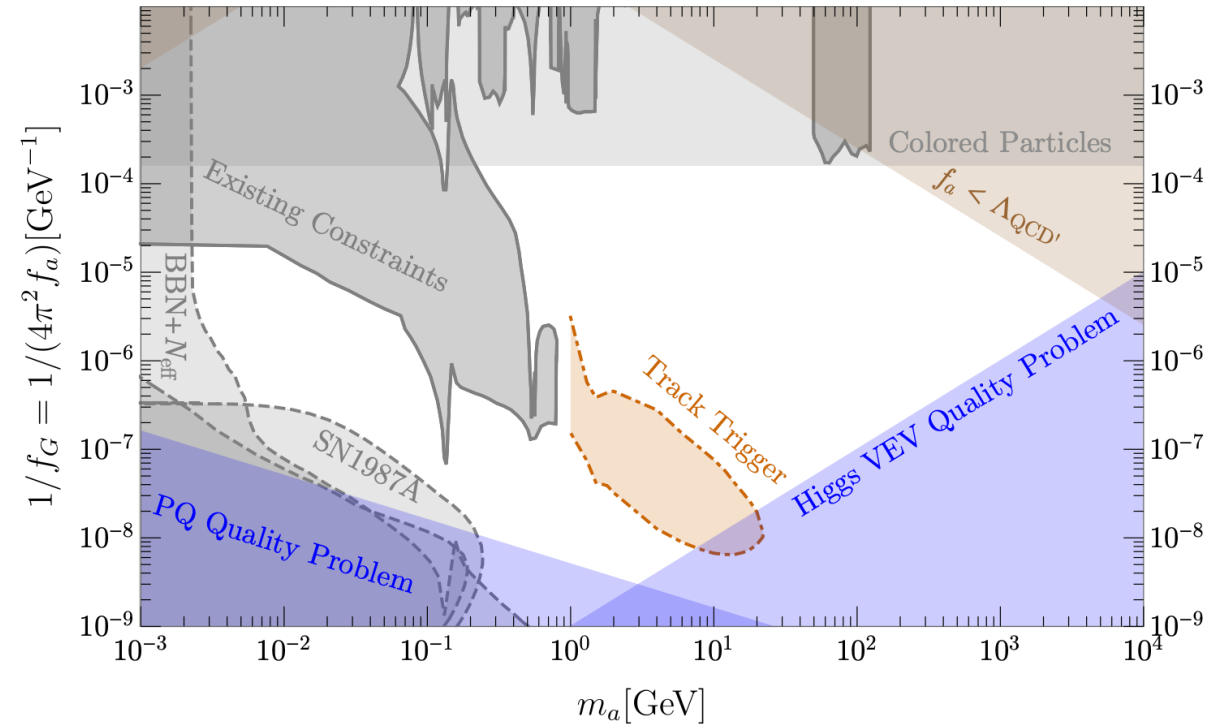
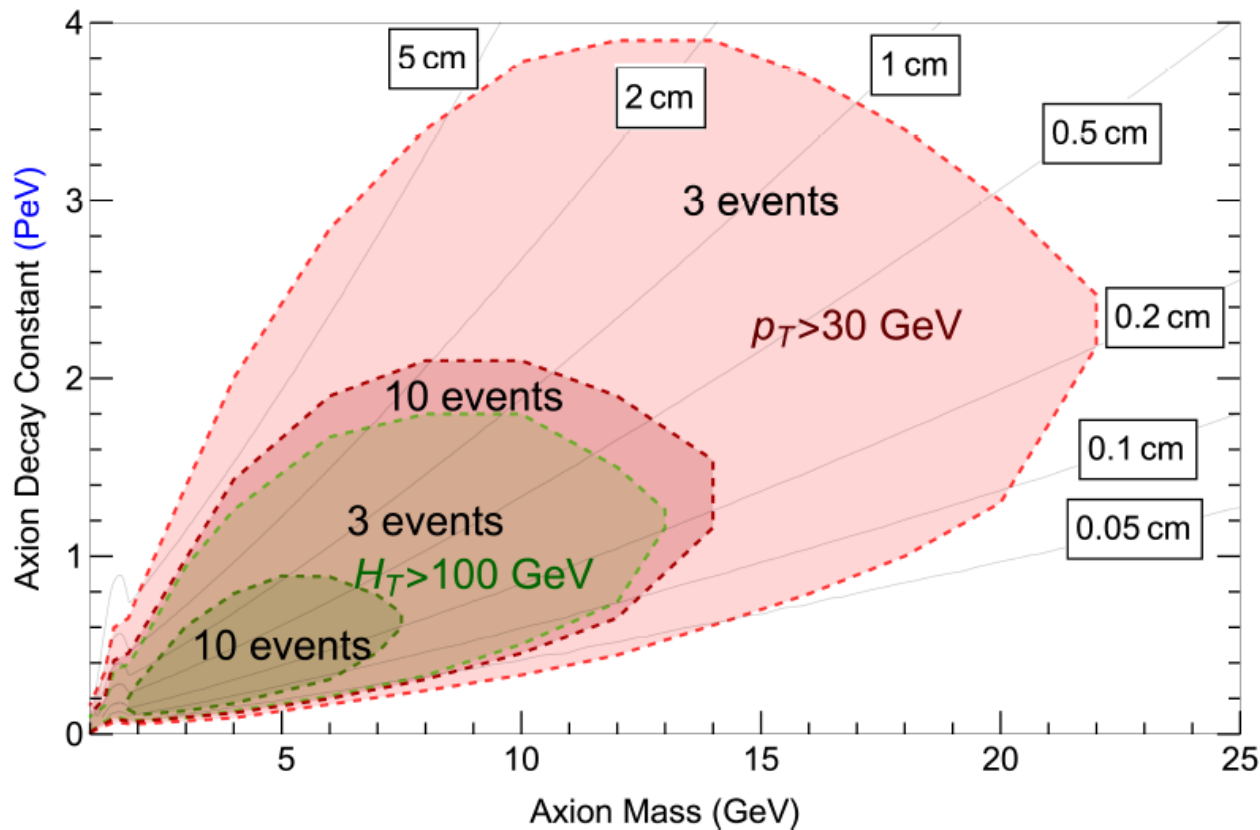
Background: 10^{12} to begin with (post triggering);
 2D-4D vertexing reduces it to 10^3
 Track/jet information matching between different
 subdetectors can further reduce it to < 1

1. The 2D common vertex has a minimal distance to the interaction point of 0.5 cm and maximal distance of 35 cm, $0.5 \text{ cm} < d_T < 35 \text{ cm}$;
2. The 2D tracks fit a common vertex with standard deviation $\Delta d_T < 1 \text{ cm}$;
3. The 2D common vertex is sufficiently displaced away from the interaction point, $d_T / \Delta d_T > 5$;
4. The corresponding 4D vertex has a standard deviation in z direction $\Delta d_z < 5 \text{ cm}$;
5. The corresponding 4D vertex has a z -direction location $d_z < 20 \text{ cm}$;
6. The corresponding 4D vertex has a standard deviation in time $\Delta d_t < 500 \text{ ps}$;
7. The corresponding 4D vertex has a time $d_t < 1000 \text{ ps}$;
8. The tracks are within 0.4 in pseudorapidity of the reconstructed displaced jet direction $|\eta_i - \eta_V| < 0.4$ for all the three tracks;
9. The tracks are within 0.4 in azimuthal angle of the reconstructed displaced jet direction $|\phi_i - \phi_V| < 0.4$ for all the three tracks;

A displaced track trigger enables searches for axions

Unique singly produced LLPs making the LHC main detectors the major place to look for them:

Long lifetime = low production rate



We employ a 2D-4D vertexing selection for displaced jet to veto the fake track background, demonstrating the plausibility of this search.

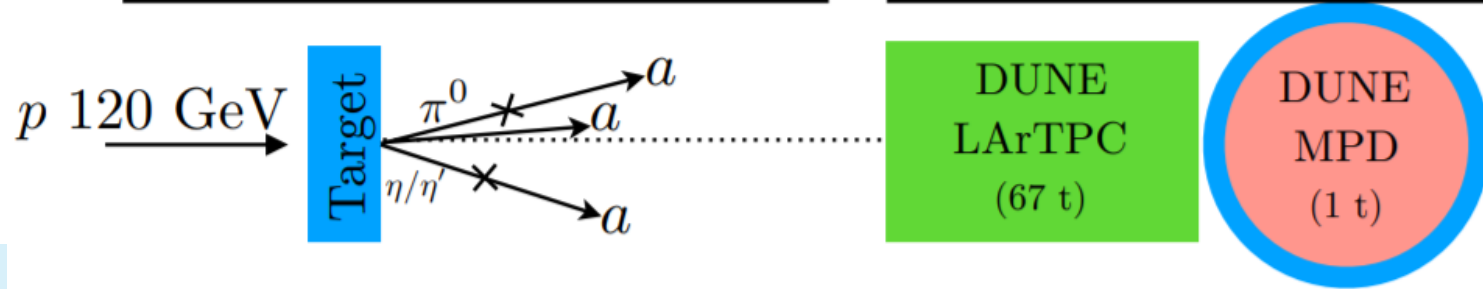
See also nice discussion on “dark pion” pheno in Cheng, Li, Salvioni, [2110.10691](https://arxiv.org/abs/2110.10691)

It is GeV, not TeV scale new physics. Can we do better?

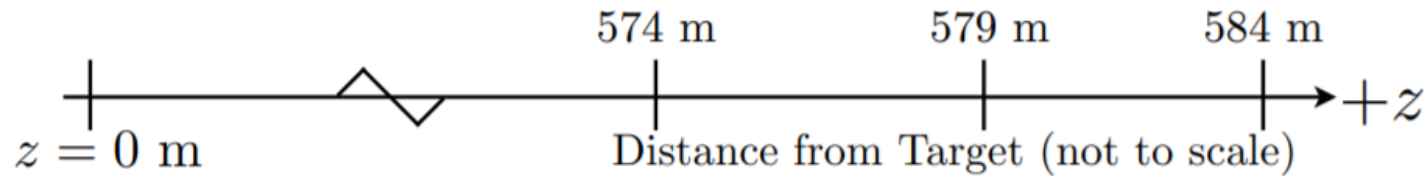
DUNE: excellent for heavy Axions

Axion Production
(meson mixing, gluon-gluon fusion)

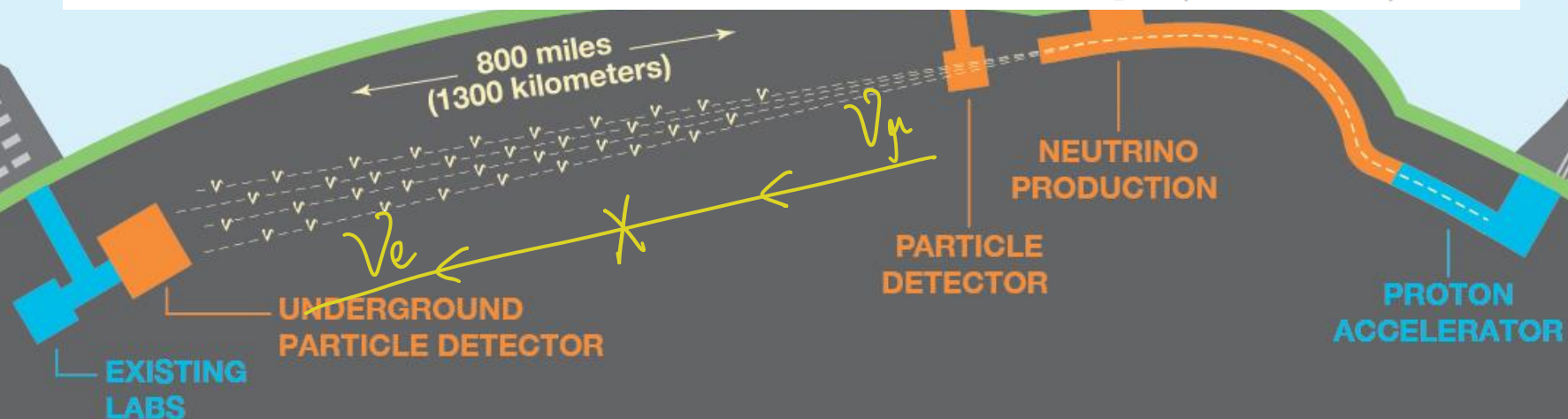
Axion Decay
(into photon pairs, hadrons)



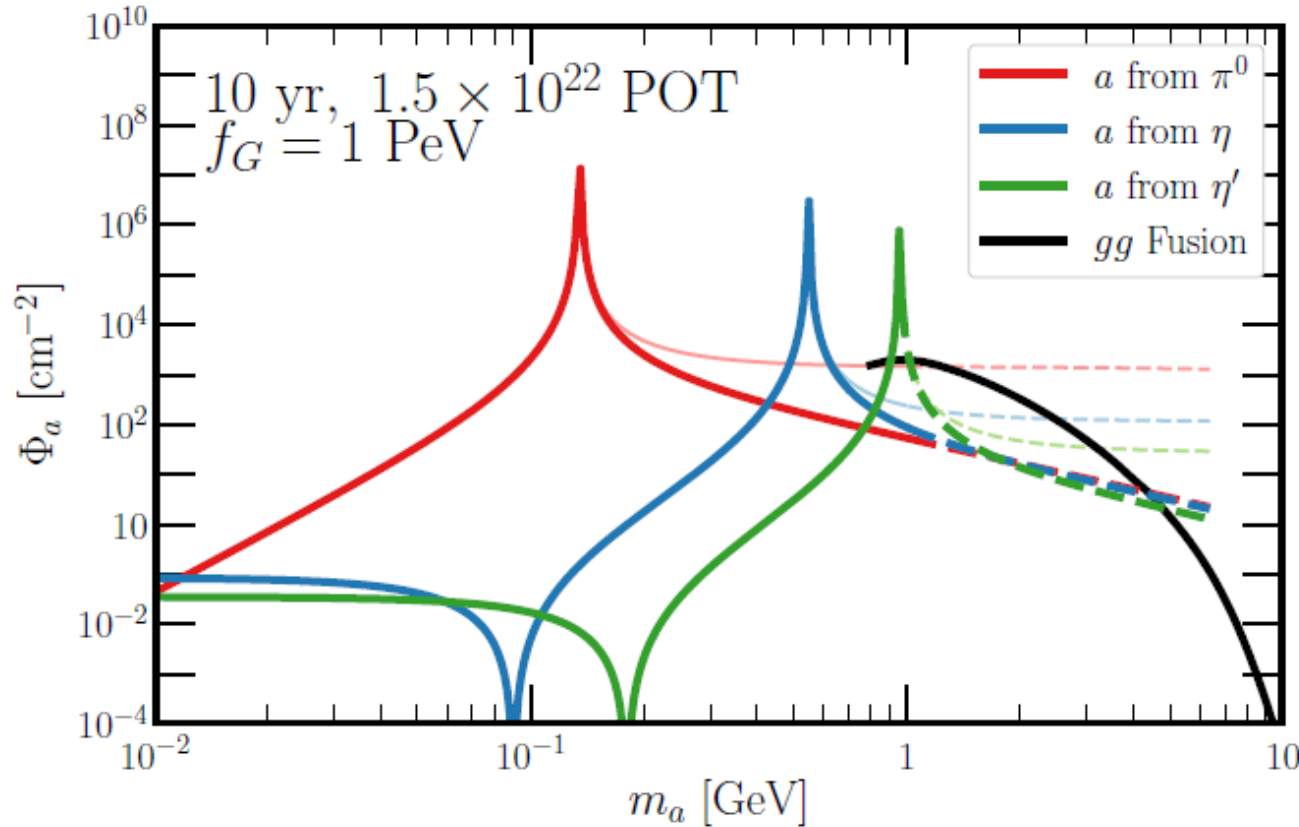
Sanford Underground
Research Facility



Fermilab



DUNE setup



large flux

1.47×10^{21} POT/yr.

$$\pi = \pi_{\text{phys}} + \theta_{a\pi} a_{\text{phys}} + \dots \approx \pi_{\text{phys}} + \frac{1}{6} \frac{f_\pi}{f_a} \frac{m_a^2}{m_a^2 - m_\pi^2} a_{\text{phys}} + \dots,$$

$$\eta = \eta_{\text{phys}} + \theta_{a\eta} a_{\text{phys}} + \dots \approx \eta_{\text{phys}} + \frac{1}{\sqrt{6}} \frac{f_\pi}{f_a} \left(\frac{m_a^2 - \frac{4}{9} m_\pi^2}{m_a^2 - m_\eta^2} \right) a_{\text{phys}} + \dots,$$

$$\eta' = \eta'_{\text{phys}} + \theta_{a\eta'} a_{\text{phys}} + \dots \approx \eta'_{\text{phys}} + \frac{1}{2\sqrt{3}} \frac{f_\pi}{f_a} \left(\frac{m_a^2 - \frac{16}{9} m_\pi^2}{m_a^2 - m_{\eta'}^2} \right) a_{\text{phys}} + \dots$$

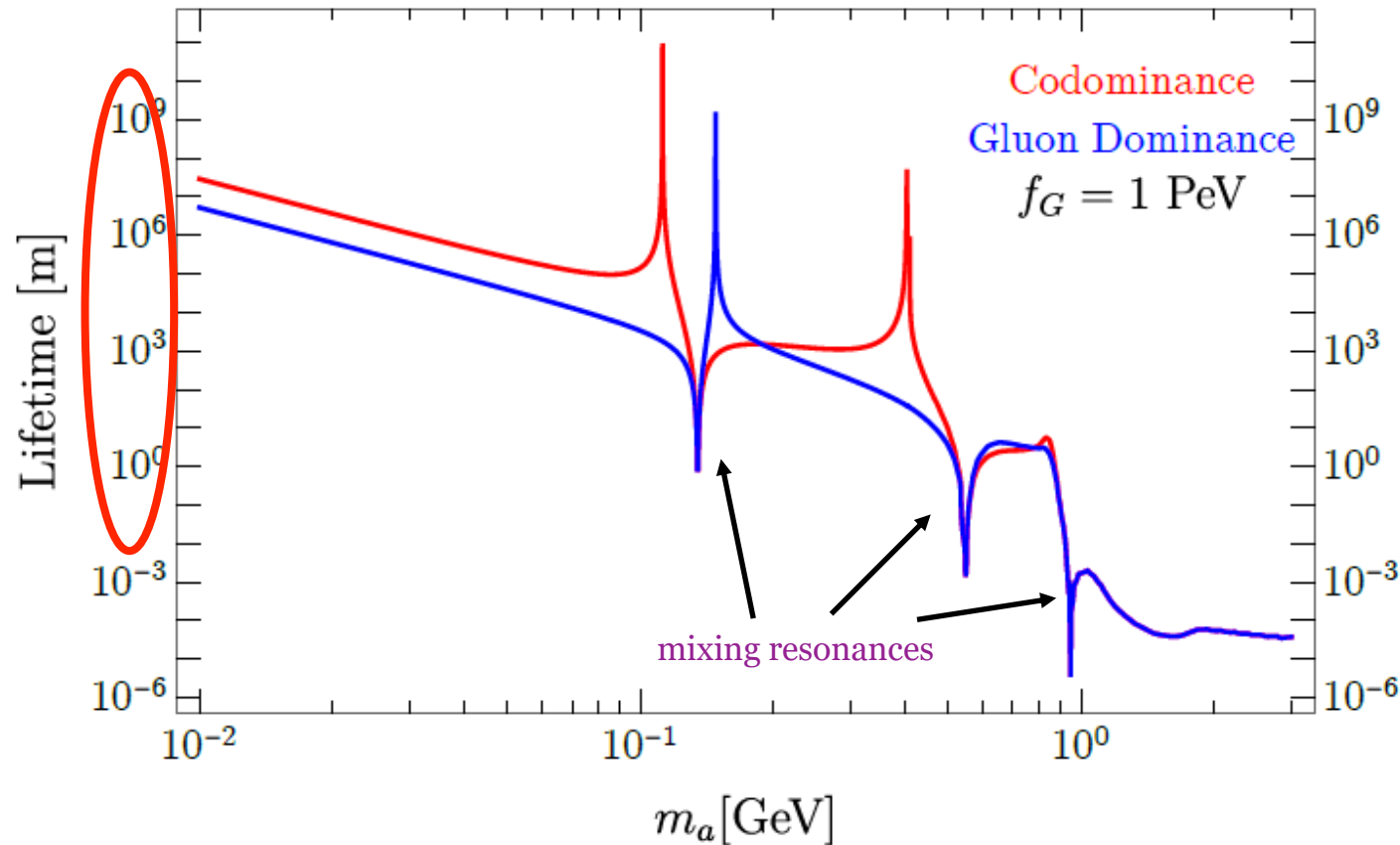
$$N_{\text{axions}} = N_{\text{POT}} \times [2.89 |\theta_{a\pi}|^2 f(m_\pi, m_a) + 0.33 |\theta_{a\eta}|^2 f(m_\eta, m_a) + 0.03 |\theta_{a\eta'}|^2 f(m_{\eta'}, m_a)]$$

$$f(m_{\text{meson}}, m_a) = \begin{cases} \left(\frac{m_a}{m_{\text{meson}}} \right)^{-1.6} & \text{if } m_a > m_{\text{meson}} \\ 1 & \text{if } m_a \leq m_{\text{meson}}. \end{cases}$$

Decay via $\gamma\gamma$ and hadronic states

- DUNE can detect both $\gamma\gamma$ and hadronic final states

Aloni et. al. '18



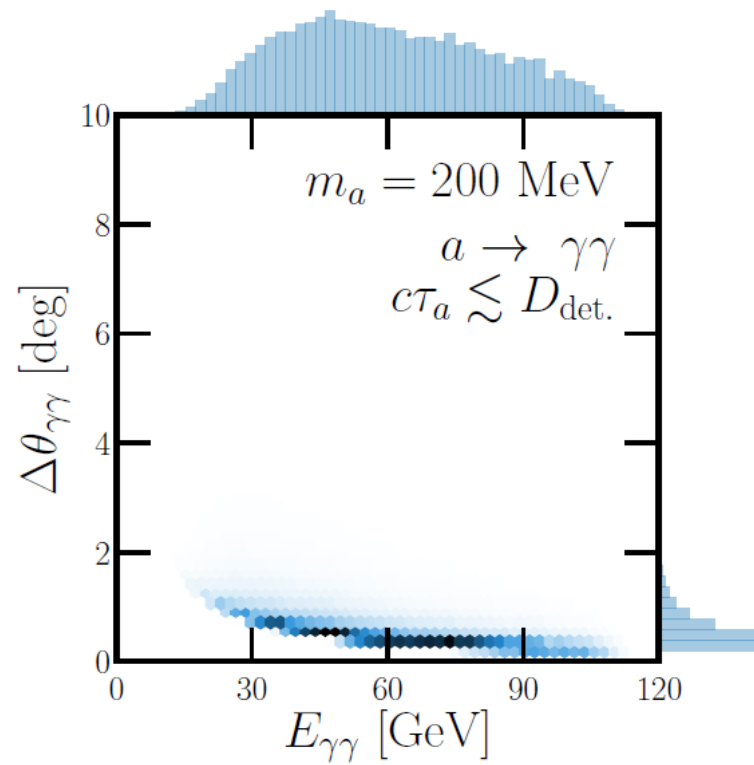
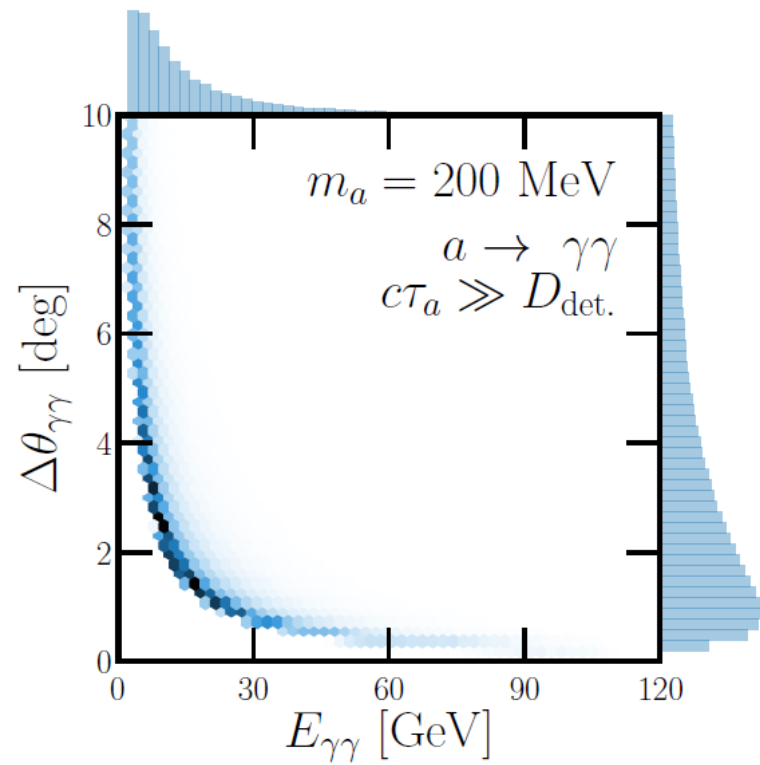
$$c_{\gamma\gamma}^{\text{eff}}(m_a \lesssim \text{GeV})$$

$$\approx c_2 + \frac{5}{3}c_1 - 1.92c_3$$

$$- c_3 \frac{m_a^2}{m_\pi^2 - m_a^2} \frac{m_d - m_u}{m_d + m_u} + \dots$$

$$\frac{a}{8\pi f_a} \left(c_3 \alpha_3 G\tilde{G} + c_2 \alpha_2 W\tilde{W} + c_1 \alpha_1 B\tilde{B} \right)$$

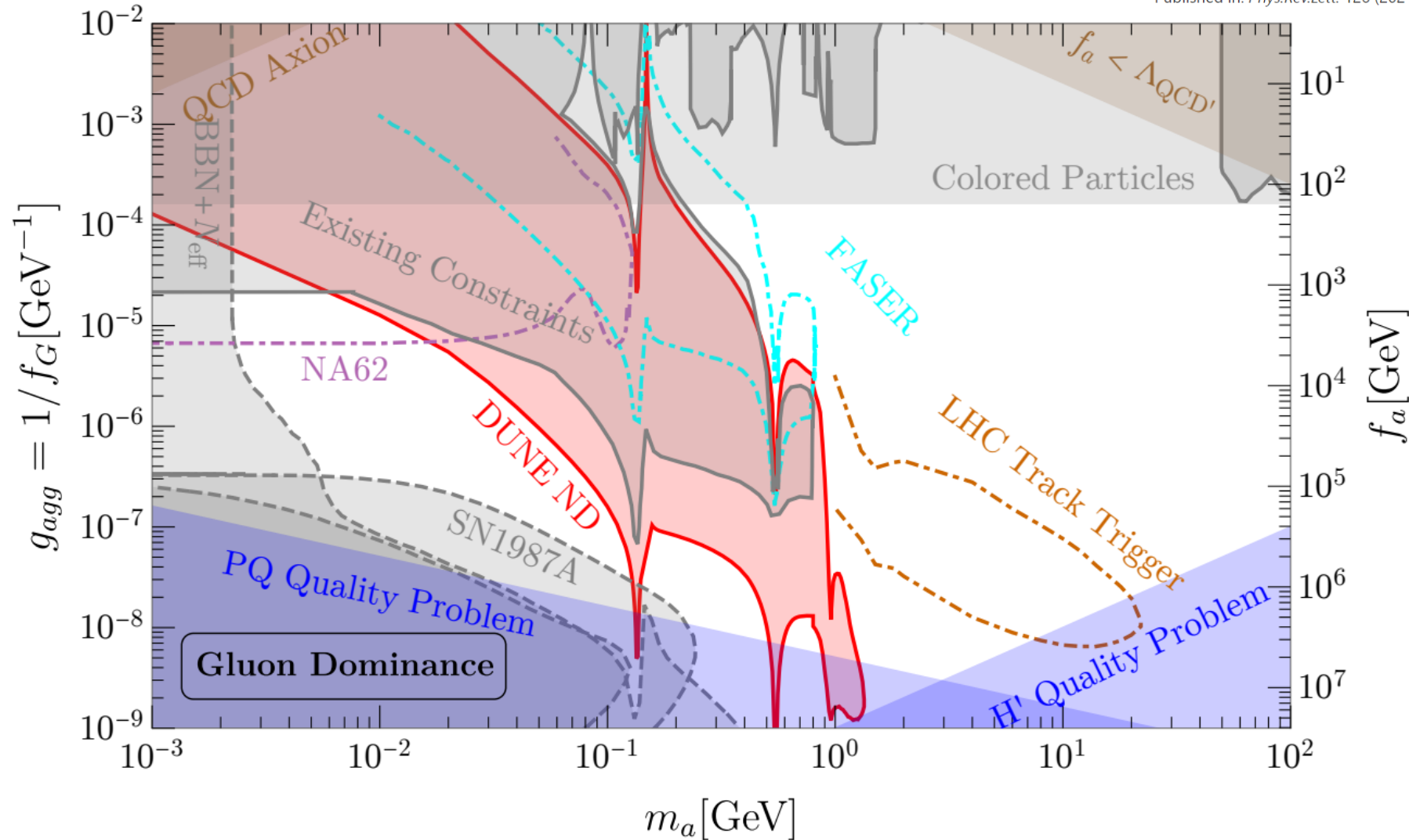
Signal and BG considerations for $a \rightarrow \gamma\gamma$



Backgrounds from neutral current π are low energy ($\lesssim 5\text{GeV}$) and isotropic

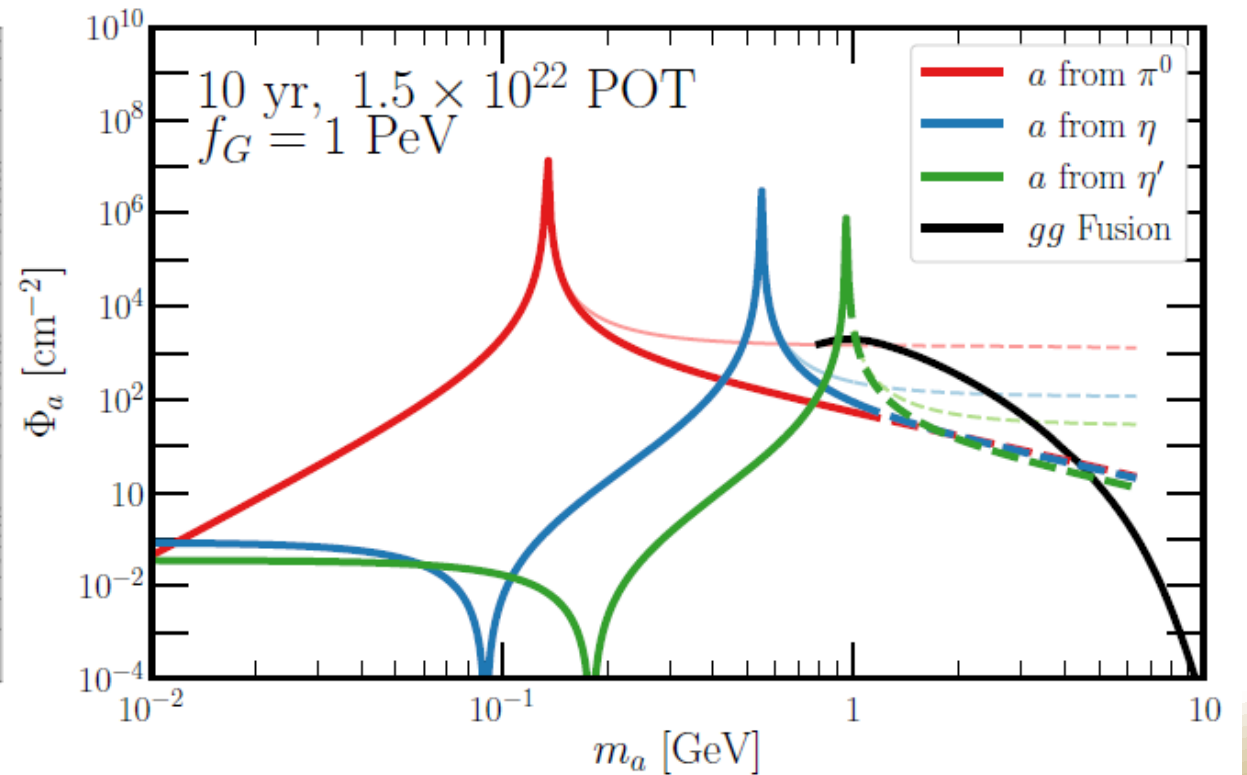
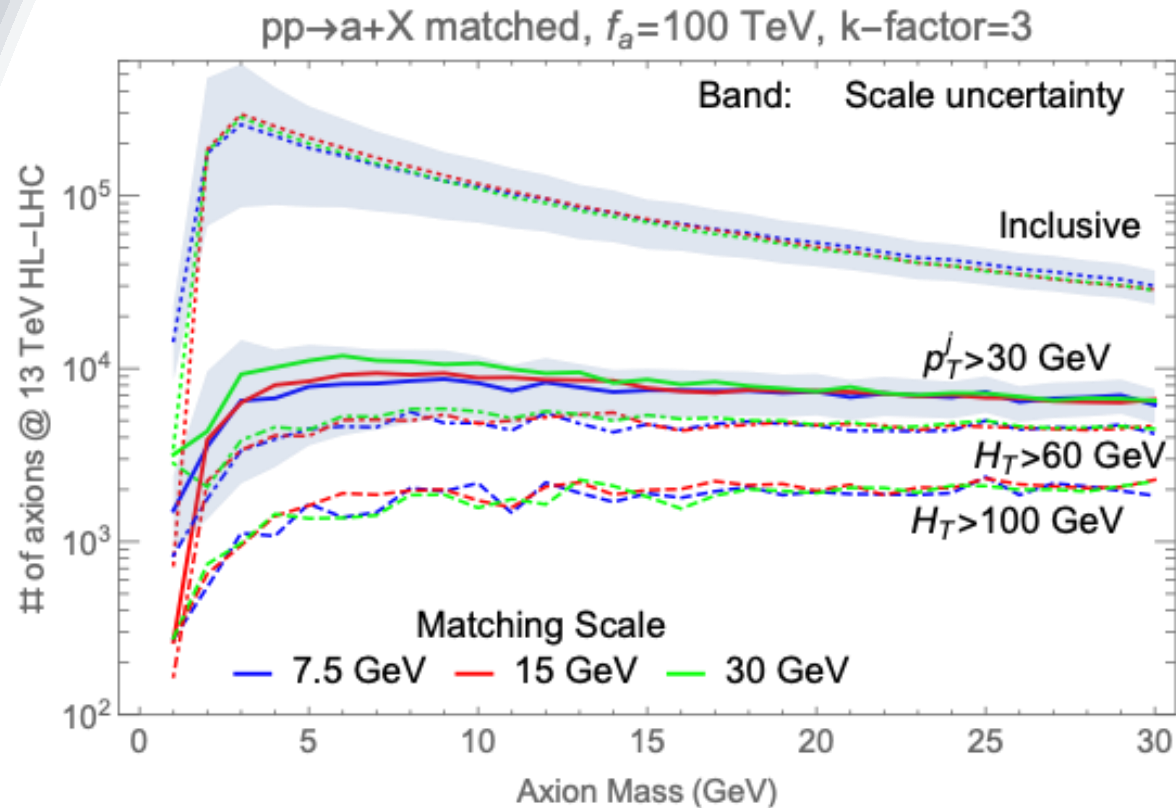
Signature	Liquid Argon ArgonCube		Gaseous Argon MPD	
	Signal	Background	Signal	Background
$a \rightarrow \gamma\gamma$	Invariant Mass	$\text{NC}\pi^0$	Invariant Mass	$\text{NC}\pi^0$
	$\gamma\gamma$ Direction	Nearly-Isotropic	$\gamma\gamma$ Direction	Nearly-Isotropic
	High-Energy	Low-Energy	High-Energy	Low-Energy
$a \rightarrow \text{hadrons}$				Low-energy recoils
	Invariant Mass	$\text{CC}1\mu 2\pi$	Invariant Mass	$\text{CC}1\mu 2\pi$
	Opening angle	DIS	Opening angle	DIS
	High-energy	Low-energy	High-Energy	Low-Energy
	gg Direction	Nearly-Isotropic	gg Direction	Nearly-isotropic
			Low-energy recoils	

DUNE coverage



High-quality axion (or generic Axion-like-particles) shows an exciting opportunity for particle physics

But, do we really know the gluonic ALP production?

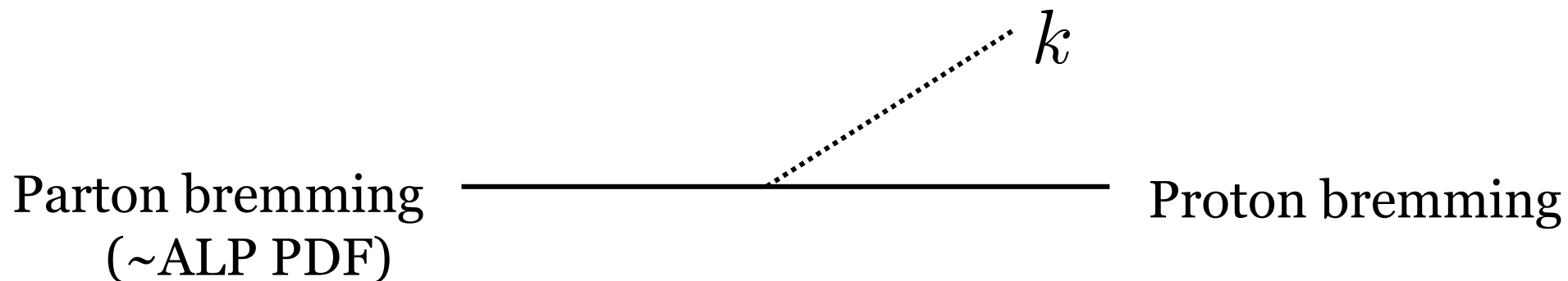


Gluonic ALP production reconsidered

Meson decays, Mixing with Mesons, Hard Scattering, Parton splitting, **Bremsstrahlung...**

Forward ALP can be viewed as an ISR

ISR from partons, proton?

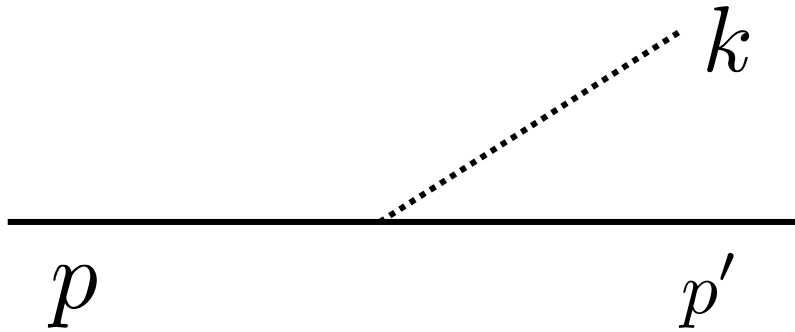


Our (preliminary) result shows that the ISR signal from gluon and quark bremsstrahlung far smaller than the parton bremsstrahlung

Primary contribution.

Splitting Function

The signal rate for emitting an axion from the proton bremsstrahlung



$$\sigma = \int dz dp_T^2 \frac{d\mathcal{P}}{dz dp_T^2} \hat{\sigma}_{\text{NSD}}(\hat{s})$$

$$\frac{d\mathcal{P}}{dz dp_T^2} = \frac{1}{16\pi^2} \frac{z}{1-z} \frac{1}{(p'^2 - m_p^2)^2} \left[\frac{1}{2} \sum |\mathcal{M}|^2 \right] F_S(k, p')^2$$

Form Factor

$$F_{pp^*a}(p'^2) = \frac{\Lambda^4}{\Lambda^4 + (p'^2 - m_p^2)^2} \quad \Lambda = 1 \sim 2 \text{ GeV}$$

$$F_S(k, p') = F_{1,p}^a(k^2) F_{pp^*a}(p'^2)$$

Nucleon Form Factor

$$F_{1,p}(k^2)|_{\text{timelike}} = \frac{1}{(1 - \gamma e^{i\delta} k^2)^2} \left[1 - \alpha + \alpha \frac{m_{a_1}^2 (m_{a_1}^2 - k^2 + i m_{a_1} \Gamma_{a_1})}{(m_{a_1}^2 - k^2)^2 + m_{a_1}^2 \Gamma_{a_1}^2} \right]$$

Proton-ALP Coupling

Effective operator $\frac{C_p(m_a)}{2f_a} \bar{p} \gamma^\mu \gamma_5 p \partial_\mu a$

$$C_p(m_a) = C_p^0 + \sum_{\phi} \frac{C_{K,\phi} m_a^2}{m_a^2 - m_\phi^2} \frac{g_{\phi pp}}{m_p} - \sum_{\phi} \frac{C_{M,\phi} m_\phi^2}{m_a^2 - m_\phi^2} \frac{g_{\phi pp}}{m_p}$$

Kinetic mixing

Mass mixing

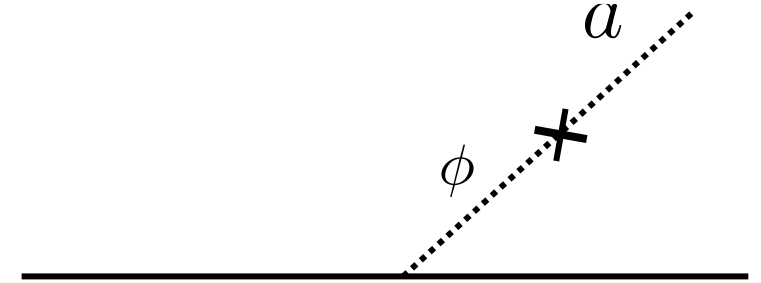
$$g_{pa} = \frac{1}{2f_a} \left[\hat{c}_{uu} (D + D_s + F) + \hat{c}_{dd} D_s + \hat{c}_{ss} (D + D_s - F) \right] + \frac{1}{2f_a} \left[(D + F) \langle \pi a \rangle - \frac{1}{\sqrt{3}} (D - 3F) \langle \eta_8 a \rangle + \sqrt{\frac{2}{3}} (2D + 3D_s) \langle \eta_1 a \rangle \right]$$

$$D \approx 0.80, F \approx 0.46, D_s \approx -0.41$$

In the massless limit, matches known results (PDG)

$$C_p = -0.47(3) + 0.88(3)C_u - 0.39(2)C_d - 0.038(5)C_s - 0.012(5)C_c - 0.009(2)C_b - 0.0035(4)C_t,$$

$$C_n = -0.02(3) + 0.88(3)C_d - 0.39(2)C_u - 0.038(5)C_s - 0.012(5)C_c - 0.009(2)C_b - 0.0035(4)C_t, \text{ ar 2022}$$



$$\mathcal{L} \supset \text{tr} \bar{B} (i \not{D}) B + \frac{D}{2} \text{tr} \bar{B} \gamma_\mu \gamma_5 \{u^\mu, B\} + \frac{F}{2} \text{tr} \bar{B} \gamma_\mu \gamma_5 [u^\mu, B] + \frac{D_s}{2} \text{tr} \bar{B} \gamma_\mu \gamma_5 B \text{tr} u^\mu$$

$$\pi^0 \rightarrow \pi^0 + \langle \pi^0 a \rangle a + \langle \pi^0 \eta_8 \rangle \eta_8 + \langle \pi^0 \eta_1 \rangle \eta_1$$

$$\eta_8 \rightarrow \cos \theta_{\eta\eta'} \eta_8 + \sin \theta_{\eta\eta'} \eta_1 + \langle \eta_8 a \rangle a + \langle \eta_8 \pi^0 \rangle \pi^0$$

$$\eta_1 \rightarrow -\sin \theta_{\eta\eta'} \eta_8 + \cos \theta_{\eta\eta'} \eta_1 + \langle \eta_1 a \rangle a + \langle \eta_1 \pi^0 \rangle \pi^0$$

Proton-ALP Coupling

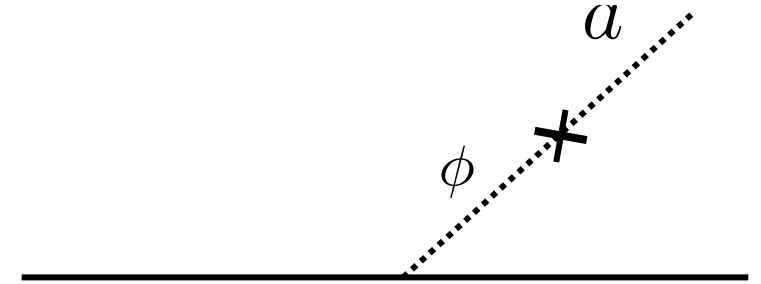
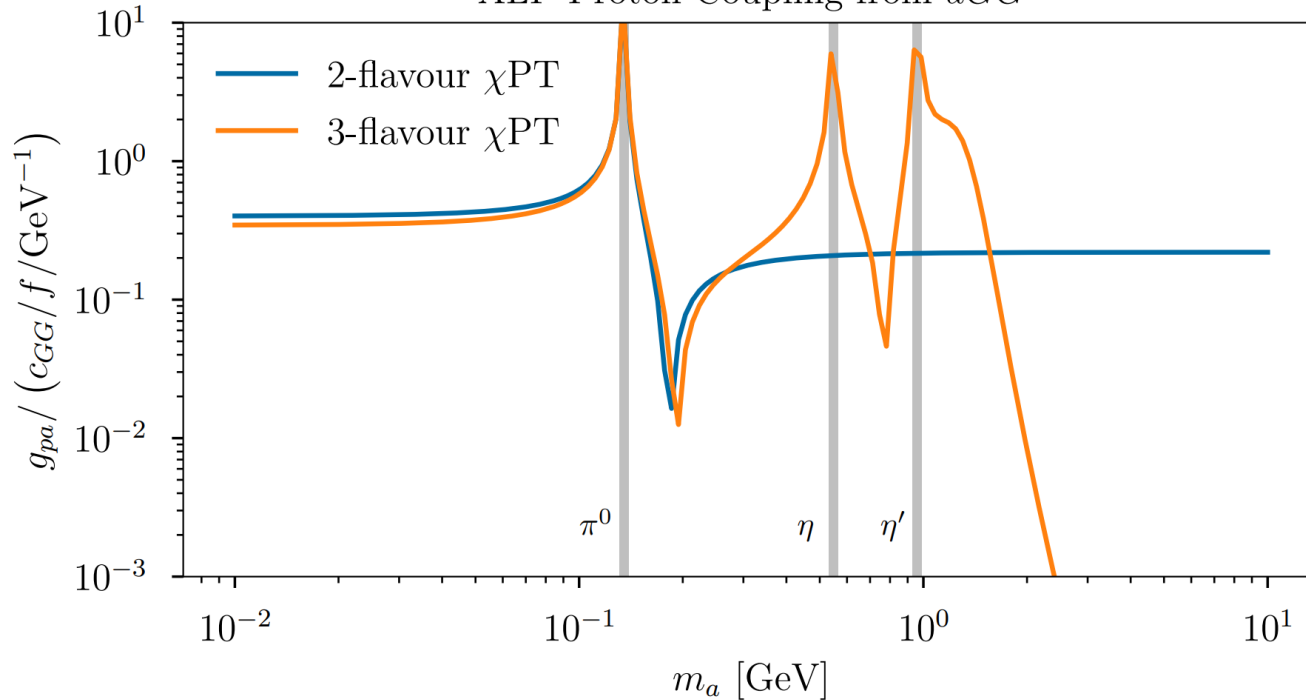
Effective operator $\frac{C_p(m_a)}{2f_a} \bar{p} \gamma^\mu \gamma_5 p \partial_\mu a$

$$C_p(m_a) = C_p^0 + \sum_{\phi} \frac{C_{K,\phi} m_a^2}{m_a^2 - m_\phi^2} \frac{g_{\phi pp}}{m_p} - \sum_{\phi} \frac{C_{M,\phi} m_\phi^2}{m_a^2 - m_\phi^2} \frac{g_{\phi pp}}{m_p}$$

Kinetic mixing

Mass mixing

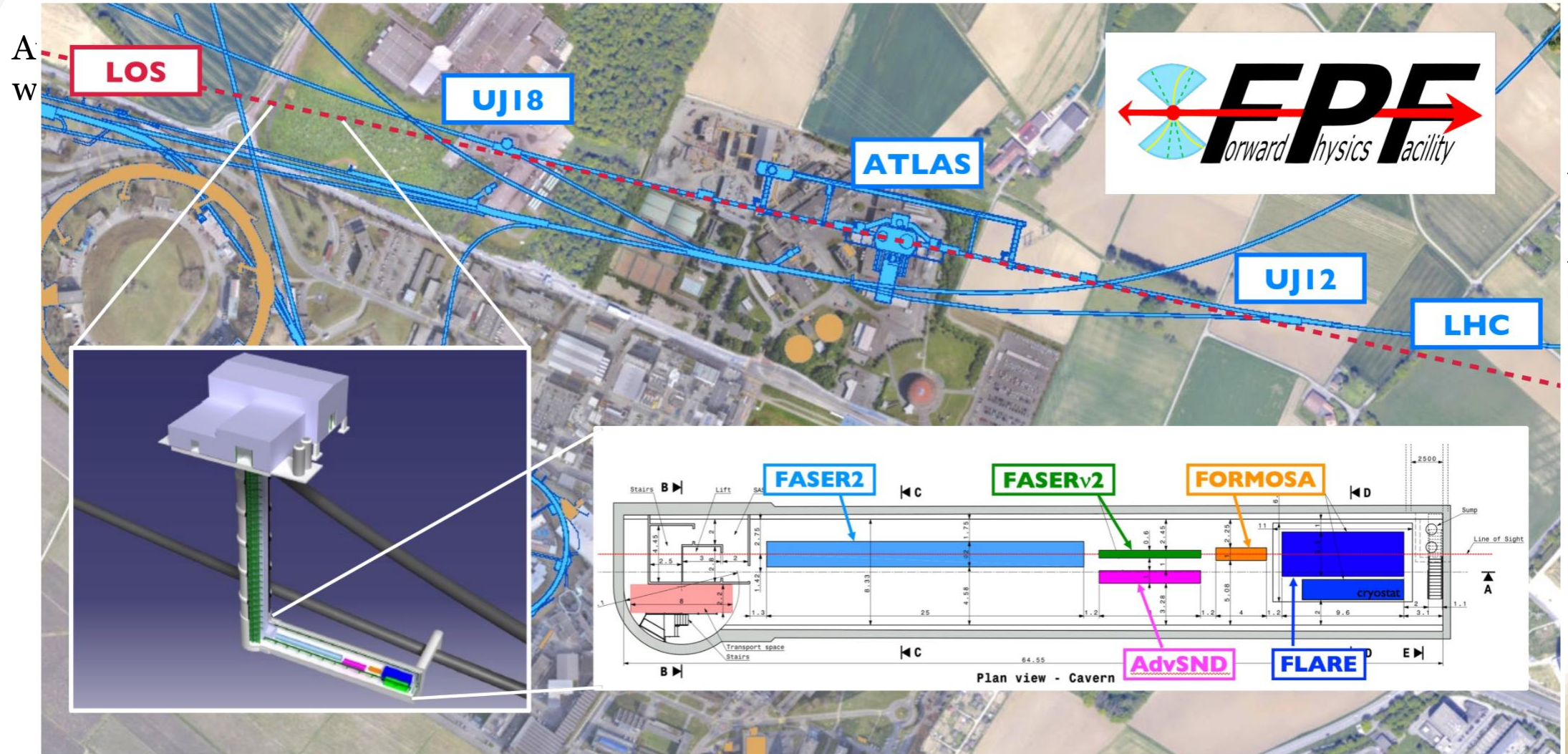
ALP Proton Coupling from $aG\tilde{G}$



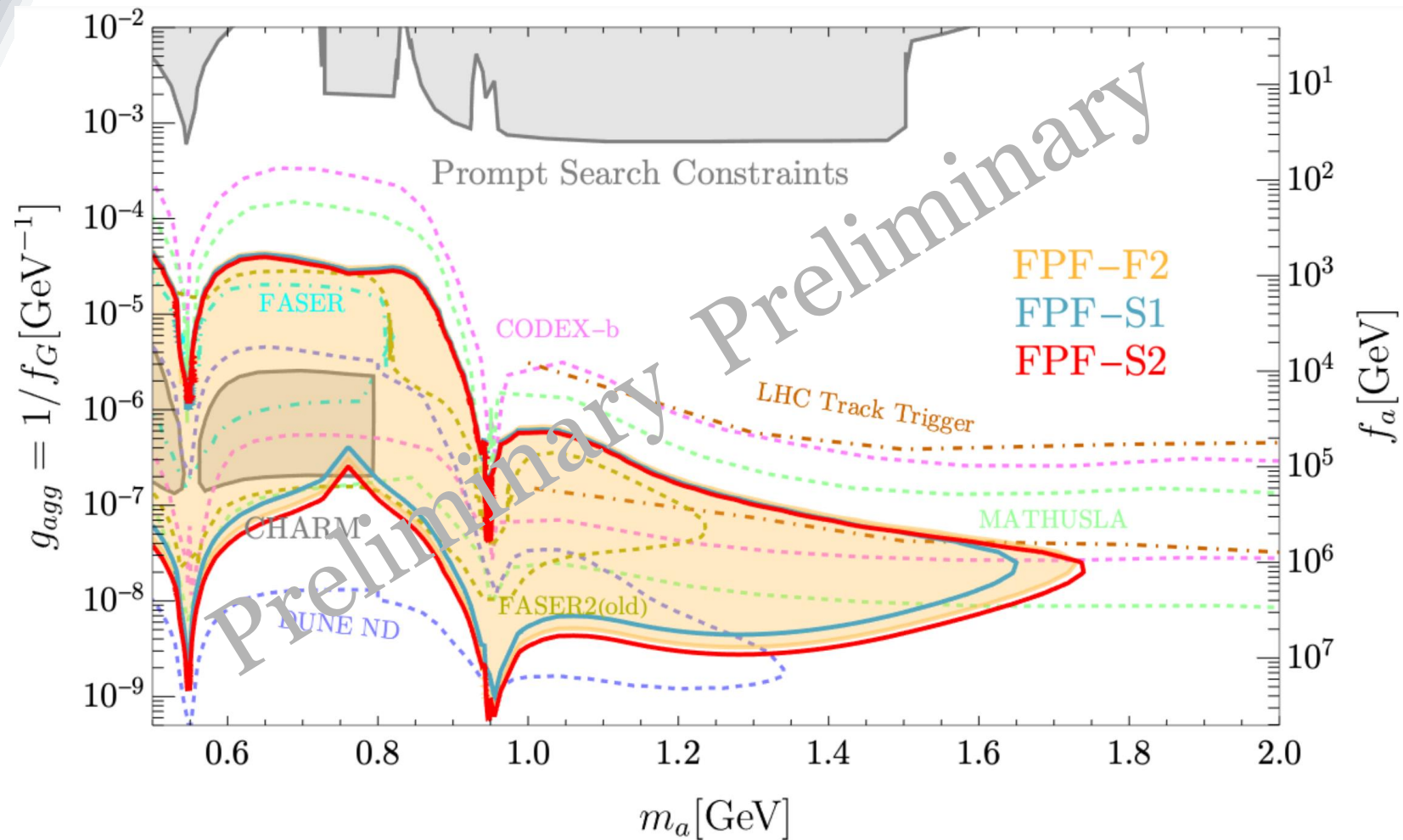
The scale-invariant part cancels with the kinetic mixing part in LO χ PT in the large m_a limit.

See the discussion in N. Blinov, E. Kowalczyk, M. Wynne, [2112.09814](https://arxiv.org/abs/2112.09814)

Forward ALP productions impact: LHC Forward Detectors



New projections using proper production



Kunfeng Lyu, ZL, in progress

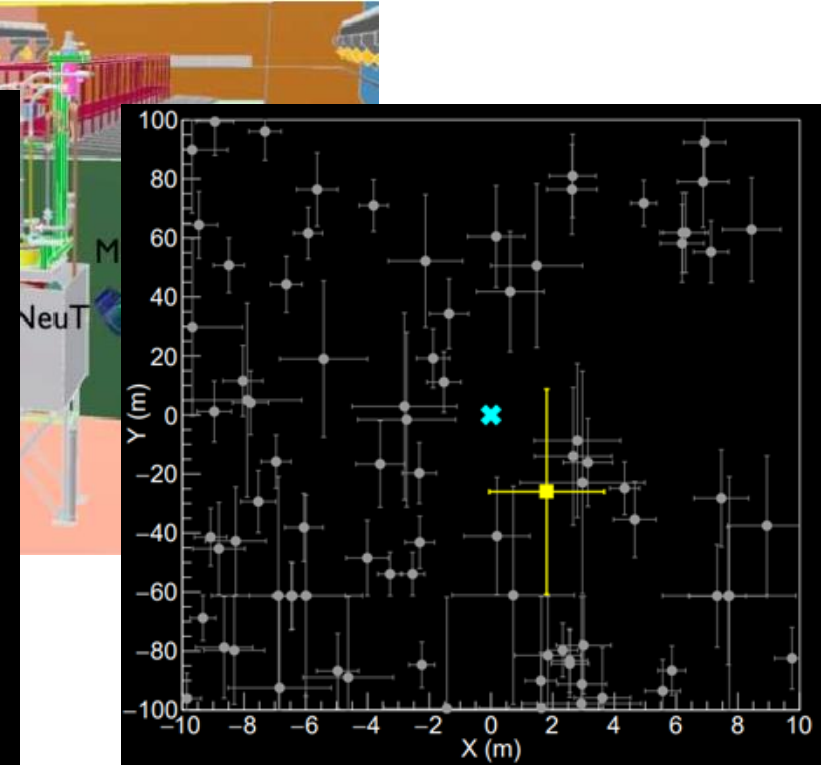
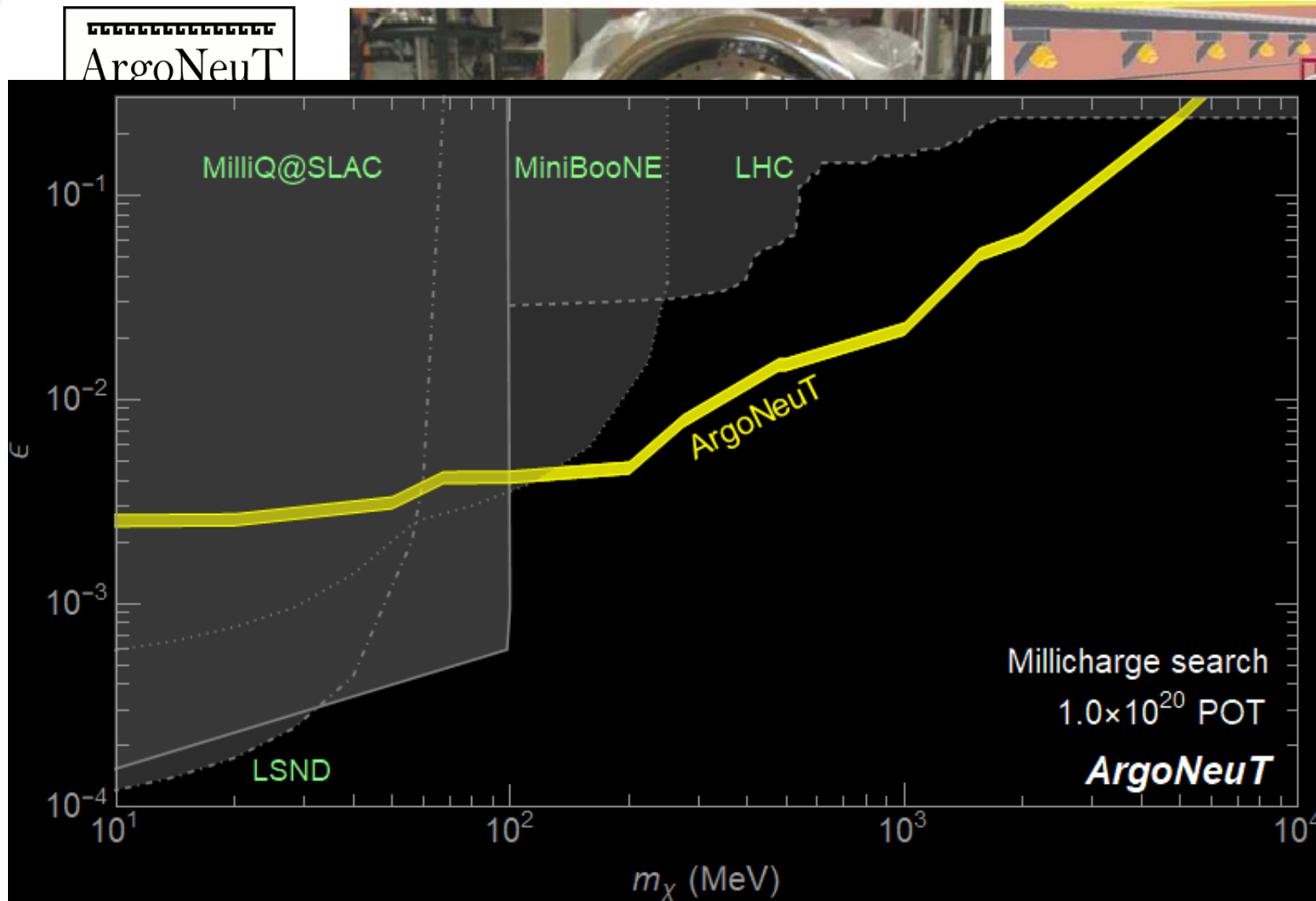
Also included as part of the FPF report

DUNE ND: 2011.05995
 FASER: 1806.02348

LHC Track: 1911.12364
 FASER2: 1811.12522

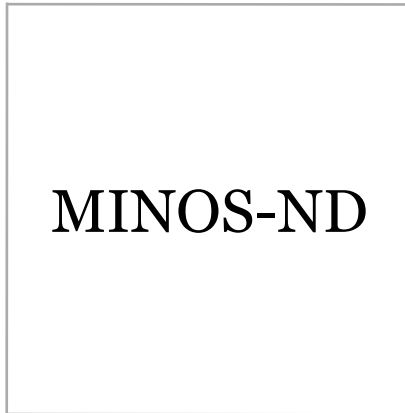
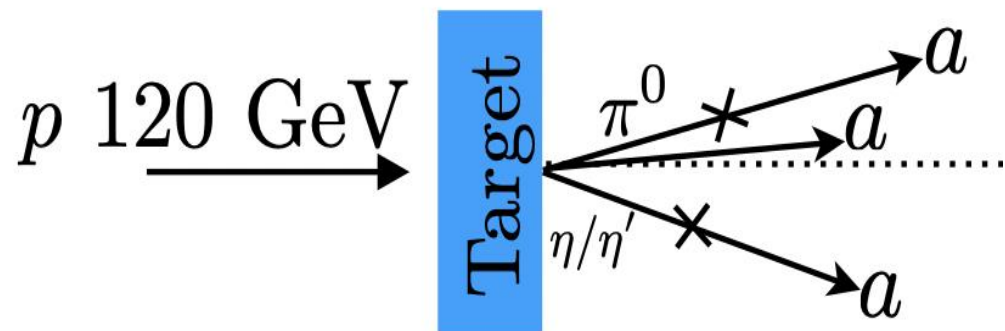
CODEX-b: 1911.00481
 MATHUSLA: 1606.06298

Can we do something with the existing data?

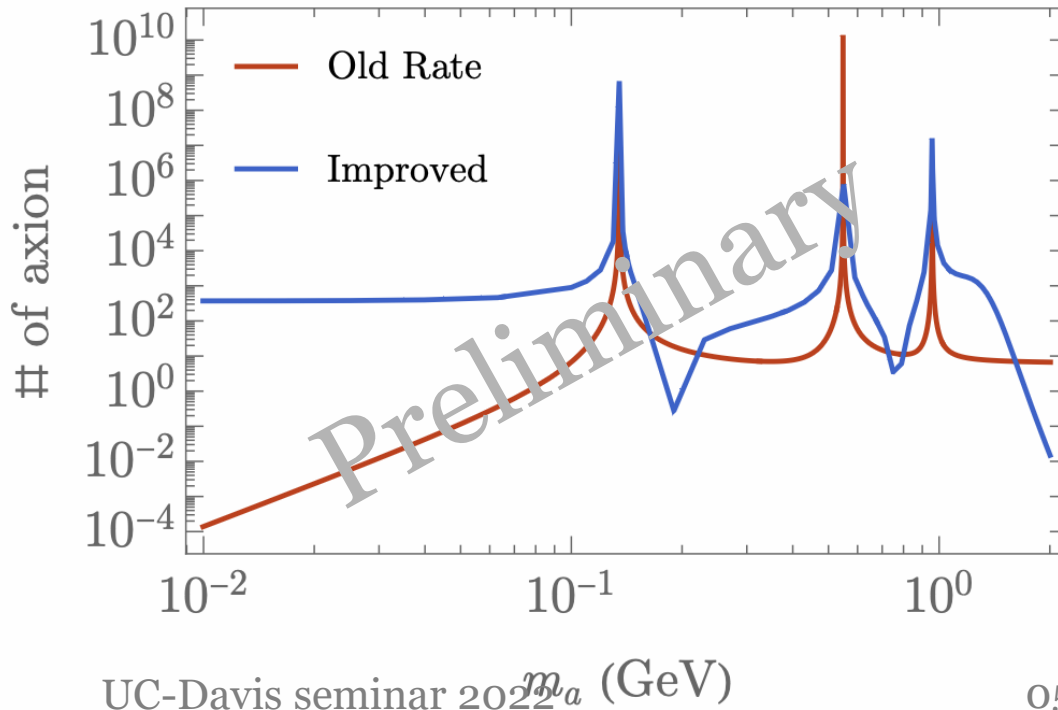
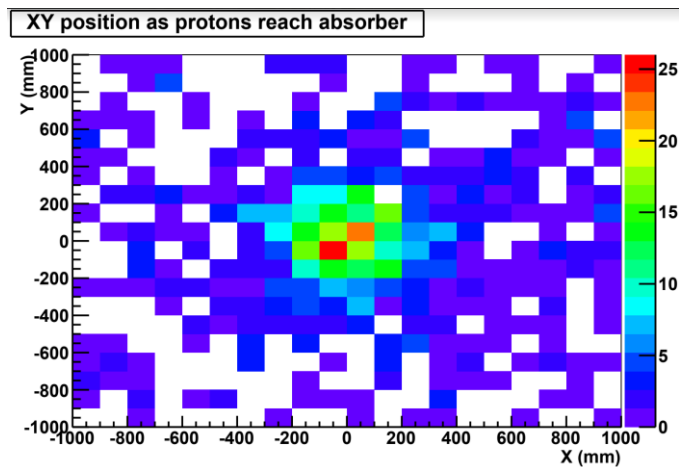


Harnik, **ZL**, Palamara, 19'
also with ArgoNeuT Collaboration, 19'

Production



Geometrically Accepted by ArgoNeuT, $f_a=1$ PeV

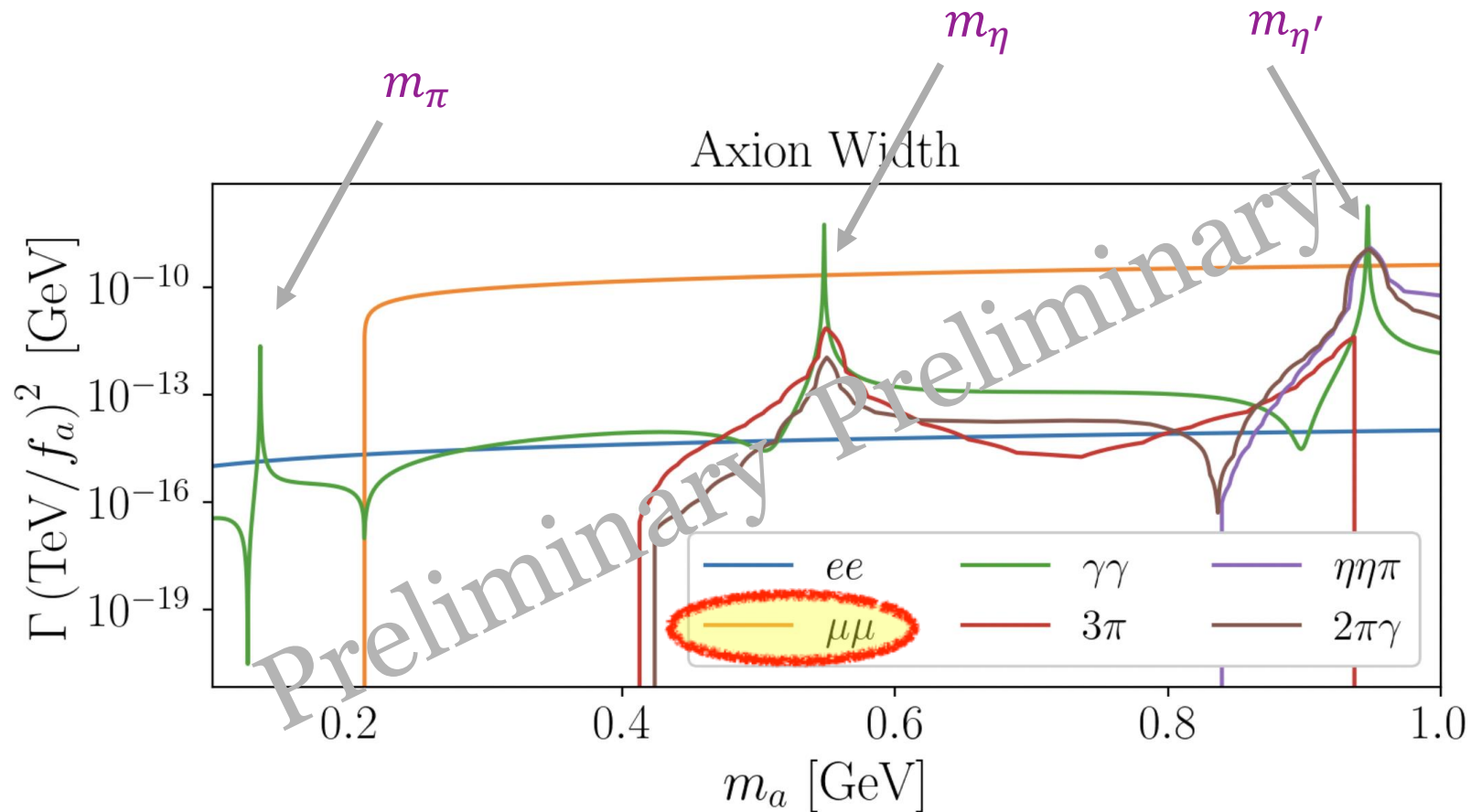


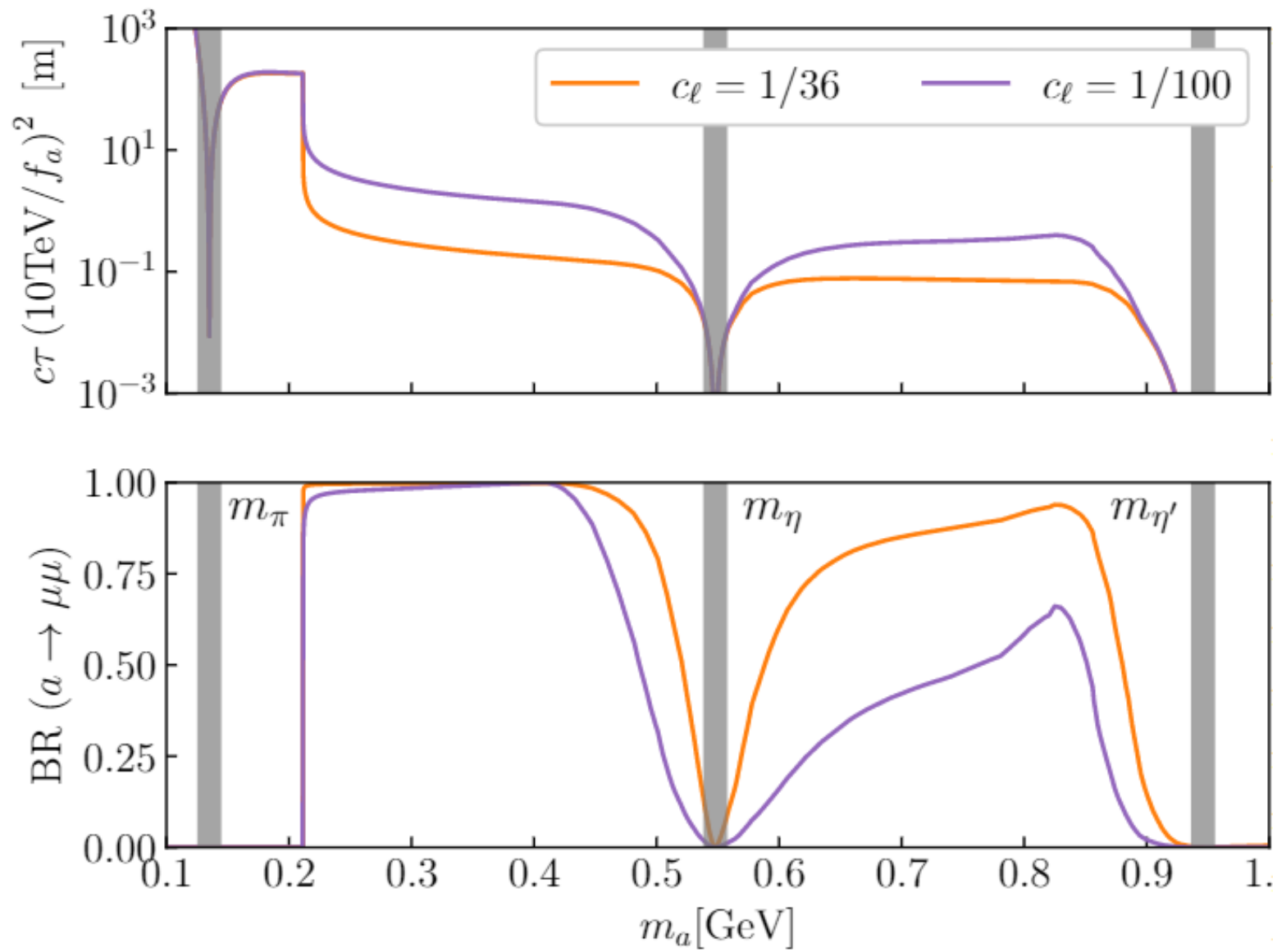
$$\mathcal{L}_{\text{gauge}} = \frac{c_3 \alpha_3}{8\pi f_a} a G \tilde{G} + \frac{c_2 \alpha_2}{8\pi f_a} a W \tilde{W} + \frac{c_1 \alpha_1}{8\pi f_a} a B \tilde{B}.$$

ArgoNeuT + MINOs can look for dimuons

- Dominated by 2μ decay

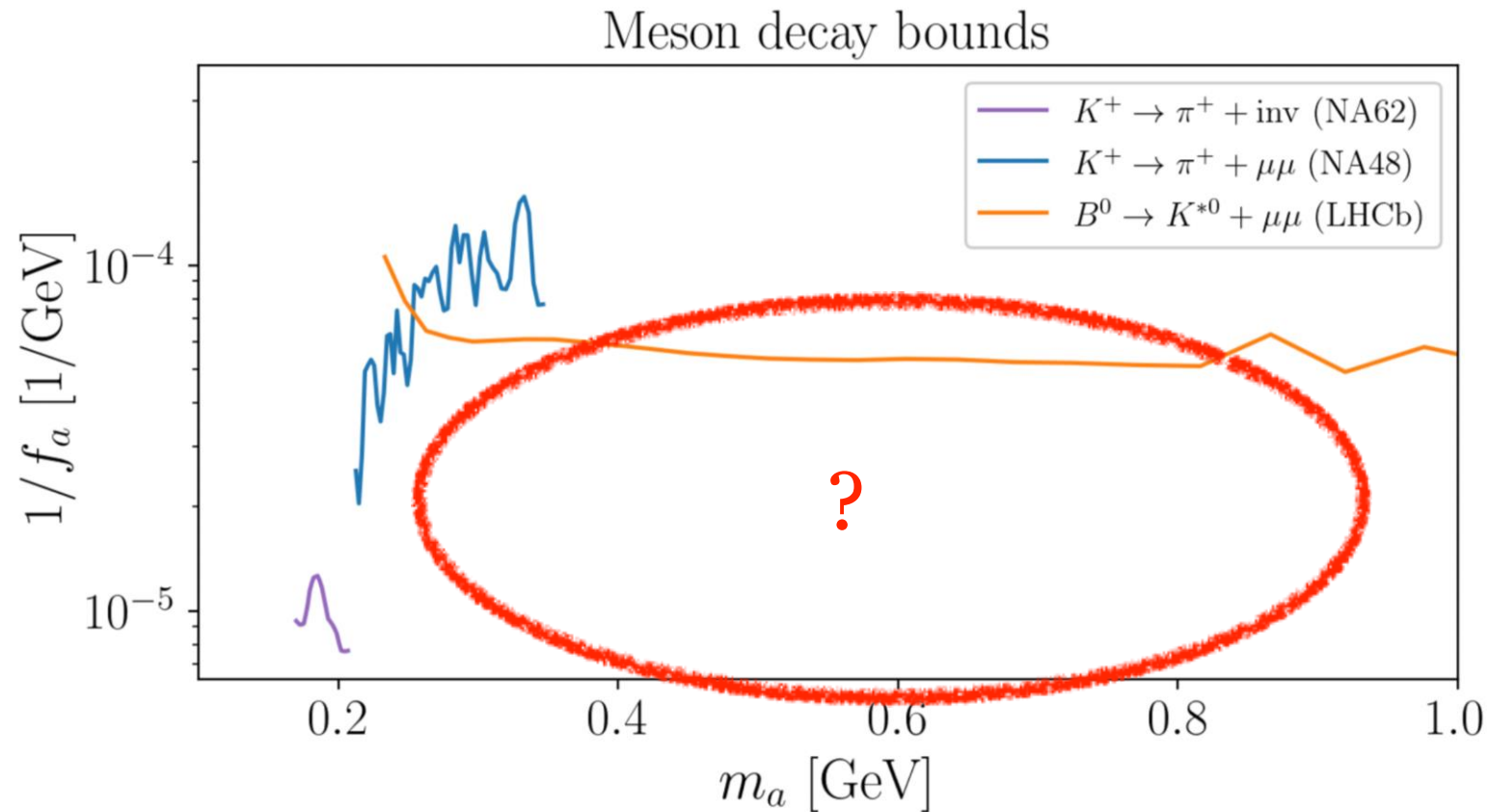
$$\mathcal{L}_{\text{lepton}} = \sum_{\ell=e,\mu,\tau} \frac{\partial_\mu a}{2f_a} (c_{V\ell} \bar{\ell} \gamma^\mu \ell + c_{A\ell} \bar{\ell} \gamma^\mu \gamma_5 \ell)$$



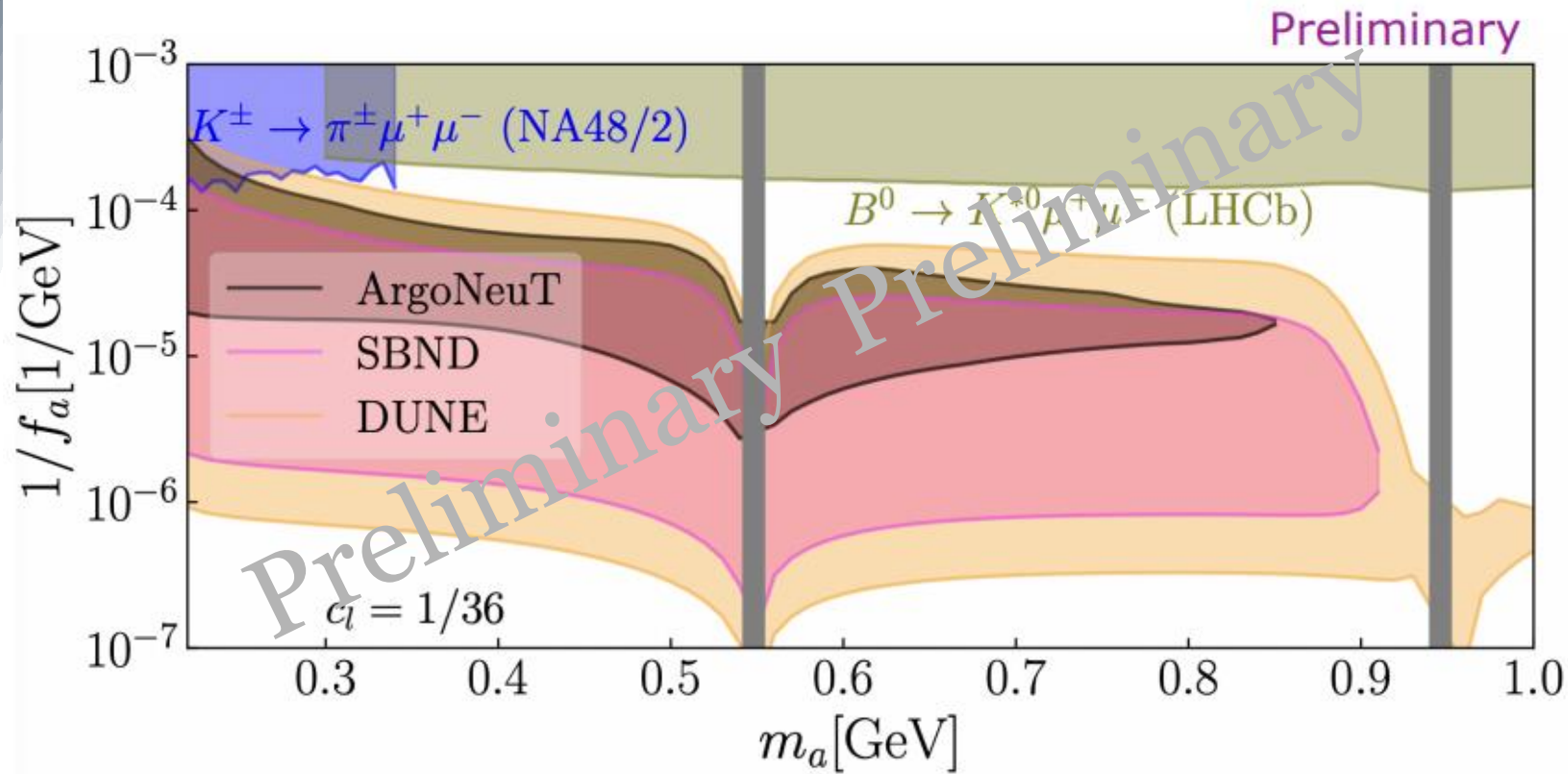


Constraints

- Renormalization group evolution induces **flavor off-diagonal quark couplings**, and strongest constraints from rare meson decay



Preliminary Results

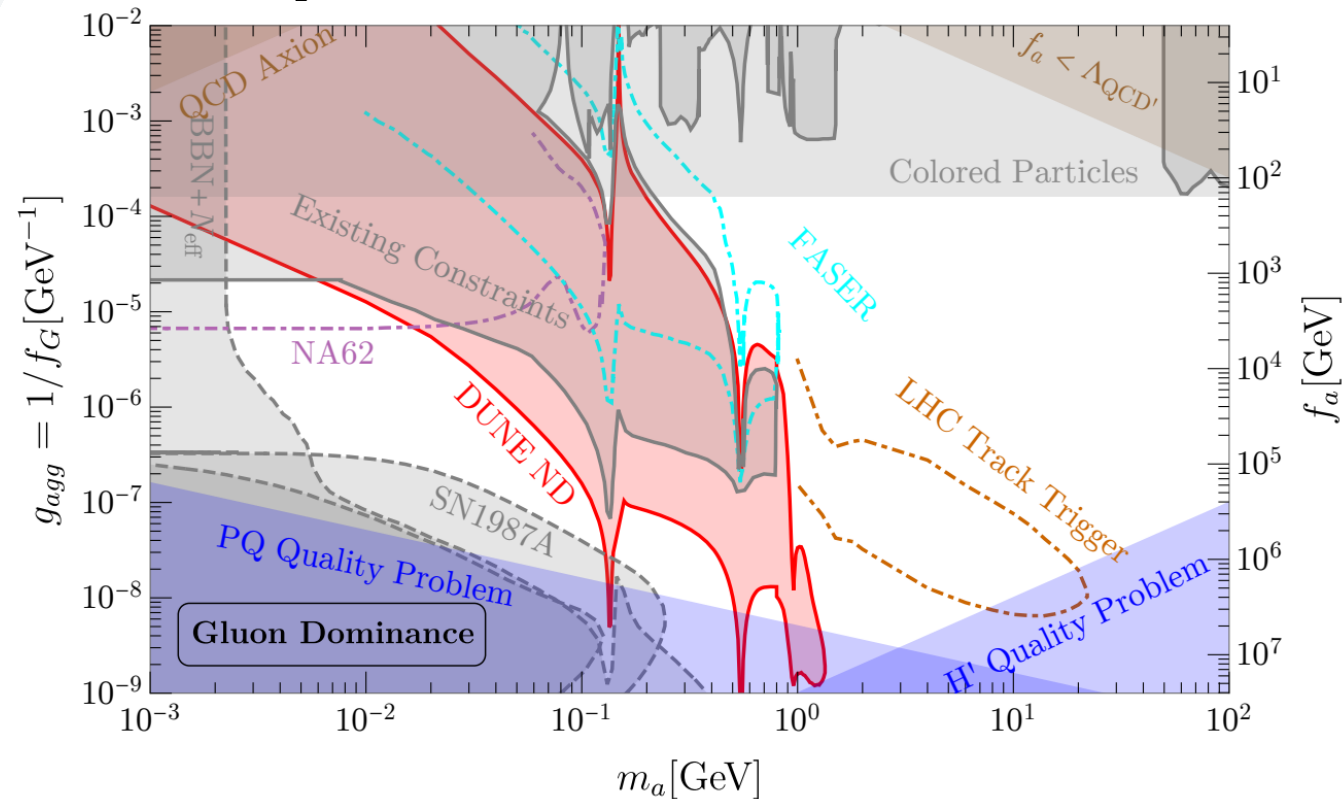


Existing data could put constraints on gluonic ALPs

Working with **S. Kumar, R. Co**, on theory studies on different future options.

Working with **ArgoNeuT** collaboration on reconstructions and putting out new results soon.

Summary and Outlook



- Gluonic ALPs are interesting to search for;
- Can be motivated by strong CP and the quality problem
- Its production, decay and search strategy all have interesting features/improvements to explore.

LHC (including FPF) and Neutrino experiments (DUNE, ArgoNeuT) will cover a large and unique parameter spaces of such motivated scenarios.

Thank you!

Matching Procedure and Momentum Shift-Scheme

pseudo-scalar case

$$|\mathcal{M}|^2 = \left(\frac{c_{pa}}{2f_a}\right)^2 [-4(k \cdot p'_m)(k \cdot p) - 2m_a^2(p'_m \cdot p) - 2m_p^2 m_a^2]$$

scalar case

$$|\mathcal{M}|^2 = \left(\frac{c_{pa}}{2f_a}\right)^2 [-4(k \cdot p'_m)(k \cdot p) - 2m_a^2(p'_m \cdot p) + 2m_p^2 m_a^2]$$

