A Causal Framework for Non-linear Quantum Mechanics

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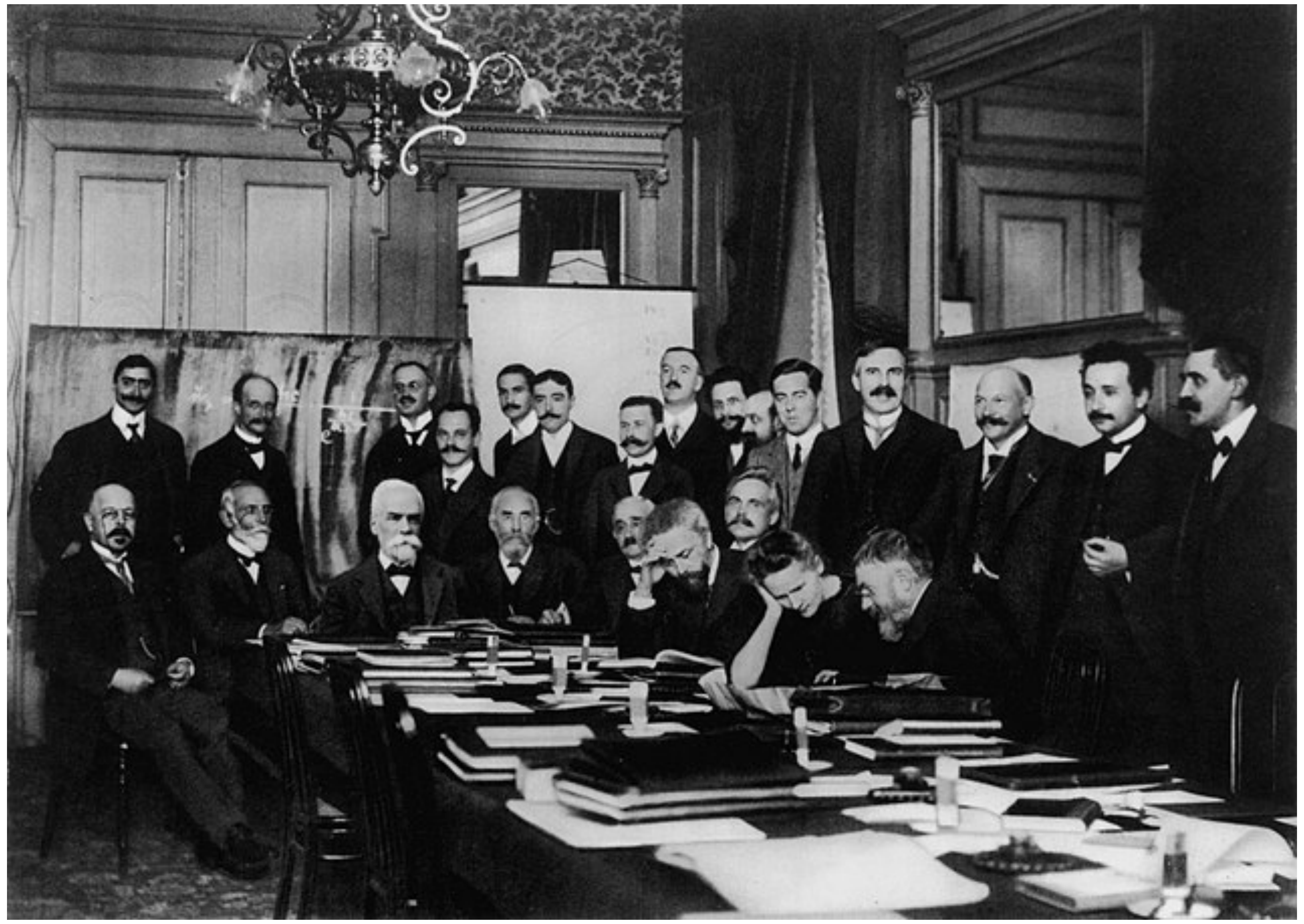
Why?



Quantum Mechanics

Theory built on observations in the 1900s
Why should it be “the absolute truth”?

What?

Two Postulates of Quantum Mechanics

Probability

Linearity

Which?

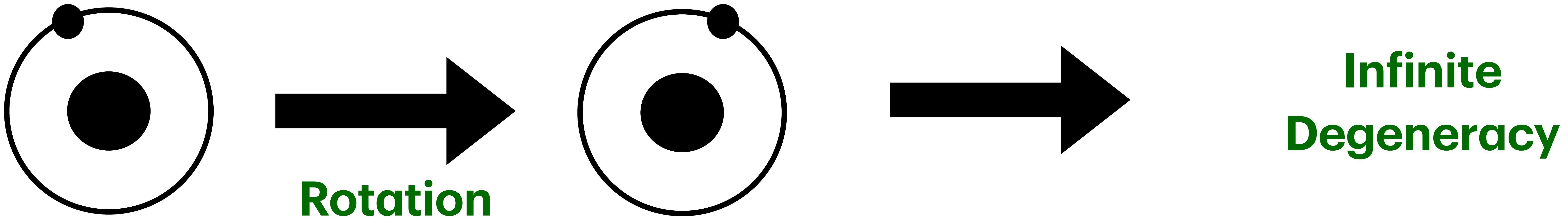
Probability

Finite system has a finite set of energies

Continuous observables and symmetries

} Deterministic Observables?

Could an electron in an atom have a well defined position?



Quantum Mechanics

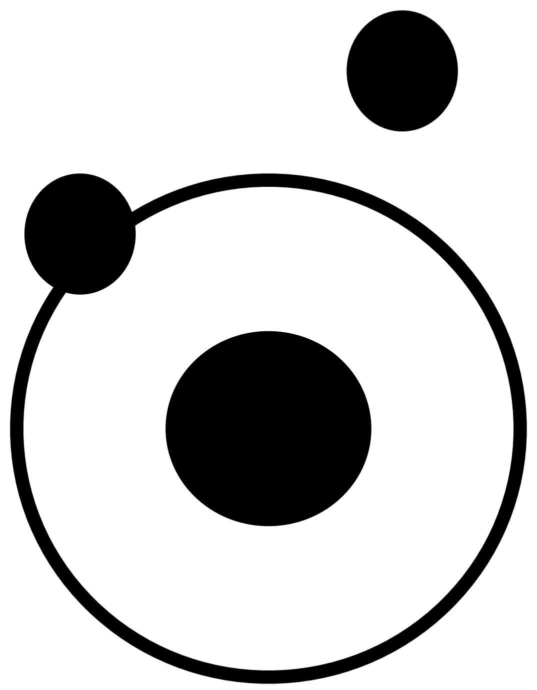
Sacrifice Determinism.

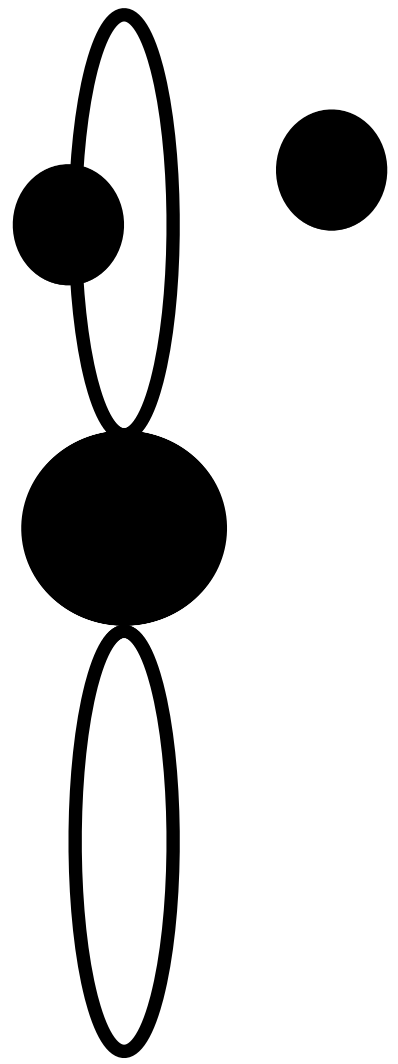
Preserve finite set of energy states, continuous symmetries and observables

Bell Inequalities, Kochen-Specker, SSC Theorems

Linearity

Wave-function does not interact with itself





Lamb Shift

Degeneracies of the Hydrogen Atom

No self-energy of electron cloud
(Two different shapes - yet, same energy)

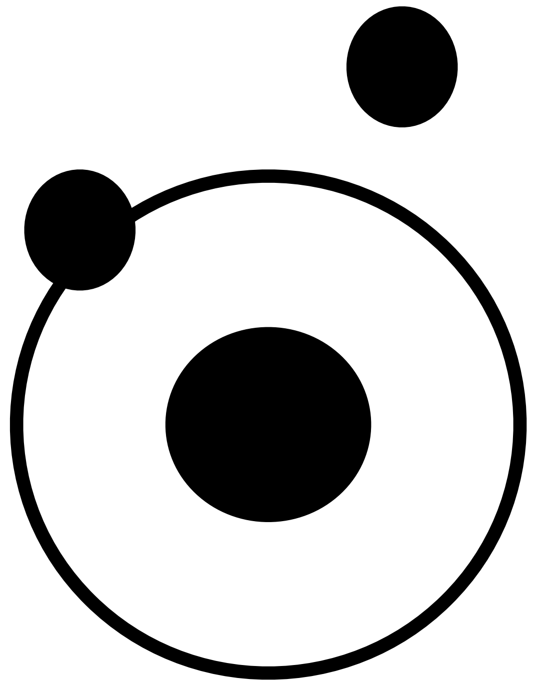
Another charged particle clearly sensitive to shape

Why Not?

Challenges

Causality? Measurement?
Quantum Field Theory?

Causality



Plenty of examples of causal non-linear theories

Causality Enforced by Green's Functions

$$\Box G\left(x;x'\right) = \delta^{(4)}\left(x-x'\right)$$

Linear Quantum Mechanics is basis independent

Local Interactions - Position Basis!

Causality issue? Use Position Basis!

Non Linear Quantum Mechanics in position basis?

Natural embedding in local field theory

Outline

1. The Framework
(Field Theory, Schrodinger, Causality, Measurement)

2. Macroscopic Effects

3. Constraints and Tests

4. Cosmological Sensitivity

5. Conclusions

The Framework

The Framework

The Schrodinger Picture of Quantum Field Theory

$$|\chi(t)\rangle$$

Quantum State of Fields (e.g. in Fock states)

$$\phi(x)$$

Time Independent Operators

$$H = \int d^3x\, \mathcal{H}(\phi(x), \pi(x))$$

Time Evolution

$$i\frac{\partial|\chi(t)\rangle}{\partial t} = H|\chi(t)\rangle$$

Action

$$S = \int dt \frac{i}{2}\left(\langle\dot{\chi}|\chi\rangle - \langle\chi|\dot{\chi}\rangle\right) + \langle\chi|H|\chi\rangle$$

The Framework

Yukawa $H \supset \int d^3x \, y \, \phi(x) \, \bar{\Psi}(x) \, \Psi(x)$

Action

$$S = \int dt \frac{i}{2} \left(\langle \dot{\chi} | \chi \rangle - \langle \chi | \dot{\chi} \rangle \right) + \langle \chi | H | \chi \rangle \supset \langle \chi(t) | \left(\int d^3x \, y \, \phi(x) \, \bar{\Psi}(x) \, \Psi(x) \right) | \chi(t) \rangle$$

$$\supset \left(\int d^3x \, y \, \langle \chi(t) | \phi(x) \, \bar{\Psi}(x) \, \Psi(x) | \chi(t) \rangle \right)$$

Quantum Field Theory $\supset \left(\int d^3x \, y \, \langle \chi(t) | \phi(x) \, \bar{\Psi}(x) \, \Psi(x) + \frac{\phi^2}{\Lambda} \bar{\Psi}\Psi + \ldots | \chi(t) \rangle \right)$

Non-linearities in the operators but not in the state

The Framework

Yukawa $H \supset \int d^3x \, y \, \phi(x) \, \bar{\Psi}(x) \, \Psi(x)$

Linear QFT: $S \supset \left(\int d^3x \, y \, \langle \chi(t) | \phi(x) \, \bar{\Psi}(x) \, \Psi(x) | \chi(t) \rangle \right)$

Non-Linear QFT: $S_{NL} \supset \epsilon \left(\int d^3x \, \langle \chi(t) | \phi(x) | \chi(t) \rangle \langle \chi(t) | \bar{\Psi}(x) \, \Psi(x) | \chi(t) \rangle \right)$

Obeys all the rules

Higher order in states - leads to state dependent quantum evolution

Analyze non-linearity perturbatively

Perturbation Theory

$$\mathcal{H} \supset y\Phi\bar{\Psi}\Psi = \left(y\phi + \epsilon\langle\chi|\phi|\chi\rangle\right)\bar{\Psi}\Psi$$

$$i\frac{\partial|\chi\rangle}{\partial t} = H|\chi\rangle$$

At zeroth order, this is just standard QFT

At first order, use zeroth order solution - expectation value is simply a background field

Perform standard QFT on this background field to compute first order correction

Applies to all orders : To compute term of given order, only need lower order terms
Lower order terms enter as background fields

Causality: Non-linearity enters via expectation value. At lowest order, causal from QFT.
Causal background field for all higher orders

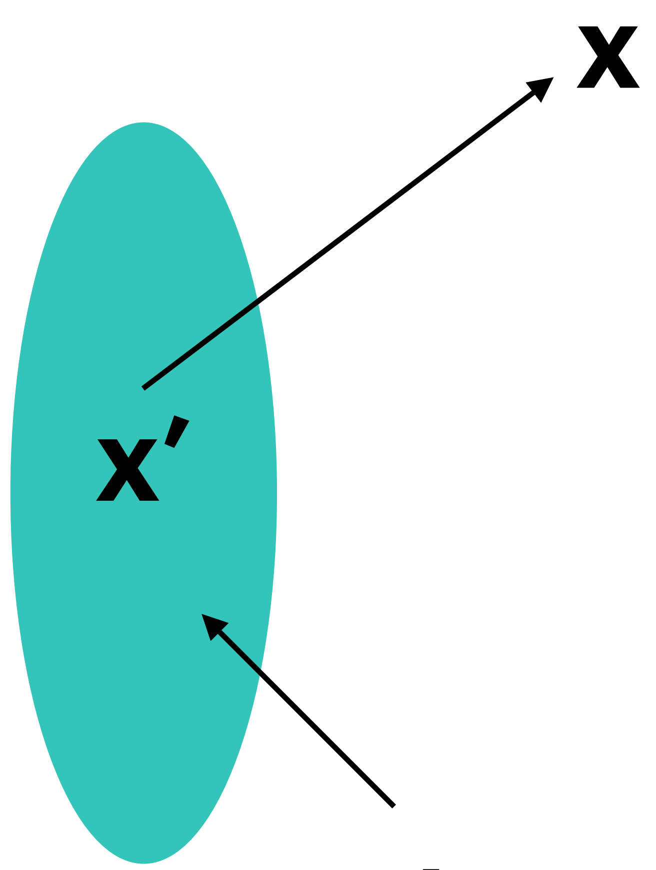
Single Particle

$$\mathcal{L} \supset y\Phi\bar{\Psi}\Psi = y\left(\phi + \tilde{\epsilon}\langle\chi|\phi|\chi\rangle\right)\bar{\Psi}\Psi$$

Suppose we have a ψ particle - how does its wave-function evolve?

To zeroth order, ψ just sources the Φ field

Straightforward Computation of Expectation Value



$$\langle\chi|\phi\left(x\right)|\chi\rangle = \int d^4x'\psi^*\left(x'\right)\psi\left(x'\right)G_R\left(x-x'\right)$$

Charge Density of ψ

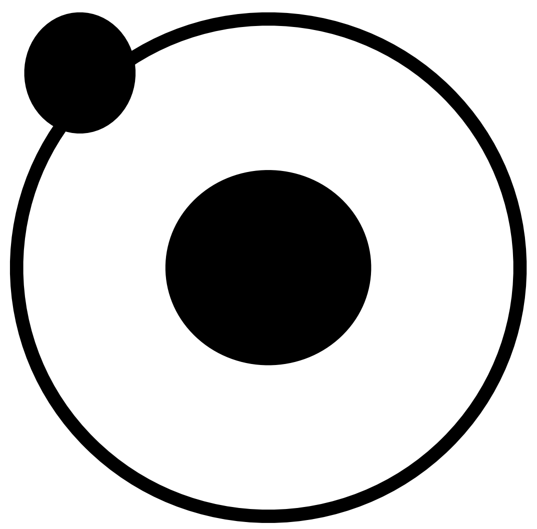
Causal Green's Function

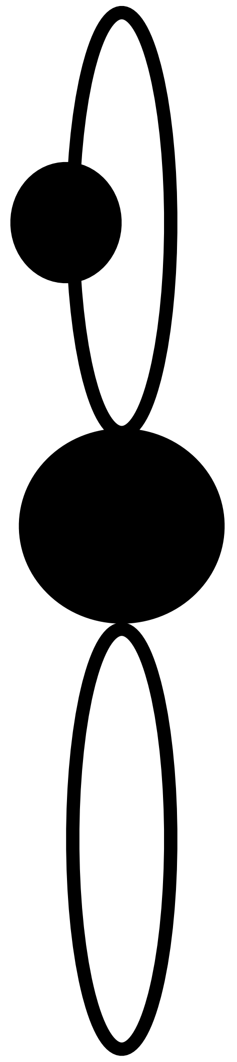
Schrodinger Equation

$$\mathcal{H} \supset y\Phi\bar{\Psi}\Psi = \left(y\phi + \epsilon\langle\chi|\phi|\chi\rangle\right)\bar{\Psi}\Psi$$

Single particle equation derived from field theory
Equation depends upon theory (Yukawa, Φ^4 etc)

$$i\frac{\partial\Psi(t,\mathbf{x})}{\partial t} = \left(H + \tilde{\epsilon}y\int d^4x'\Psi^*(x)\,\Psi(x')\,G_R(x;x')\right)\Psi(t,\mathbf{x})$$





Fixed Central particle

Self interaction of wave-function breaks degeneracy of levels

Hermitean Form of Hamiltonian implies conserved norm

Maintain Probabilistic Interpretation

Entangled Systems

$$\Psi\left(x,y;t\right)=\sum_{i,j}c_{ij}\left(t\right)\alpha_i\left(x\right)\beta_j\left(y\right)$$

How do multi-particle systems evolve?

$$\mathcal{H}\supset y\Phi\bar{\Psi}\Psi=\left(y\phi+\epsilon\langle\chi|\phi|\chi\rangle\right)\bar{\Psi}\Psi$$

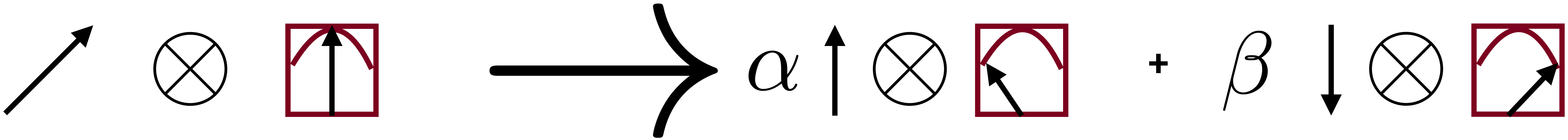
$$\langle\chi|\phi|\chi\rangle=\int d^3x_1d^3y_1d\tau|\Psi\left(x_1,y_1;\tau\right)|^2\left(G_R\left(t,x;\tau,x_1\right)+G_R\left(t,y;\tau,x_1\right)+G_R\left(t,x;\tau,y_1\right)+G_R\left(t,y;\tau,y_1\right)\right)$$

Additive form demanded by Polchinski - Natural in Field theory!

Measurement in Quantum Mechanics

Not some mysterious process

Interaction between quantum state and measuring device



$|\Psi\rangle$ $\quad |A_0\rangle \quad\quad |i\rangle \otimes |A_i\rangle$

$$|\Psi\rangle \otimes |A_0\rangle \to \sum_i c_i |i\rangle \otimes |A_i\rangle$$

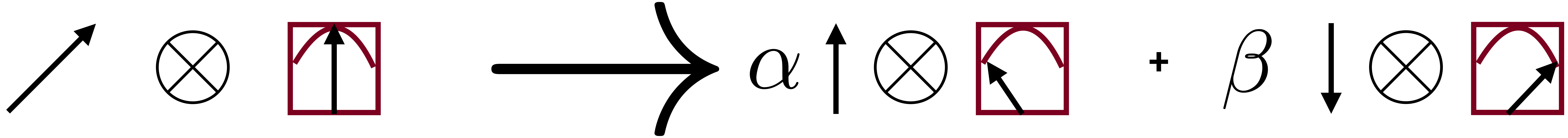
Prediction of Quantum Mechanics ("Many Worlds"), Not an interpretation

Pick: $\langle A_j | A_i \rangle = \delta_{ij} \quad \Longrightarrow \quad \rho_{|\Psi\rangle} = \sum_i c_i c_i^* |i\rangle\langle i|$

"Interpret" as direct sum of "worlds"

Measurement in Non-Linear Quantum Mechanics

Interaction between quantum state and measuring device



$|\Psi\rangle$ $|A_0\rangle$ $|i\rangle \otimes |A_i\rangle$

In linear QM, just need to know the basis vectors

Interaction Hamiltonian independent of unknown quantum state

$$\text{Pick: } \langle A_j | A_i \rangle = \delta_{ij}$$

Key Point: Non-linear Hamiltonian depends upon unknown quantum state

$$\text{No Guarantee:} \langle A_j | A_i \rangle = 0$$

$$|\Psi\rangle \otimes |A_0\rangle \rightarrow \sum_i c_i |i\rangle \otimes |A_i\rangle + \epsilon \sum_{i,j} d_{i,j} |i\rangle \otimes |A_j\rangle$$

Measurement Noise

Gauge Theories and Gravitation

Linear QFT Lagrangian, Shift bosonic field by expectation value

To Path Integral, add:

$$e^{iS_0+i\int d^4x\left(e\frac{(A_\mu+\epsilon_\gamma\langle\chi|A_\mu|\chi\rangle)}{1+\epsilon_\gamma}J^\mu+\epsilon_{\tilde{\gamma}}\langle\chi|F_{\mu\nu}|\chi\rangle F^{\mu\nu}\right)}$$

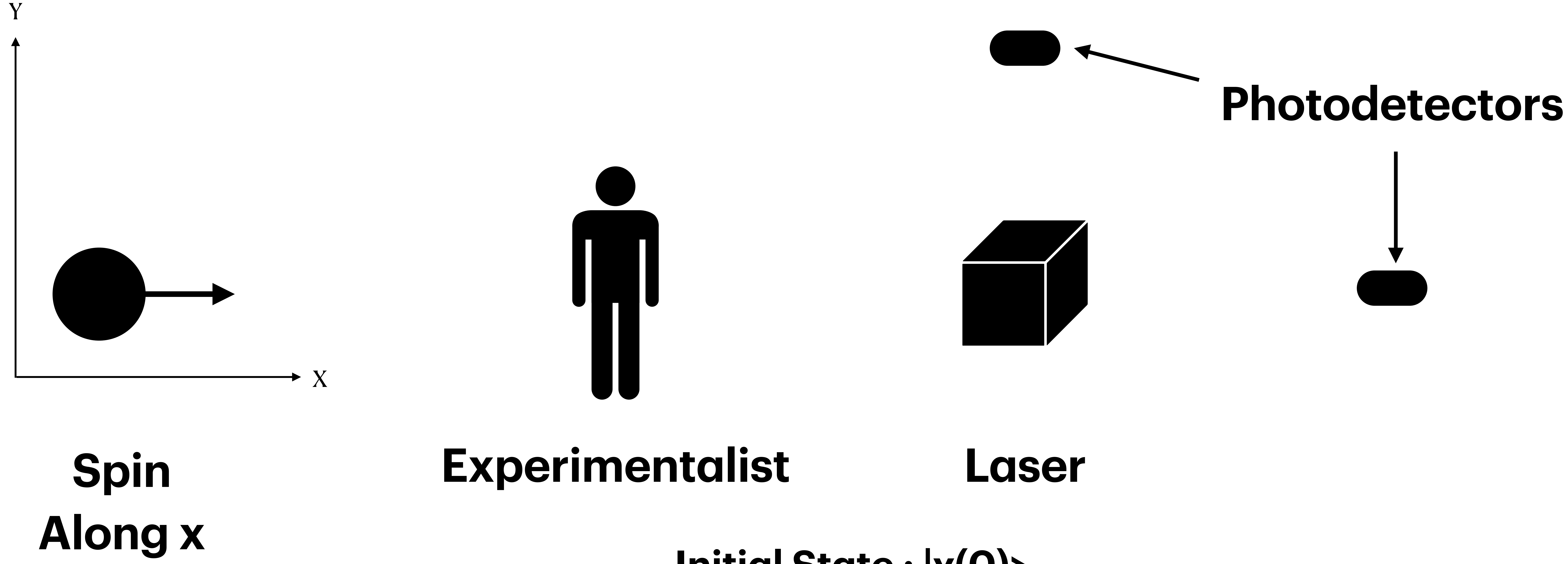
Background Field

Gravitation

$$e^{iS_0+i\int d^4x(\epsilon_G\langle\chi|g_{\mu\nu}|\chi\rangle\partial^\mu\phi\partial^\nu\phi)}$$

Macroscopic Effects

Linear Quantum Mechanics



Initial State : |χ(0)>

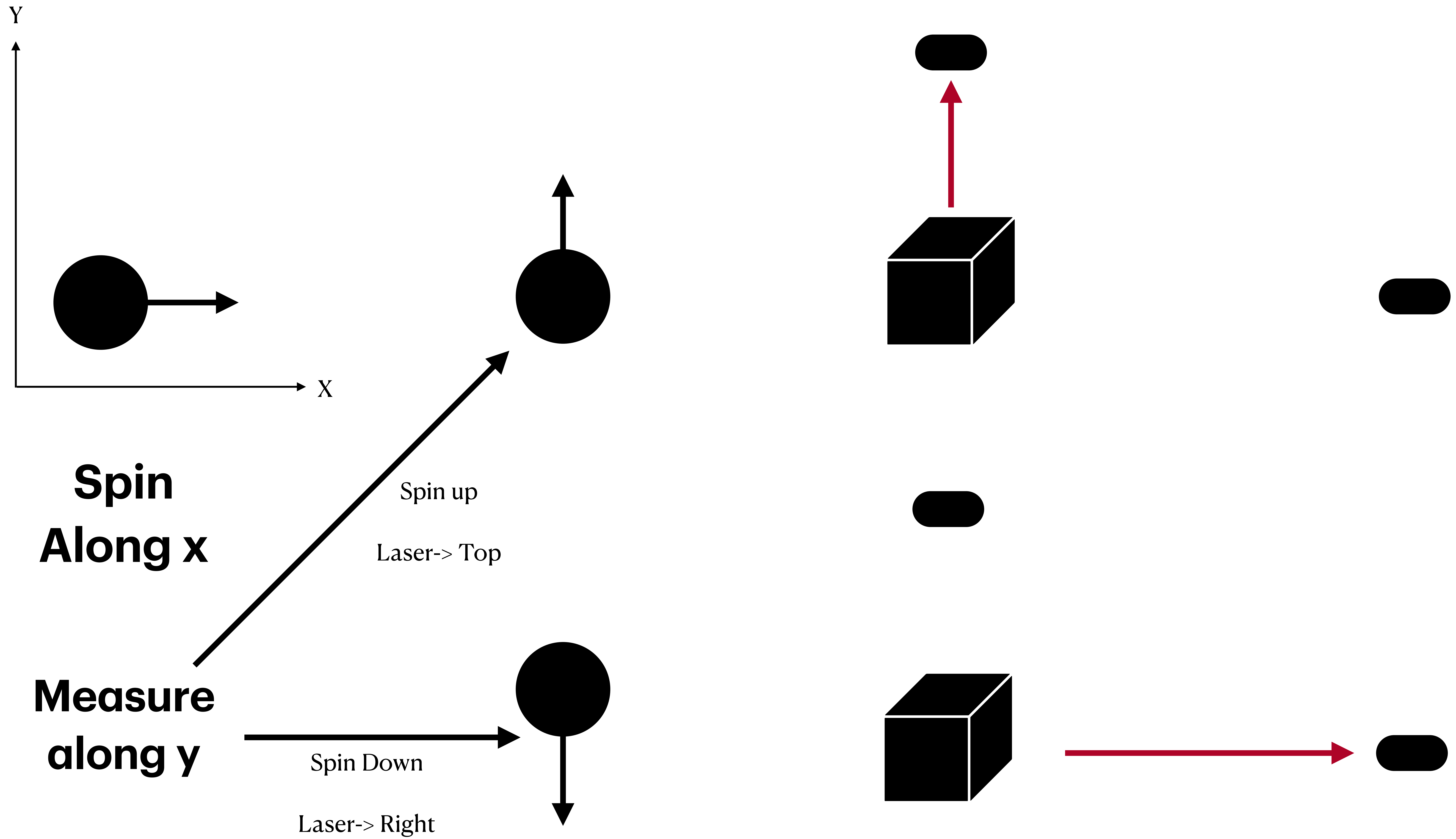
Represents Full Quantum State (spin, experimentalist...)

Goal: Create Macroscopic Superposition

Method: Measure spin along y.

Depending upon outcome, send laser along different directions

Macroscopic Superposition



Final State: $|\chi\rangle = |U\rangle|T\rangle|E_T\rangle + |D\rangle|R\rangle|E_R\rangle$

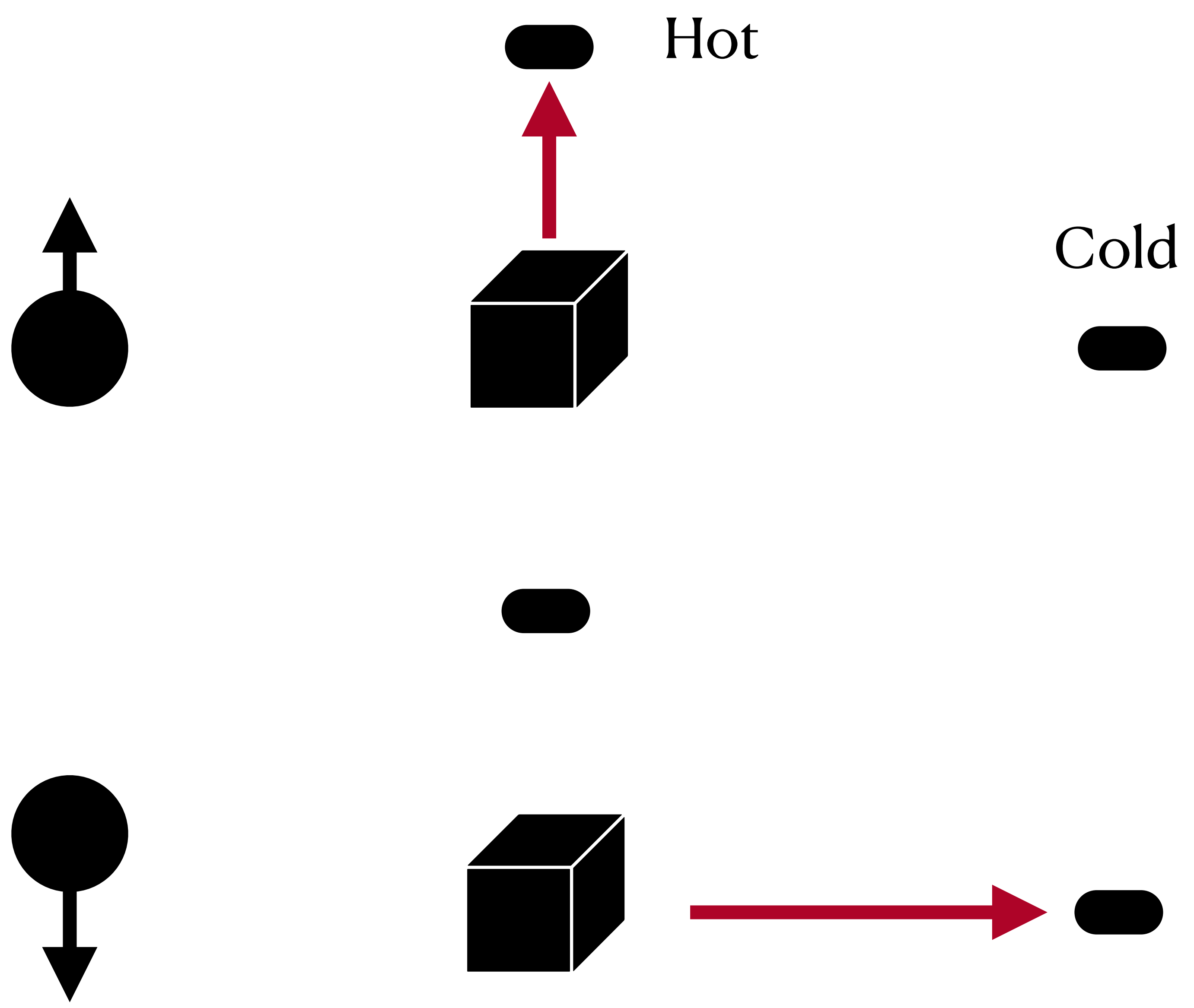
Prediction of QM

Linear Quantum Mechanics

Which photodetectors light up?

|χ> = |U>|T>|E$_T$> + |D>|R>|E$_R$>

$$\mathcal{L} \supset e A_\mu \bar{\Psi} \gamma^\mu \Psi$$



Compute Transition Matrix Elements

$$\langle U| \langle T| \langle E_T| e A_\mu (x_T) \bar{\Psi} (x_T) \gamma^\mu \Psi (x_T) |\chi\rangle \neq 0$$

$$\langle U| \langle T| \langle E_T| e A_\mu (x_R) \bar{\Psi} (x_R) \gamma^\mu \Psi (x_R) |\chi\rangle = 0$$

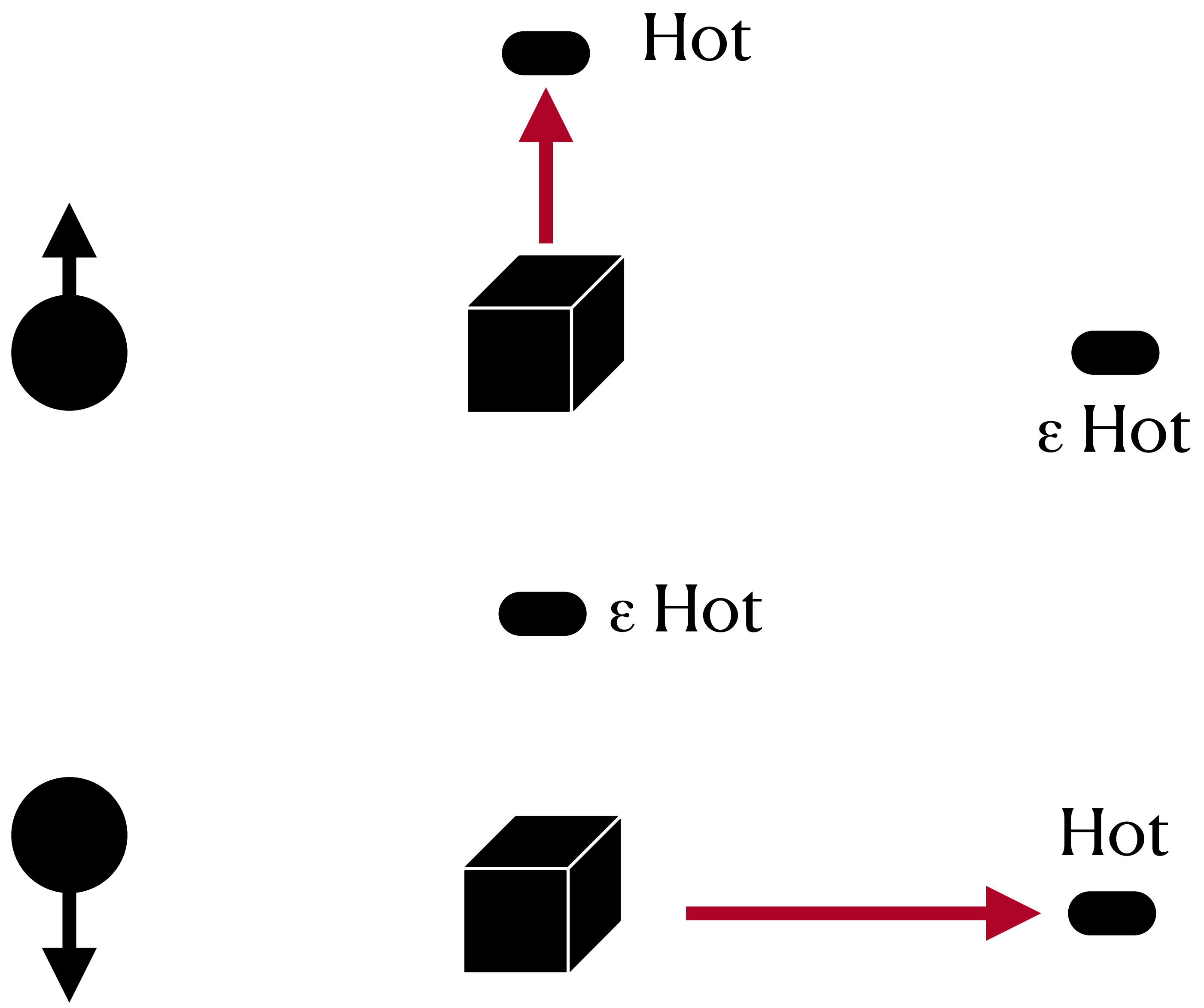
$$\langle T| A_\mu (x_R) |T\rangle = 0$$

But in both |E$_T$>, |E$_R$>: $\langle \chi | A_\mu (x_T) | \chi \rangle \neq 0, \langle \chi | A_\mu (x_R) | \chi \rangle \neq 0$

Non-Linear Quantum Mechanics

$$\mathcal{L} \supset eA_\mu \bar{\Psi}\gamma^\mu \Psi + \epsilon_\gamma e \langle\chi|A_\mu|\chi\rangle \bar{\Psi}\gamma^\mu\Psi$$

State Dependent Non-linear Term



But in both $|E_T>$, $|E_R>$:

$$\langle\chi|A_\mu\left(x_T\right)|\chi\rangle \neq 0, \langle\chi|A_\mu\left(x_R\right)|\chi\rangle \neq 0$$

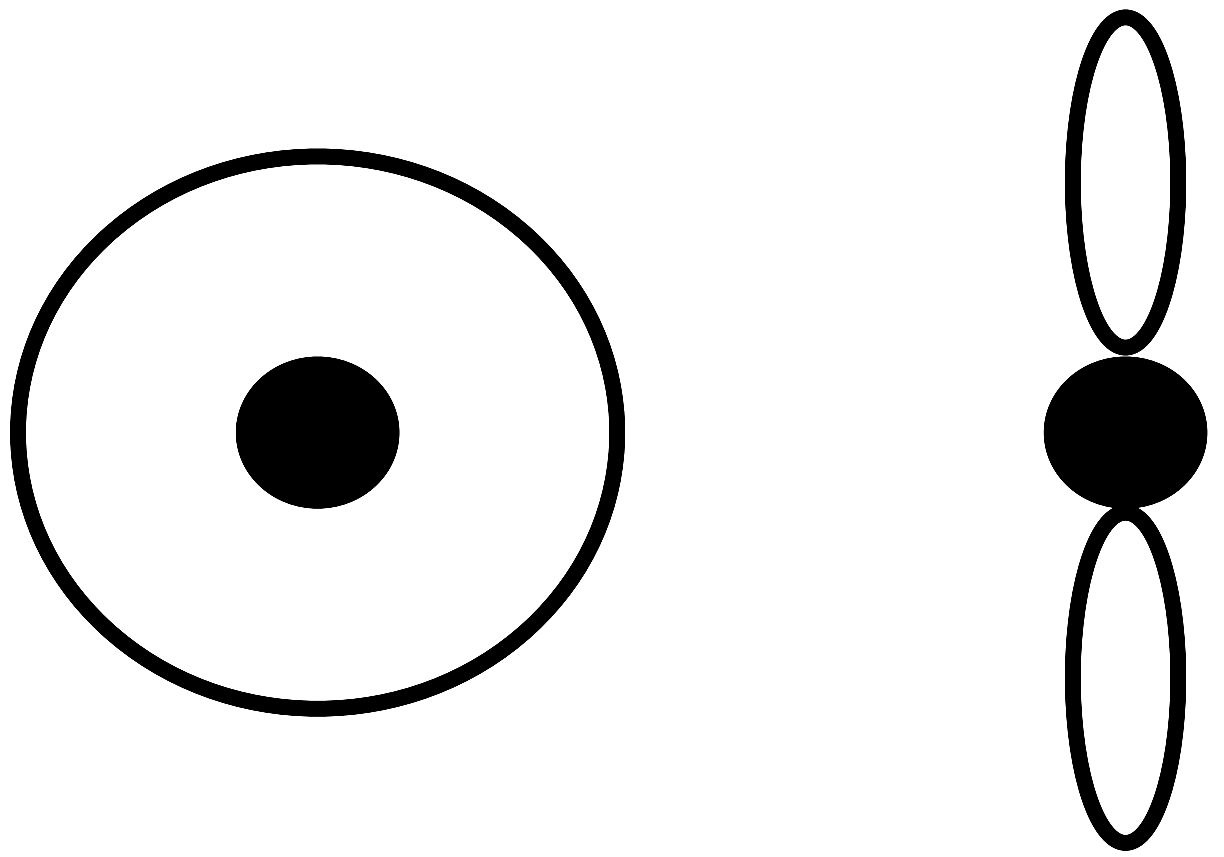
Communication between "worlds"
Consequence of Causality - trace over entangled particles

Non-linearity visible despite Environmental De-coherence!
Polchinski: "Everett Phone"

Constraints and Tests

Constraints

What does this do to the Lamb Shift?



Proton at Fixed Location

2S and 2P electron have different charge distribution

Different expectation value of electromagnetic field

Level Splitting!

BUT: Cannot decouple center of mass and relative co-ordinates

Proton wave-function is spread over some region (e.g. some trap size ~ 100 nm),
Expectation value of electromagnetic field diluted
In neutral atom - heavily suppressed, except at edges!

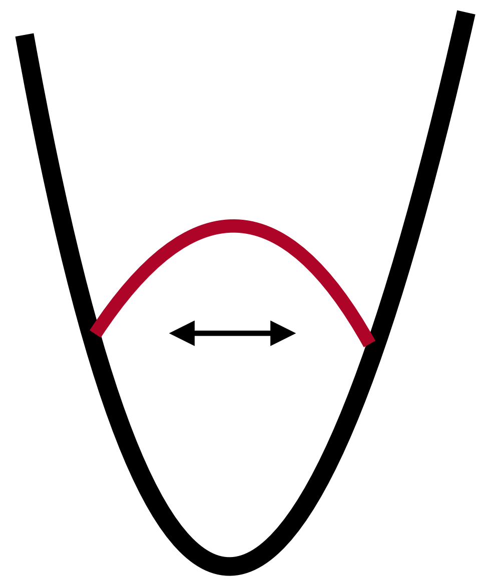
$\varepsilon < 10^{-2}$

Similarly, kills possible bounds on QCD and gravity

Constraints

Leading Constraint?

For $\varepsilon > 0$ (repulsive interaction)



Too large a repulsion, Cant trap ion in trap

$\varepsilon < 10^{-5}$

No direct limit on $\varepsilon < 0$ (attractive interaction)

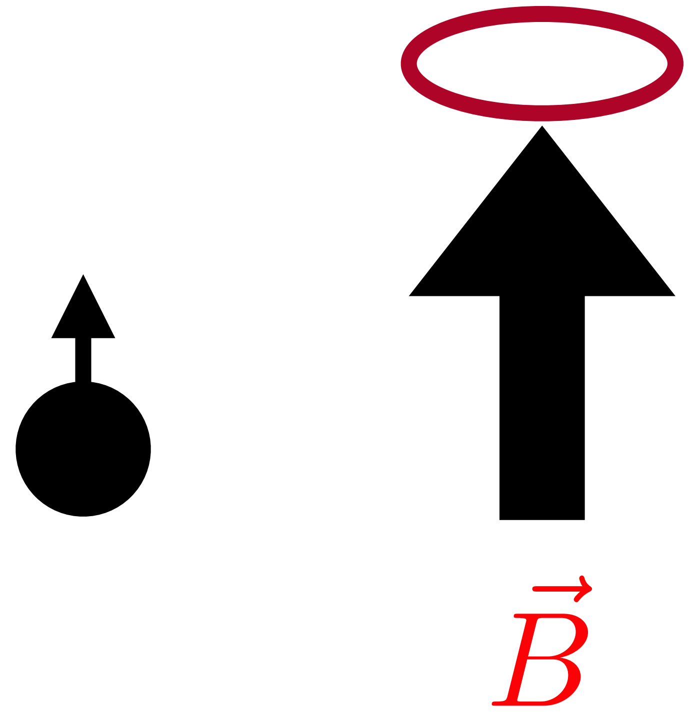
Perhaps from mapping of ion in trap?

Experimental Tests

Key Point: Create macroscopic superposition

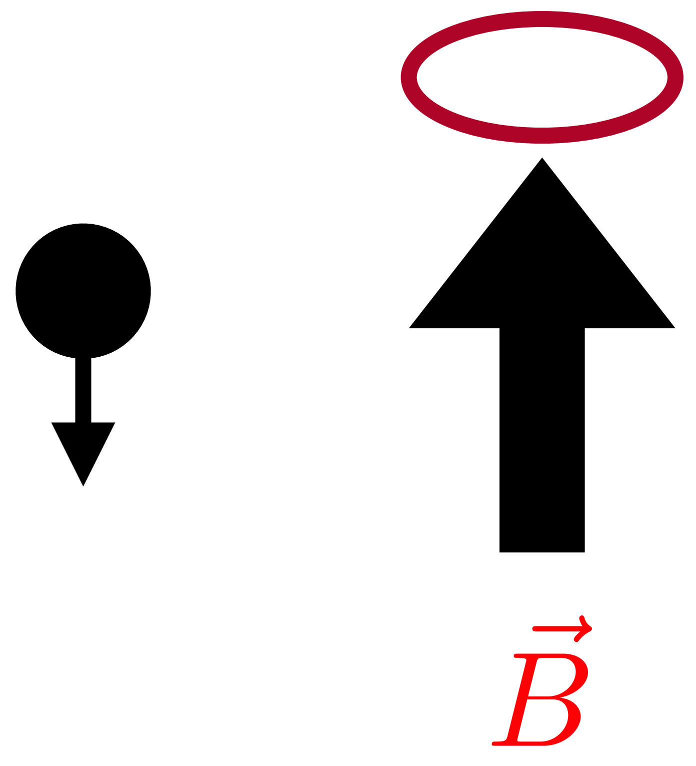
Create Expectation value of EM/Gravity

Search for Expectation value



On: {T, T + dT}
Off: {T + dT, T + 2 dT}

SQUID
ε B in {T + dT, T+ 2 dT}?



Off: {T, T + dT}
On: {T + dT, T + 2 dT}

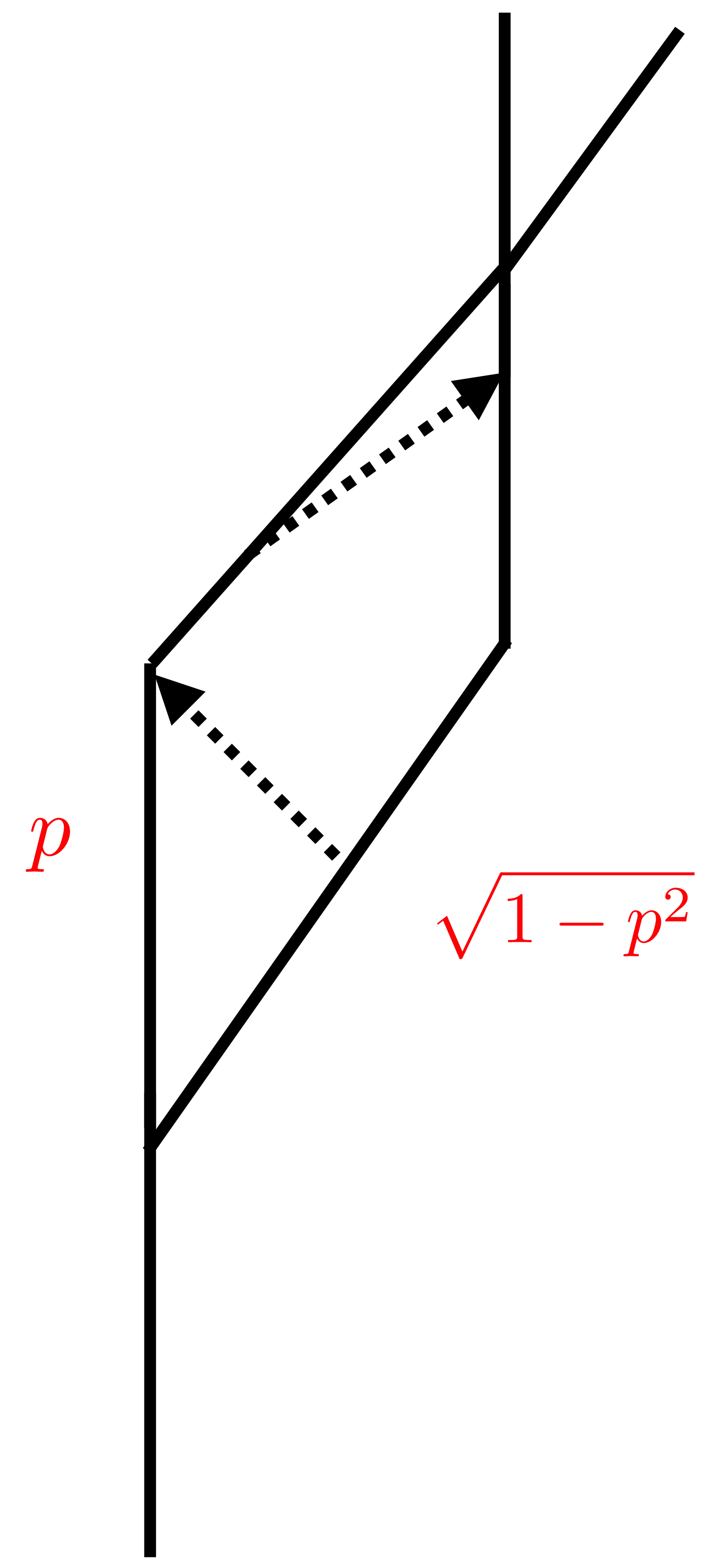
SQUID
ε B in {T, T+ dT}?

Similar tests for gravity, QCD

Experimental Tests

Interferometry - interaction between paths

Take an ion - split its wave-function



Coulomb Field of one path interacts with the other path

Gives rise to phase shift that depends on the intensity p of the split

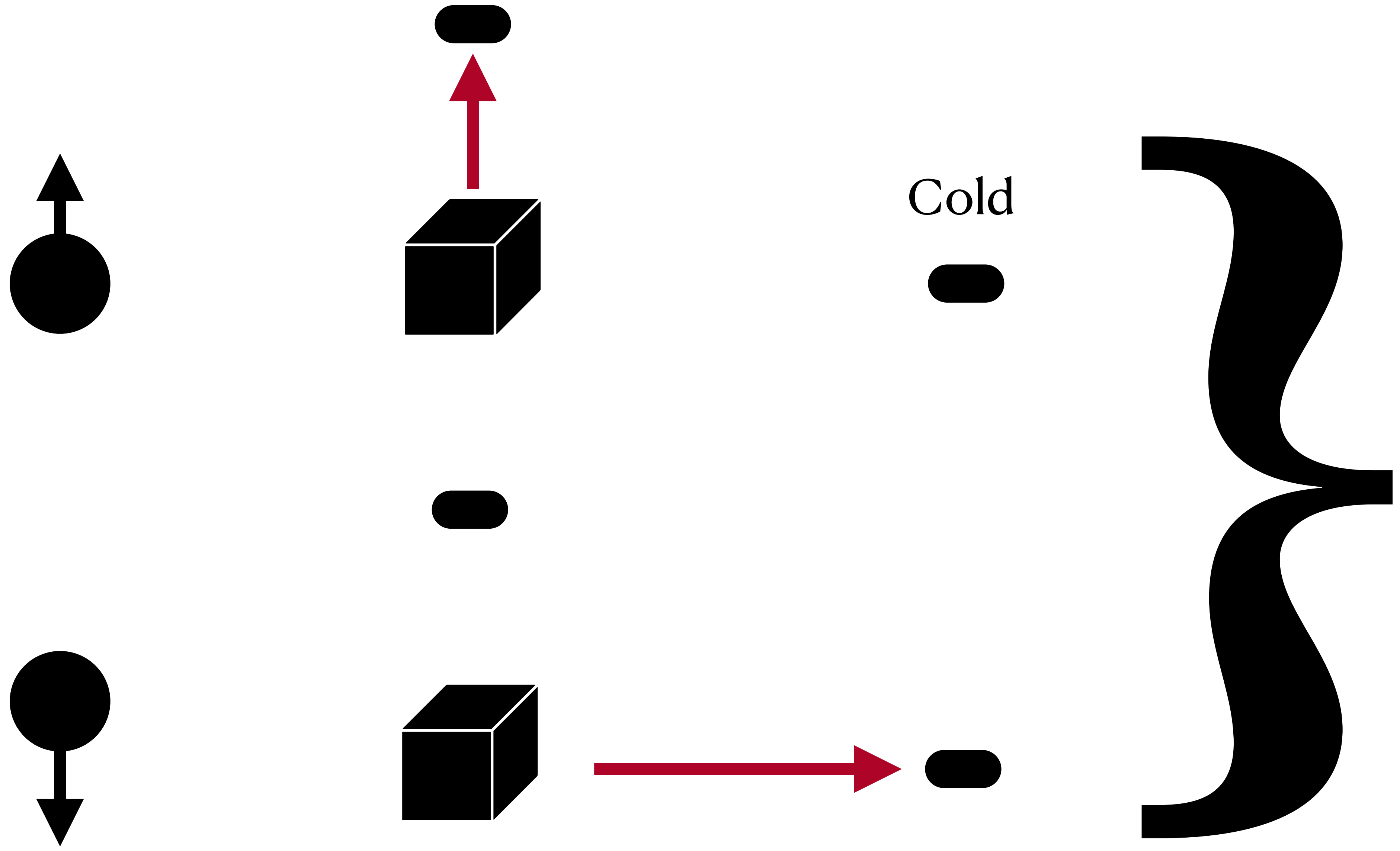
Use intensity dependence to combat systematics

Cosmological Sensitivity

Non-Linearity and Cosmology

$$\mathcal{L} \supset eA_\mu \bar{\Psi}\gamma^\mu \Psi + \epsilon_\gamma e \langle \chi | A_\mu | \chi \rangle \bar{\Psi}\gamma^\mu \Psi$$

$$|\chi\rangle = \alpha|U\rangle + \beta|M\rangle \implies \langle\chi|A_\mu|\chi\rangle = |\alpha|^2\langle U|A_\mu|U\rangle + |\beta|^2\langle M|A_\mu|M\rangle$$



All occurs in |U>

Cannot undo small α - hide large ε

For large β entirely dominated by |M>

Local Exploitability completely determined by unchangeable initial conditions

Stark Difference from Linear Quantum Mechanics

Non-Linearity and Cosmology

$$\mathcal{L} \supset eA_\mu \bar{\Psi}\gamma^\mu \Psi + \epsilon_\gamma e \langle \chi | A_\mu | \chi \rangle \bar{\Psi}\gamma^\mu \Psi$$

$$|\chi\rangle = \alpha|U\rangle + \beta|M\rangle \implies \langle\chi|A_\mu|\chi\rangle = |\alpha|^2\langle U|A_\mu|U\rangle + |\beta|^2\langle M|A_\mu|M\rangle$$

Could we boost by “Projection Operation”?

$$\frac{\langle\chi|A_\mu|\chi\rangle}{\boxed{\langle U|O|U\rangle}}$$

→ **Environment is not local**
No local projection operator!

In linear quantum mechanics, we use this non-local projection operator - but degeneracy with coupling implies quantum phenomena understood without needing it

Likely a generic feature of local non-linear quantum mechanics

Conclusions

Conclusions

1. Quantum Field Theory can be generalized to include non-linear, state dependent time evolution

2. Conventional tests of quantum mechanics in atomic and nuclear systems do NOT probe causal non-linear quantum mechanics

3. Straightforward set of experimental tests possible to probe non-linear quantum mechanics

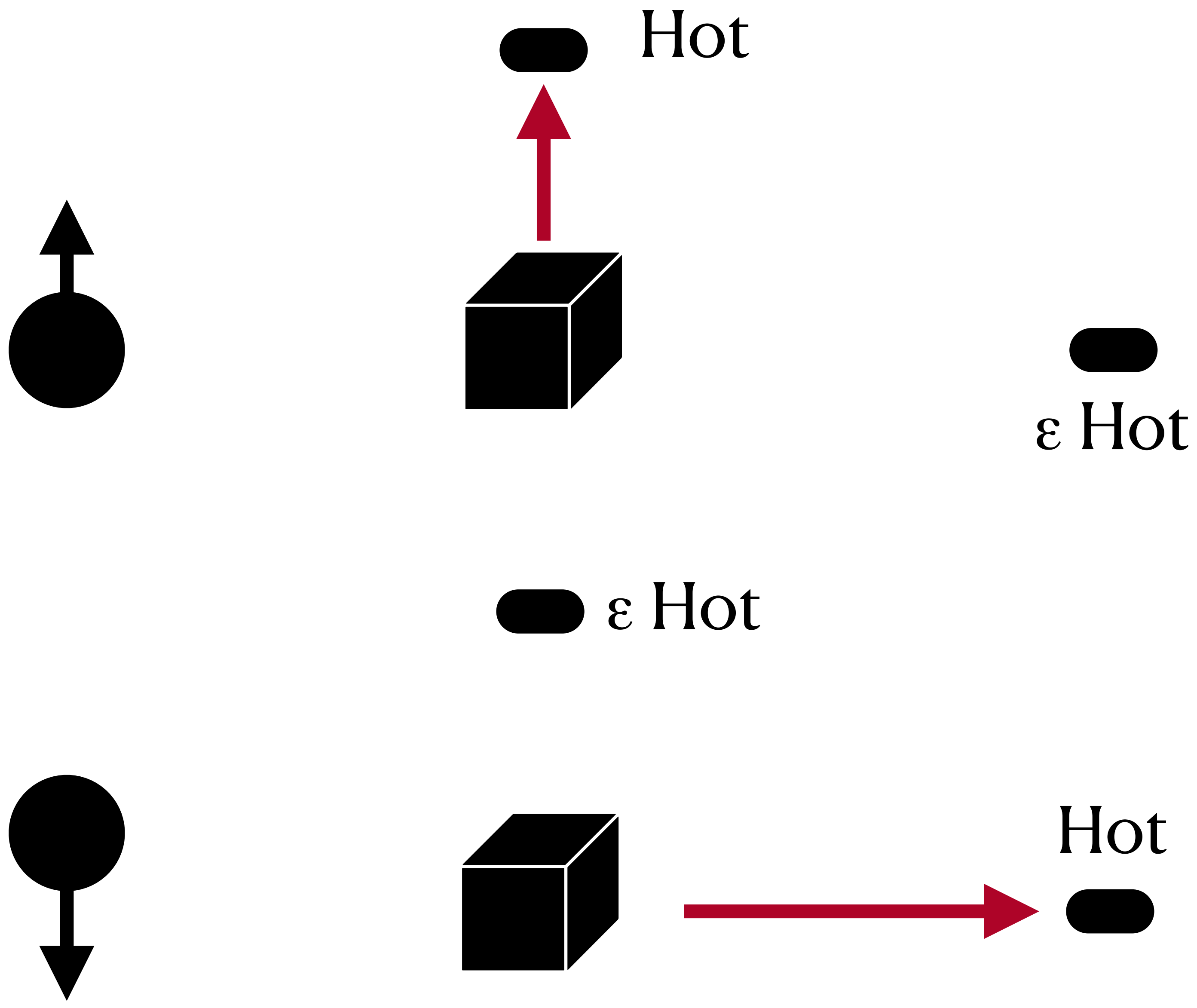
4. Cosmological Sensitivity requires many more experimental probes

Backup

Delicate Non-Linearity

Suppose |X> = |U>

O performs Laser experiment on July 6 - discovers non-linear quantum mechanics!



$$|\chi\rangle = \frac{1}{\sqrt{2}}\left(|U\rangle|O_U\rangle + |D\rangle|O_D\rangle\right)$$

O wants to repeat experiment

Suppose $|O_U\rangle$ decides to run experiment at 9 AM on July 10

But $|O_D\rangle$ runs experiment on 9 AM on July 20

State on 9 AM on July 10

$$|\chi\rangle = \frac{1}{\sqrt{2}}\left(|U\rangle|O_U\rangle\frac{(|U\rangle|T\rangle + |D\rangle|R\rangle)}{\sqrt{2}} + |D\rangle|O_D\rangle\right)$$

Delicate Non-Linearity

State on 9 AM on July 10

$$|\chi\rangle = \frac{1}{\sqrt{2}}\left(|U\rangle|O_U\rangle\frac{(|U\rangle|T\rangle + |D\rangle|R\rangle)}{\sqrt{2}} + |D\rangle|O_D\rangle\right)$$

Compare with State on July 6

$$|\chi\rangle = \frac{1}{\sqrt{2}}\left(|U\rangle|T\rangle + |D\rangle|R\rangle\right)$$

$$\langle U|\langle O_U|\langle U\langle T|\langle E_T|eA_\mu\left(x_R\right)\bar{\Psi}\left(x_R\right)\gamma^\mu\Psi\left(x_R\right)|\chi\rangle = \frac{1}{2}\langle U|\langle T|\langle E_T|eA_\mu\left(x_R\right)\bar{\Psi}\left(x_R\right)\gamma^\mu\Psi\left(x_R\right)|\chi\rangle$$

Effect is 1/2 of prior effect!

But, full effect if O_U and O_D perform experiment at same time!

Quantum Pollution: Without adequate care, superpositions may diverge wildly, preventing exploitability. Not automatic - but need careful protocols!

Particles have been scattering for 13 billion years. Cosmological dilution?

Cosmological Relaxation of Non-Linear QM?

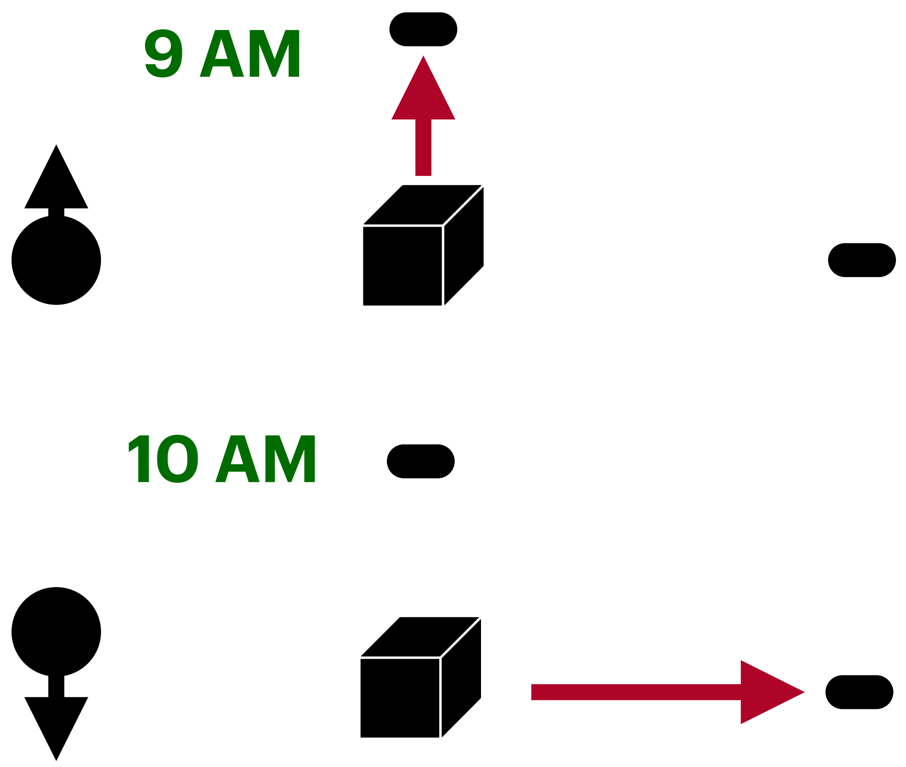
$$\mathcal{L} \supset eA_\mu \bar{\Psi}\gamma^\mu\Psi + \epsilon_\gamma e\langle\chi|A_\mu|\chi\rangle\bar{\Psi}\gamma^\mu\Psi$$

All we need is the expectation value. Non-Linear effects are resistant to decoherence.

For e.g. when we repeat the experiment, it is ok for O_U and O_D to be two different individuals - all we care is that the fields are turned on at the same space-time points

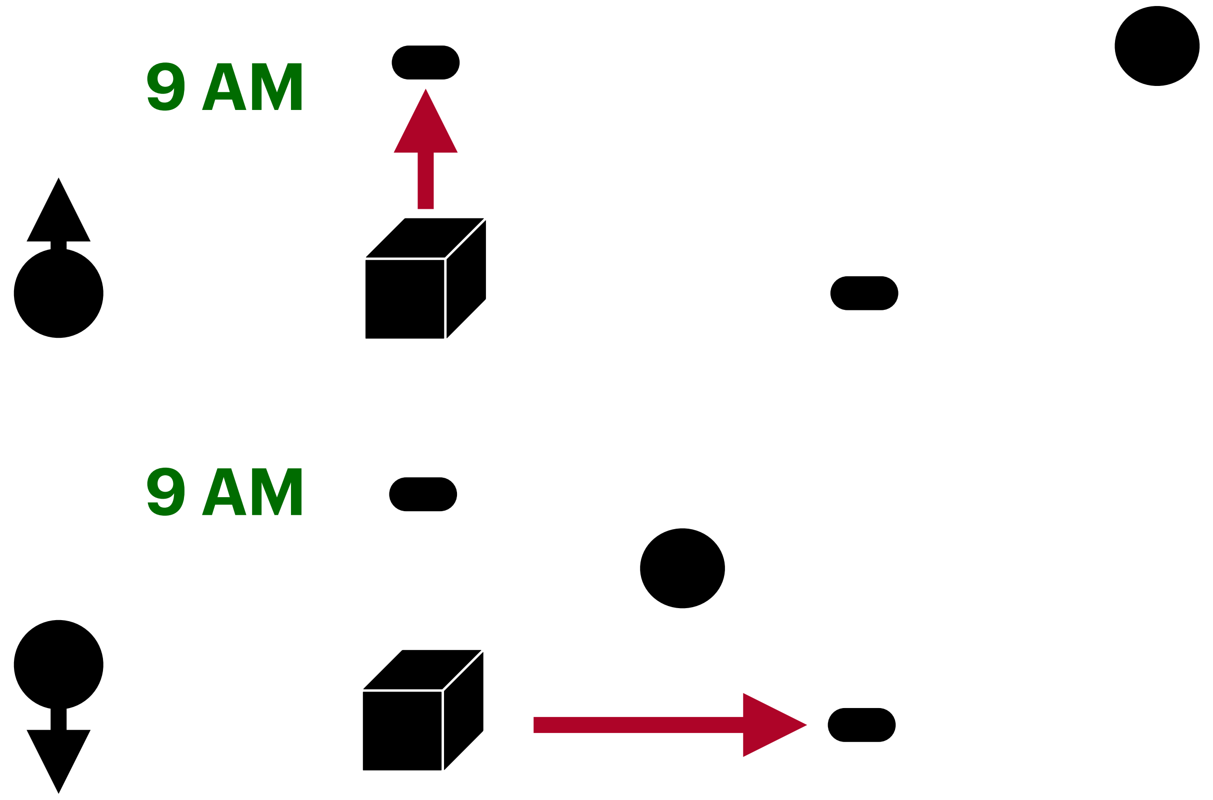
Relevant

Superpositions where expectation values of fields are very different



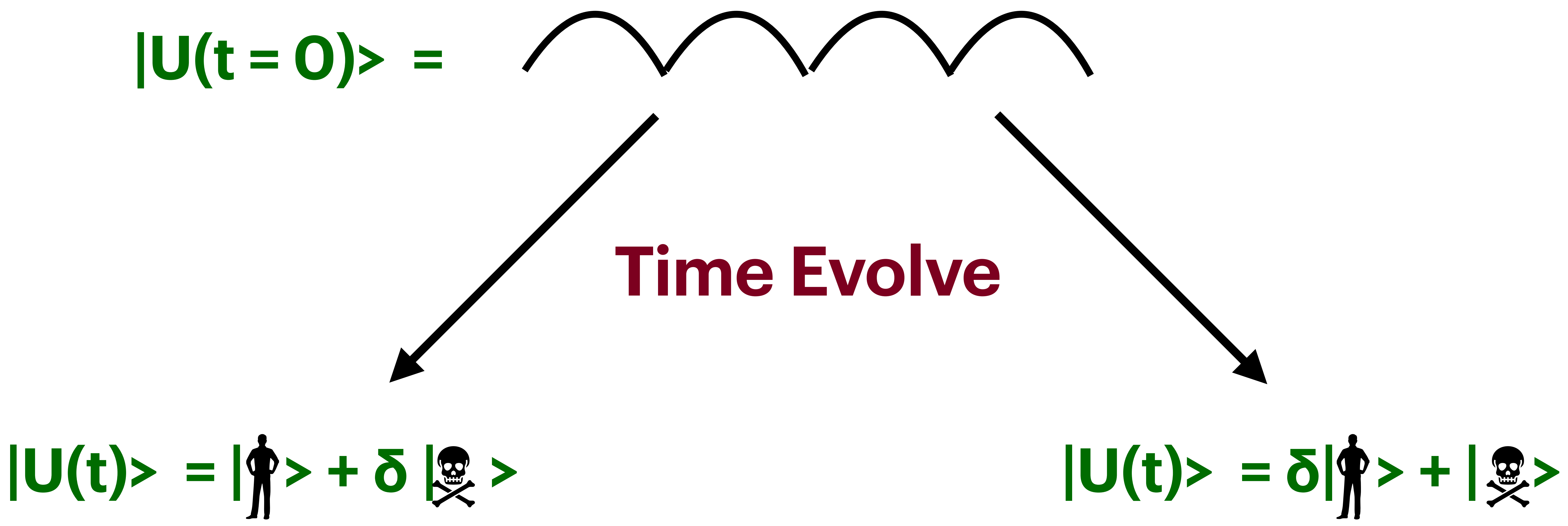
Irrelevant

Scattering where expectation values are not significantly changed



Classical Universe?

Suppose |X> = |U>



Can quantum events (scattering, decay etc.) lead to wildly different classical outcomes?

Clearly Possible - e.g. Human choosing to act differently based on quantum event

But, fundamentally - this is because humans can be quantum amplifiers

Are there natural quantum amplifiers, for e.g. in chaotic systems?

Classical Universe?

|U(t)> = |🧍> + δ |☠ >   **Or** |U(t)> = δ|🧍> + |☠>

Are there natural quantum amplifiers, for e.g. in chaotic systems?

Key Point: Changing classical evolution of a system requires coherent motion of N atoms

Probability that N atoms coherently move in some way: p^N
With p ~ O(1) scattering probability

Even with N ~ 100, these are very small probabilities
For typical chaotic examples, e.g. Butterfly effect, N >> 100

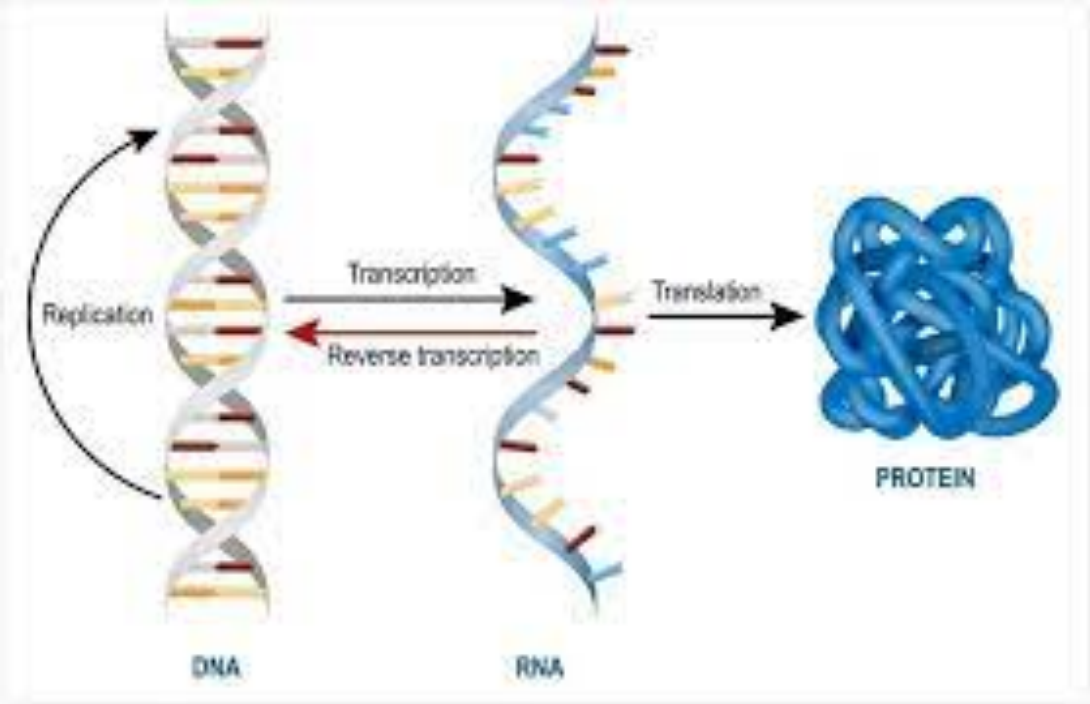
|U(t)> = |🧍> + δ |☠ >   **Reasonable!**

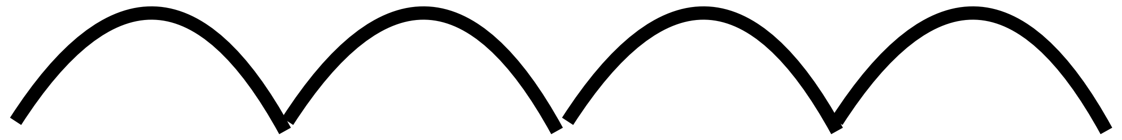
Quantum Amplifiers are Hard!

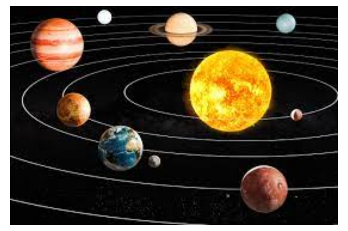
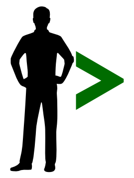
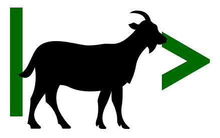
Evolutionary Dilution?

Humans can be quantum amplifiers

Is N ~ O(few) for biology?
Maybe for RNA/DNA?



|U(t = 0)> = 

|U(t)> = |> (|> + |> + ...)

Dilutes Laboratory Effects!

Inflation

Quantum Amplifiers are Hard!

Except in Cosmic Inflation!

Inflation rapidly places quantum state in a homogenous and isotropic state (Bunch - Davies Vacuum)

$\langle\chi|\phi|\chi\rangle$ **is homogeneous and isotropic BUT evolves in time**

But, our universe is clearly inhomogeneous
How could homogeneous state become inhomogeneous?

Answer: Massive Superposition of Statistically Similar Universes!

$$|\chi\rangle = \sum_i c_i |U_i\rangle, \ c_i \sim e^{-N}$$

Most of the Universe,
The space-time point where the Earth is located is in intergalactic space!

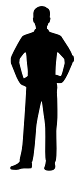
Probes of Non-Linear Quantum Mechanics

Non-Linear effects deeply tied to unchangeable initial and evolution of total quantum state

Probe not just non-linear quantum mechanics but full evolutionary history of quantum state!

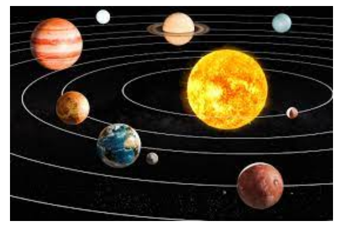
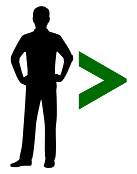
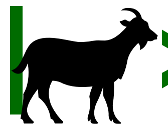
Three Scenarios

Classical Universe

|U(t)> = |> + δ |>

E.g. warm inflation, bouncing cosmologies

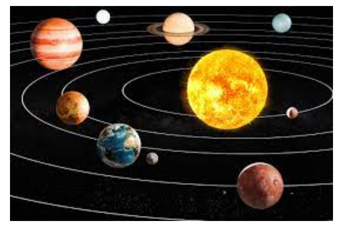
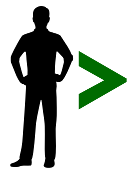
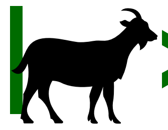
Quantum Biology

|U(t)> = |> (|> + |> + ...)

Canonical Inflation

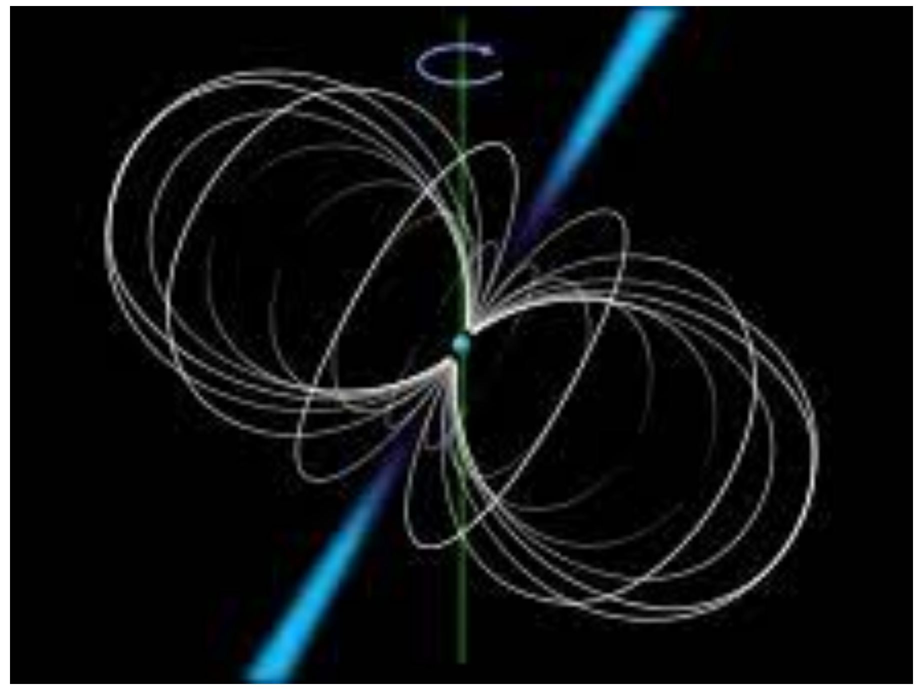
$|\chi\rangle = \sum_i c_i |U_i\rangle, \; c_i \sim e^{-N}$

Quantum Biology

|U(t)> = |> (|> + |> + ...)

The large scale structure of the universe is the same

Local structures (e.g. buildings) are vastly different



Shield

Look for coherent astrophysical or geological source (e.g. radio source/ magnetic field) inside shield!

$\epsilon_\gamma \lesssim 10^{-5}$ **Magnetic Shields Work!**

For gravity, perhaps look for gravitational effects of waves on man-made islands?

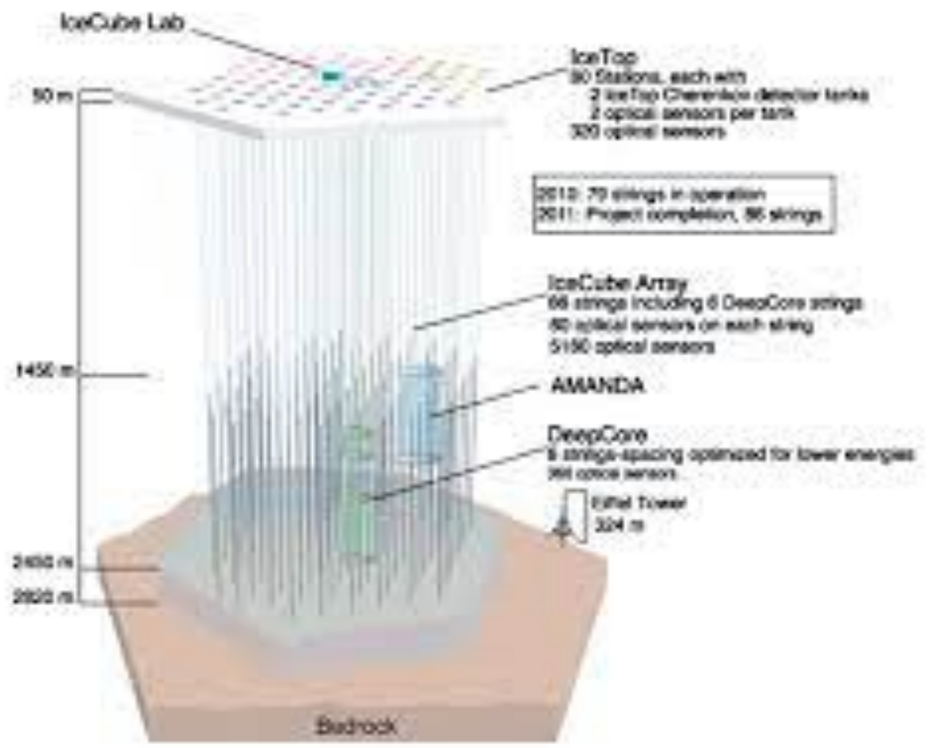
Inflationary Universe

$$|\chi\rangle = \sum_i c_i |U_i\rangle, \;\; c_i \sim e^{-N}$$

Earth is in the middle of nowhere in the vast majority of the universes

Look for cosmic rays inside shields!

Shield



e.g. Coherent Radio Sources

e.g. protons in IceCube

$$\text{Current:} \;\; \propto \frac{1}{e^{\frac{N}{2}}}$$ **Maybe visible if N is not too big**

Inflationary Universe

Are there effects that persist in the large N limit?

$\langle\chi|\phi|\chi\rangle$ **is homogeneous and isotropic BUT evolves in time**

Inhomogeneous effects such as electromagnetic waves will average down

Homogeneous fields will not - but field needs to have a non-zero VEV

The Metric has a non-zero vev across the superposition!

$$\tilde{g}_{\mu\nu} = \frac{g_{\mu\nu} + \epsilon_G \frac{\langle\chi|g_{\mu\nu}|\chi\rangle}{\langle\chi|\chi\rangle}}{1+\epsilon_G}$$

Inflationary Metric Interference

$$\tilde{g}_{\mu\nu} = \frac{g_{\mu\nu} + \epsilon_G \frac{\langle \chi | g_{\mu\nu} | \chi \rangle}{\langle \chi | \chi \rangle}}{1 + \epsilon_G}$$

Inflation: Expectation value is the average homogeneous FRW metric

$$g_s = -\left(1 - \frac{r_s}{r}\right) dt^2 + \frac{dr^2}{\left(1 - \frac{r_s}{r}\right)} + r^2 d\Omega^2 \qquad \langle g \rangle = -dt^2 + dr^2 + r^2 d\Omega^2$$

Renormalize and Expand

$$g_{\text{eff}} \simeq \left[-(1 - \frac{R_s}{r}) dt^2 + \left(1 + \frac{R_s}{r} + \left(\frac{R_s}{r}\right)^2 (1 + \epsilon_G) \right) dr^2 \right] + r^2 d\Omega^2$$

Long Distance Modification of Gravity!

Corrects Second Order GR Term!

Strong Field tests of GR!

Black Hole Horizon

$$\tilde{g}_{\mu\nu} = \frac{g_{\mu\nu} + \epsilon_G \frac{\langle \chi | g_{\mu\nu} | \chi \rangle}{\langle \chi | \chi \rangle}}{1 + \epsilon_G}$$

Inflation: Expectation value is the average homogeneous FRW metric

$$g_s = -\left(1 - \frac{r_s}{r}\right) dt^2 + \frac{dr^2}{\left(1 - \frac{r_s}{r}\right)} + r^2 d\Omega^2$$

$$\langle g \rangle = -dt^2 + dr^2 + r^2 d\Omega^2$$

$$g_{\text{eff}} = \frac{1}{1 + \epsilon_G}\left[-(1 - \frac{r_s}{r} + \epsilon_G)dt^2 + \left(\frac{1}{1 - \frac{r_s}{r}} + \epsilon_G\right) dr^2\right] + r^2 d\Omega^2$$

$g_{tt} \to 0, g_{rr} \to \infty$ at different values of r!

Creates a firewall!