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VACUUM ENERGY IN FRAMIDS

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FRAMIDS

From relativistic QFT perspective:

"CM" = QFT state that breaks boosts
(and possibly other symmetries)

Fluids, solids, superfluids, supersolids: well understood EFTs, SSB.

Fermi liquids: peculiar relationship to SSB of boosts.

"Framids": simplest possibility. Where are they?

WHY BOOST GOLDSTONES ARE SPECIAL

1) Suppose $\cancel{\vec{K}}$, but P^μ, $\vec{J}$ unbroken.

2) $\langle T^{00} \rangle = \rho$, $\langle T^{ij} \rangle = p\, \delta^{ij}$

3) Boost w/ $\vec{\eta}(x)$ (Goldstone fields)

$$\Rightarrow \begin{cases} \delta P^0 \simeq \displaystyle\int d^3x \, \langle T^{0i} \rangle \eta^i = 0 \\ \\ \delta P^i \simeq \displaystyle\int d^3x \left(\langle T^{00} \rangle \eta^i + \langle T^{ij} \rangle \eta^j \right) = \int d^3x \, (\rho + p) \, \eta^i \end{cases}$$

$\sim a + a^\dagger \neq a^\dagger a$

but excitations <u>have</u> to diagonalize P^μ!

POSSIBLE WAYS OUT

1) Fermi liquids: no local Goldstone fields $\vec{\eta}(x)$

2) Solids: $\cancel{\vec{P}}$ $\Longrightarrow$ excitations are not eigenstates of $\vec{P}$

3) Superfluids: $\vec{\eta}(x) = \vec{\nabla}\pi(x)$

$$\Longrightarrow \int d^3x \, (\rho + p)\, \vec{\eta}(x) = 0$$

4) Framids: $\boxed{\rho + p = 0}$ i.e. $\boxed{\langle T^{\mu\nu} \rangle \propto \eta^{\mu\nu}}$

FRAMID EFT

Order parameter for $\cancel{\vec{K}}$: $\langle V^\mu(x) \rangle = \delta^\mu_0$

Goldstone fields $\vec{\eta}(x)$: $V^\mu(x) \equiv \left(e^{i\vec{\eta}(x)\cdot\vec{K}}\right)^\mu{}_\nu \, \delta^\nu_0$

$\left(\text{cf.} \quad \Phi(x) = e^{i\pi(x)} \, v \right)$

$\mathcal{L}_{EFT} \supset (\partial_\mu V^\mu)^2, \; (\partial_\mu V_\nu)^2, \; (V^\mu \partial_\mu V_\nu)^2 \qquad (V_\mu V^\mu = 1)$

$+$ higher ∂'s.

$- \Lambda$

$T_{\mu\nu} \supset (\partial V \cdot \partial V)_{\mu\nu}, \; (V \cdot \partial\partial V)_{\mu\nu}, \; (V \cdot \partial V \, V \cdot \partial V)_{\mu\nu}$

$+ \Lambda \, \eta_{\mu\nu}$

MYSTERY

On background / ground state: $V^\mu \to \delta^\mu_0 = \text{const}$

$$T^{\mu\nu} \longrightarrow \Lambda \eta^{\mu\nu}$$

Still: background / ground state breaks Lorentz

$\Rightarrow$ No selection rule enforcing $\langle T^{\mu\nu} \rangle \propto \eta^{\mu\nu}$

$\sim$ C.C. problem: $\langle T^{\mu\nu} \rangle = 0 \quad (\ll M^4)$

w/ NO selection rule enforcing it.

DOES IT SURVIVE QUANTUM CORRECTIONS ?

$$\mathcal{L}_{EFT} \longrightarrow \left.\frac{1}{2}\dot{\vec{\eta}}^2 - \frac{1}{2}c_L^2(\vec{\nabla}\cdot\vec{\eta})^2 - \frac{1}{2}c_T^2(\vec{\nabla}\times\vec{\eta})^2\right\}$$

$$+ \dot{\eta}\dot{\eta}\nabla\eta + \dot{\eta}\nabla\eta\nabla\eta + \nabla\eta\nabla\eta\nabla\eta$$

$$+ \ldots$$

Why should it ? Lorentz <u>not</u> manifest (SSB)

Our work: compute $\langle T^{\mu\nu} \rangle$ at 1-loop.

Technical complications : 1) measure $\longrightarrow$ <u>not</u> a problem

2) UV divergences $\longrightarrow$ DR, PV

WARM-UP : RELATIVISTIC SCALAR

How to check $\langle T^{\mu\nu} \rangle \propto \eta^{\mu\nu}$ without using L.I. ?

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2$$

$$\langle T^{\mu\nu} \rangle = \int \frac{d^4k}{(2\pi)^4}\, k^\mu k^\nu \, \overbrace{\theta(k^0)\,\delta(k^2-m^2)}^{\tilde{G}_W(k)}$$

$$\longrightarrow \int \frac{d^3k}{(2\pi)^3}\, \frac{1}{2}\, \frac{k^\mu k^\nu}{\omega_k} \qquad \omega_k = \sqrt{\vec{k}^2 + m^2}$$

So :

$$\rho = \int \frac{d^3k}{(2\pi)^3}\, \frac{1}{2}\omega_k \qquad p = \frac{1}{3}\int \frac{d^3k}{(2\pi)}\, \frac{1}{2}\, \frac{k^2}{\omega_k}$$

At face value: $\rho, p > 0 \implies \rho + p \neq 0$

REGULARIZATIONS

1) Dim-Reg : $\quad 3 \to d$

$$\rho \longrightarrow -m^{d+1} \frac{\Gamma\left(-\frac{d+1}{2}\right)}{2(4\pi)^{\frac{d+1}{2}}} = -p$$

$$\Longrightarrow \quad \rho + p = 0$$

2) 3-field Pauli-Villars :

$$\tilde{G}_F(k;m) = \frac{i}{k^2 - m^2 + i\varepsilon} \longrightarrow \tilde{G}_F^{PV}(k;m) + \sum_{a=1}^{3} c_a \tilde{G}_F(k;\alpha_a \cdot M)$$

$M \gg m, \quad c_a, \alpha_a \sim 1 \quad$ s.t. $\quad \tilde{G}_F^{PV} \sim \frac{1}{k^8} \quad$ for $\; k \gg M$

$$\rho \longrightarrow \rho(m) + \sum_{a=1}^{3} c_a\, \rho(\alpha_a M)$$

α - independent

$$= f(\alpha) M^4 + g(\alpha)\, m^2 M^2 + \frac{1}{32\pi^2} m^4 \log\frac{m}{M} + h(\alpha)\, m^4$$

$$p \longrightarrow p(m) + \sum_{a=1}^{3} c_a\, p(\alpha_a M) = -\rho$$

$$\Longrightarrow \quad \rho + p = 0$$

Note: PV = higher derivative additions to $\mathcal{L}$

e.g.: $$\frac{i}{k^2 - m^2} - \frac{i}{k^2 - M^2} \longrightarrow \frac{i}{k^2 - k^4/M^2 - m^2} \longrightarrow \Delta\mathcal{L} = -\frac{1}{M^2}(\Box\phi)^2$$

FOR THE FRAMID

$$\rho \equiv \langle T^{00} \rangle = \langle \dot{\vec{\eta}}^{\,2} - \mathcal{L} \rangle = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2} \left(\omega_L + 2\omega_T \right)$$

$$p \equiv \frac{1}{3} \langle T^{ii} \rangle = \langle \frac{1}{3} c_L^2 (\vec{\nabla} \cdot \vec{\eta})^2 + \frac{1}{3} c_T^2 (\vec{\nabla} \times \vec{\eta})^2 + \mathcal{L} \rangle$$

$$= \frac{1}{3} \int \frac{d^3k}{(2\pi)^3} \frac{1}{2} \left(\frac{c_L^2 k^2}{\omega_L} + \frac{c_T^2 k^2}{\omega_T} \right)$$

w/ $\quad \omega_{L,T} = c_{L,T} \cdot k$

very similar to relativistic case, but what about regularization?

PAULI - VILLARS

IF one can do PV separately for $\vec{\eta}_L$ & $\vec{\eta}_T$,

then everything works out:

$\vec{\nabla}\times \;\; = 0 \qquad \vec{\nabla}\cdot \;\; = 0$

$$\tilde{G}_F^L(\vec{k},\omega) = \tilde{G}_F^{rel.}(k_L^\mu) \qquad k_L^\mu \equiv (\omega, c_L\vec{k})$$

$$\tilde{G}_F^T(\vec{k},\omega) = \tilde{G}_F^{rel}(k_T^\mu) \qquad k_T^\mu \equiv (\omega, c_T\vec{k})$$

$$PV_L,\ PV_T \longrightarrow \quad \rho = \left(\frac{1}{c_L^3} + \frac{2}{c_T^3}\right)\rho_{rel}$$

$$p = \left(\frac{1}{c_L^3} + \frac{2}{c_T^3}\right)p_{rel}$$

$$\Longrightarrow \quad \rho + p = 0$$

CAN WE ?

PV should correspond to local, Lorentz-invariant $\Delta\mathcal{L}$.

Tricky, because:

1) splitting $\vec{\eta} \to \vec{\eta}_L + \vec{\eta}_T$ is not local

2) $\vec{\eta}$ transforms non-linearly under Lorentz

$\Longrightarrow$ $\langle \eta \eta \rangle$ has complicated transformation rules.

YES WE CAN

We need:

$$\frac{i}{k_L^2} \longrightarrow \frac{i}{k_L^2 - k_L^4/M_1^2 - k_L^6/M_2^2 - k_L^8/M_3^2} \qquad k_L^2 = \omega^2 - c_L^2 \vec{k}^2$$

$$\Rightarrow \Delta\mathcal{L} = \vec{\eta}_L \cdot \left(-\frac{\Box_L^2}{M_1^2} + \frac{\Box_L^3}{M_2^2} - \frac{\Box_L^4}{M_3^2} \right) \vec{\eta}_L \qquad \Box_L \equiv \partial_t^2 - c_L^2 \vec{\nabla}^2$$

+ same for $\vec{\eta}_T$

locality: possible to write $\Delta\mathcal{L}$ as a sum of terms

$$\vec{\eta} \cdot \partial_t^{2a} (\vec{\nabla}^2)^b D^c \cdot \vec{\eta} \qquad w/ \qquad D_{ij} \equiv \partial_i \partial_j$$

Lorentz invariance:

$$V^\mu(x) = \left(e^{i\vec{\eta}(x)\cdot\vec{K}}\right)^\mu{}_\nu \, \delta^\nu_0$$

$$\Longrightarrow \quad V^0 = \cosh|\vec{\eta}| \cong 1 + \frac{1}{2}\vec{\eta}^{\,2}$$

$$\vec{V} = \sinh|\vec{\eta}|\,\hat{\eta} \cong \vec{\eta}$$

$$\Longrightarrow \quad \vec{\eta}\cdot \partial_t^{2a}(\vec{\nabla}^2)^b D^c \cdot \vec{\eta} \cong \begin{cases} \left((\partial^{\parallel})^a \partial^\perp_{\mu_1}\cdots\partial^\perp_{\mu_b} V_\nu\right)^2 & (c=0) \\ \\ \left((\partial^{\parallel})^a \partial^\perp_{\mu_1}\cdots\partial^\perp_{\mu_b} (\partial_\nu V^\nu)\right)^2 & (c=1) \end{cases}$$

$$\partial^{\parallel} = V^\mu \partial_\mu \qquad \partial^\perp_\mu = \partial_\mu - V_\mu V^\nu \partial_\nu$$

DIMENSIONAL REGULARIZATION

$3 \to d$

Lorentz invariant? YES : equivalent to theory for V^μ
in $d+1$ dimensions, manifestly
L.I.

$$\rho \to \int \frac{d^dk}{(2\pi)^d} \, \frac{1}{2}\left(c_L k + 2c_T k\right) \longrightarrow 0$$

$$p \to \frac{1}{d}\int \frac{d^dk}{(2\pi)^d} \, \frac{1}{2}\left(c_L k + 2c_T k\right) \longrightarrow 0$$

$\Longrightarrow \quad \rho + p = 0$, but trivial.

MORE INTERESTING

"Matter" scalar field ϕ :

$$\mathcal{L} = \mathcal{L}_{\text{framid}} + \mathcal{L}_\phi$$

$$\mathcal{L}_\phi \supset \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}M^2\phi^2 \;,\; V^\mu\partial_\mu\phi \;,\; V^\mu V^\nu \partial_\mu\partial_\nu\phi \;,\; (V^\mu\partial_\mu\phi)^2$$

$$\begin{aligned}\mathcal{L} \longrightarrow \frac{1}{2}\Big[& \dot{\vec{\eta}}^{\,2} - c_L^2 (\vec{\nabla}\cdot\vec{\eta})^2 - c_T^2 (\vec{\nabla}\times\vec{\eta})^2 \\ & + (\partial\phi)^2 - M^2\phi^2 \\ & + 2b_1\,\phi\,\vec{\nabla}\cdot\vec{\eta} + 2b_2\,\dot{\phi}\,\vec{\nabla}\cdot\vec{\eta} + b_3\,\dot{\phi}^2 \Big]\end{aligned}$$

$$\rho = \langle\, \dot{\vec{\eta}}^{\,2} + 2 b_2\, \dot{\phi}\, \vec{\nabla}\cdot\vec{\eta} + (1+b_3)\, \dot{\phi}^2 \,\rangle \qquad (\text{up to } \langle \mathcal{L} \rangle)$$

$$p = \frac{1}{d} \langle c_L^2 (\vec{\nabla}\cdot\vec{\eta})^2 + c_T^2 (\vec{\nabla}\times\vec{\eta})^2 - b_1\, \phi\, \vec{\nabla}\cdot\vec{\eta} \qquad (\quad " \quad)$$
$$- b_2\, \dot{\phi}\, \vec{\nabla}\cdot\dot{\vec{\eta}} + (\vec{\nabla}\phi)^2 \rangle$$

(minor) simplification : $b_3 \to 0$ without affecting p/ρ

After this:

$$\rho + p = \frac{1}{d} \int \frac{d^d k}{(2\pi)^d} \frac{d\omega}{(2\pi)} \Big[(d+1)\, \omega^2\, \tilde{G}^{\eta\eta}_{\omega} + \big((d+1)\omega^2 - m^2\big)\, \tilde{G}^{\phi\phi}_{\omega}$$
$$+ \big(i b_1 - (d+1)\, b_2\, \omega\big)\, |\vec{k}|\, \tilde{G}^{\eta\phi}_{\omega} \Big]$$

w/ $$\tilde{G}^{ab}_{W}(\omega,\vec{k}) = \int \frac{d\omega'}{(2\pi)} \left[\frac{i}{\omega-\omega'+i\varepsilon}\, \tilde{G}^{ab}_{F}(\omega',\vec{k}) + h.c. \right]$$

Impossible (for us + Mathematica) to do final $\int d^d k$

$\Longrightarrow$ expand for small b_1, b_2:

$$\rho + p = \frac{1}{d} \sum_{A=1}^{11} C_A \underbrace{\int \frac{d^d k}{(2\pi)^d} \frac{k^{\alpha_A}}{\left((1-c_L^2)k^2+m^2\right)^{\gamma_A}} \sqrt{k^2+m^2}^{\,\beta_A}}$$

$$\frac{1}{2} m^{d+\alpha_j+\beta_j-2\gamma_j} \Bigg[\frac{(1-c_L^2)^{\frac{-d-\alpha_j}{2}}\, \Gamma\!\left(\frac{d+\alpha_j}{2}\right) \Gamma\!\left(\frac{-d-\alpha_j+2\gamma_j}{2}\right)\, {}_2F_1\!\left(\frac{d+\alpha_j}{2}, -\frac{\beta_j}{2}; \frac{2+d+\alpha_j-2\gamma_j}{2}; \frac{1}{1-c_L^2}\right)}{\Gamma(\gamma_j)}$$
$$+ \frac{(1-c_L^2)^{-\gamma_j}\, \Gamma\!\left(\frac{d+\alpha_j-2\gamma_j}{2}\right) \Gamma\!\left(\frac{-d-\alpha_j-\beta_j+2\gamma_j}{2}\right)\, {}_2F_1\!\left(\gamma_j, \frac{-d-\alpha_j-\beta_j+2\gamma_j}{2}; \frac{2-d-\alpha_j+2\gamma_j}{2}; \frac{1}{1-c_L^2}\right)}{\Gamma\!\left(-\frac{\beta_j}{2}\right)} \Bigg],$$

$\boldsymbol{\alpha}$	$\boldsymbol{\beta}$	$\boldsymbol{\gamma}$	$\boldsymbol{C}$
1	0	0	$\frac{1}{2}c_L(d+1)$
2	-1	0	$\frac{1}{2}$
0	1	0	$\frac{d}{2}$
6	-1	2	$\frac{b_2^2}{4}(1-c_L^2)(d+1)$
4	-1	2	$\frac{b_1^2}{4}(d-1)+\frac{b_2^2}{4}m^2(3+d(2-c_L^2))$
2	-1	2	$\frac{b_1^2}{4}m^2(d+2)+\frac{b_2^2}{4}m^4(d+2)$
5	0	2	$-\frac{b_2^2}{4}c_L(1-c_L^2)(d+1)$
3	0	2	$-\frac{b_1^2}{4c_L}(1-c_L^2)(d-1)-\frac{b_2^2}{4}c_Lm^2(d+3)$
1	0	2	$-\frac{b_1^2}{4c_L}m^2(d+1)$
6	-3	2	$-\frac{b_1^2}{4}c_L^2(d-1)$
4	-3	2	$-\frac{b_1^2}{4}c_L^2m^2d$

Putting everything together:

$$\boxed{\rho + P = 0}$$!!

SUPERFLUID CHECK

For other "CM" systems, we expect $\rho + p \neq 0$

$$\left(\begin{array}{l} \rho \cong \rho_m c^2 \\ p \ll \rho \end{array} \right)$$

Superfluid: $\mathcal{L} = P\left((\partial\phi)^2 \right)$

$$\phi = \mu t + \pi(x)$$

$\uparrow$ phonon

Couple to massive φ: $\partial_\mu \phi \, \partial^\mu \varphi$, $\varphi (\partial\phi)^2$, etc.

Follow same steps ... $\Rightarrow$ $\rho + p \neq 0$.

CONCLUSIONS

Framid provides an example of "unnatural" $\langle T_{\mu\nu} \rangle$:

1) $\langle T^{\mu\nu} \rangle \propto \eta^{\mu\nu}$ enforced by symmetries, but in a contrived way.

2) No standard selection rules.

3) From EFT viewpoint, miraculous cancellations.

4) Implications for c.c. ?