Signals of new dynamics during the inflation era

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Work in collaboration with Zhong-zhi Xianyu 1910.12876, 2004.02887, with Zhong-zhi Xianyu, Yiming Zhong, 2109.14635 with Haipeng An, KunFeng Lyu, and Siyi Zhou, 2009.12381, 2111.xxxx

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Inflation: a stage for new dynamics

* High energy: H can be 10¹³ GeV.

- * Can produce heavy new physics particles.
- * Inflaton can travel a large distance in field space.
 - Can trigger dramatic changes in spectator sectors which couple to the inflaton

This talk

- Signal of particle production. "Cosmological collider physics".
- * Drama in the spectator sector, signal of a first order phase transition during the inflation.

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Signal of particle production





 $k_1 \sim k_2 \sim k_{\text{short}}$ $k_3 \sim k_{\text{long}}$

Time of propagation:

$$t = \frac{1}{H} \log \frac{k_{\rm short}}{k_{\rm long}}$$

oscillatory signal

$$\sim \sin(mt) = \sin\left(\frac{m}{H}\log\frac{k_{\text{short}}}{k_{\text{long}}}\right)$$

"Cosmological collider" Chen, Wang, 2009, 2012 Arkani-Hamed, Maldacena, 2015



 $e^{-\pi\nu}$ Boltzmann suppressed if m_o >> H

 $\left(\frac{k_{\text{long}}}{k_{\text{short}}}\right)^{\frac{1}{2}+i\nu}$

No oscillation if $m_{\sigma} < 3/2$ H

The presence of oscillation signal requires $m_{\sigma} \approx H$

Example:

Relevant scales for inflation

 $V_{\rm inf}^{1/4}\simeq 10^{16}~{\rm GeV}$

 $\Lambda_{\rm GUT}, \Lambda_{\rm string}, \ldots$

SUSY

Strong dynamics

 $\Lambda_{\rm TC}, \ldots$

 $(\dot{\phi}_0)^{1/2} \simeq 10^{14 \div 15} \; {\rm GeV}$

 m_{ρ},\ldots

 $H \simeq 10^{13} \text{ GeV}$

 $m_{3/2}, m_{\tilde{q}}, \ldots$

 m_{π},\ldots

Example:

Relevant scales for inflation

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 $\Lambda_{\text{GUT}}, \Lambda_{\text{string}}, \dots$

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m_{2} , m_{z} ,	m_{π}, \ldots
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Can be probed by cosmological collider

Sensitivity of the cosmological collider

 What kind of inflaton - new physics coupling lead to observable signals.

Many recent works:

X. Chen, Z. Xianyu, Y. Wang, et. al.H. An, M. McAneny, A. Ridgway, M. WiseH. Lee, D. Baumann, G. L. Pimentel.S. Kumar, R. SundrumA. Hook, J. Huang, D. Racco



Size of the signal

$$\langle \zeta_1 \zeta_2 \zeta_3 \rangle = (2\pi)^4 P_{\zeta}^2 \frac{1}{(k_1 k_2 k_3)^2} S(k_1, k_2, k_3)$$

CMB:
$$P_{\zeta} = \frac{H^2}{\dot{\phi}_0^2} \left(\frac{H}{2\pi}\right)^2 \simeq 2 \times 10^{-9} \qquad \dot{\phi}_0^{1/2} \simeq 60H$$

$$S(k_1, k_2, k_3) \sim f_{\rm NL} \sim \frac{1}{2\pi P_{\zeta}^{1/2}} \times \text{ interactions}$$

interactions = couplings, loops, propagators

For comparison. Planck: $f_{NL} \leq a$ few; LSS: $f_{NL} \sim 1$; 21cm: 10⁻²

1: non-derivative coupling

$\mathscr{L} \supset \lambda \phi^2 \sigma^2 + \mu \phi \sigma^2, \quad \sigma : \text{scalar}$

None-zero value of the inflaton field will generate a mass for σ .

$$\delta m_{\sigma} = \lambda \phi_0^2, \quad \phi_0 = \langle \phi \rangle$$

For the mass of σ to be around H, without fine-tuning, we need

$$\delta m_{\sigma}^2 < H^2 \to \lambda < \frac{H^2}{\phi_0^2}$$

For a similar reason, we need $\mu < \frac{H^2}{\phi_0}$

1: non-derivative coupling



3



Generið size of ϕ_0 :

 $N \simeq \int \mathrm{d}\phi \, \frac{H}{\dot{\phi}_0} \sim \frac{H}{\dot{\phi}_0} \Delta \phi = \mathcal{O}(10) \qquad \phi_0 \sim \Delta \phi \sim N \dot{\phi}_0 / H \ \Rightarrow \ \frac{H}{\phi_0} \sim \frac{2\pi P_{\zeta}^{1/2}}{N} \sim 10^{-5}$



Generid size of ϕ_0 :

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Hence:
$$f_{\rm NL}^{\rm (osc)} \sim \frac{1}{4N^3} P_{\zeta}$$
 tiny!

Lesson learned

- * Sizable coupling to the inflaton can generate a mass correction to the matter field σ .
- Requiring this not to make σ too heavy requires small coupling, and small signal.
- Can't avoid by fine-tuning. Inflaton has a sizable excursion, can at most fine tune for a very narrow range.

2. Derivative coupling

- * A consequence of an approximate $\phi \rightarrow \phi + c$
- Well motivated from the flatness of inflaton potential.
- It turns out this is also favored for giving a sizable signal.

2. Derivative coupling

Dim 5:
$$\frac{1}{\Lambda} \partial_{\mu} \phi J^{\mu}, \quad \frac{\phi}{\Lambda} F \wedge F$$

Naive estimate:

$$f_{\rm NL}^{\rm (osc)} \sim \frac{1}{16\pi^2} \frac{1}{2\pi P_{\zeta}^{1/2}} \left(\frac{H}{\Lambda}\right)^3 < \frac{1}{16\pi^2} \frac{1}{2\pi P_{\zeta}^{1/2}} \left(\frac{H}{\dot{\phi}_0^{1/2}}\right)^3 = \frac{1}{16\pi^2} \cdot \sqrt{2\pi} P_{\zeta}^{1/4}$$

At the same time, there is a further enhancement.

Modified dispersion relation

Particle production proportional to $\frac{\dot{\omega}}{\omega^2}$

A modified dispersion relation in the inflationary background of the form

$$\omega^2 = k_{\text{phys}}^2 + 2\mu k_{\text{phys}} + m^2 + \dots$$
, where $k_{\text{phys}} = \frac{k}{a(t)}$

 $\rightarrow \frac{\dot{\omega}}{\omega^2} \simeq \frac{\mu}{\omega^2}$ Enhancement possible for sizable μ

Coupling:

$$\omega^2 = k_{\text{phys}}^2 + 2\mu k_{\text{phys}} + m^2 + \dots, \quad k_{\text{phys}} = \frac{k}{a(t)}$$

Comes from $\vec{k} \cdot \hat{n}$, $\hat{n} =$ another vector

Without broken rotational invariance, \hat{n} comes from spin.

Possible couplings:

$$\frac{1}{\Lambda}\partial_{\mu}\phi J^{\mu}, \qquad J^{\mu} = \bar{\psi}\gamma_{5}\gamma^{\mu}\psi, \quad \epsilon^{\mu\nu\rho\sigma}A_{\nu}\partial_{\rho}A_{\sigma}$$

 $\rightarrow \mu = \frac{\phi_0}{\Lambda}$

An alternative scenario with scalars: Bodas, Kumar, Sundrum, 2010.04727

Chiral fermion



Chiral fermion





$$\exp(\pi\mu)\exp\left(-\frac{\pi}{H}\sqrt{m^2+\mu^2}\right) \simeq \exp\left(-\frac{\pi m^2}{2H\mu}\right) \sim O(1), \quad \text{for } \mu \gg m \gg H$$
No $e^{-m/H}$ -like suppression!

Enhanced mode peaks in a region around $k_{\text{phys}} \sim \mu$, with size $\Delta k_{\text{phys}} \sim m$. \rightarrow phase space = a shell with volume $4\pi\mu^2 m$

Putting it together:



Overall exp factor: $e^{-\pi m^2/\mu H}$

Phase space: $4\pi m\mu^2$

$$f_{\rm NL}^{\rm (osc)} \left(\begin{array}{c} & & \\ & \psi \\ & & \\ &$$

Signal can be large for $\mu > m > H$

General form of signal

analytic piece / "background part"

$$\mathcal{S} \approx A\left(\frac{k_1}{k_3}\right)^{-N} + B\left(\frac{k_1}{k_3}\right)^{-L} \sin\left(\omega\log\frac{k_1}{k_3} + \varphi\right)$$

nonanalytic piece / "signal part"

		В	L	ω
$s = 0, \ m > \frac{3}{2}, \ \mu = 0 \ [7]$	tree	$e^{-\pi m}$	$\frac{1}{2}$	$\sqrt{m^2 - \frac{9}{4}}$
$s = 0, \ 0 < m < \frac{3}{2}, \ \mu = 0 \ [7]$	tree	-	$\frac{1}{2} - \sqrt{\frac{9}{4} - m^2}$	0
$s > 0, \ m > s - \frac{1}{2}, \ \mu = 0 \ [13]$	tree	$e^{-\pi m}$	$\frac{1}{2}$	$\sqrt{m^2 - (s - \frac{1}{2})^2}$
$s > 0, \ 0 < m < s - \frac{1}{2}, \ \mu = 0 \ [13]$	tree	-	$\frac{1}{2} - \sqrt{(s - \frac{1}{2})^2 - m^2}$	2 0
$s = 0, \ m > \frac{3}{2}, \ \mu = 0 \ [7]$	1-loop	$e^{-2\pi m}$	2	$2\sqrt{m^2 - \frac{9}{4}}$
Dirac fermion, $m > 0, \mu = 0$ [8]	1-loop	$e^{-2\pi m}$	3	2m
Dirac fermion, $m > 0, \mu > 0$ [8]	1-loop	$e^{2\pi\mu-2\pi\sqrt{m^2+\mu}}$	$\overline{\iota^2}$ 2	$2\sqrt{m^2+\mu^2}$
$s = 1, \ m > \frac{1}{2}, \ \mu \ge 0 \ [10]$	1-loop	$e^{2\pi\mu-2\pi m}$	2	$2\sqrt{m^2 - \frac{1}{4}}$

With µ enhancement

Zhong-Zhi Xianyu and LTW. 1910.12876, 2004.02887



Full numerical calculation

Zhong-Zhi Xianyu, Yiming Zhong, and LTW. 2109.14635



Comparing with analytic est.



* Good agreement for $k_1/k_3 > 20$.

* Some shift in frequency for lower k_1/k_3

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The excursion of the inflaton

 $\Delta \phi \sim N_{\rm efold} \sqrt{\epsilon} M_{\rm Planck}$

Large excursion of the inflaton field plausible, even if we restrict ourselves to the case where $\Delta \phi < M_{Planck}$

This is the case even for a small part of inflation with $N_{efold} \approx O(1)$

Any physics/observable effect?

Example: Inflaton + spectator



Inflation sector Single field slow roll... Spectator, less energy, not driving spacetime evolution

Suppose the coupling is weak, suppressed by some high scale M $\sim \phi \sim \Delta \phi$

For example:
$$f\left(\frac{\phi}{M}\right)m_{\sigma}^{2}\sigma^{2}$$
, $g\left(\frac{\phi}{M}\right)b\sigma^{4}$, etc.

Field excursion of inflaton can change the mass and couplings in the spectator sector, leading to interesting dynamics.



Rolling inflaton \rightarrow (1st order?) phase transition in the spectator sector

1st order phase transition

Phase transition is 1st order, and spectator sector does not dominate energy density:

 $H^4 \ll m_{\sigma}^4 \ll 3M_{\rm Pl}^2 H^2$ This is possible to arrange.



 $r_{\rm bubble} \ll H^{-1}$

 $t_{\rm bubble\ collision} \sim r_{\rm bubble} \ll H^{-1}$

An instantaneous source of GW.





During inflation:

Mode starts inside horizon, oscillates till horizon exit.

- Amplitude depends on k.
- Leads to oscillatory pattern in frequency.

After reheating, modes re-enter the horizon

Oscillations

$\tau_*^{-1} < k < \Delta_\tau^{-1}$





Time scale of bubble collision $\approx \Delta_{\tau}$.

Oscillation pattern in frequency smeared out in this regime.



Mode outside horizon at the time of phase transition

No oscillation. Can treat the GW as if it is from a point source.





 $\frac{d\rho_{\rm GW}^{\rm osc}}{d\log k} = \frac{2G_N |\tilde{T}_{ij}(0,0)|^2}{\pi V a^4(\tau) a^2(\tau_\star)} \left\{ \left[\tilde{\mathcal{E}}_0^i(k) \tilde{\mathcal{G}}_0^f(k) \right]^2 k^3 \left[1 + \mathcal{S}(k\Delta_\tau) \cos 2k(\tau_\star - \tau_0) \right] \right\}$

$$\begin{split} \frac{d\rho}{d\log k} & \left[\tilde{\mathcal{E}}_{0}^{i}(k)\tilde{\mathcal{G}}_{0}^{f}(k)\right]^{2}k^{3}\left[1+\mathcal{S}(k_{p}\Delta_{p})\cos 2k(\tau_{\star}-\tau_{0})\right] \\ & \left[\tilde{\mathcal{E}}_{0}^{i}(k)\right]^{2}k^{5} & \left[\tilde{\mathcal{E}}_{0}^{i}(k)\right]^{2}k^{5} \\ & \left[\tilde{\mathcal{E}}_{0}^{i}(k)\right]^{2}k^{5} & \left[\tilde{\mathcal{E}}_{0}^{i}(k)\tilde{\mathcal{G}}_{0}^{f}(k)\right]^{2}k^{3} \\ & \left[\tilde{\mathcal{E}}_{0}^{i}(k)\tilde{\mathcal{G}}_{0}^{f}(k)\right]^{2}k^{5} & \left[\tilde{\mathcal{E}}_{0}^{i}(k)\tilde{\mathcal{G}}_{0}^{f}(k)\right]^{2}k^{3} \\ & \left[\tilde{\mathcal{E}}_{0}^{osc}\right] \\ & k_{p} \approx H & k_{p} \approx \Delta_{p}^{-1} & \mathbf{f} \\ \\ & \frac{d\rho_{GW}^{osc}}{d\log k} = \frac{2G_{N}|\tilde{T}_{ij}(0,0)|^{2}}{\pi Va^{4}(\tau)a^{2}(\tau_{\star})} \left\{\left[\tilde{\mathcal{E}}_{0}^{i}(k)\tilde{\mathcal{G}}_{0}^{f}(k)\right]^{2}k^{3}\left[1+\mathcal{S}(k\Delta_{\tau})\cos 2k(\tau_{\star}-\tau_{0})\right]\right\} \\ & Occillation \end{split}$$

$$\begin{split} \frac{\mathrm{d}\rho}{\mathrm{dlogk}} & \left[\tilde{\mathcal{E}}_{0}^{i}(k)\tilde{\mathcal{G}}_{0}^{f}(k)\right]^{2}k^{3}\left[1+\mathcal{S}(k_{p}\Delta_{p})\cos 2k(\tau_{\star}-\tau_{0})\right] \\ & \left[\tilde{\mathcal{E}}_{0}^{i}(k)\right]^{2}k^{5} \\ & \left[\tilde{\mathcal{E}}_{0}^{i}(k)\tilde{\mathcal{G}}_{0}^{f}(k)\right]^{2}k^{5} \\ & \left[\tilde{\mathcal{E}}_{0}^{i}(k)\tilde{\mathcal{G}}_{0}^{f}(k)\right]^{2} \\ & \left[\tilde{\mathcal{E}}_{0}^{i}(k)\tilde{\mathcal{G}}_{0}^{f}(k)\right]^{2} \\ & \left[\tilde{\mathcal{E}}_{0}^{i}(k)\tilde{\mathcal{G}}_{0}^{f}(k)\right]^{2} \\ & \left[\tilde{\mathcal{E}}_{0}^{i}(k)\tilde{\mathcal{G}}_{0}^{f}(k)\right]^{2}k^{3}\left[1+\left(\tilde{\mathcal{E}}_{0}^{i}(k)\tilde{\mathcal{G}}_{0}^{i}(k)\tilde{\mathcal{G}}_{0}^{f}(k)\right)\right]^{2} \\ & \left[\tilde{\mathcal{E}}_{0}^{i}(k)\tilde{\mathcal{G}}_{0}^{f}(k)\right]^{2} \\ & \left[\tilde{\mathcal{E}}_{0}^{i}(k)\tilde{\mathcal{G}}_{0}^{f}(k)\right]^{2}k^{3}\left[1+\left(\tilde{\mathcal{E}}_{0}^{i}(k)\tilde{\mathcal{G}}_{0}^{i}(k)\tilde{\mathcal{G}}_{0}^{i}(k)\right)\right]^{2} \\ & \left[\tilde{\mathcal{E}}_{0}^{i}(k)\tilde{\mathcal{G}}_{0}^{i}(k)\right]^{2} \\ & \left[\tilde{\mathcal{E}}_{0}^{i}(k)\tilde{\mathcal{G}}_{0}^{i}(k)\right]^{2}k^{3}\left[1+\left(\tilde{\mathcal{E}}_{0}^{i}(k)\tilde{\mathcal{G}}_{0}^{i}(k)\tilde{\mathcal{G}}_{0}^{i}(k)\right)\right]^{2} \\ & \left[\tilde{\mathcal{E}}_{0}^{i}(k)\tilde{\mathcal{G}}_{0}^{i}(k)\right]^{2} \\ & \left[\tilde{\mathcal{E}}_{0}^{i}(k)\tilde{\mathcal{G}}_{0}^{i}(k)\right]^{2}k^{3}\left[1+\left(\tilde{\mathcal{E}}_{0}^{i}(k)\tilde{\mathcal{G}}_{0}^{i}(k)\right)^{2}\tilde{\mathcal{E}}_{0}^{i}(k)\tilde{\mathcal{G}}_{0}^{i}(k)\right]^{2} \\ & \left[\tilde{\mathcal{E}}_{0}^{i}(k)\tilde{\mathcal{G}}_{0}^{i}(k)\right]^{2}k^{3} \\ & \left[\tilde{\mathcal{E}}_{0}^{i}(k)\tilde{\mathcal{G}}_{0}^{i}(k)\right]^{2}k^{3}\left[1+\left(\tilde{\mathcal{E}}_{0}^{i}(k)\tilde{\mathcal{G}}_{0}^{i}(k)\right)^{2}\tilde{\mathcal{E}}_{0}^{i}(k)\tilde{\mathcal{G}}_{0}^{i}(k)\right]^{2} \\ & \left[\tilde{\mathcal{E}}_{0}^{i}(k)\tilde{\mathcal{E}$$



Dependence on later evolution



- $\tilde{\mathcal{G}}_{0}^{f}(k)$ Depends on the evolution of the background spacetime during inflation
- $\tilde{\mathcal{E}}_0^i(k)$ Depends on the evolution of the background spacetime after inflation

Alternative scenarios can change the shape of the GW signal!

Scenarios of inflation and its aftermath

Scenarios of inflation

Quasi de Sitter:

 $a(\tau) = -\frac{1}{H\tau}$

Power law:

 $a(t) = a_0 (t/t_0)^p$

Lucchin and Matarrese, 1985

p→∞, quasi de Sitter

Scenarios after inflation:

Parameterized as

$$a(t) \sim t^{j}$$

	w	$\rho(a)$	\tilde{p}	$\tilde{\alpha}$
MD	0	a^{-3}	2/3	-3/2
RD	1/3	a^{-4}	1/2	-1/2
Λ	-1	a^0	∞	3/2
Cosmic string	-1/3	a^{-2}	1	∞
Domain wall	-2/3	a^{-1}	2	5/2
kination	1	a^{-6}	1/3	0

Impact on spectrum



Intermediate

	RD	MD	$t^{ ilde{p}}$
dS	k^{-5}	k^{-7}	$k^{-3+2\frac{\tilde{p}}{\tilde{p}-1}}$
t^p	$k^{-3+2\frac{p}{1-p}}$	$k^{-5+2\frac{p}{1-p}}$	$k^{-1+2\left(\frac{p}{1-p}+\frac{\tilde{p}}{\tilde{p}-1}\right)}$

Intermediate

	RD	MD	$t^{ ilde{p}}$
dS	k^{-1}	k^{-3}	$k^{1+2\frac{\tilde{p}}{\tilde{p}-1}}$
t^p	$k^{1+2\frac{p}{1-p}}$	$k^{-1+2\frac{p}{1-p}}$	$k^{3+2\left(\frac{p}{1-p}+\frac{\tilde{p}}{\tilde{p}-1}\right)}$

IR

	RD	MD	$t^{ ilde{p}}$
dS	k^3	k^1	$k^{5+2\frac{\tilde{p}}{\tilde{p}-1}}$
t^p	k^3	k^1	$k^{5+2\frac{\tilde{p}}{\tilde{p}-1}}$

Comparing scenarios



 $dS \times (MD-)RD$ $\beta/H_{inf} = 20$ 10⁻¹³ RD $\begin{aligned} \tau_2 &= |\tau_*| \\ \tau_2 &= 5|\tau_*| \end{aligned}$ $\tau_2 = 20 |\tau_*|$ 10⁻¹⁵ ອ ປັ C 10⁻¹⁷ k^3 10⁻¹⁹ 10⁻²¹ 10⁻⁷ 10⁻⁸ 10^{-6} 10⁻⁵ 10⁻⁴ 0.001 f(Hz)

→ different slope in UV part.

Scenarios after reheating. $\tau_2 = MD-RD$ transition

Conclusions

- Cosmological observations can reveal new dynamics in the inflationary era.
- * Production of new particles with m \sim H leads to distinct signals.
 - Non-derivative coupling does not lead to large signal due to correction to the mass of new particle
 - Well motivated derivative couplings → observable signals.

Conclusions

- Cosmological observations can reveal new dynamics in the inflationary era.
- Potentially large inflaton excursion can trigger new dynamics in a spectator sector.
 - * Can trigger 1st order phase transition \rightarrow GW.
 - Can probe an era invisible from CMB/LSS observables.

Extra



2. Derivative coupling

Dim-6 (quasi single field):

$$\mathcal{L} \supset \frac{c_6}{\Lambda^2} (\partial \phi)^2 Q^{\dagger} Q - m_Q^2 Q^{\dagger} Q - \lambda_Q |Q|^4$$

Validity of EFT in inflationary background: $\Lambda^2 > \dot{\phi}_0$

 $\dot{\phi}_0 \neq 0$ contributes to the mass of Q. To avoid fine tuning:

$$\operatorname{Max}\left\{m_Q^2, \ \lambda_Q Q_0^2, \ \delta m_Q^2 = c_6 \left(\frac{\dot{\phi}_0}{\Lambda}\right)^2\right\} \lesssim H^2 \qquad \frac{c_6}{\Lambda^2} \lesssim \frac{H^2}{\dot{\phi}_0^2} = \frac{(2\pi)^2 P_{\zeta}}{H^2}$$

Can be sizable (O(1)) if $\lambda_Q \sim (2\pi)^2 P_\zeta \simeq 8 \times 10^{-8}$

Observing the signal

Ne: efold till the end of inflation = time of the phase transition

1st order phase transition during inflation

Bubble nucleation rate:

$$\frac{1}{V} \simeq m_{\sigma}^4 e^{-S_4}$$

Efficient phase transition:

$$\int_{-\infty}^{t} dt' \frac{\Gamma}{V H^3} \simeq O(1) \rightarrow S_4 \sim \log\left(\frac{\phi H}{\dot{\phi}} \frac{m_{\sigma}^4}{H^4}\right) \sim \log\left(\frac{\phi}{\epsilon^{1/2} M_{\rm Pl}} \frac{m_{\sigma}^4}{H^4}\right)$$

Phase transition is 1st order ($S_4 \gg 1$), and spectator sector does not dominate energy density:

$$H^4 \ll m_\sigma^4 \ll 3M_{\rm Pl}^2 H^2$$

This is possible to arrange.

$$\frac{dS_4 \prod_{\substack{\phi \in \mathrm{ff} \\ \varphi \in \mathrm{ff} \\ \varphi \in \mathrm{ff} \\ \varphi = -(\mu^{2} \frac{1}{2^{2} \frac{1}{2} \frac{1}{2^{2} \frac{1}{2} \frac{1}{2^{2} \frac{1}{2^{2} \frac{1}{2} \frac{1}{2^{2} \frac{1}{2} \frac{1}{2^{2} \frac{1}{2} \frac{1}{2} \frac{1}{2^{2} \frac{1}{2} \frac{1}{2^{2} \frac{1}{2} \frac{1}{2$$