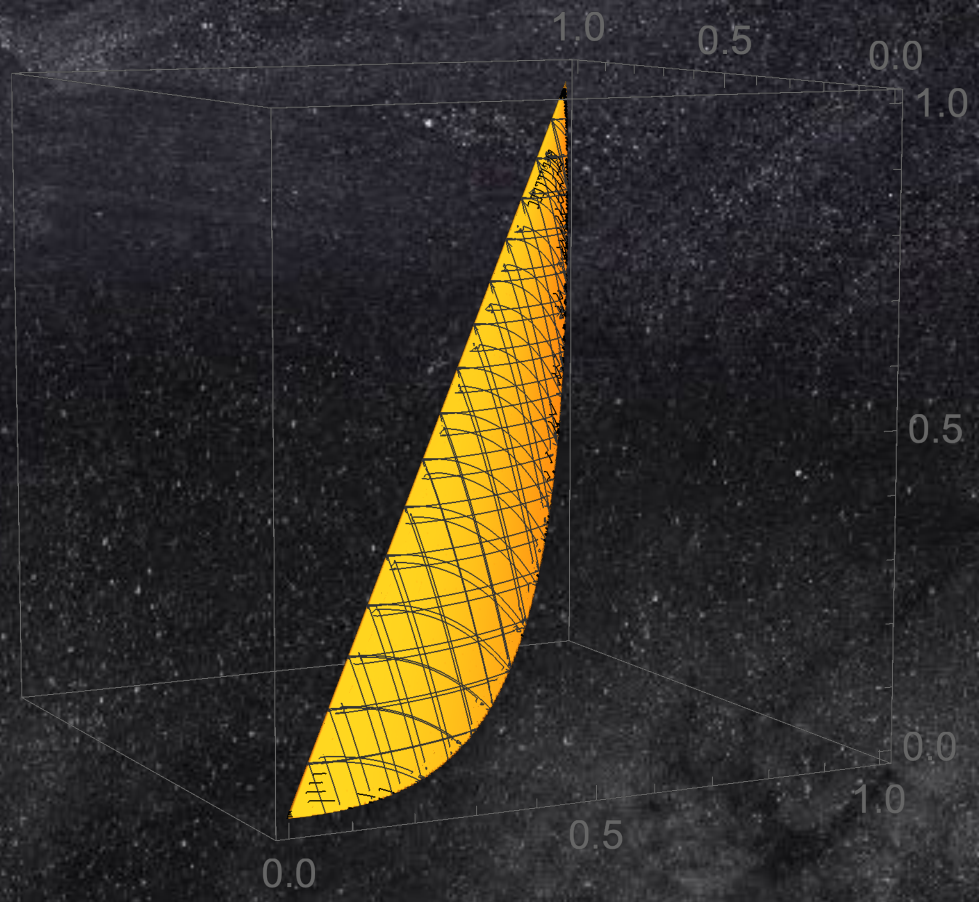
# Positive Moments for Scattering Amplitudes



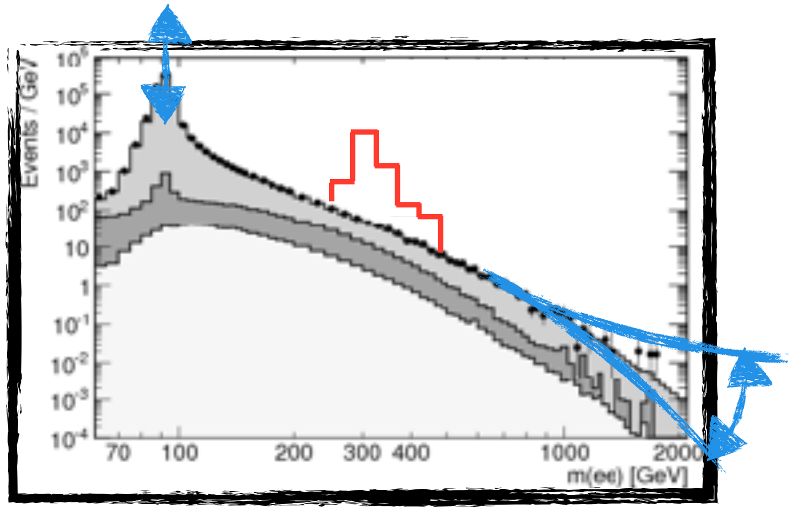
Francesco Riva

(Geneva University)

UC Davis - 8.11.21

# Precision Measurements

At the frontier of experimental capabilities:



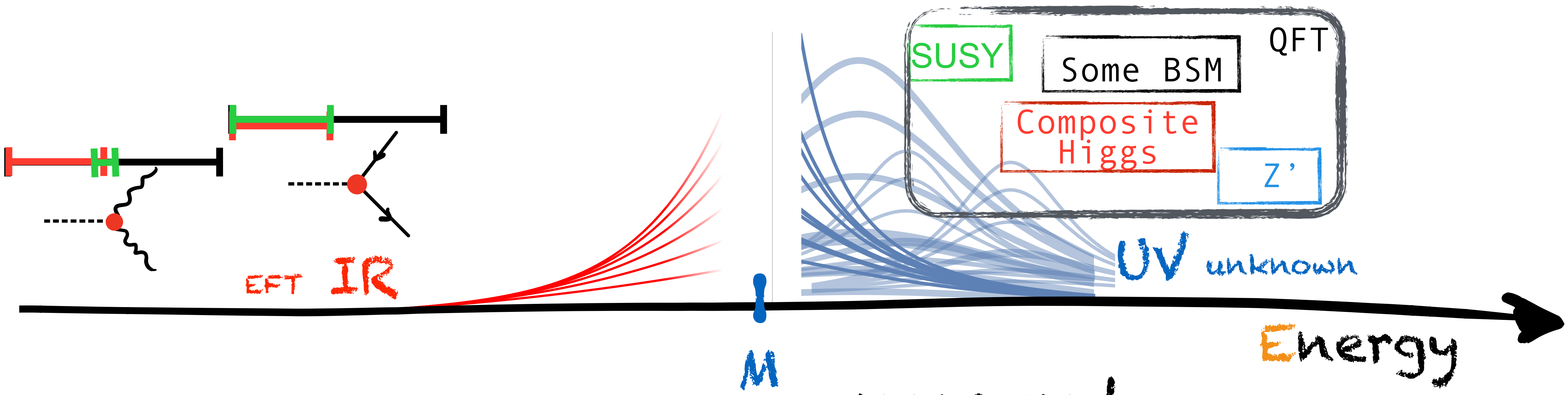
Departures from SM = Series in E/M

↑ Heavy New Physics

**Effective Field Theories ↔ BSM Searches**

(Colliders, Quantum Gravity, Cosmology,...)

# Effective Field Theories



What can be ~~learned~~ measured?

IR Predictions ← UV Hypotheses

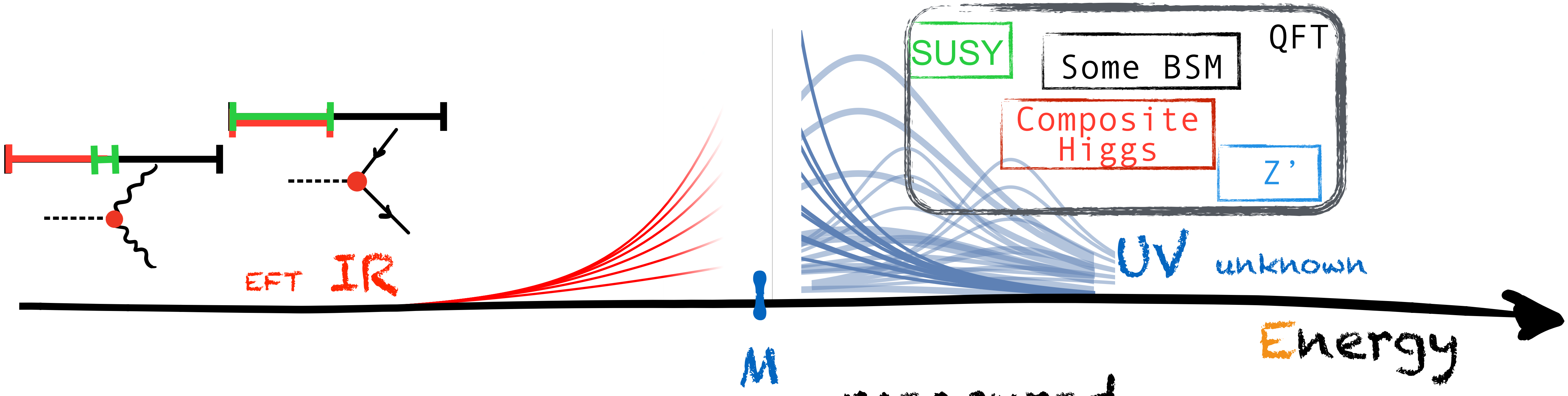
broader... broader... broader...

Lore: all E-behaviours IR consistent!

$$A_{2\to 2}(s,t) = c_0 + c_2 s^2 + c_{2,1} s^2 t + c_4 s^4 + \cdots + c_{n,m} s^n t^m$$

(e.g. tree-level)

# Effective Field Theories



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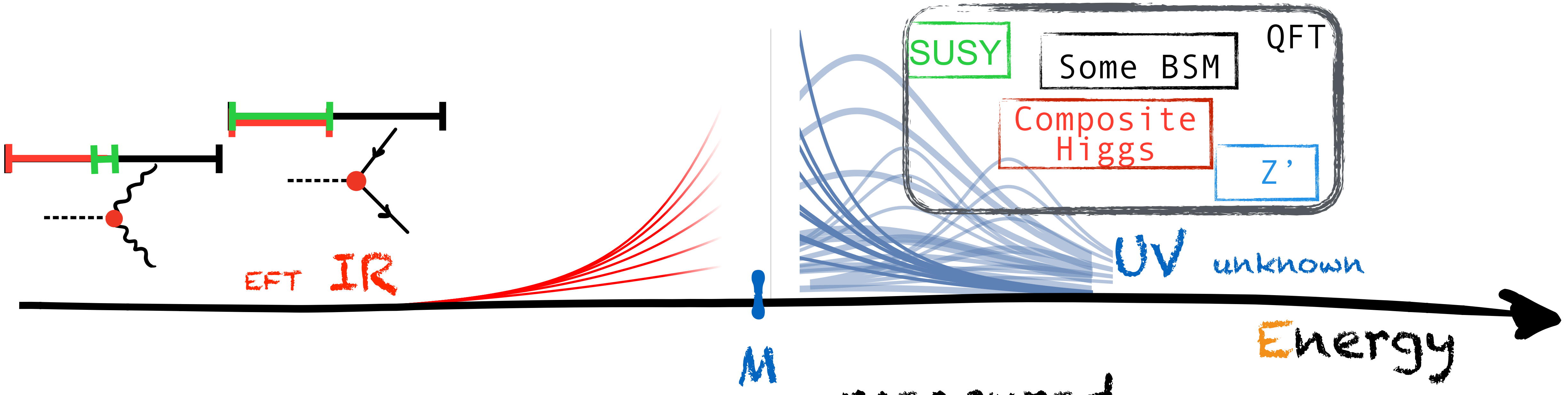

measured

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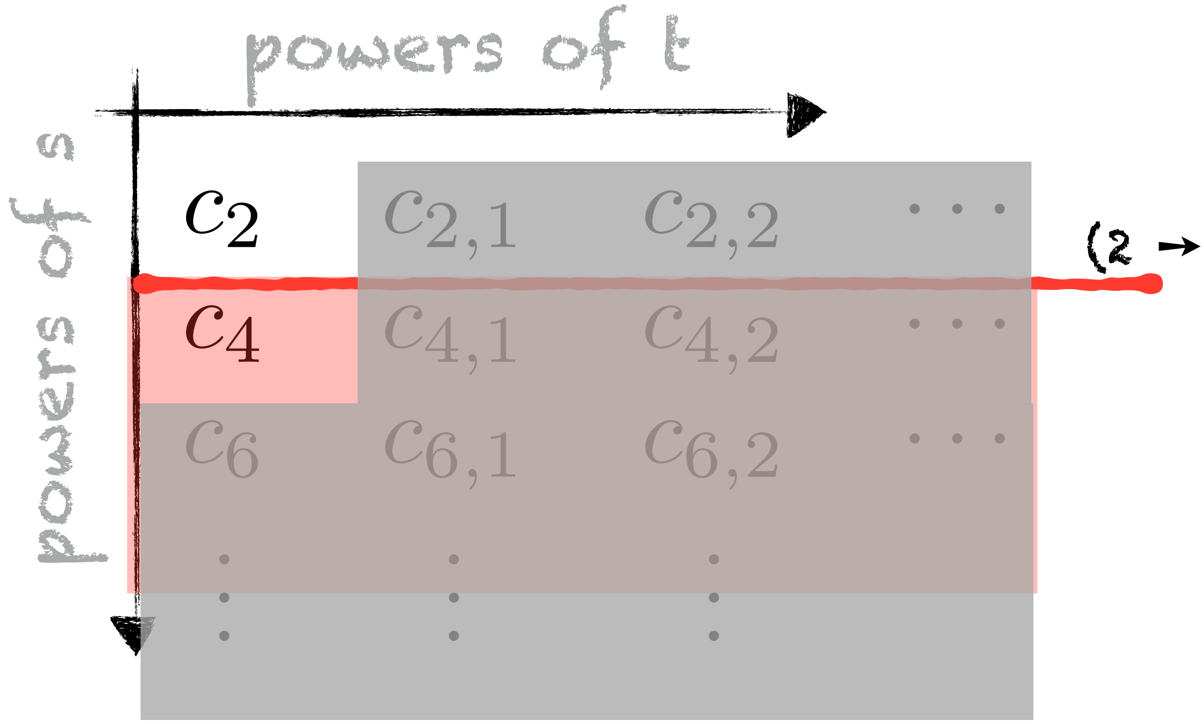
(e.g. tree-level)

Which IR theories are Causal and Unitary in UV?

# Notation/Outline



| | | | |
|---|---|---|---|
| $c_2$ | $c_{2,1}$ | $c_{2,2}$ | $\cdots$ |
| $c_4$ | $c_{4,1}$ | $c_{4,2}$ | $\cdots$ |
| $c_6$ | $c_{6,1}$ | $c_{6,2}$ | $\cdots$ |
| $\vdots$ | $\vdots$ | $\vdots$ | |

1. UV, IR

(2 →2 amplitude, dispersion relation, arcs, moments)

2. Implications
   - weak coupling
   - strong coupling

3. Non forward & Gravity

$$A_{2\to 2}(s,t) = c_0 + c_2 s^2 + c_{2,1} s^2 t + c_4 s^4 + \cdots + c_{n,m} s^n t^m$$

(e.g. tree-level)

1. UV → IR

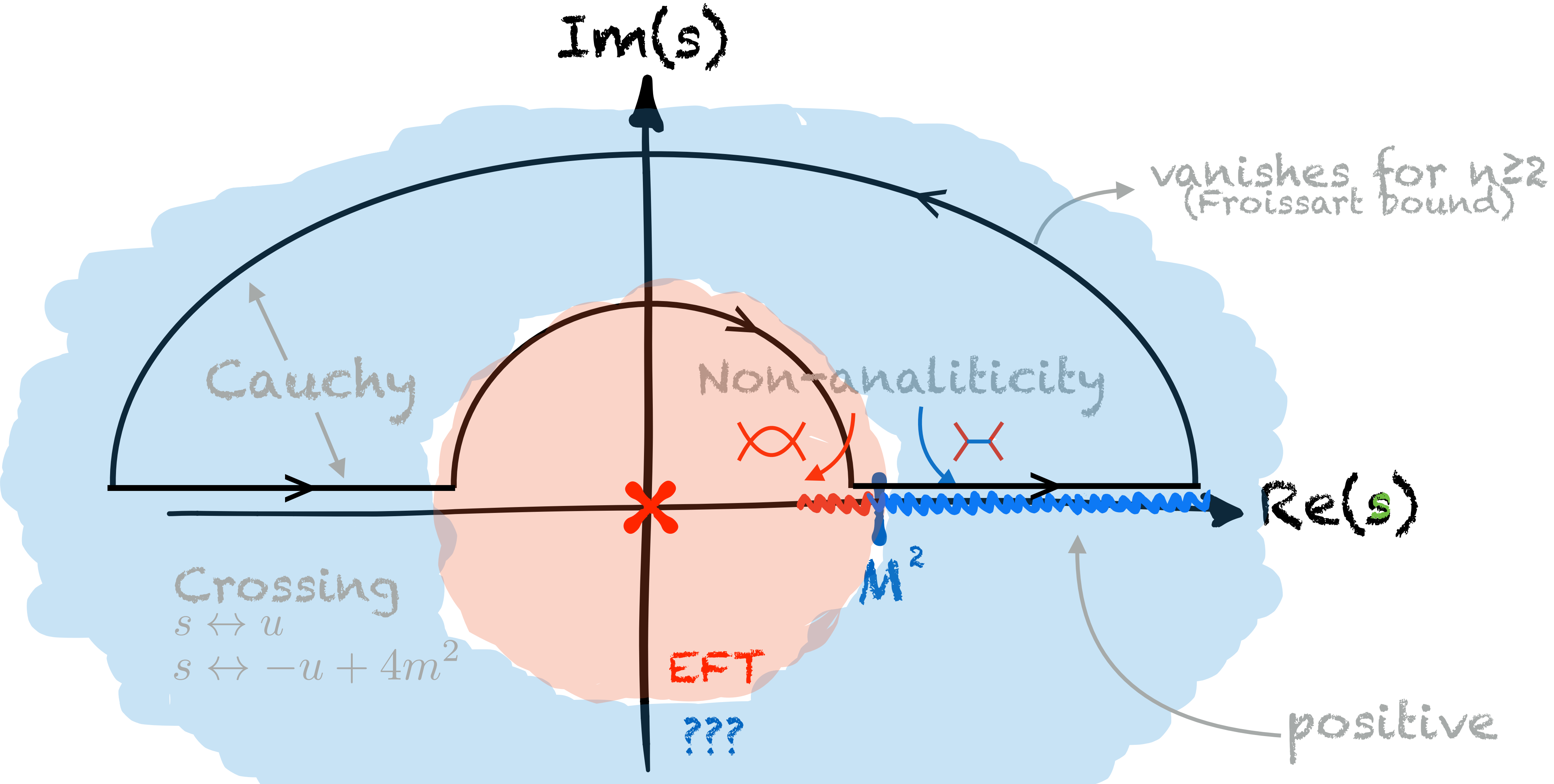
# UV-IR Connection

Froissart,Martin',...60s
Adams,Arkani-Hamed,Dubovsky,
Nicolis,Rattazzi'06,
...

Forward Scattering $t \sim \theta^2 = 0 \longrightarrow$

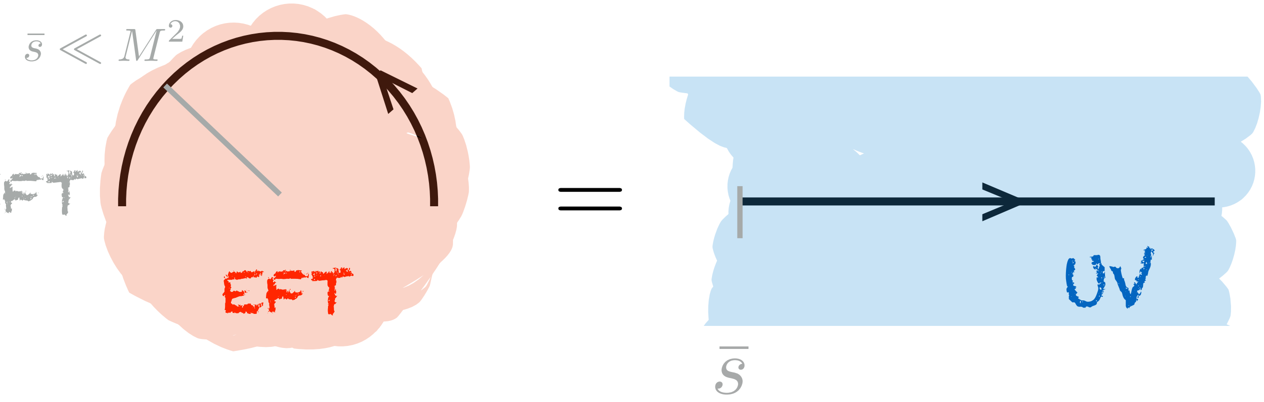
Total energy $s = E^2$

| Physical Properties | | Mathematical Properties of 2→2 forward amplitude $A(s)/s^n$ |
|---|---|---|
| Causality | ▶ | Analytic in $s \in \mathbb{C}/phys$ |
| Unitarity | ▶<br>(optical theorem) | Positive across $s \in \mathbb{R}$ |



# Arcs: UV-IR Connection

$\bar{s}$ = any energy within EFT



positive

$$\text{Arcs: } \mathcal{A}_n(\bar{s}) \equiv \int_{\cap_{\bar{s}}} \frac{ds}{\pi i}\frac{A(s)}{s^{n+1}} = \frac{2}{\pi}\int_{\bar{s}}^{\infty} ds \frac{\mathrm{Im}A(s)}{s^{n+1}}$$

**Calculable in EFT**

(e.g at tree-level $\mathcal{A}_n = c_n$, the energy coefficients in EFT)

$\mathcal{A}_n > 0 \quad (n \geq 2)$

$\mathcal{A}_n > 0 \quad (n \geq 2)$

Adams,Arkani-Hamed,Dubovsky,
Nicolis,Rattazzi'06,

- Consistency condition for EFTs

# More UV-IR Connections

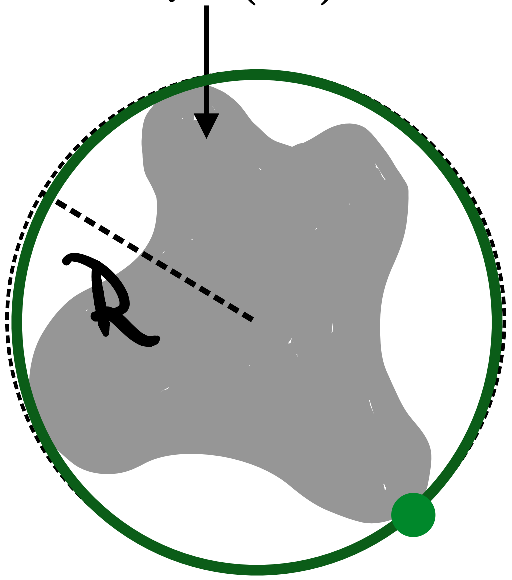
Bellazzini,Elias-Miro,Rattazzi,Riembau,FR'20

Moments

$$\int_0^1 d\mu(x) x^n \xleftarrow{\text{variables change}} \frac{2}{\pi}\int_{\bar{s}}^{\infty} ds \frac{\overset{\text{positive}}{\overset{\downarrow}{\mathrm{Im} A(s)}}}{s^{n+1}}$$

Moments appear everywhere in physics...

e.g. stones $d\mu(x)$ = mass distributions



n=0: total mass M (sets units)

n=1: centre of mass <RM

n=2: moment of inertia <R²M

→ **Bounded**

What bounds do moments satisfy?

# Bounds ⟷ Positive Polynomials

Bellazzini,Elias-Miro,Rattazzi,Riembau,FR'20

**Bounds on EFT arcs** ⟷ **Bounds on Moments** ⟷ **Positive polynomials in [0,1]**

∼ Wilson coef.

$\mathcal{A}_n = \int_0^1 d\mu(x) x^n$

$$\sum_{i=0}^{N} \alpha_i \mathcal{A}_i > 0 \longleftarrow \int_0^1 p_N(x) d\mu(x) > 0 \longleftarrow p_N(x) = \sum_{i=0}^{N} \alpha_i x^i > 0$$

## Characterisations of positive polynomials

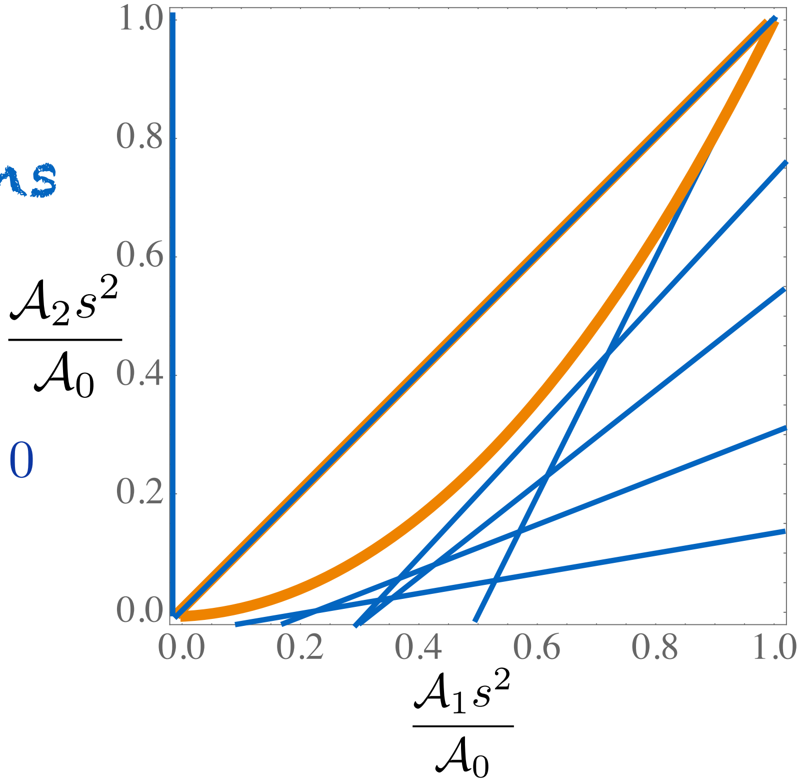
$$p = x^n (1-x)^m$$

- linear
- Always ∞ conditions

$p = x(1-x) \geq 0$

$p = x \mapsto \mathcal{A}_1 - s^2 \mathcal{A}_2 \geq 0$

...up to 3 arcs...



generic polynomials of fixed order

$$p = q_1^2 + x q_2^2 + (1-x) q_3^2 + x(1-x) q_4^2$$

- Non-Linear (semidefinite)
- Finite conditions for Finite number of arcs

$$q_1 = \sum^{N} \beta_i x^i \Rightarrow \sum^{N} \beta_i \mathcal{A}_{i+j} \beta_j > 0$$

Hankel Matrix Must be Positive Definite

# Two-Sided Bounds

Bellazzini,Elias-Miro,Rattazzi,Riembau,FR'20

All conditions involving N arcs only, written as Hankel Matrices:

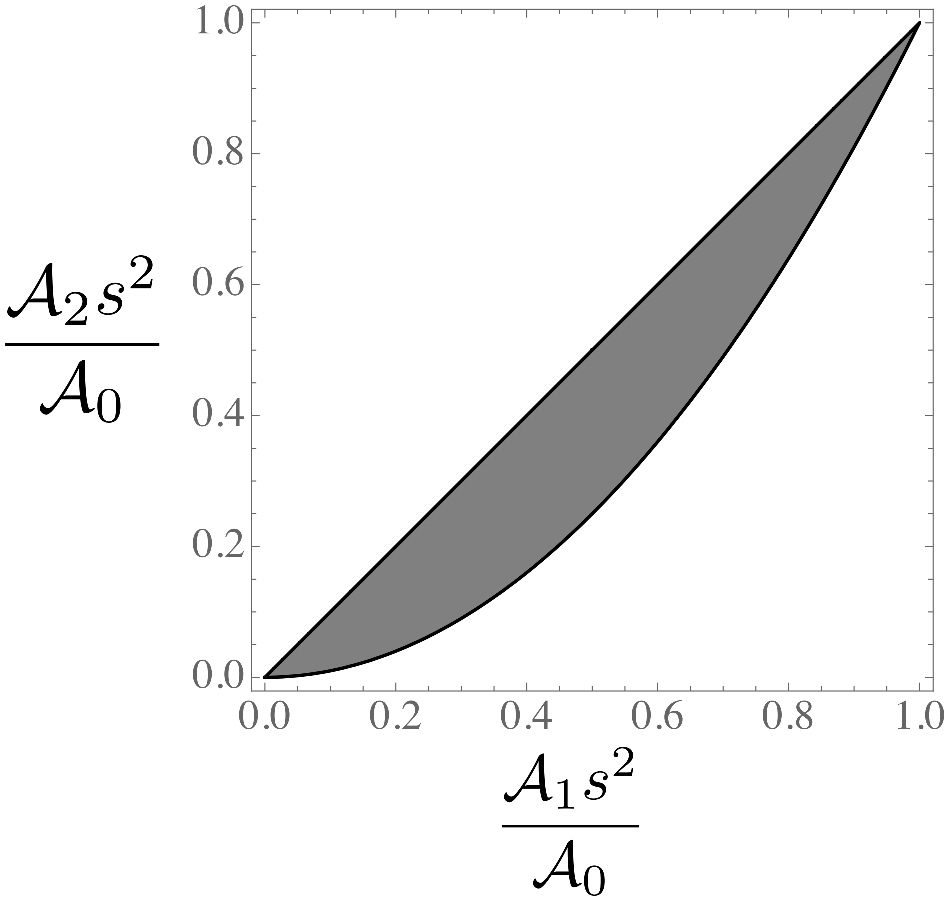
$$\boxed{\begin{aligned} H_N^0 &\succ 0 \\ H_N^1 &\succ 0 \\ H_{N-1}^0 - \hat{s}^2 H_N^1 &\succ 0 \\ H_{N-1}^1 - \hat{s}^2 H_N^2 &\succ 0 \end{aligned}}$$

e.g. $H_4^0 \equiv \begin{pmatrix} \mathcal{A}_0 & \mathcal{A}_1 & \mathcal{A}_2 \\ \mathcal{A}_1 & \mathcal{A}_2 & \mathcal{A}_3 \\ \mathcal{A}_2 & \mathcal{A}_3 & \mathcal{A}_4 \end{pmatrix}$

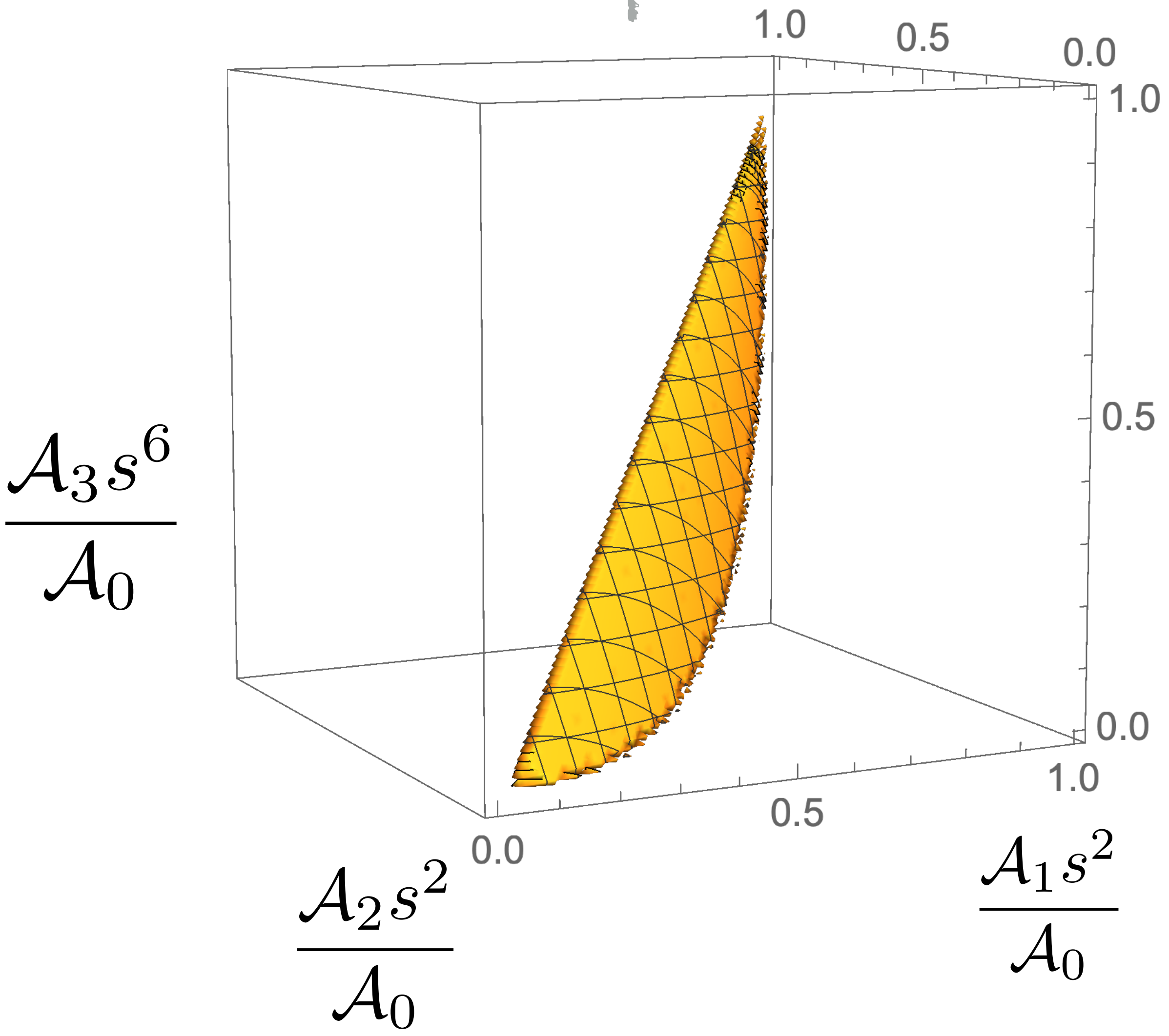
s: any energy within EFT

...up to 3 arcs...

$\mathcal{A}_0 > s^2 \mathcal{A}_1 \qquad \mathcal{A}_1 > s^2 \mathcal{A}_2 \qquad \mathcal{A}_1^2 < \mathcal{A}_2 \mathcal{A}_0$



...up to 4 arcs...



Any moment two-sided bounded in terms of $\mathcal{A}_0$ and $\hat{s}$

1. UV → $\boxed{\text{IR}}$

What are arcs in the IR EFT?

Bellazzini, Elias-Miro, Rattazzi, Riembau, FR'20

# IR arcs

**EFT amplitude** (forward)

$$\beta_4 = \frac{7c_2^2}{160\pi^2}$$

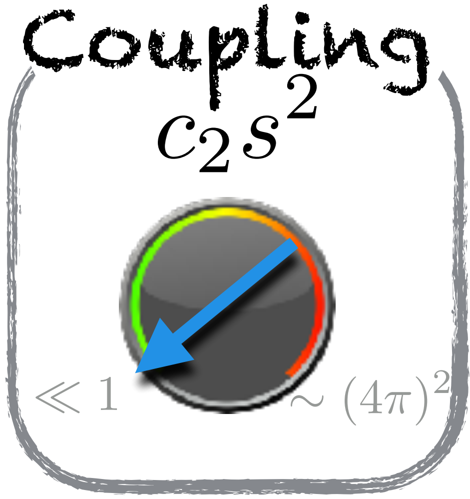
$$\downarrow$$

$$A(s) = c_0 + c_2 s^2 + c_4 s^4 + \quad c_6 s^6 + \cdots$$

- **Arcs** $\mathcal{A}_n \equiv \int_{\cap_{\hat{s}}} \frac{ds}{\pi i} \frac{A(s)}{s^{2n+3}}$

Smaller window
in which theory
looks tree-level

Coupling $c_2 s^2$



Assume higher loops
small, e.g. $c_4 c_6 s^8$

$$\mathcal{A}_0 = \quad c_2 \quad + \cdots$$

$$\mathcal{A}_1 = \quad c_4 \qquad\qquad + \cdots$$

$$\mathcal{A}_2 = \quad c_6 \qquad\qquad + \cdots$$

**Weak Coupling**: $\mathcal{A}_n = c_{2n+2}$, all $\mathcal{L}$ couplings captured by arcs

**Strong Coupling**: high arcs dominated by $c_2$ loop effects!
(e.g ChiPT)

- Information unaccessible

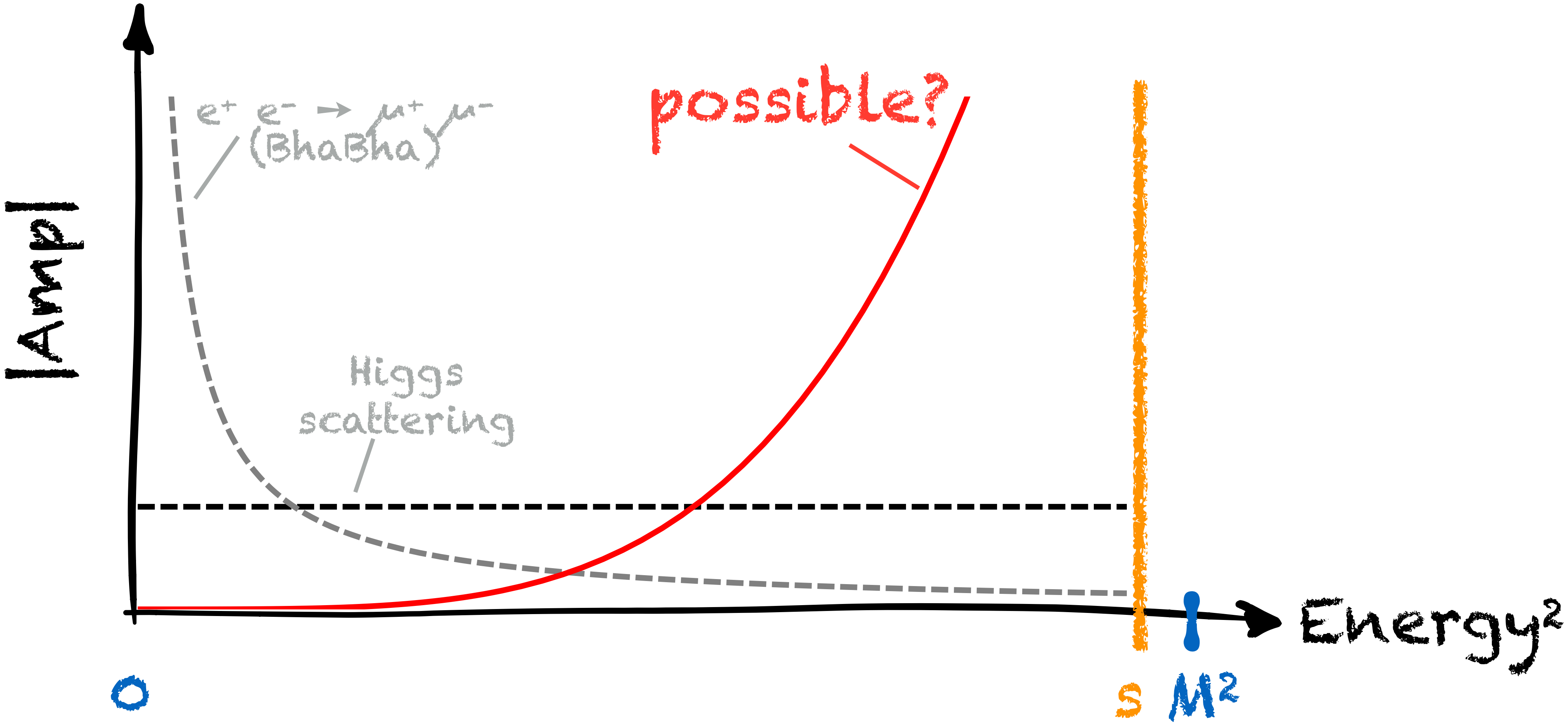
# 2. Applications

(at weak coupling)

# 1. How soft can EFTs be?

(weak coupling)

$$A(s) = c_0 + c_2 s^2 + \underline{c_4} s^4 + \underline{c_6} s^6 + c_8 s^8 + \cdots$$



Naively:

- all coefficients comparable (units of M²)
- indeed, c's mixed by running

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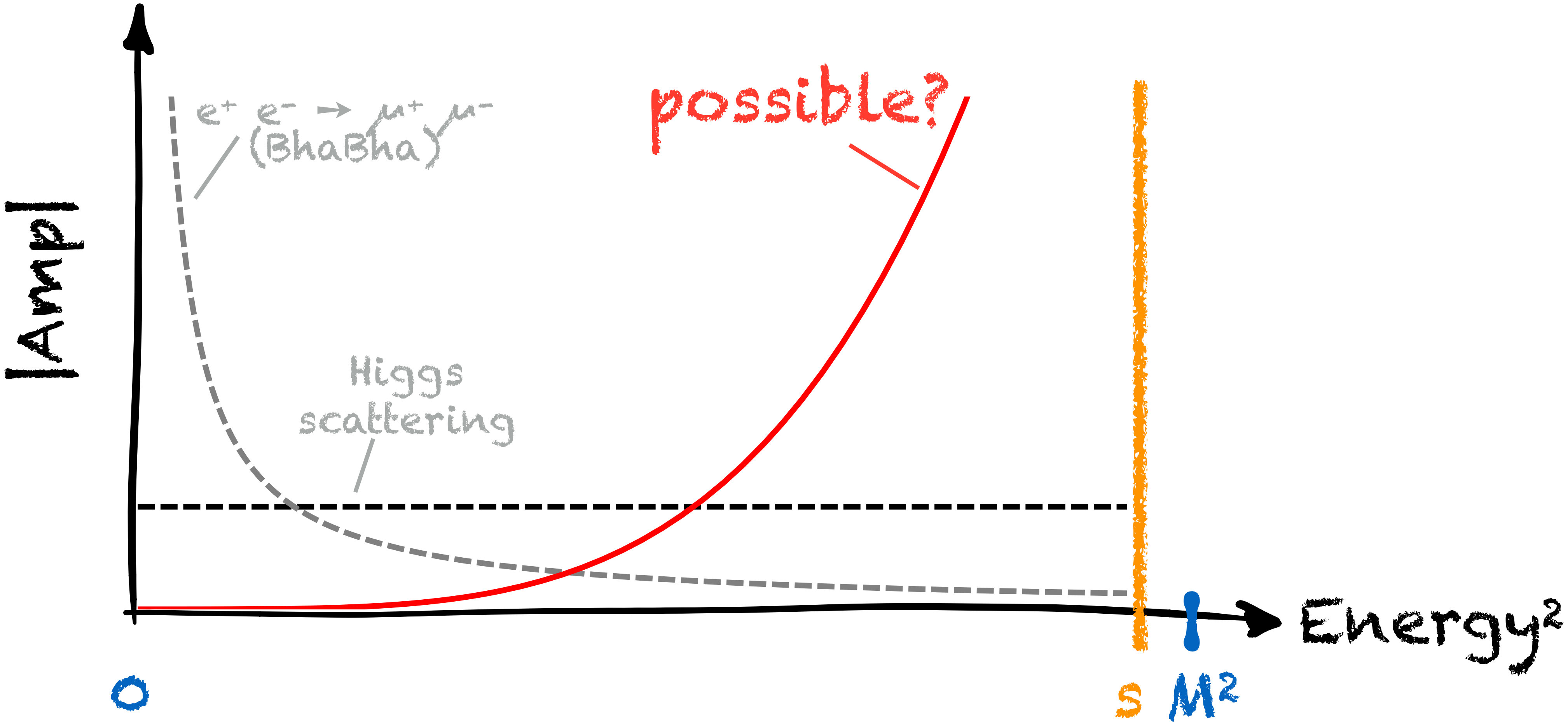

**Naively:**

- all coefficients comparable (units of M²)
- indeed, c's mixed by running

...but...

**Symmetries:**

make hyerarchies natural

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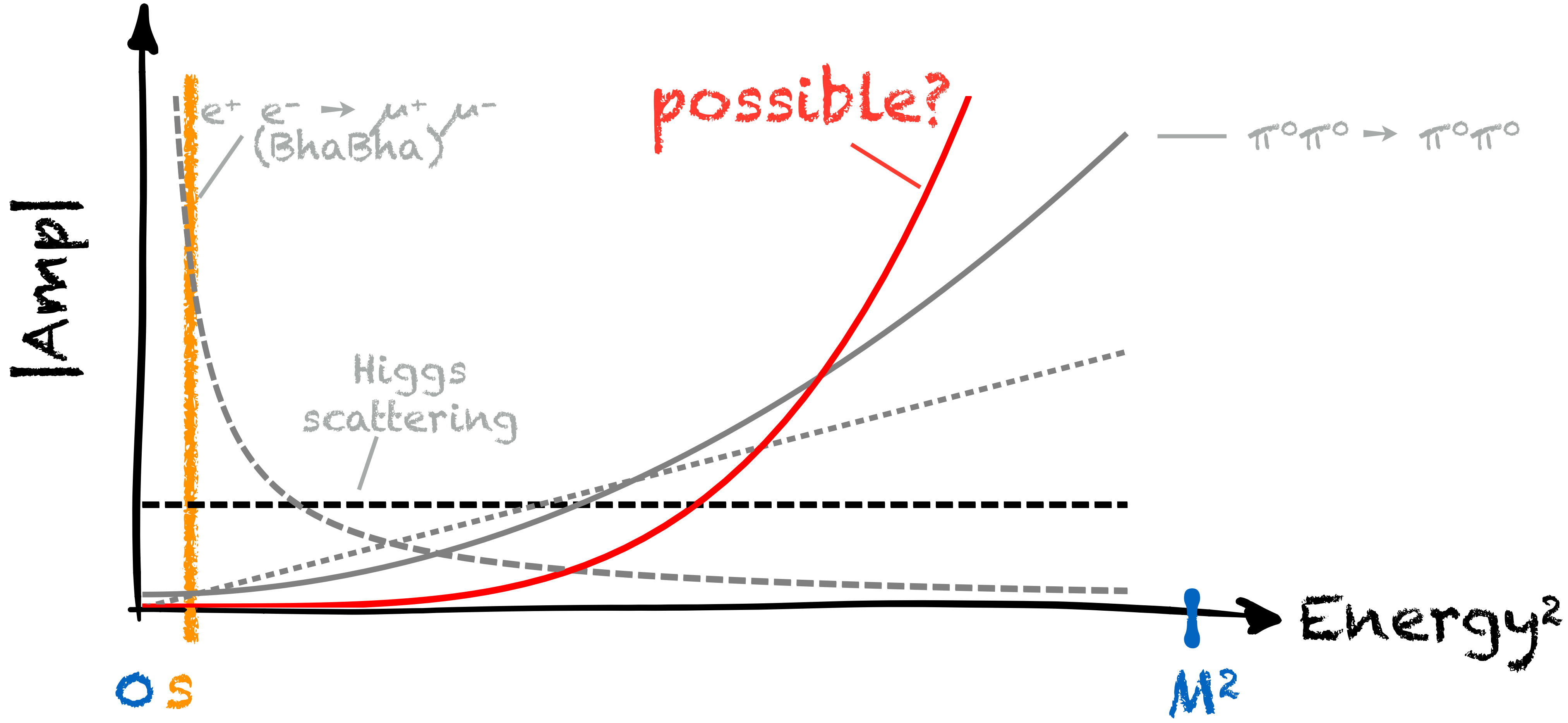

Symmetries:

make hyerarchies natural

**Goldstone** $\pi \to \pi + \alpha$

$\mathcal{L} = c_2(\partial\pi)^4$ ▶ $A \sim c_2 s^2 + \cdots$

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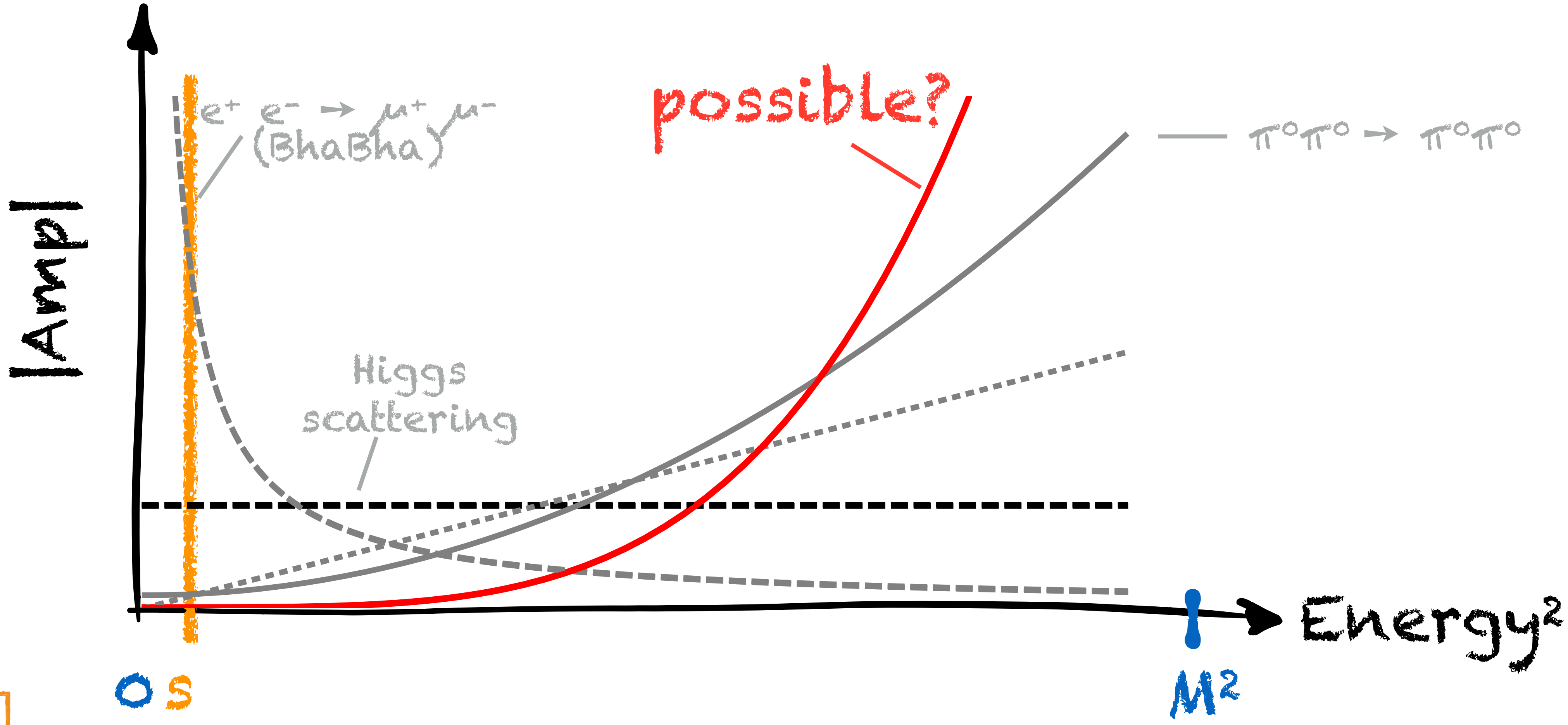

**Galileons** Nicolois,Rattazzi,Trincherini'08

$\pi \to \pi + \alpha + \beta_\mu x^\mu$

$\mathcal{L} = c_4(\partial\partial\pi)^4$ ▶ $A \sim c_4 s^4 + \cdots$

**Super-Soft** $\pi \to \pi + \alpha + \beta x \cdots + \gamma x^n$

▶ $A \sim c_N s^{2N}$

Symmetries:

make hyerarchies natural

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# 1. How soft can EFTs be?

(weak coupling)

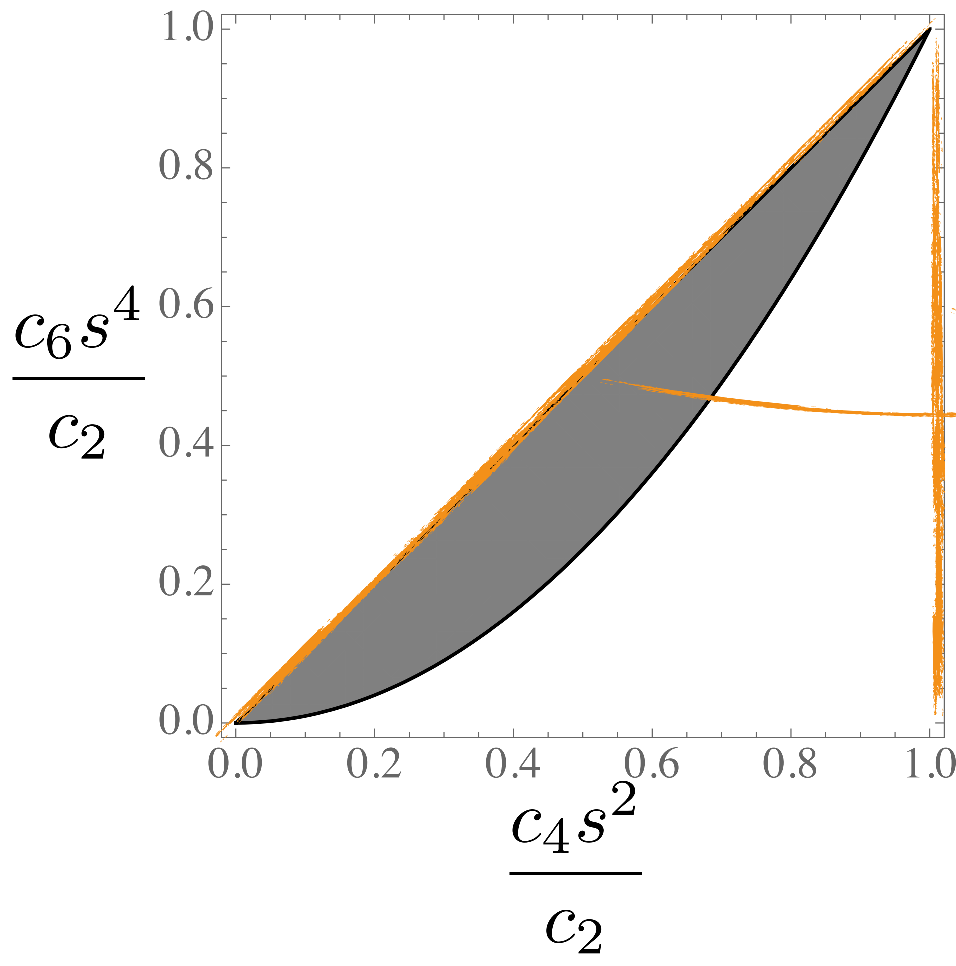
Bellazzini, Elias-Miro, Rattazzi, Riembau, FR'20

see also
Englert, Giudice, Greljo, McCullough'19,
Bellazzini, Serra, Sgarlata, FR'19

## Arcs in tree-level approximation:

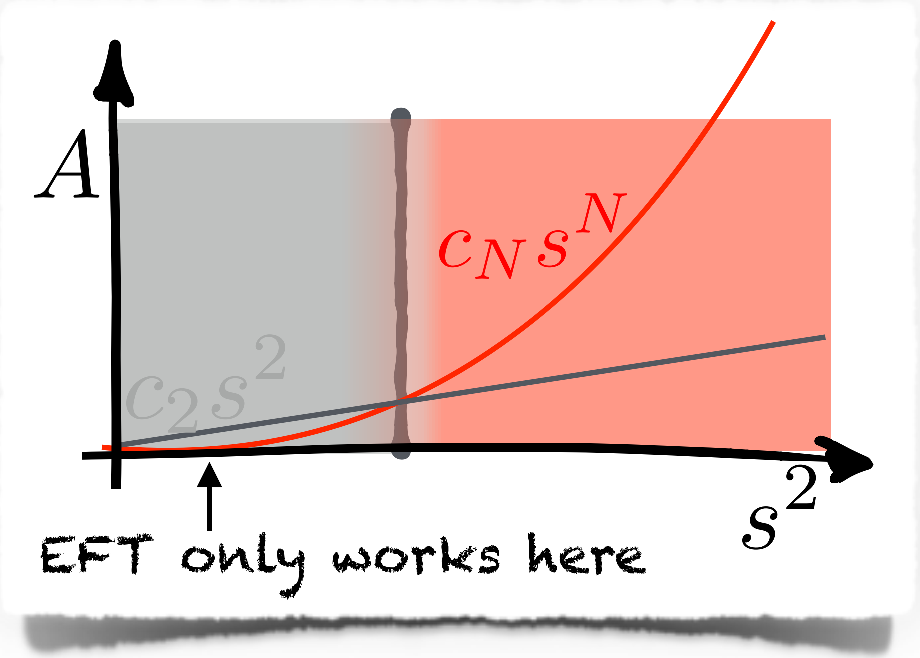
$$A(s) = \cancel{c_0} + c_2 s^2 + c_4 s^4 + c_6 s^6 + \cdots$$

▶ $$\mathcal{A}_n \equiv \int_{\cap_s} \frac{ds}{\pi i} \frac{A(s,t)}{s^{2n+3}} = c_{2n+2}$$



$$c_2 s^2 > c_4 s^4 > c_6 s^6 > \cdots$$

Froissart



▶ info on theory cutoff

Supersoft theories have low cutoff...

... so low that supersoftness unobservable!

(dimension>8 operators cannot dominate)

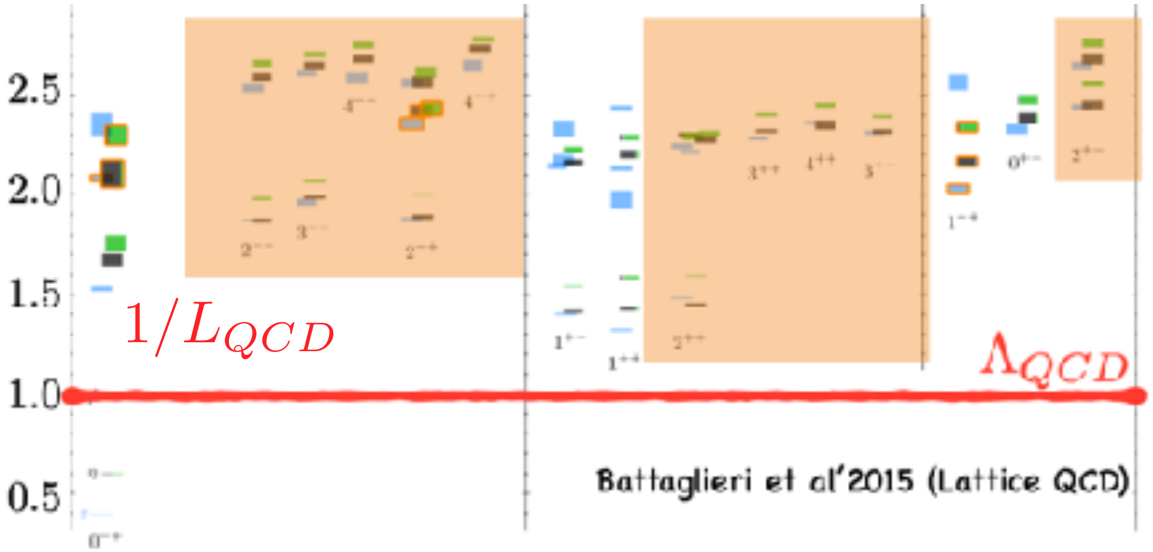
# 2. Massive Higher Spin

Bellazzini,Serra,Sgarlata,FR'19

$$\Phi^{\mu_1\cdots\mu_J}$$

Higher Spin resonances exist in QCD, Nuclei/atoms, Strings,...
(J>2)

$$m_{HS} \gtrsim \frac{1}{L_{HS}}$$



Battaglieri et al'2015 (Lattice QCD)

Can there be lighter HS states?

Interactions grow with $E \lesssim \dfrac{1}{L_{HS}}$

$$A(\Phi\Phi \to \Phi\Phi) \propto \frac{s^2}{m_{HS}^4} + \cdots + \frac{s^{2J}}{m_{HS}^{4J}}$$

$$m_{HS} \gtrsim \frac{1}{L_{HS}} \longleftarrow \propto \;\; c_2 \;\; > \bar{s}^{2J-2} \;\; c_{2J}$$

Higher Spin always heavier than their size$^{-1}$

# 3. Non forward & Gravity

Bellazzini,Riembau,FR'to appear
Tolley,Wang,Zhou'20; Caron-Huot,Mazac,
Rastelli,Simons-Duffin',vanDuong `20-'21

# Finite-t

NON-Forward = no bounds?

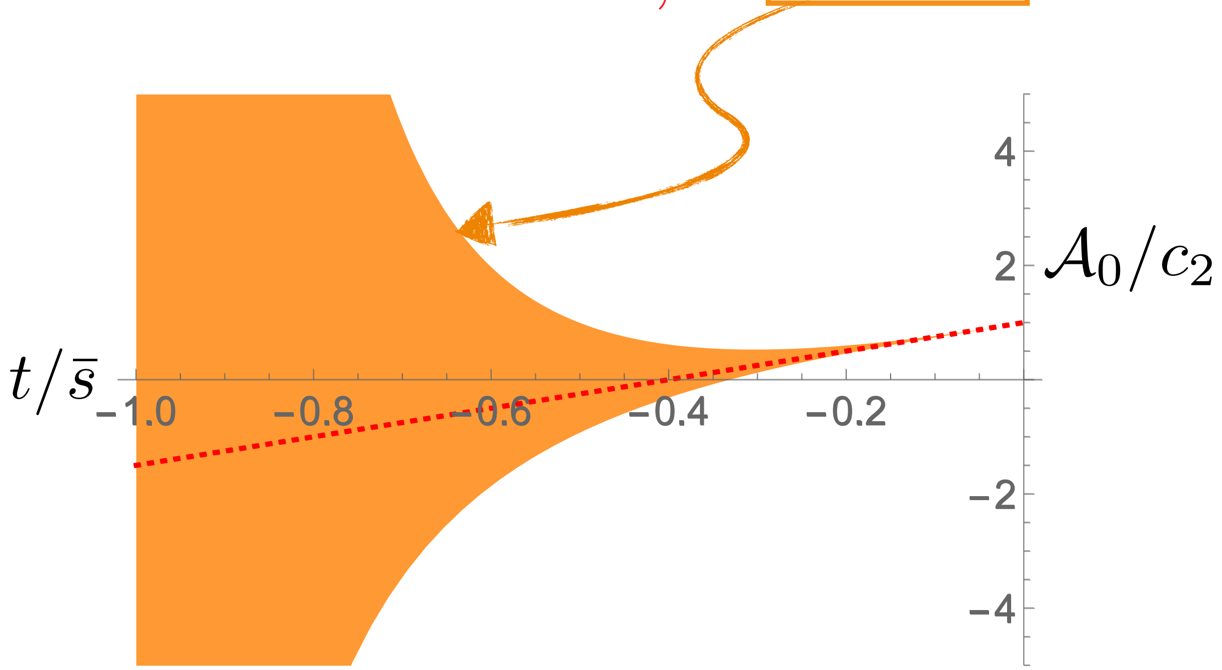
$$A(s,t) = \cancel{c_0} + c_2 s^2 + c_4 s^4 + \cdots + c_{2,1} s^2 t + c_{2,2} s^2 t^2 + \cdots$$

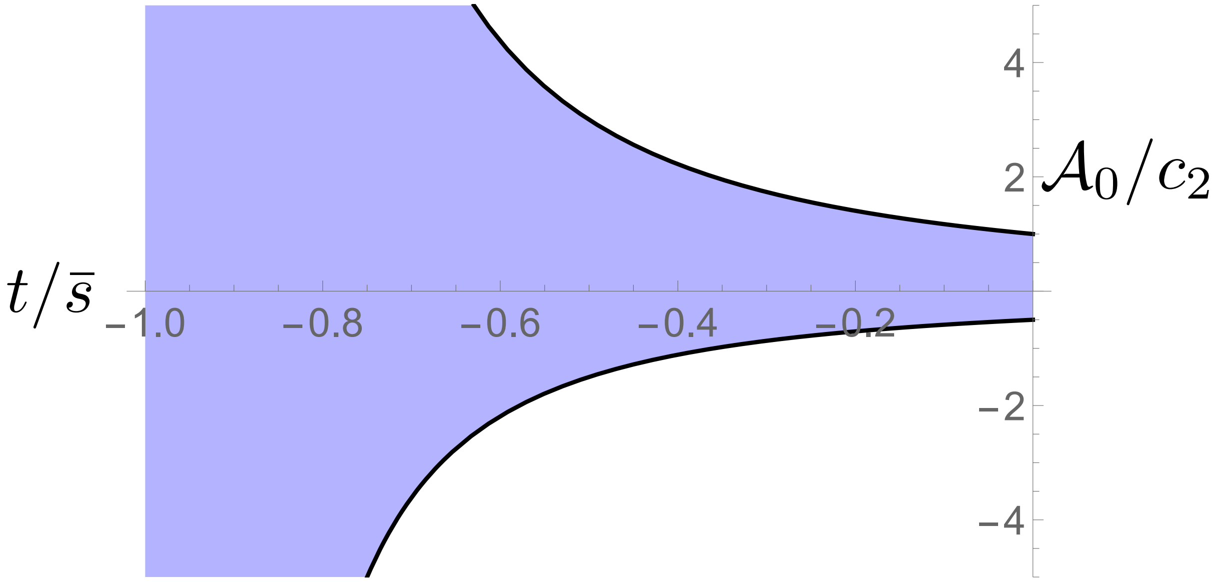
Galileon Nicolois,Rattazzi,Trincherini'08
(appears in massive/modified gravity)

If A(s,t) analytic*: arcs at t≠0 !

$$\mathcal{A}_0(t) \equiv \int_{\cap_{\bar{s}_t}} \frac{ds}{\pi i} \frac{A(s,t)}{(s+\frac{t}{2})^3} = \frac{2}{\pi}\int_{\bar{s}}^{\infty} ds \sum_\ell \overset{>0}{\mathrm{Im} f_\ell(s)} \frac{\overset{-1/2<\,\bullet\,<1}{P_\ell\left(1+\frac{2t}{s}\right)}}{(s+\frac{t}{2})^3}$$

$$c_2 + c_{2,1} t \boxed{+\cdots}$$





*Proved for lightest particle in theory, not in general

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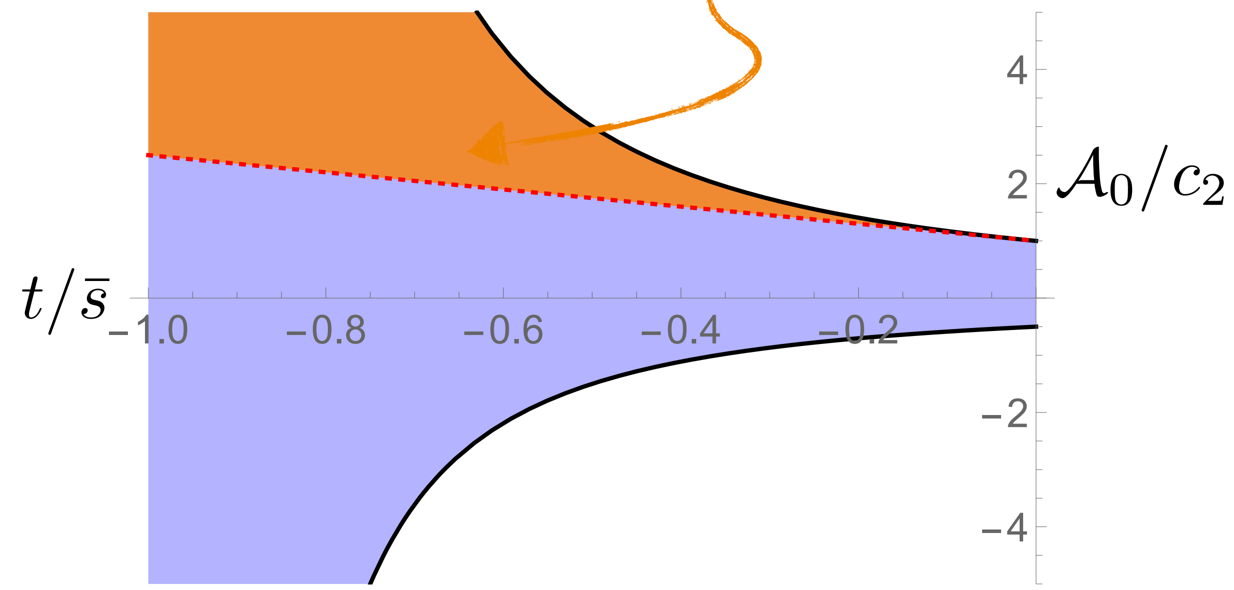

$$-\frac{3}{2} c_2 < c_{2,1} s$$

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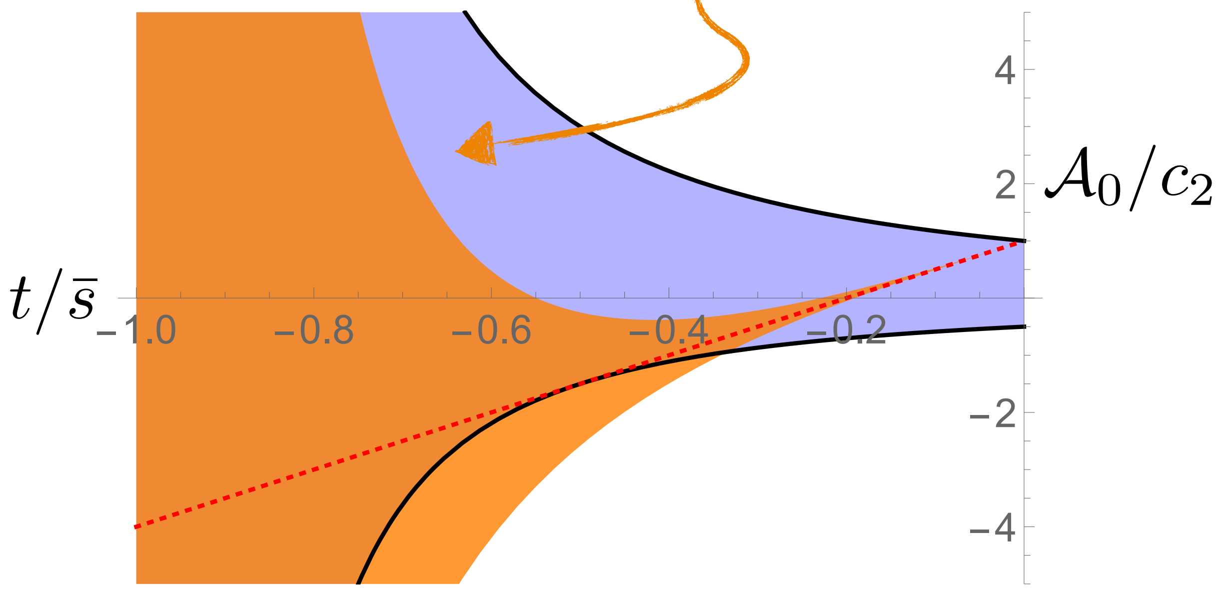
$$c_2 + c_{2,1} t \boxed{+ \cdots}$$



$$-\frac{3}{2}c_2 < c_{2,1} s \lesssim 5c_2$$

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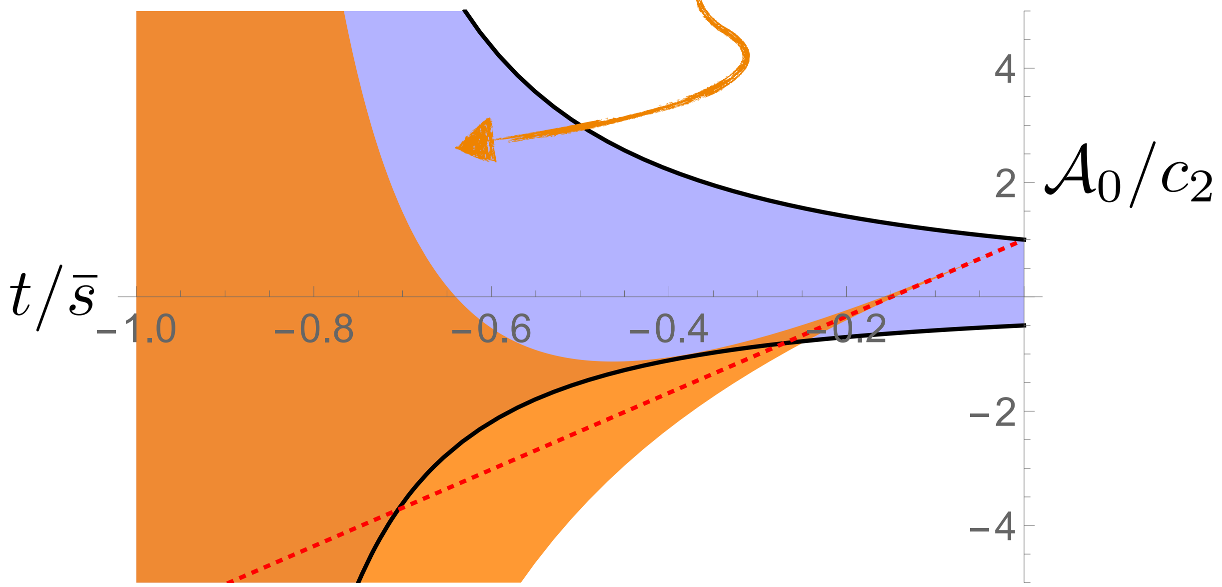

$$-\frac{3}{2} c_2 < c_{2,1} s < 5.17 c_2$$

Galileons have small cutoff!

*Proved for lightest particle in theory, not in general

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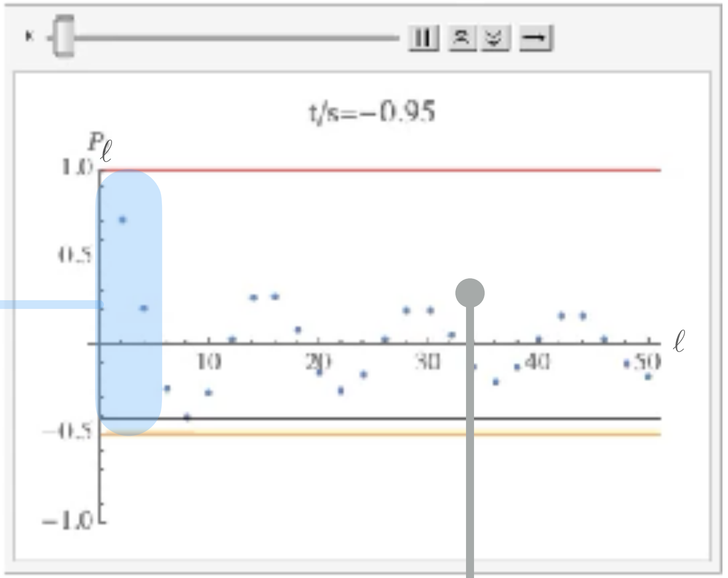
# Finite-t

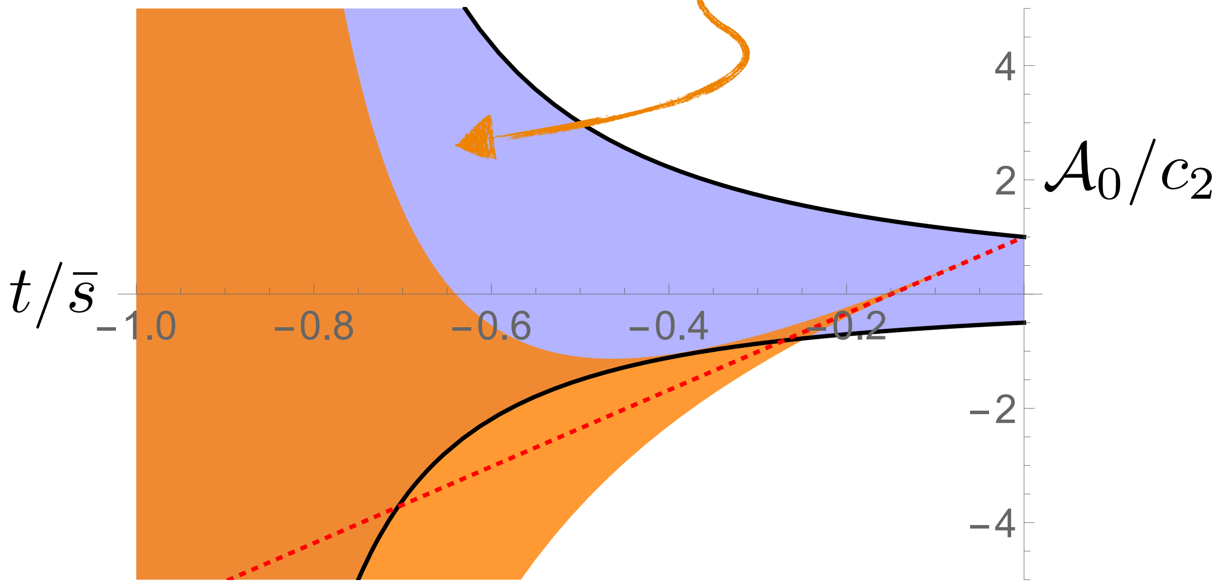
- Lower bound t-dependent
- Saturated by $\ell = 2,4$



$-1/2< \qquad <1$

$$\mathcal{A}_0(t) \equiv \int_{\cap_{\bar{s}_t}} \frac{ds}{\pi i}\frac{A(s,t)}{(s+\frac{t}{2})^3} = \frac{2}{\pi}\int_{\bar{s}}^{\infty} ds \sum_{\ell} \mathrm{Im} f_\ell(s)\frac{P_\ell(1+\frac{2t}{s})}{(s+\frac{t}{2})^3}$$

$$c_2 + c_{2,1}t \boxed{+\cdots}$$



$$-\frac{3}{2}c_2 < c_{2,1}s < 5.17c_2$$

Galileons have small cutoff!

(even smaller without particles of spin 2,4 at cutoff)

*Proved for lightest particle in theory, not in general

# Finite-t

Alternatively, close to t=0 ...

$$\mathcal{A}_0(t) \equiv \int_{\cap_{\bar{s}_t}} \frac{ds}{\pi i} \frac{A(s,t)}{(s+\frac{t}{2})^3} = \frac{2}{\pi} \underbrace{\int_{\bar{s}}^{\infty} ds \sum_{\ell}}_{\text{2D moment problem}} \mathrm{Im} f_\ell(s) \frac{\overbrace{P_\ell(1+\frac{2t}{s})}^{\text{at t=0 expandable in powers of } \ell^n}}{(s+\frac{t}{2})^3}$$

at tree-level: same result

Chiang, Huang, Rodina, Weng `21
Bellazzini,Riembau,FR' never to appear

...but beyond tree-level amplitude not analytic at t=0!

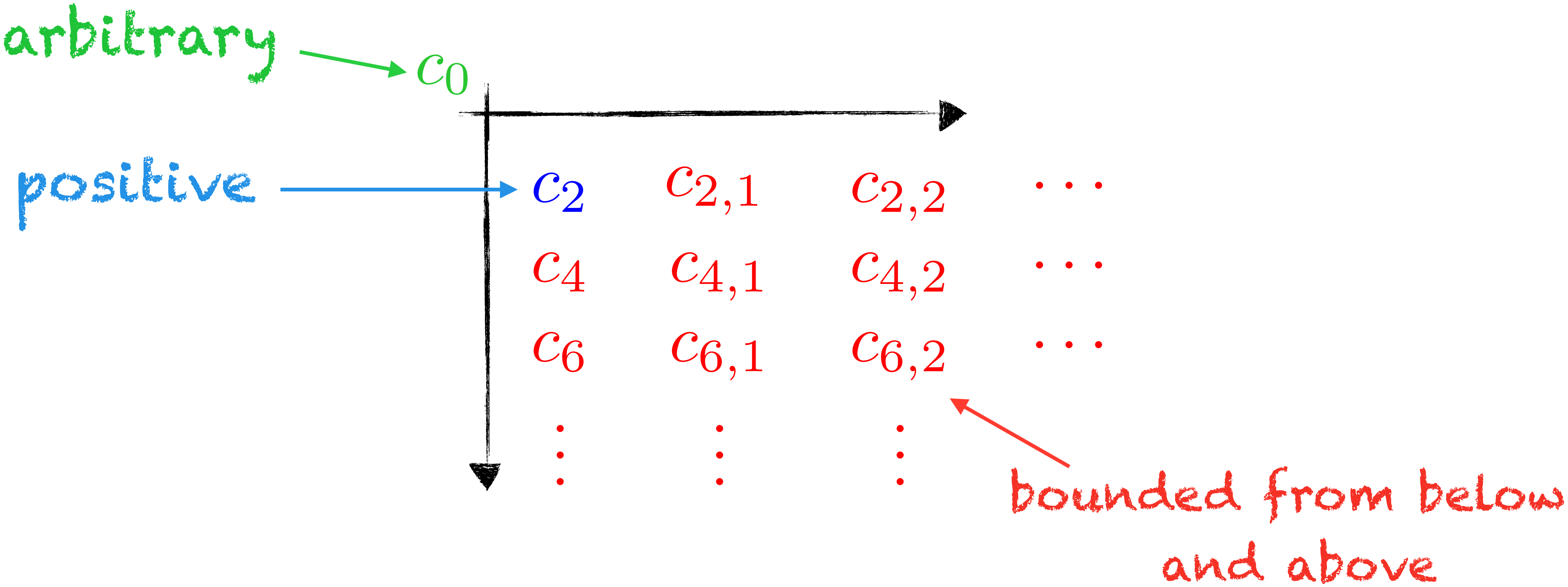
- bound with this approach disappears for massless particles

# EFTs

Tree-level, beyond forward:

$$A(s,t) = \sum_{p,q} c_{p,q} s^p t^q = \quad c_0 \ + \ c_2 s^2 \ + \ c_{2,1} s^2 t \ + \cdots$$

Of $\infty$ many coefficients, only 2 can lead the amplitude:



arbitrary → $c_0$

positive → $c_2$

$$\begin{array}{cccc} c_2 & c_{2,1} & c_{2,2} & \cdots \\ c_4 & c_{4,1} & c_{4,2} & \cdots \\ c_6 & c_{6,1} & c_{6,2} & \cdots \\ \vdots & \vdots & \vdots & \end{array}$$

bounded from below and above

# 3. Massive Gravity

Massive Gravity EFT $\xleftrightarrow{g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu}}$ spin-2, $m_g \cancel{=} 0$ → 2 +3 d.o.f.

$$S = \int d^4x \sqrt{g} \frac{M_{Pl}^2}{2} \left[ R \right]$$

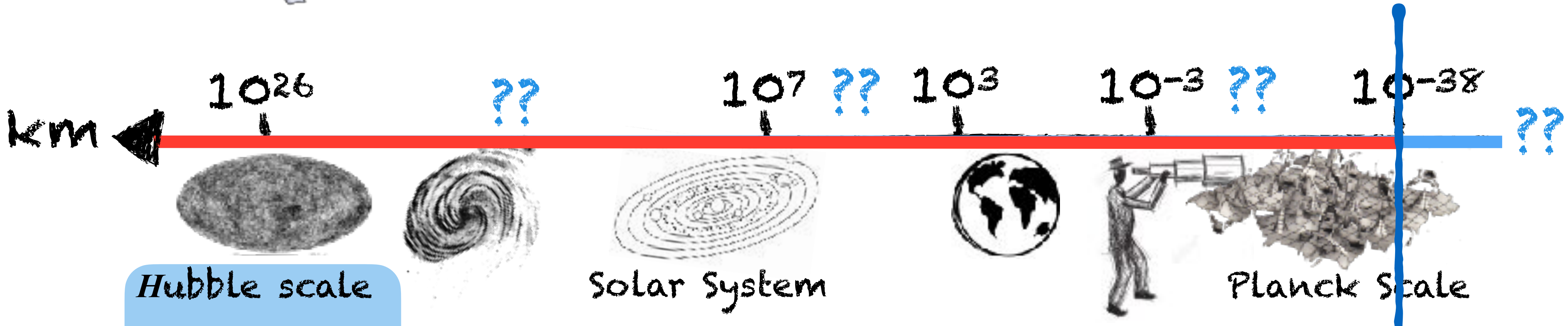
$$\boldsymbol{A(hh \to hh)} \sim \left( \boxed{\frac{m_g^2 s^2}{\Lambda_3^6}}_{c_2} + \boxed{\frac{s^2 t}{\Lambda_3^6}}_{c_{2,1}} \right)$$

$\Lambda_3 \equiv \sqrt[3]{M_{Pl} m_g^2}$

Tuning of coefficients

Fierz,Pauli'1930s, Arkani-Hamed,Georgi,Schwartz'02, deRham,Gabadatze,Tolley'10

$c_{2,1} s_{max} \lesssim 5 c_2$ ▶ $s_{max} \lesssim 5 m_g^2$



$m_g^{-1}$

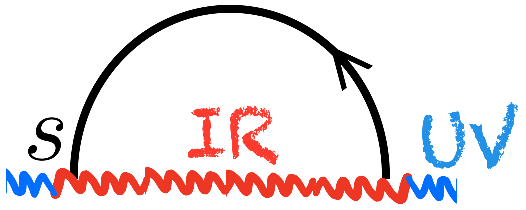
$\Lambda_3^{-1}$

$M^{-1} = M_{Pl}^{-1}$

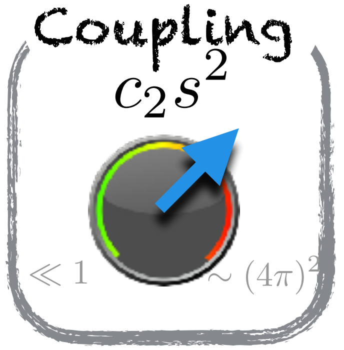
- Might Solve the c.c. problem
- Exp bound $m_g^{-1} \gtrsim 0.1 H$

▶ Massive Gravity not compatible with unitarity bounds

# 4. Strong Coupling within the EFT



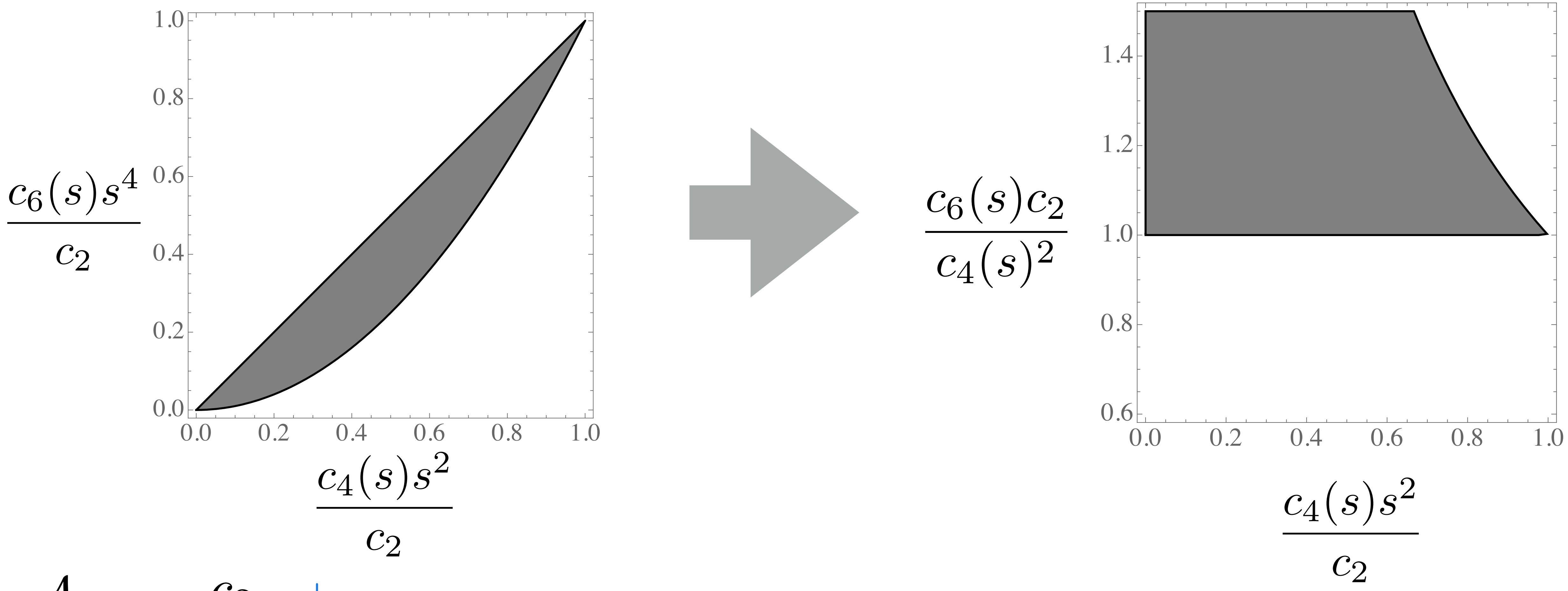
# Strongish Coupling



Arcs: suitable to access running

Do bounds apply for running coefficients $c_n(s)$?

$$A(s) = c_2 s^2 + s^4 \underbrace{[c_4 + \beta_4 \log(-is)]}_{c_4(s)} - i\pi s^5 \beta_5/2 + s^6 \underbrace{[c_6 + \beta_6 \log(-is) + \beta_6' \log^2(-is)]}_{c_6(s)} + \ldots$$

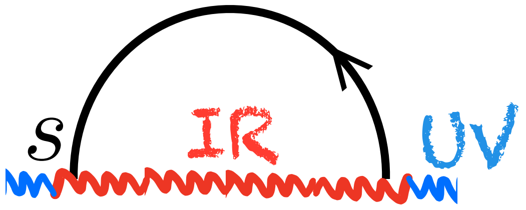


$$\mathcal{A}_0 = c_2 + \cdots$$

$$\mathcal{A}_1 = c_4(s) + \cdots$$

$$\mathcal{A}_2 = -\frac{\beta_4}{2s^2} + c_6(s) + \cdots$$

$\beta_4 = \frac{7c_2^2}{160\pi^2}$



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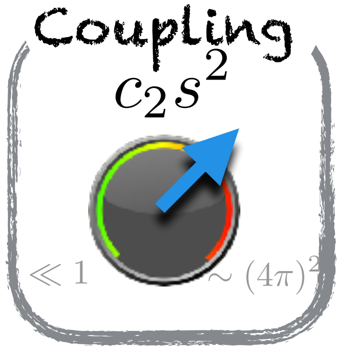

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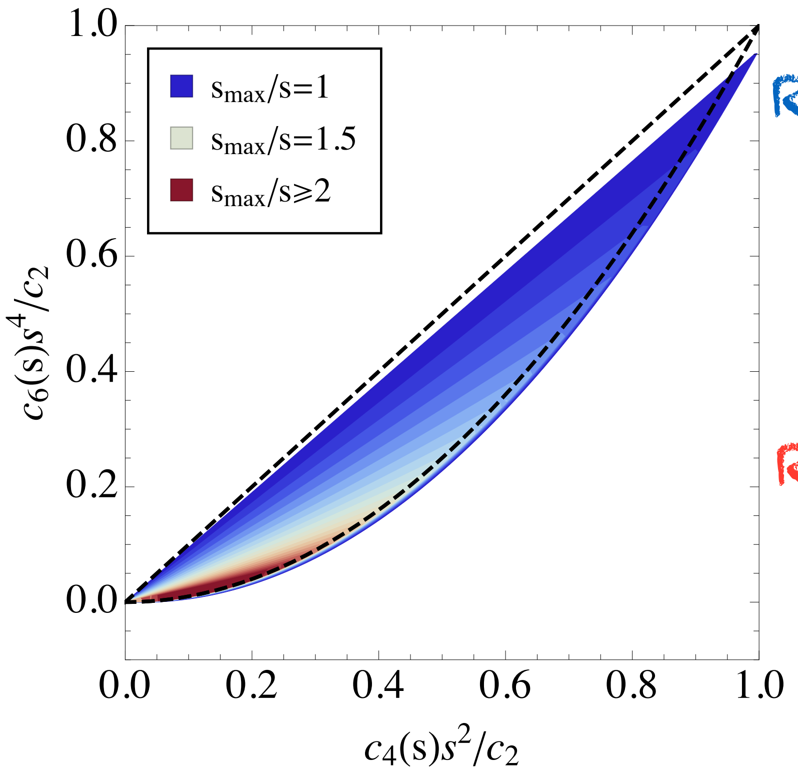
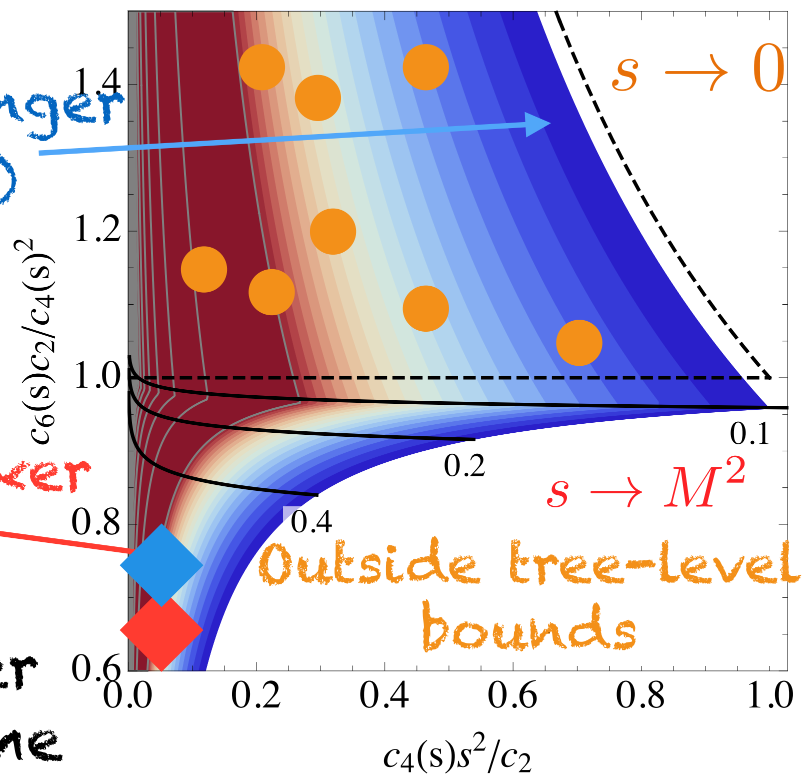


Real bounds little stronger (supersoftness still dead)

Real bounds much weaker e.g. $c_6(s) < 0$ ok

Some theories never have tree-level regime



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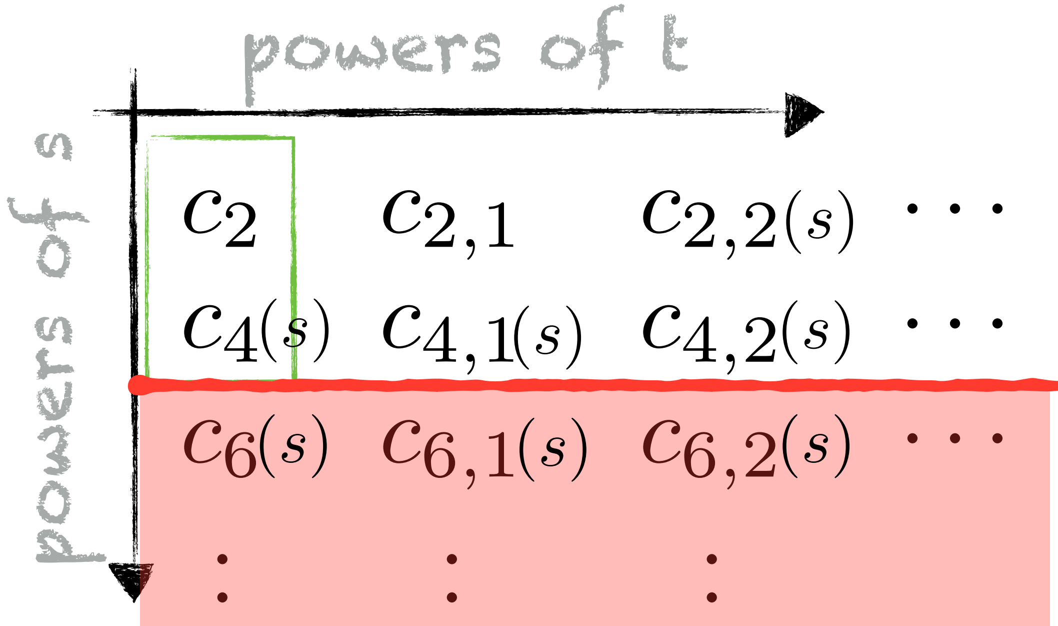
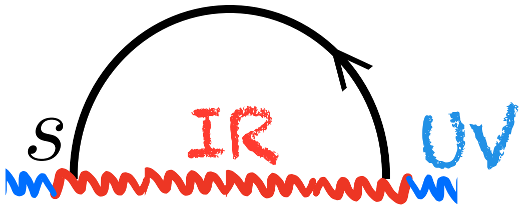
$$\mathcal{A}_2 = -\frac{\beta_4}{2s^2} + c_6(s) + \cdots$$

Assumed small

$\frac{c_4 c_6 s^8}{16\pi^2} \ll c_2$



| | | | |
|---|---|---|---|
| $c_2$ | $c_{2,1}$ | $c_{2,2}(s)$ | $\cdots$ |
| $c_4(s)$ | $c_{4,1}(s)$ | $c_{4,2}(s)$ | $\cdots$ |
| $c_6(s)$ | $c_{6,1}(s)$ | $c_{6,2}(s)$ | $\cdots$ |
| $\vdots$ | $\vdots$ | $\vdots$ | |



# Strongish Coupling

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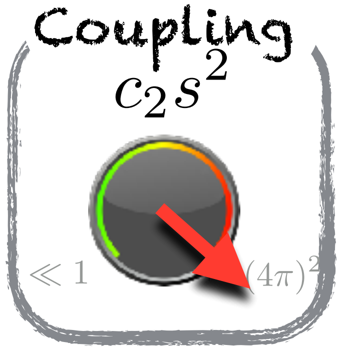
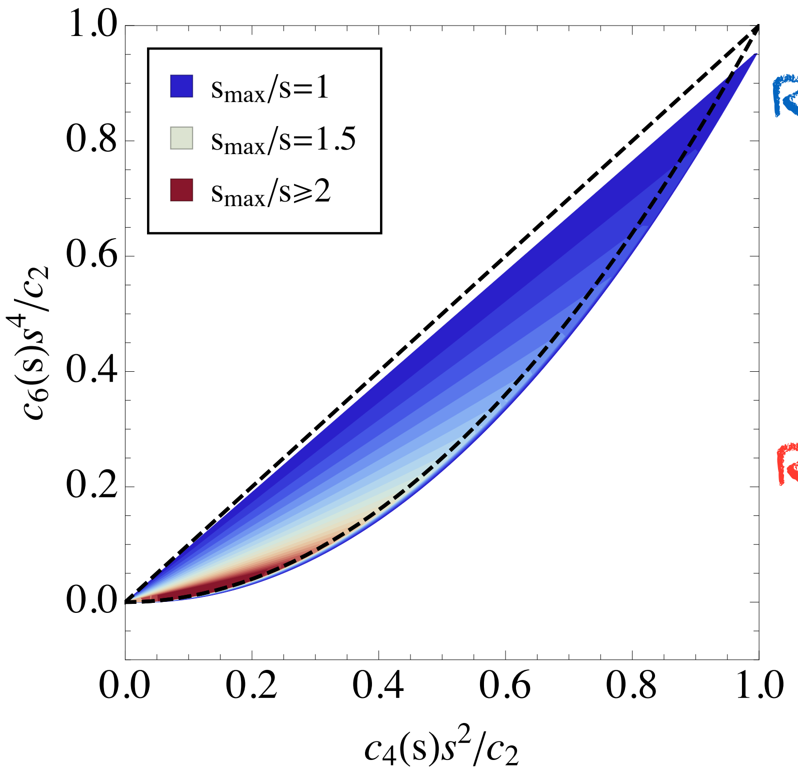


$$A(s) = c_2 s^2 + s^4 \left[c_4 + \beta_4 \log(-is)\right] - i\pi s^5 \beta_5/2 + s^6 \left[c_6 + \beta_6 \log(-is) + \beta_6' \log^2(-is)\right] + \ \ldots$$



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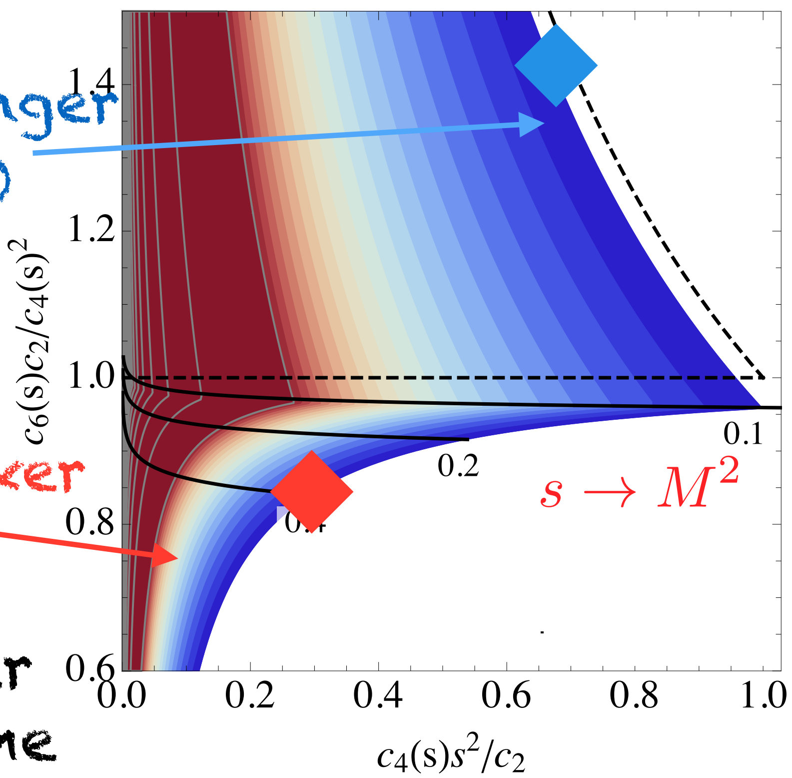


Some theories never have tree-level regime

e.g XPt

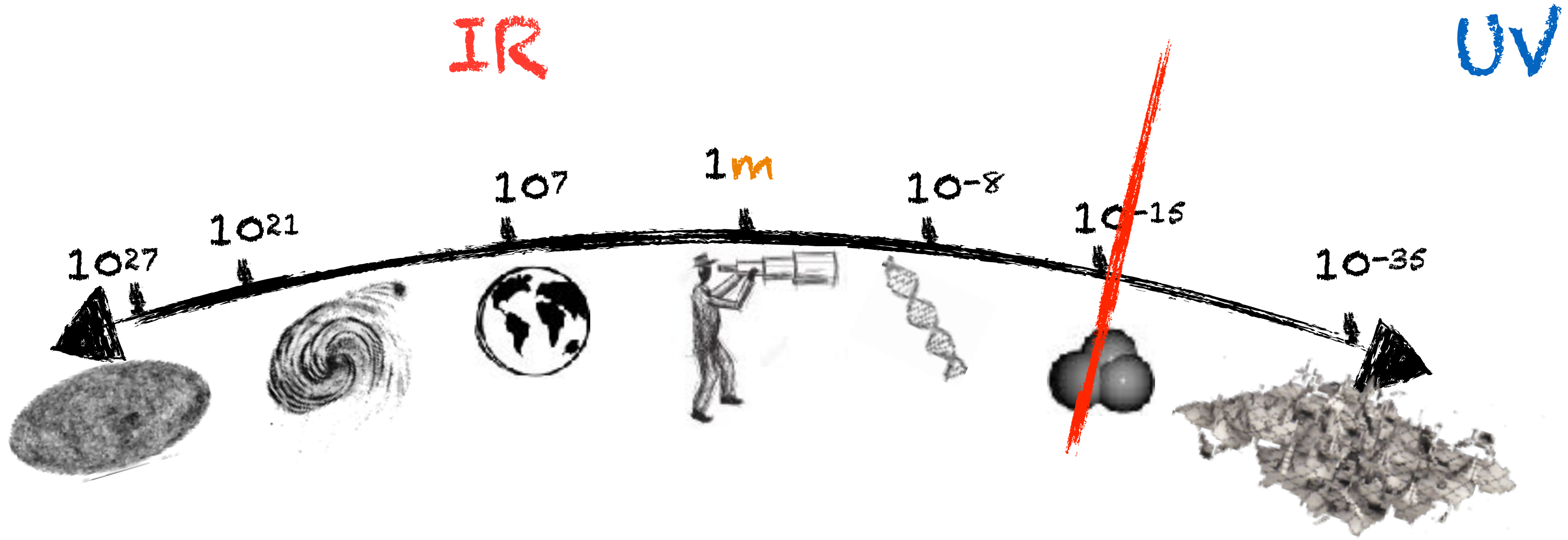
$$\mathcal{A}_0 = \quad c_2 \quad + \cdots$$

$$\mathcal{A}_1 = \quad c_4(s) + \cdots$$

$$\mathcal{A}_2 = -\frac{\beta_4}{2s^2} + c_6(s) + \cdots$$

$$\frac{c_4 c_6 s^8}{16\pi^2} \sim c_2$$

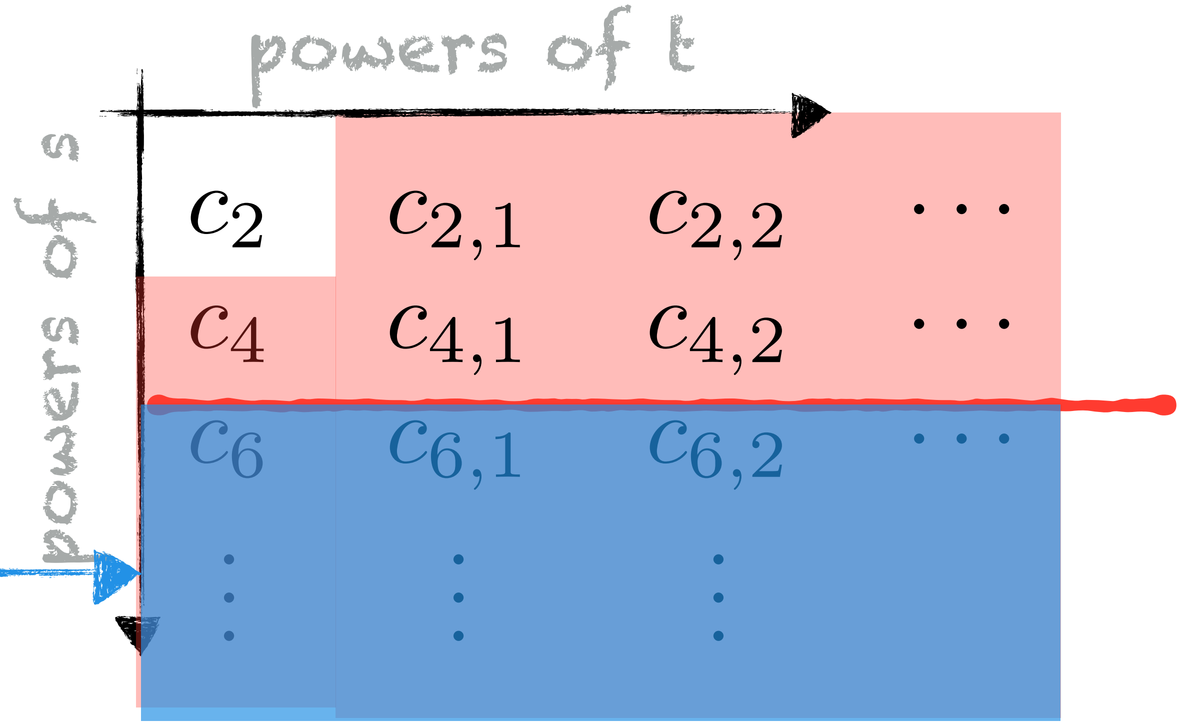
# Summary



**Constrained EFTs** ← **moments** ← Causality, Unitarity, Lorentz invariance, Locality

- Only 3(2) coefficients can dominate
- ~~Supersoftness~~
- $m_{HS}>1/L_{HS}$
- massive gravity X

IR running important →



| | | | |
|---|---|---|---|
| $c_2$ | $c_{2,1}$ | $c_{2,2}$ | $\cdots$ |
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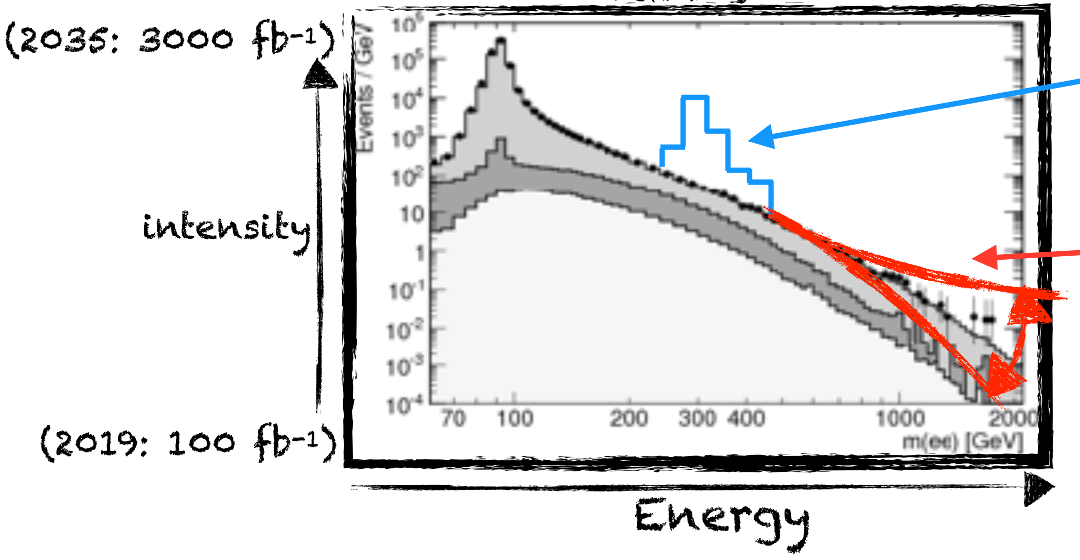
# SM Precision tests

LHC



New Physics searches (so far)

New Physics searches (future) : EFTs

Operator Dimension

Standard Model - 4

Experimental bounds

5

6

Experiments target leading effects
(generically dim-6)

7

Overlap?

Froissart Bound

8

BSM

9

10

11

12

violate B,L

...

2-sided bounds
(can't be the leading effect)

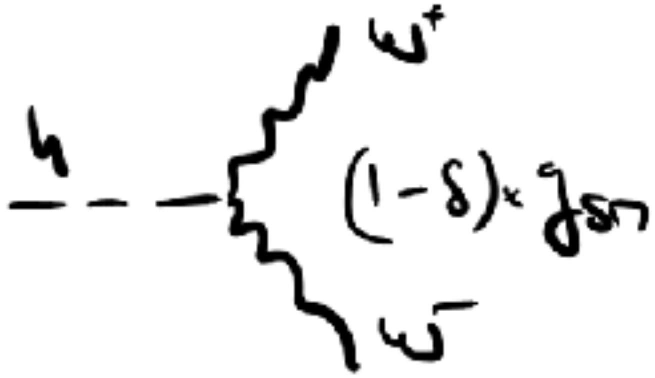
Theory Positivity Bounds

# SM Precision tests

**UV Assumptions**

Low,Rattazzi,Vichi'12
Falkowski,Rychkow,Urbano'12
Remmen,Rodd'20
Zhang,Zhou'20

Stronger UV convergence → Bounds/sum rules on dimension-6



$\delta<0$ → Isospin-2 light resonance

Standard Model - 4

Experimental bounds

5

6 Experiments target leading effects

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Froissart Bound

BSM

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...

violate B,L

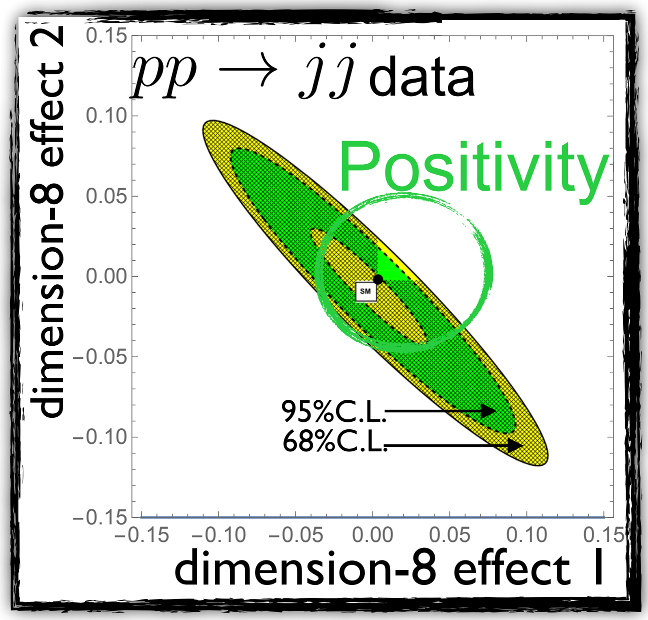
2-sided bounds
(cant be the leading effect)

Theory Positivity Bounds

# SM Precision tests

**IR Features** Non-linear IR symmetries: suppress dim-6

→experiments target dim-8



Quarks as PseudoGoldstini:

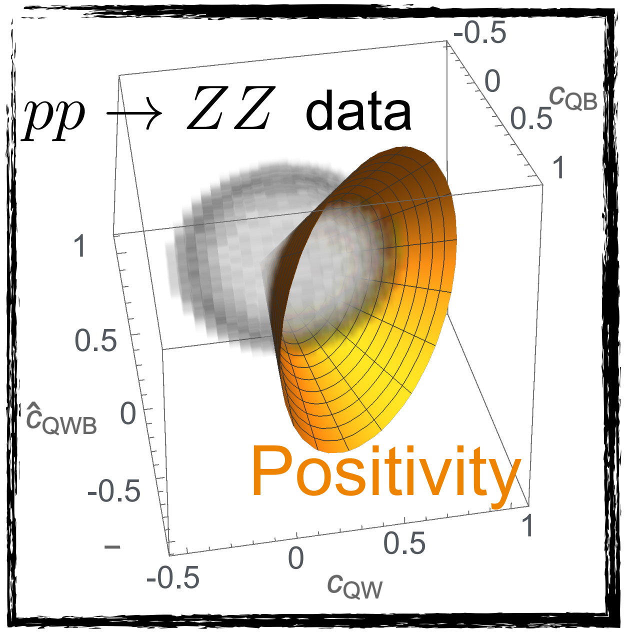
Bellazzini,FR,Sgarlata,Serra'17

$$\mathcal{L}_{int} \sim (\bar{\psi}\gamma^{\mu}\partial^{\nu}\psi)^2$$

Pseudo-Goldstone Higgs
(flat coset):

Bellazzini,FR'18
Liu,Pomarol,Rattazzi,FR'16

$$\mathcal{L}_{int} \sim D_{\mu}H^{\dagger}D_{\nu}HW^{\mu\tau}W_{\tau}^{\nu}$$



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Bellazzini,FR'18

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...

violate B,L

2-sided bounds
(cant be the leading effect)

Theory Positivity Bounds

# SM Precision tests

**IR Features** processes without dim-6

→experiments target dim-8

- $e^+e^-/\bar{q}q \to ZZ$ no dimension-6! Bellazzini,FR'18 Gu,Wang,Zhang'20
- Helicity selection rules:

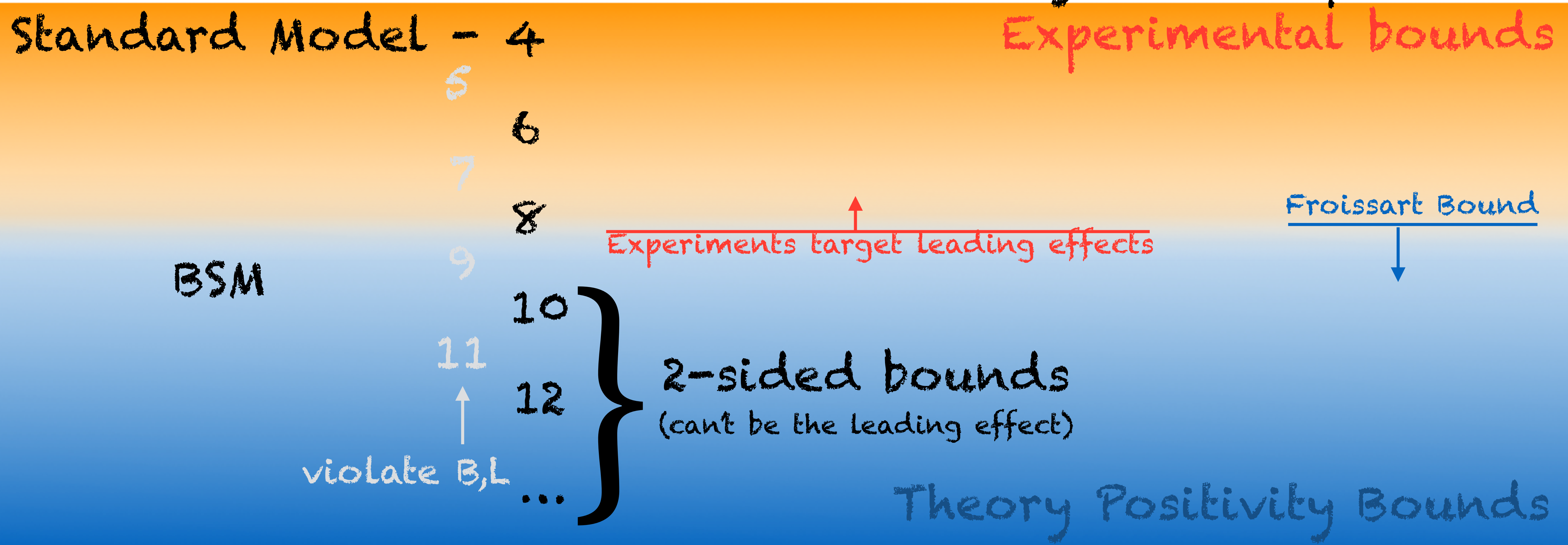
Azatov,Contino,Machado,FR'16

| $A_4$ | $h(SM)$ | $h(\mathcal{O}^6)$ | $h(\mathcal{O}^8)$ |
|---|---|---|---|
| $VVVV$ | 0 | 4,2 | 4,0 |
| $VV\phi\phi$ | 0 | 2 | 2 |
| $VV\psi\psi$ | 0 | 2 | 2 |
| $V\psi\psi\phi$ | 0 | 2 | 2 |

← total helicity

← dim-6 dont interfere with SM, dim-8 do

- Positivity guide experiment



Standard Model - 4

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Experiments target leading effects

Froissart Bound

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...

violate B,L

2-sided bounds

(cant be the leading effect)

Theory Positivity Bounds

# IR Effects alter Bounds

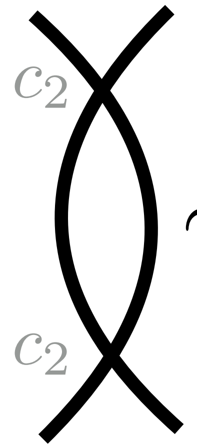
Change relation Wilson coeff. ⟷ arcs (on which bounds apply)

**Running**

$$\sim \frac{c_2^2 s^4}{16\pi^2}\log\frac{s}{\mu}$$

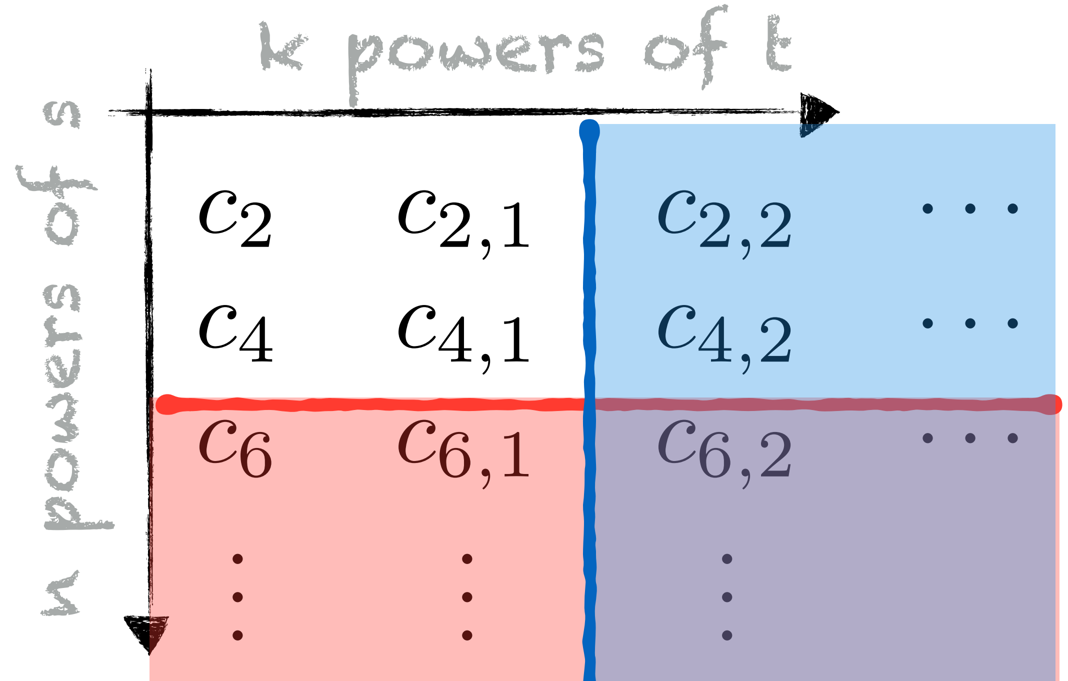
$$\delta\mathcal{A}_n \sim \frac{c_2^2}{16\pi^2 s^{2n-2}}$$

**Collinear Divergences**



$$\sim \frac{c_2^2 s^2 t^2}{16\pi^2}\log\frac{s}{t}$$

$$\delta\partial_t^k \mathcal{A}_n \sim \left(\frac{c_2^2}{16\pi^2}\log\frac{s}{m^2}\right)^{k-2}$$



| | | | |
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| $c_6$ | $c_{6,1}$ | $c_{6,2}$ | $\cdots$ |
| $\vdots$ | $\vdots$ | $\vdots$ | |

# Polygons vs Polynomials

Arkani-Hamed,Huang$^2$,2020

Bellazzini, Elias-Miro, Rattazzi, Riembau, FR'20

Positivity from geometry

Different "functional"approach

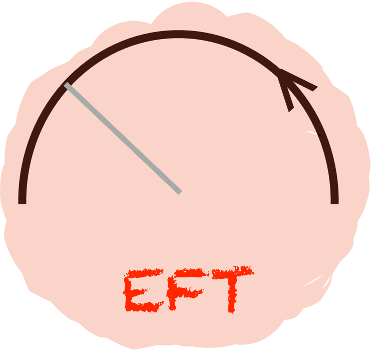
Forward Bounds for infinite arcs same

$$1 - x/2 - x^2/8 + \cdots = q(x) = \sqrt{1-x}$$

- Focus on "Optimal" bounds for finite many arcs, (both forward and at finite-t)
- Two-sided bounds

Residues

Arcs



- Suitable for EFT cutoff estimate
- Ideal for running

# 3. Finite-t supersoftness and Galileons

**Beyond forward:**

Galileon Nicolois,Rattazzi,Trincherini'08

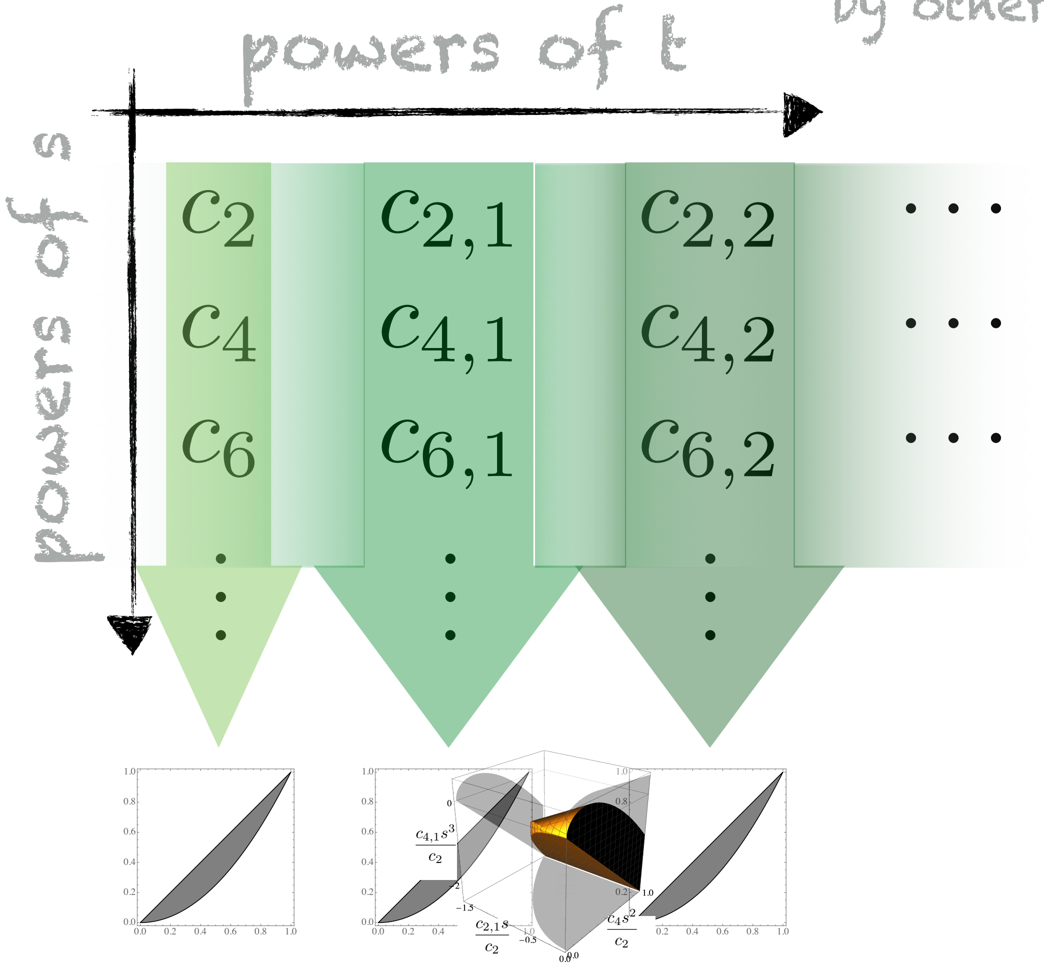
$$A_{2\to2}(s,t) = c_0 + c_2 s^2 + c_{2,1}s^2 t + c_4 s^4 + \cdots + c_{n,m}s^n t^m$$

**At tree level:**

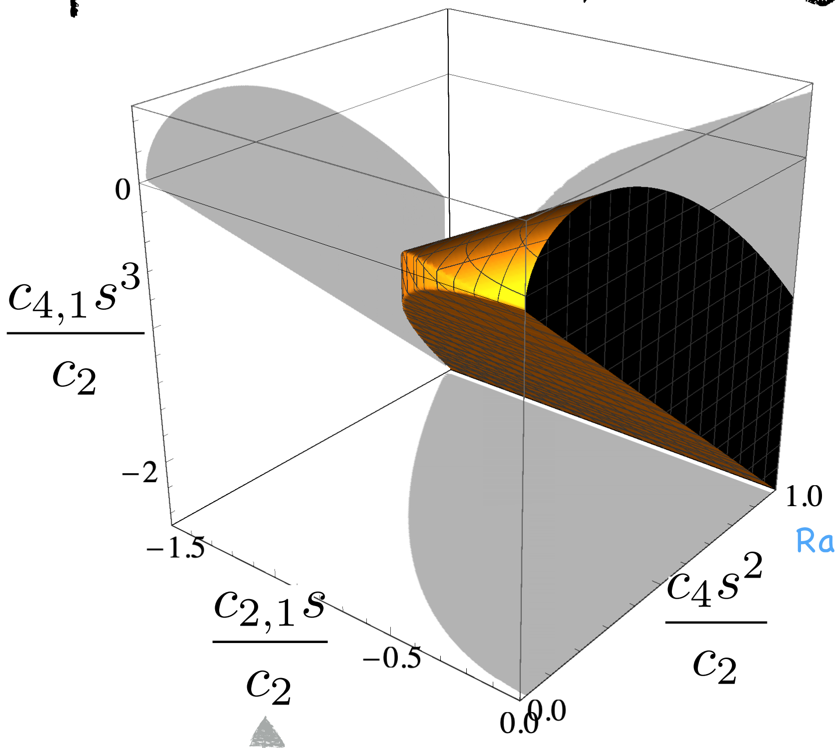
- $$c_{p,q} = \partial_t^q A_n(s,t) = \partial_t^q \frac{2}{\pi}\int_s^\infty ds' \frac{\mathrm{Im}A(s',t)}{(\hat{s}'+\frac{t}{2})^{2n+3}}$$

$\partial_t^q \mathrm{Im}A|_{t=0} > 0$ Martin'65

Negative, but limited by other moments



**Optimal bounds for single t-derivative:**



Bellazzini,Elias-Miro, Rattazzi,Riembau,FR'20

Can be slightly negative $c_{2,1} > -\frac{3}{2}\sqrt{c_4 c_2}$

also deRham,Melville,Tolley,Zhou'17 : $> -3c_2/2s$

# 3. Finite-t supersoftness and Galileons

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$$A_{2\to 2}(s,t) = c_0 + c_2 s^2 + c_{2,1} s^2 t + c_4 s^4 + \cdots + c_{n,m} s^n t^m$$

At tree level:

$$\partial_t^q \mathrm{Im} A|_{t=0} = \sum_{\ell=0}^{\infty} \frac{(\ell+q)!}{(\ell-q)!q!} \mathrm{Im} f_\ell(s)$$

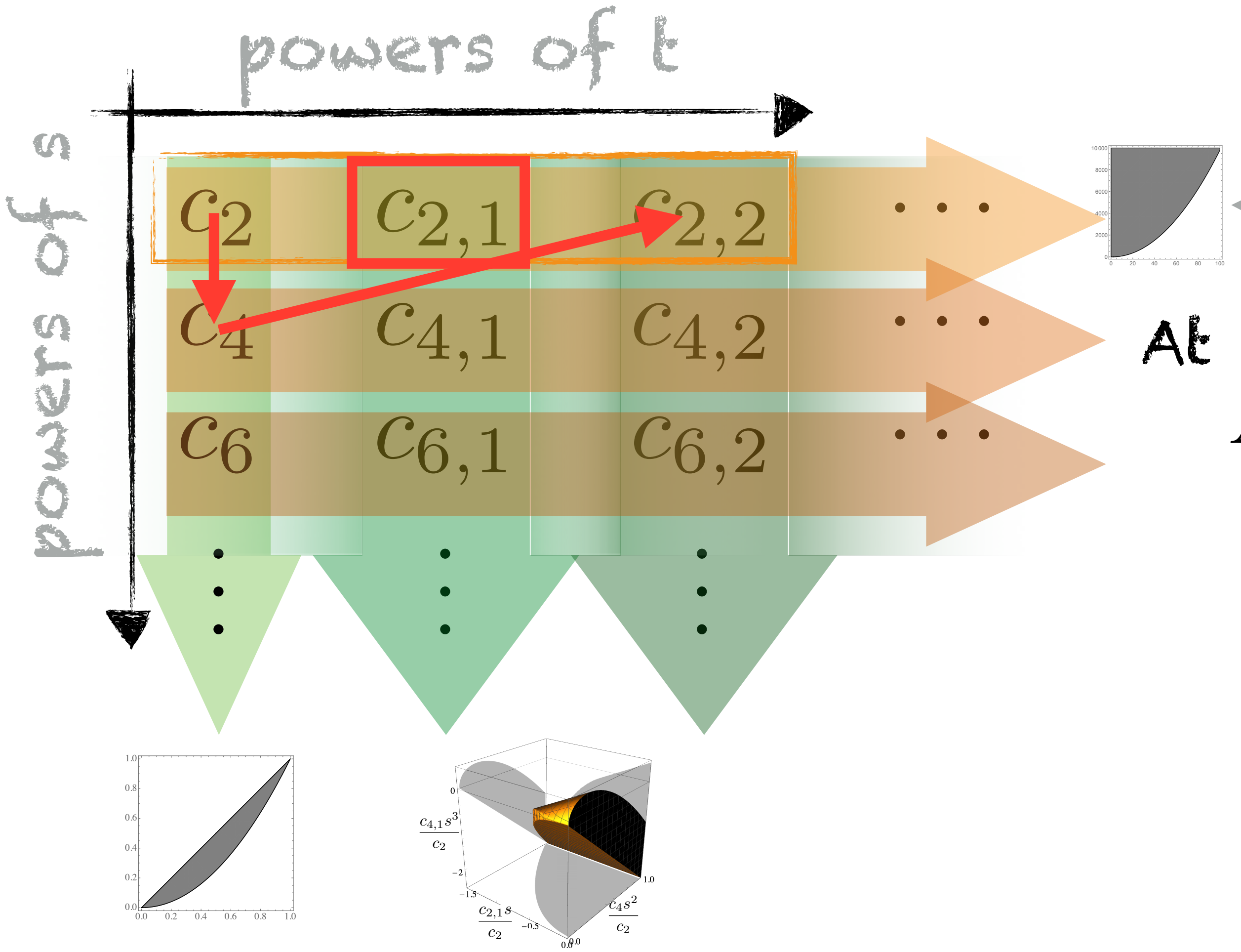
Arkani-Hamed,Huang²,2020

- $$c_{p,q} = \partial_t^q A_n(s,t) = \partial_t^q \frac{2}{\pi}\int_s^\infty ds' \frac{\mathrm{Im}A(s',t)}{(\hat{s}'+\frac{t}{2})^{2n+3}}$$

$$\sim \int_0^{\infty} d\mu(l) l^{2q}$$

Moments in l

Bellazzini et al, to appear



At tree-level* x-ing symmetry implies

$$A(s,t) = \cdots + g_4\underbrace{(s^2+t^2+u^2)^2}_{c_{2,2}=2c_4} + \cdots$$

- $$\boxed{-\frac{3}{2}c_2 < c_{2,1}s < 8c_2}$$

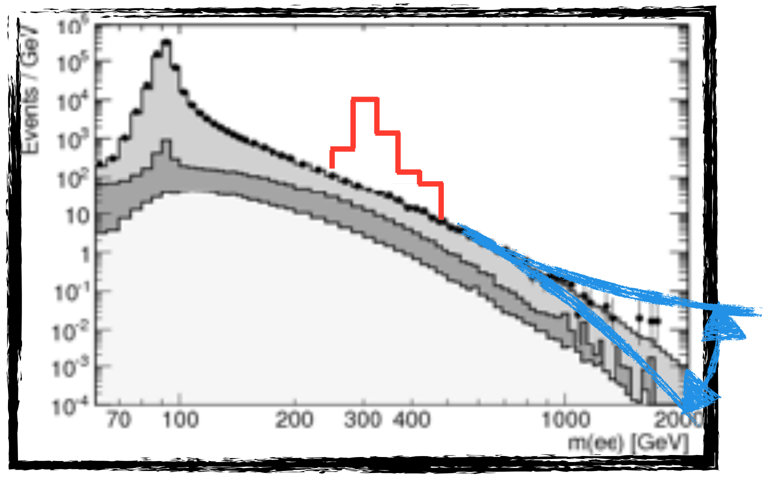
Tolley,Wang,Zhou'20
Caron-Huot,vanDuong'20

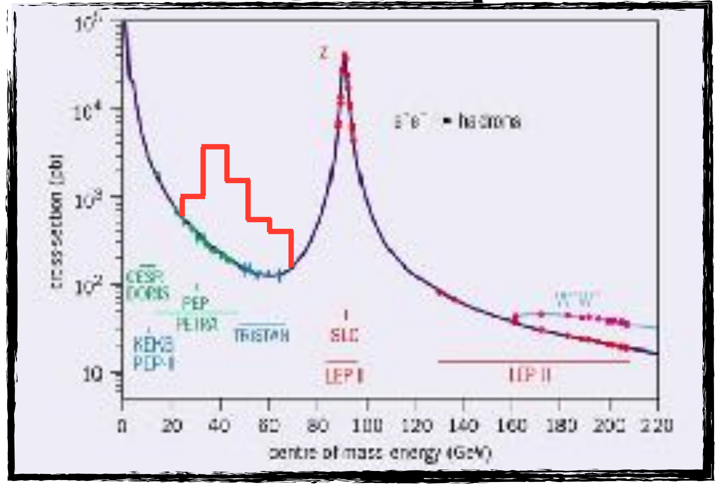
**Galileons tightly bounded!**

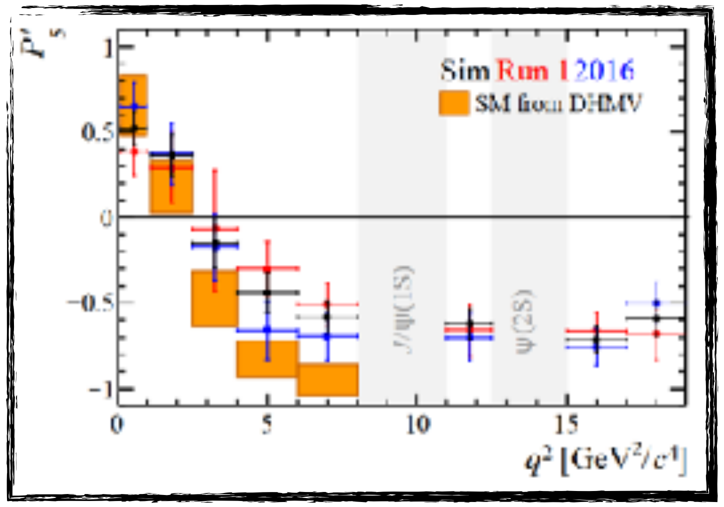
* See loop effects later

# Precision Measurements

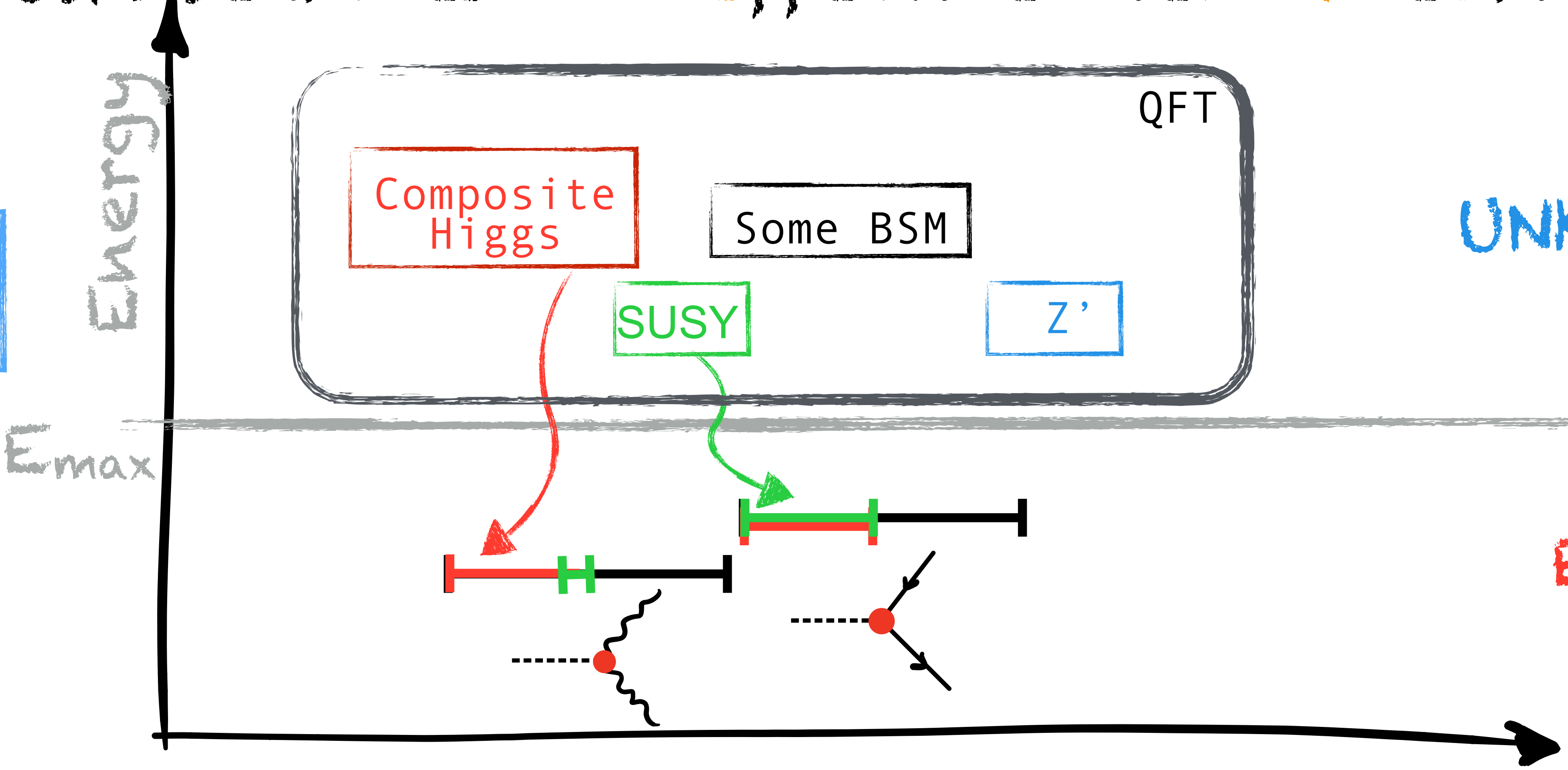
At the edge of experimental capabilities:







BSM Searches ↔ **E**ffective **F**ield **T**heories



QFT

Composite Higgs

Some BSM

SUSY

Z'

UNKnown UV

EFT

What can be learned?

What can be measured?

Are there generic predictions common to all BSM?

# Precision Measurements

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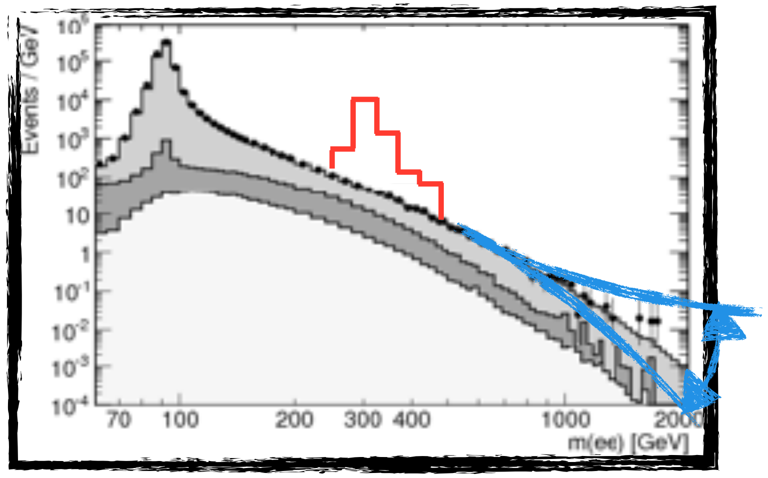
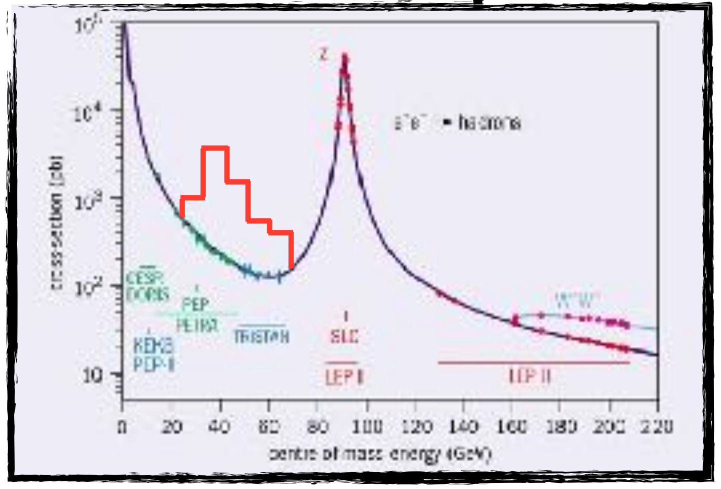
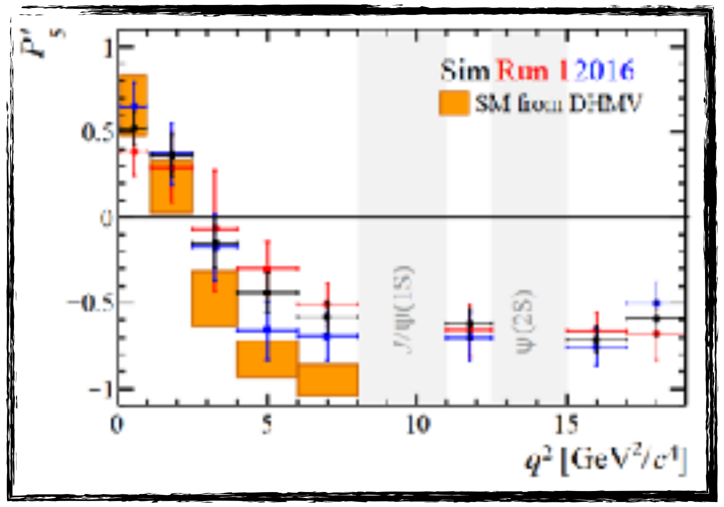
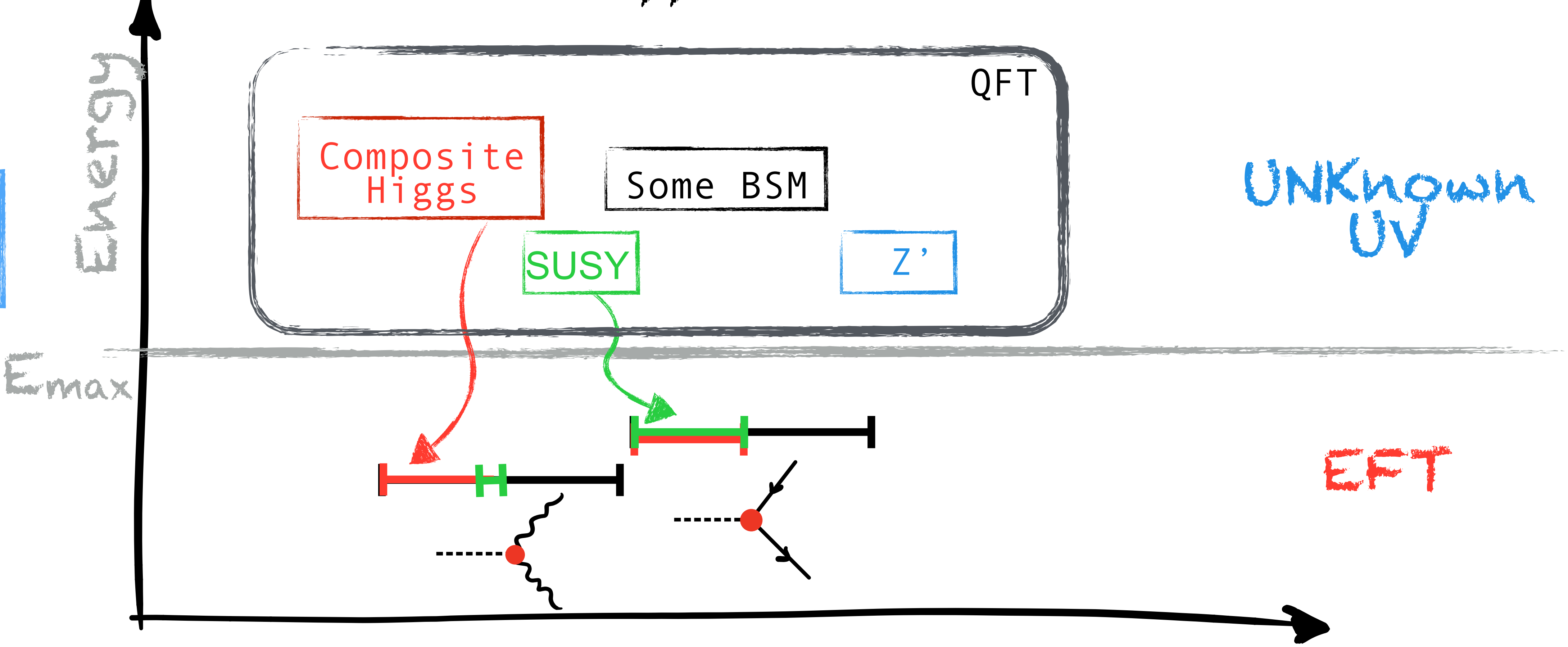






BSM Searches ↔ Effective Field Theories

What can be learned?



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Arkani-Hamed,Huang²,2020
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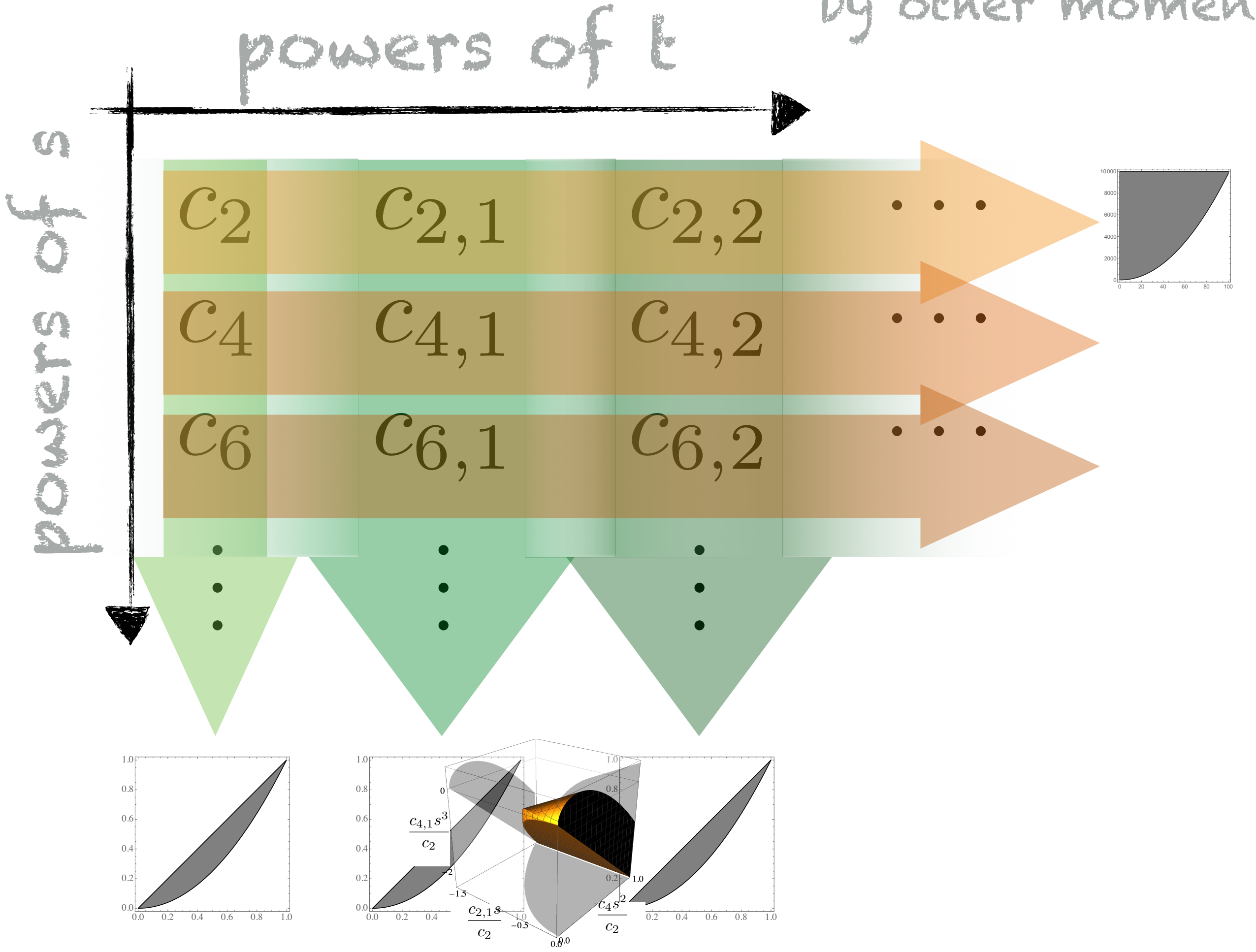
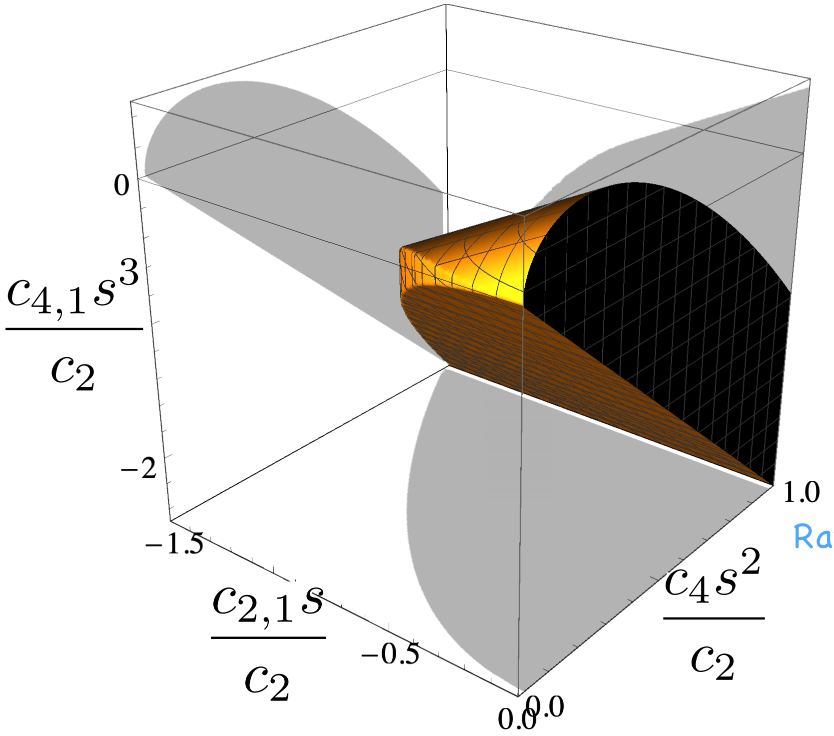
Tolley,Wang,Zhou'20
Caron-Huot,vanDuong'20





Bellazzini,Elias-Miro,
Rattazzi,Riembau,FR'20