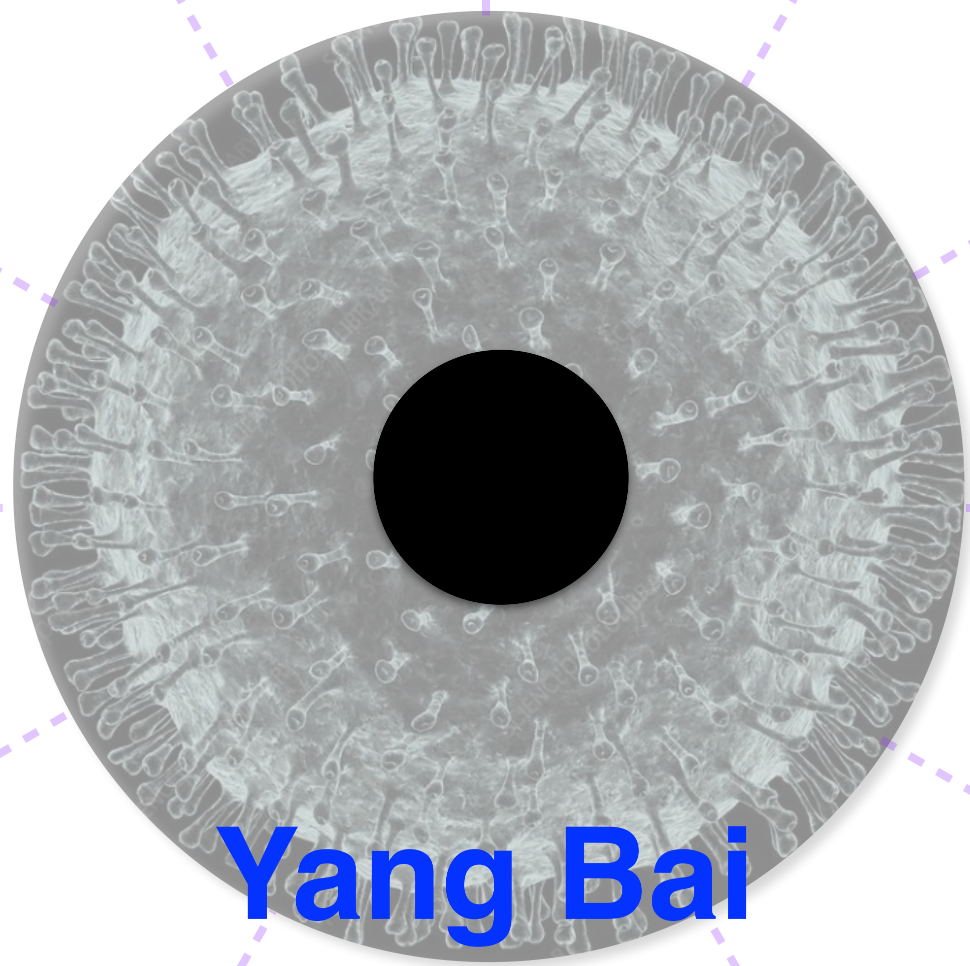
Magnetic Black Holes with Electroweak-Symmetric Coronas



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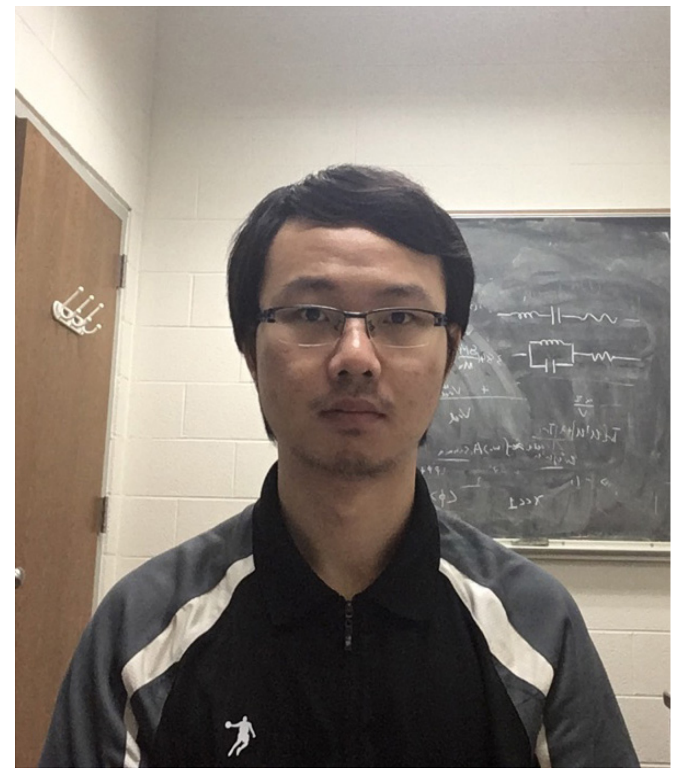

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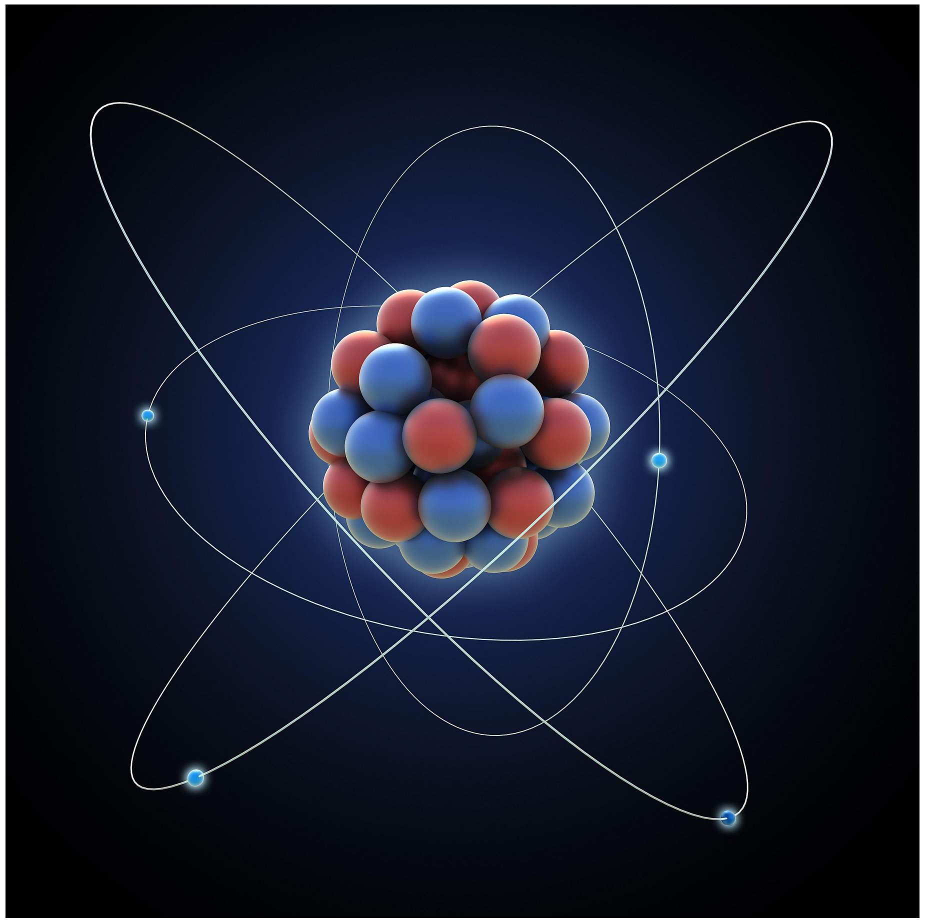
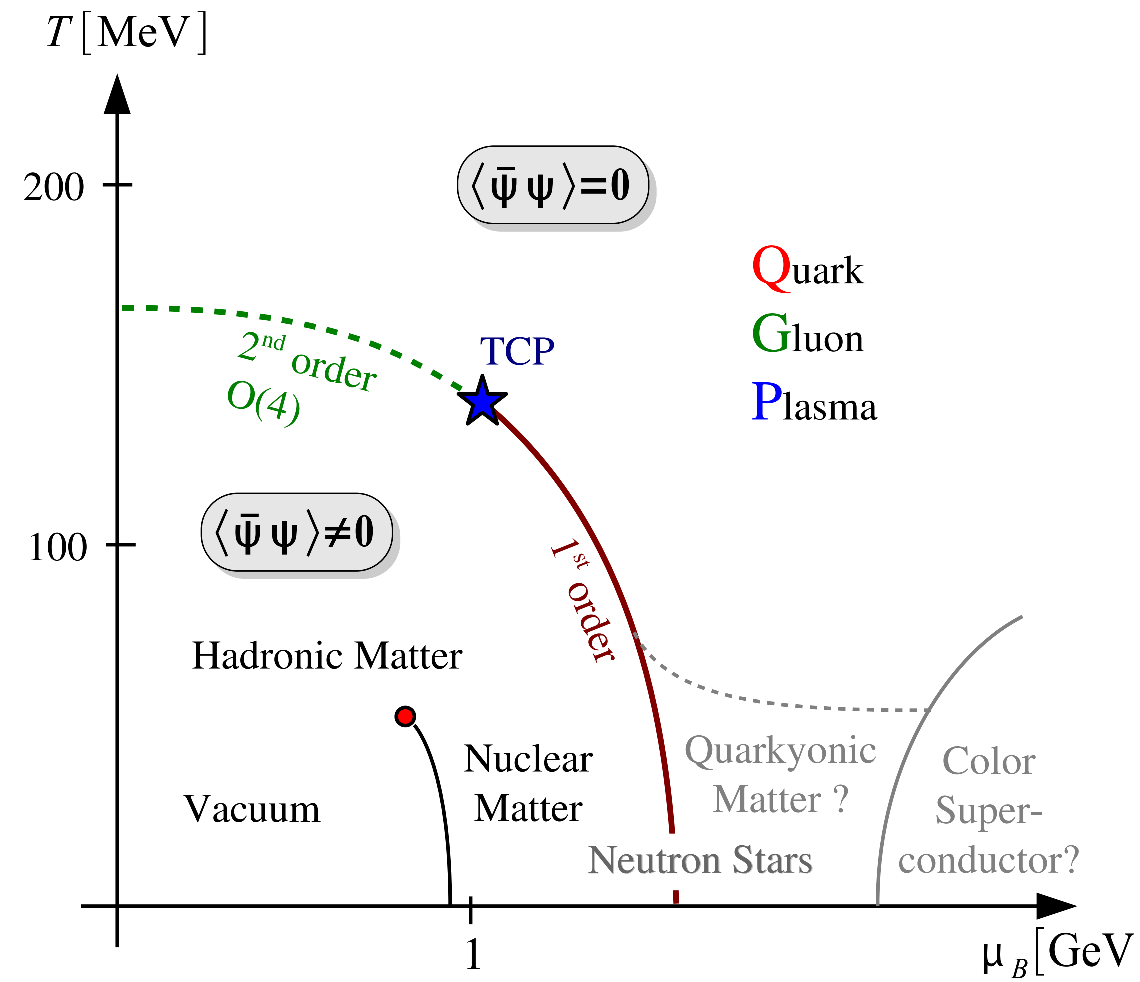
arxiv: 2007.03703, 2012.15430, 2103.06286

Motivation



ANDRZEJ WOJCICKI/Getty Images

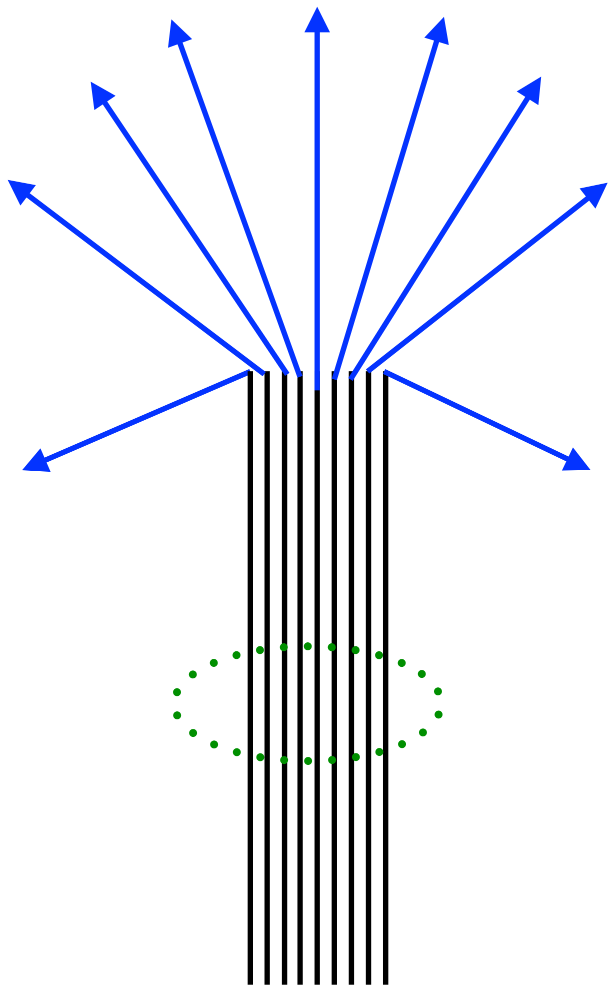


Forcrand, et. al., arXiv:1503.08140

- **Any interesting (stable) states in the electroweak sector?**

Dirac Monopole

- In E&M, we have learned that there is no monopole
- Dirac in 1931 proposed the possible existence of monopole



$$\mathbf{B} = Q\,\frac{h\,\hat{\mathbf{r}}}{4\pi r^2}$$

$$Q = 1$$

$$h = \frac{2\pi}{e} \approx 68.5\,e$$

t 'Hooft-Polyakov Monopole

- **Based on spontaneously broken gauge theory: SU(2)/U(1)**

$$\mathscr{L} = \frac{1}{2}(D_\mu \Phi)^2 - \frac{1}{4}\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}) - \frac{\lambda}{4}\left(|\Phi|^2 - f^2\right)^2$$

triplet

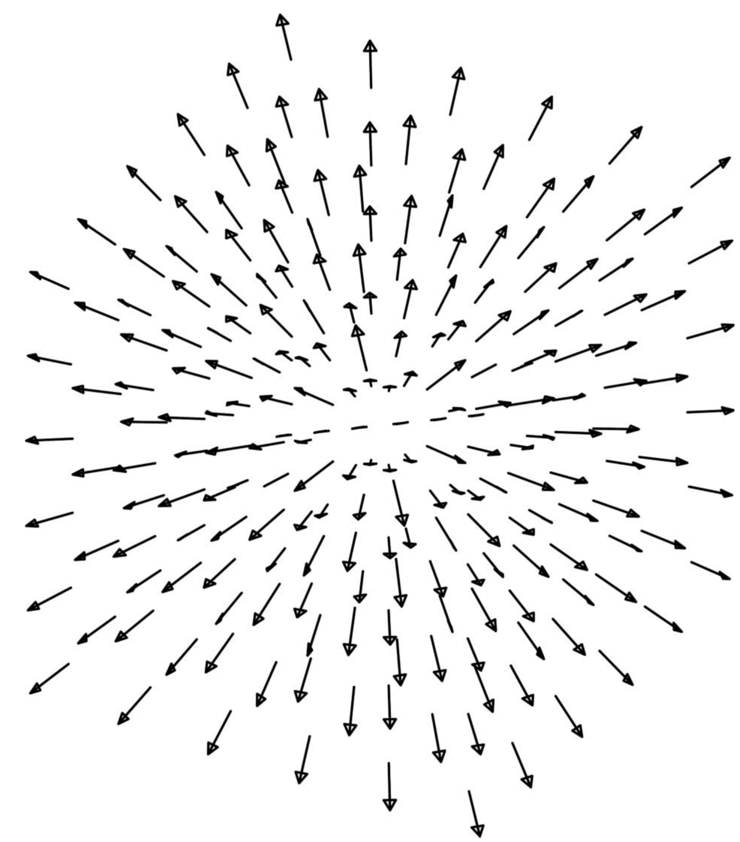
$$D_\mu \Phi^a = \partial_\mu \Phi^a + g\,\epsilon^{abc} A^b_\mu \Phi^c \qquad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\,\epsilon^{abc} A^b_\mu A^c_\nu$$

- **In the "hedgehog gauge" with $A^a_0 = 0$ (spherically symmetric)**

$$\Phi^a = \hat{r}^a f \phi(r)$$

$$A^a_i = \frac{1}{g}\,\epsilon^{aij}\,\hat{r}^j \left(\frac{1-u(r)}{r}\right)$$

$Q = 2$



t ‘Hooft-Polyakov Monopole

- **Total energy or mass (finite)**

$$M_{\mathscr{M}} = \int 4\,\pi\, r^2 \left(\frac{1}{2}\, B_i^a\, B_i^a + \frac{1}{2}\,(D_i\Phi^a)(D_i\Phi^a) + V(\Phi) \right)$$

$$= \frac{4\pi f}{g} \int d\bar{r}\bar{r}^2 \left(\frac{\bar{r}^2\, \phi'^2 + 2\, u^2 \phi^2}{2\, \bar{r}^2} + \frac{(1-u^2)^2 + 2\,\bar{r}^2\, u'^2}{2\,\bar{r}^4} + \frac{\lambda}{4g^2}(\phi^2 - 1)^2 \right)$$

- **Classical equations of motion ($\bar{r} \equiv g\, f\, r = m_W\, r$)**

$$\frac{d^2\phi}{d\bar{r}^2} + \frac{2}{\bar{r}}\frac{d\phi}{d\bar{r}} = \frac{2\, u^2\, \phi}{\bar{r}^2} + \frac{\lambda}{g^2}\phi\,(\phi^2 - 1)$$

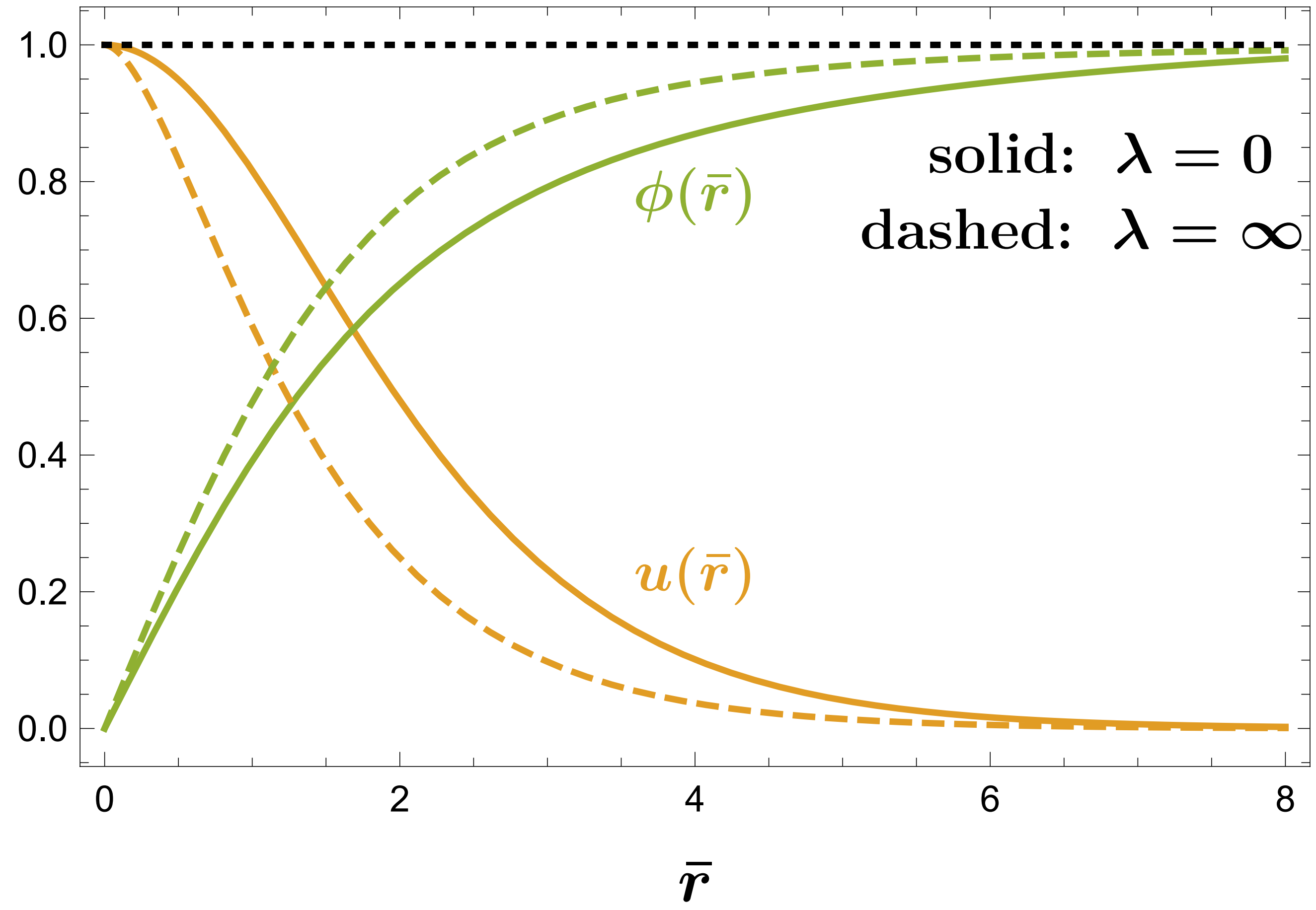
$$\frac{d^2u}{d\bar{r}^2} = \frac{u\,(u^2 - 1)}{\bar{r}^2} + u\,\phi^2$$

- **Boundary conditions**

$$\phi(0) = 0\,, \quad \phi(\infty) = 1\,, \quad u(0) = 1\,, \quad u(\infty) = 0$$

t 'Hooft-Polyakov Monopole

$$M_{\mathscr{M}} \equiv \frac{4\pi f}{g} Y(\lambda/g^2) \qquad Y(0) = 1 \qquad Y(\infty) \approx 1.787$$



- **Topological reason:** $\pi_2[G/U(1)] = \pi_1[U(1)] = \mathbb{Z}$
- **GUT monopole:** $SU(5) \to SU(3) \times SU(2) \times U(1)$

$$M_{\mathscr{M}}^{\text{GUT}} \sim 10^{17}\,\text{GeV}$$

Monopole in the Standard Model

- **In the SM:** $SU(2)_W \times U(1)_Y \to U(1)_{\rm EM}$ **with a Higgs doublet**
- **Topological reason:** $\pi_2[SU(2)_W \times U(1)_Y / U(1)_{\rm EW}] = 0$ **, no finite-energy EW monopole**
- **In more detail and again making a spherical configuration**

$$H = \frac{v}{\sqrt{2}}\,\phi(r)\,\xi\,, \qquad \xi = i \begin{pmatrix} \sin(\frac{\theta}{2})\, e^{-i\phi} \\ -\cos(\frac{\theta}{2}) \end{pmatrix} \qquad H^\dagger\,\vec{\sigma}\,H = -\frac{v^2}{2}\,\phi(r)^2\,\hat{r}$$

↑ as the triplet case

$$A_i^a = \frac{1}{g}\,\epsilon^{aij}\,\hat{r}^j \left(\frac{1-u(r)}{r}\right) \longleftarrow SU(2)_W$$

$$B_i = -\frac{1}{g_Y}(1-\cos\theta)\,\partial_i\phi \longleftarrow U(1)_Y$$

Nambu, NPB130 (1977) 505

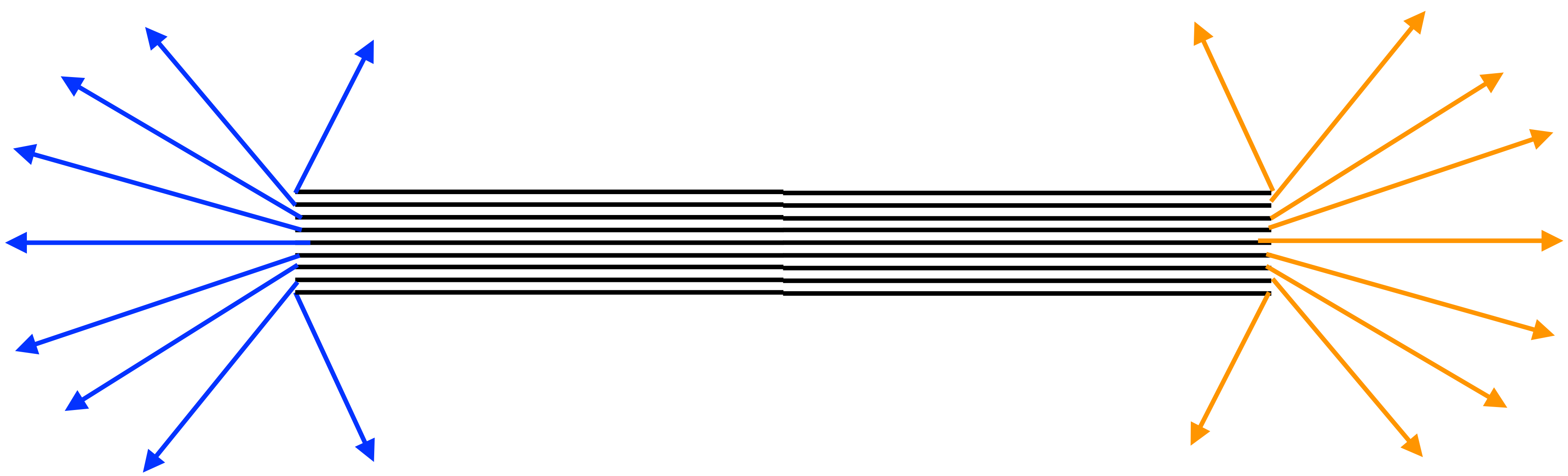
Cho, Maison, hep-th/9601028

Monopole in the Standard Model

$$S = -4\pi \int dt\, dr\, r^2\, (K + U)$$

$$K = \frac{(u')^2}{g^2 r^2} + \frac{1}{2} v^2 (\phi')^2 \qquad U = \frac{(u^2-1)^2}{2 g^2 r^4} + \frac{v^2 u^2 \phi^2}{4 r^2} + \frac{\lambda_h v^4}{8}\left(\phi^2 - 1\right)^2 + \frac{1}{2 g_Y^2 r^4}$$

- **The spherical EW monopole has an infinite mass**
- **Nambu's monopole-anti-monopole dumbbell configuration**



- **Unstable! May be produced at a future collider**

- **Introduce BSM physics to have a finite-energy monopole**

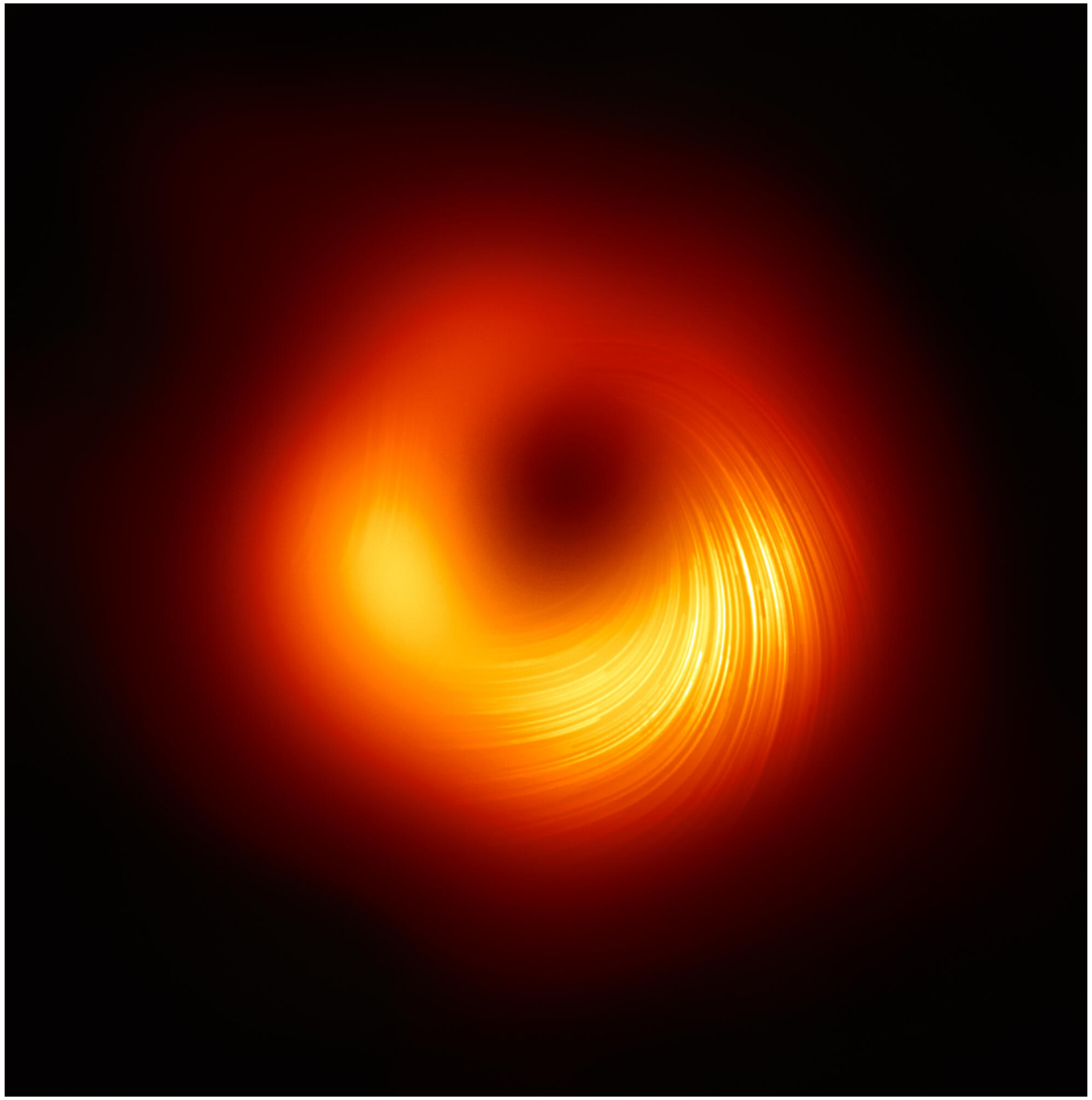
  for instance, $U(1)_Y \subset SU(2)_R$

- **Or hide the divergent part behind the event horizon of a black hole**

- **For the second avenue, no new BSM physics is needed. We just need to study the possible states based on**

Standard Model + General Relativity

Black Hole



Credit: EHT Collaboration

Black Holes

- **Schwarzschild black hole**

$$ds^2 = -\left(1 - \frac{2GM}{r}\right)dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

($2GM = r_{\mathrm{H}}$, circled in the second term)

- **Charged or Reissner-Nordstrom (RN) black hole**

$$ds^2 = -B_{\mathrm{RN}}(r)dt^2 + B_{\mathrm{RN}}(r)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

$$B_{\mathrm{RN}}(r) = 1 - \frac{2GM}{r} + \frac{G\sqrt{Q_{\mathrm{E}}^2 e^2 + Q_{\mathrm{M}}^2 h^2}}{4\pi r^2}$$

- **The outer horizon radius is**

$$r_+ = \frac{\left(M_{\mathrm{eBH}} + \sqrt{M_{\mathrm{eBH}}^2 - (Q_{\mathrm{E}}^2 e^2 + Q_{\mathrm{M}}^2 h^2)M_{\mathrm{pl}}^2/4\pi}\right)}{M_{\mathrm{pl}}^2}$$

$$M_{\mathrm{eBH}} = \frac{\sqrt{Q_{\mathrm{E}}^2 e^2 + Q_{\mathrm{M}}^2 h^2}}{\sqrt{4\pi}} M_{\mathrm{pl}}$$

Hawking Radiation and PBH Lifetime

- **According to the first law of the black hole thermal dynamics, the thermal radiation temperature has (for non-extremal BH)**

$$T = \frac{M_{\rm pl}^2}{8\pi M_{\rm BH}}$$

- **Using the black body radiation formula, $P \propto R^2 T^4$, the lifetime of a Schwarzschild black hole is**

$$\tau \approx \frac{5120\pi}{g_*} \frac{M_{\rm BH}^3}{M_{\rm pl}^4}$$

- **Requiring it to be longer than the age of our universe, one has a lower bound on PBH mass**

$$M_{\rm PBH} \gtrsim 10^{15}\,\text{g}$$

Extremal Black Hole

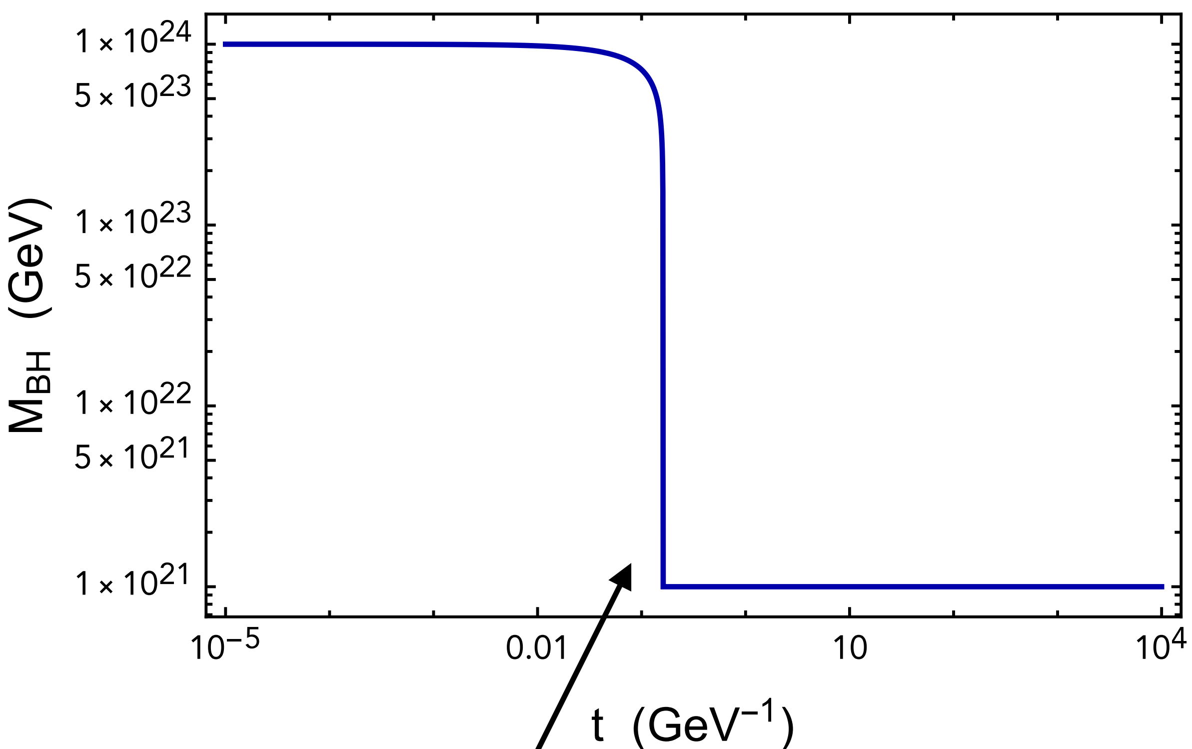
- **The Hawking radiation is fourth power of T . One way to suppress T is to make it extremal**

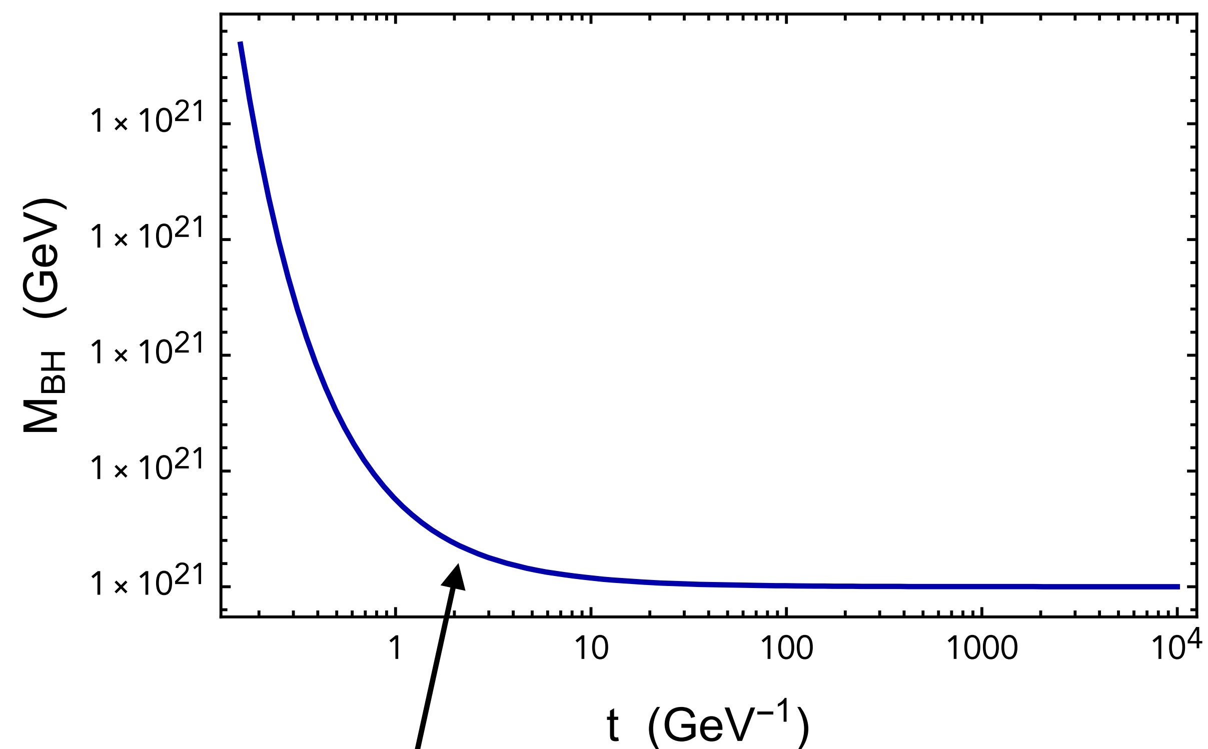
$$T(M_{\rm BH}, M_{\rm eBH}) = \frac{M_{\rm pl}^2}{2\pi} \frac{\sqrt{M_{\rm BH}^2 - M_{\rm eBH}^2}}{\left(M_{\rm BH} + \sqrt{M_{\rm BH}^2 - M_{\rm eBH}^2}\right)^2}$$

- **A PBH with a charge Q will evolve towards a near extremal one, which has suppressed T**

$$\frac{dM_{\rm BH}}{dt} \approx -\frac{\pi^2}{120} g_* 4\pi r_+^2 \left[T(M_{\rm BH}, M_{\rm eBH})\right]^4$$

Evolution of the Black Hole Mass





$$\tau \approx \frac{5120\pi}{g_*} \frac{M_{\rm BH}^3}{M_{\rm pl}^4}$$

$$M_{\rm BH}(t) = M_{\rm eBH} + \frac{120\pi\, M_{\rm eBH}^4}{g_*\, M_{\rm pl}^4\, t}$$

$$T_{\rm eBH} = \sqrt{\frac{60\, M_{\rm eBH}}{\pi\, g_*\, t}}$$

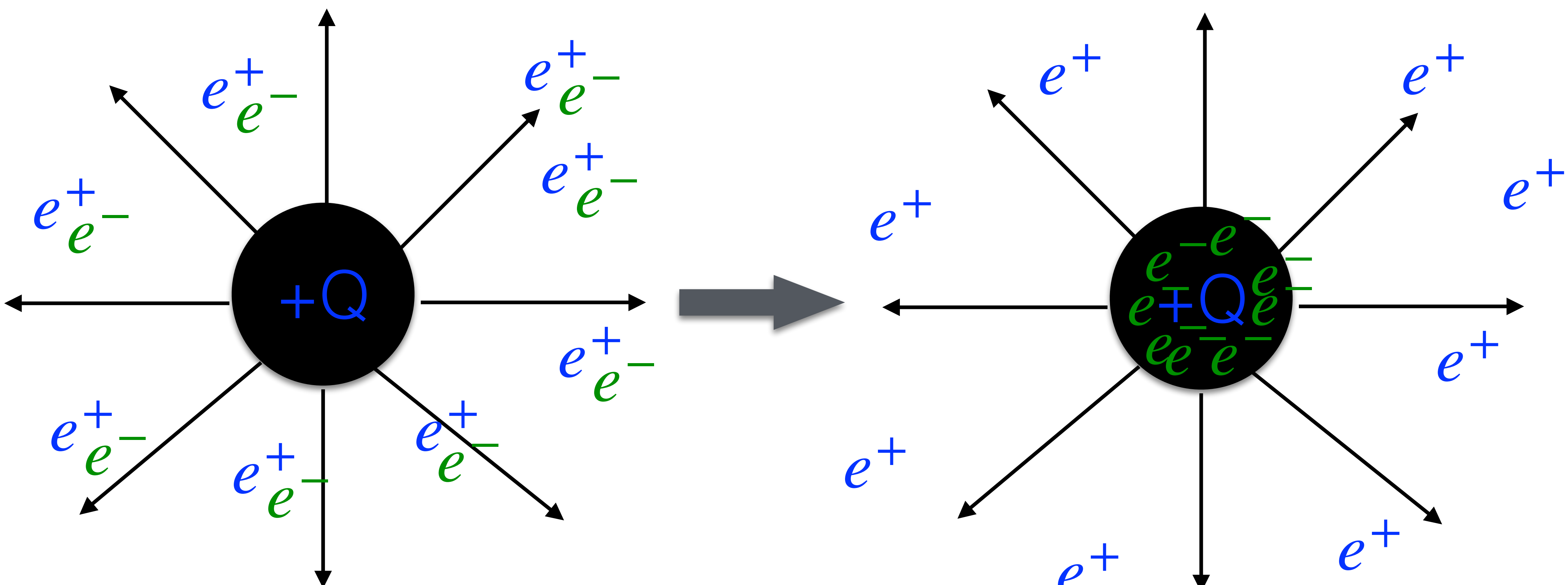
- **The initial BH evaporation still generates lot of Hawking radiations**

Electrically-Charged BH in SM

- **The charged BH has a large electric field close to the event horizon**

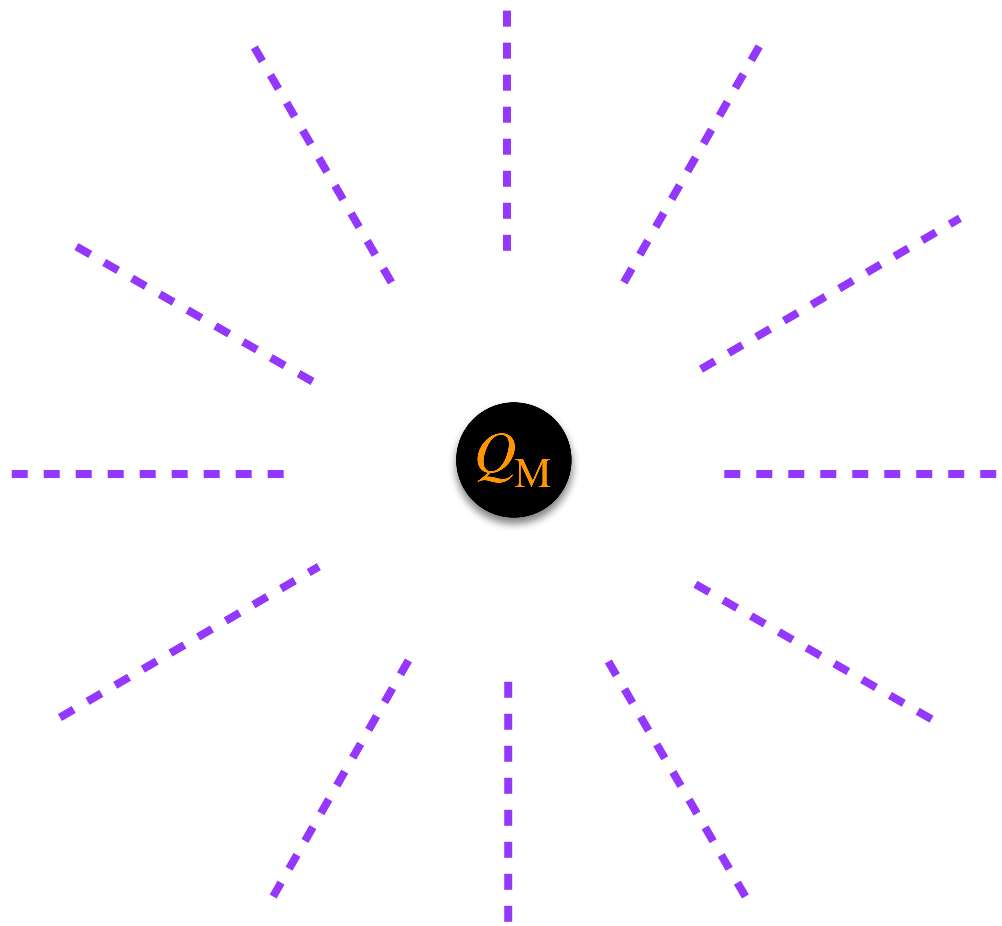
$$E = \frac{M_{\rm pl}^3}{\sqrt{4\pi}\, M_{\rm eBH}}$$

- **The Schwinger effects can generate electrons and positrons from vacuum and discharge the eBH**



Magnetically-Charged BH in SM

- **Since there is no finite-energy magnetic monopole in the SM, no worry about Schwinger discharge**
- **If the GUT exists, it may worry its emission of GUT monopole, which is very heavy**



$$B(R_{\rm eBH}) = \frac{Q}{2\,e\,R_{\rm eBH}^2} \approx \frac{e\,M_{\rm pl}^2}{2\,\pi\,Q}$$

$$Q \gtrsim 10^6$$

Electroweak Symmetry Restoration

- **In a large B field background, the electroweak symmetry is restored**

Salam and Strathdee, NPB90 (1975) 203

Ambjorn and Olesen, NPB330 (1990) 193

$$\mathscr{E} \supset \frac{1}{2}|D_iW_j - D_jW_i|^2 + \frac{1}{4}F_{ij}^2 + \frac{1}{4}Z_{ij}^2 + \frac{1}{2}g^2\varphi^2 W_i W_i^\dagger + (g^2\varphi^2/4\cos^2\theta_W)Z_i^2$$
$$+ig(F_{ij}\sin\theta_W + Z_{ij}\cos\theta_W)W_i^\dagger W_j + \frac{1}{2}g^2\left[(W_iW_i^\dagger)^2 - (W_i^\dagger)^2(W_j)^2\right]^2$$
$$+(\partial_i\varphi)^2 + \lambda(\varphi^2 - \varphi_0^2)^2$$

$$(W_1^\dagger, W_2^\dagger)\begin{pmatrix} \frac{1}{2}g^2\varphi_0^2 & i\,e\,F_{12} \\ -i\,e\,F_{12} & \frac{1}{2}g^2\varphi_0^2 \end{pmatrix}\begin{pmatrix} W_1 \\ W_2 \end{pmatrix}$$

- **For a large $|f_{12}|$, a negative determinant leads to W-condensation and electroweak restoration. This happens when**

$$e\,B \gtrsim m_h^2$$

Electroweak Symmetry Restoration

$$B(R_{\rm eBH}) = \frac{Q}{2\,e\,R_{\rm eBH}^2} \approx \frac{e\,M_{\rm pl}^2}{2\,\pi\,Q} \qquad\qquad e\,B(R_{\rm eBH}) \gtrsim m_h^2$$

- **Electroweak symmetry restoration happens for**

$$Q \lesssim Q_{\rm max} \equiv \frac{e^2\,M_{\rm pl}^2}{2\pi\,m_h^2} \approx 1.4 \times 10^{32}$$

Lee, Nair, Weinberg, PRD45(1992) 2751

Maldacena, arXiv:2004.06084

- **For Q=2, one can obtain the spherically symmetric configuration**

- **For Q > 2, a non-spherically symmetric configuration is anticipated, and requires complicated numerical calculations**

Guth, Weinberg, PRD14(1976) 1660

Q=2: spherical solution

$$ds^2 = P^2(r)\, N(r)\, dt^2 - N(r)^{-1}\, dr^2 - r^2\, d\theta^2 - r^2\, \sin^2\theta\, d\phi^2$$

$$S_{\rm E} = -\frac{1}{16\pi\, G}\int d^4x\sqrt{-g}\, R = -\frac{1}{2\, G}\int dt\, dr\, r\, P'(1-N)$$

$$S_{\rm matter} \supset \int d^4x\sqrt{-g}\, \mathcal{L}_{\rm EW}$$

$$N(r) = 1 - \frac{2\, G\, F(r)}{r} + \frac{4\,\pi\, G}{g_{\rm Y}^2\, r^2}$$

- **The asymptotic mass of the system has**

$$M = F(\infty)$$

Q=2: spherical solution

$$\mathcal{L}_{\rm SM} \supset \mathcal{L}_{\rm EW} = -\frac{1}{4} W^a_{\mu\nu} W^{a\,\mu\nu} - \frac{1}{4} Y_{\mu\nu} Y^{\mu\nu} + |D_\mu H|^2 - \frac{\lambda}{2}\left(H^\dagger H - \frac{v^2}{2}\right)^2$$

$$D_\mu H = \left(\partial_\mu - i\,\frac{g}{2}\,\sigma^a\,W^a_\mu - i\,\frac{g_Y}{2}\,Y_\mu\right) H$$

$$H = \frac{v}{\sqrt{2}}\,\rho(r)\,\xi \qquad \xi = i\begin{pmatrix} \sin\left(\frac{\theta}{2}\right) e^{-i\,\phi} \\ -\cos\left(\frac{\theta}{2}\right)\end{pmatrix}$$

$$W^a_i = \epsilon^{aij}\,\frac{r^j}{r^2}\left(\frac{1-f(r)}{g}\right) \qquad Y_i = -\frac{1}{g_Y}(1-\cos\theta)\,\partial_i\phi$$

- **Change from the hedgehog gauge to the unitary gauge**

$$\xi \longrightarrow U\xi = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad \text{with} \qquad U = -i\begin{pmatrix} \cos\left(\frac{\theta}{2}\right) & \sin\left(\frac{\theta}{2}\right) e^{-i\phi} \\ \sin\left(\frac{\theta}{2}\right) e^{i\phi} & -\cos\left(\frac{\theta}{2}\right)\end{pmatrix}$$

$$A_\mu = -\frac{1}{e}(1-\cos\theta_W)\partial_\mu\phi$$

$$Z_\mu = 0$$

Cho and Maison, hep-th/9601028

Q=2: EOMs and BCs

$$S_{\rm matter} \;=\; -4\pi \int dt\, dr\, r^2\, \left[P(r)\, N(r)\, \mathcal{K} + P(r)\, \mathcal{U}\right]$$

$$\mathcal{K} \;=\; \frac{v^2\,\rho'^2}{2} + \frac{f'^2}{g^2\,r^2}\,,$$

$$\mathcal{U} \;=\; \frac{v^2\,f^2\,\rho^2}{4\,r^2} + \frac{(1-f^2)^2}{2\,g^2\,r^4} + \frac{\lambda}{8}\,v^4\,(\rho^2-1)^2 + \frac{1}{2\,g_Y^2\,r^4} \equiv \mathcal{U}_1 + \frac{1}{2\,g_Y^2\,r^4}$$

$$F' \;=\; 4\pi\, r^2\,(\mathcal{U}_1 + N\,\mathcal{K})\;,$$

$$(N\,f')' + 8\pi\,G\,r\,N\,f'\,\mathcal{K} \;=\; \frac{f(f^2-1)}{r^2} + \frac{g^2}{4}\,v^2\,f\,\rho^2\;,$$

$$\left(r^2\,N\,\rho'\right)' + 8\pi\,G\,r^3\,N\,\rho'\,\mathcal{K} \;=\; \frac{1}{2}\,\rho\,f^2 + \frac{\lambda\,v^2}{2}\,r^2\,\rho(\rho^2-1)\;.$$

$$N' = \frac{1}{r} - 8\pi\,G\,r\,\mathcal{U}\,, \qquad \text{at } r = r_H$$

$$N'\,f' = \frac{f(f^2-1)}{r^2} + \frac{g^2}{4}\,v^2\,f\,\rho^2\,, \qquad \text{at } r = r_H\;,$$

$$N'\,\rho' = \frac{1}{2}\,\frac{f^2\,\rho}{r^2} + \frac{\lambda\,v^2}{2}\,\rho(\rho^2-1)\,, \qquad \text{at } r = r_H\;.$$

$$f(\infty) = 0$$

$$\rho(\infty) = 1$$

Q=2: solutions

- **Setting $f(r) = 0$ and $\rho(r) = 1$, one has the ordinary RN magnetic black hole solution**

$$M_{\rm BH}^{\rm RN} = \frac{r_H}{2\,G} + \frac{2\pi}{e^2\,r_H} \qquad \geq \qquad M_{\rm eBH}^{\rm RN} = \frac{\sqrt{4\pi}\,M_{\rm pl}}{e}$$

- **For the hairy magnetic black hole solution:**

$$M_{\rm hMBH} = F(\infty) = \int_{r_H}^{\infty} dr' 4\pi\,r'^2\,[\mathcal{K}(r') + \mathcal{U}_1(r')] + \frac{r_H}{2\,G} + \frac{2\pi}{g_Y^2\,r_H}$$

Hair mass $\ll$ **Black hole mass**

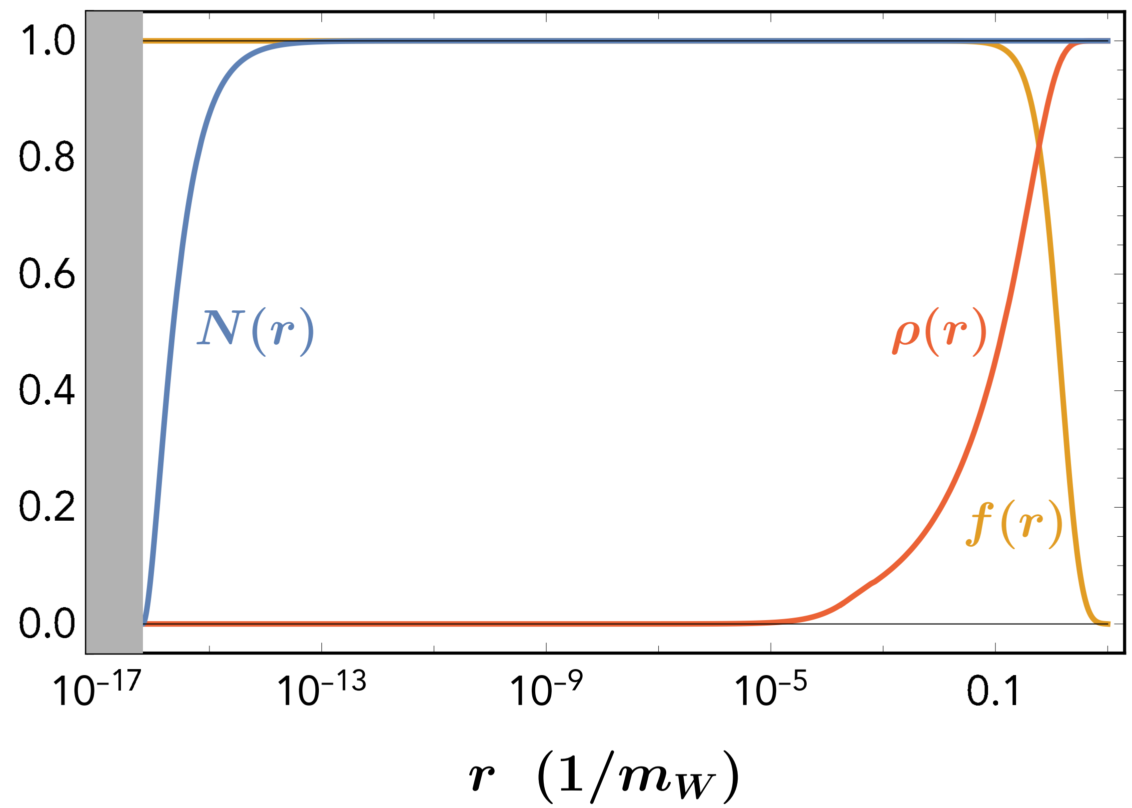
- **Ignoring the hair mass, one has**

$$M_{\rm hMBH} \approx \frac{r_H}{2\,G} + \frac{2\pi}{g_Y^2\,r_H} \geq M_{\rm ehMBH} = \cos\theta_W\,\frac{\sqrt{4\pi}\,M_{\rm pl}}{e}$$

Hyper-magnetic black hole!

Q=2: profiles

extremal hMBH

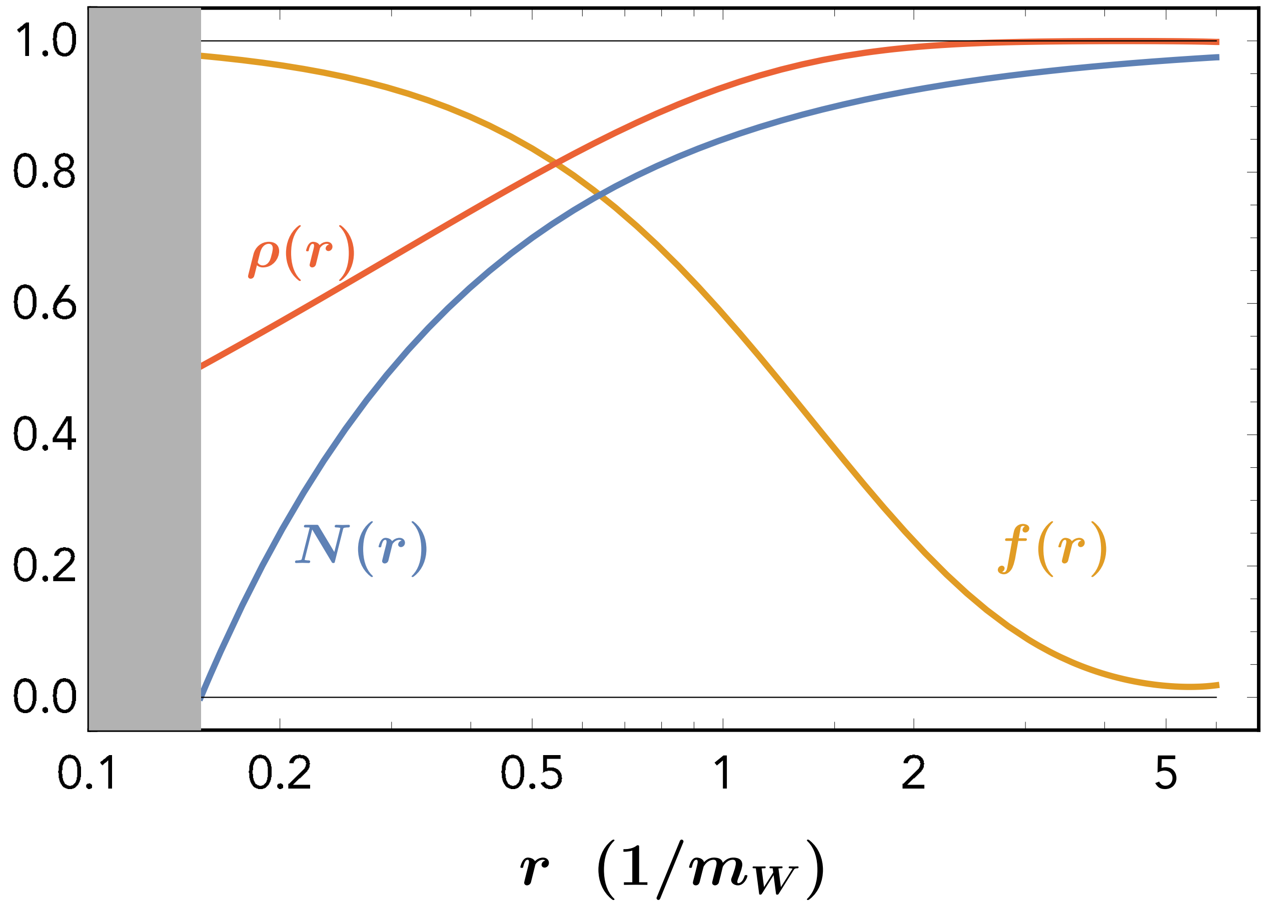


$$M_{\text{ehMBH}} \approx \cos\theta_W \frac{\sqrt{4\pi}\, M_{\text{pl}}}{e} + 0.75 \times \frac{2\pi\, v^2}{m_W} = (1.2 \times 10^{20} + 3.6 \times 10^3)\,\text{GeV}$$

The electroweak symmetry is restored inside

Q=2: profiles

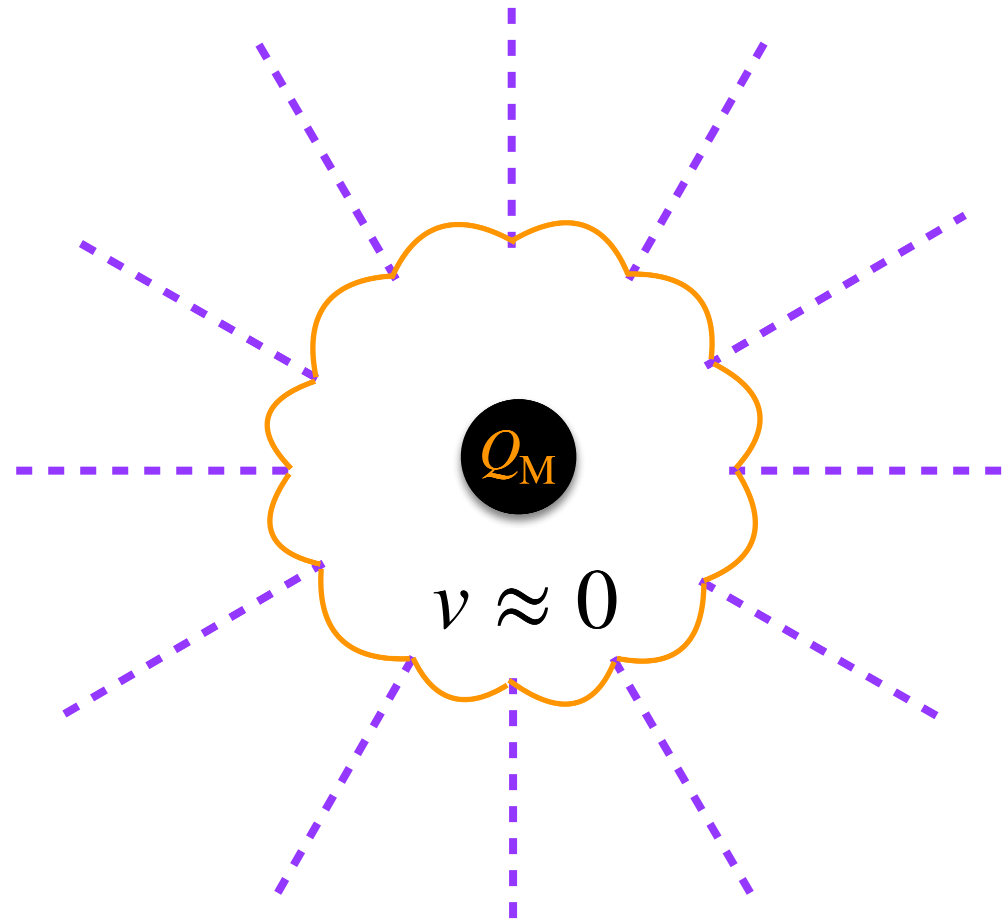
non-extremal hMBH



$r_H = 0.15/m_W$

$$M_{\rm hMBH} \leq M_{\rm hMBH}^{\rm max} = \frac{1}{2\,G\,m_W} + \mathcal{O}(m_W) \approx 9.3 \times 10^{35}\ \text{GeV}$$

Q>2: non-spherical



$v = 246\,\mathrm{GeV}$

$R_{\rm EW} \simeq \sqrt{\frac{Q}{2}}\,\frac{1}{m_h}$

$$M^{\rm tot}_{\rm MeBH}(Q) \;\simeq\; c_W\,\frac{\sqrt{\pi}\,Q}{e}\,M_{\rm pl} + \frac{4\pi}{3}\,R^3_{\rm EW}\,\frac{m_h^2\,v^2}{8} = c_W\,\frac{\sqrt{\pi}\,Q}{e}\,M_{\rm pl} + \frac{\pi}{12\sqrt{2}}\,Q^{3/2}\,\frac{v^2}{m_h}$$

$$\equiv\; M_{\star}(Q) + \frac{\pi}{12\sqrt{2}}\,Q^{3/2}\,\frac{v^2}{m_h}\,, \qquad\qquad M_{\star}(Q) = c_W\,M^{\rm RN}_{\rm eBH}$$

$r_H \simeq R_{\rm EW}$

- For $Q < Q_{\rm max} \simeq 10^{32}$, $\quad M_{\star} \lesssim 9\times 10^{51}\,\mathrm{GeV} \sim M_{\oplus} \qquad R^{\rm max}_{\rm EW} \sim 1\,\mathrm{cm}$

2d Modes

- **In the existence of magnetic field, the massless 2d modes exist for a Dirac 4D massless fermion**

$$ds^2 = e^{2\sigma(t,x)}\left(-dt^2 + dx^2\right) + R^2(t,x)\left(d\theta^2 + \sin^2\theta\, d\phi^2\right) \qquad A_\phi = \tfrac{Q}{2}\cos\theta$$

$$dx = \frac{dr}{f(r)}, \qquad e^{2\sigma(t,x)} = f(r) \equiv (1 - R_{\rm e}/r)^2, \qquad R(t,x) = r$$

$$\not{D}\widetilde{\chi} = m_\chi\,\widetilde{\chi} \qquad \widetilde{\chi}_{\alpha\beta} = \frac{e^{-\frac{1}{2}\sigma}}{R}\,\psi_\alpha(t,x)\,\eta_\beta(\theta,\phi)$$

$$\left[\sigma_y \frac{\partial_\phi - iA_\phi}{\sin\theta} + \sigma_x\left(\partial_\theta + \frac{\cot\theta}{2}\right)\right]\eta = 0,$$

$$(i\sigma_x\partial_t + \sigma_y\partial_x)\,\psi = m_\chi\, e^\sigma \psi\ .$$

← **2d fermion**

2d Modes

Kazama, Yang, Goldhaber, '1977

- **Solutions for Q > 0,**

$$\propto {}_qY_{q,-m}(\theta,\phi)$$

$$\eta_1 = 0\ ,$$

$$\eta_2 = \left(\sin\frac{\theta}{2}\right)^{j-m}\left(\cos\frac{\theta}{2}\right)^{j+m} e^{im\phi} = \frac{(1-\cos\theta)^{\frac{q-m}{2}}(1+\cos\theta)^{\frac{q+m}{2}}}{2^{q-\frac{1}{2}}(\sin\theta)^{\frac{1}{2}}}\, e^{im\phi}$$

$$j = (|Q|-1)/2 \equiv q-1/2 \text{ and } -j \le m \le j$$

- **There are |Q| massless modes for** $m_\chi = 0$

Field	$SU(3)\times SU(2)\times U(1)$	Number of 2d modes (left - right)
q_L	$(\mathbf{3},\mathbf{2})_{\frac{1}{6}}$	Q
u_R	$(\mathbf{3},\mathbf{1})_{\frac{2}{3}}$	- 2 Q
d_R	$(\mathbf{3},\mathbf{1})_{-\frac{1}{3}}$	Q
l_L	$(\mathbf{1},\mathbf{2})_{-\frac{1}{2}}$	- Q
e_R	$(\mathbf{1},\mathbf{1})_{-1}$	Q

Maldacena, arXiv:2004.06084

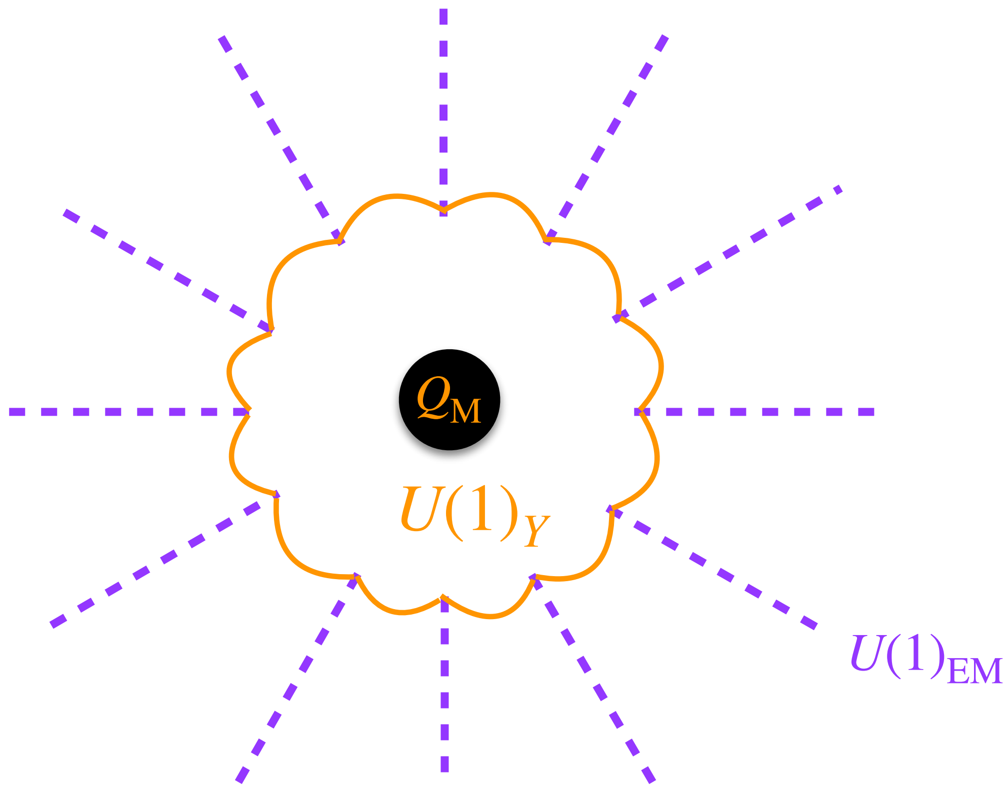
2d Hawking radiation

- **Fermions are massless (ignoring QCD vacuum) inside the EW-corona region**

$$P_2 = \frac{dE}{dt} = \frac{\pi\, g_*}{24} T^2(M_{\rm BH}, M_{\star})$$

- **For high T, $g_* = 18\,|\,Q\,|$ for three-family fermions**

- **The 2d radiation is very fast; it reaches extremal very quickly**



- 2d neutrino modes can not escape
- EM charged states can travel outside of coronas

2d Hawking radiation

- **For $T < m_e$, the 2d radiation is suppressed. The 4D radiation dominants**

$$P_4 = \frac{dE}{dt} \approx \frac{\pi^2 g_*}{120} (4\pi R_{\rm EW}^2)\, T^4(M_{\rm BH}, M_{\star})$$

with $g_* = 2$ for photon and $g_* = 21/4$ for neutrinos

- **For $T > m_e$, the 2d radiation usually dominants over 4D**

$$\tau_{\rm BH} \approx \frac{24\pi^{3/2}\, c_W\, M_{\star}^2}{e\, M_{\rm pl}^3} \log\left[\frac{M_{\rm pl}^4\,(M_{\rm BH} - M_{\star})}{2\pi^2\, m_e^2\, M_{\star}^3}\right]$$

shorter than the 4D time scale by a factor of $M_{\rm pl}/M_{\rm BH}$

Primordial MBHs ?

- There are various ways to form primordial black holes
  - Large primordial fluctuations
  - Phase transitions, boson stars, ……
- Produce large number of monopoles and anti-monopoles (maybe Nambu's dumbbell configurations)
- The formation of black holes eat totally N objects
- Anticipate the net BH magnetic charge: $\sim \sqrt{N}$

YB, Orlofsky, arXiv: 1906.04858

- To be studied more. Let's discuss how to search for them

Parker Limits

- **Requiring the domains of coherent magnetic field are not drained by magnetic monopoles**

- **PMBH flux:** $F_{\circledast} \approx (9.5 \times 10^{-21}\,\mathrm{cm}^{-2}\mathrm{sr}^{-1}\mathrm{s}^{-1})\, f_{\circledast} \left(\frac{10^{26}\,\mathrm{GeV}}{M_{\circledast}}\right) \left(\frac{\rho_{\mathrm{DM}}}{0.4\ \mathrm{GeV\,cm}^{-3}}\right) \left(\frac{v}{10^{-3}}\right)$

- **Mean energy gained by PMBHs for the regeneration time is smaller than the energy stored in B**

Turner, Parker, Bogdan, PRD26(1982) 1296

$$\Delta E \times F_{\circledast} \times (\pi \ell_c^2) \times (4\pi\ \mathrm{sr}) \times t_{\mathrm{reg}} \lesssim \frac{B^2}{2} \frac{4\pi\, \ell_c^3}{3}$$

$$\Delta E \simeq M_{\circledast} \Delta v^2/2 \qquad \Delta v \simeq B\, h_Q\, \ell_c/(M_{\circledast} v)$$

$$f_{\circledast} \lesssim 50 \times \frac{v_{-3}}{\rho_{0.4}\, \ell_{21}\, t_{15}}$$

$\rho_{0.4} = \rho_{\mathrm{DM}}/(0.4\,\mathrm{GeV\,cm}^{-3})$

$v_{-3} = v/(10^{-3})$

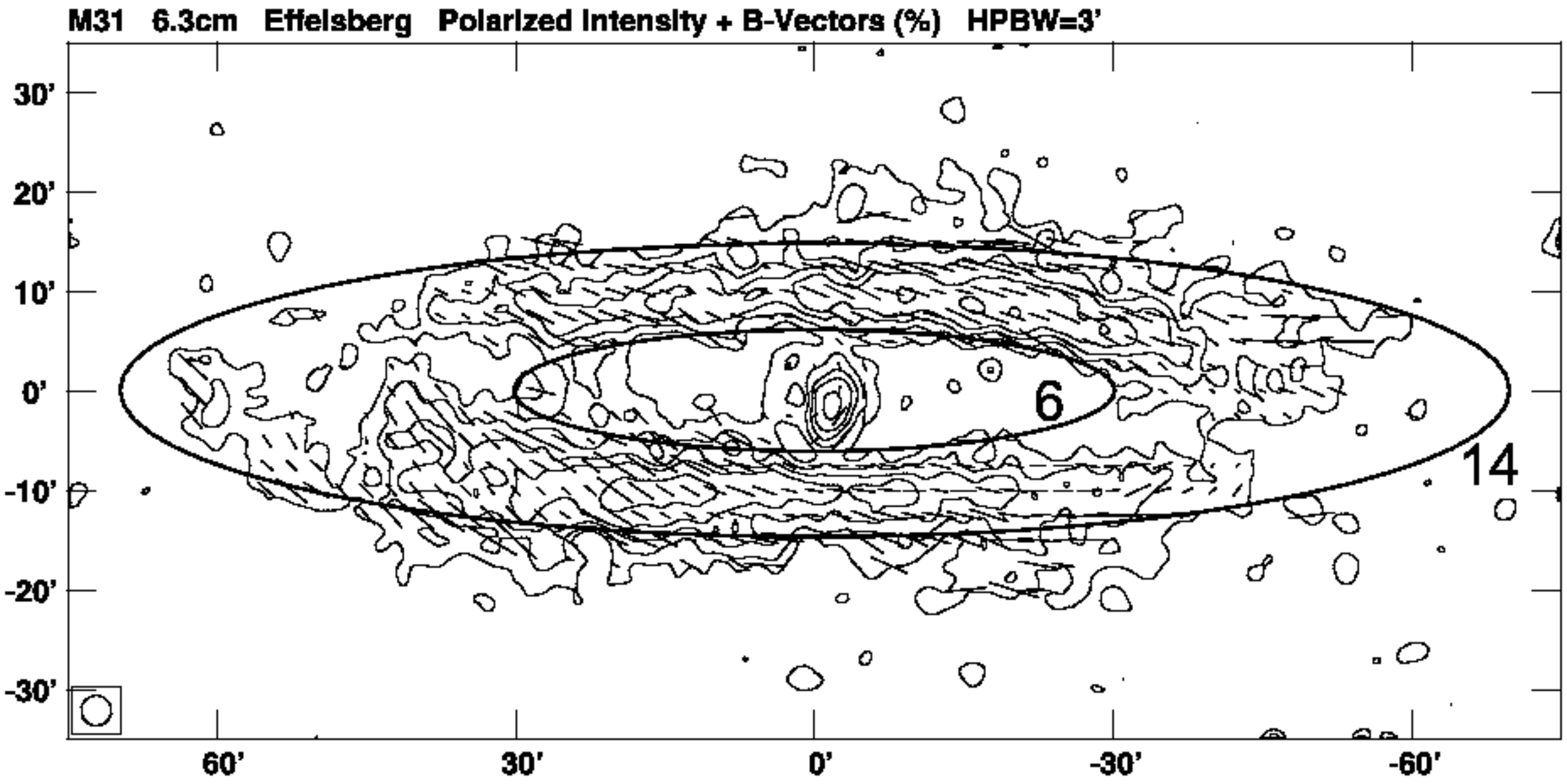
$t_{15} = t_{\mathrm{reg}}/(10^{15}\,\mathrm{s})$

$\ell_{21} = \ell_c/(10^{21}\ \mathrm{cm})$

Parker Limit from M31

A. Fletcher et al.: The magnetic field in M 31

astro-ph/0310258



$$\ell_c \sim 10 \text{ kpc} \Rightarrow \ell_{21} \sim 30 \text{ and } t_{\rm reg} \sim 10 \text{ Gyr} \Rightarrow t_{15} \sim 300$$

$$f_{\bullet} \lesssim 6 \times 10^{-3}$$

which is independent of PMBH mass

PMBHs inside the Sun

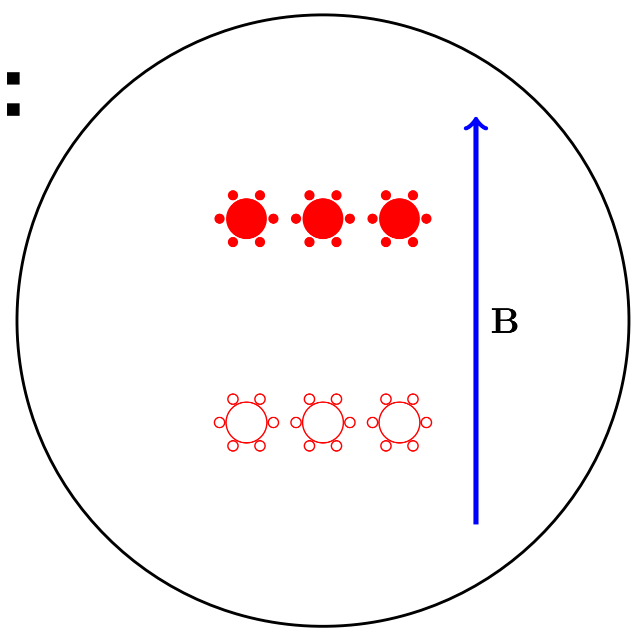
- **The capture rate is**

$$C_{\rm cap} \approx \epsilon\,\pi\,R_\odot^2\left[1+(v_{\rm esc}/v)^2\right]\,4\,\pi F_{\star} \approx \left(9.2\times 10^3\,{\rm s}^{-1}\right)\,\epsilon\,f_{\star}\,M_{26}^{-1}$$

- **Then, it drifts to the center region with a time scale**

$$t_{\rm drift} \sim \frac{R_\odot}{v_{\rm drift}} \sim \frac{R_\odot^3}{M_\odot}\frac{n_e\,e^2}{c_W^2\,m_e\,v_{\rm th}}M_{\star} \sim (8\times 10^4\,{\rm s})\,M_{26}$$

- **Force-balance equation:**



$$0 = F = B\,\frac{2\pi Q}{e} - \frac{4\pi}{3}\,G\,\rho_c\,M_{\star}\,z - \frac{G\,N_{\star}\,M_{\star}^2}{(2\,z)^2}$$

PMBHs inside the Sun

$$0 = F = B\,\frac{2\pi Q}{e} - \frac{4\pi}{3}\,G\,\rho_c\,M_\odot\,z - \frac{G\,N_\odot\,M_\odot^2}{(2\,z)^2}$$

- **For** $N < N_\odot^{\rm crit} \simeq \dfrac{18\,M_{\rm pl}^3\,B^3}{\sqrt{\pi}\,c_W^3\,M_\odot\,\rho_c^2} = (3.8\times 10^{10})\,B_{100}^3\,M_{26}^{-1}$ **the first two terms are important**

$$z_B \simeq \frac{3\,B\,M_{\rm pl}}{2\sqrt{\pi}\,c_W\,\rho_c} = (2.0\times 10^3\,{\rm cm})\,B_{100}$$

- **For** $N > N^{\rm crit}$, **an equilibrium is quickly reached between capture and annihilation rates with**

$$\Gamma_A = \frac{1}{2}\,C_A\,N_\odot^2 \approx \frac{1}{2}\,C_{\rm cap} = \left(4.6\times 10^3\,{\rm s}^{-1}\right)\,f_\odot\,M_{26}^{-1}$$

Annihilation Products

- **For two eBHs with Q_1 and $-Q_2$ charges, the merge product has**

$$Q = Q_1 - Q_2$$

$$M_{\rm BH} \approx c_W \sqrt{\pi}(Q_1 + Q_2)M_{\rm pl}/e$$

- **It is a non-extremal MBH with**

$$T_{\rm BH} \simeq \frac{M_{\rm pl}^2}{2\pi} \frac{1}{8\,M_{\star}(Q_1)} = (2.8 \times 10^{10}\,{\rm GeV})\,M_{26}^{-1}$$

- **For $T_{\rm BH} > m_e$, it has quick 2d Hawking radiation to reach the extremal state**
- **The radiated charged particles can decay into photons, neutrinos and protons; only (not too high-energy) neutrinos can easily propagate out of the Sun**

Solar ν from PMBH Annihilation

- **To satisfy the neutrino energy cut,**

$$M_{\star} \lesssim M_{\max,E} = (2.8 \times 10^{35}\ \mathrm{GeV}) \left(\frac{10\,\mathrm{GeV}}{E_\nu^{\mathrm{cut}}}\right)$$

- **To have the time interval of two events shorter than the experimental operation time**

$$M_{\star} \lesssim M_{\max,t} = (2.1 \times 10^{37}\ \mathrm{GeV})\, f_{\star} \left(\frac{t_{\mathrm{exp}}}{532\,\mathrm{day}}\right)$$

- **The generated neutrino flux is** $E_\nu \simeq \langle E_f \rangle / \eta_\nu \approx (1.19/\eta_\nu)\, T_{\mathrm{BH}}$

$$I_\nu \approx \frac{N_\nu\, \Gamma_A}{4\pi\, d_\oplus^2} \approx (5.5 \times 10^{-9}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1})\, M_{26}\, \eta_\nu\, f_{\star}$$

$$f_{\star} \lesssim \begin{cases} 1.4 \times 10^{-7}\,, & 2 \times 10^{21}\ \mathrm{GeV} \lesssim M_{\star} \lesssim 2.9 \times 10^{30}\ \mathrm{GeV}\,, \\ M_{\star}/(2.1 \times 10^{37}\ \mathrm{GeV})\,, & 2.9 \times 10^{30}\ \mathrm{GeV} \lesssim M_{\star} \lesssim 2.8 \times 10^{35}\ \mathrm{GeV}\,, \end{cases} \quad \text{(IceCube)}$$

- **Super-K probes even heavier masses because a smaller energy cut**

PMBH inside Earth

- **Similar story as the Sun, the capture rate is**

$$C_{\rm cap} \approx \epsilon \, \pi \, R_\oplus^2 \, 4\,\pi F_{\bullet} \approx \left(0.15\,{\rm s}^{-1}\right) \, \epsilon \, f_{\bullet} \, M_{26}^{-1}$$

- **Other than the neutrino signals, the total power generated from BH annihilation is**

$$P_A \simeq (2.4 \times 10^{15}\,{\rm W})\, f_{\bullet}$$

- **The internal heat of the Earth is $P_\oplus \approx 4.7 \times 10^{13}$ W, so**

$$f_{\bullet} \lesssim 0.02 \qquad\qquad \text{(Earth heat)}$$

for $1.2 \times 10^{23}\ {\rm GeV} \ \lesssim\ M_{\bullet} \lesssim 1 \times 10^{37}\ {\rm GeV}$

Stop PMBH ↗ (lower bound)

$t_{\rm drift} < t_\oplus$ ↖ (upper bound)

PMBH inside Neutron Stars

- **The capture rate is**

$$C_{\rm cap} \approx \epsilon\,\pi\,R^2 \left[\frac{1+(v_{\rm esc}/v)^2}{1-v_{\rm esc}^2}\right] 4\,\pi F_{\star} \approx (0.11\,{\rm s}^{-1})\, f_{\star}\, R_{10}^2\, M_{26}^{-1}$$

$$N_{\star}^{\rm NS} = C_{\rm cap}\,\tau_{\rm NS} \sim \left(3.3\times 10^{16}\right)\; f_{\star}\, R_{10}^2\, M_{26}^{-1}\,\tau_{10} \qquad \tau_{\rm NS} = \tau_{10}\times 10^{10}\ {\rm yr}$$

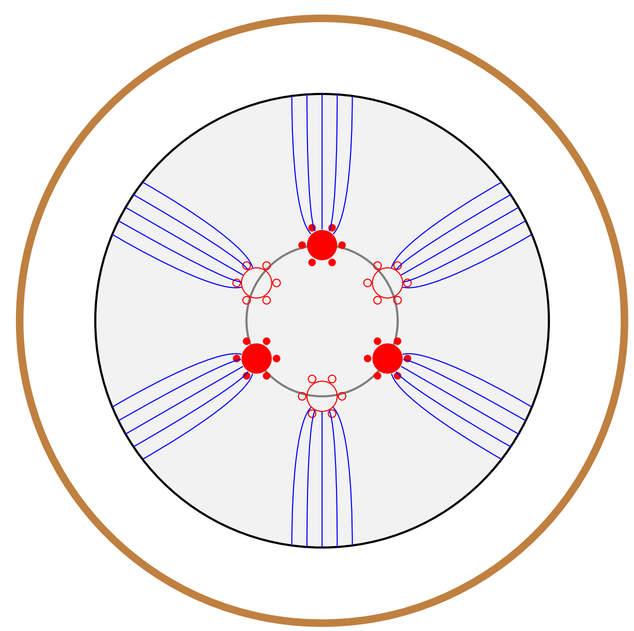
- **The inner core of a neutron star is anticipated to be a proton superconductor** Gezerlis, et. al, arXiv:1406.6109

- **The magnetic field of PMBH is confined to flux tubes with**

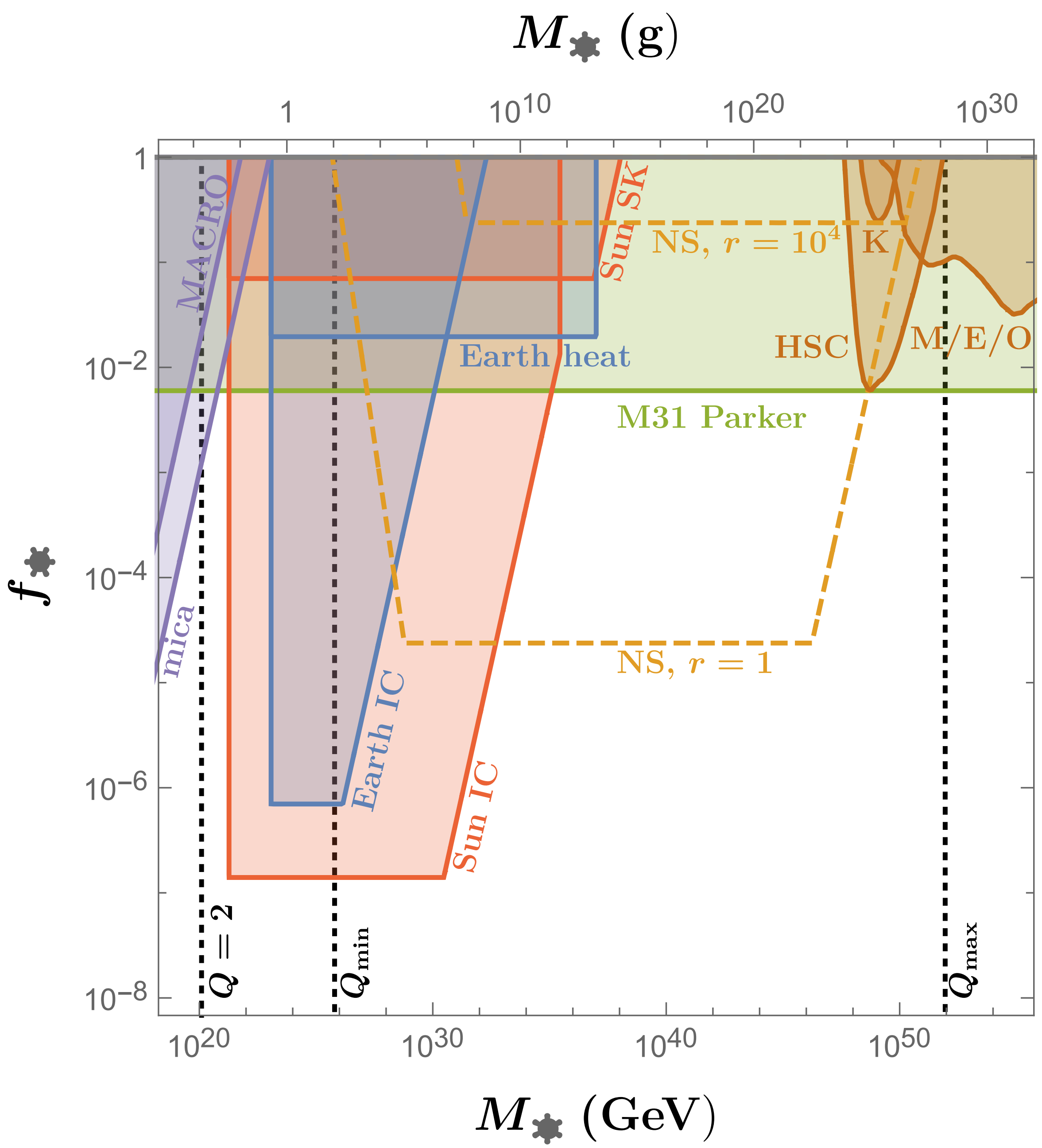
$$\lambda = \left(\frac{m_p}{e^2\,n_p}\right)^{1/2} \sim 10^{-12}\ {\rm cm}$$

$$B_\Phi \sim \frac{\Phi}{\pi\,\lambda^2} \sim 10^{16}\ {\rm gauss}$$

$$F_{\rm T} \sim B_\Phi^2\,\pi\,\lambda^2\,\ln\left(\lambda/\xi\right) \sim 10^4\ {\rm N}$$



Fraction of PMBH over dark matter



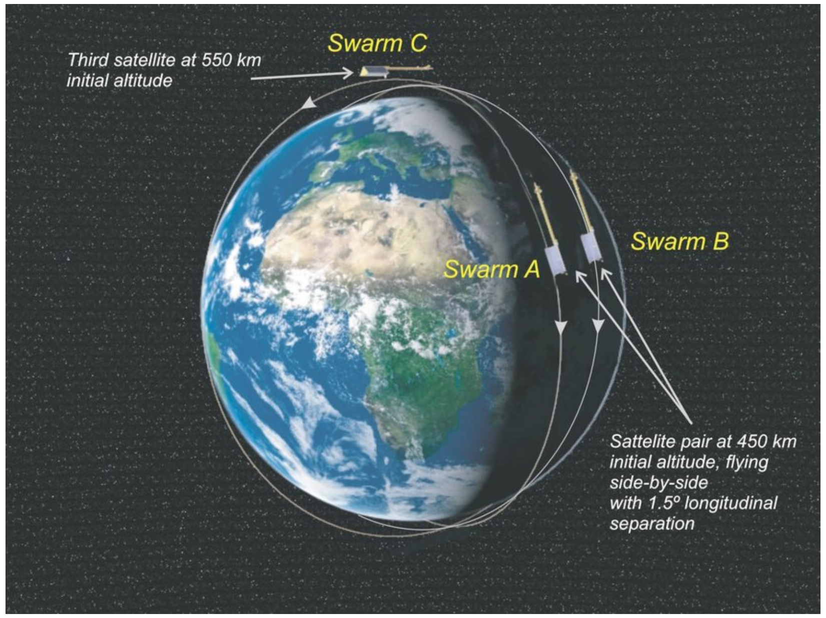
Other searches: Ghosh, Thalapillil, Ullah, 2009.03363

Diamond and Kaplan, 2103.01850

Monopole Moment of Earth Magnetic Field

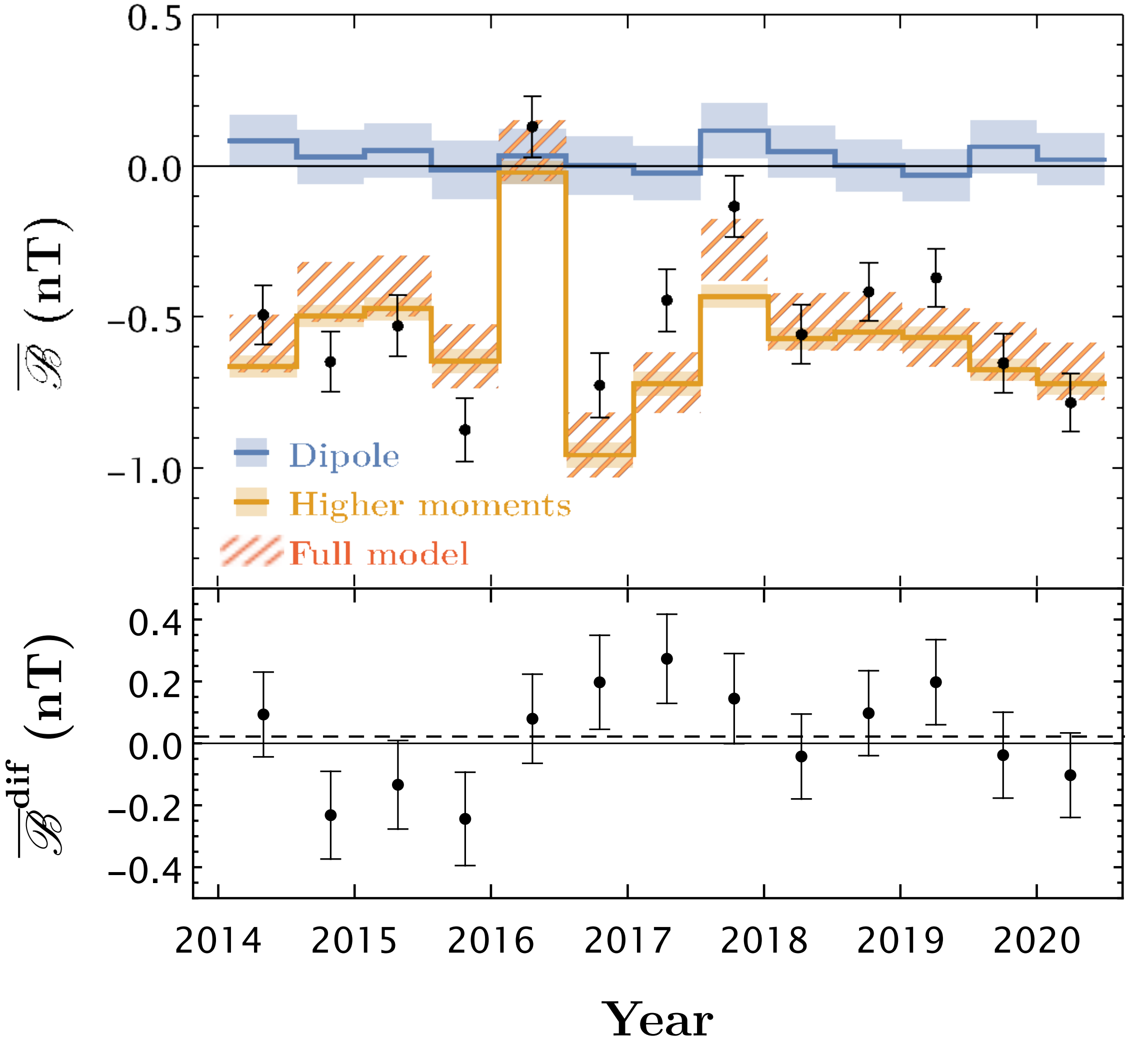
- Using Gauss law to search for monopoles

$$\overline{B}_{\rm m} \equiv \frac{1}{4\pi}\oint \boldsymbol{B}_{\rm m}(r,\theta,\phi)\cdot\hat{\mathbf{n}}\,d\Omega = Q\,h\,\frac{1}{4\pi\,R^2}$$



$$\overline{\mathscr{B}} = \frac{1}{4\pi}\int\left[\frac{r(\theta,\phi)}{R_{\rm ref}}\right]^3 \boldsymbol{B}(r,\theta,\phi)\cdot\hat{\mathbf{r}}\,d\Omega$$

Monopole Moment of Earth Magnetic Field



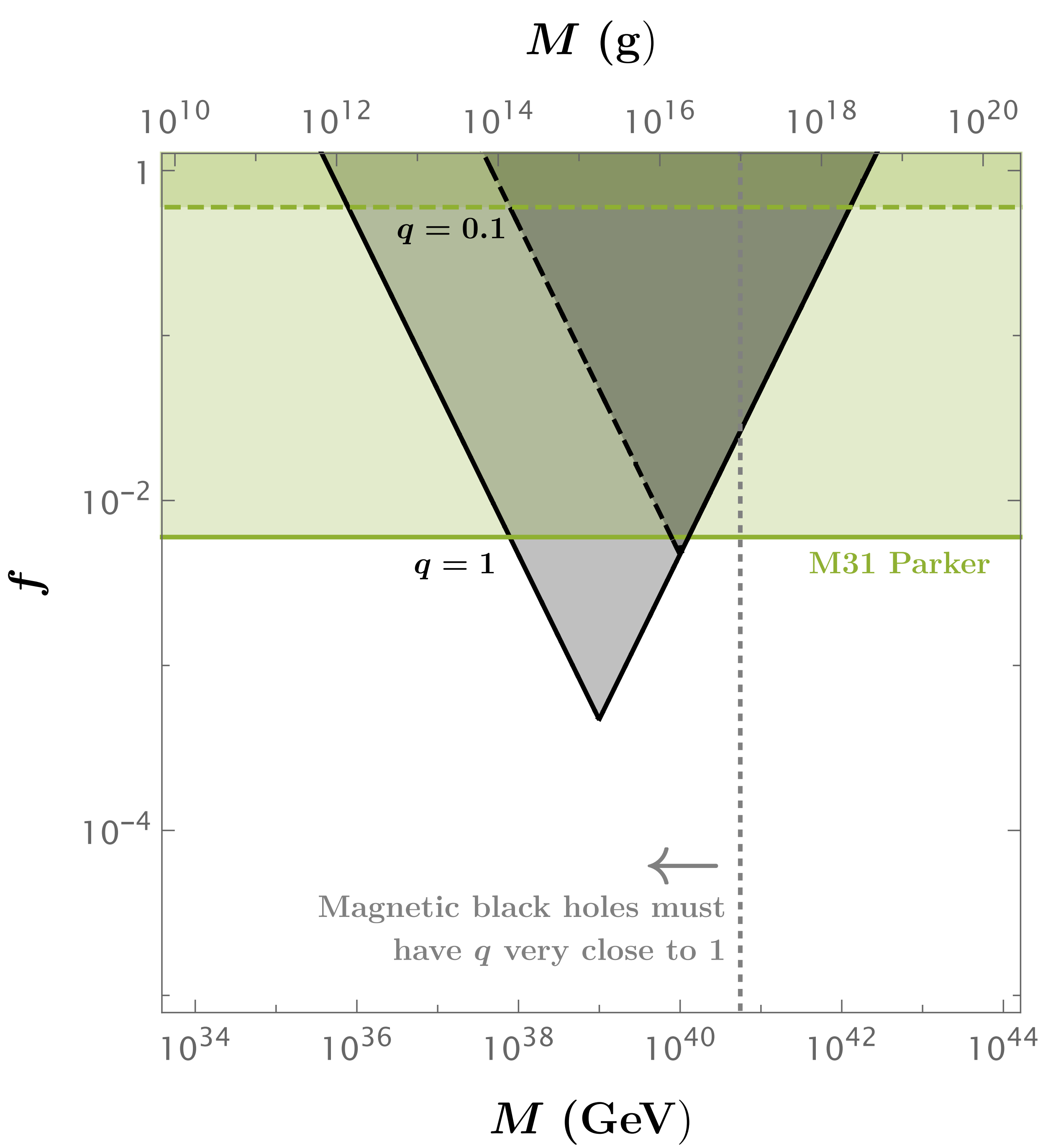
$$|B_m(r = R_\oplus)| < 0.13\,\mathrm{nT}$$

or $$|Q_{\rm net}| < 1.6 \times 10^{19}$$

YB, Lu, Orlofsky

2103.06286

Monopole Moment of Earth Magnetic Field



YB, Lu, Orlofsky

2103.06286

Conclusions

- Magnetic black holes with $Q < 10^{32}$ have electroweak-symmetric coronas
- It has a fast 2d Hawking radiation rate and can reach the extremal state quickly
- Because of their heavy masses, they require astrophysical objects or environment to infer their existence
- Its abundance should be 10^{-2} smaller than the dark matter abundance because of the Parker limit (M31)
- The existence of such objects only requires the known physics, SM+GR, and they deserve more studies