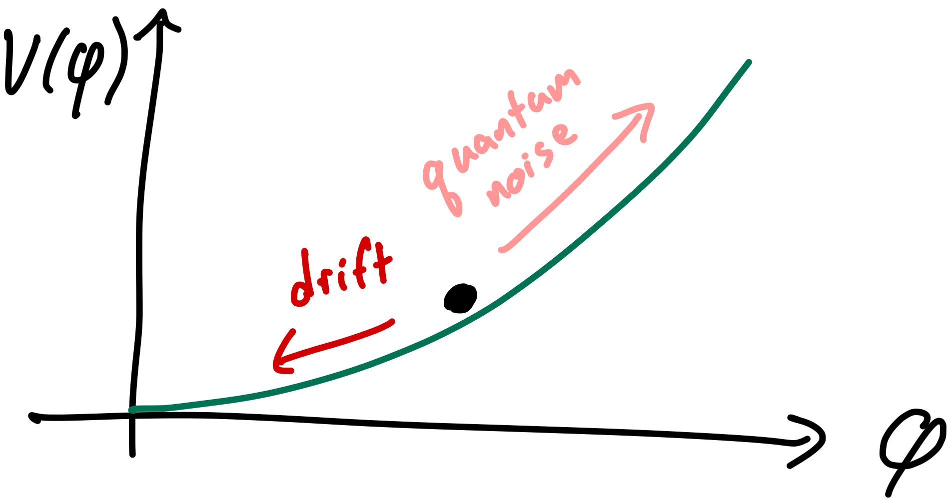
Stochastic Inflation at NNLO



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QMAP Seminar, June 7

The IR of QFT in de Sitter

Conceptual

What DOFs emerge?

What governs their dynamics?

What symmetries persist?

Practicle

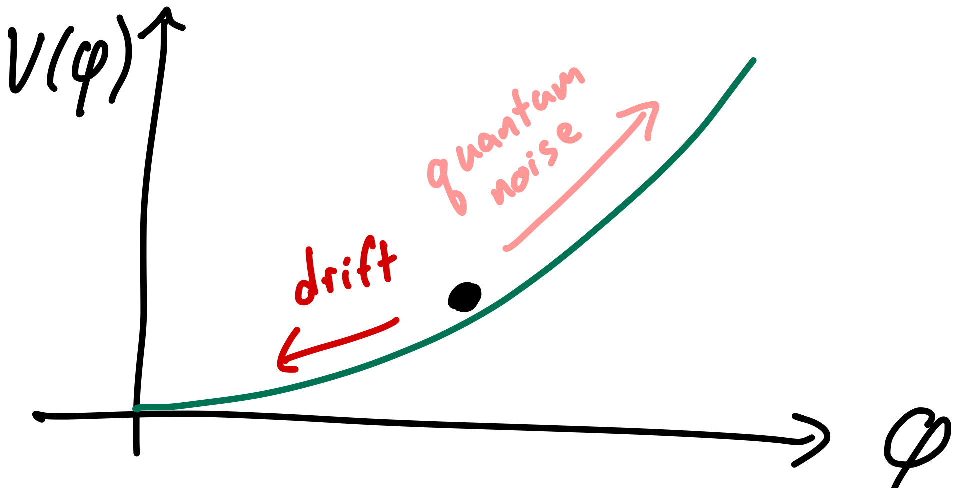
How are operators organized?

Can divergent integrals be tamed?

Can IR logs be systematically summed?

Starobinsky's Stochastic Inflation (1986)

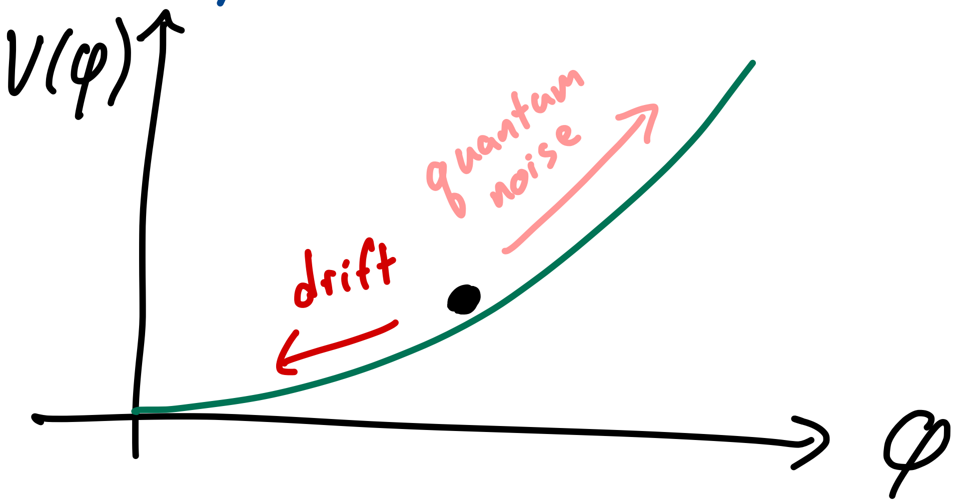
Massless scalar field in dS



$\Rightarrow$ Focker - Planck equation:

$$\frac{\partial}{\partial t} P(\varphi, t) = \frac{H^3}{8\pi^2} \frac{\partial^2}{\partial \varphi^2} P(\varphi, t) + \frac{1}{3H} \frac{\partial}{\partial \varphi}\left[V'(\varphi) P(\varphi, t)\right]$$

Starobinsky's Stochastic Inflation

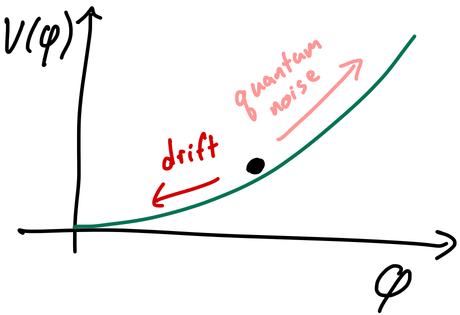


Gaussian noise

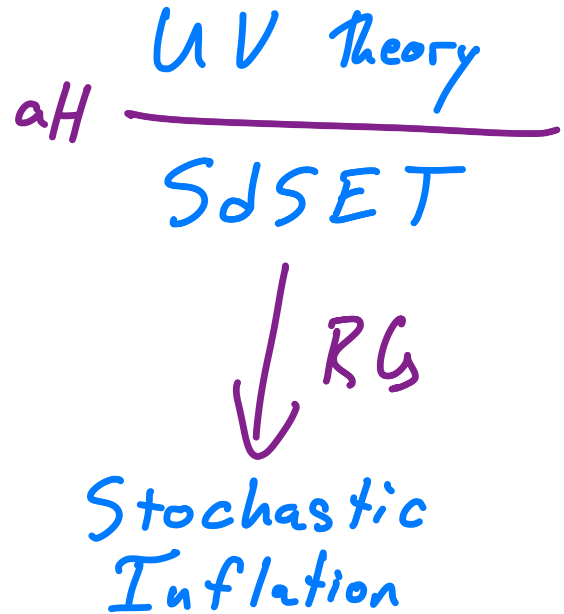
Tree-level potential

Systematic corrections?

Stochastic Inflation $\Longleftrightarrow$ RG flow



$\Longleftrightarrow$



Outline:

I. Stochastic Inflation

II. Soft de Sitter EFT

III. Light Scalars in dS

IV. Matching and Running

V. Outlook

Stochastic Inflation

Leading Order

Probability distribution $P(\varphi, t)$

$$\frac{\partial}{\partial t} P(\varphi,t) = \underbrace{\frac{H^3}{8\pi^2}\frac{\partial^2}{\partial\varphi^2} P(\varphi,t)}_{\text{noise}} + \underbrace{\frac{1}{3H}\frac{\partial}{\partial\varphi}\left[V'(\varphi)P(\varphi,t)\right]}_{\text{drift}}$$

H = Hubble and $V' = \frac{\partial V}{\partial \varphi}$

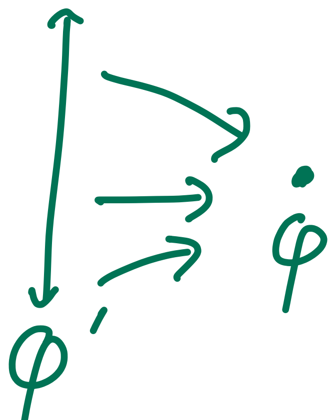
Fixed point solution ($\partial P/\partial t = 0$)

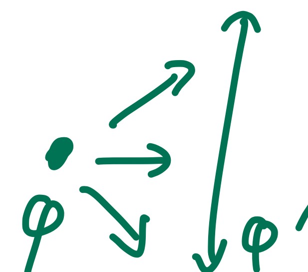
$$P_{eq} \sim \exp\left(-8\pi V/3H^4\right)$$

Beyond Leading Order

Assume "Markovian" fluctuations (no memory)

$$\frac{\partial}{\partial t}P(\varphi,t)=\int d\varphi'\left[P(\varphi',t)\,W(\varphi|\varphi') - P(\varphi,t)\,W(\varphi'|\varphi)\right]$$





$W(\varphi|\varphi')$ is transition rate $\varphi' \to \varphi$

Beyond Leading Order

Perform Kramers-Moyal "local" expansion

$$\frac{\partial}{\partial t} P(\varphi, t) = \sum_{n=1}^{\infty} \frac{1}{n!} \frac{\partial^n}{\partial \varphi^n} \Omega_n(\varphi) P(\varphi, t)$$

w/ $\Omega_n(\varphi) = \int d\Delta\varphi \, (-\Delta\varphi)^n \, W(\varphi + \Delta\varphi | \Delta\varphi)$

Beyond Leading Order

$$\frac{\partial}{\partial t} P(\varphi, t) = \sum_{n=1}^{\infty} \frac{1}{n!} \frac{\partial^n}{\partial \varphi^n} \Omega_n(\varphi) P(\varphi, t)$$

$\Omega_n(\varphi)$ has polynomial expansion

$$\Omega_n(\varphi) = \sum_{m=0}^{\infty} \frac{1}{m!} \Omega_n^{(m)} \varphi^m$$

LO Stochastic Inflation

$$V = \sum_{\ell} \frac{1}{\ell!} c_\ell \varphi^\ell \Rightarrow \Omega_1^{(m)} = \frac{1}{3H} c_{m+1} \quad \Bigg| \quad \Omega_2^{(0)} = \frac{H^3}{8\pi^2}$$

Beyond Leading Order

Generic structure

$$\frac{\partial}{\partial t} P(\varphi,t) = \sum_{n=2}^{\infty} \frac{1}{n!} \frac{\partial^n}{\partial \varphi^n} \left[\sum_{m=0}^{\infty} \frac{1}{m!} \Omega_n^{(m)} \varphi^m P(\varphi,t) \right]$$

Higher order noise

$$+ \frac{1}{3H} \frac{\partial}{\partial \varphi} \left[V'(\varphi) P(\varphi,t) \right]$$

Beyond Leading Order

Assume $\lambda\varphi^4$ in UV

$$P_{eq} \sim \exp\left(8\pi\lambda\varphi^4 / 3H^4 \right)$$

$$\Rightarrow \varphi_{eq} \sim H\lambda^{-1/4}$$

Perturbative expansion

$$\Rightarrow \Omega_n^{(m)} \sim \lambda^{n+m} \quad + \quad c_\ell \sim \lambda^{\ell-3}$$

Corrections About Fixed Point

LO $(\lambda^{1/2})$

$$\frac{\partial P}{\partial t} = \frac{H^3}{8\pi^2}\frac{\partial^2 P}{\partial \varphi^2} + \frac{1}{3H}\frac{\partial}{\partial \varphi}\left[\frac{1}{3!}\lambda \varphi^3 P\right]$$

NLO (λ)

$$\frac{\partial P}{\partial t} = \cdots + \frac{\partial^2}{\partial \varphi^2}\left[\Omega_2^{(2)}\varphi^2 P\right] + \frac{1}{3H}\frac{\partial}{\partial \varphi}\left[\frac{1}{5!}c_6\varphi^5 P\right]$$

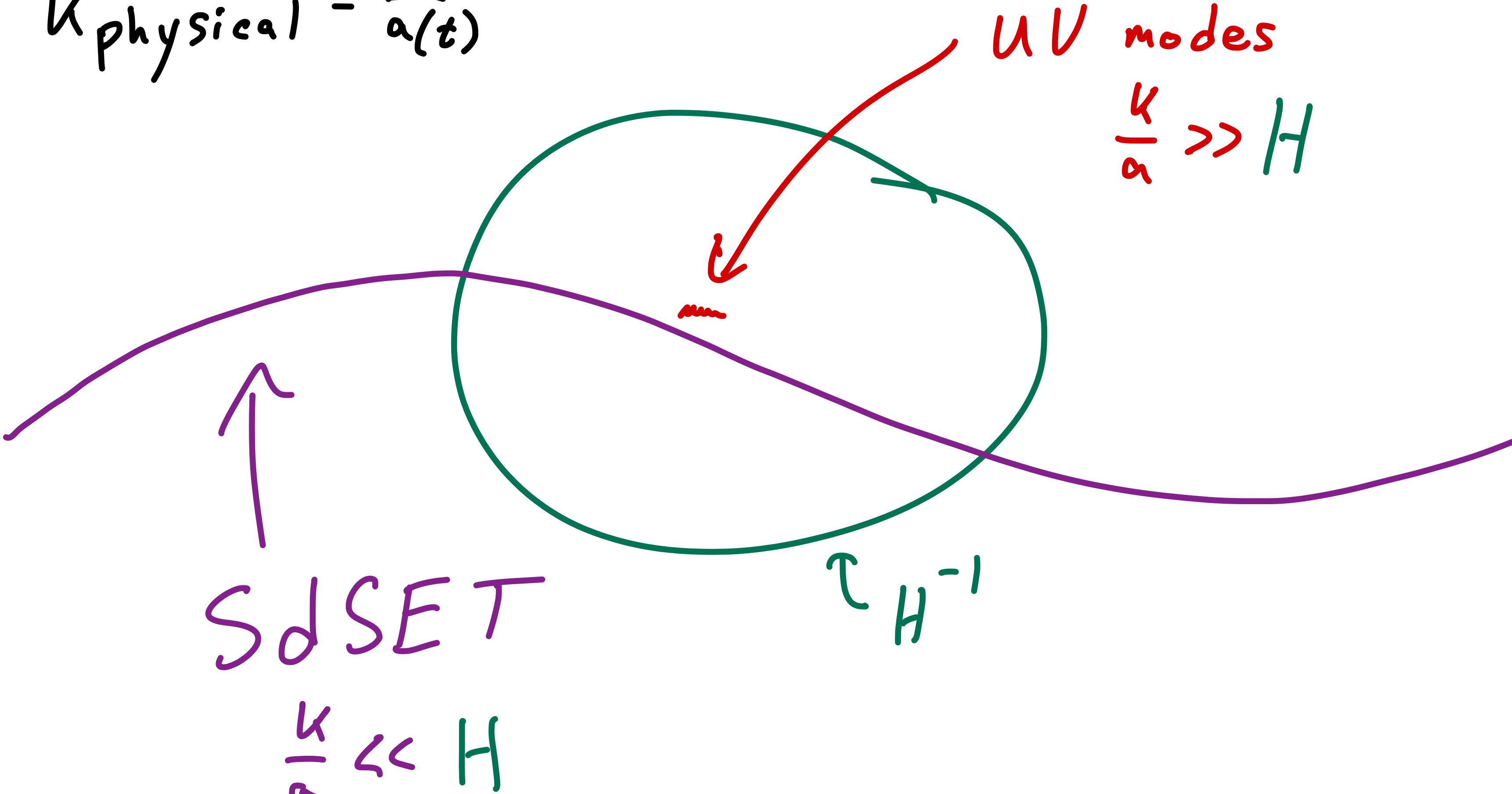
NNLO $(\lambda^{3/2})$

$$\frac{\partial P}{\partial t} = \cdots + \frac{\partial^2}{\partial \varphi^2}\left[\Omega_2^{(4)}\varphi^4 P\right] + \frac{1}{3H}\frac{\partial}{\partial \varphi}\left[\frac{1}{7!}c_8\varphi^7 P\right] + \frac{\partial^3}{\partial \varphi^3}\left[\Omega_3^{(1)}\varphi P\right]$$

Soft de Sitter Effective Theory

arXiv: 2007.03693

$$k_{physical} = \frac{k}{a(t)}$$



Confusions Abound

- Want to expose late time and long wavelength behavior of (in-in) correlation functions

- Calculate in a frame
  $\Rightarrow$ space + time treated independently

- Full theory calculations use hard cutoffs

Applications of SdSET

1) Correlators for massive scalars in dS
"Physics beyond the horizon is irrelevant"

2) Starobinsky's stochastic inflation
"Resum marginal operators using RG"

3) Metric fluctuations during inflation
"Power counting $\Rightarrow$ superhorizon modes freeze out"

4) Eternal inflation
"Tower of relevant operators appear $\Rightarrow$ novel phase"

Why EFT?

dS provides natural "ruler":
the inverse comoving horizon $\Lambda_{UV} = aH$

Interested in long wavelengths $k \ll aH$

Large seperation of scales
$\Longrightarrow$ power counting $k/(aH)$

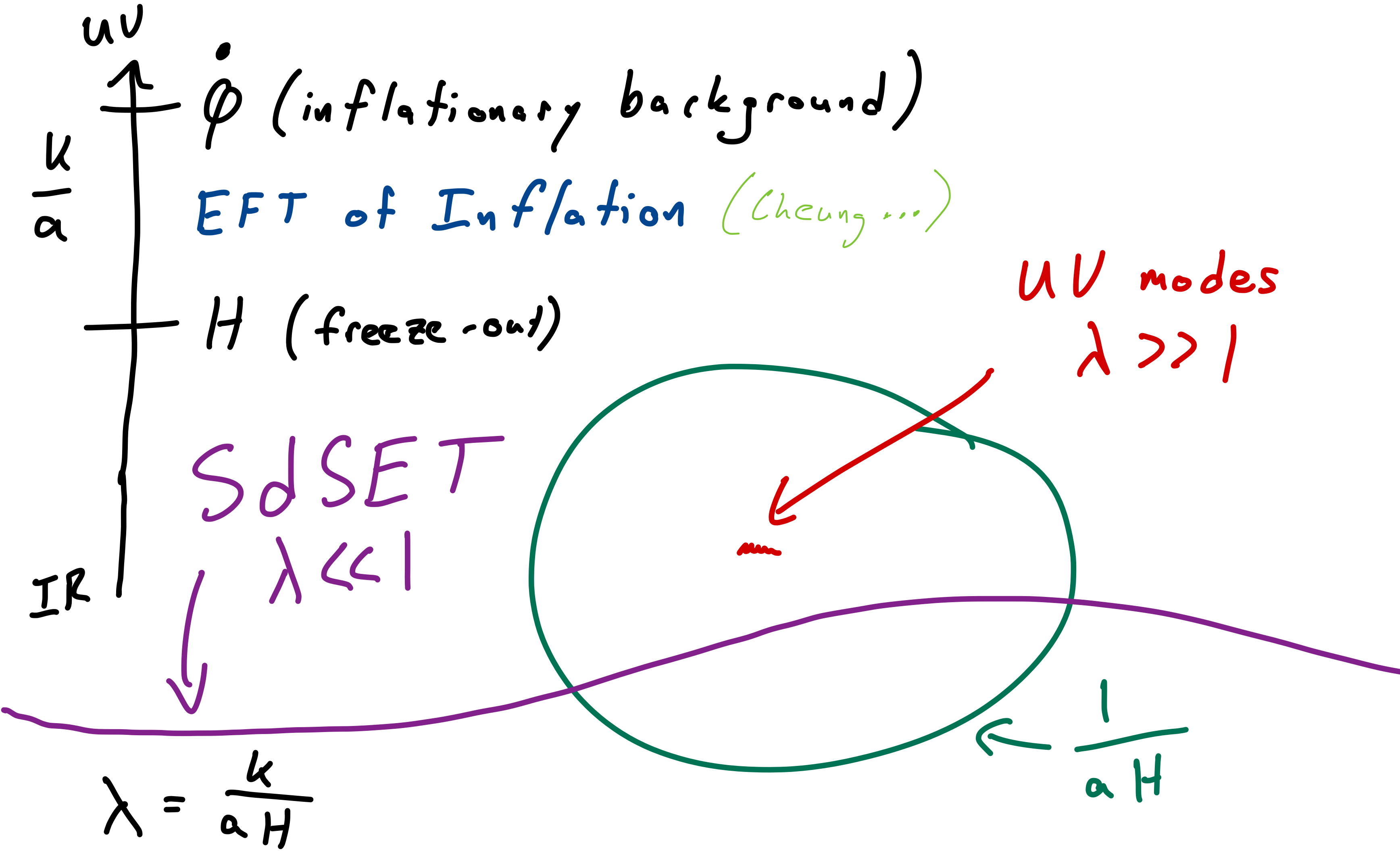
(Continuum) EFT

Conceptual

- Isolate propagating DOFs
- Quadratic action $\Rightarrow$ dynamics
- Expose symmetries

Practical

- Power counting $\simeq$ dim analysis
- Regulate integrals w/o breaking symmetries
- RG sums full theory IR logs


UV
$\dot{\phi}$ (inflationary background)
$\frac{k}{a}$
EFT of Inflation (Cheung ...)
H (freeze-out)
SdSET
$\lambda \ll 1$
IR
UV modes
$\lambda \gg 1$
$\frac{1}{aH}$
$\lambda = \frac{k}{aH}$

One-to-many Mode Expansion

Factorize into soft and hard modes

$$\phi(\vec{x},t) = \phi_S(\vec{x},t) + \Phi_H(\vec{x},t)$$

Integrate out hard modes

$\Rightarrow$ Local operator expansion

Observables order-by-order in power counting

dS Spacetime

dS metric: $ds^2 = -dt^2 + a^2(t)\,d\vec{x}^2$

proper time

Notation: $\underline{t} \equiv Ht$

& $\tau = -\exp(-\underline{t})/H$

conformal time

Soft de Sitter Effective Theory

Full theory

$$S_\phi = \int d^3x\, d\underline{t}\, \frac{(a(\underline{t})H)^3}{H^4}\left[-\frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4\right]$$

SdSET

$$S_{\underline{\pm}} = \int d^3x\, d\underline{t}\left[-\nu\left(\dot{\varphi}_+\varphi_- - \varphi_+\dot{\varphi}_-\right) - \sum_{n\geq 2}(aH)^{3-n\alpha-\beta}\frac{c_{n,1}}{n!}\varphi_+^n\varphi_-\right]$$

Scalar fields in dS

EOM $\quad \ddot{\phi} + 3\dot{\phi} + \frac{k^2}{(aH)^2}\phi + \frac{m^2}{H^2}\phi = 0$

Soft limit $\quad \phi_s = (aH)^{-3/2+\nu}\varphi_s$

w/ $\quad \nu = \pm\sqrt{\frac{9}{4} - \frac{m^2}{H^2}}$

or $\quad \alpha = \frac{3}{2} - \nu \quad \beta = \frac{3}{2} + \nu \quad$ s.t. $\quad \alpha + \beta = 3$

WLOG $\quad \alpha < \beta$

SdSET Fields

Two IR degrees of freedom

- "Growing" mode φ_+ ← Correlators of interest
- "Decaying" mode φ_-

w/ $\phi_s = H\left((aH)^{-\alpha}\varphi_+ + (aH)^{-\beta}\varphi_-\right)$

Time dependence factorizes :)

Canonical Quantization

Full theory quantum field $\phi = \int \left(\bar{\phi} a^{\dagger} + \bar{\phi}^{*} a \right)$

w/ $\left[a^{\dagger}_{\vec{k}} , a_{\vec{k}'} \right] = (2\pi)^3 \, \delta(\vec{k} - \vec{k}')$

Bunch-Davies $\bar{\phi} = -i e^{i\left(\nu + \frac{1}{2}\right)\frac{\pi}{2}} \frac{\sqrt{\pi}}{2} H(-\tau)^{3/2} H^{(1)}_{\nu}(-k\tau)$

Take Soft Limit

Expand ϕ for $k\tau \ll 1$

$$\phi_s \simeq \int \Big[(aH)^{-\alpha}\, \bar{\varphi}_+ \overbrace{\left(e^{i\delta_r} a^\dagger + e^{-i\delta_r} a\right)}^{\tilde{a}} + (aH)^{-\beta}\, \bar{\varphi}_- \underbrace{\left(i e^{-i\delta_v} a^\dagger - i e^{i\delta_v} a\right)}_{\tilde{b}} \Big]$$

w/ $\bar{\varphi}_+ \sim \dfrac{1}{k^{3/2-\alpha}}$ + $\bar{\varphi}_- \sim \dfrac{1}{k^{3/2-\beta}}$

Stocastic Random Variables

$\tilde{a}_{\vec{k}}$ and $\tilde{b}_{\vec{k}}$ are real

Satisfy $[\tilde{a}^\dagger, \tilde{a}] = [\tilde{b}^\dagger, \tilde{b}] = 0$

$$\langle \tilde{a}_{\vec{k}} \tilde{a}_{\vec{k}'} \rangle = \langle \tilde{b}_{\vec{k}} \tilde{b}_{\vec{k}'} \rangle = (2\pi)^3 \delta(\vec{k} + \vec{k}')$$

$a_{\vec{k}} |0\rangle$

Initial Conditions

Identify operators

$$\varphi_+ = \int \bar{\varphi}_+ \, \tilde{a}$$

$$\varphi_- = \int \bar{\varphi}_- \, \tilde{b}$$

Endowed with classical power spectra

$$\left\langle \varphi_+(\vec{k})\, \varphi_+(\vec{k}') \right\rangle \sim \frac{1}{k^{3-2\alpha}} (2\pi)^3 \delta(\vec{k}+\vec{k}')$$

Augmented by non-Gaussian corrections from UV

Defining SdSET

- DOF φ_+ and φ_-
- Power counting $\sim \dfrac{k}{\Lambda_{uv}}$ w/ $\Lambda_{uv} = aH$
- Symmetries
  (1) "spacetime"
  (2) "reparametrization"
- Initial conditions

* Very close analogy w/ Heavy Quark EFT

Leading Interactions

$$S'_{int} \supset -\int (aH)^{3-n\alpha} \frac{c_n}{n!} \varphi_+^n$$

can be removed with

$$\varphi_- \to \varphi_- + \frac{n c_n}{3\nu(3-n\alpha)n!} (aH)^{3-n\alpha} \varphi_+^{n-1}$$

Powercounting Interactions

$$S_{int} \supset -\int (aH)^{3-n\alpha-m\beta} \frac{c_{n,m}}{n!\,m!} \varphi_+^n \varphi_-^m$$

$$\sim \left(\frac{k_1}{aH}\right)^{n\alpha+m\beta-3} \quad \text{w/} \quad m \geq 1$$

relevant $n\alpha + m\beta < 3$

marginal $n\alpha + m\beta = 3$

irrelevant $n\alpha + m\beta > 3$

Light Scalars in dS

Light Scalars in dS

As $m^2 \to 0$, $\alpha \to 0$

$$S_{int} = -\int \sum_{n>1} \frac{c_n}{n!} (aH)^{(1-n)\alpha} \varphi_+^n \varphi_-$$

$$\sim \lambda^{(n-1)\alpha} \to \lambda^0$$

$\Rightarrow$ Tower of marginal interactions

EOM: $$2\nu \dot{\varphi}_+ = -\sum_{n>1} (aH)^{(2-n)\alpha} \frac{c_n}{n!} \varphi_+^n$$

Dynamical Dimensional Regularization

Compute correlators of composite operators

Often encounter $\int \frac{d^d p}{p^d}$

$\Rightarrow$ dim reg fails

Introduce "dyn dim reg"

Trick: analytically continue in α

$$\Rightarrow \int \frac{d^d p}{p^d} \longrightarrow \int \frac{d^d p}{p^{d+\delta\alpha}}$$

Light Scalars in dS

Composite operators

$$\mathcal{O}_n = \varphi_+^n \sim \left(k/aH\right)^{n\alpha} \to \mathcal{O}(1)$$

RG mixing expected

Contract any two legs

$$\langle \mathcal{O}_n \ldots \rangle \supset \langle \mathcal{O}_{n-2} \ldots \rangle \binom{n}{2} \frac{C_\alpha^2}{2} \int \frac{d^3p}{(2\pi)^3} \frac{H^{2-2\alpha}}{p^{3-2\alpha}}$$

Light Scalars in dS

$$\int \frac{d^3p}{(2\pi)^3} \frac{H^{2-2\alpha}}{p^{3-2\alpha}}$$ is scaleless and diverges as $\alpha \to 0$

Isolate UV divergence

$$p^2 \to p^2 + \bar{k}_{IR}^2$$

$$\langle \mathcal{O}_n \ldots \rangle \supset \langle \mathcal{O}_{n-2} \cdots \rangle \binom{n}{2} \frac{C_\alpha^2}{4\pi^2} \left(-\frac{1}{2\alpha} - \gamma_E - \log \frac{aH}{\bar{k}_{IR}} \right)$$

Dynamical RG $\Leftrightarrow$ Stocastic Inflation

Resum time dependent logs:

$$\frac{\partial}{\partial t}\langle \mathcal{O}_n \ldots \rangle = -\frac{n}{3}\sum_{m\geq 1}\frac{c_m}{m!}\langle \mathcal{O}_{n-1}\,\mathcal{O}_m \ldots \rangle + \frac{n(n-1)}{8\pi^2}\langle \mathcal{O}_{n-2}\ldots \rangle$$

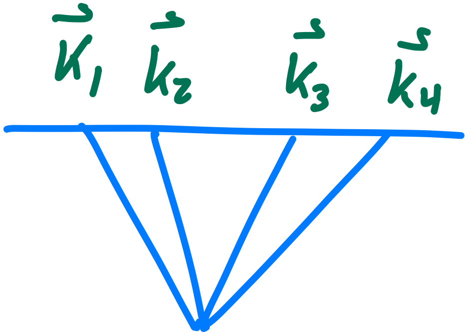
(Starobinsky; Starobinsky, Yokoyama)

Is equivalent to a Fokker-Planck eq

for $P(\varphi,t)$ w/ $\langle \varphi^n \rangle = \int d\varphi\, P(\varphi,t)\,\varphi^n$ (Baumgart + Sundrum)

Matching and Running

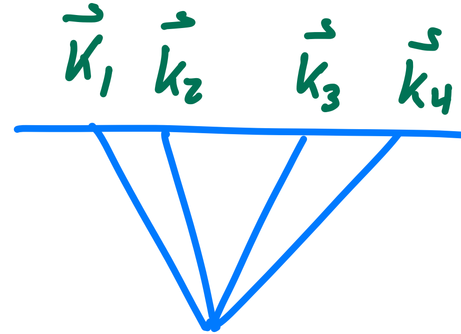
Tree Matching



ϕ^4

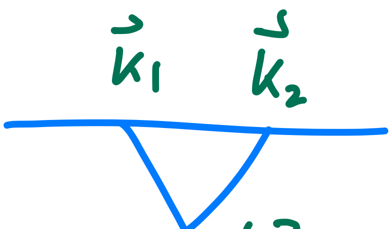
$$=$$

$$\lambda = C_{3,1}$$



$\varphi_+^3 \varphi_1$

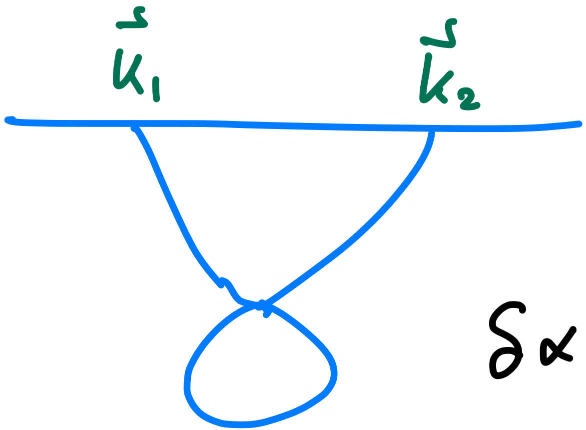
Initial Conditions



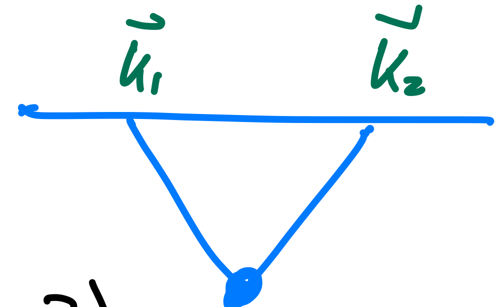
ϕ^2

$$= \langle \varphi_+(k_1)\, \varphi_+(k_2) \rangle_{IC}^{Gauss}$$

One Loop Matching



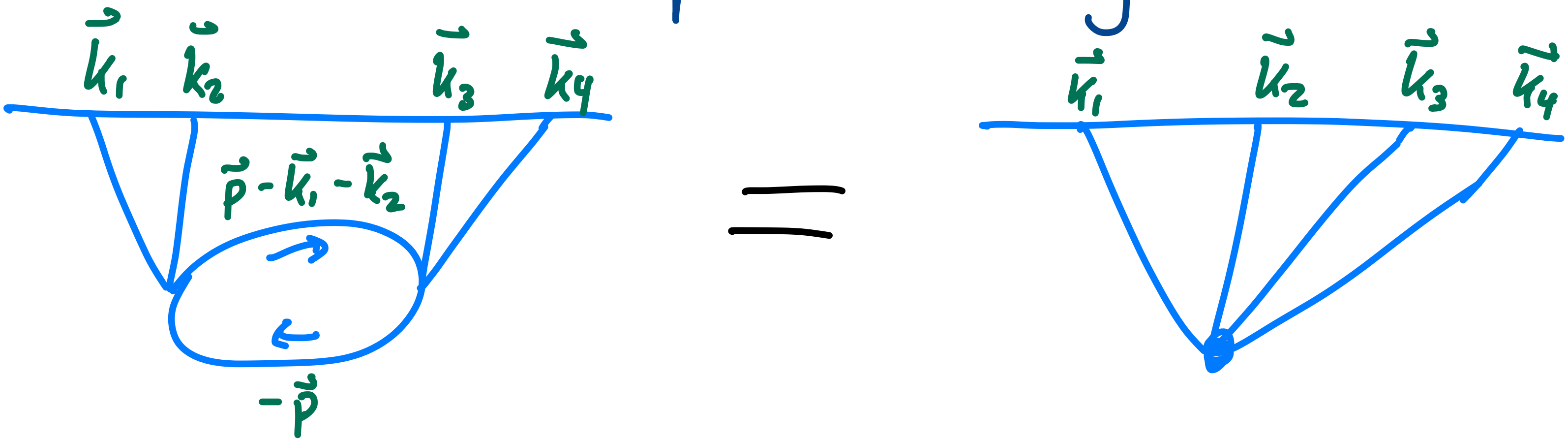
$$=$$



$$\delta\alpha = \frac{\lambda}{8\pi^2}\,\frac{1}{3}\left(\gamma_E - \frac{7}{3}\right)$$

Other terms removed by $\varphi_- \to \varphi_- + \frac{\lambda}{9}(aH)^3\varphi_+^2$

One Loop Matching

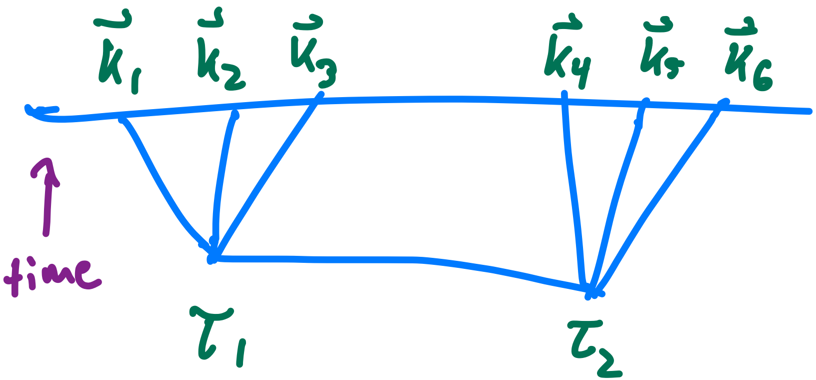


$$C_{3,1} = \lambda - \frac{\lambda^2}{4\pi^2}\left(\frac{1}{9}\gamma_E(2+3\gamma_E) + \frac{5}{12}\pi^2\right)$$

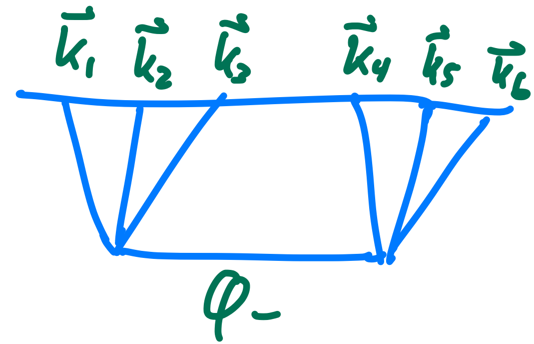
$+\ \mathcal{O}(\lambda^2)$ impact on initial conditions

(contributes to NNNLO RG)

Initial Conditions to λ^2

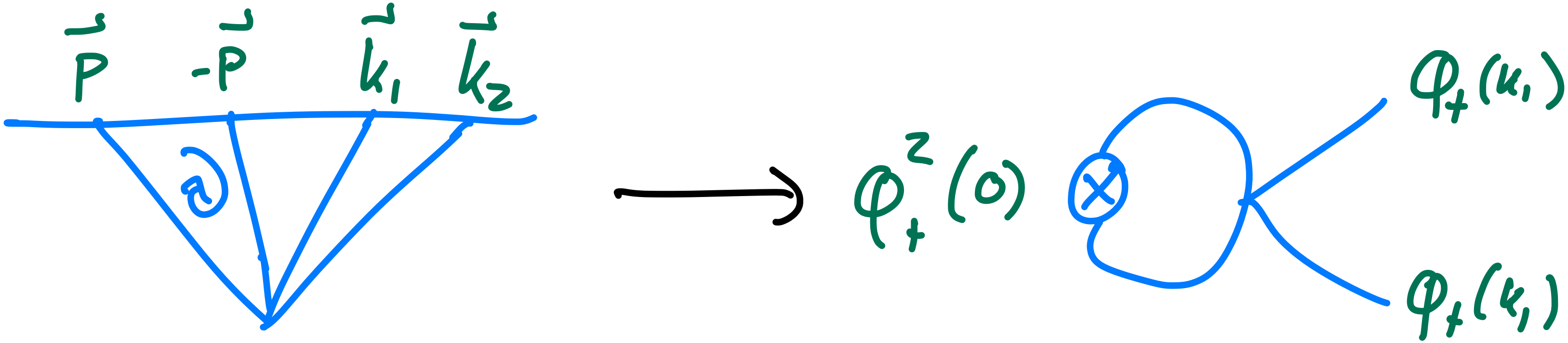


$$= \atop \mathcal{O}(\lambda^2)$$



$$+ \left\langle \varphi_+(\vec{k}_1) \dots \varphi_+(\vec{k}_6) \right\rangle_{IC}$$

NLO $\varphi_+^2(0)$ RG

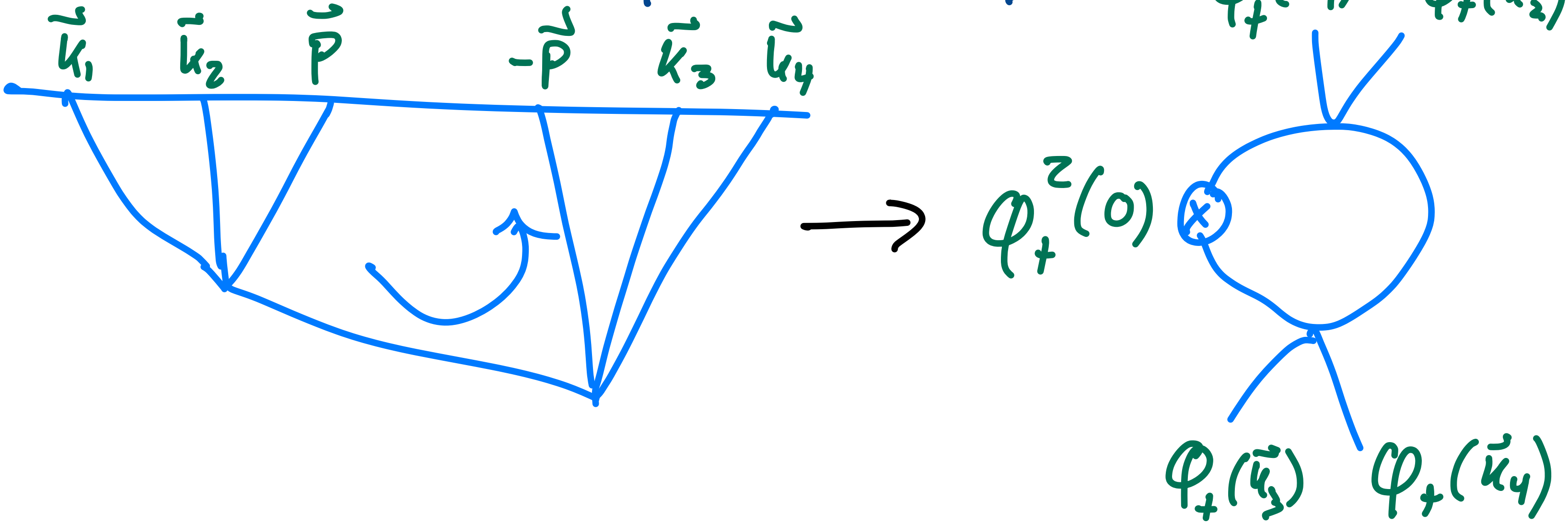


IR

$$\left\langle \varphi_+^2[\vec{x}=0]\,\varphi_+(\vec{k}_1)\,\varphi_+(\vec{k}_2) \right\rangle$$

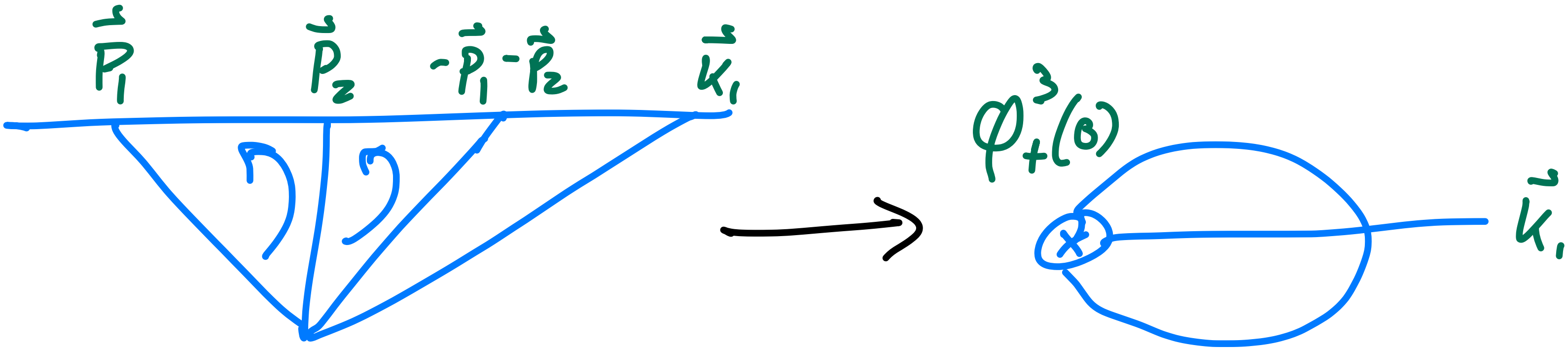
$$= \lambda P(k_1) P(k_2) \left(\frac{1}{48\pi^2\alpha^2} + \frac{(4 - 3\gamma_E - 3\log K)}{72\pi^2\alpha} + \text{finite} \right)$$

NNLO $\varphi_+^2(0)$ RG



$$\left\langle \varphi_+^2[\vec{x}=0]\,\varphi_+(k_1)\ldots\varphi_+(k_4)\right\rangle \sim \frac{\lambda^2}{8\pi^2\alpha}\,\frac{1}{27}\Big[16+4\gamma_E(-11+3\gamma_E)+3\pi^2+12(\log 2)^2 + \ldots\Big]$$

NNLO $\varphi^3(0)$ RG



$$\left\langle \varphi_+^3[\vec{x}=0]\,\varphi_+(\vec{k}_1)\right\rangle = \frac{\lambda}{16\pi^2}\frac{1}{12}P(k_1)\left[\frac{1}{\alpha}+\cdots\right]$$

Outlook

Stochastic Inflation at NNLO

$$\frac{\partial}{\partial \underline{t}} P(\varphi_+, \underline{t}) = \frac{1}{3}\frac{\partial}{\partial \varphi_+}\left[\partial_{\varphi_-} V(\varphi_+, \varphi_-)\Big|_{\varphi_-=0} P(\varphi_+, \underline{t})\right]$$

$$+ \frac{\partial^2}{\partial \varphi_+^2}\left\{\left(b_0 + b_1 \varphi_+^2 + b_2 \varphi_+^4\right) P(\varphi_+, \underline{t})\right\}$$

$$+ \frac{\partial^3}{\partial \varphi_+^3}\left(d_0 \varphi_+ P(\varphi_+, \underline{t})\right)$$

w/ $V = \frac{\lambda}{3}\varphi_-\left(\varphi_+^3 + \frac{\lambda}{9}\varphi_+^5 + \frac{\lambda^2}{81}\varphi_+^7 + \cdots\right)$

$b_1 = \frac{\lambda}{72\pi^2}(4 - 3\gamma_E)$ | $b_2 = \frac{1}{8\pi^2}\frac{\lambda^2}{27}[\cdots]$ | $d_0 = \frac{\lambda}{192\pi^2}$

One more field redefinition...

$$\varphi_+ \to \varphi_+ - \frac{b_1}{6b_0}\varphi_+^3 + \frac{3b_1^2 - 4b_0 b_2}{4b_0^2}\varphi_+^5$$

$$\frac{\partial}{\partial \underline{t}} P = \frac{1}{3}\frac{\partial}{\partial\varphi_+}\left[V_{eff}' P\right] + \frac{1}{8\pi^2}\frac{\partial^2}{\partial\varphi_+^2}P + \frac{\lambda_{eff}}{192\pi^2}\frac{\partial^3}{\partial\varphi_+^3}(\varphi_+ P)$$

$$V_{eff} = \frac{\lambda_{eff}}{3!}\left(\varphi_+^3 + \frac{\lambda_{eff}}{18}\varphi_+^5 + \frac{\lambda_{eff}}{162}\varphi_+^7 + \dots\right)$$

$$\lambda_{eff} = \lambda - 12b_2 - \frac{\lambda^2}{2\pi^2}\left(\frac{1}{3}\gamma_E(2+3\gamma_E) + \frac{5\pi^2}{4}\right)$$

Outlook

It works!!

$$S_{\pm} = \int d^3x \, dt \left[-\nu \left(\dot{\varphi}_+ \varphi_- - \varphi_+ \dot{\varphi}_- \right) - \sum_{n \geq 2} (aH)^{3-n\alpha-\beta} \frac{c_{n,1}}{n!} \varphi_+^n \varphi_- \right]$$

Outlook

In forthcoming paper:

- $P_{eq}(\varphi, t)$ perturbatively to order $\lambda^{3/2}$
- Relaxation eigenvalues to order $\lambda^{3/2}$

Future work:

Long wavelength limit of inflationary correlators

Applications to pheno

Explore SdSET

Backup

Summary of SdSET

1) Correlators for massive scalars in dS

"Physics beyond the horizon is irrelevant"

2) Starobinsky's stochastic inflation

"Resum marginal operators using RG"

3) Metric fluctuations during inflation

"Power counting $\Rightarrow$ superhorizon modes freeze out"

4) Eternal inflation

"Tower of relevant operators appear $\Rightarrow$ novel phase"

Power Counting

$$\lambda \sim \frac{k}{aH}$$

$+$

$$S \sim \lambda^0$$

$\Longrightarrow$

$$t \sim 1$$

$$\vec{x} \sim 1/\lambda$$

$$\vec{k} \sim \lambda$$

$$\varphi_+ \sim \lambda^\alpha$$

$$\varphi_- \sim \lambda^\beta$$

Symmetries

(1) Spacetime

$$t \to t \qquad \vec{k} \to \eta \vec{k}$$

$$\vec{x} \to \frac{1}{\eta} \vec{x} \qquad \varphi_+ \to \eta^{\alpha} \varphi_+$$

$$a \to \eta\, a \qquad \varphi_- \to \eta^{\beta} \varphi_-$$

(+ additional isometry transformation for static dS)

(2) Reparametrization invariance (RPI)

$$\varphi_+ \to \varphi_+ + (aH)^{\alpha-\beta} \varphi_-$$

$$\varphi_- \to (1-\varepsilon)\, \varphi_-$$

Free SdSET from Top Down

Plug $\phi_s = H\left(a^{-\alpha}\varphi_+ + a^{-\beta}\varphi_-\right)$ into S'_ϕ

Combine terms using int by parts and e.g. $H^2(\alpha^2 - 3\alpha) + m^2 = 0$, $\alpha + \beta = 3$, ...

$$\Rightarrow \quad S_{2,\pm} = \int \mathrm{d}^3x\,\mathrm{d}\underset{\uparrow}{t}\,\frac{1}{2}\Big[[aH]^{2\nu}\dot{\varphi}_+^2 + [aH]^{-2\nu}\dot{\varphi}_-^2 + 2\dot{\varphi}_+\dot{\varphi}_- - 2\nu\left(\dot{\varphi}_+\varphi_- - \varphi_+\dot{\varphi}_-\right)$$
$$- [aH]^{2\nu-2}\partial_i\varphi_+\partial^i\varphi_+ - [aH]^{-2\nu-2}\partial_i\varphi_-\partial^i\varphi_- - 2[aH]^{-2}\partial_i\varphi_+\partial^i\varphi_-\Big]$$

$\underline{t}$

Free SdSET from Top Down

Drop subleading terms

$$S_{2,\pm} = \int \mathrm{d}^3x\,\mathrm{d}t\,\frac{1}{2}\Big[[aH]^{2\nu}\dot\varphi_+^2 + \cancel{[aH]^{-2\nu}\dot\varphi_-^2} + 2\dot\varphi_+\dot\varphi_- - 2\nu\left(\dot\varphi_+\varphi_- - \varphi_+\dot\varphi_-\right)$$
$$- [aH]^{2\nu-2}\partial_i\varphi_+\partial^i\varphi_+ - \cancel{[aH]^{-2\nu-2}\partial_i\varphi_-\partial^i\varphi_-} - 2[aH]^{-2}\partial_i\varphi_+\partial^i\varphi_-\Big]$$

Int by parts and let $\varphi_- \to \varphi_- + \frac{1}{2}(aH)^{2\nu}\varphi_+$

$$S_{2,\pm} = \int \mathrm{d}^3x\,\mathrm{d}t\,\Big[\dot\varphi_+\dot\varphi_- - \nu\left(\dot\varphi_+\varphi_- - \varphi_+\dot\varphi_-\right) - [aH]^{-2}\partial_i\varphi_+\partial^i\varphi_-\Big]$$

Treat as interaction $\Rightarrow \mathcal{O}(\lambda^4)$

(Weinberg)

Free SdSET

$$S_{2,\pm} = \int \mathrm{d}^3x\,\mathrm{d}\mathtt{t}\left[-\nu\left(\dot{\varphi}_+\varphi_- - \varphi_+\dot{\varphi}_-\right) - \frac{1}{[a\,H]^2}\partial_i\varphi_+\partial^i\varphi_-\right] + \mathcal{O}(\lambda^4)$$

$$\Downarrow$$

$$\dot{\varphi}_+ = \frac{1}{2\nu[a\,H]^2}\partial^2\varphi_+ \qquad\qquad \dot{\varphi}_- = -\frac{1}{2\nu[a\,H]^2}\partial^2\varphi_-$$

Leading power solutions are constant

Locality

- $\varphi_\pm$ & Φ_H are momentum eigenstates
- Momentum conservation $\Rightarrow$ $\cancel{\mathcal{L} \supset \varphi_\pm^3 \Phi_H}$
- Leading effect of integrating out Φ_H occurs at 1-loop

$$\delta S \sim \int \Phi_H^2(\vec{x},\tau)\, \Phi_H^2(\vec{y},\tau') \underset{\substack{\uparrow \\ FT + k \ll p}}{\sim} e^{-p|\tau-\tau'|}$$

$\Rightarrow$ Local interaction

Powercounting Interactions

$$S_{int} \supset -\int (aH)^{3-n\alpha-m\beta} \frac{c_{n,m}}{n!\,m!} \varphi_+^n \varphi_-^m$$

$$\sim \lambda^{(n-1)\alpha+(m-1)\beta} \quad \text{w/} \quad m \geq 1$$

$$m^2 \neq 0 \iff 0 < \alpha < 3/2 \quad \text{and} \quad 3/2 < \beta < 3$$

Interactions are $\mathcal{O}(\lambda) \Rightarrow$ irrelevant

$$m^2 \to 0 \Rightarrow \alpha \to 0 \ \ldots$$

Applications

Massive Scalars in dS

- Massive Scalars are "free"

  Interactions are irrelevant

- See paper for variety of calculations

  (Green and Premkumar)

- Non-trivial role of initial conditions

Metric Fluctuations During Inflation

Power counting + symmetries

$\Rightarrow$ no time dependance

$$ds^2 = -N^2 \mathrm{d}t^2 + a^2(t)\, e^{2\zeta(\vec{x},t)} \left(e^{2\gamma(\vec{x},t)}\right)_{ij} \left(\mathrm{d}x^i + N^i \mathrm{d}t\right)\left(\mathrm{d}x^j + N^j \mathrm{d}t\right)$$

ζ is adiabatic scalar fluctuation

γ_{ij} is tensor fluctuation

N + N^i are Lagrange multipliers (Arnowitt Deser Misner)

Metric Fluctuations During Inflation

Full theory quadratic Lagrangian

$$\mathcal{L}_{2,\zeta} = -\frac{M_{\rm pl}^2 \partial_t H}{H^2 c_s^2}\left(\partial_t \zeta^2 - a^{-2} c_s^2 \partial_i \zeta \partial^i \zeta\right)$$

Many non-linear symmetries, e.g.

$$\vec{x} \to e^{-\eta}\vec{x} \implies \zeta \to \zeta(e^{-\eta}\vec{x}) - \eta$$

Can take SdSET limit w/ $\zeta \to \zeta_+ + \ldots$

Metric Fluctuations During Inflation

What types of operators could cause time evolution?

$$\mathcal{L}_{int} \supset \cancel{\zeta_+^{n-1}\zeta_-} + \cdots$$

Violate shift symmetries

$\Rightarrow$ Only allowed terms involve $\frac{\partial_i}{aH}\zeta_\pm$

$\Rightarrow$ Power suppressed!

Full theory argument: Salopek + Bond; Maldacena (tree)
Assassi, Baumann, Green; Senatore, Zaldarriaga (all orders)

Eternal Inflation

Metric dynamics are important

$$\Rightarrow \quad S'_{int} \supset \int \sqrt{-g}\,(aH)^{-n\alpha}\,\frac{c_n}{n!}\,\varphi_+^n$$

Can not be removed by field redef

(int by parts generates gravitional contributions)

As $\alpha \to 0$, $\varphi_+^n \sim 1/\lambda^3 \Rightarrow$ relevant operators

$\Rightarrow$ Non-perturbative!

Summary