Direct detection of sub-GeV dark matter with the Migdal effect in semiconductors

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November 30, 2020
UC Davis seminar

Motivation

Traditional approach to direct detection of dark matter: DM-nucleus scattering

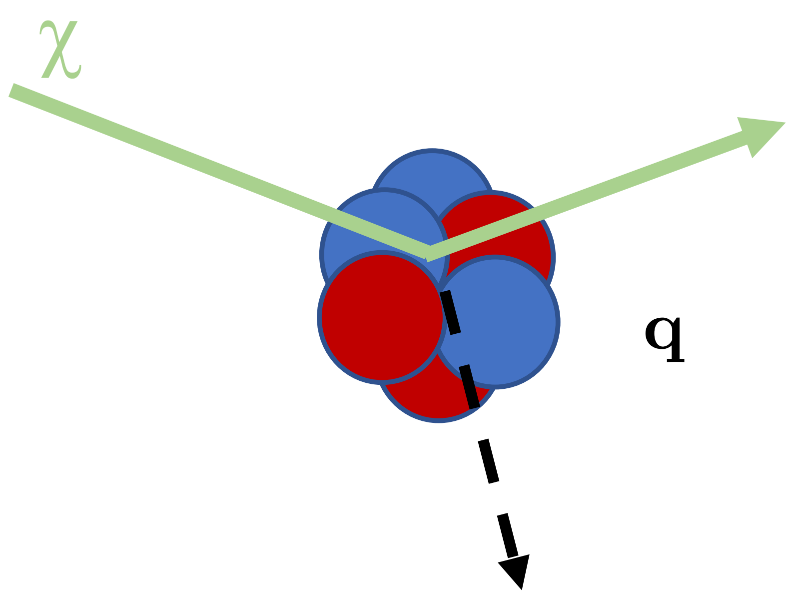
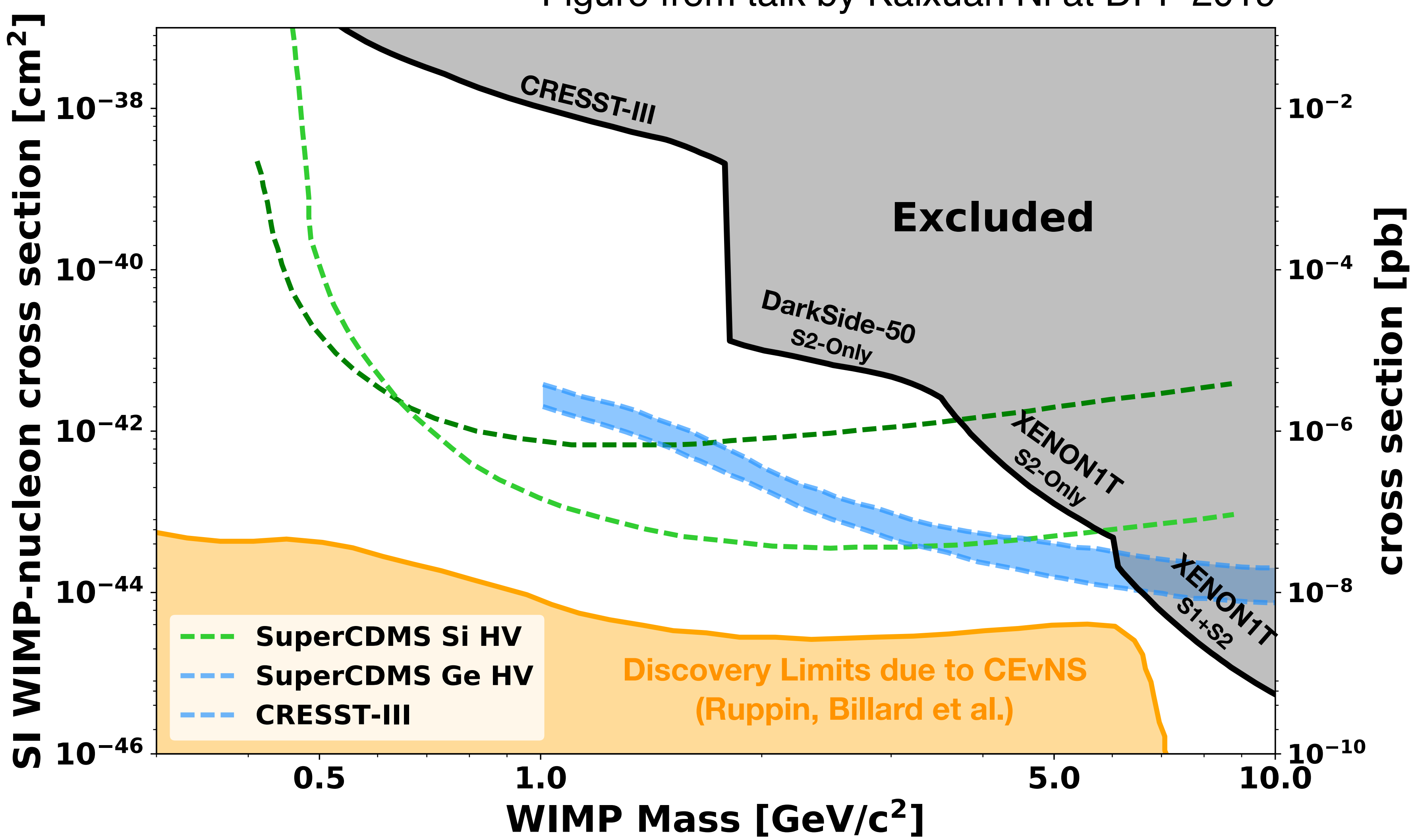


Figure from talk by Kaixuan Ni at DPF 2019



Challenges for sub-GeV DM

Kinematics of nuclear recoils from light dark matter

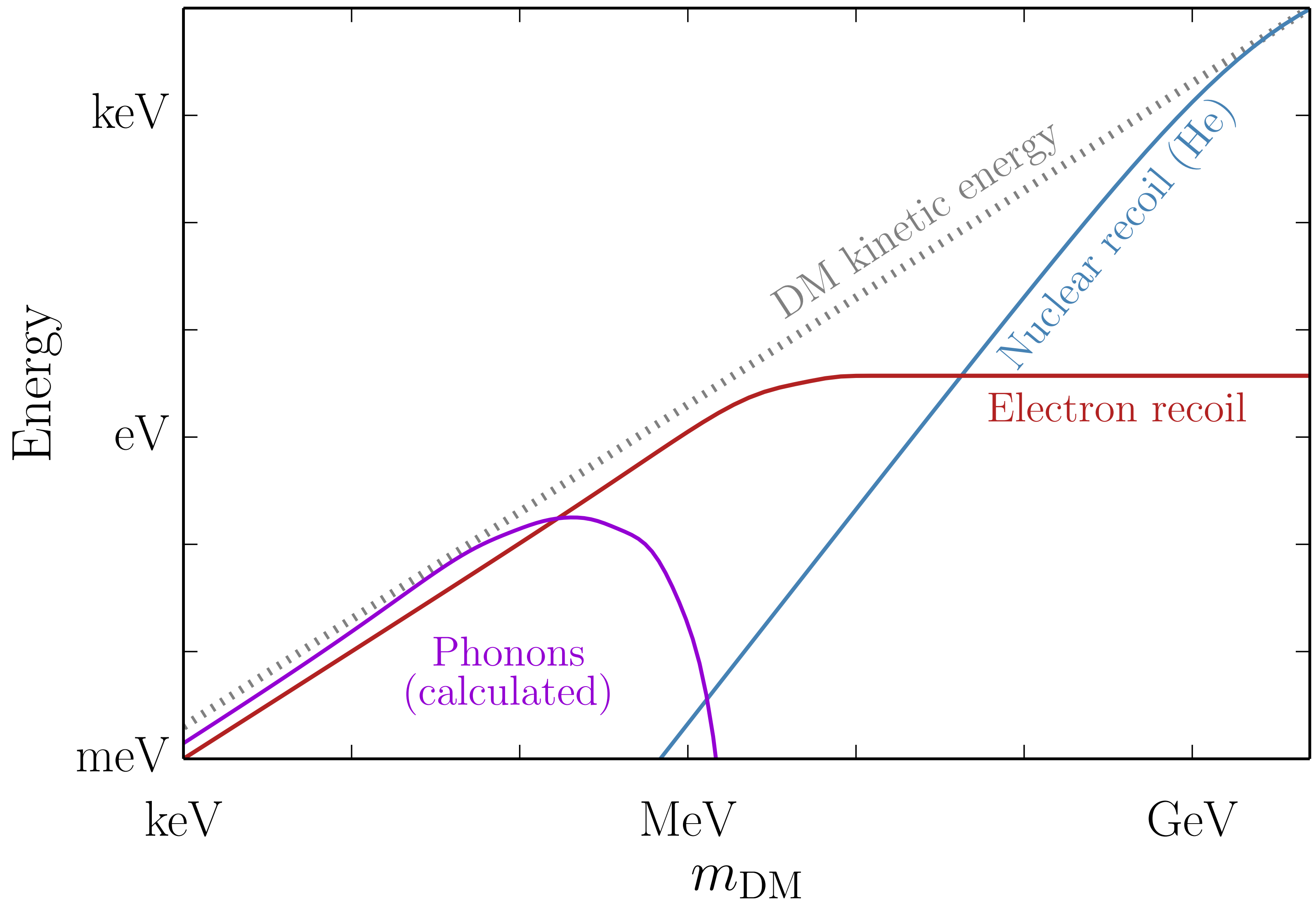
$$E_R = \frac{|\mathbf{q}|^2}{2m_N} \leq \frac{2\mu_{\chi N}^2 v^2}{m_N}$$

Drops quickly below $m_\chi \sim 10$ GeV

Best nuclear recoil threshold is currently $E_R > 30$ eV (CRESST-III) with DM reach of $m_\chi > 160$ MeV.

The kinematics of DM scattering against **free** nuclei is inefficient, and it does not always describe target response accurately.

Material properties matter

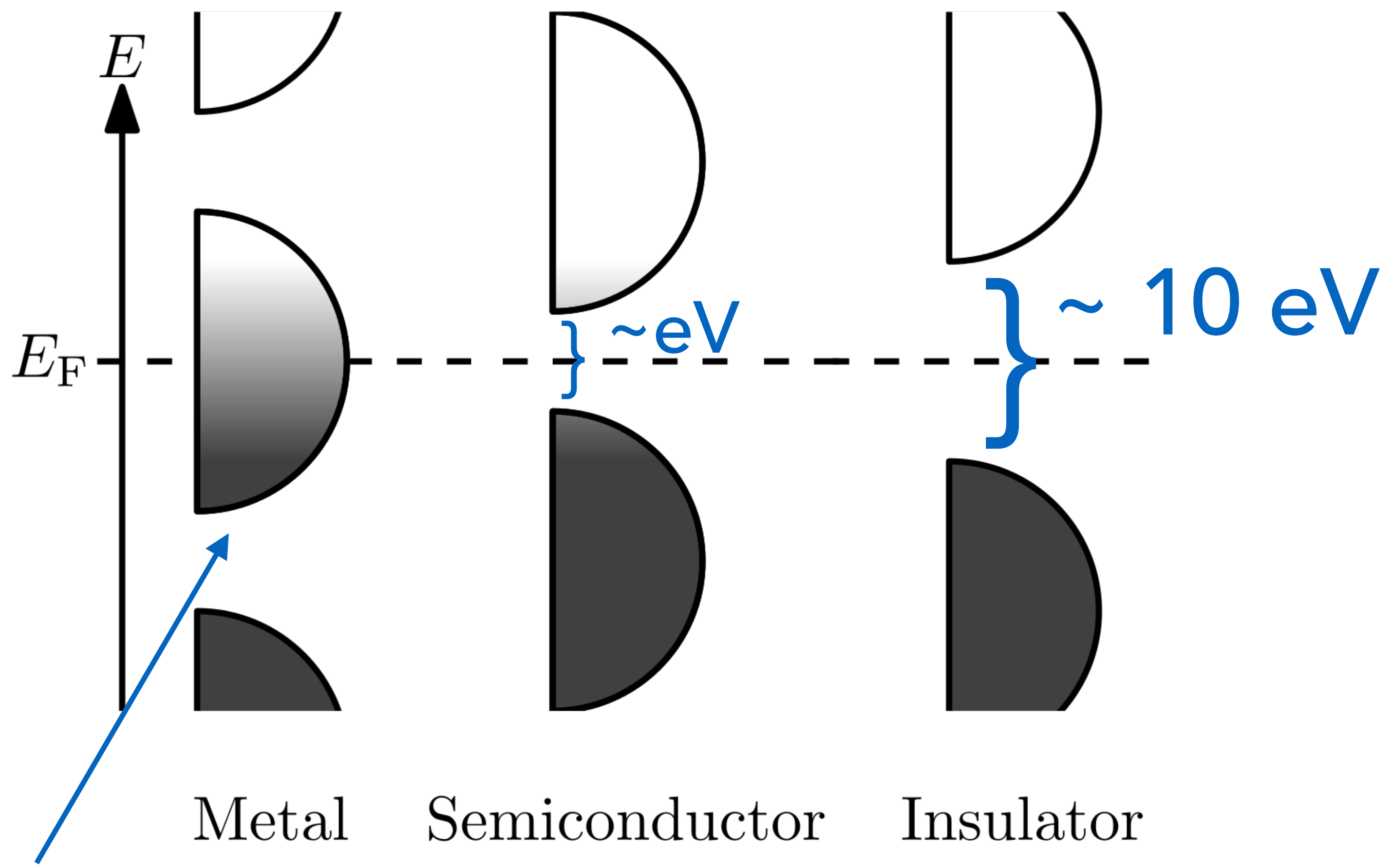


Nuclear response is phonon-dominated at low energies.

Electronic response depends on details of band structure/eigenstates.

Electron recoils

Electronic band structure

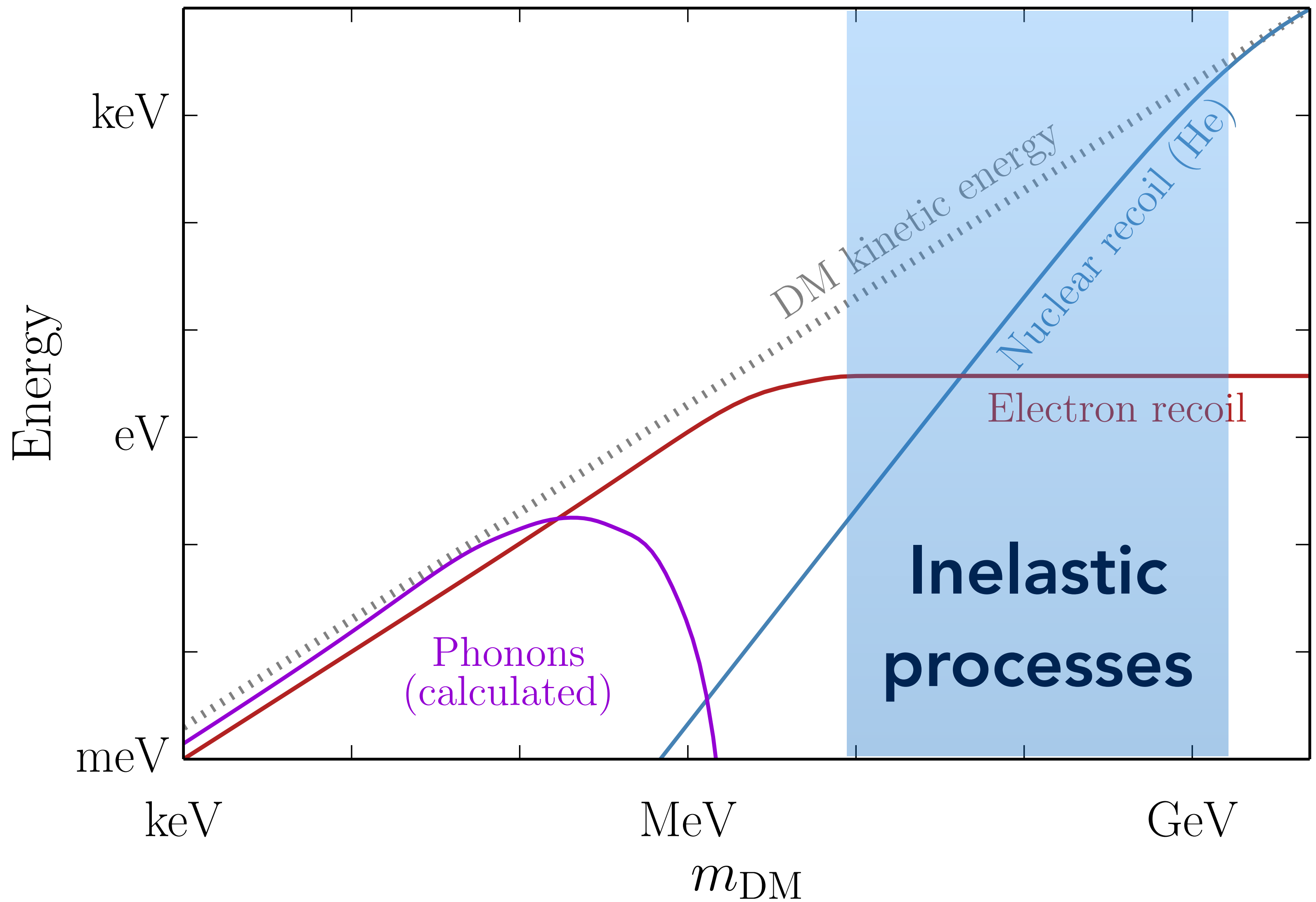


sub-eV gap in superconductor, Dirac material, etc

Ge, Si

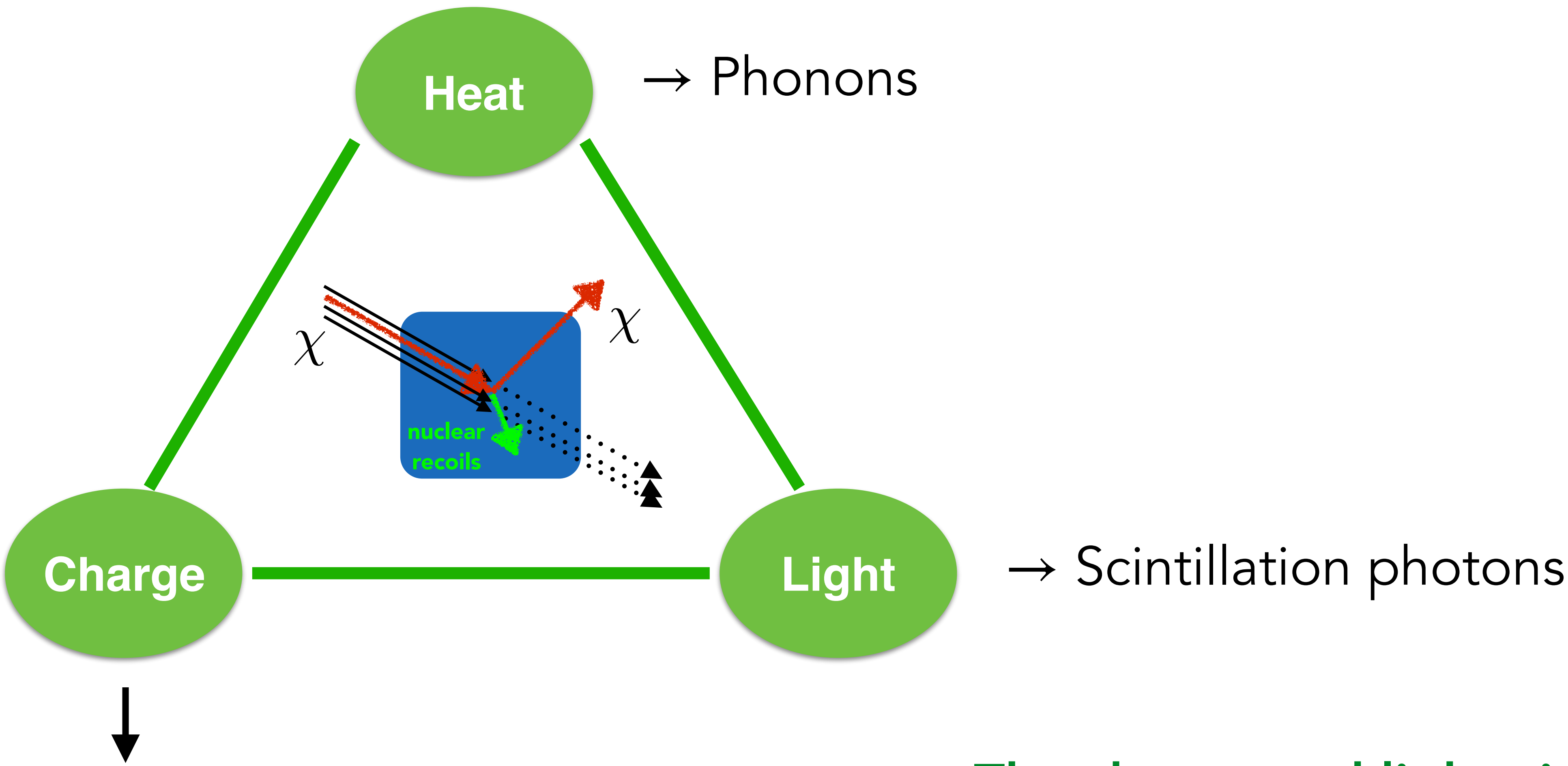
(Xe)

Material properties matter



Inelastic nuclear recoils or 2 → 3 processes can also extract more DM kinetic energy.

Challenges for sub-GeV DM



ionized atoms or electron-hole pairs in semiconductors

The charge and light yield for nuclear recoils below few hundred eV is not well understood, but expected to be ~0 on average.

Graphic from talk by Kaixuan Ni

Strategies for detecting nuclear recoils from sub-GeV DM

1. Decreasing the heat threshold

- Detectors in development to reach heat/phonon thresholds of ~ eV and below (e.g. SuperCDMS SNOLAB)

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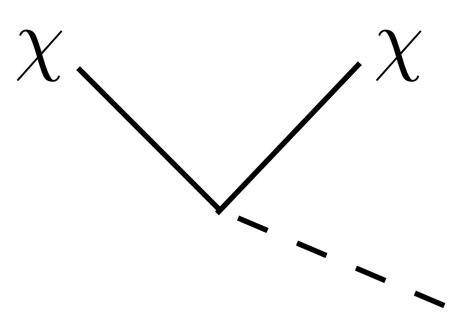
- **Direct phonon excitations from DM scattering**
  At low enough energies, cannot treat as free nucleus; harmonic potential matters. $\omega \approx 1 - 100 \text{ meV}$ for acoustic and optical phonons in crystals. (many works, e.g. Griffin, Knapen, TL, Zurek 2018; Cox, Melia, Rajendran 2019)

DM-phonon scattering

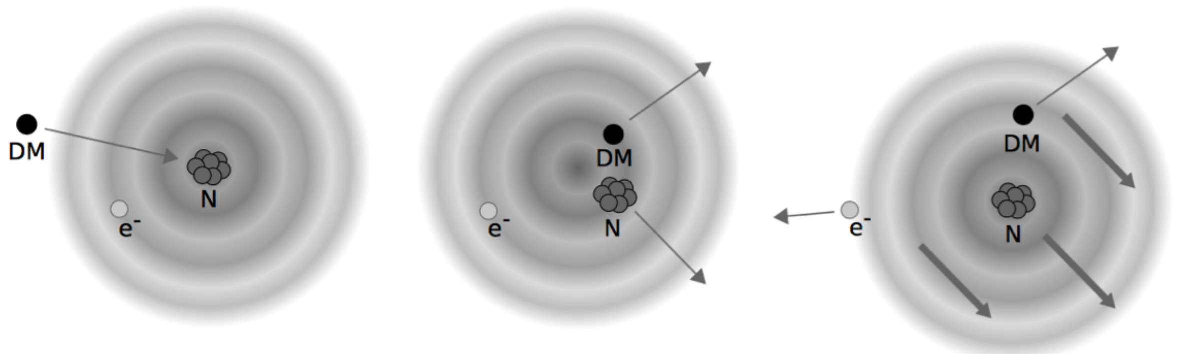


Kinematics of phonons relevant (and advantageous) for sub-MeV dark matter

Strategies for detecting nuclear recoils from sub-GeV DM

2. Increasing the charge signal

- Atomic Migdal effect
  Ionization of electrons
  which have to 'catch up'
  to recoiling nucleus
  (e.g. Ibe, Nakano, Shoji, Suzuki 2017)

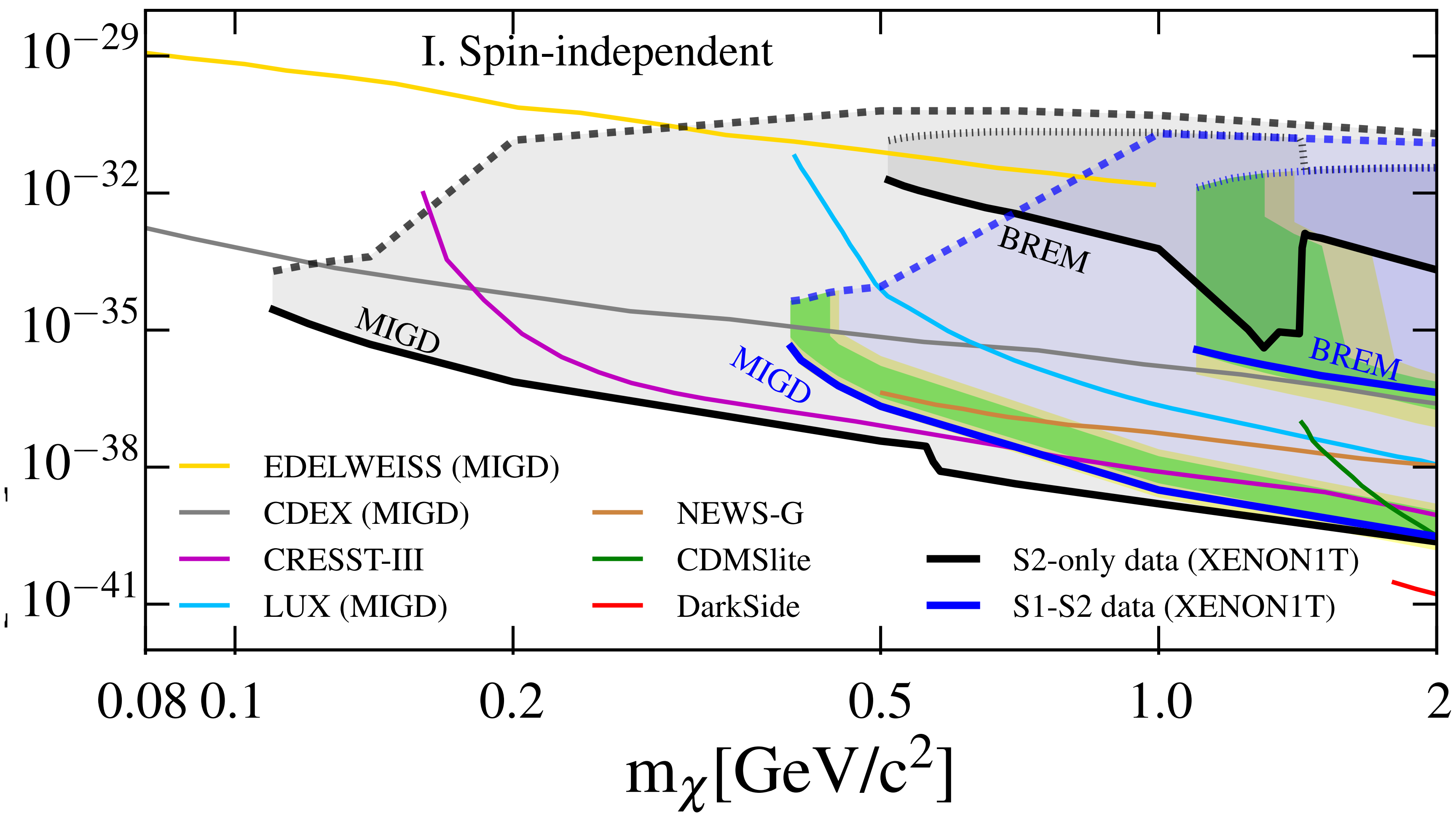


From 1711.09906

- Bremsstrahlung of (transverse) photons in LXe
  Kouvaris & Pradler 2016

Strategies for detecting nuclear recoils from sub-GeV DM

2. Increasing the charge signal



Results from XENON1T search (PRL 2019)

Strategies for detecting nuclear recoils from sub-GeV DM

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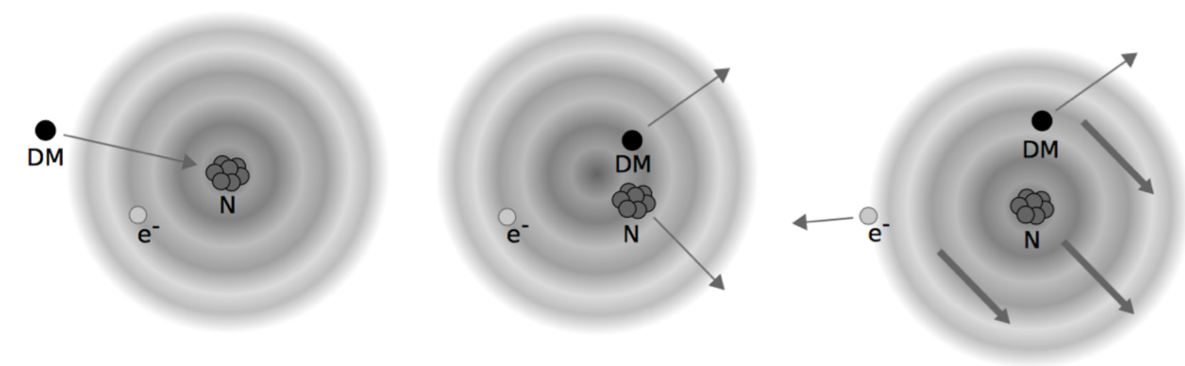


From 1711.09906

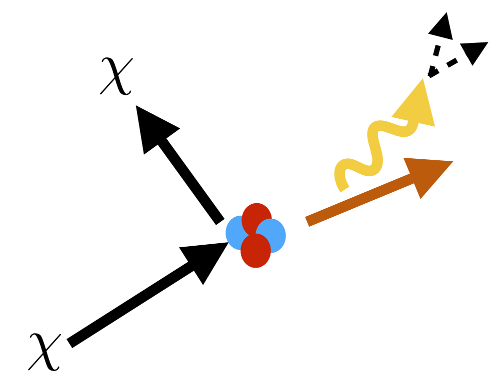
- Bremsstrahlung of (transverse) photons in LXe
  Kouvaris & Pradler 2016
- **Migdal effect (including plasmon emission) in semiconductors**

Many-body effects are relevant in many of these cases!

Searching for nuclear recoils in crystal targets

GeV

Migdal effect (and plasmon emission) in semiconductors



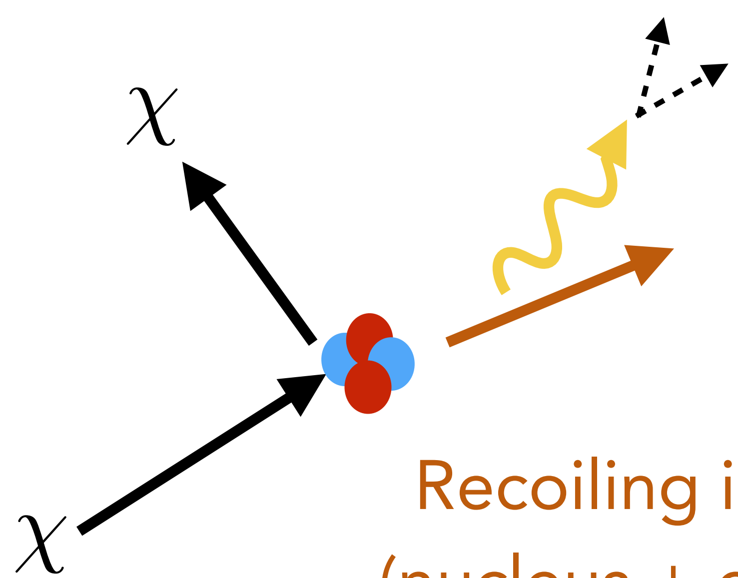
MeV

Phonon excitations in polar crystals

keV

m_{DM}

Detecting nuclear recoils via the Migdal effect

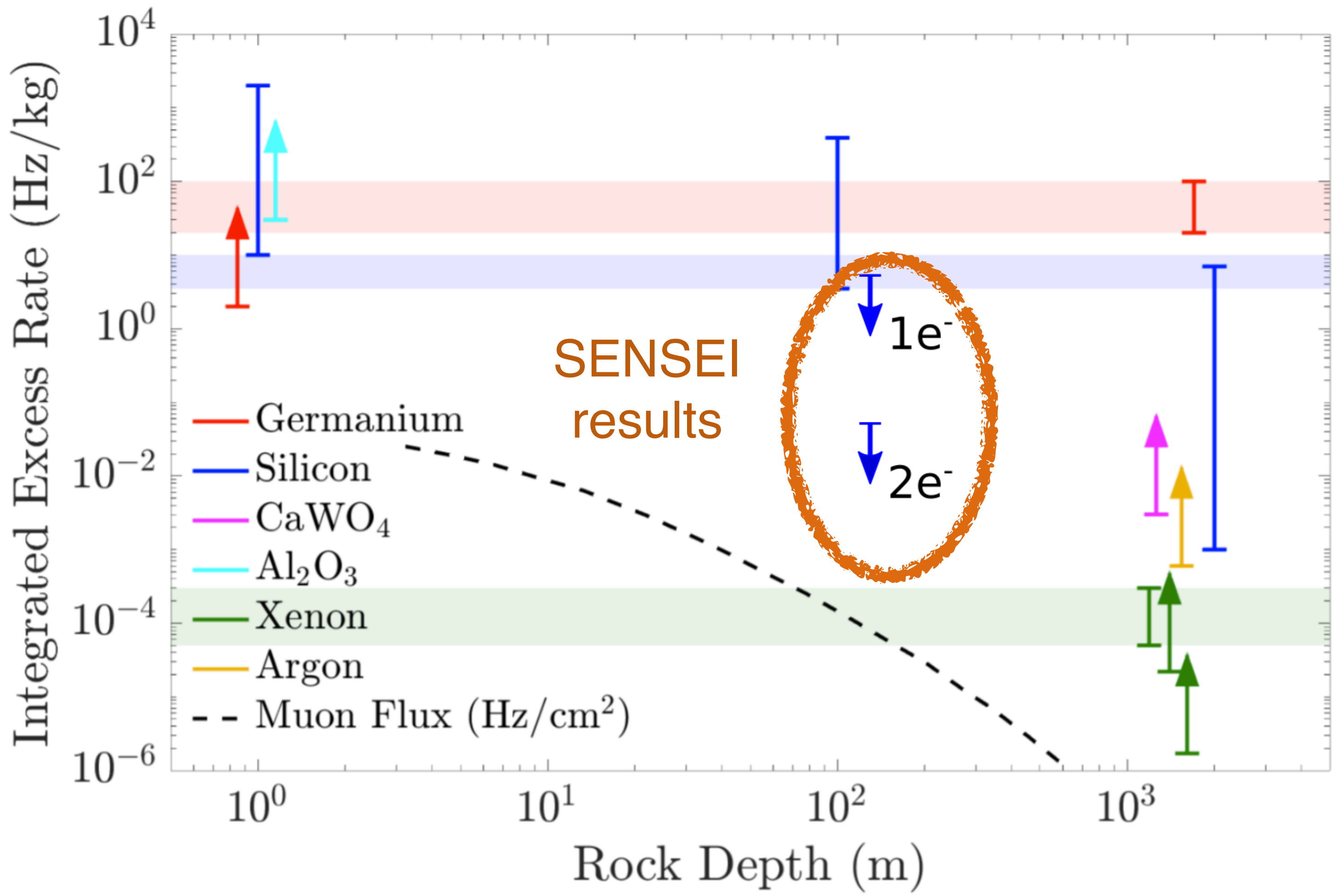


Recoiling ion
(nucleus + core
electrons)

With Jonathan Kozaczuk (2003.12077)
+ with Jonathan Kozaczuk and Simon Knapen (2011.09496)

Plasmons from dark matter?

Proposed by Kurinsky, Baxter, Kahn, Krnjaic (2002.06937) as an explanation of low-energy rates in semiconductor DD experiments.

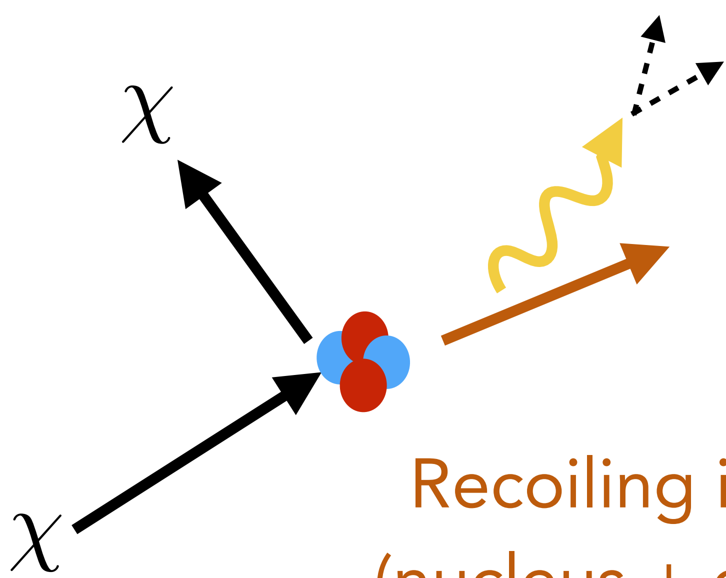


Slide from SENSEI talk, based on figure from Kurinsky et al.

- Excess in 1e- or 2e- bins (assumption requiring plasmon decays to phonons)
- If nuclear recoil, requires O($10^{-3} - 1$) probability to produce plasmons
- Could also be excited by large flux of fast-moving millicharged DM

Plasmons from dark matter?

Our goal in 2003.12077: calculate the plasmon excitation rate from nuclear recoils in semiconductors. This is an additional charge signal that can improve reach for sub-GeV DM.



Assumptions

For nuclear recoil energy
$\omega_{\text{phonon}} \ll E_R \lesssim E_{\text{core}}$
treat as a free nucleus with tightly bound core electrons. Valid for
$10 \text{ MeV} \lesssim m_\chi \lesssim 1 \text{ GeV}$.

Plasmons

- Simple picture: uniform displacement of electrons by $\mathbf{r}$

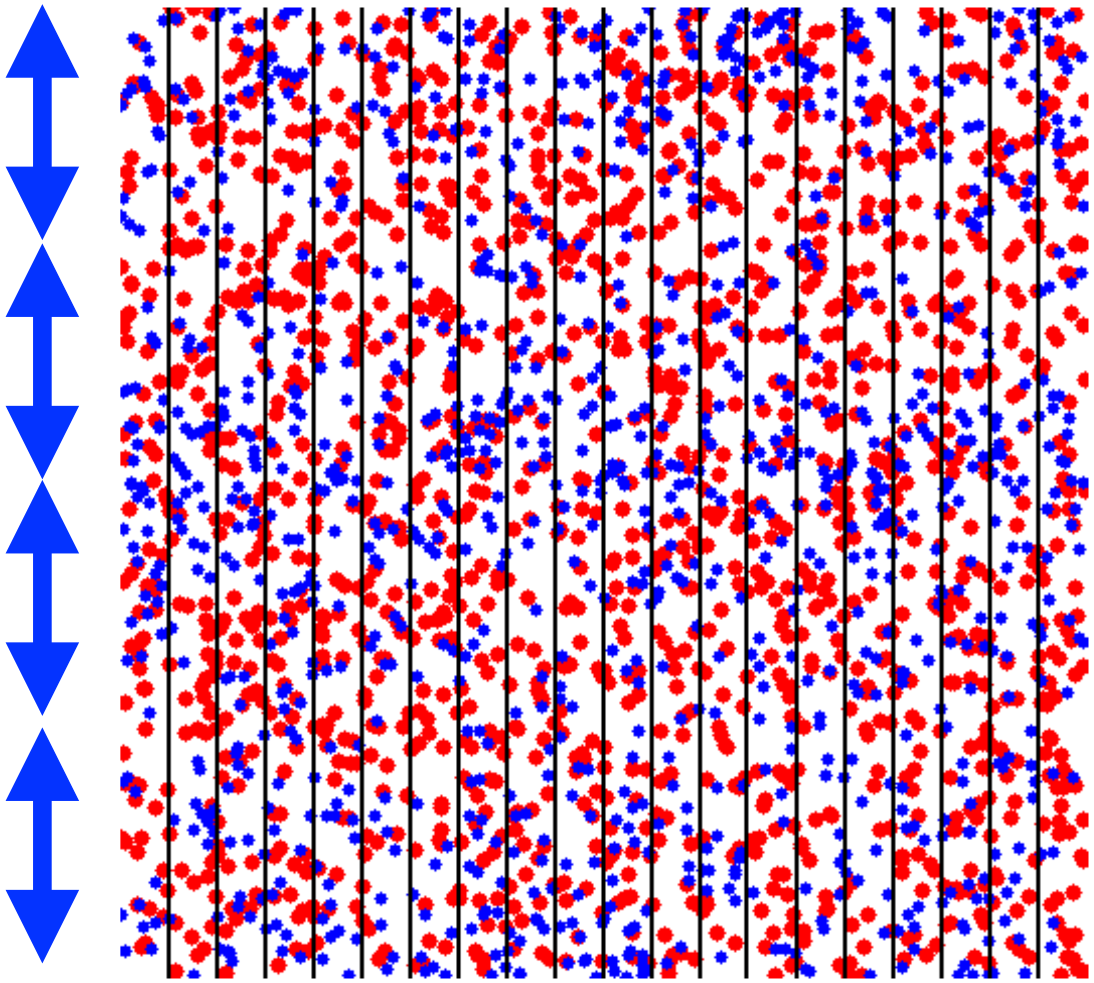
$$-e\mathbf{E} = 4\pi\alpha_{em} n_e \mathbf{r}$$

$$\ddot{\mathbf{r}} = -\omega_p^2 \mathbf{r}$$

Plasma frequency $\qquad \omega_p^2 \equiv \frac{4\pi\alpha_{em} n_e}{m_e}$

- Plasmons are quantized longitudinal E-field excitations in the medium (contrast with "transverse photons")

Electron gas in fixed ion background



red: ion blue: electron

Electron gas model

- Toy model: bremsstrahlung of a longitudinal mode in a metal (degenerate electron gas in fixed ion background)

- Plasmon appears as a zero of the dielectric function

Gauss's law without external source

$$\hat{\epsilon}_L(\omega, \mathbf{k})\mathbf{k} \cdot \mathbf{E} = 0 \to \mathbf{k} \cdot \mathbf{E} \neq 0 \text{ when } \hat{\epsilon}_L(\omega, \mathbf{k}) = 0$$

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- Or as a pole in the longitudinal propagator

$$D^{00}(\omega, \mathbf{k}) = \frac{1}{k^2 \hat{\epsilon}_L(\omega, \mathbf{k})} = \frac{1}{k^2 - \Pi_L(\omega, \mathbf{k})}$$

(Coulomb gauge)

$$\hat{\epsilon}_L(\omega, \mathbf{k}) = 1 - \frac{\Pi_L(\omega, \mathbf{k})}{k^2}$$

Electron gas model

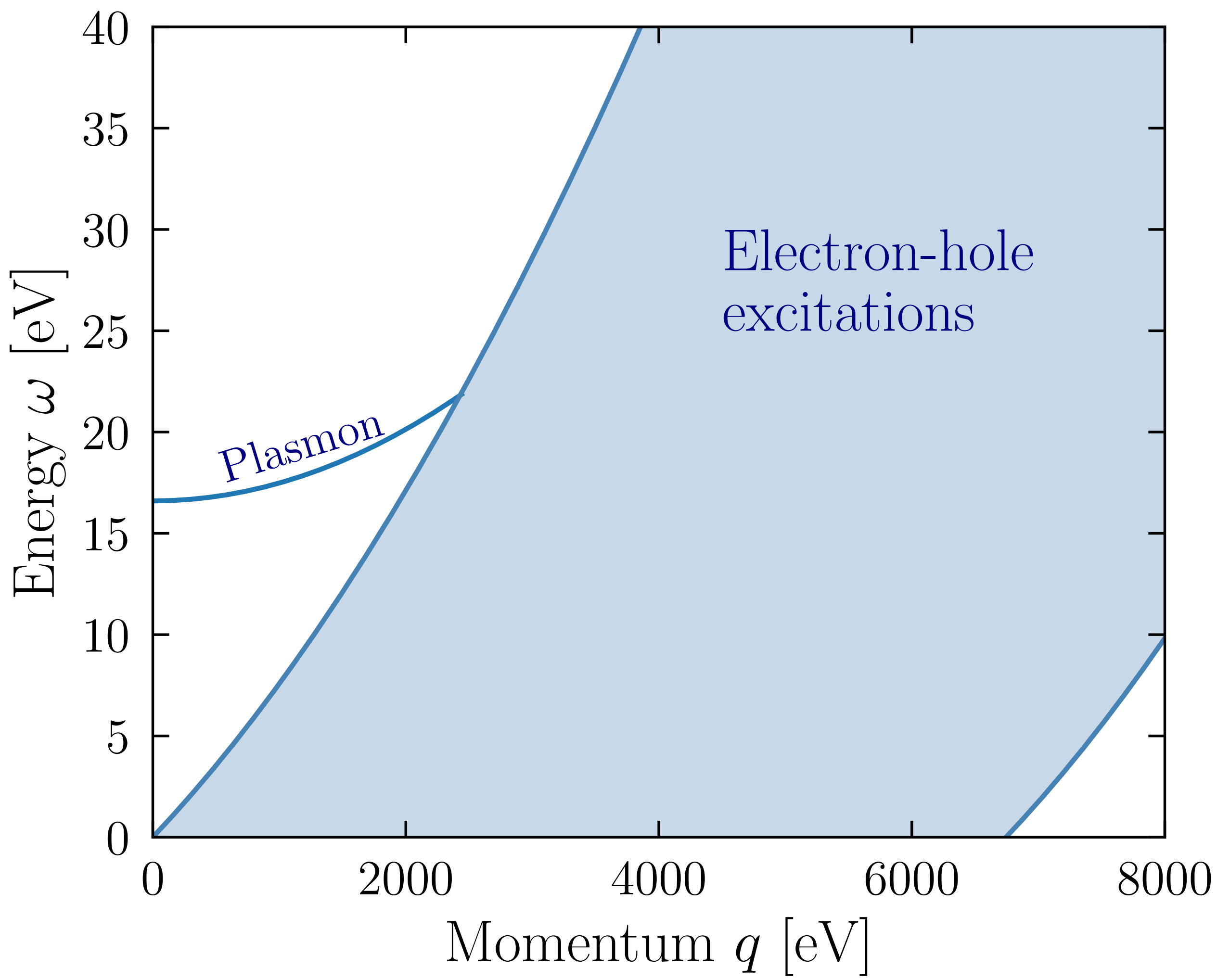
- Plasmon is infinitely long lived for small *k* in this toy model

- For $k \gtrsim \omega_p / v_F$ (~2.4 keV in Si,Ge) there is a large plasmon decay width into electron-hole pairs.

- Plasmons cannot be directly produced by DM with typical halo velocities v ~ 1e-3:

$$\omega = \mathbf{k} \cdot \mathbf{v} - \frac{k^2}{2m_\chi} \quad \rightarrow \quad k \geq \frac{\omega}{v} \sim 16\,\mathrm{keV}$$

Spectrum of longitudinal excitations in the electron gas



Electron gas model

Standard bremsstrahlung calculation in QFT but with final longitudinal mode

$$\chi(p) + N \to \chi(p') + N(q_N) + \omega_L(k)$$

In the limit of soft brem, $k \ll \sqrt{2m_N E_R}$ (valid for us):

$$\frac{d^2\sigma_{\text{plasmon}}}{dE_R dk} = \frac{2Z_{\text{ion}}^2 \alpha_{em}}{3\pi} \frac{Z_L(k)k^2}{\omega_L(k)^3} \frac{E_R}{m_N} \times \left.\frac{d\sigma}{dE_R}\right|_{\text{el}}$$

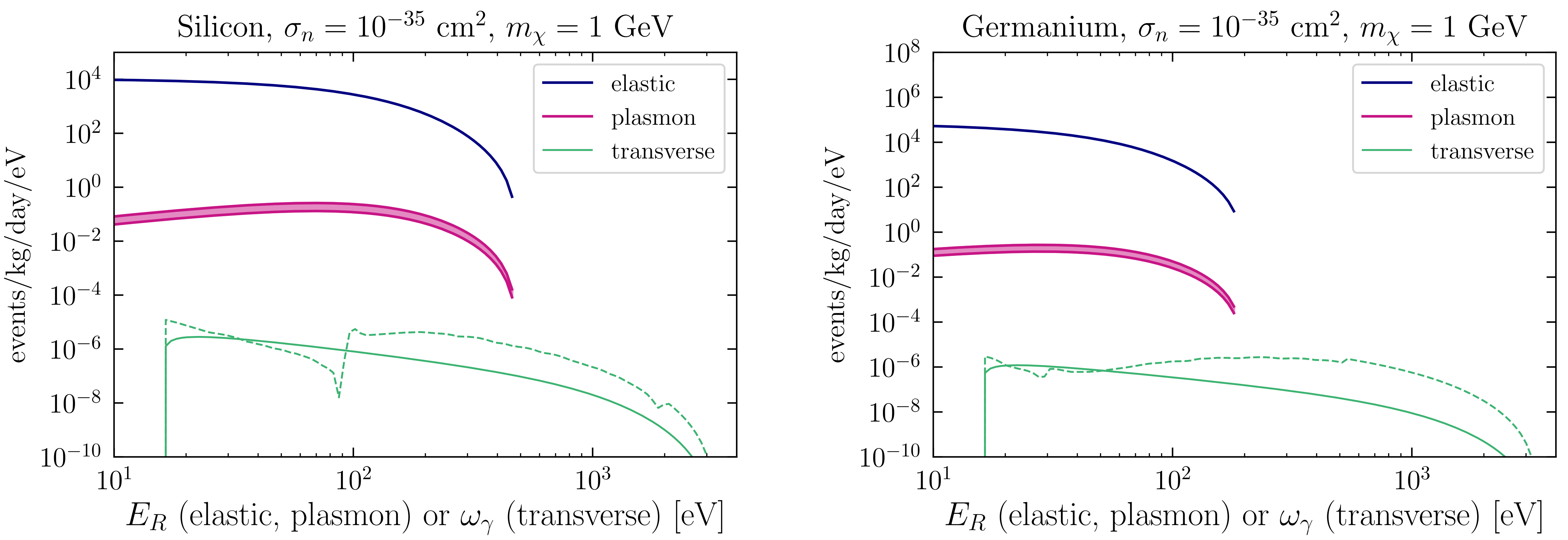
Elastic DM-nucleus scattering cross section

Roughly 4-6 orders of magnitude larger than brem of transverse photons

Bremsstrahlung of plasmons is low-probability, but allows low-energy nuclear recoils to be detected with charge signals in semiconductors.

Plasmon production rate

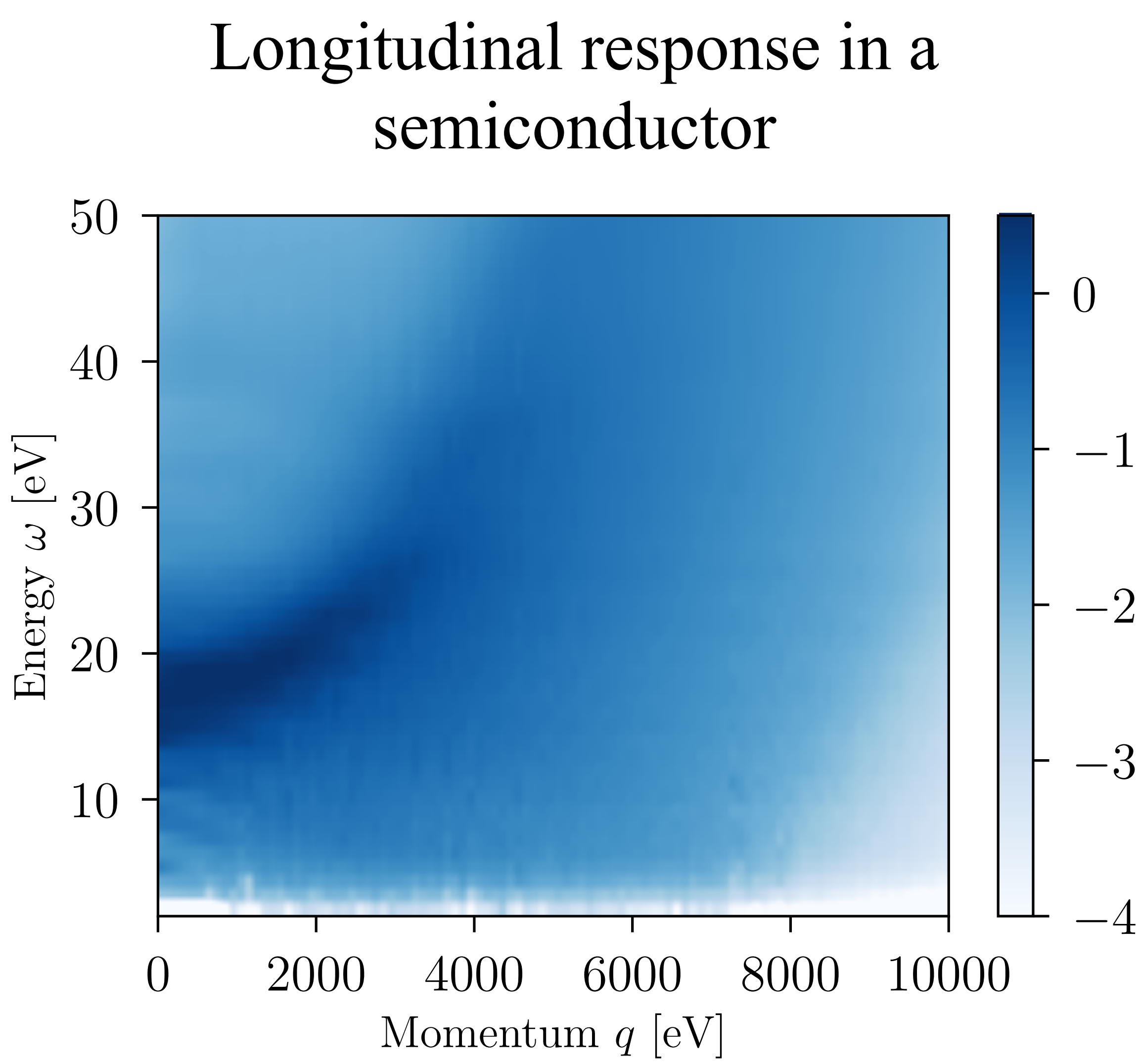
Assume universal coupling to nucleons $$\left.\frac{d\sigma}{dE_R}\right|_{\text{el}} = \frac{A^2 m_N \sigma_n}{2\mu_{\chi n}^2 v^2}$$



Plasmon production in semiconductors

Differences from electron gas model:

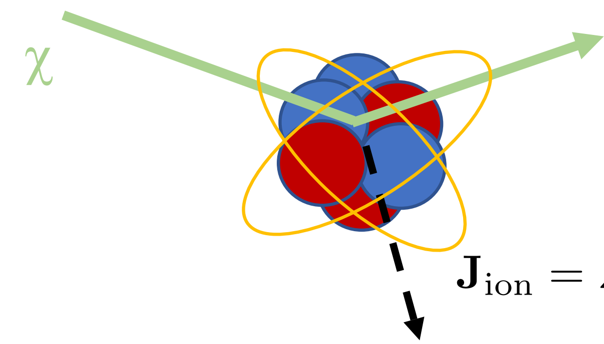
- Band gap: $\omega_g \sim O(1)$ eV (but $\omega_g \ll \omega_p$)
- Electron wavefunctions: plane waves → Bloch waves
- Plasmon decays by interband transitions.



We deal with this by rewriting plasmon production in terms of $\hat{\epsilon}_L$

Plasmon production in semiconductors

Current sourced by ion recoiling against DM:



$$\mathbf{J}_{\rm ion} = Z_{\rm ion}\, e\, \mathbf{v}_{\rm ion}\, \Theta(t)\, \delta^3(\mathbf{x} - \mathbf{v}_{\rm ion} t)$$

Energy transfer to material:

$$W = -\int d^3k \int_0^\infty \frac{d\omega}{(2\pi)^4} 2\,\mathrm{Re}\left[\mathbf{J}^*_{\rm ion}(\omega, \mathbf{k}) \cdot \mathbf{E}(\omega, \mathbf{k})\right]$$

Longitudinal part of Maxwell's equations:

$$J_{{\rm ion},L}(\omega, \mathbf{k}) = \frac{i}{\omega} Z_{\rm ion}\, e\, \mathbf{v}_{\rm ion} \cdot \frac{\mathbf{k}}{k}$$

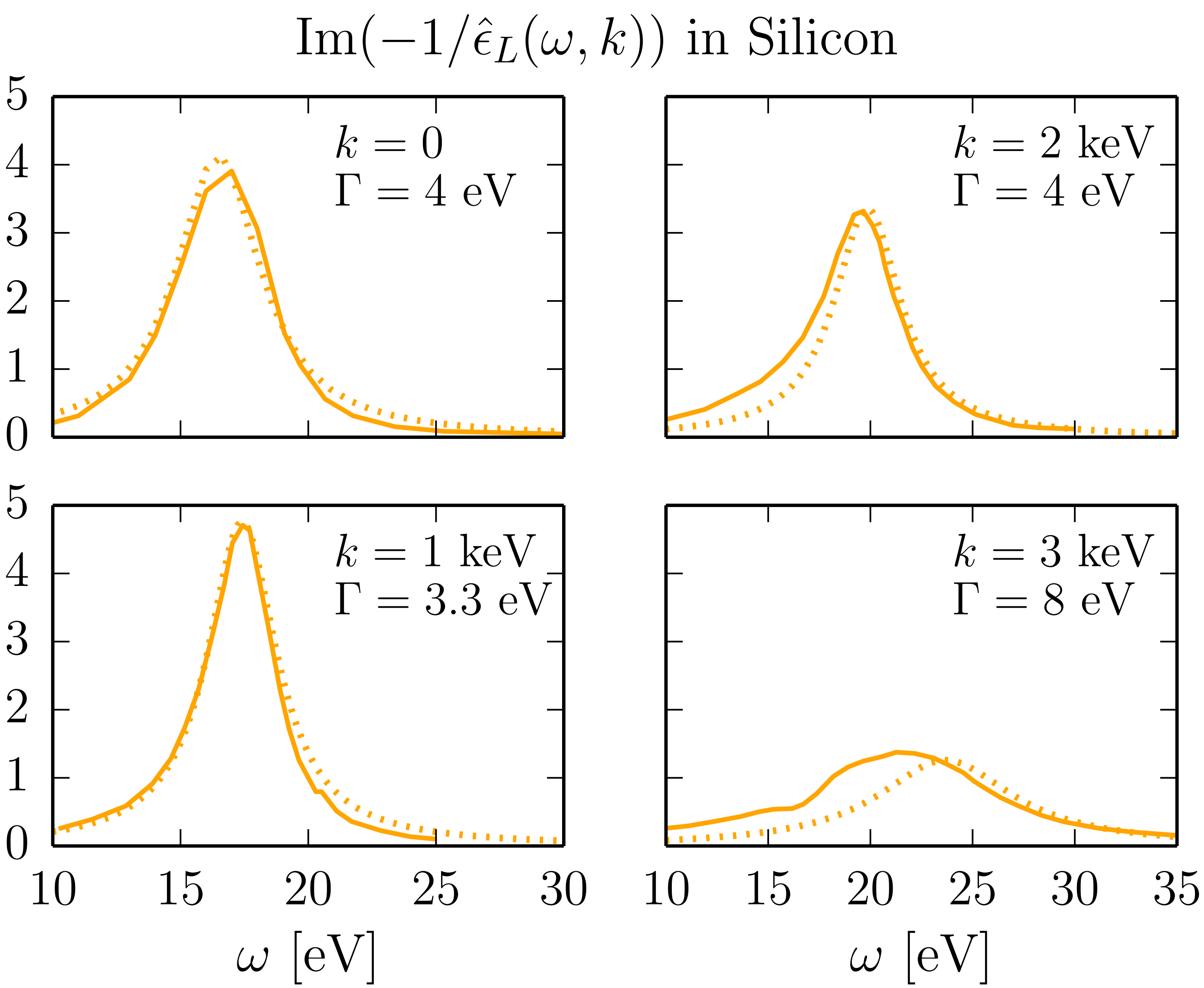
$$i\,\omega D_L(\omega, \mathbf{k}) = i\,\omega \hat{\epsilon}_L(\omega, \mathbf{k}) E_L(\omega, \mathbf{k}) = J_{{\rm ion},L}(\omega, \mathbf{k})$$

Energy loss rate to longitudinal electronic excitations (not only plasmons)

$$\frac{dW_L}{dk} = \int_0^\infty d\omega \frac{2 Z_{\rm ion}^2 \alpha_{em}}{3\pi^2} |\mathbf{v}_{\rm ion}|^2 \frac{k^2}{\omega^3} \mathrm{Im}\left(\frac{-1}{\hat{\epsilon}_L(\omega, \mathbf{k})}\right)$$

Energy loss function

Energy loss function



Solid: X-ray scattering from Weissker et al. 2010
Dashed: Modified electron gas model

Electron gas picture provides a reasonable approximation of the plasmon pole for simple semiconductor like Si.

Including a finite width Γ for electron gas

$$\mathrm{Im}\left(\frac{-1}{\hat{\epsilon}_L(\omega, \mathbf{k})}\right) \simeq Z_L(\omega, k) \frac{\omega_L(k)^2 \omega \Gamma}{\left(\omega^2 - \omega_L(k)^2\right)^2 + \omega^2 \Gamma^2}$$

Ionization signals from nuclear recoils

Probability for inelastic process with plasmon production:

$$\frac{dN_L}{d\omega dk} = \frac{4Z_{\text{ion}}^2\alpha_{em}}{3\pi^2}\frac{E_R}{m_N}\frac{k^2}{\omega^3}\,\mathrm{Im}\left(\frac{-1}{\hat{\epsilon}_L(\omega,\mathbf{k})}\right)$$

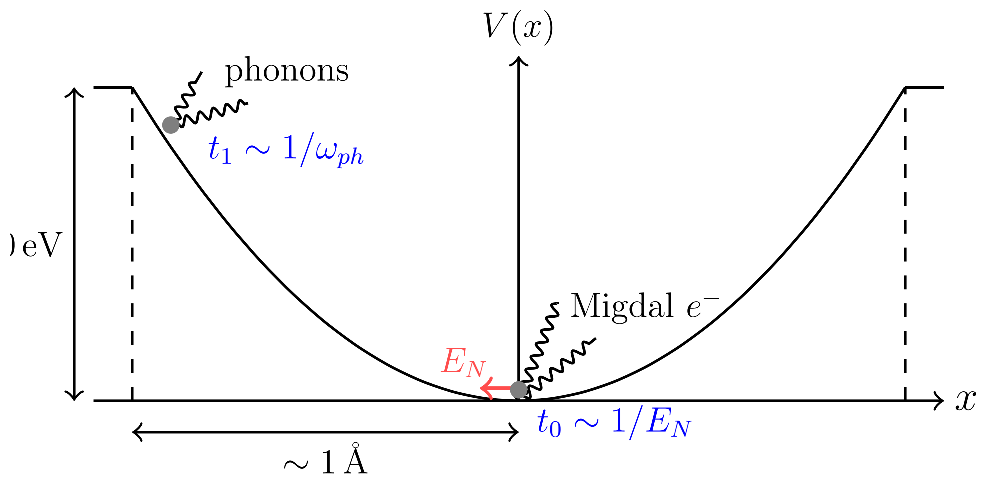
Expect a plasmon resonance at ~16 eV (5-6 electrons). Possible even when expected nuclear recoil is well below 16 eV.

But energy loss function contains **all** electronic excitations (charge signals), even away from plasmon pole.

We can use density functional theory (DFT) codes to numerically compute the full energy loss function.

Full rate in semiconductors

Newer work, with Knapen and Kozaczuk:



Usual DM-nucleus scattering

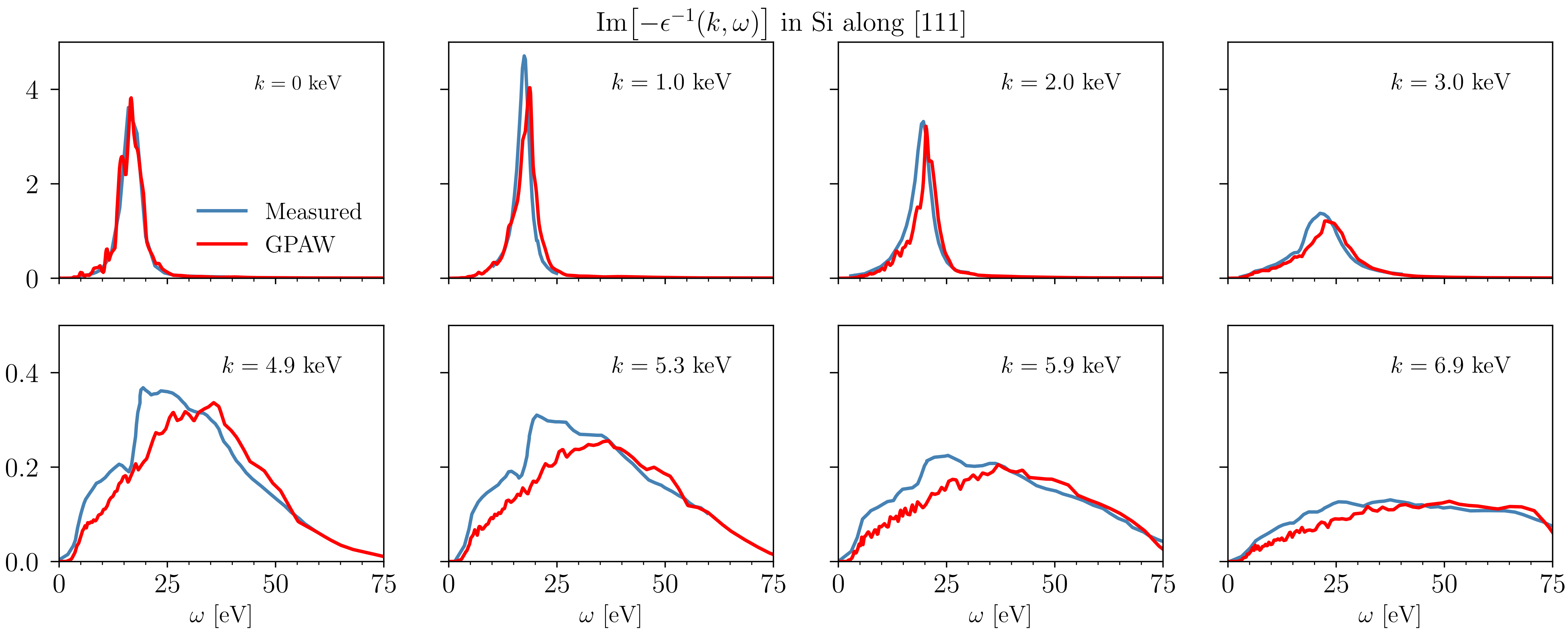
$$\frac{d\sigma}{d\omega} = \frac{2\pi^2 A^2 \sigma_n}{m_\chi^2 v_\chi} \int \frac{d^3\mathbf{q}_N}{(2\pi)^3} \int \frac{d^3\mathbf{p}_f}{(2\pi)^3}\, \delta(E_i - E_f - \omega - E_N) \times 4\alpha Z_{\text{ion}}^2 \sum_{\mathbf{K}} \int \frac{d^3\mathbf{k}}{(2\pi)^3} \left[\frac{1}{\omega - \mathbf{q}_N \cdot (\mathbf{k}+\mathbf{K})/m_N} - \frac{1}{\omega} \right]^2$$

$$\times \frac{F(\mathbf{p}_i - \mathbf{p}_f - \mathbf{q}_N - \mathbf{k} - \mathbf{K})^2}{|\epsilon_{\mathbf{KK}}(\mathbf{k},\omega)|^2} \times \underbrace{\frac{4\pi^2\alpha}{V} \sum_{\mathbf{p}_e} \frac{|[\mathbf{p}_e + \mathbf{k}|e^{i\mathbf{r}\cdot\mathbf{K}}|\mathbf{p}_e]_\Omega|^2}{|\mathbf{k}+\mathbf{K}|^2} \left(f(\mathbf{p}_e) - f(\mathbf{p}_e + \mathbf{k})\right) \delta(\omega_{\mathbf{p}_e+\mathbf{k}} - \omega_{\mathbf{p}_e} - \omega)}_{\text{Im}\,[\epsilon_{\mathbf{KK}}(\mathbf{k},\omega)]}$$

Form factor accounting
for multiphonon response
in a harmonic crystal

Energy loss function (ELF) with momentum $\mathbf{k}+\mathbf{K}$
and energy ω deposited to electrons

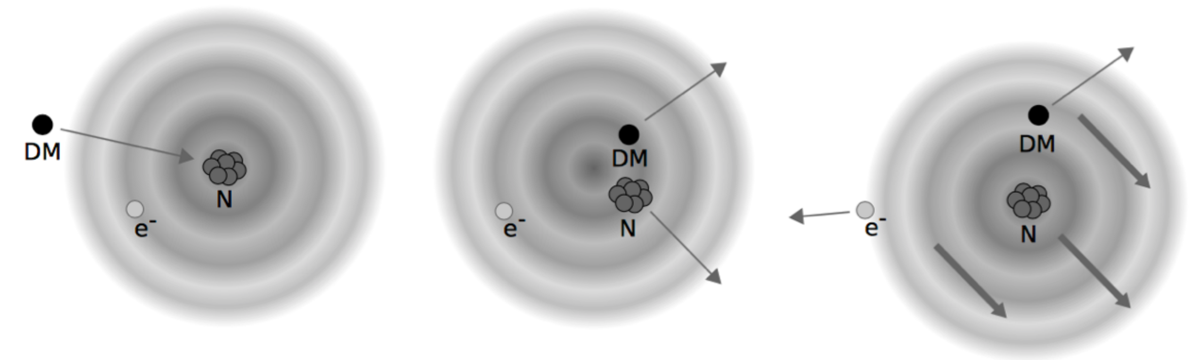
Energy loss function



Relation with atomic Migdal effect

Boost initial state to frame of moving nucleus:

$$|i\rangle \to e^{im_e \mathbf{v}_N \cdot \sum_\beta \mathbf{r}_\beta}|i\rangle$$



Nucleus recoils with velocity $\mathbf{v}_N$

Transition probability $|\mathscr{M}_{if}|^2$

$$\mathcal{M}_{if} = \langle f|\, e^{im_e \mathbf{v}_N \cdot \sum_\beta \mathbf{r}_\beta}\, |i\rangle \approx im_e \langle f|\, \mathbf{v}_N \cdot \sum_\beta \mathbf{r}_\beta \, |i\rangle$$

Problem with applying this to semiconductors: boosting argument does not apply because of crystal lattice.

Our result provides a generalization of the atomic Migdal effect with a simple physical interpretation.

Relation with atomic Migdal effect

$$
\begin{aligned}
& i m_e \langle f | \, \mathbf{v}_N \cdot \textstyle\sum_\beta \mathbf{r}_\beta \, | i \rangle \\
& \quad = \mathbf{v}_N \cdot \frac{1}{\omega} \langle f | \Sigma_\beta \mathbf{p}_\beta | i \rangle \\
& \quad = -\mathbf{v}_N \cdot \frac{1}{\omega^2} \langle f | \Sigma_\beta [\mathbf{p}_\beta, H_0] | i \rangle = \frac{-i}{\omega^2} \langle f | \sum_\beta \frac{Z_N \alpha \mathbf{v}_N \cdot \hat{\mathbf{r}}_\beta}{|\mathbf{r}_\beta - \mathbf{r}_N|^2} | i \rangle .
\end{aligned}
$$

Fourier transform (in time) of dipole potential from recoiling nucleus

Relation with atomic Migdal effect

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\end{aligned}
$$

Fourier transform (in time) of dipole potential from recoiling nucleus

Atomic Migdal effect

$$
\frac{dP(E_N)}{d\omega} \approx \left(\frac{4\pi Z_N \alpha}{\omega^2} \right)^2 \sum_{i,f} \left| \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{\mathbf{v}_N \cdot \mathbf{k}}{k^2} \langle f | e^{i \mathbf{k} \cdot \mathbf{r}} | i \rangle \right|^2 \delta\left(E_i + \omega - E_f \right)
$$

Semiconductor Migdal effect

$$
\frac{dP}{d\omega} \approx \frac{(4\pi Z_{\text{ion}} \alpha)^2}{\omega^4 V} \sum_{\mathbf{p}_e} \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{|\mathbf{v}_N \cdot \mathbf{k}|^2}{k^4} \frac{|[\mathbf{p}_e + \mathbf{k} | \mathbf{p}_e]_\Omega|^2}{|\epsilon(\mathbf{k}, \omega)|^2} \times \left(f(\mathbf{p}_e) - f(\mathbf{p}_e + \mathbf{k}) \right) \delta(\omega_{\mathbf{p}_e + \mathbf{k}} - \omega_{\mathbf{p}_e} - \omega)
$$

Relation with atomic Migdal effect

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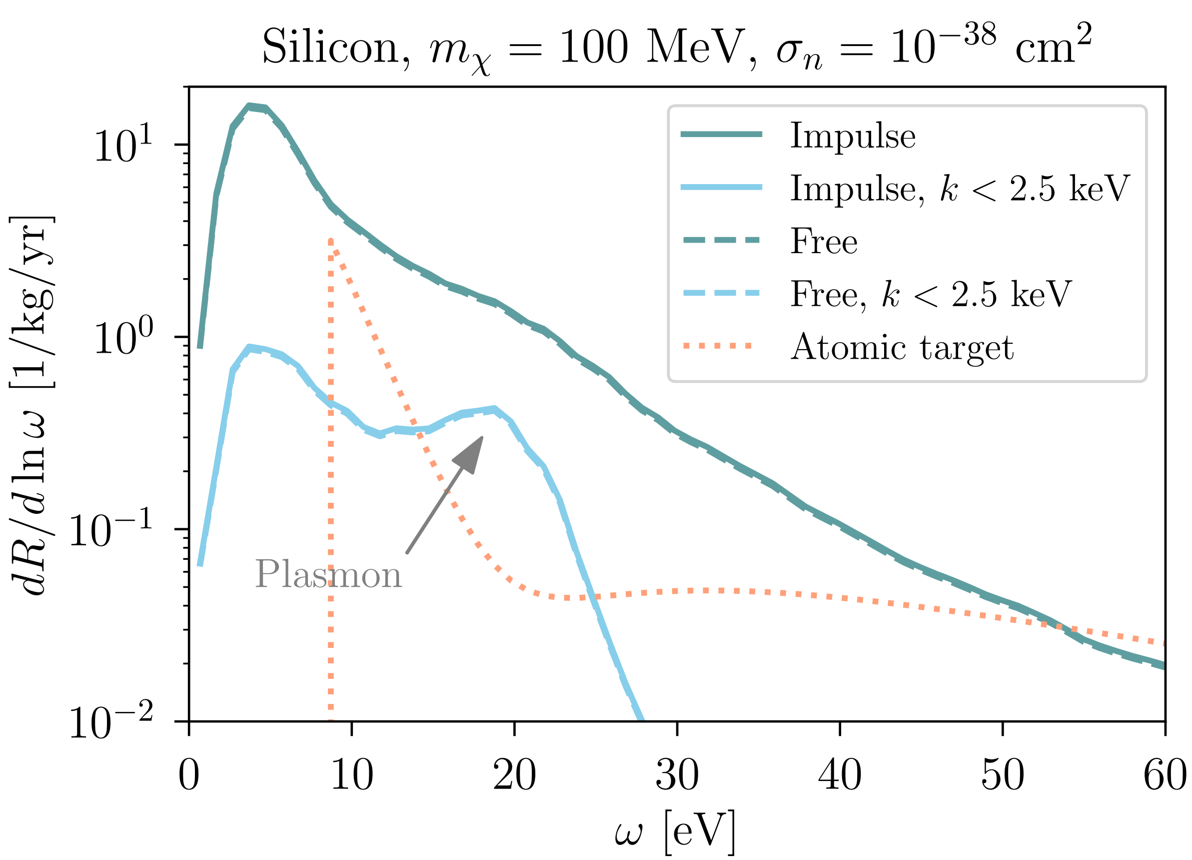
\end{aligned}
$$

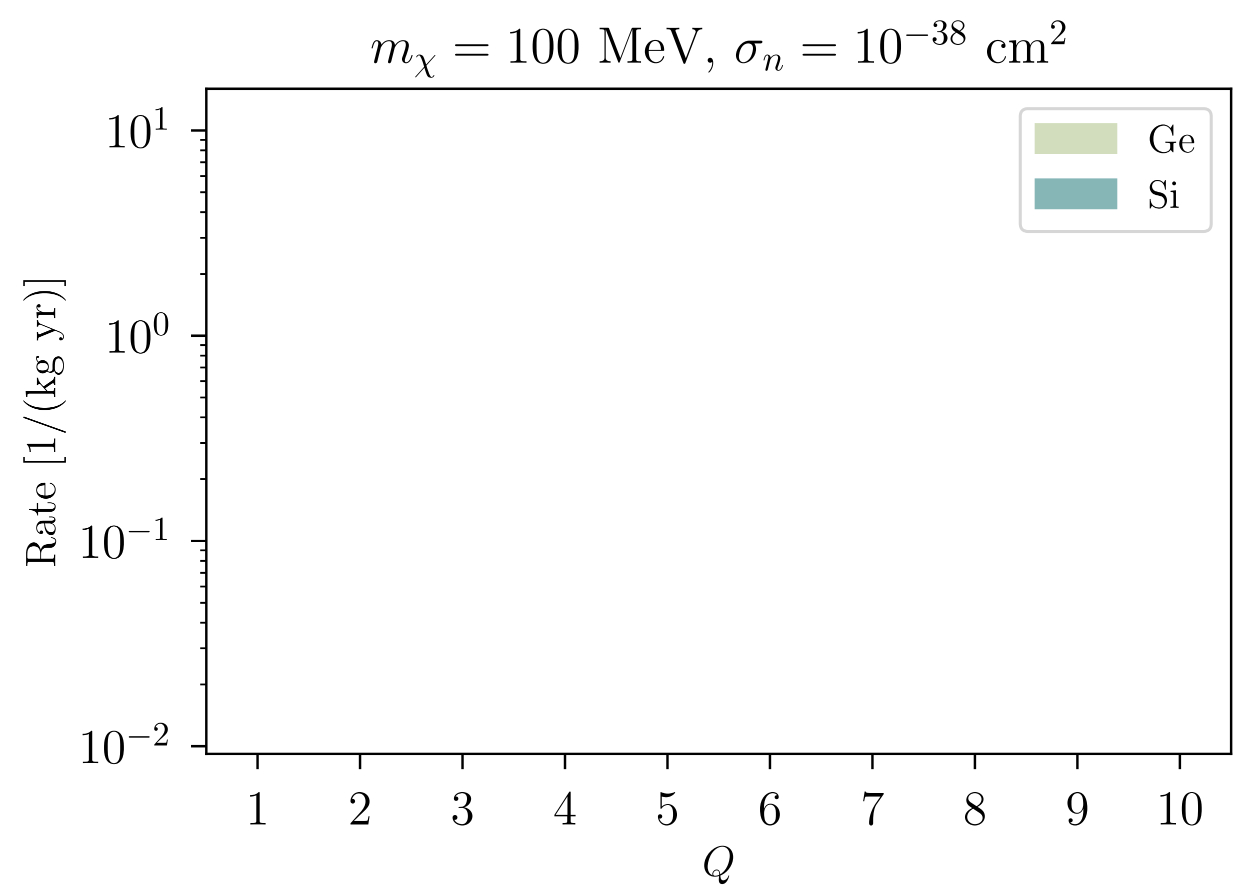
Fourier transform (in time) of dipole potential from recoiling nucleus

Interpretation: the Migdal effect is just an in-medium analog of bremsstrahlung. The moving nucleus generates an electric field, which can excite an electron.

This operator relation does NOT hold in semiconductors. Starting from $\langle f | \, \mathbf{v}_N \cdot \mathbf{r} \, | i \rangle$ would generate the dipole potentials of all nuclei (that is, boosting all nuclei). We argue for starting from the dipole form.

Full rate in semiconductors

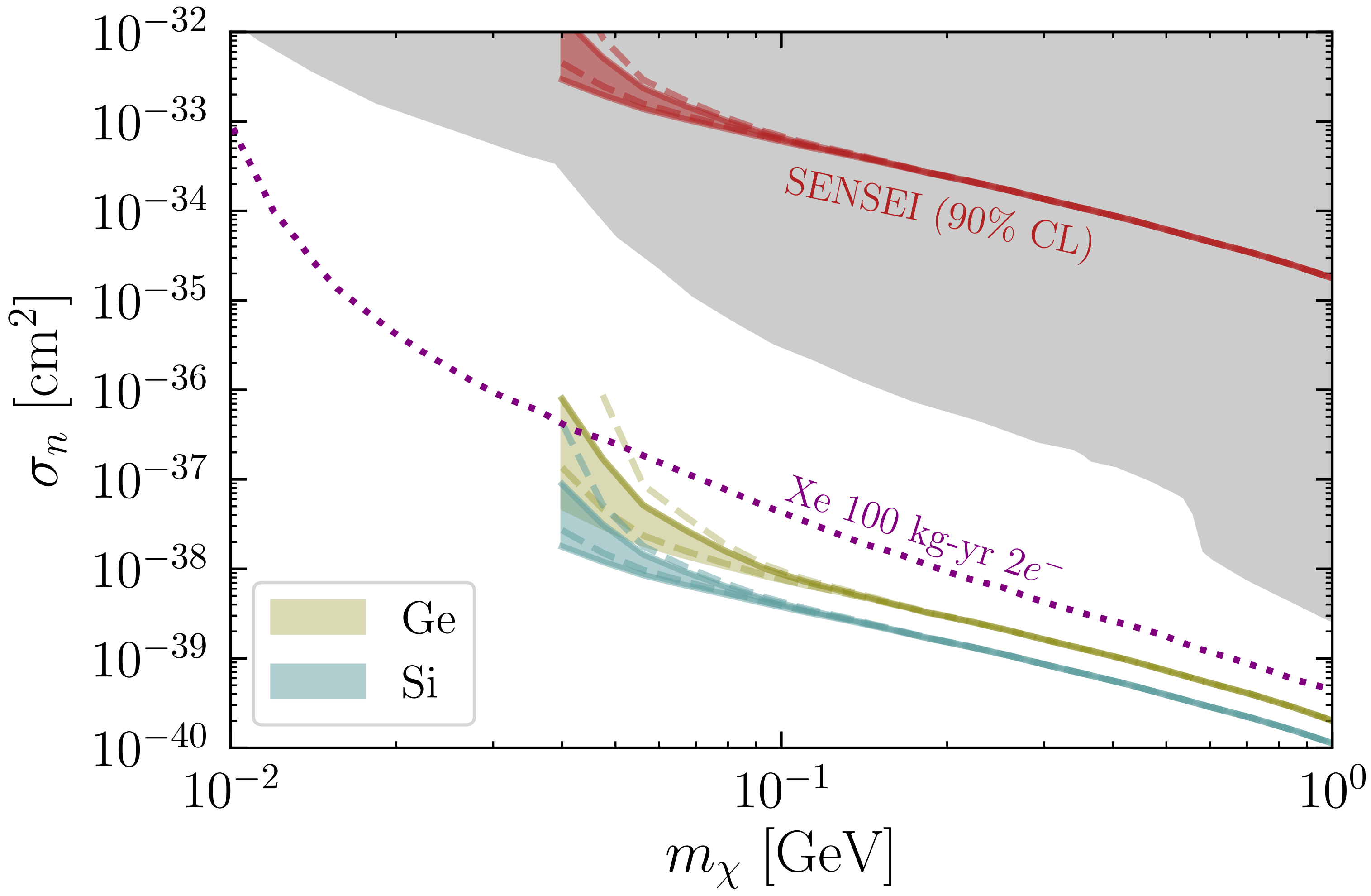




Rate in semiconductors is much larger due to lower gap for excitations.

Sensitivity in semiconductors

1 kg-year exposure, with Q > 2 (similar to proposed experiments)



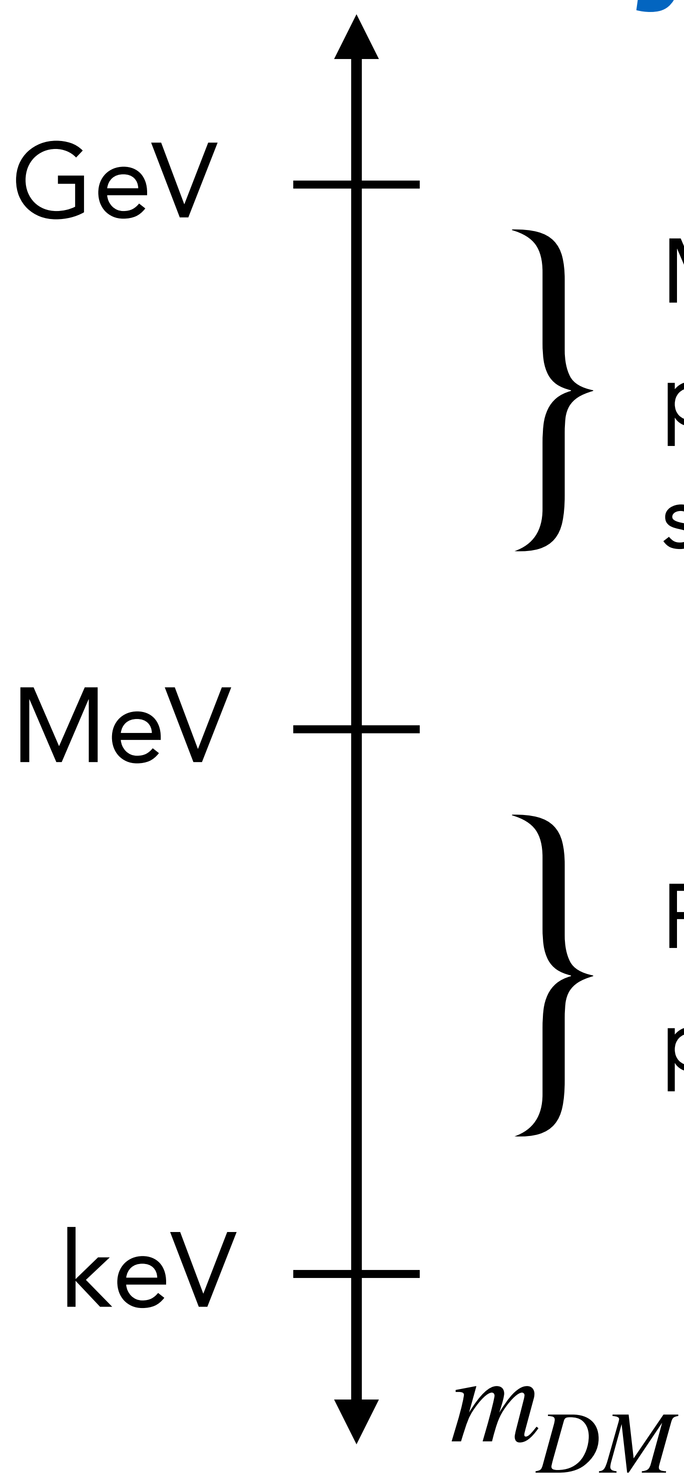
The Migdal effect in semiconductors can enhance sensitivity to nuclear recoils from sub-GeV dark matter

Summary

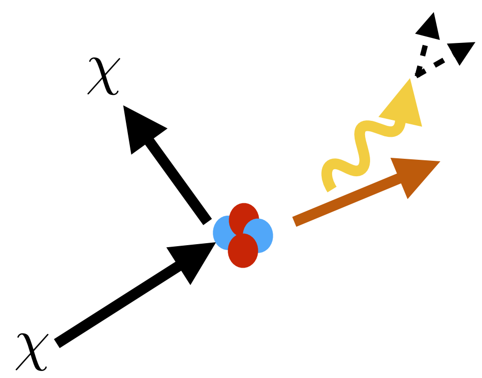
We presented the first derivation and calculation of the Migdal effect in semiconductors, which had previously been studied primarily in atomic targets.

To understand sub-GeV DM scattering in materials, we need to understand the material response, accounting for in-medium properties and collective excitations.

Searching for nuclear recoils in crystal targets



Migdal effect (and plasmon emission) in semiconductors



Phonon excitations in polar crystals

Thanks!