

# Constraining EFTs with Machine Learning

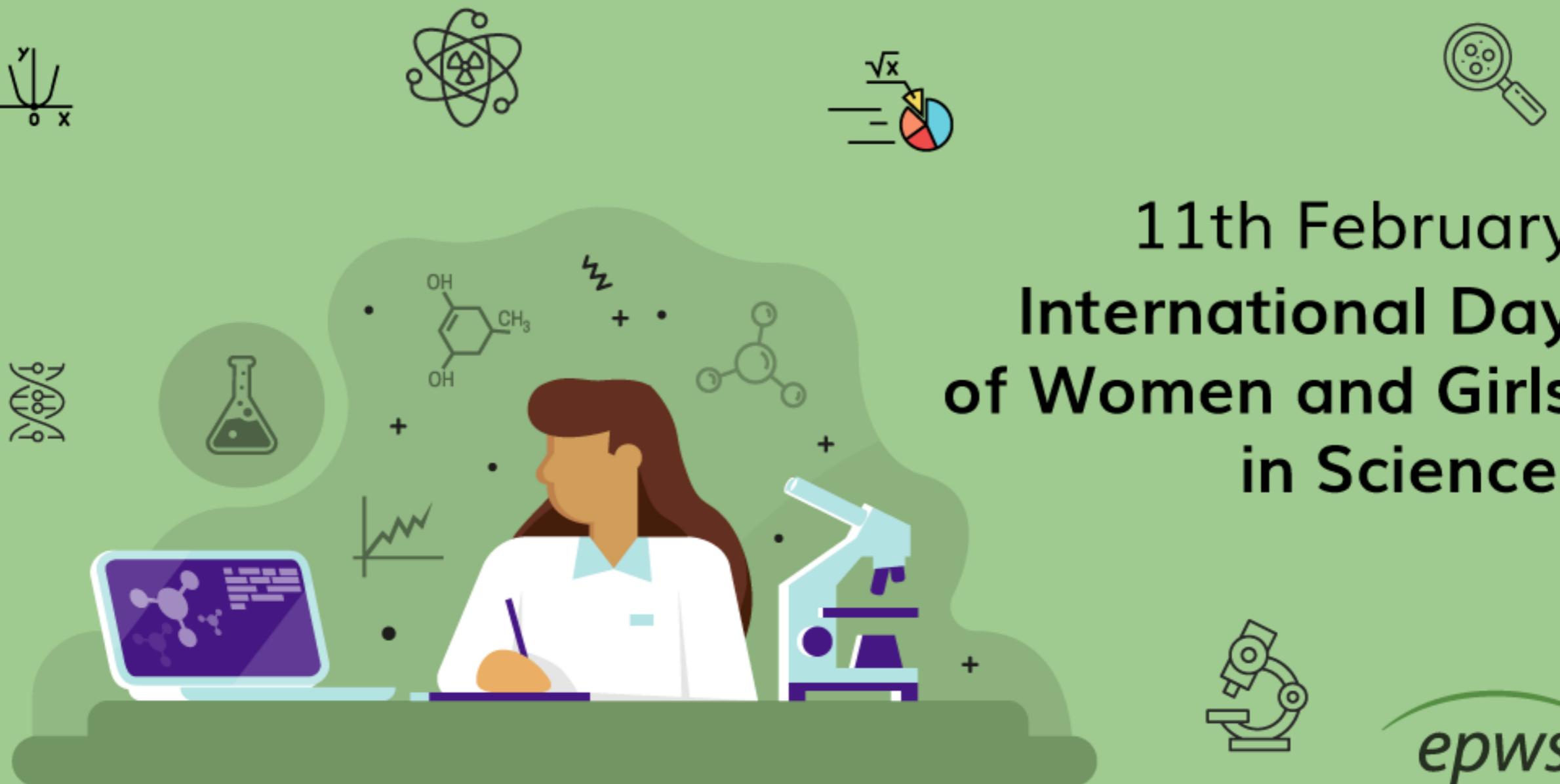


## MadMiner

Felix Kling



II February 2020  
UC Davis



11th February  
**International Day  
of Women and Girls  
in Science.**



epws

# Motivation

## Status of the Field:

- Higgs discovery: Standard Model complete
- no discovery of new physics yet

THE  
HIGGS  
BOSON



# Motivation

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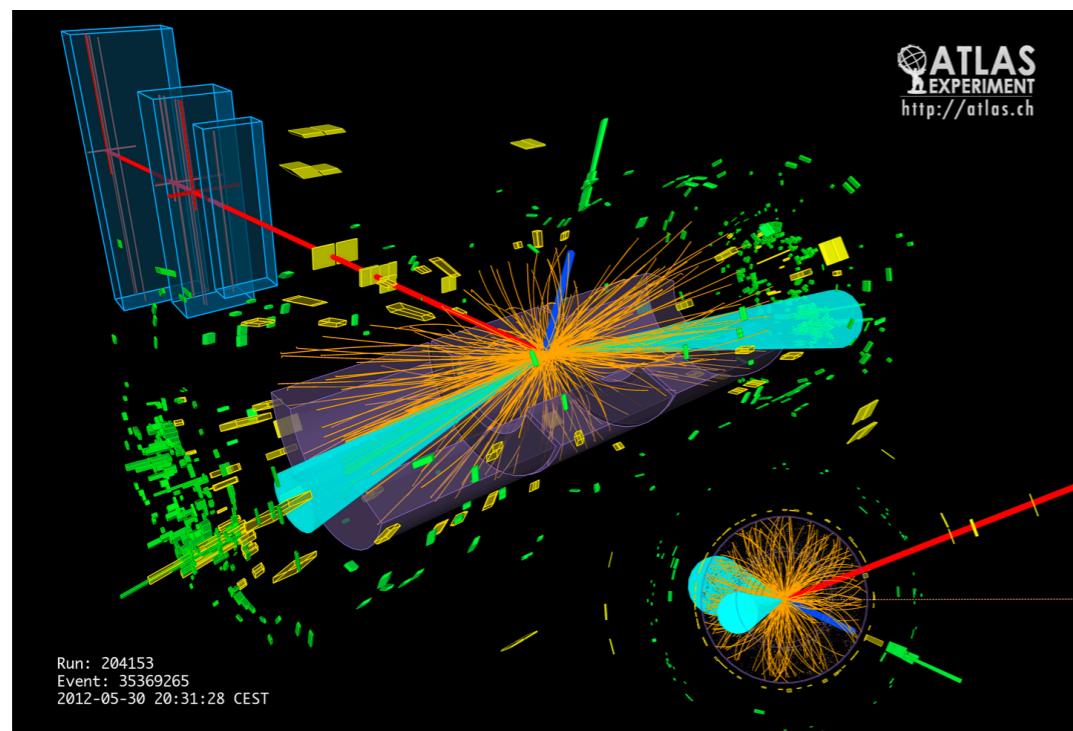
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- maybe BSM physics will be obvious (di-X resonance at XXX GeV)
- maybe BSM physics is more subtle (Higgs couplings in dim6 SMEFT)



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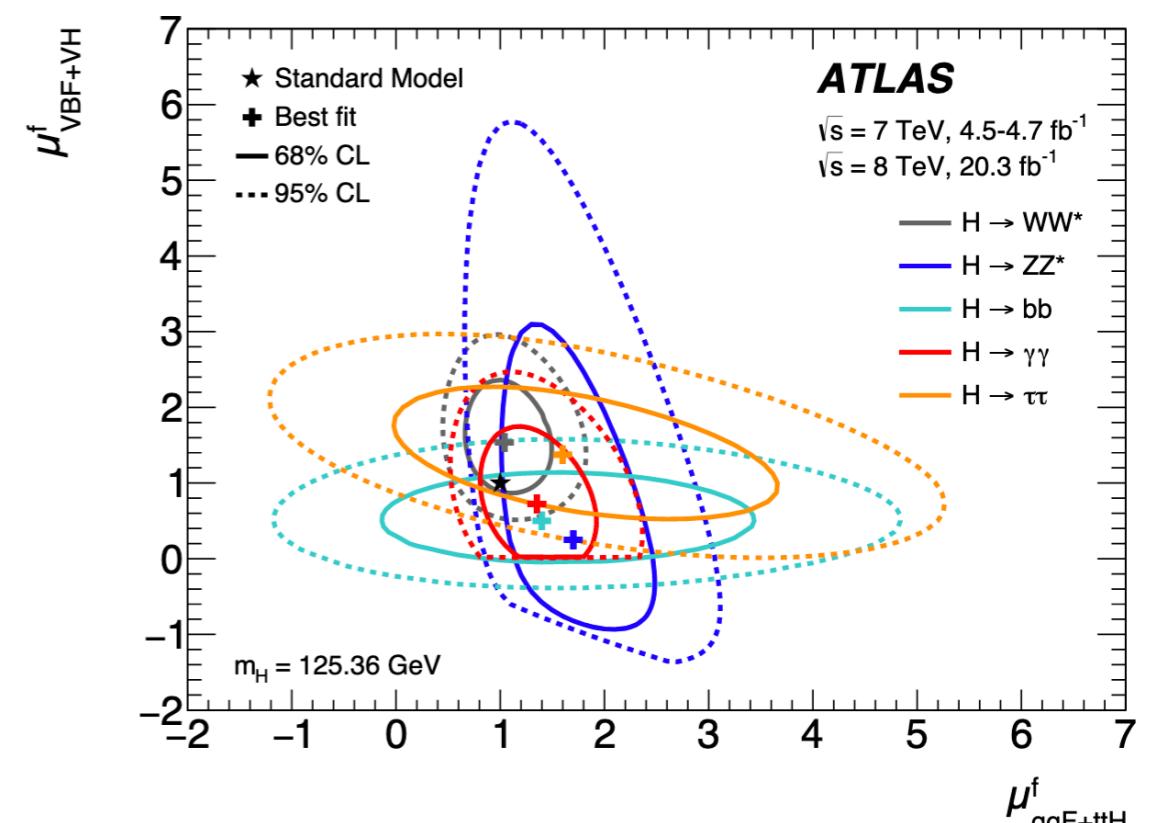
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## Era of Data

high statistics, many distributions,  
multivariate analyses

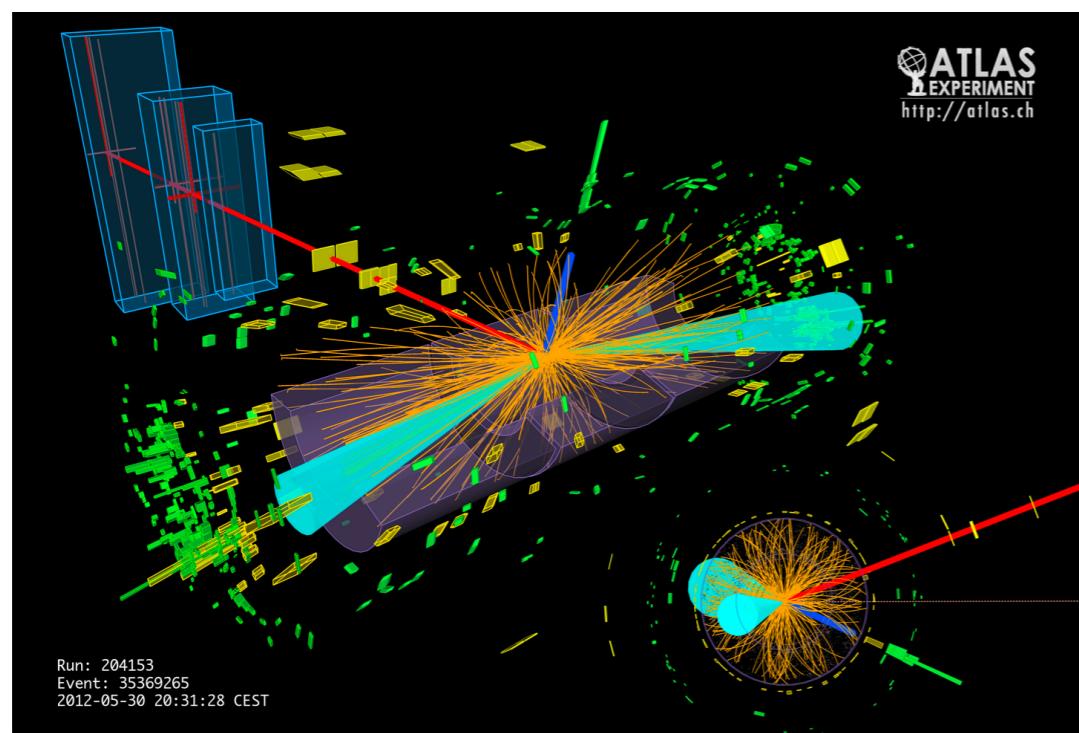


## Complex Theories

large number of theory parameters,  
predict subtle kinematic features

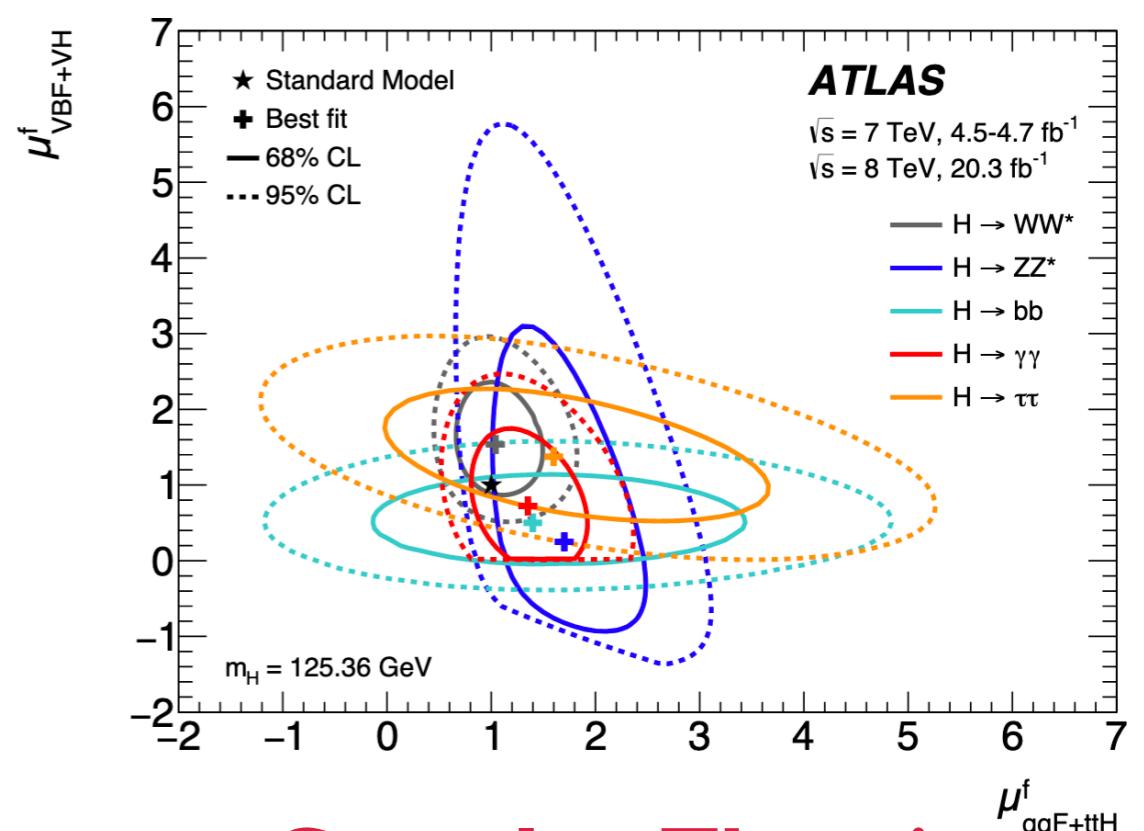
# Motivation

## How to test Theories in an Era of Data?



### Era of Data

high statistics, many distributions,  
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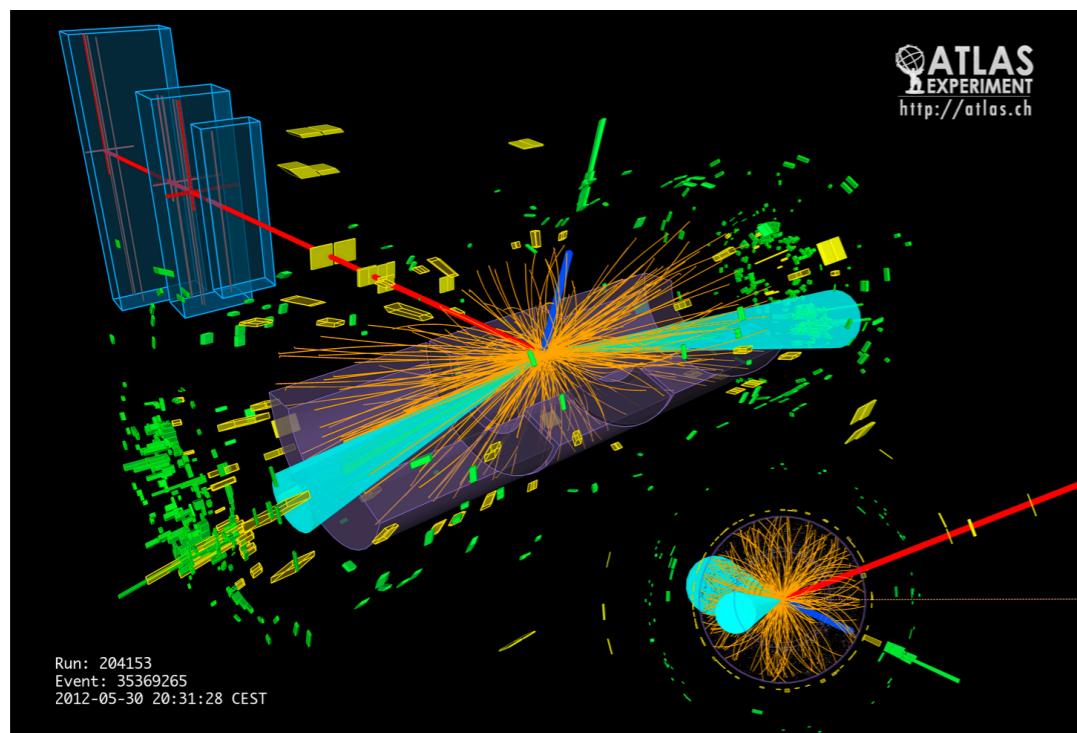


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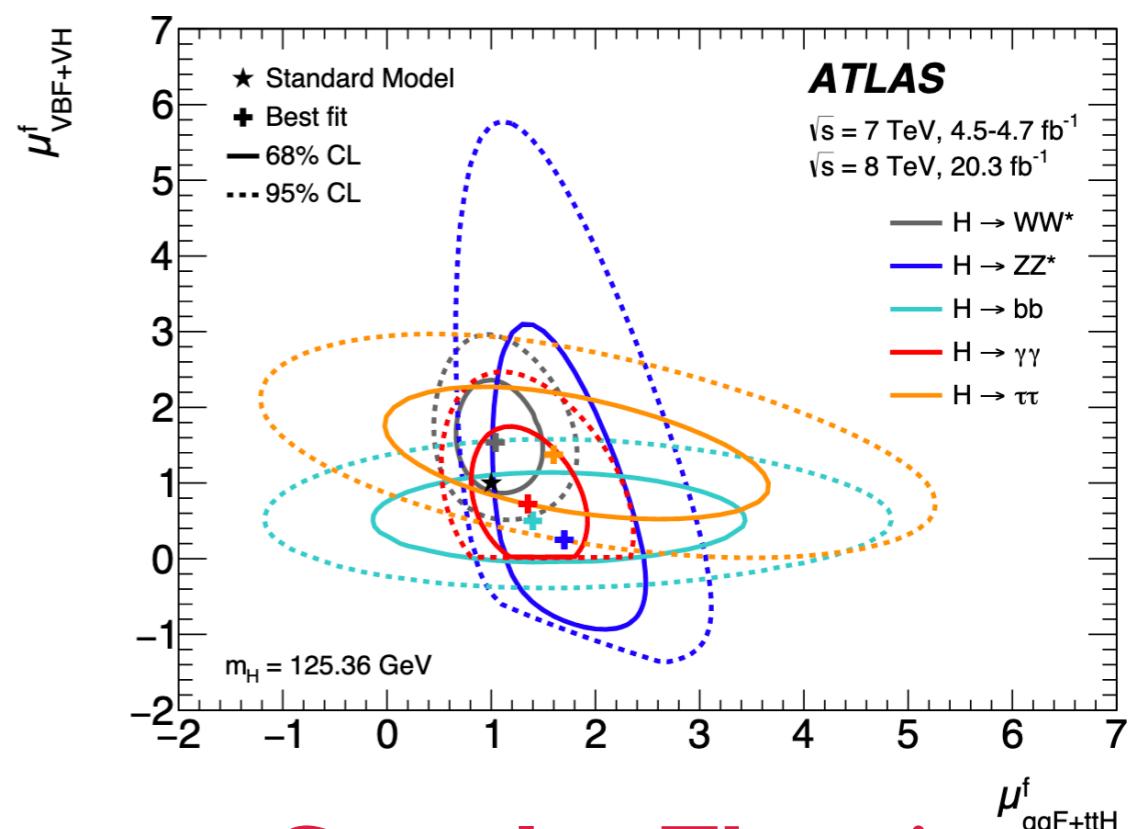
# Motivation

## How to extract all Information from the Data?



### Era of Data

high statistics, many distributions,  
multivariate analyses



### Complex Theories

large number of theory parameters,  
predict subtle kinematic features

# Outline

## **Introduction: Inference**

What's is the Problem?

## **Review: Inference Techniques**

What did we do so far?

## **The MadMiner Approach**

What do we do?

## **Optimal Observables and Fisher Information**

This will turn out to be useful.

## **The MadMiner Tool**

Using these methods is super easy!

## **A Realistic Physics Example**

Probing SMEFT in  $t\bar{t}$

## **Summary and Conclusion**

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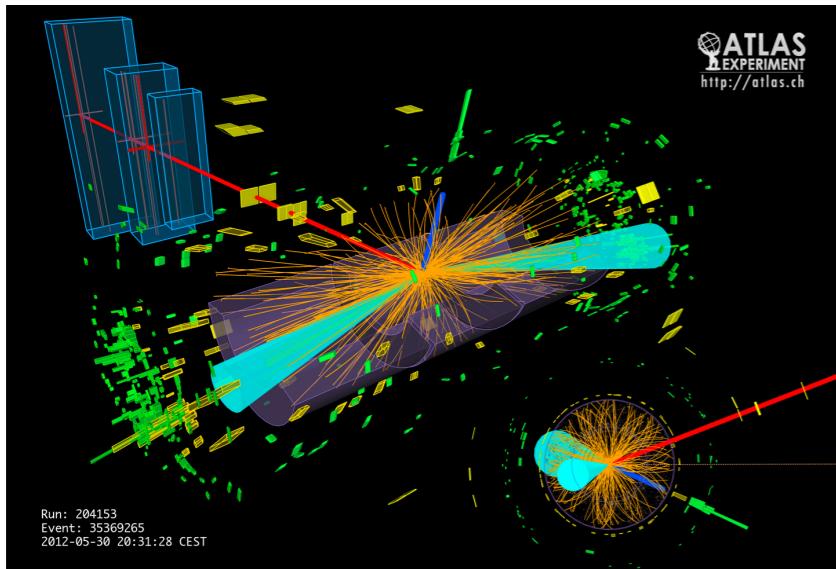
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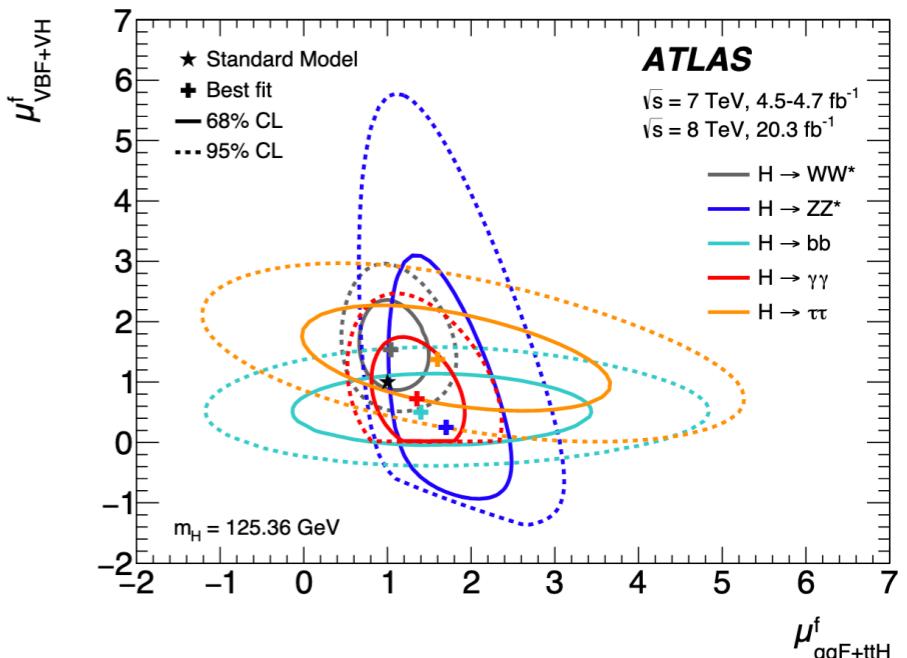
# Terminology



## Observables $x$

anything that can be measured

Example:  $x = \{E, pT, m, \Delta\varphi, \dots\}$ .

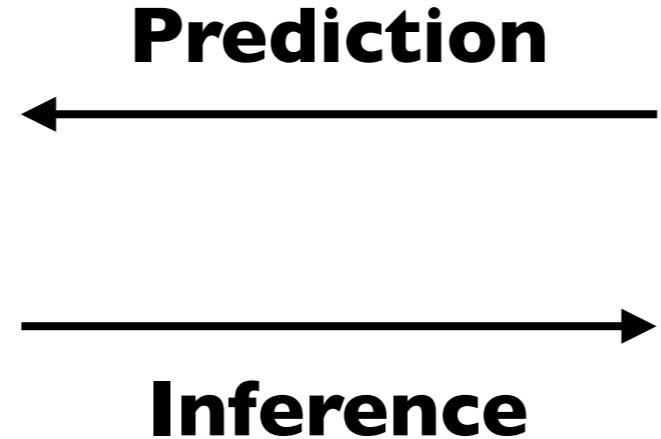
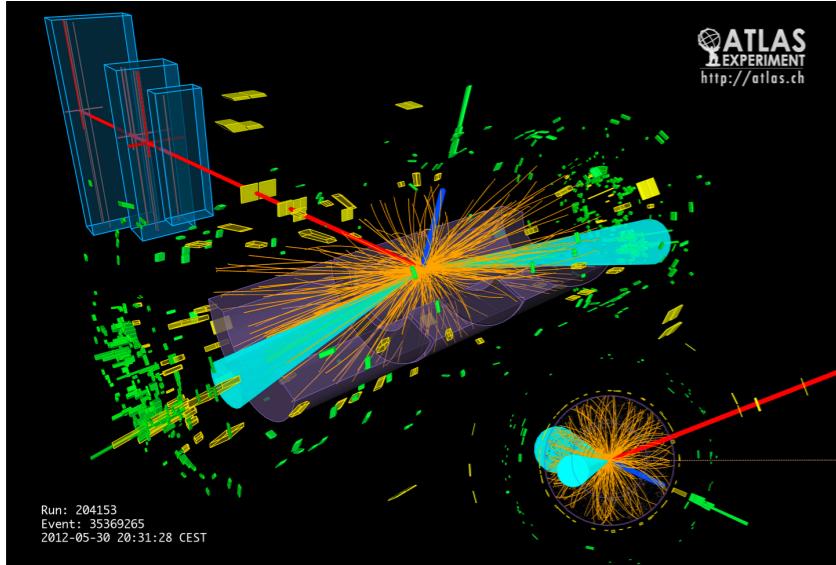


## Theory Parameters $\theta$

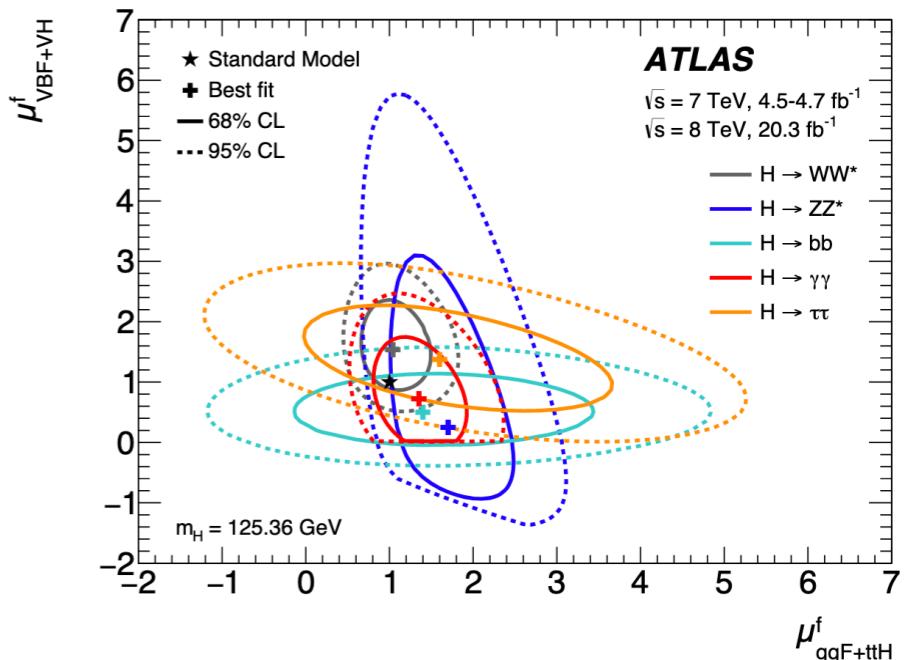
describes the theory

Example:  $\mathcal{L} = \sum \theta_i \mathcal{O}_i$ .

# Terminology

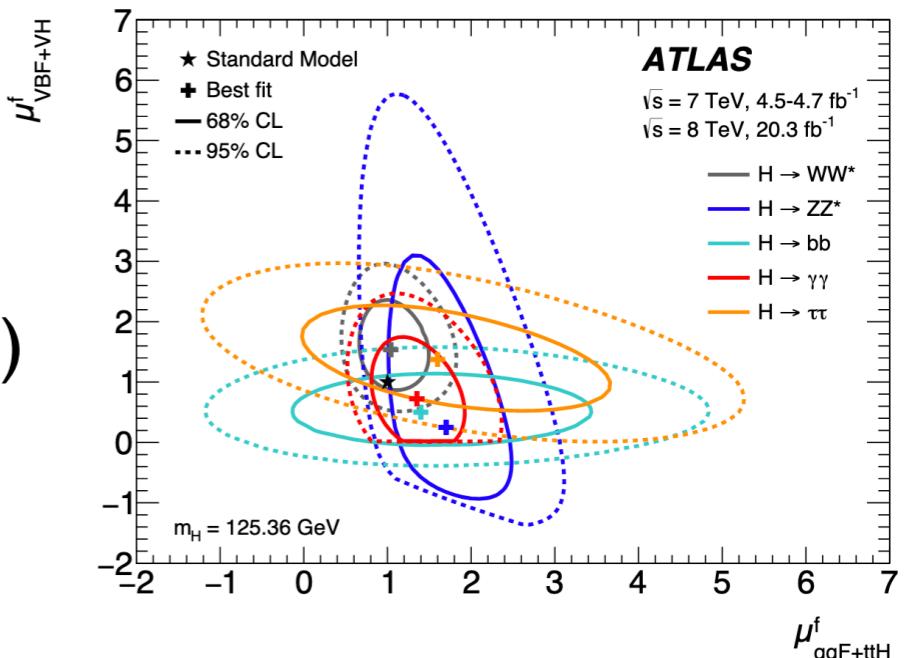
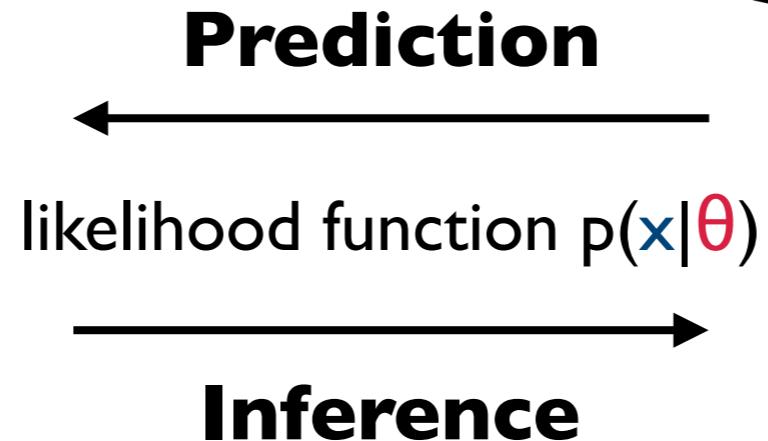
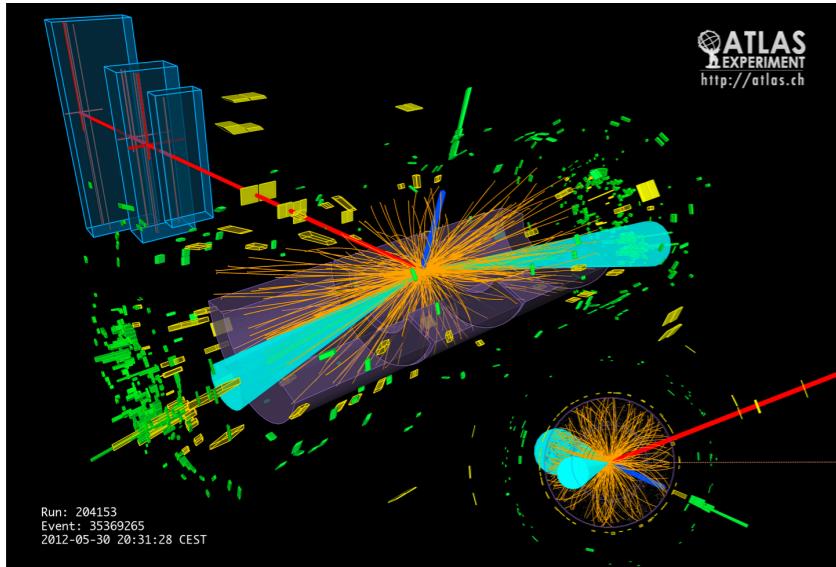


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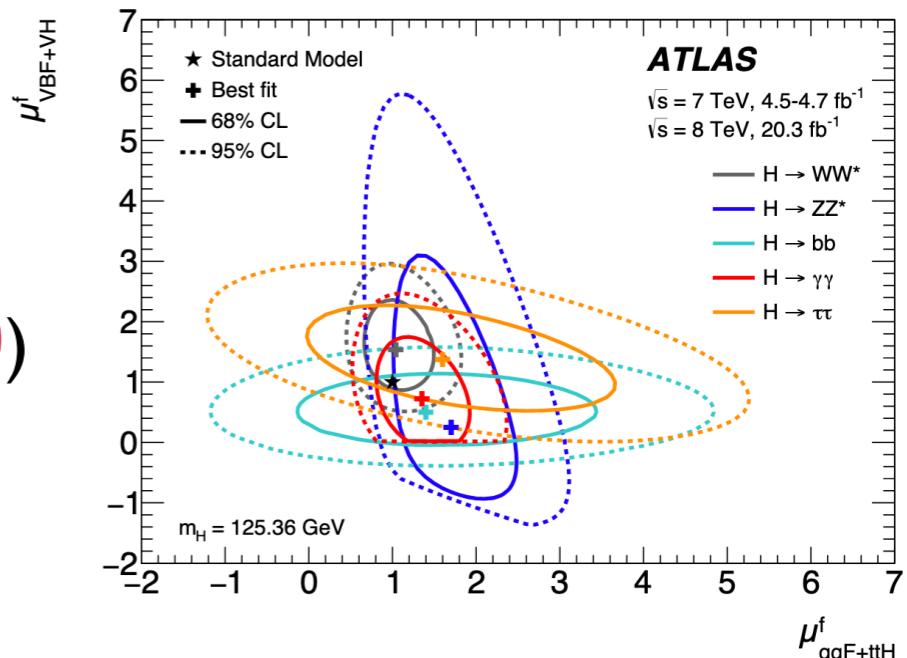
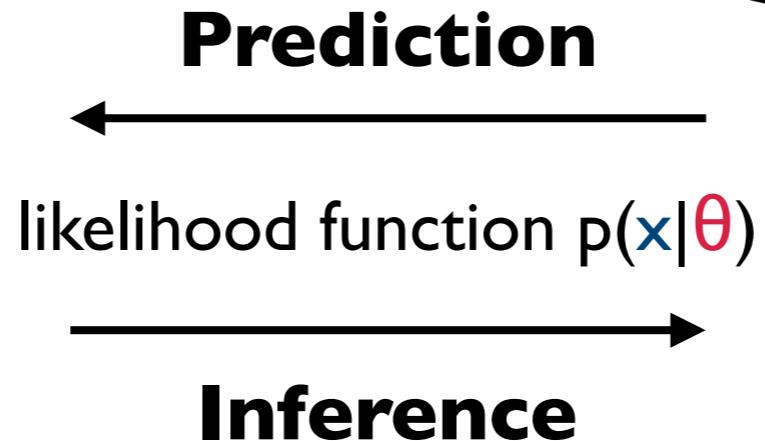
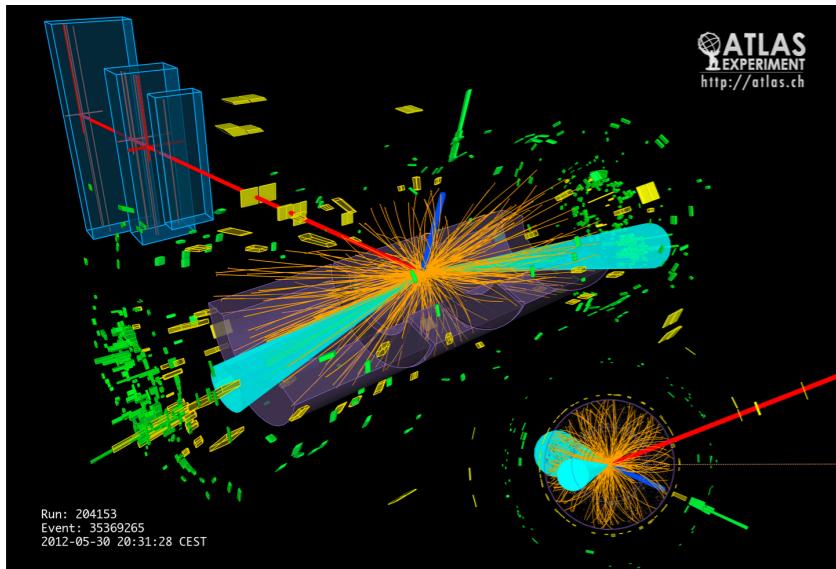
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### Likelihood Function $p(\mathbf{x}|\boldsymbol{\theta})$

likelihood of an observation  $\mathbf{x}$  as a function of the theory parameter  $\boldsymbol{\theta}$

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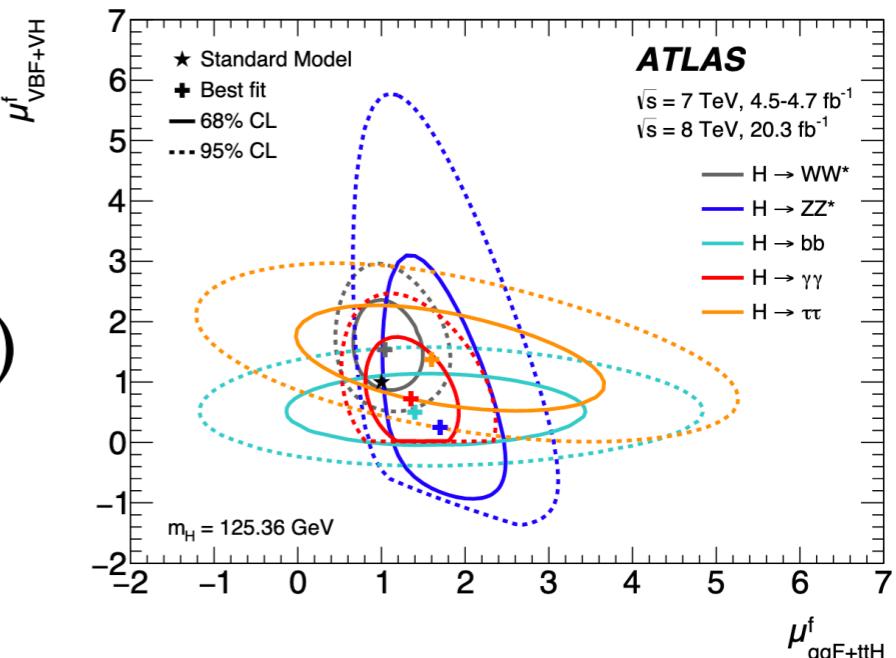
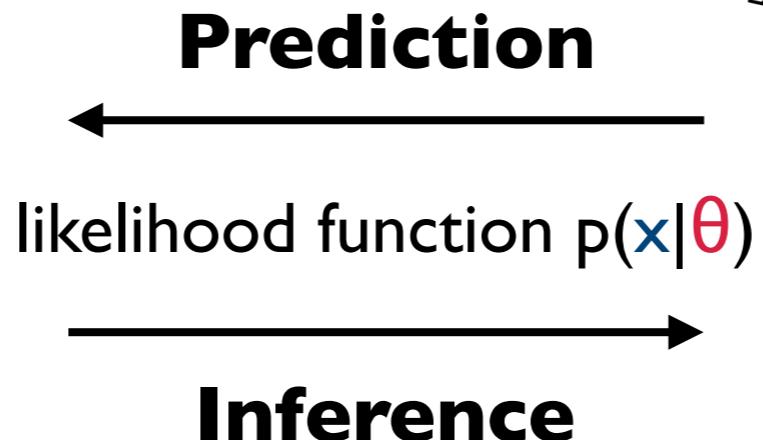
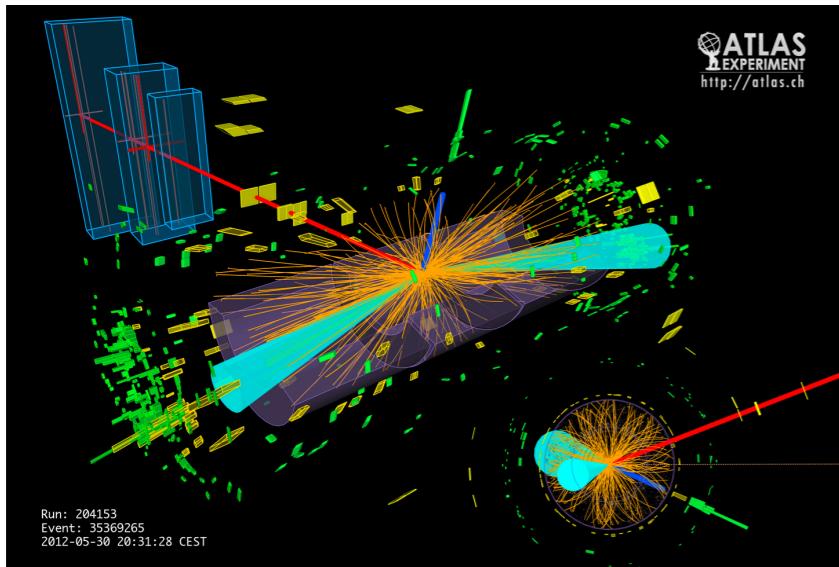
### Likelihood Ratio $r(\mathbf{x}|\boldsymbol{\theta}_{\text{ref}}, \boldsymbol{\theta}) = p(\mathbf{x}|\boldsymbol{\theta})/p(\mathbf{x}|\boldsymbol{\theta}_{\text{ref}})$

“how much more likely is data  $\mathbf{x}$  described by theory  $\boldsymbol{\theta}$  than  $\boldsymbol{\theta}_{\text{ref}}$ “

### Neyman-Pearson Lemma:

The log-likelihood ratio  $\log r(\mathbf{x}|\boldsymbol{\theta}_{\text{ref}}, \boldsymbol{\theta})$  is the most powerful test statistic to discriminate between two hypotheses  $\boldsymbol{\theta}_{\text{ref}}$  and  $\boldsymbol{\theta}$ .

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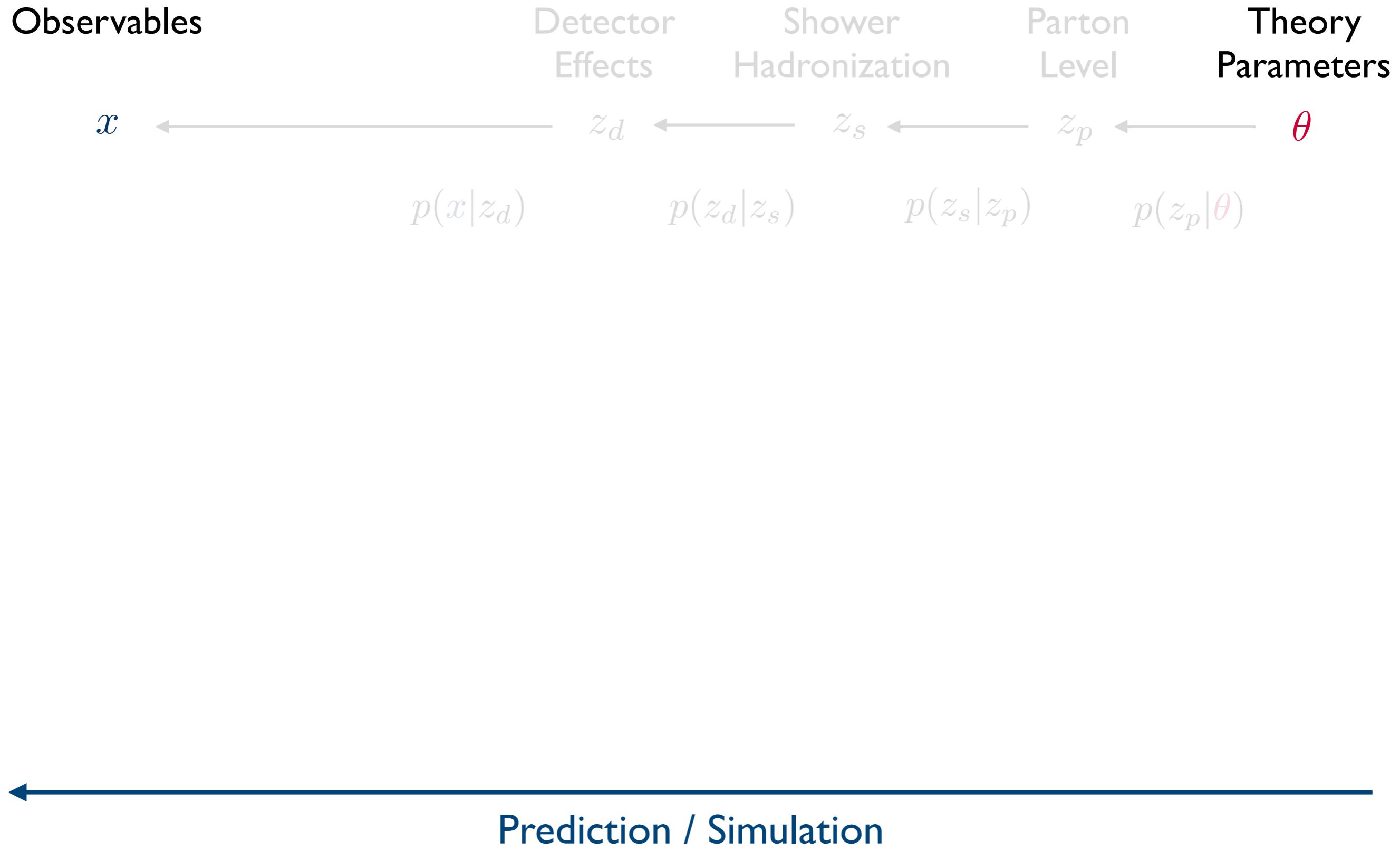
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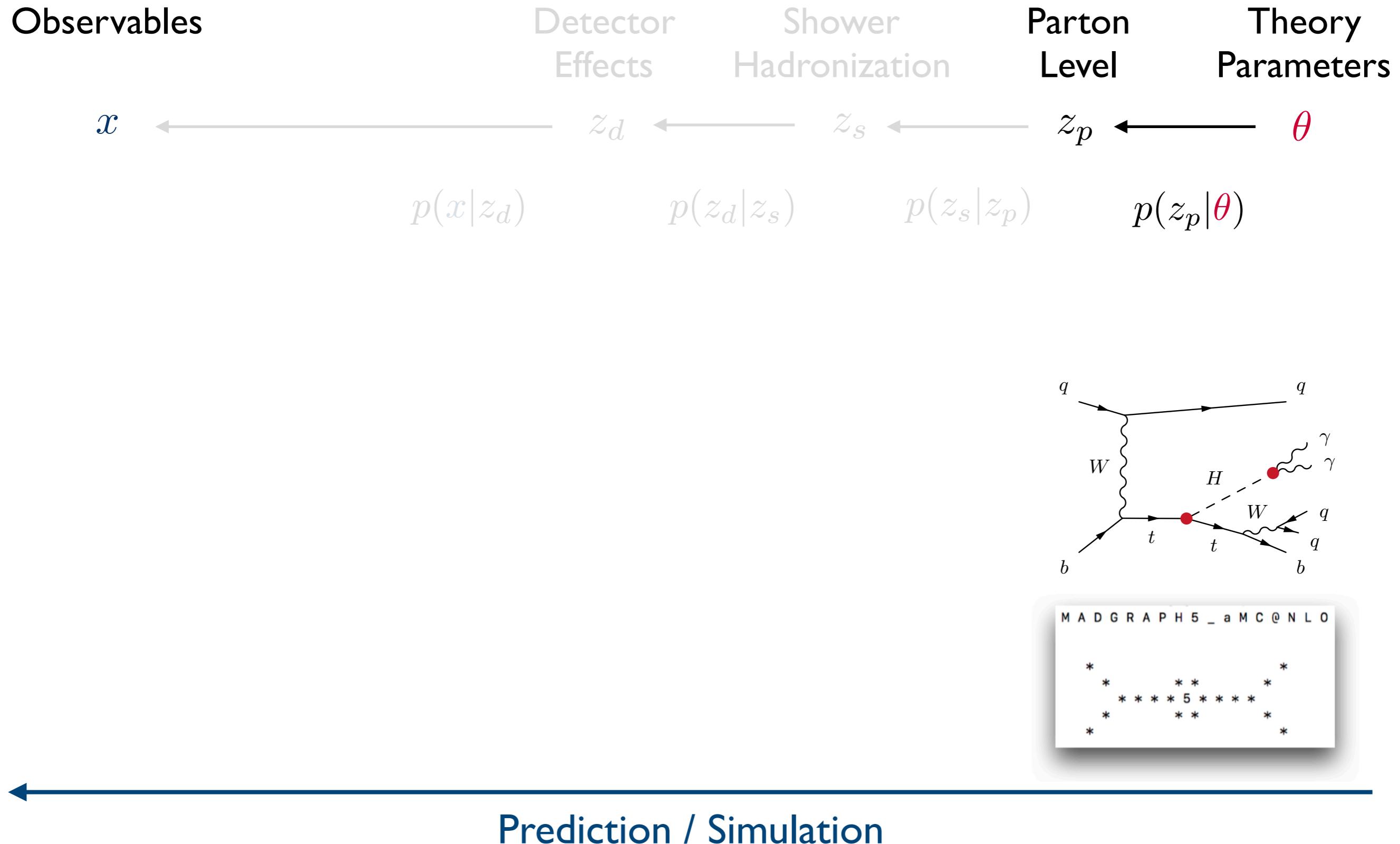
likelihood of an observation  $\mathbf{x}$  as a function of the theory parameter  $\boldsymbol{\theta}$

How can we obtain  $p(\mathbf{x}|\boldsymbol{\theta})$ ?

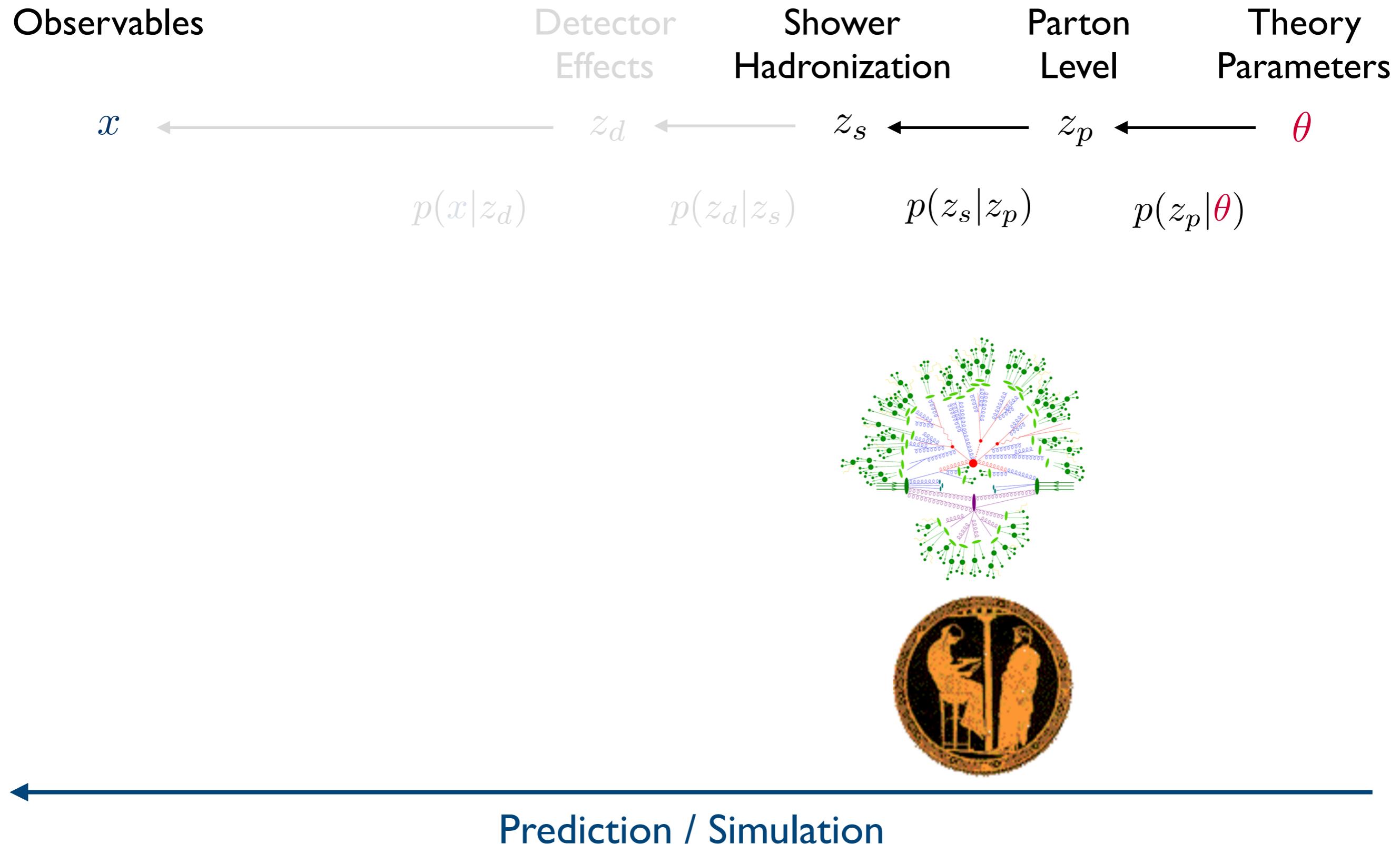
# Particle Physics Simulations



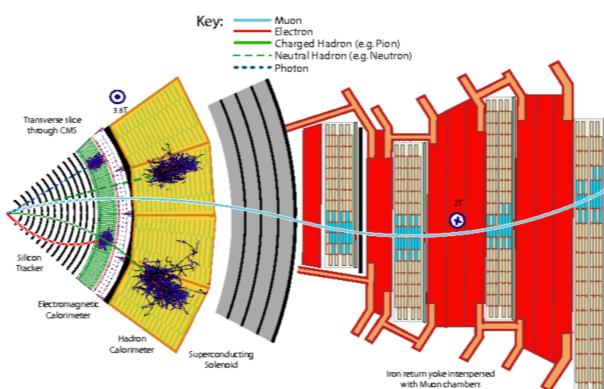
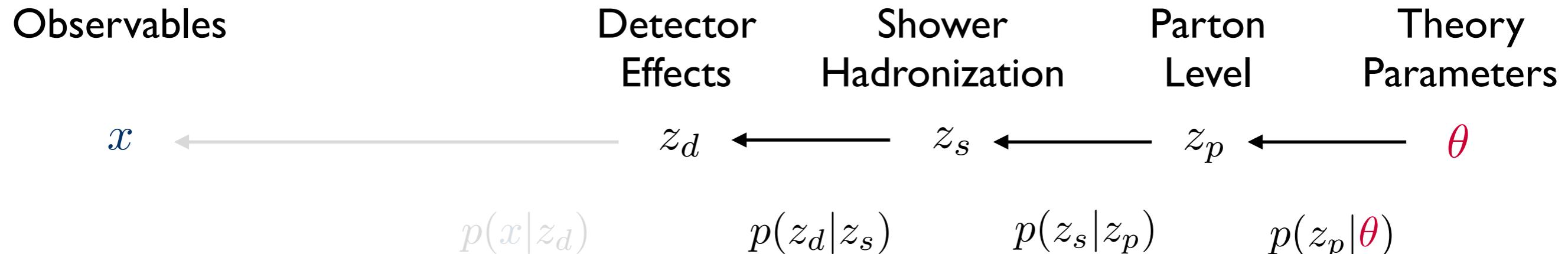
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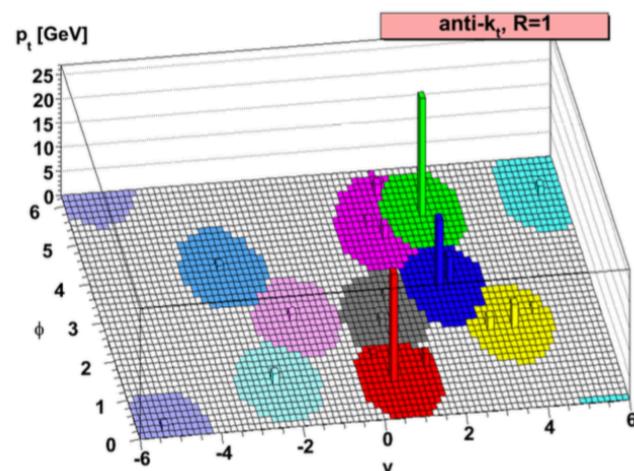
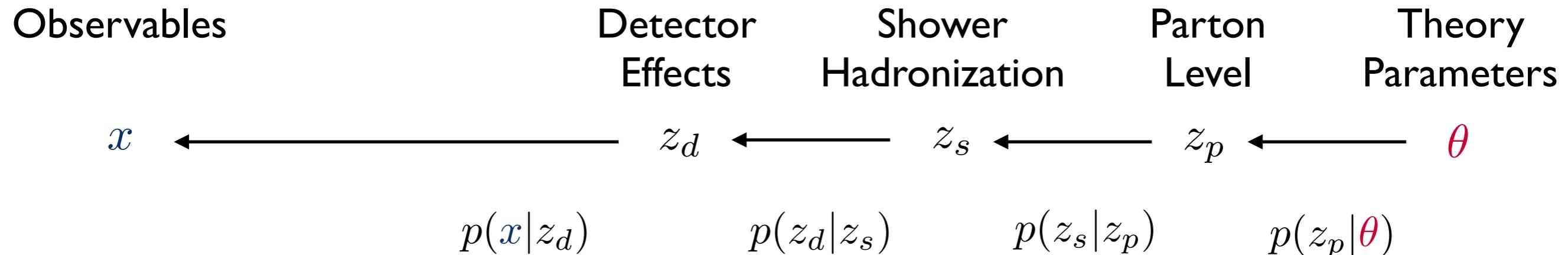


**DELPHES**  
fast simulation



Prediction / Simulation

# Particle Physics Simulations



← Prediction / Simulation

# Particle Physics Simulations

Observables

Latent Variables  $z$ : internal, not observable

Detector  
Effects

Shower  
Hadronization

Parton  
Level

Theory  
Parameters

$x$

$z_d$

$z_s$

$z_p$

$\theta$

$$p(x|\theta) = \int dz_d \ dz_s \ dz_p \ p(x|z_d)$$

$$p(z_d|z_s)$$

$$p(z_s|z_p)$$

$$p(z_p|\theta)$$

obtain MC event sample

Prediction / Simulation

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$$p(z_d|z_s)$$

$$p(z_s|z_p)$$

$$p(z_p|\theta)$$

likelihood function needed

It's infeasible to calculate the integral  
over this enormous latent space



likelihood function is *intractable*



use *estimator*  $\hat{p}(x|\theta)$

Inference

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# Inference Methods

$$\log r_{\text{full}}(\{x\}|\theta, \theta_{\text{ref}}) = \log r_{\text{rate}}(n|\theta, \theta_{\text{ref}}) + \sum_{i=1}^n \log r(x_i|\theta, \theta_{\text{ref}})$$

## likelihood function

$$p_{\text{full}}(\{x\}|\theta) = \text{Pois}(n|L\sigma(\theta)) \times \prod_{i=1}^n p(x_i|\theta)$$

full likelihood  
(all events)

rate  
(event number)

kinematics  
(single event)

# Inference Techniques

$$\log r_{\text{full}}(\{x\}|\theta, \theta_{\text{ref}}) = \log r_{\text{rate}}(n|\theta, \theta_{\text{ref}}) + \sum_{i=1}^m \log r_i(x_i|\theta, \theta_{\text{ref}})$$

ignore this term

## Option I: Rate Only

- only consider rate: “cut and count”
- $\log r_{\text{full}}(n|\theta, \theta_{\text{ref}}) = \log r_{\text{rate}}(n|\theta, \theta_{\text{ref}}) = \text{Pois}(n|L\sigma(\theta))/\text{Pois}(n|L\sigma(\theta_{\text{ref}}))$
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## Option 1: Rate Only

Histogram

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## Option 2: Summary Statistics

- few hand-picked observables  $x'$
- estimate  $p(x'|\theta)$
- information loss
- problem dependent
- Example:
  - \* histograms
  - \* STXS

# Inference Techniques

$$\log r_{\text{full}}(\{x\}|\theta, \theta_{\text{ref}}) = \log r_{\text{rate}}(n|\theta, \theta_{\text{ref}}) + \sum_{i=1}^n \log r(x_i|\theta, \theta_{\text{ref}})$$



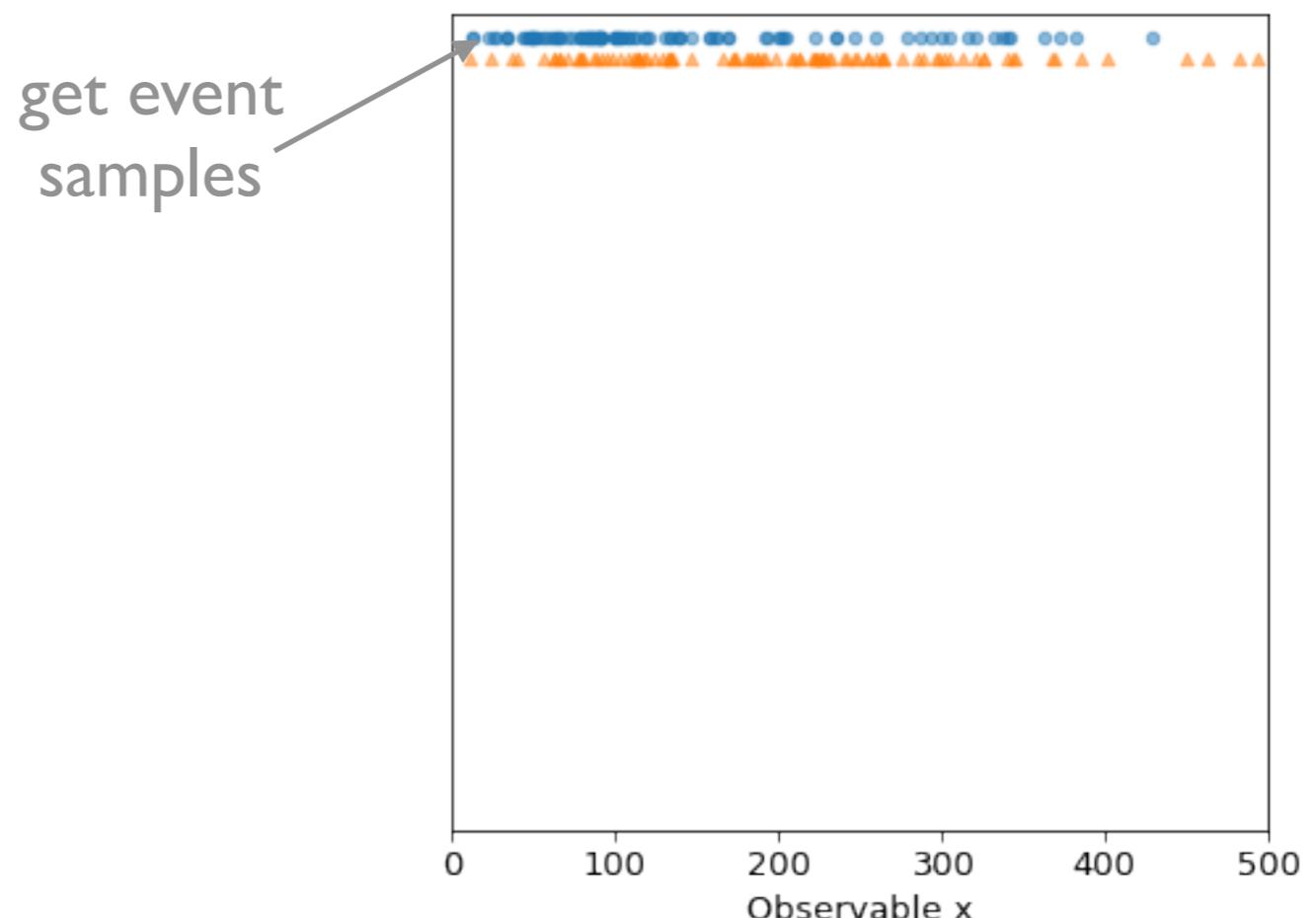
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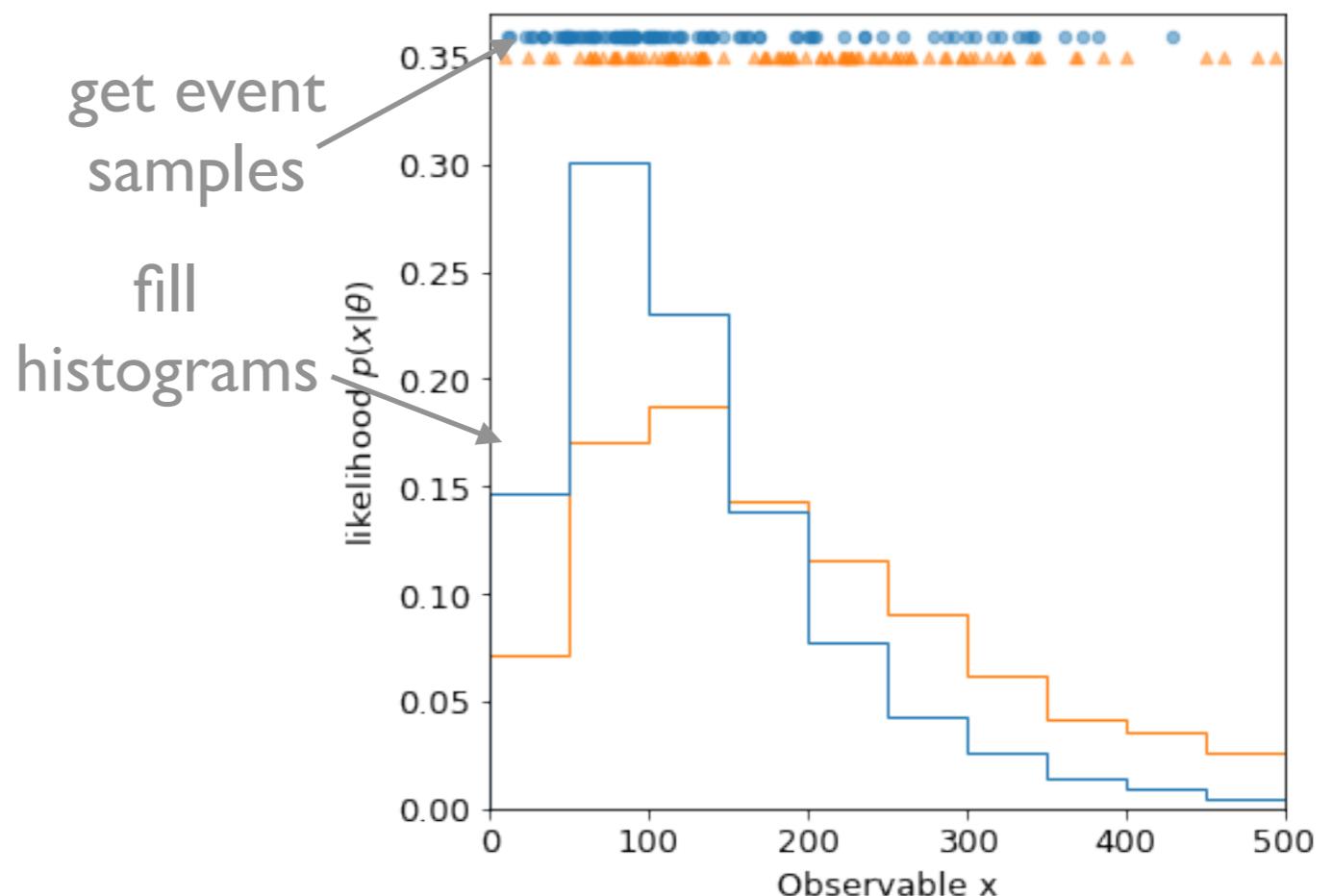
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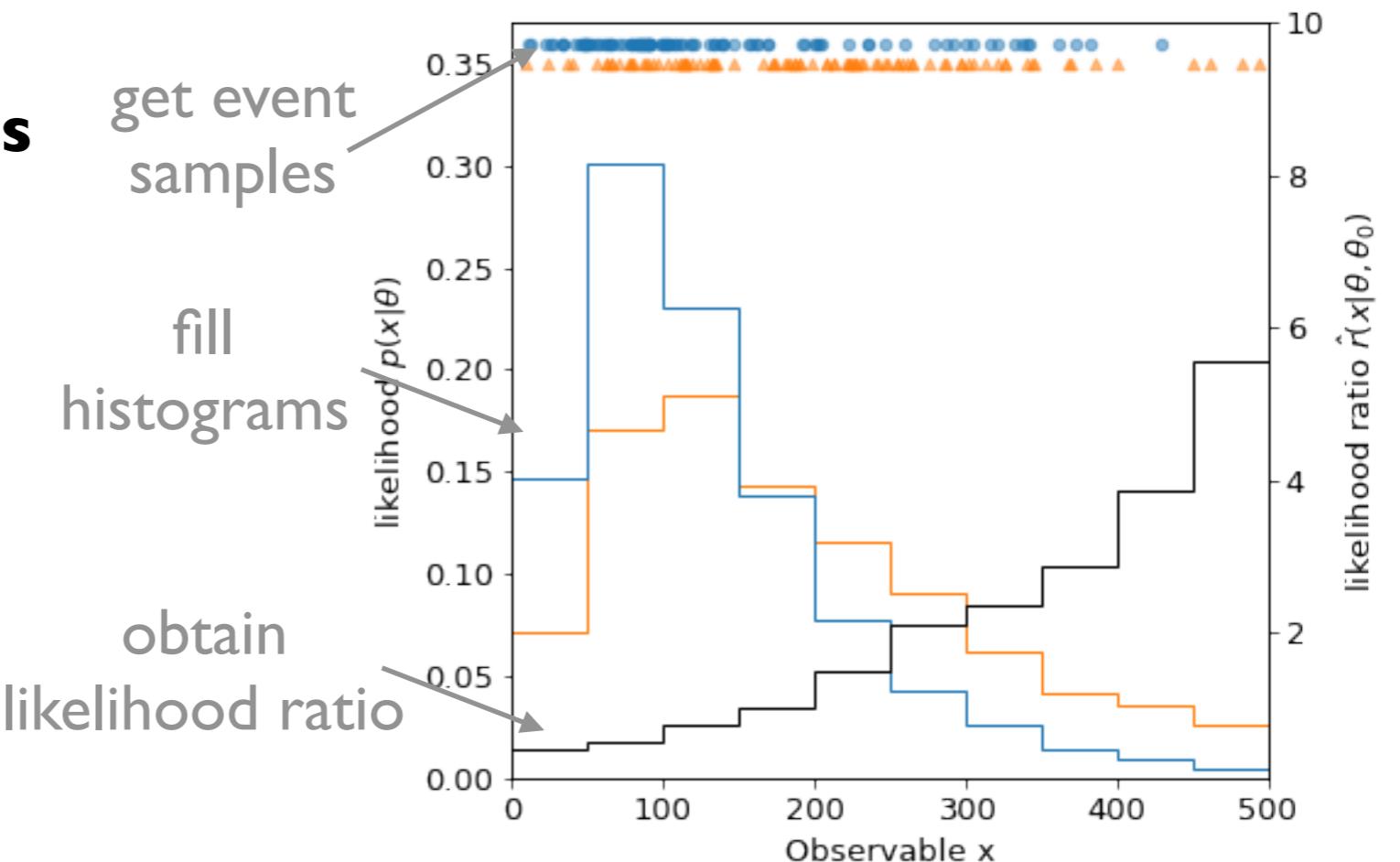
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## Option 3: Machine Learning

- estimate  $r(x|\theta)$  from multivariate analysis
- works great for S vs BG
- struggles with S' vs S:
  - \*large number of S'
  - \* very similar S', S
- physics not always clear
- Example: ML Classifier

use ML Classifier

# Inference Techniques

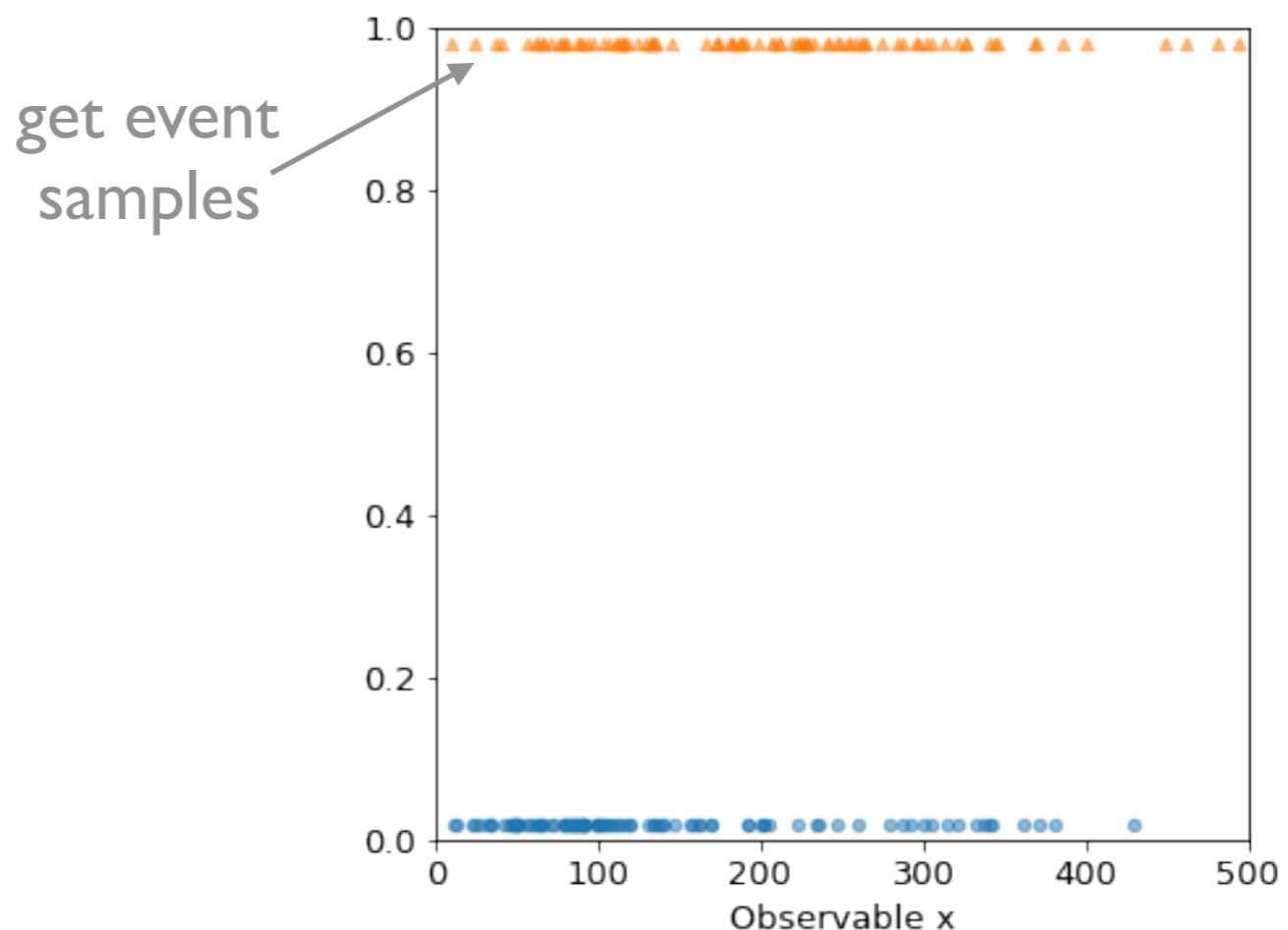
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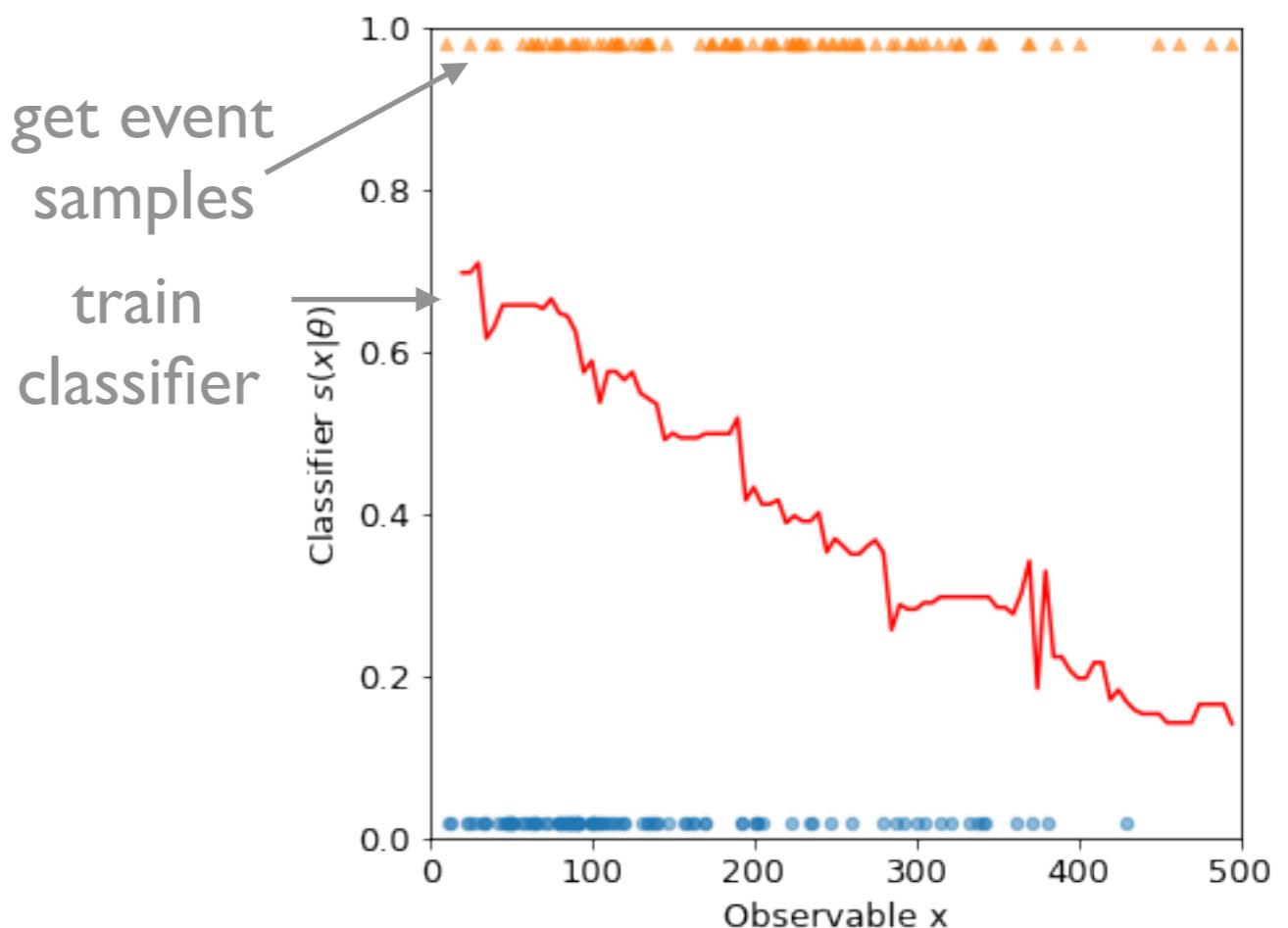
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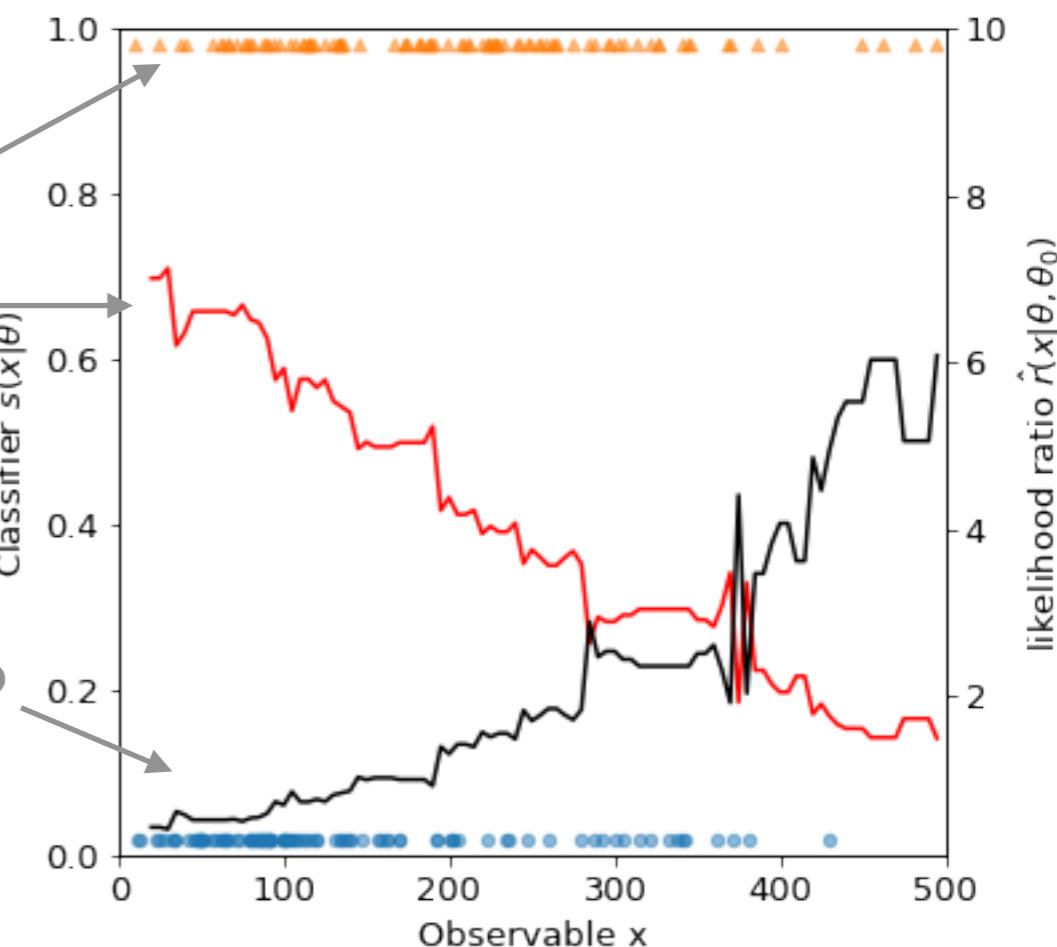
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- optimal decision function:

$$s(x|\theta) = \frac{p(x|\theta_{\text{ref}})}{p(x|\theta) + p(x|\theta_{\text{ref}})}$$

use ML Classifier

get event samples  
train classifier  
obtain likelihood ratio



# Inference Techniques

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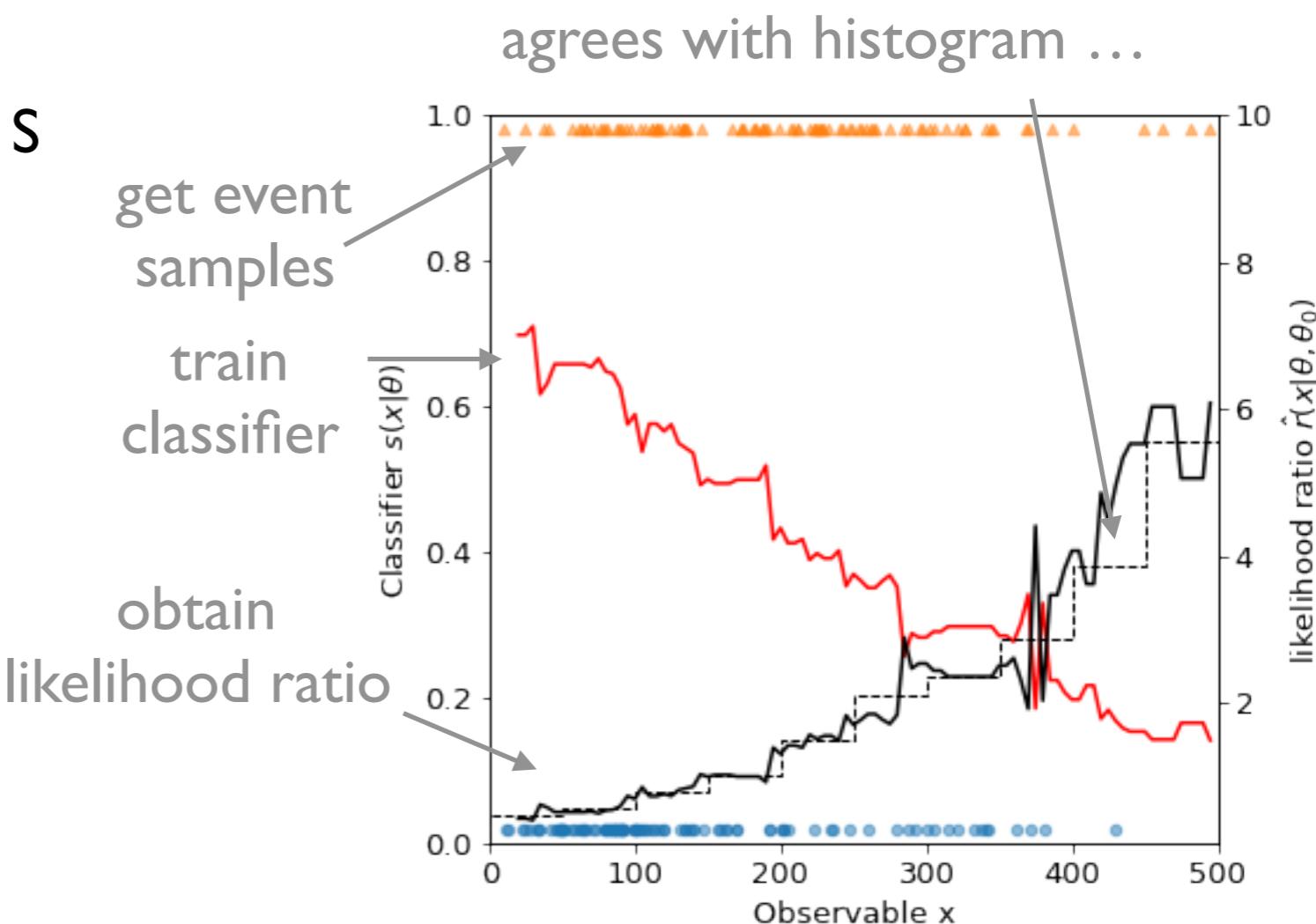


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**use ML Classifier**



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use Matrix Element

## Option 4: Matrix Element Based

- uses  $p(x|\theta) \sim |\mathcal{M}(x|\theta)|^2$
- multivariate analysis, direct physics insight
- requires approximations of detector response:  $x=z_p$
- works great at parton level: S' vs S is easy
- S vs BG can be hard
- Example: Matrix Element Method (MEM), Optimal Observables (OO)

# Inference Techniques

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**Can we have MEM  
at detector level?**

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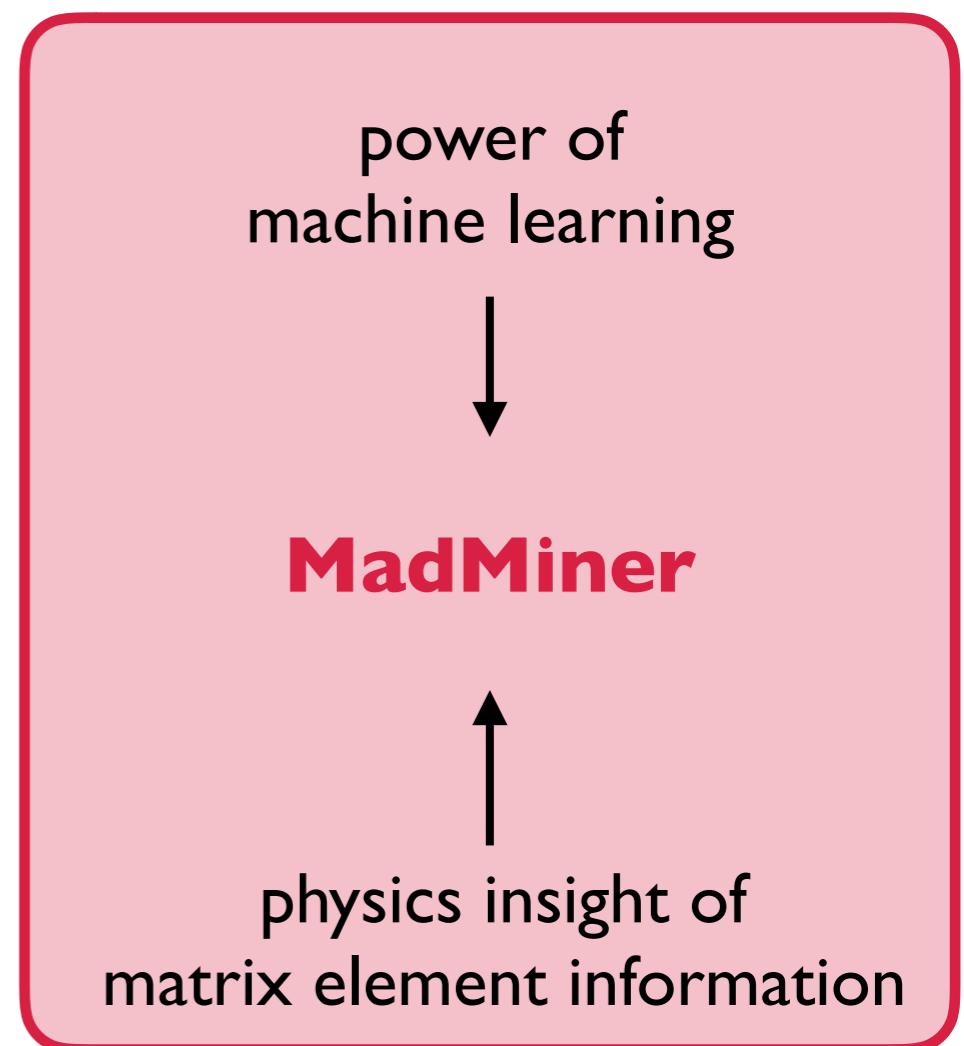
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- works great for S vs BG
- struggles with S' vs S:
  - \*large number of S'
  - \* very similar S', S
- physics not always clear
- Example: ML Classifier



## Option 4: Matrix Element Based

- uses  $p(x|\theta) \sim |M(x|\theta)|^2$
- multivariate analysis, direct physics insight
- requires approximations of detector response:  $x=z_p$
- works great at parton level: S' vs S is easy
- S vs BG can be hard
- Example: Matrix Element Method (MEM), Optimal Observables (OO)

# Outline

## **Introduction: Inference**

What's is the Problem?

## **Review: Inference Techniques**

What did we do so far?

## **The MadMiner Approach**

What do we do?

## **Optimal Observables and Fisher Information**

This will turn out to be useful.

## **The MadMiner Tool**

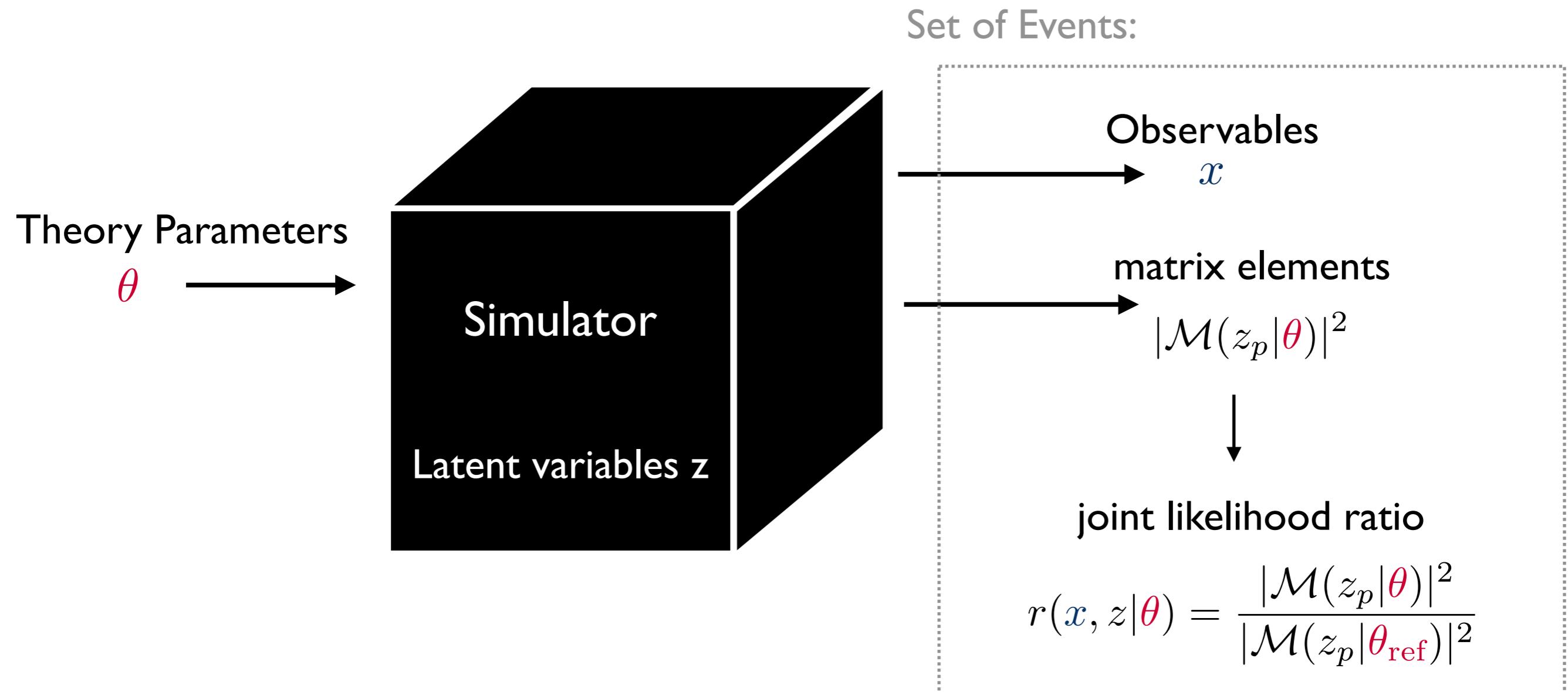
Using these methods is super easy!

## **A Realistic Physics Example**

Probing SMEFT in  $t\bar{t}$

## **Summary and Conclusion**

# The MadMiner Approach



# The MadMiner Approach

## Problem Setup:

We want: Likelihood-ratio

$$r(\mathbf{x}|\theta)$$

data      theory

---

Generators give us: Joint Likelihood-ratio

$$r(\mathbf{x}, \mathbf{z}|\theta)$$

# The MadMiner Approach

## Problem Setup:

We want: Likelihood-ratio

$$r(\mathbf{x}|\theta)$$

data      theory

A diagram illustrating the likelihood ratio. At the top is the expression  $r(\mathbf{x}|\theta)$ . Below it, two arrows point upwards from the words "data" and "theory".

Generators give us: Joint Likelihood-ratio

$$r(\mathbf{x}, \mathbf{z}|\theta)$$

## A Pedagogical Example for Illustration

- processes

\* signal: fully leptonic  $t\bar{t}h$   $pp \rightarrow t\bar{t}h \rightarrow (b\ell^+) (\bar{b}\ell^-) (\gamma\gamma) E_T^{\text{miss}}$

\* background :  $t\bar{t}\gamma\gamma$  continuum

- theory

\* SMEFT model:  $\mathcal{L} = \mathcal{L}_{SM} + c_G \mathcal{O}_G$  with  $\mathcal{O}_G = g_s^2/m_W^2 (H^\dagger H) G_{\mu\nu}^a G_a^{\mu\nu}$

\* theory parameter:  $\theta = 100 \times c_G$

- simulation: MadGraph5 + Pythia8 + Delphes3 + HL-LHC setup

[HL-LHC WG: 1902.00134]

- one observable:  $x = p_{T,\gamma\gamma}$

- latent variables:

\* neutrino  $p_z$

\* jet charge+flavor

\* all the hadron momenta

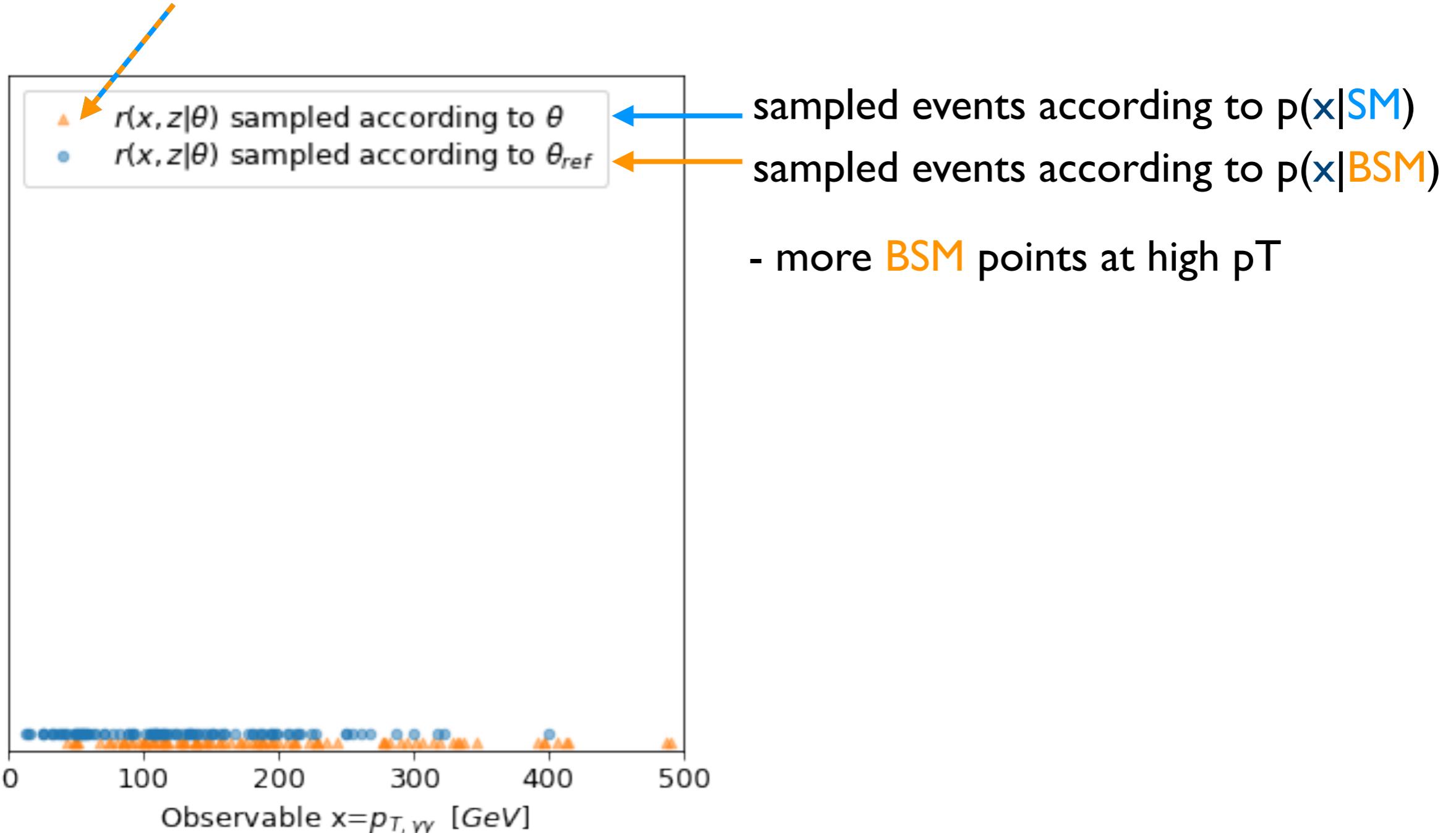
\* random numbers in detector simulation

\* all other observables

# The MadMiner Approach

## How is Likelihood Estimated?

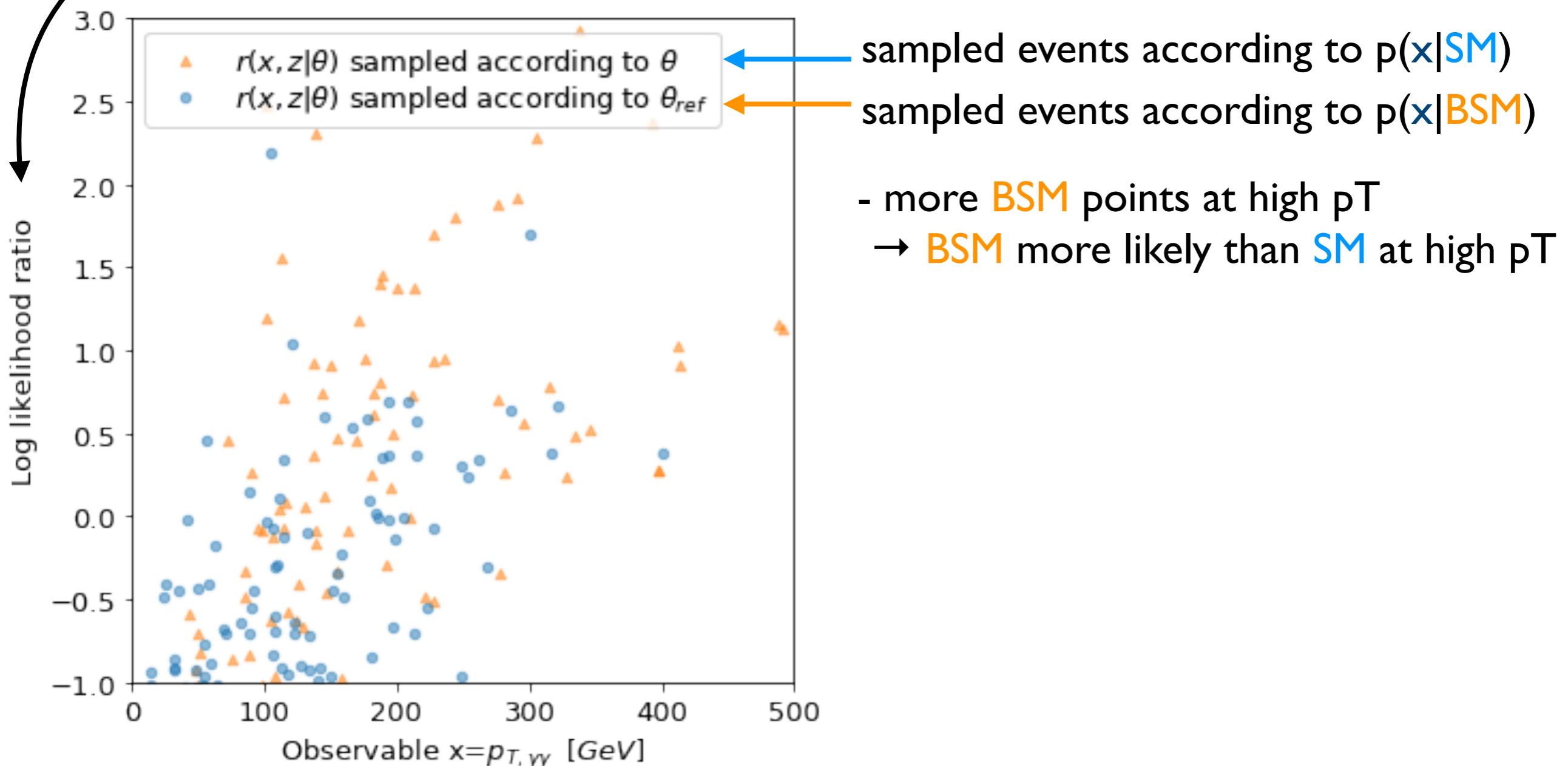
- consider two models **BSM ( $\theta=1$ )** vs **SM ( $\theta_{\text{ref}}=0$ )**
  - \* sampled events according to  $p(x|\text{SM})$ ,  $p(x|\text{BSM})$



# The MadMiner Approach

## How is Likelihood Estimated?

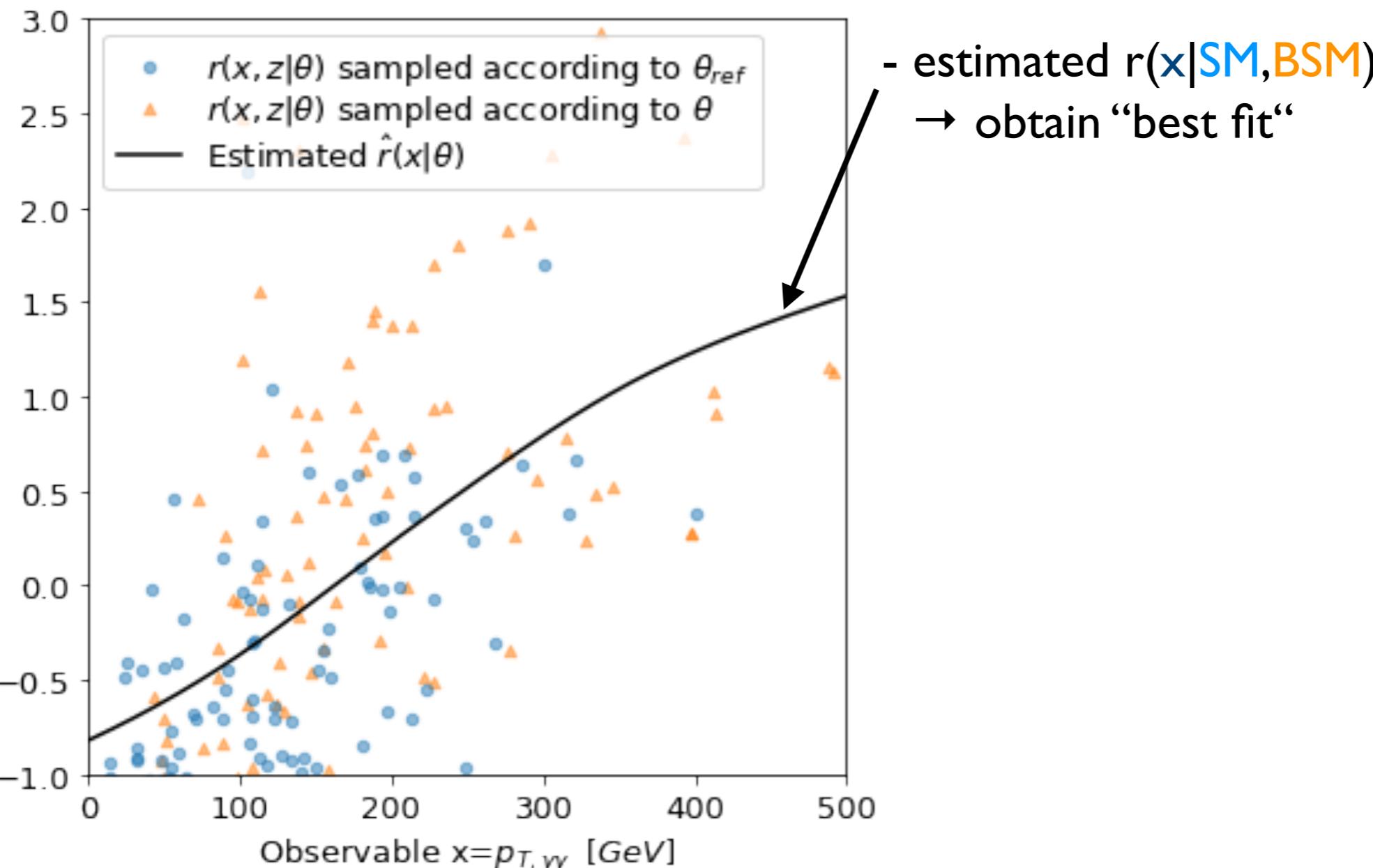
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  - \* sampled events according to  $p(x|\text{SM})$ ,  $p(x|\text{BSM})$
  - \* y-axis: joint likelihood ratio  $r(x,z|\text{BSM},\text{SM})$



# The MadMiner Approach

## How is Likelihood Estimated?

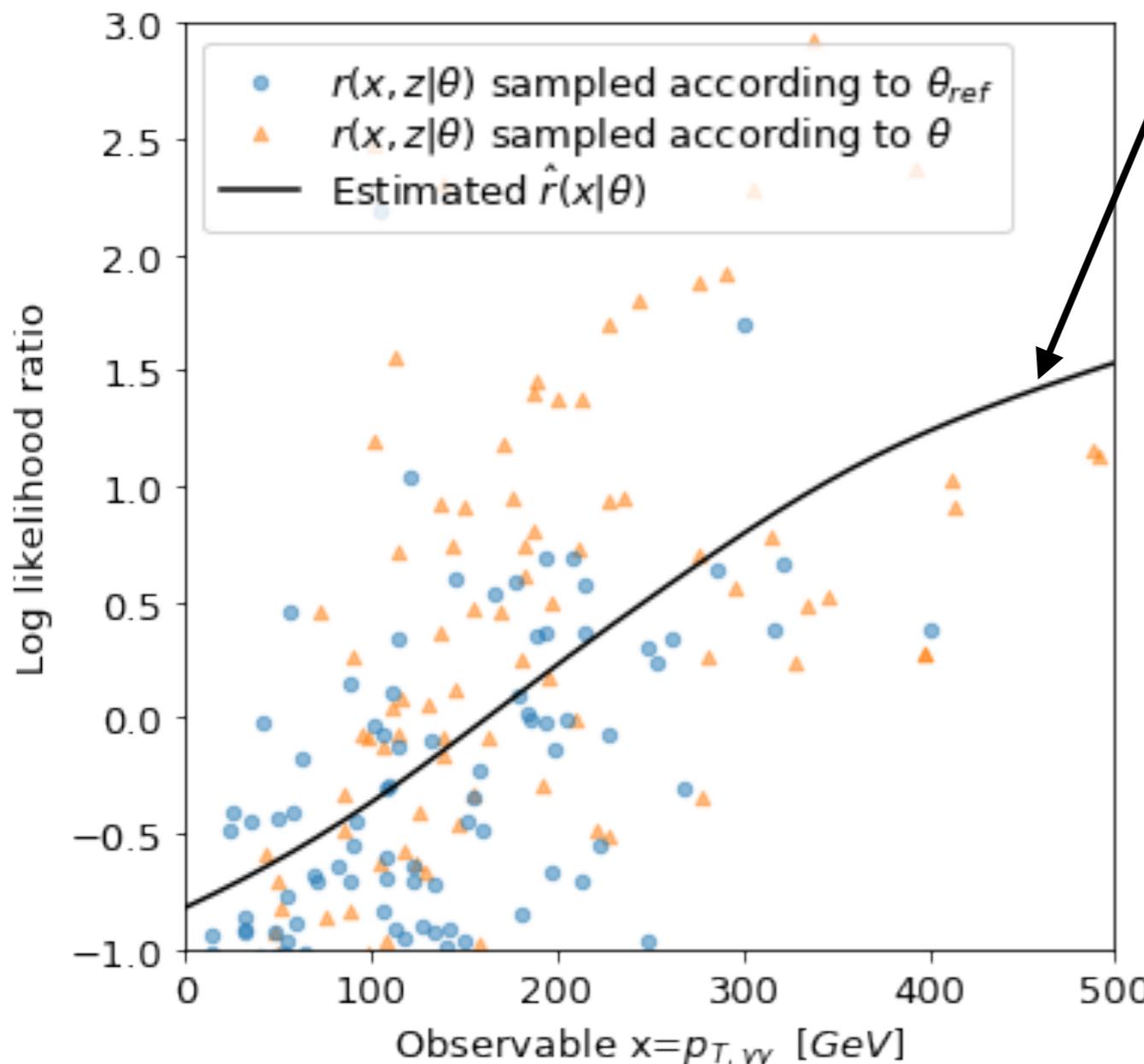
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# The MadMiner Approach

## How is Likelihood Estimated?

- consider two models BSM ( $\theta=1$ ) vs SM ( $\theta_{\text{ref}}=0$ )
  - \* sampled events according to  $p(x|\text{SM})$ ,  $p(x|\text{BSM})$
  - \* y-axis: joint likelihood ratio  $r(x,z|\text{BSM,SM})$



- estimated  $r(x|\text{SM,BSM})$ 
  - obtain “best fit”
- define functional (loss function)

$$L[\hat{r}(x|\theta)] \sim \sum |r(x|\theta) - \hat{r}(x|\theta)|^2$$

and minimize it

$$r(x|\theta) = \arg \min \hat{r}(x|\theta)$$

neural network

loss function

stochastic gradient descent

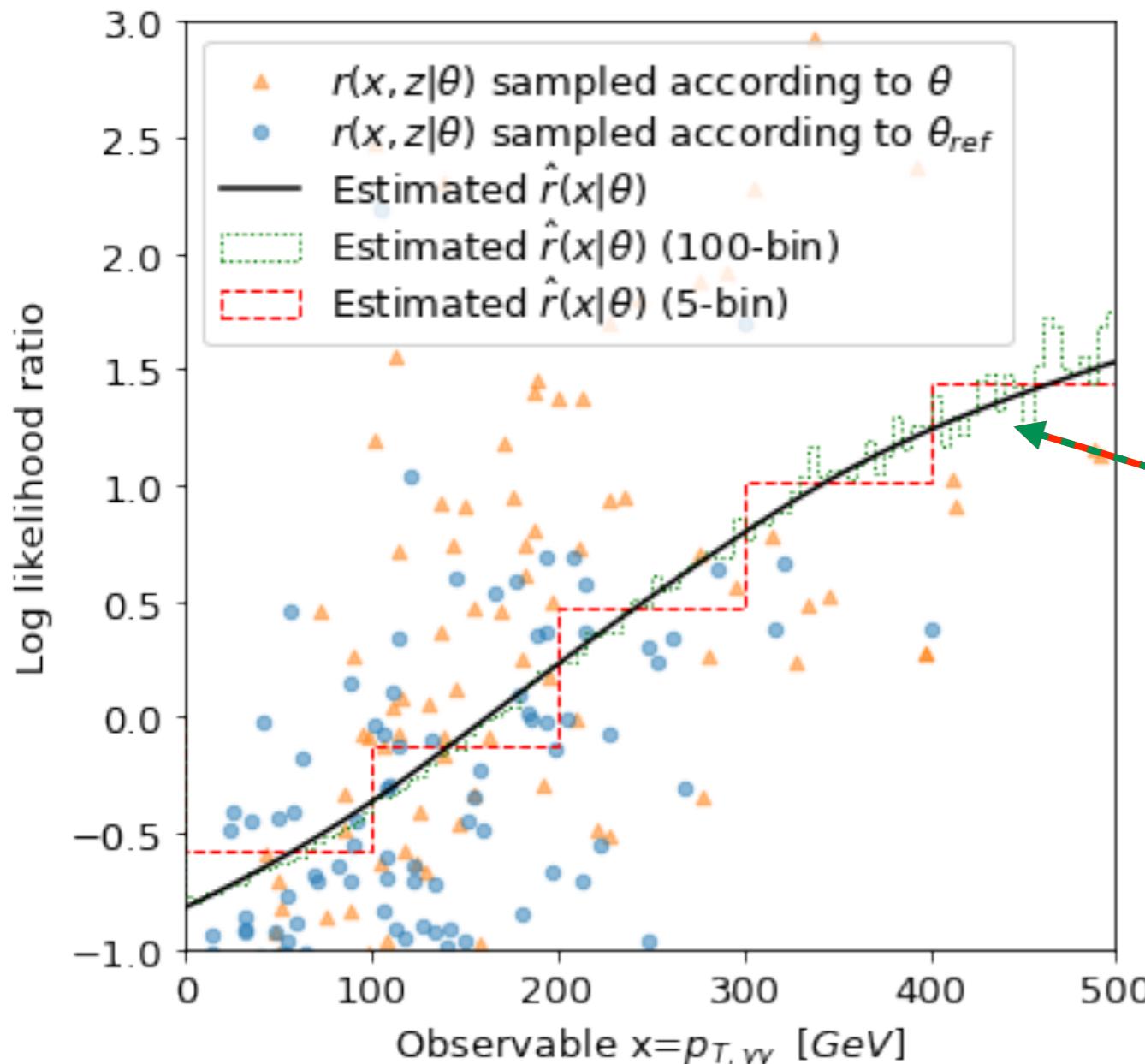
A *sufficiently expressive network, efficiently trained in this way with enough data will learn the likelihood ratio function  $r(x|\theta)$ !*

[Proof: J. Brehmer, K. Cranmer, G. Louppe, J. Pavez 1805.00020]

# The MadMiner Approach

## How is Likelihood Estimated?

- consider two models **BSM ( $\theta=1$ )** vs **SM ( $\theta_{\text{ref}}=0$ )**
  - \* sampled events according to  $p(x|\text{SM})$ ,  $p(x|\text{BSM})$
  - \* y-axis: joint likelihood ratio  $r(x,z|\text{BSM,SM})$

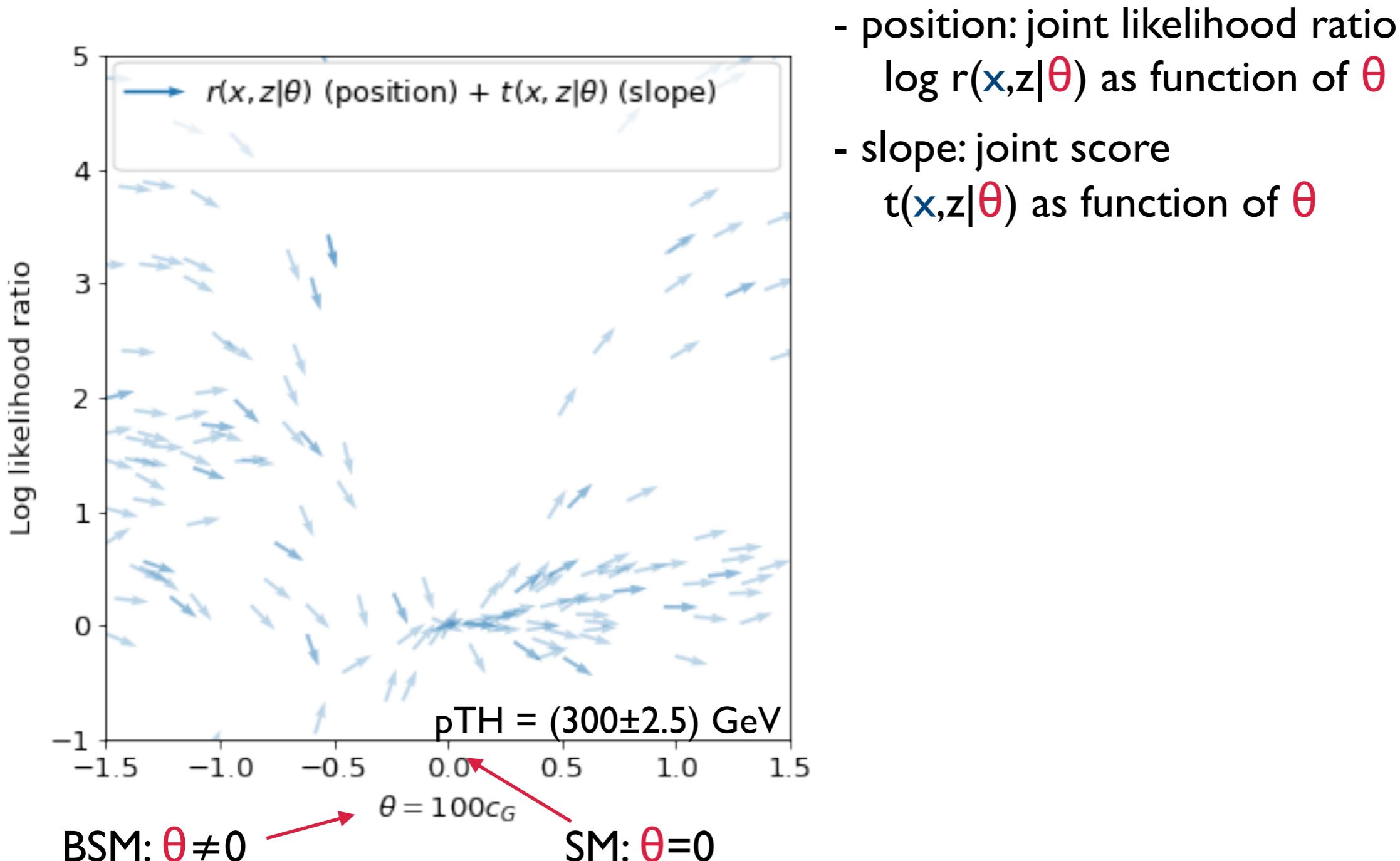


- estimated  $r(x|\text{SM,BSM})$ 
  - obtain “best fit”
- LLR obtained using histogram
  - agrees well :)
  - “continuum limit of for large # of bins”

# The MadMiner Approach

## Useful: the Score!

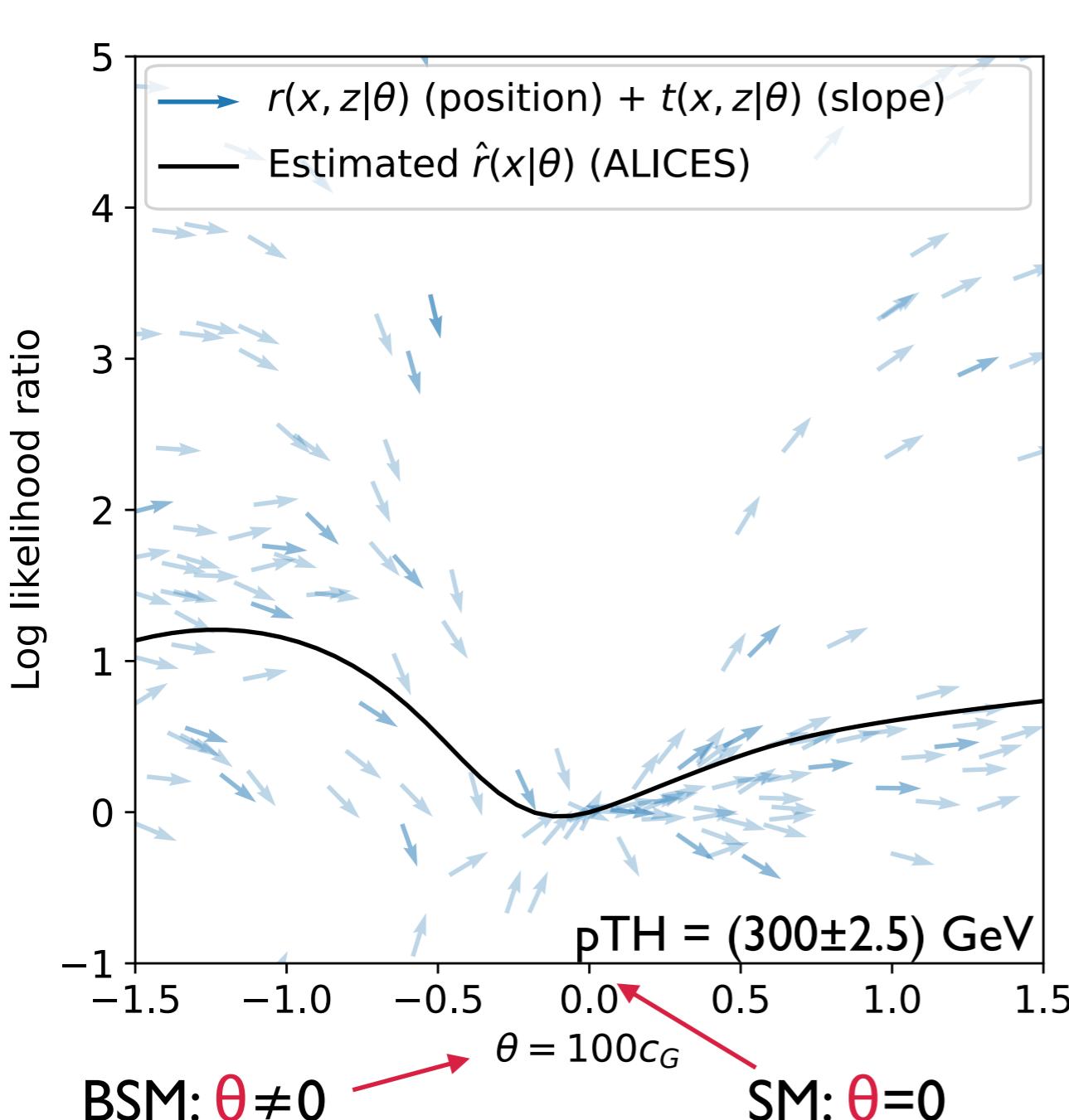
- knowing the derivative often helps: “How does data  $x$  change, when theory  $\theta$  is changed?”  
→ Score:  $t(x|z|\theta) = d \log p(x|z|\theta) / d\theta$



# The MadMiner Approach

## Useful: the Score!

- knowing the derivative often helps: “How does data  $x$  change, when theory  $\theta$  is changed?”  
→ Score:  $t(x|\theta) = d \log p(x|\theta) / d\theta$



- position: joint likelihood ratio  $\log r(x, z|\theta)$  as function of  $\theta$
  - slope: joint score  $t(x, z|\theta)$  as function of  $\theta$
- ↓
- obtain “best fit”
- estimate  $r(x, \theta)$  and  $t(x, \theta)$

### Likelihood Ratio Estimator (ALICES)

- learn LLR as function of  $x$  and  $\theta$
- use  $r(x, z|\theta)$  and  $t(x, z|\theta)$  as input

$$\text{NN} : (x, \theta) \rightarrow \hat{r}(x|\theta) \approx p(x|\theta) / p(x|\theta_{\text{ref}})$$

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# Optimal Observables

## Scores are Optimal Observables

- expand LLR around  $\theta_{\text{ref}}$

- \*  $\log r(\mathbf{x}|\theta) = \log r(\mathbf{x}|\theta_{\text{ref}}) + t(\mathbf{x})|_{\theta_{\text{ref}}} \cdot (\theta - \theta_{\text{ref}}) + \dots$

- score:  $t(\mathbf{x}|\theta) = d \log p(\mathbf{x}|\theta) / d\theta$

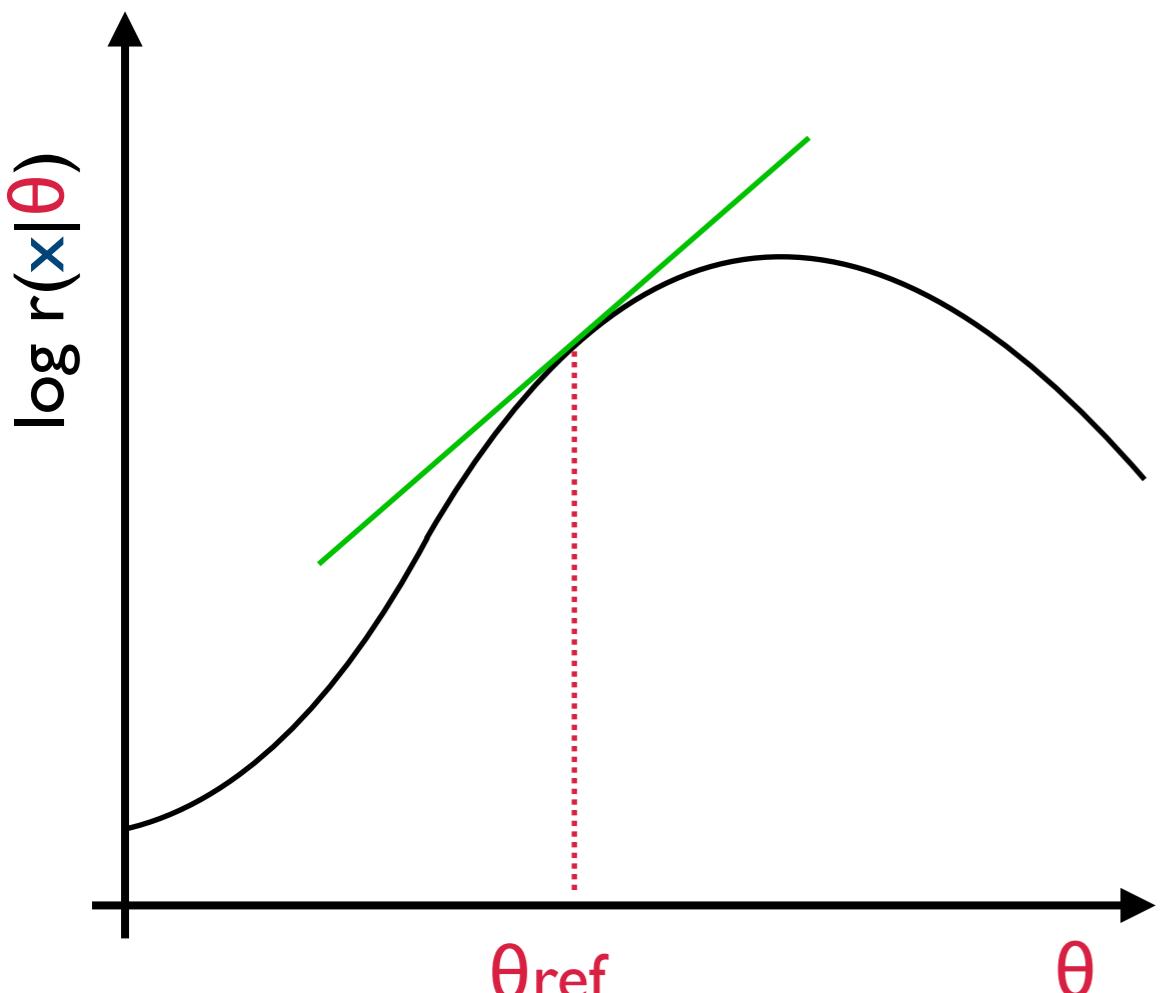
- close to  $\theta_{\text{ref}}$

- \* score is sufficient statistics

- \* knowing  $t(\mathbf{x})|_{\theta_{\text{ref}}}$  is as powerful as knowing  $r(\mathbf{x}|\theta)$

- \*  $t(\mathbf{x})|_{\theta_{\text{ref}}}$  are optimal observables

- in SMEFT:  $t(\mathbf{x})|_{\text{SM}}$  is sensitive to interference



### Score Estimator (SALLY)

- learn score as function of  $\mathbf{x}$  at  $\theta_{\text{ref}}$

- $t(\mathbf{x}, \mathbf{z}|\theta_{\text{ref}})$  as input

$$\text{NN} : \mathbf{x} \rightarrow \hat{t}(\mathbf{x}) \approx \nabla_{\theta} \log(\mathbf{x}|\theta)|_{\theta_{\text{ref}}}$$

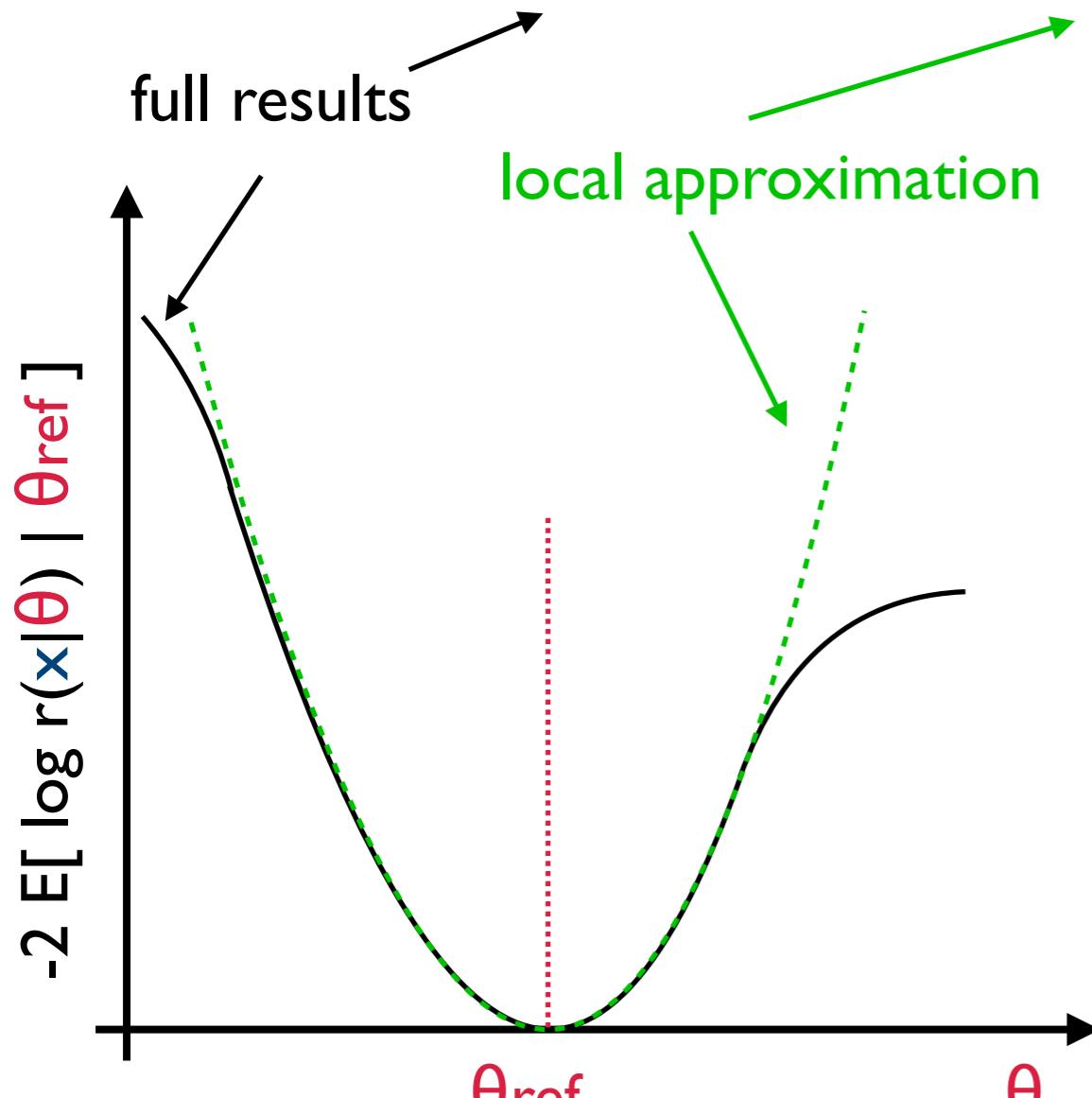
# Fisher Information

## Fisher Information

See [J. Brehmer, K. Cranmer, FK, T. Plehn 1612.05261]

- expand expected LLR around  $\theta_{\text{ref}}$

$$\mathbb{E}[-2 \log r_{\text{full}}(\mathbf{x}|\boldsymbol{\theta})|\theta_{\text{ref}}] = I_{ij}(\theta_{\text{ref}}) \times (\theta - \theta_{\text{ref}})_i (\theta - \theta_{\text{ref}})_j + \dots$$



- Fisher Information:

$$I_{ij} = \mathcal{L} \frac{\partial_i \sigma(\boldsymbol{\theta}) \partial_j \sigma(\boldsymbol{\theta})}{\sigma(\boldsymbol{\theta})} + \frac{\mathcal{L} \sigma(\boldsymbol{\theta})}{n} \sum_{\mathbf{x} \sim p(\mathbf{x}|\theta_{\text{ref}})} t_i(\mathbf{x}) t_j(\mathbf{x})$$

- useful properties:

- \* simple:  $n \times n$  matrix (for  $n$  theory parameters)
- \* Cramer-Rao bound:  $\text{cov}[\hat{\boldsymbol{\theta}}|\boldsymbol{\theta}_0] \leq I^{-1}_{ij}(\boldsymbol{\theta}_0)$
- \* independent of parameterizations of  $\mathbf{x}$
- \* covariant under  $\boldsymbol{\theta} \rightarrow \boldsymbol{\theta}'$
- \* additive between experiments / phase-space
- \* easy to include systematics
- \* defines metric on theory parameter space

*The Fisher information encodes the maximum sensitivity of observables to theory parameters for a given experiment*

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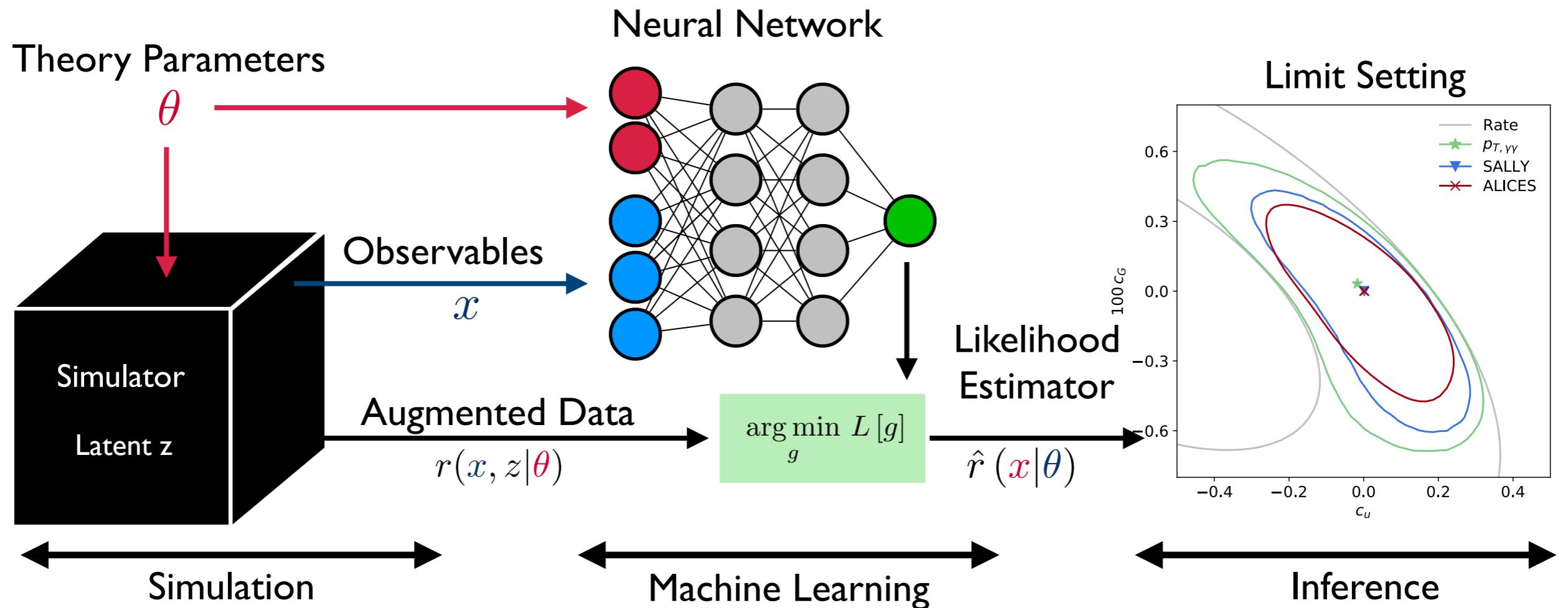
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# MadMiner: The Tool

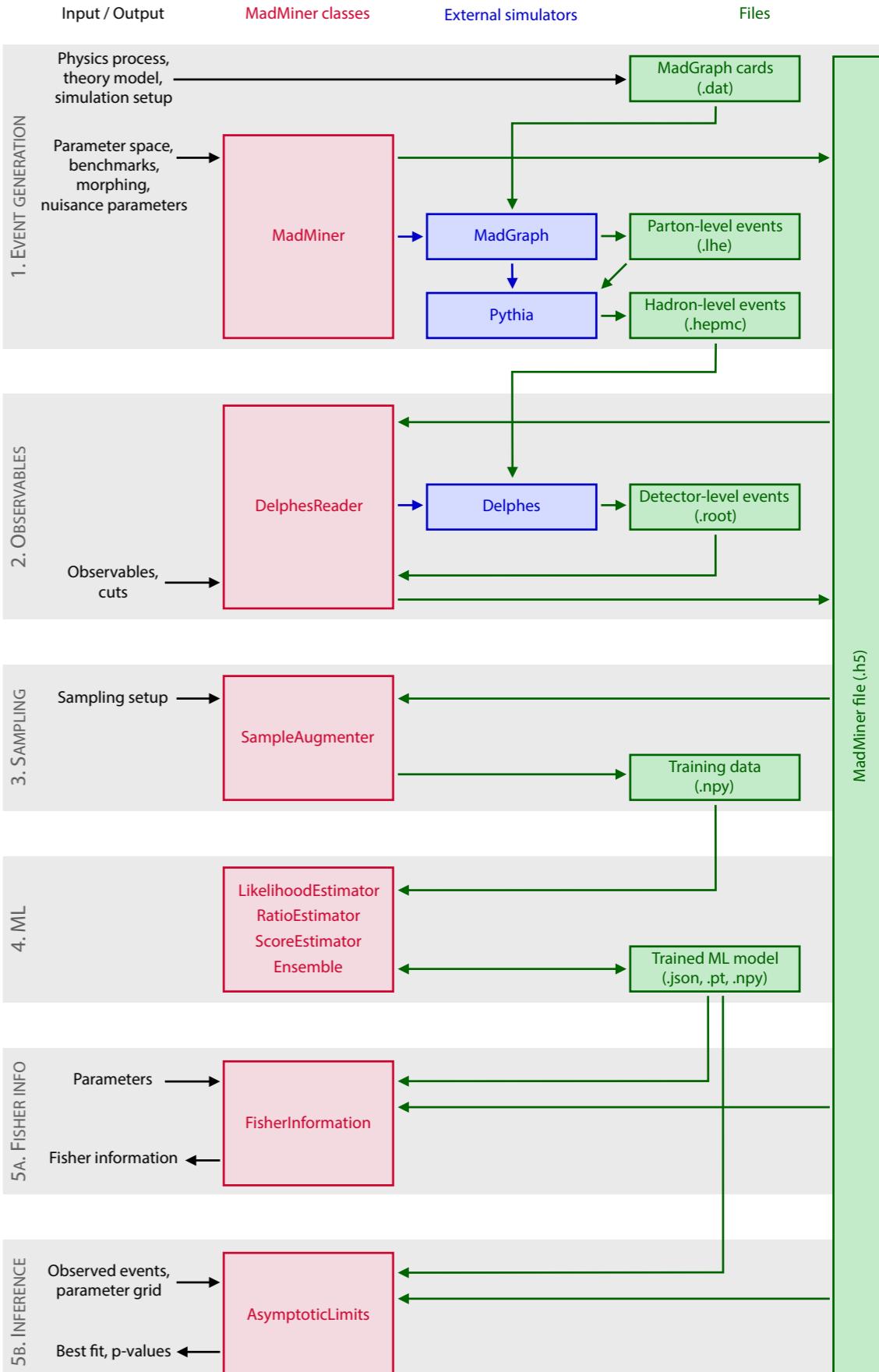
## A short summary



# MadMiner: The Tool

**MadMiner** [J. Brehmer, FK, I. Espejo, K. Cranmer [1907.10621]]

- automizes these techniques
- straightforward to apply them to LHC problems
- out of the box: Pheno-level analysis
  - \* MadGraph, Pythia, Delphes
  - \* backgrounds
  - \* PDF/scale uncertainties
  - \* ML uncertainties
  - \* morphing
  - \* many inference techniques (SALLY, ALICES ...)
- scalable to state-of-the-art experimental tools
- python package
  - \* modular interface
  - \* extensive documentation
  - \* on GitHub  
[github.com/diana-hep/madminer](https://github.com/diana-hep/madminer)
  - \* easy to install  
`pip install madminer`



# MadMiner: The Tool

## MadMiner Team



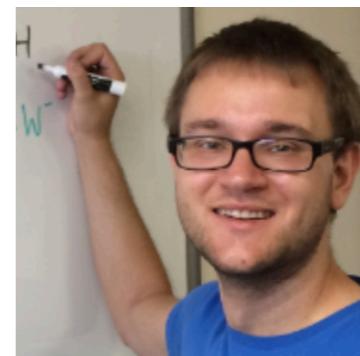
Johann Brehmer



Kyle Cranmer



Irina Espejo



Felix Kling

**Already used by many people**



Thank you a lot for testing!

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## **Summary and Conclusion**

# Probing SMEFT in tth

## The Physics Process

- processes

\* signal: fully leptonic tth  $pp \rightarrow t\bar{t}h \rightarrow (b\ell^+) (\bar{b}\ell^-) (\gamma\gamma) E_T^{\text{miss}}$

\* background: tt $\gamma\gamma$  continuum

\* 24.5 tth and 33.6 tt $\gamma\gamma$  events

Disclaimer: This is not the most sensitive process, but it demonstrates many features in MadMiner.

- theory

\* SMEFT model:  $\mathcal{L} = \mathcal{L}_{SM} + c_u \mathcal{O}_u + c_G \mathcal{O}_G + c_{uG} \mathcal{O}_{uG}$

$$\mathcal{O}_u = -\frac{1}{v^2} (H^\dagger H) (H^\dagger \bar{Q}_L) u_R$$

$$\mathcal{O}_G = \frac{g_s^2}{m_W^2} (H^\dagger H) G_{\mu\nu}^a G_a^{\mu\nu}$$

$$\mathcal{O}_{uG} = -\frac{4g_s}{m_W^2} y_u (H^\dagger \bar{Q}_L) \gamma^{\mu\nu} T_a u_R G_{\mu\nu}^a$$

- rescales top Yukawa:  $y_t \rightarrow y_t (1 + 3/2 c_u)$
- Higgs-gluon coupling:  $g_{gg\gamma} \rightarrow g_{gg\gamma} (1 + 192\pi^2/g^2 c_G)$
- chromo-dipole moment: gtt, ggtt, gtth, ggtth

- simulation: MadGraph5 + Pythia8 + Delphes3 + HL-LHC setup

\* PDF4LHC: scale + PDF uncertainties

- 48 observable:  $x = \{px, py, pz, E, pT, \eta, \Delta\Phi, \text{MET}, m_{ij}, \dots\}$

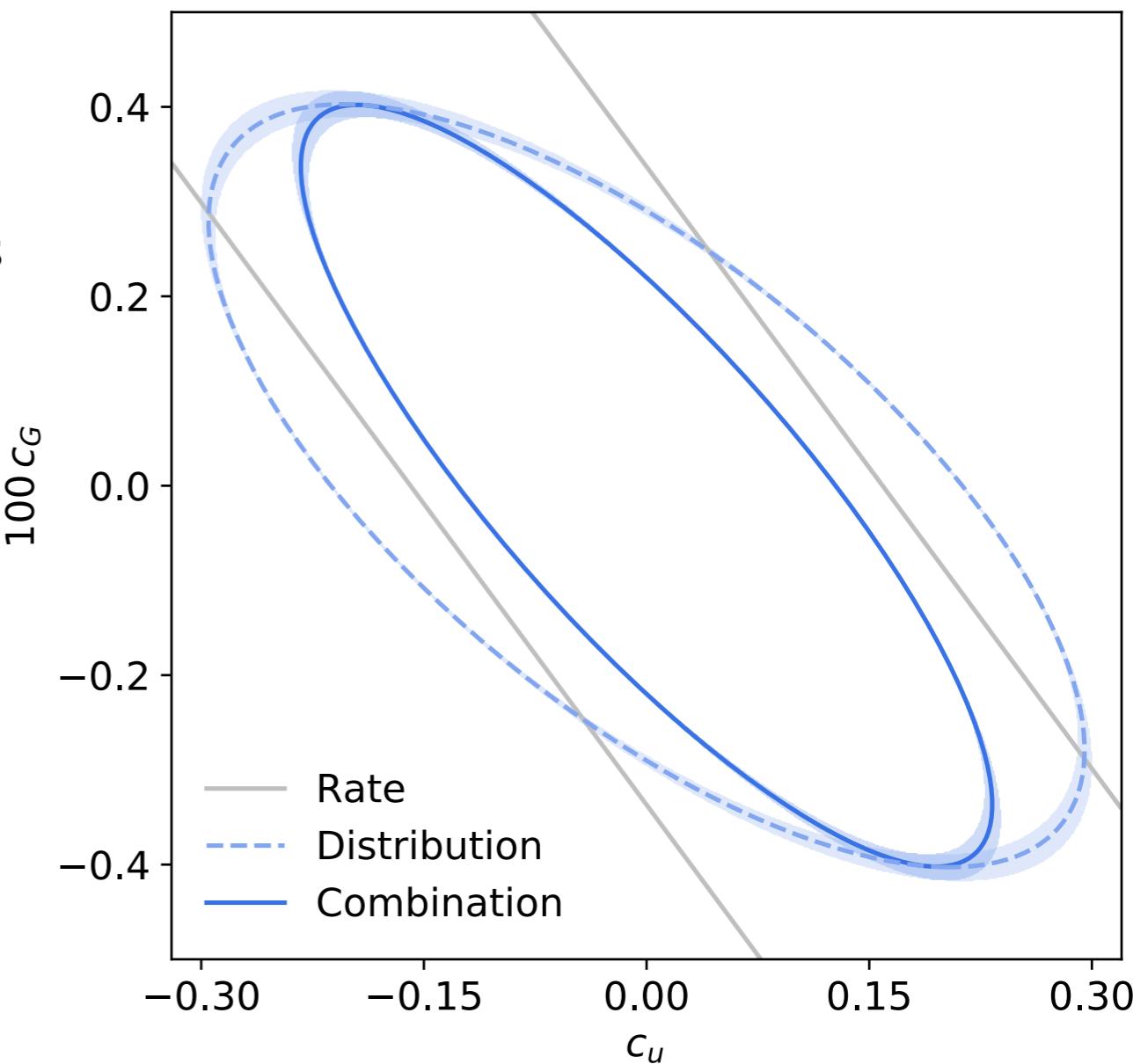
# Probing SMEFT in tth

## Fisher Information Analysis:

- Fisher Information matrix:
  - \* simple: **3x3** matrix
  - \* describes all operators simultaneously
  - \* only sensitive to interference effects

- translate into limits
  - \* 68% CL limits
  - \* separate information in rate+distributions
- results
  - \* rate: constrains only one direction
  - \* distributions: complementary information
  - \* shaded band:  $2\sigma$  ML ensemble variance

$$I_{ij} = \begin{pmatrix} c_u & 100c_G & 100c_{uG} \\ 140.5 & 68.1 & 170.6 \\ 68.1 & 47.1 & 105.7 \\ 170.6 & 105.7 & 283.3 \end{pmatrix} \begin{matrix} c_u \\ 100c_G \\ 100c_{uG} \end{matrix}$$



# Probing SMEFT in tth

## How about Systematics?

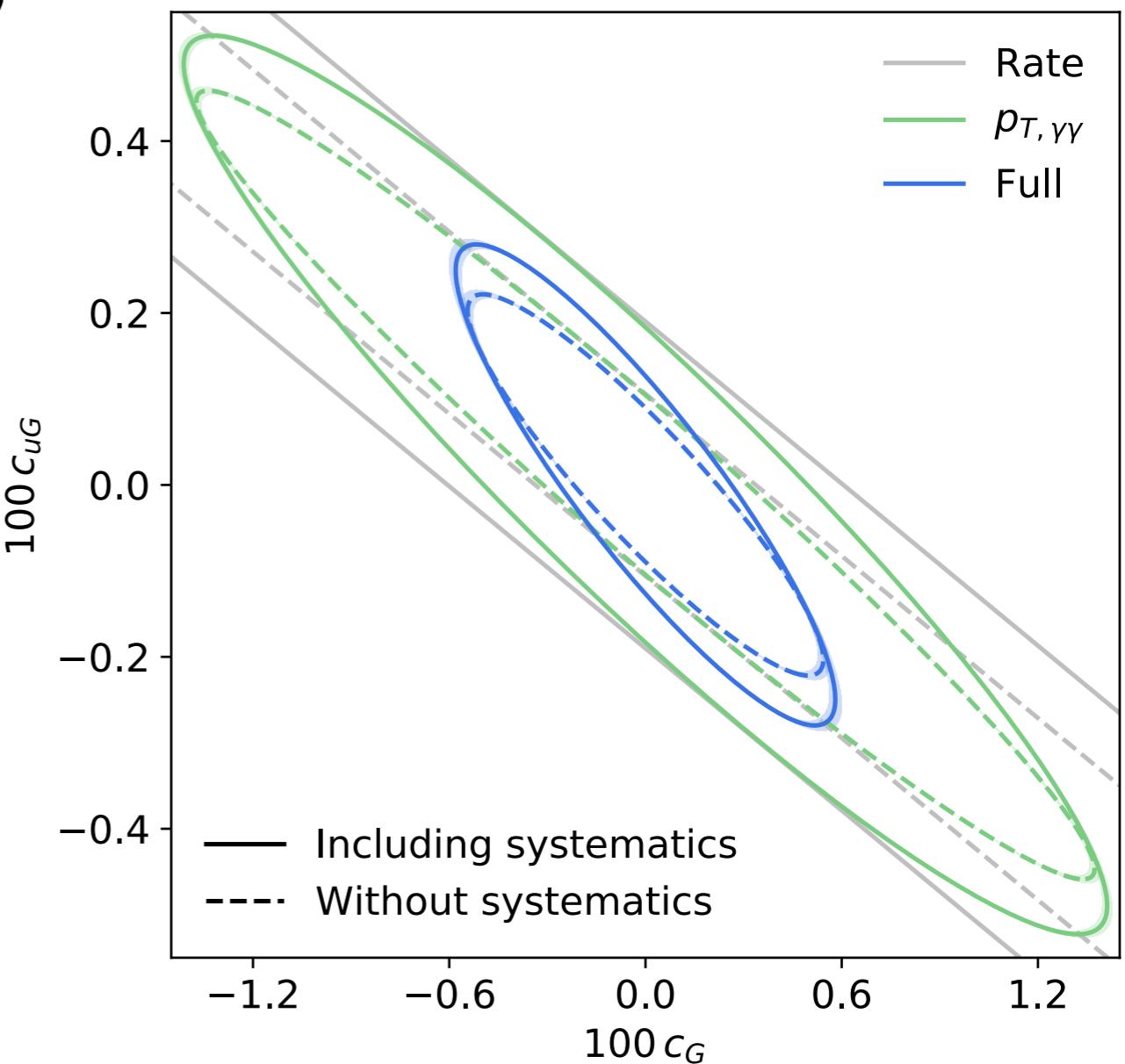
- strategy
  - \* introduce nuisance parameter  $\nu$  for scale (2) and PDF (30) uncertainties

- \* replace  $r(x|\theta) \rightarrow r(x|\theta, \nu)$
  - \* learn score, obtain Fisher Info (3+32 dim.)

- \* profile over  $\nu$

- results:

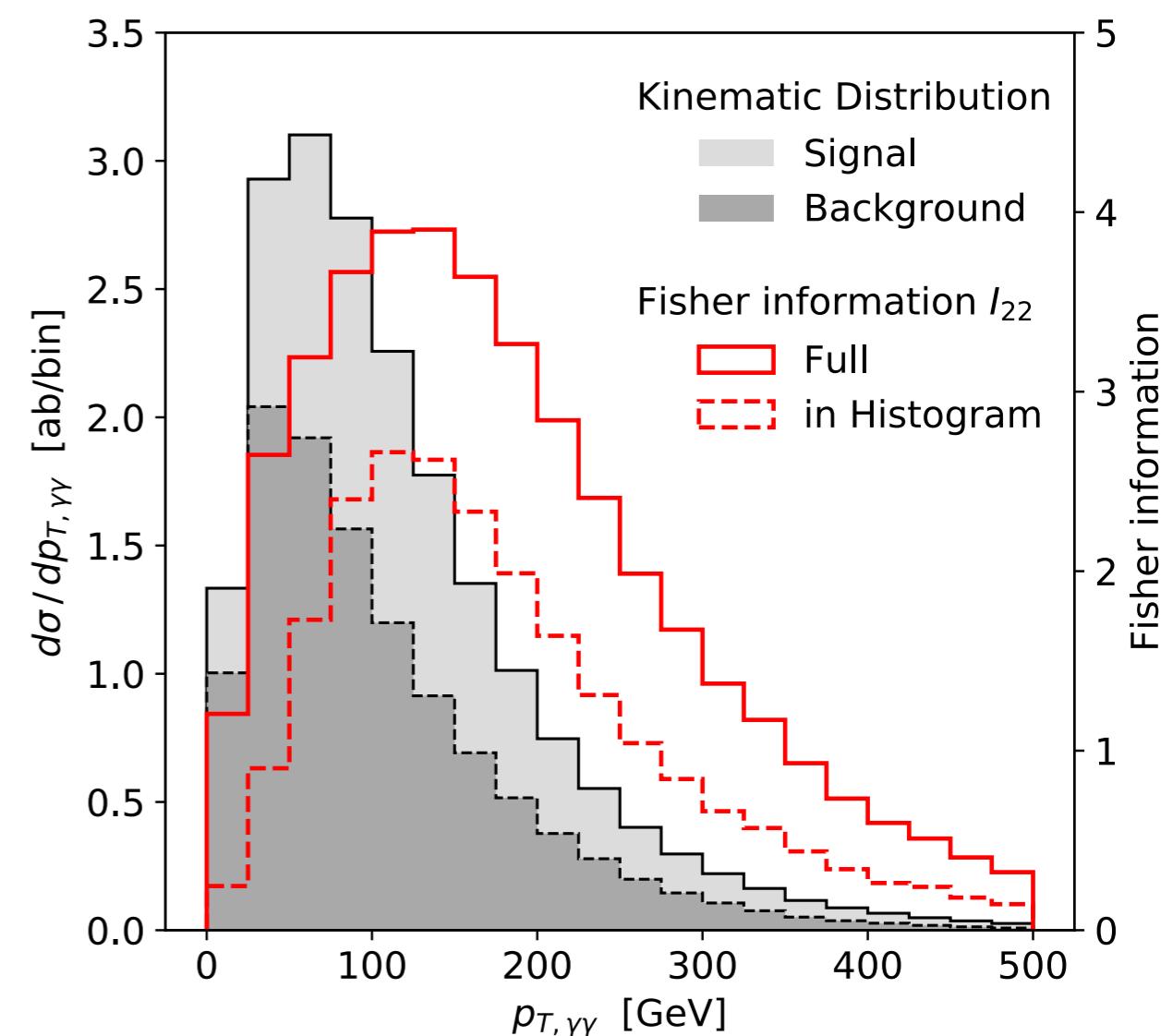
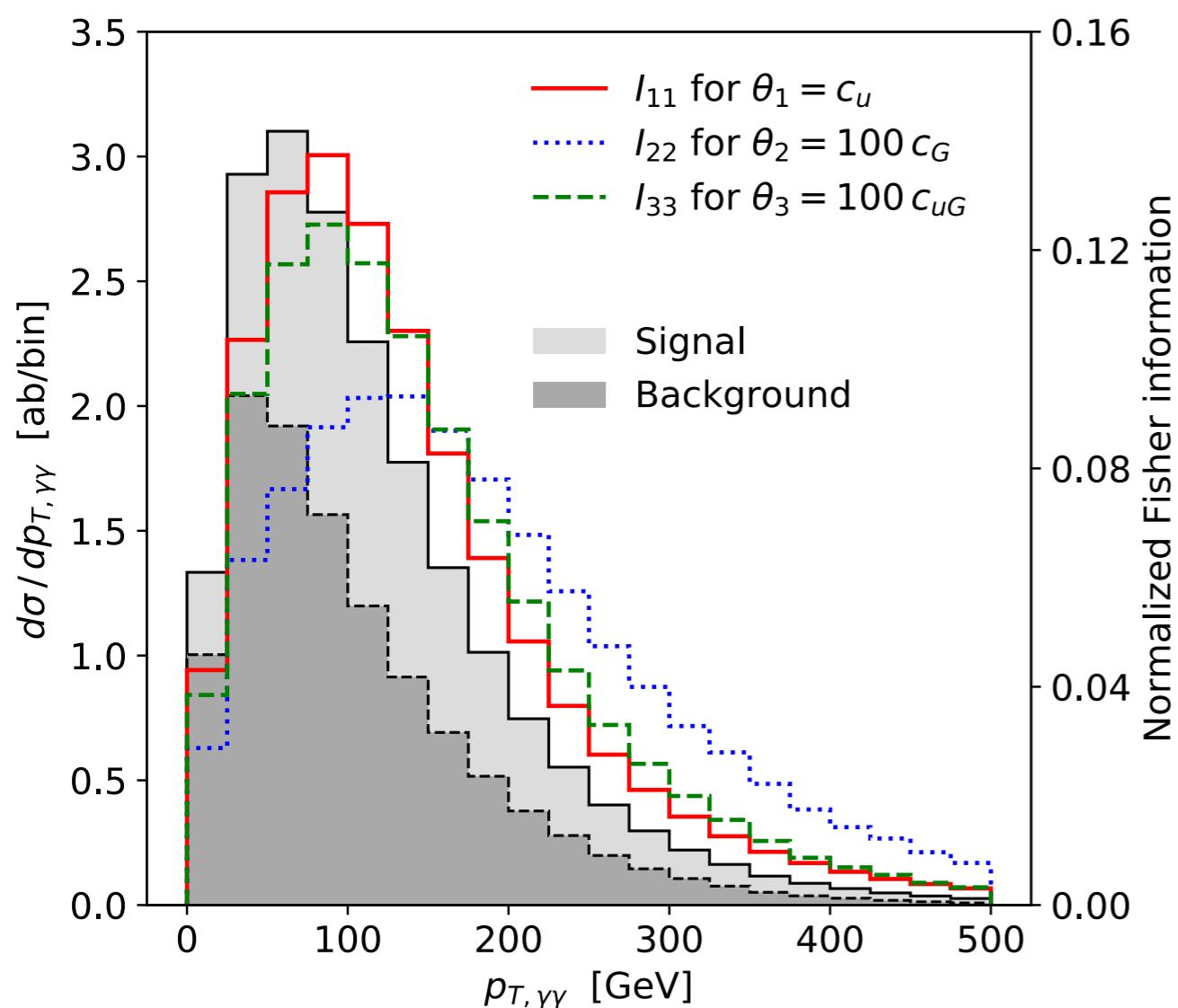
- \* mainly scale uncertainties
  - \* systematic reduce reach in rate-sensitive direction
  - \* multivariate analysis less affected



# Probing SMEFT in tth

## Kinematic distribution of the Fisher information

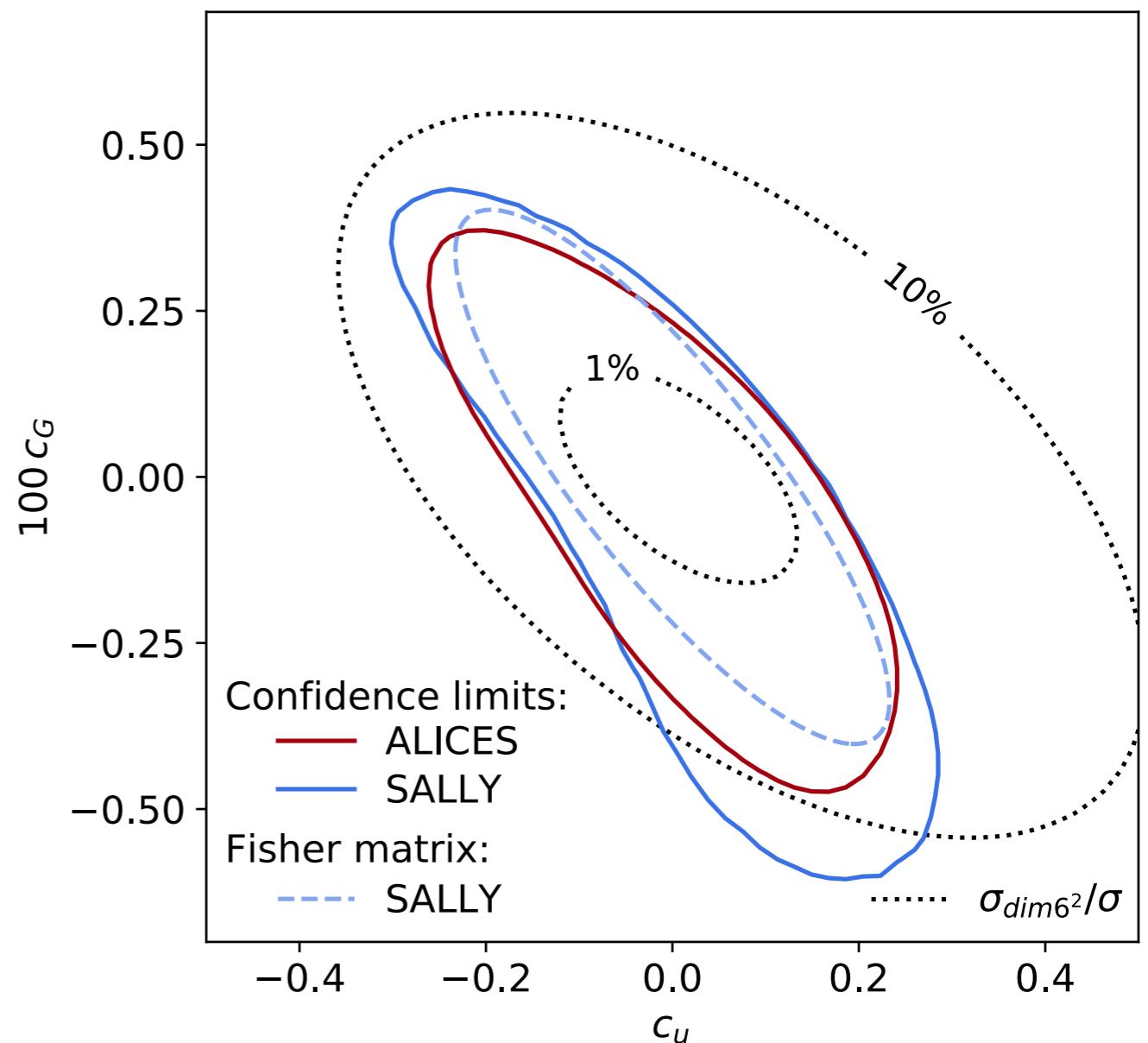
- identify the important phase-space regions



# Probing SMEFT in tth

## How good is local approximation?

- Fisher Information matrix
  - \* estimate
  - \* only sensitive to interference effects by construction
  - \* symmetric limits (ellipses)
- SALLY
  - \* estimates score  $t(\mathbf{x})|_{\text{SM}}$
  - \* use score as optimal observable (filled in histograms)
  - \* optimal only close to SM
- ALICES:
  - \* estimate  $r(\mathbf{x}|\theta)$
  - \* optimal limits in whole parameter space
- this example analysis:
  - \* few data, weak constraints
  - \* dim6 squared terms important in probed parameter space
  - \* local / full limits differ



# Probing SMEFT in tth

## Full Results

- inference methods

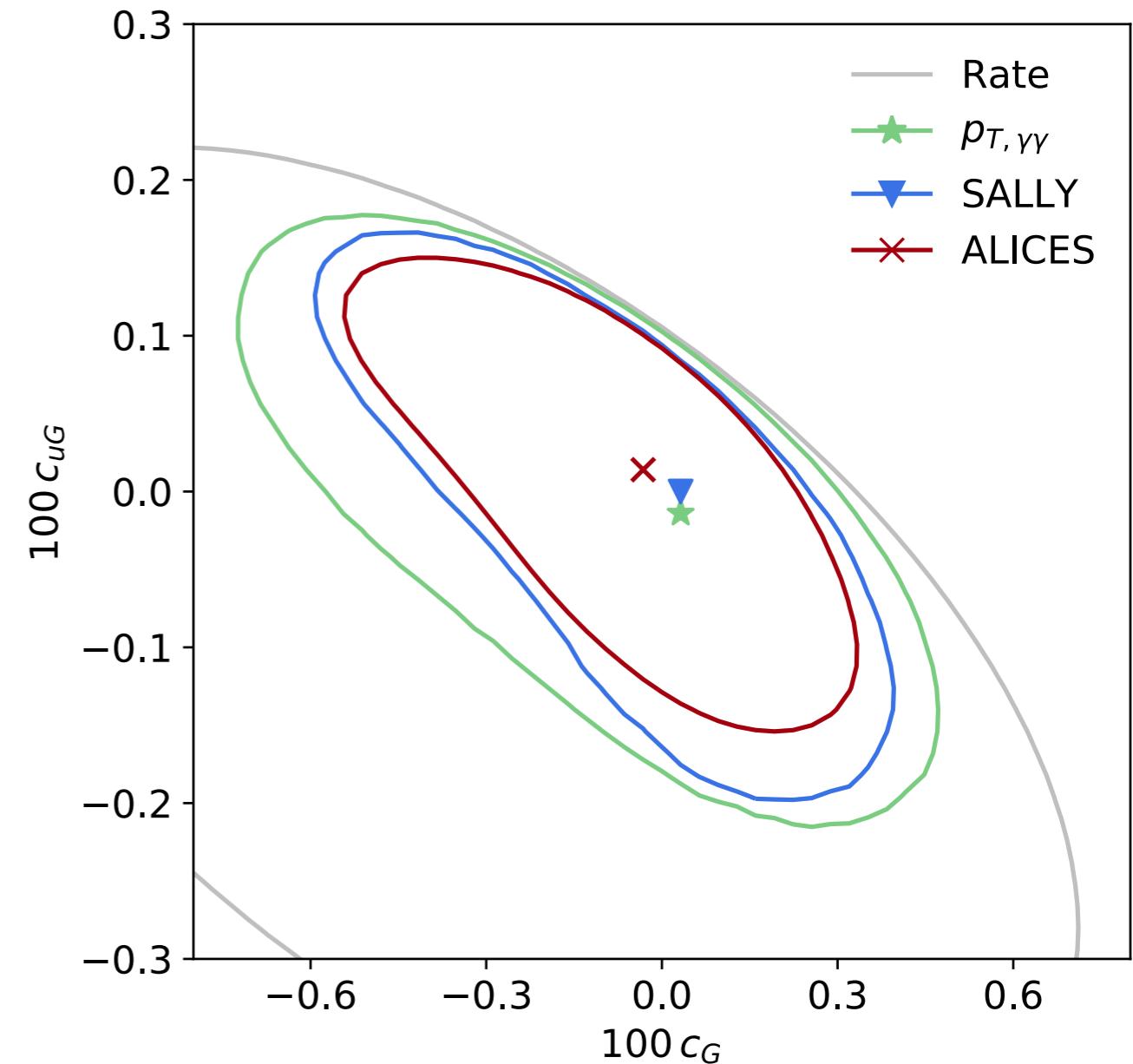
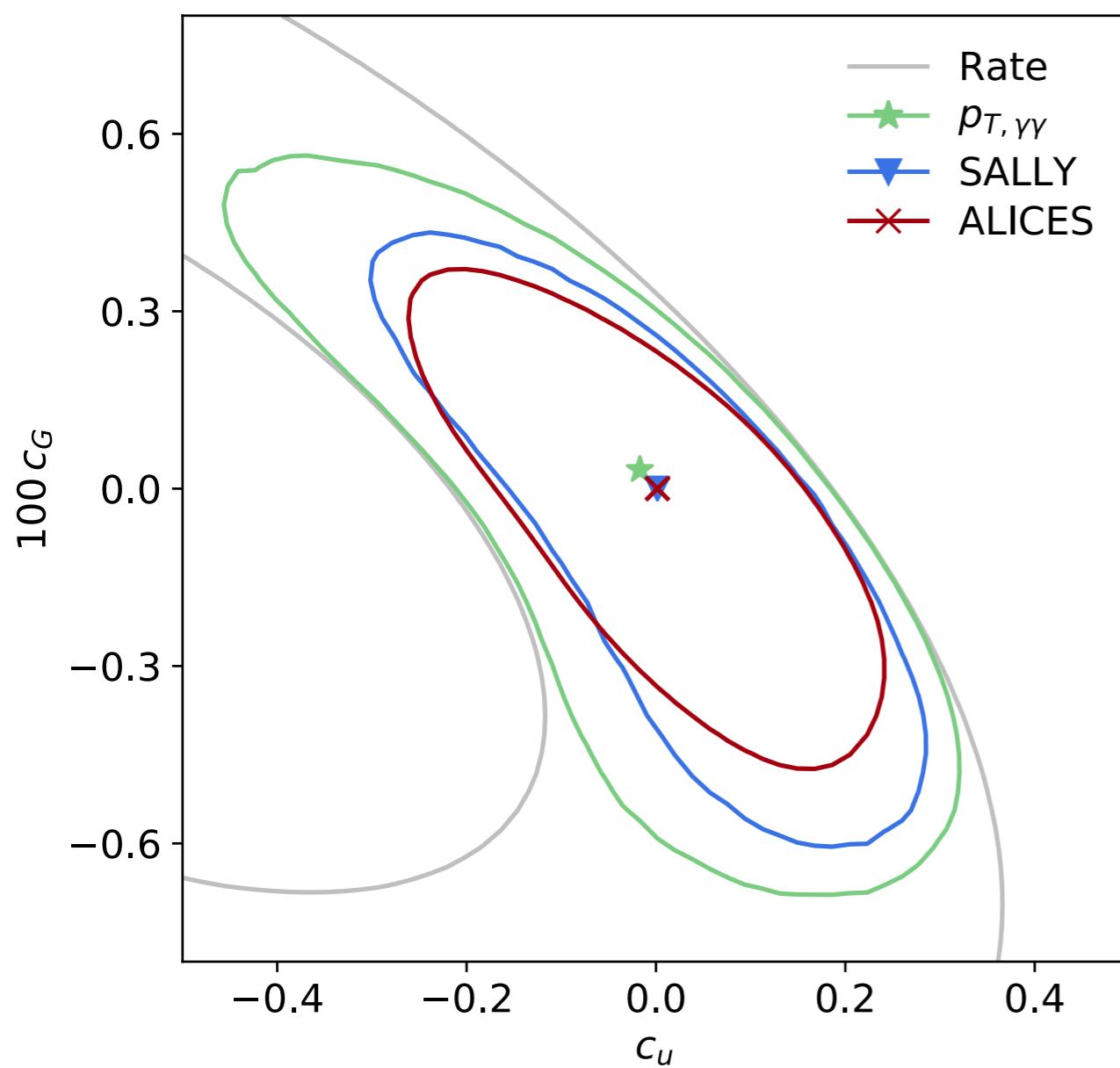
total rate

SALLY: score  $t(\mathbf{x})|_{\text{SM}}$  as locally optimal observables

pTH histogram

ALICES: use full  $r(\mathbf{x}|\theta)$

- multivariate methods significantly improve reach



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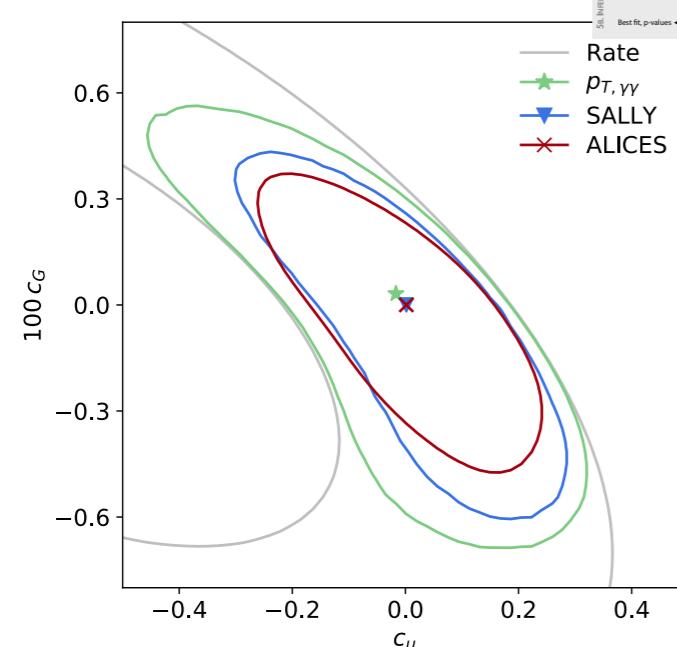
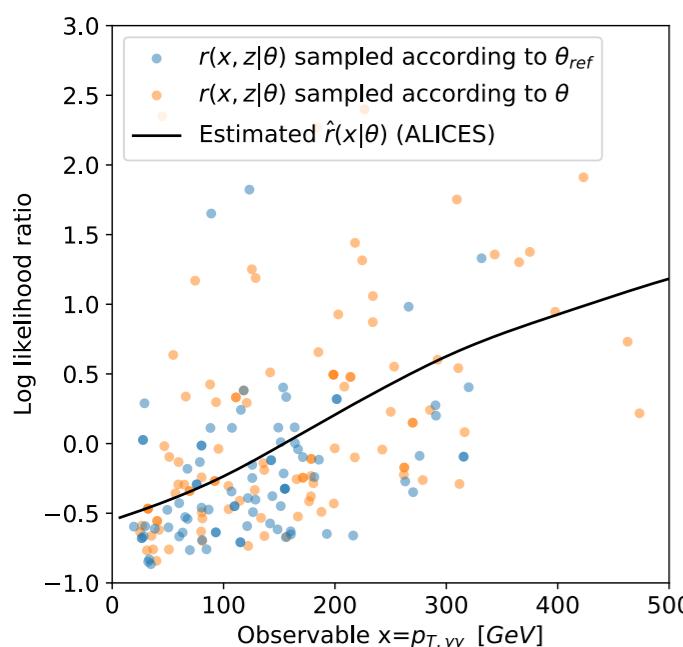
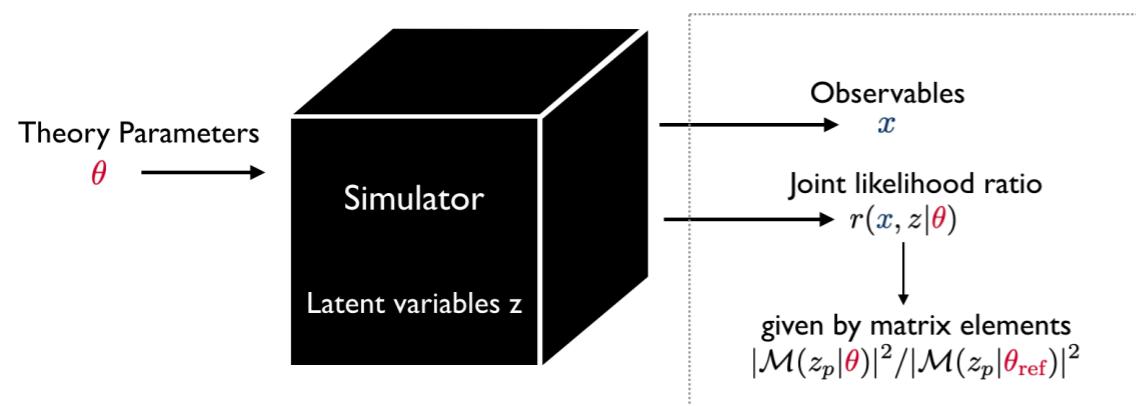
Using these methods is super easy!

## **A Realistic Physics Example**

Probing SMEFT in  $t\bar{t}$

## **Summary and Conclusion**

# Summary and Conclusion



**Motivation:** LHC probe high-dimensional theory space  $\theta$  with high dimensional data  $x$   
 \* task: determine likelihood function  $p(x|\theta)$

**Method:** uses multivariate inference techniques leveraging information in matrix elements and power of machine learning

- \* estimate full likelihood ratio including correlations + systematics
- \* learn optimal summary statistics
- \* ideal for SMEFT measurements

**Tool:** MadMiner [Brehmer, Cranmer, Espejo, FK 1907.10621]

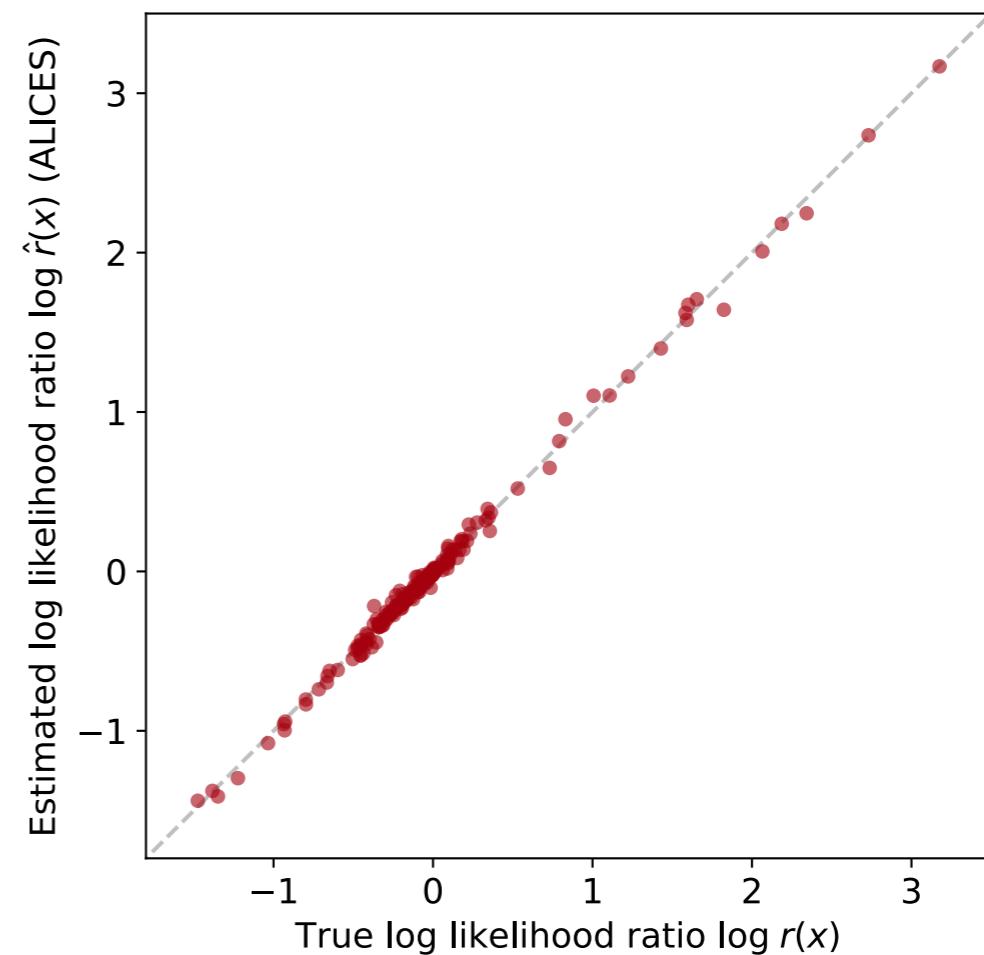
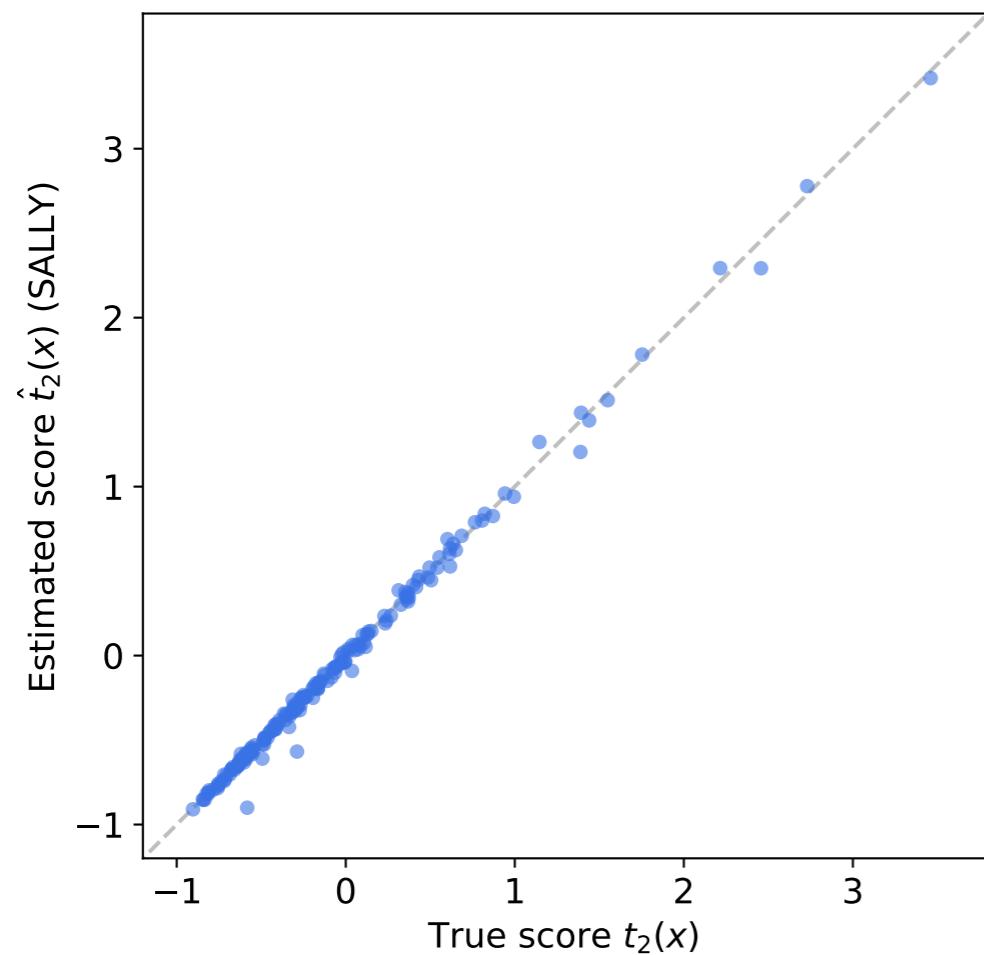
- \* python package
- \* MadGraph add-on
- \* automizes all steps of analysis
- \* already used for Pheno studies

# Outline

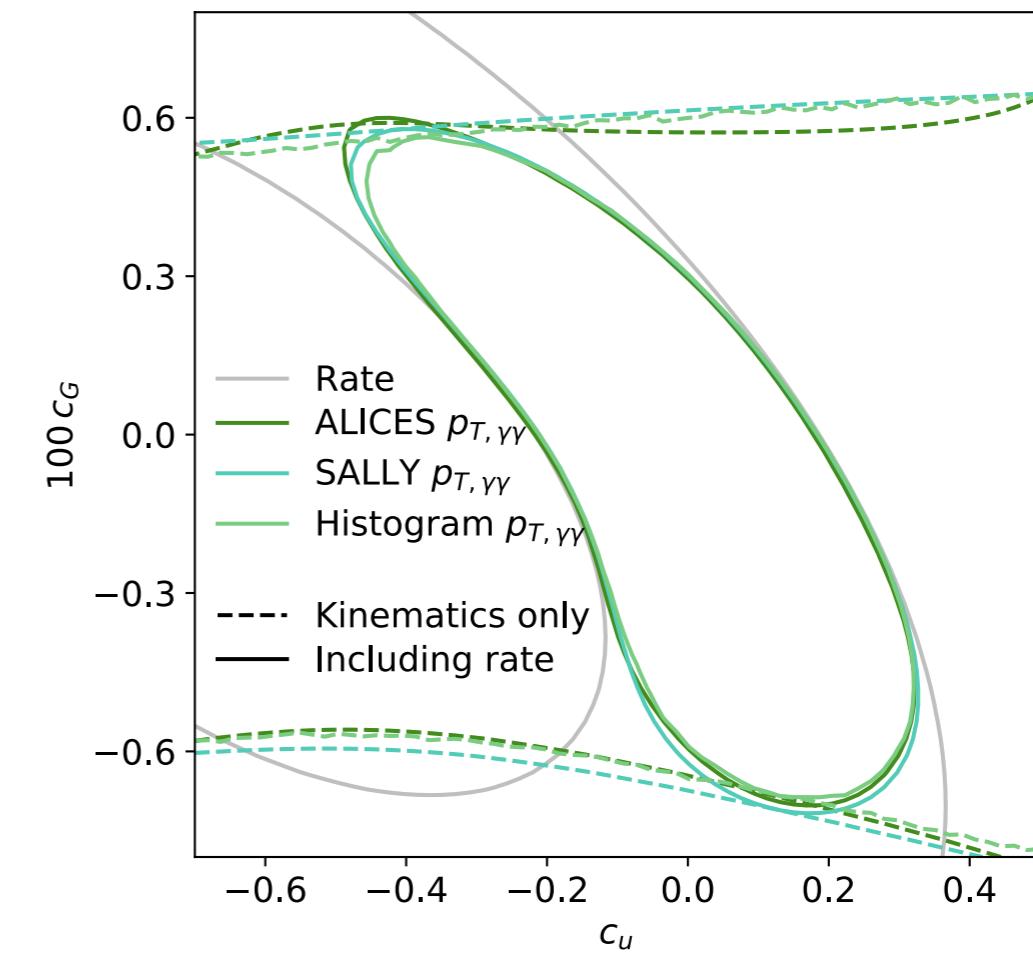
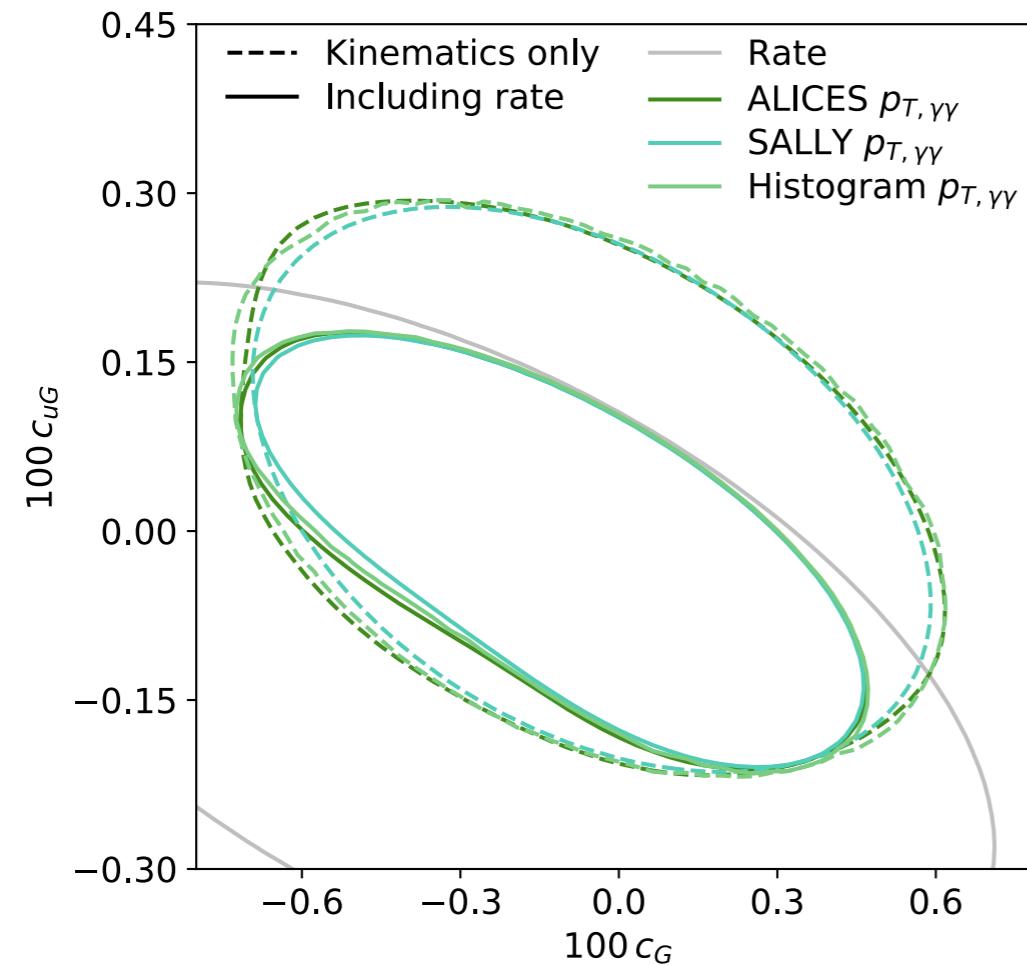
## Backup

Cool stuff that didn't make it into the main part

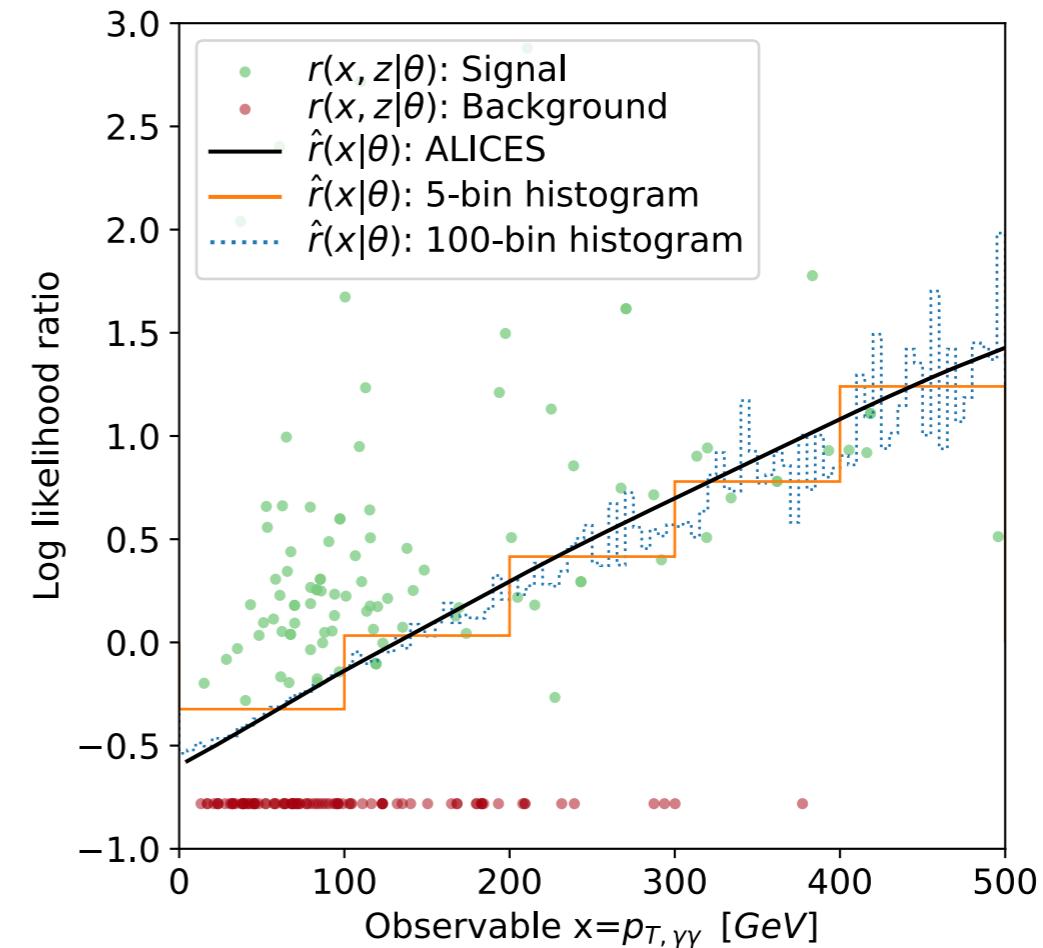
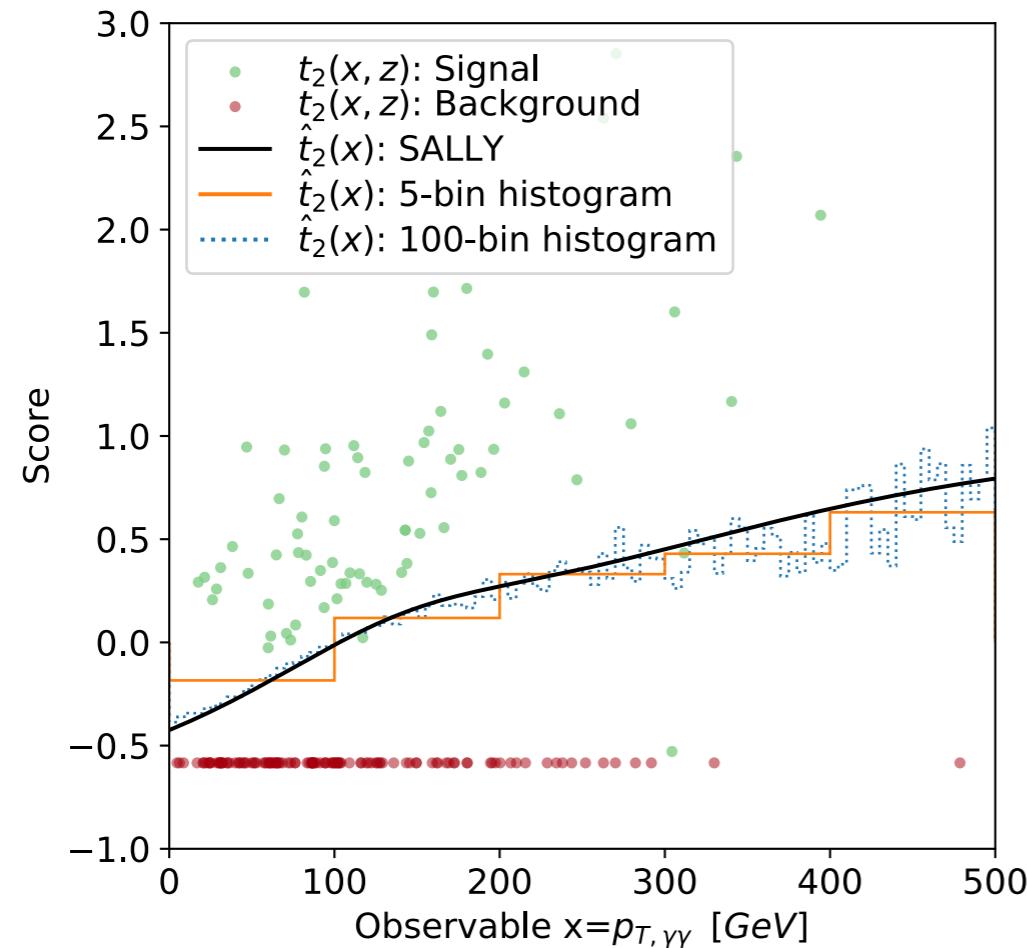
# Validation



# Validation



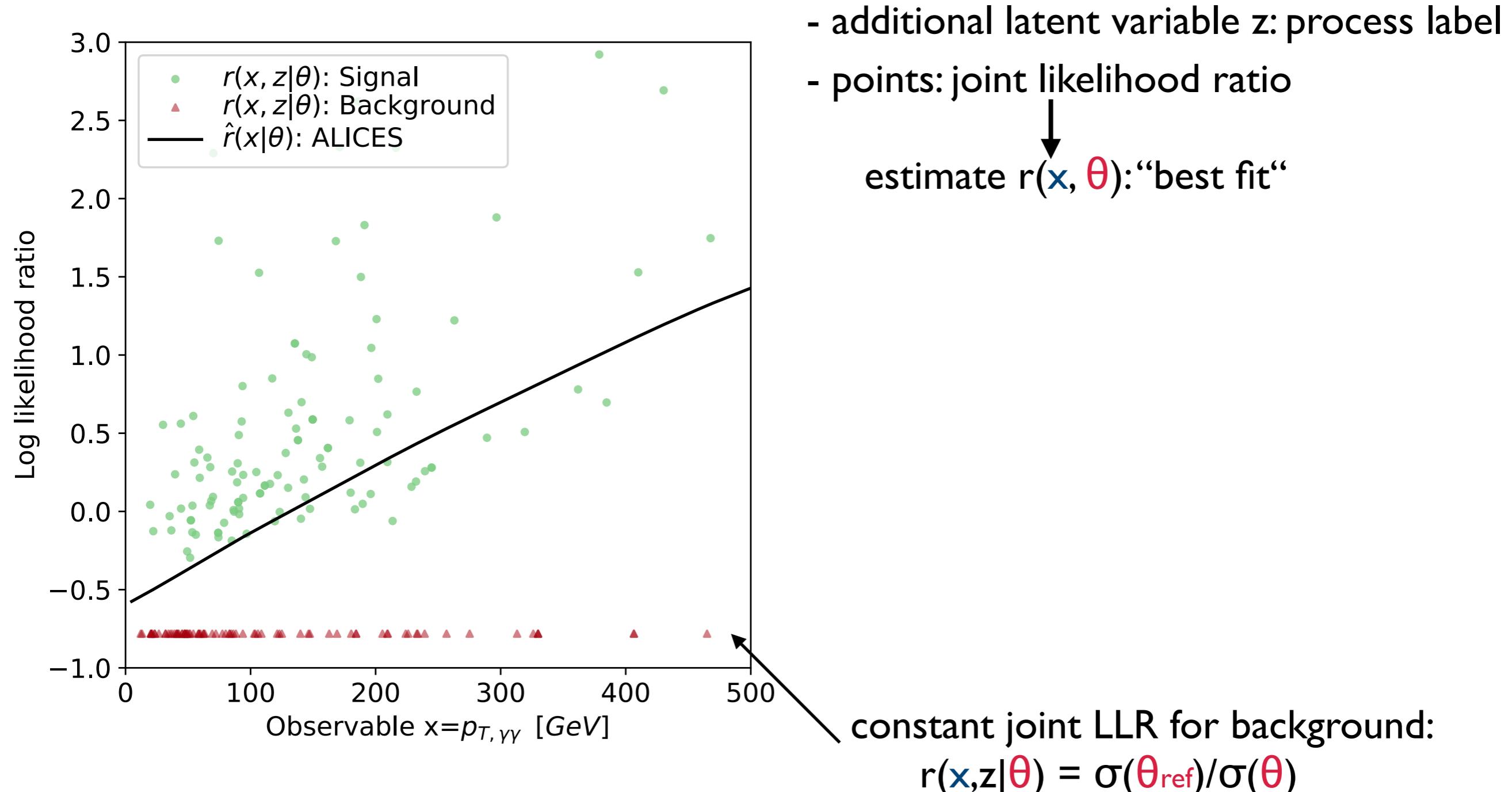
# Validation



# MadMiner with Backgrounds

## How about Backgrounds?

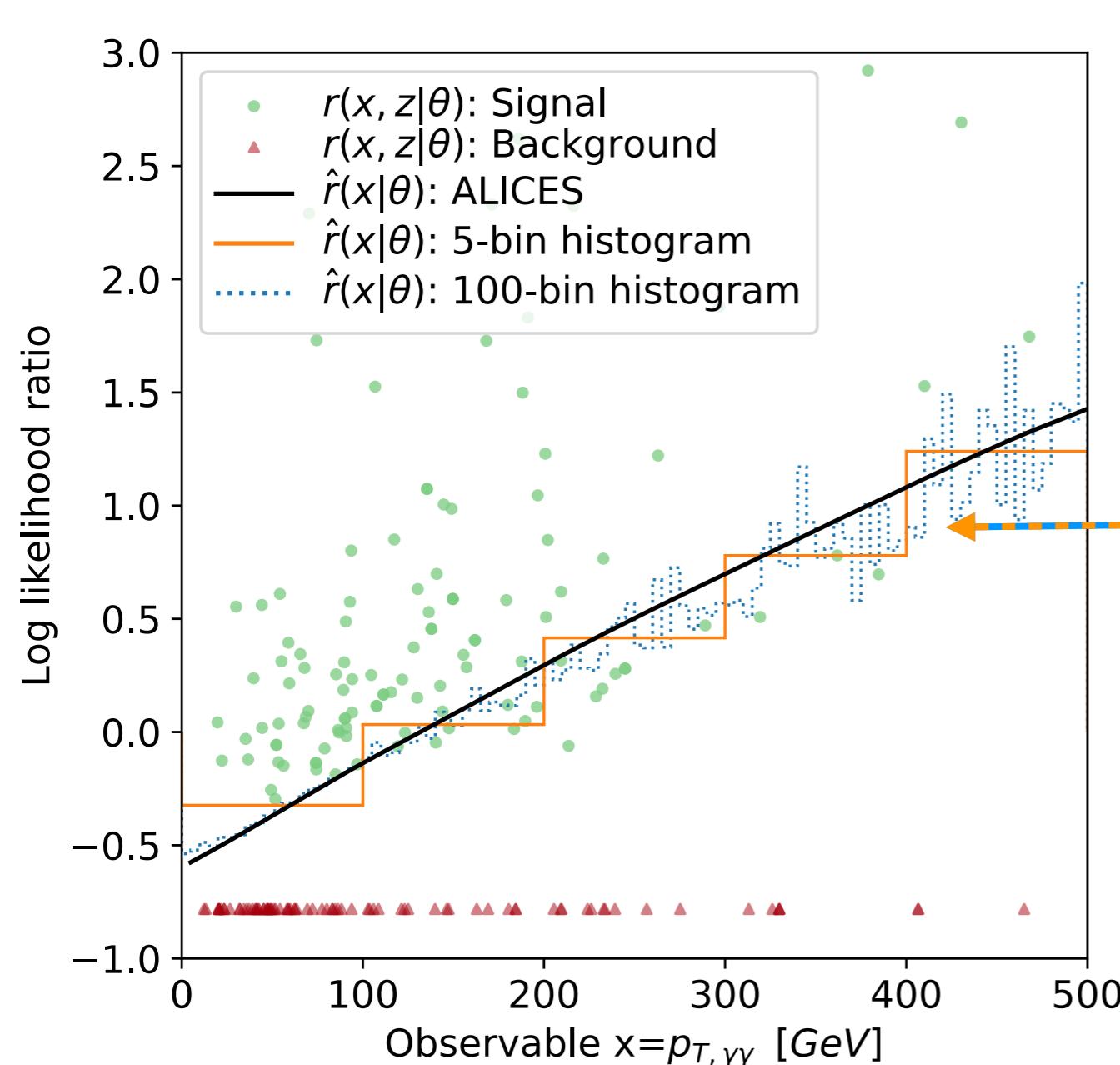
- consider two models: BSM ( $\theta=1$ ) vs SM ( $\theta_{\text{ref}}=0$ )
- include  $t\bar{t}H$  signal and  $t\bar{t}\gamma\gamma$  background



# MadMiner with Backgrounds

## How about Backgrounds?

- consider two models: BSM ( $\theta=1$ ) vs SM ( $\theta_{\text{ref}}=0$ )
- include  $t\bar{t}H$  signal and  $t\bar{t}\gamma\gamma$  background



- additional latent variable  $z$ : process label
  - points: joint likelihood ratio
- ↓
- estimate  $r(x, \theta)$ : “best fit”

## Comparison with Histogram

- LLR obtained using histogram
  - agrees well :)
  - ML “continuum limit of for large # of bins”
- realistic problem:  $x$  and  $\theta$  high dimensional

# Probing SMEFT in WH

## Another example: WH production in SMEFT

- process  $W H \rightarrow l v b b$
- 3 operators contribute

$$\tilde{\mathcal{O}}_{HD} = (\phi^\dagger \phi) \square (\phi^\dagger \phi) - \frac{1}{4} (\phi^\dagger D^\mu \phi)^* (\phi^\dagger D_\mu \phi)$$

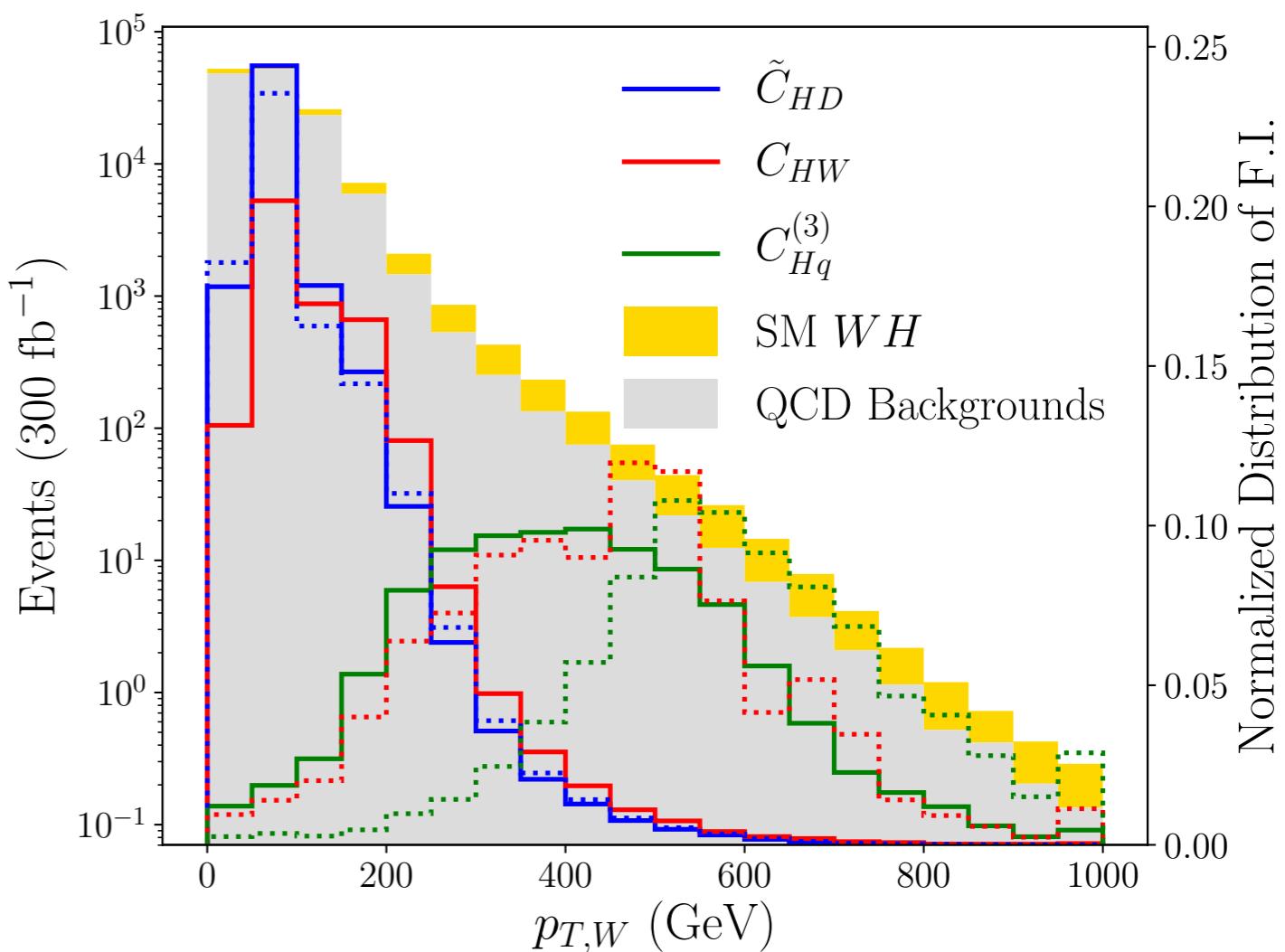
$$\mathcal{O}_{HW} = \phi^\dagger \phi W_{\mu\nu}^a W^{\mu\nu a}$$

$$\mathcal{O}_{Hq}^{(3)} = (\phi^\dagger \overset{\leftrightarrow}{D}_\mu^a \phi) (\bar{Q}_L \sigma^a \gamma^\mu Q_L)$$

- 2 main observables:  $pT H$  and  $mT, \text{tot}$
- where is the information
  - \* identify sensitive phase space regions

**Distribution of Information:**

- interference term
- quadratic term



# Probing SMEFT in WH

## Another example: WH production in SMEFT

- process  $WH \rightarrow l\nu bb$
- 3 operators contribute
- 2 main observables:  $pT,H$  and  $m_{T,tot}$
- how good can simplified template cross sections probe WH?
  - \* quantify performance using information geometry
  - \* compare to full information
  - \* additional high  $pT$  bin essential
  - \* include 2nd observable
  - \* multivariate analysis potentially much more powerful

